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Current rectification in junctions with spin-split superconductors

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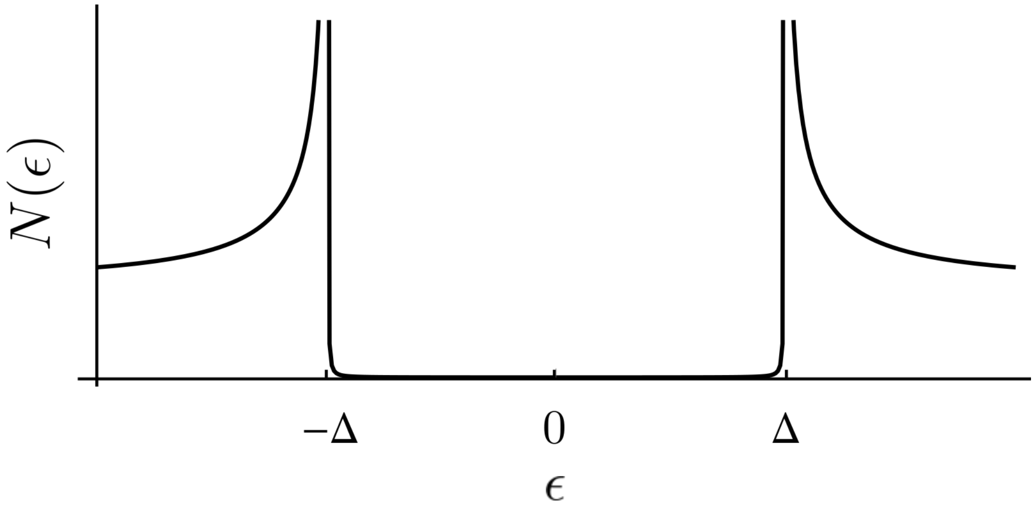
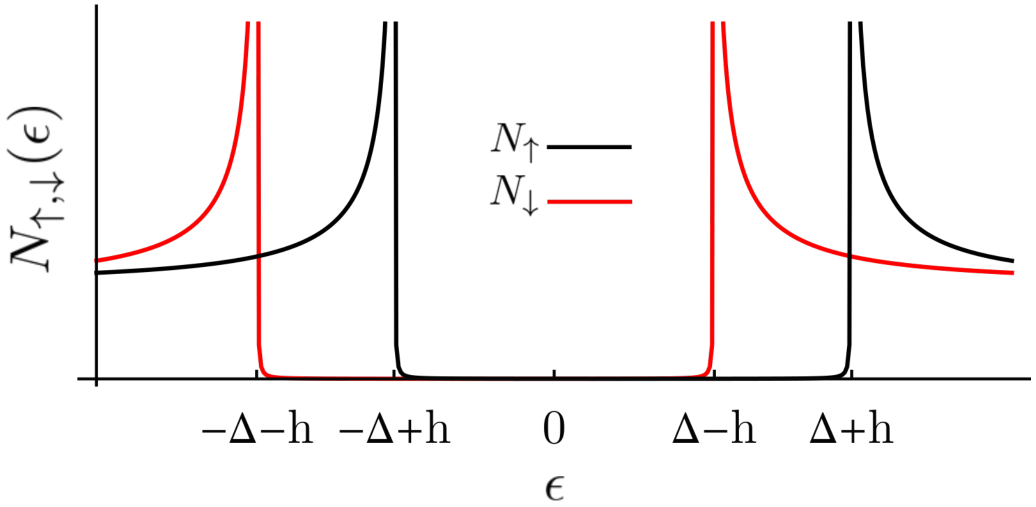
DoS of a superconductor



$$N(\epsilon) = \mathrm{Re}\,\frac{\epsilon + i\Gamma}{\sqrt{(\epsilon + i\Gamma)^2 - \Delta^2}}$$

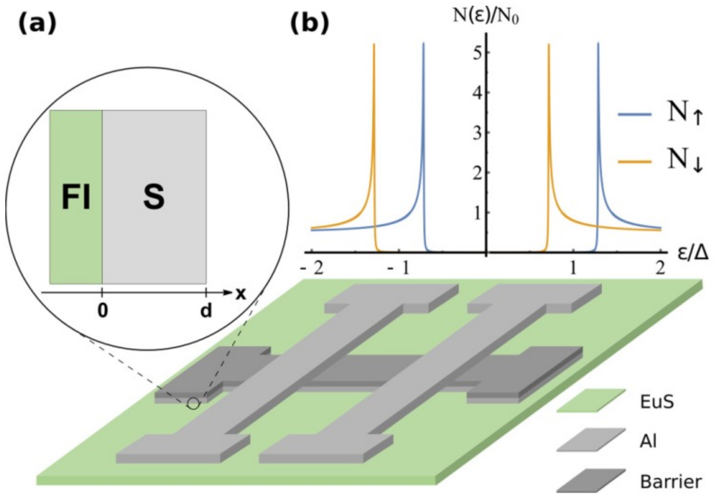
Spin-split superconductors

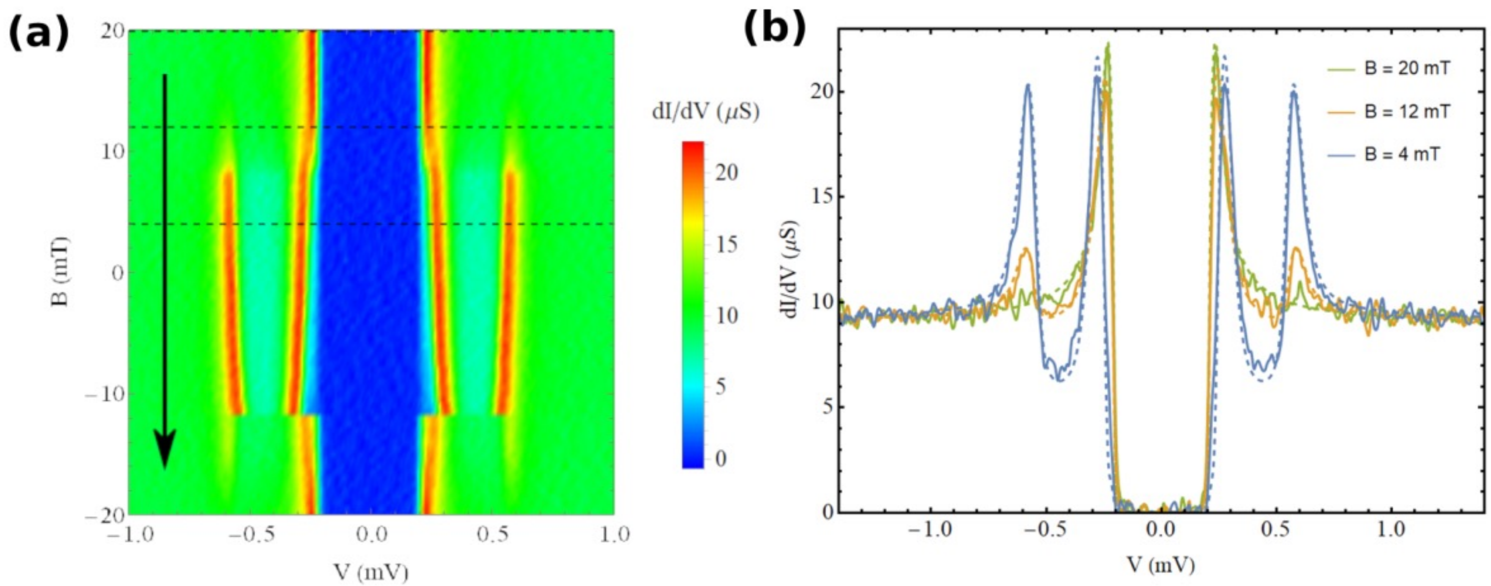


$$N_{\uparrow,\downarrow}(\epsilon) = \mathrm{Re}\,\frac{\epsilon \pm h + i\Gamma}{\sqrt{(\epsilon \pm h + i\Gamma)^2 - \Delta^2}}$$

Spin splitting in EuS/Al

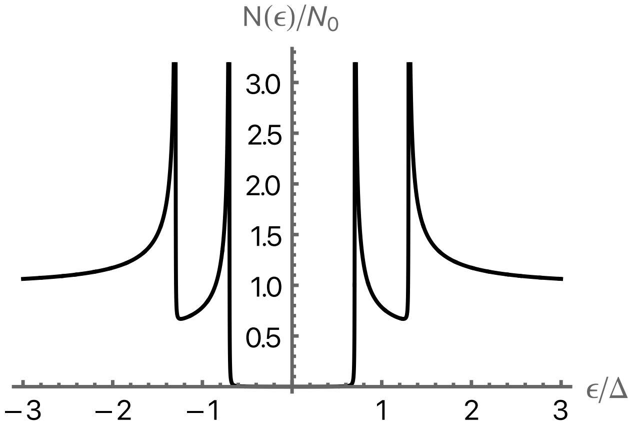
Tunneling conductance in EuS/Al/AlO/Al





Hijano et al, PR Res. 3 (2021)
Moodera et al, PRL 61 (1988)

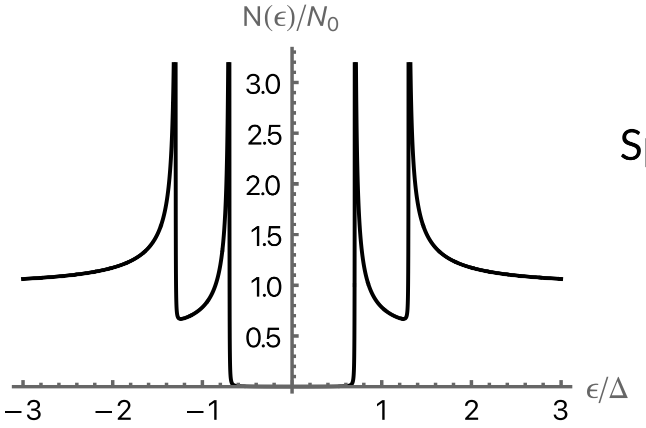
Spin-filtering and breaking electron-hole symmetry



e-h symmetric

$$N(\epsilon) = N(-\epsilon)$$

Spin-filtering and breaking electron-hole symmetry



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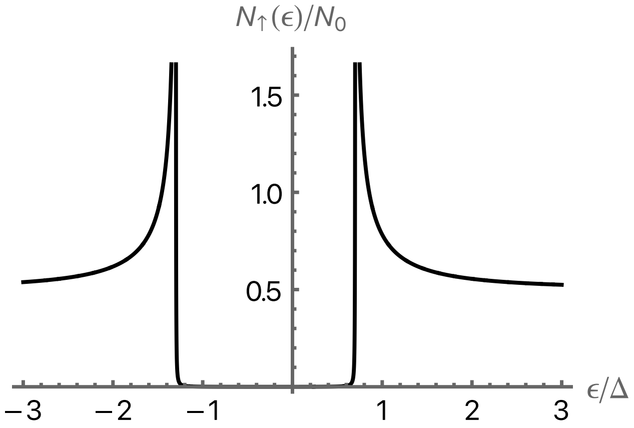
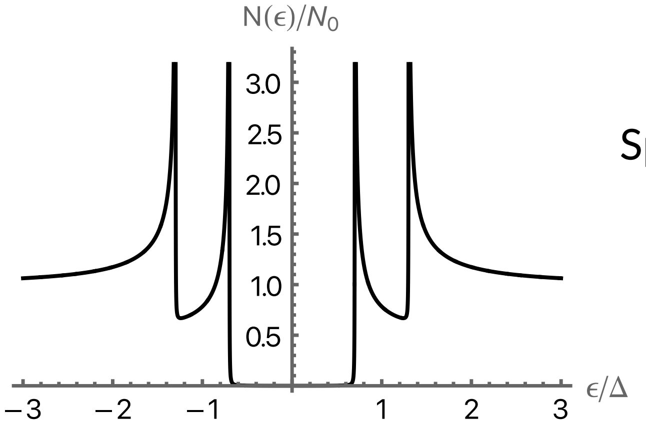
Spin filtering



Breaking of e-h symmetry

$N(\epsilon) \neq N(-\epsilon)$

Spin-filtering and breaking electron-hole symmetry



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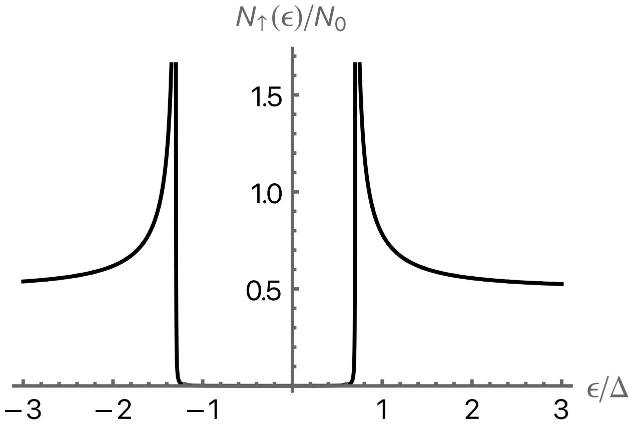
Spin filtering



Breaking of e-h symmetry

$N(\epsilon) \neq N(-\epsilon)$

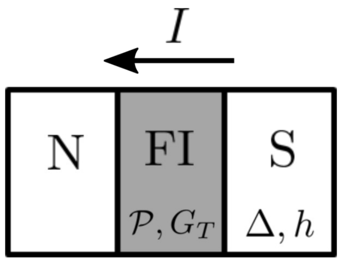
Thermoelectric effect
Ozaeta et al, PRL 112 (2014)

Current Rectification (Diode effect)
Ilic et al, PR Applied 17 (2022)
Strambini et al, Nat. Commun. 13 (2022)

Outline

- DC transport in N/FI/S junctions
  - Thermoelectric effects
  - Current rectification
  - Experimental results
- AC transport in N/FI/S junctions
- Application: radiation detectors

DC transport in N/FI/S tunnel junctions



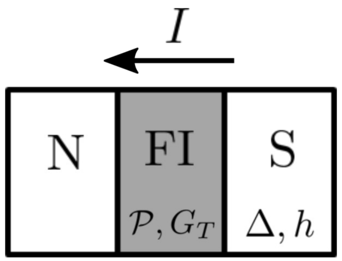
Spin-split DoS

$$N_{\uparrow,\downarrow} = \mathrm{Re}\,\frac{\epsilon \pm h + i\Gamma}{\sqrt{(\epsilon \pm h + i\Gamma)^2 - \Delta^2}}$$

Spin-polarized tunneling

$$G_\uparrow \neq G_\downarrow$$

DC transport in N/FI/S tunnel junctions



Spin-split DoS

$$N_{\uparrow,\downarrow} = \mathrm{Re}\,\frac{\epsilon \pm h + i\Gamma}{\sqrt{(\epsilon \pm h + i\Gamma)^2 - \Delta^2}}$$

Spin-polarized tunneling

$$G_{\uparrow} \neq G_{\downarrow}$$

Charge transport

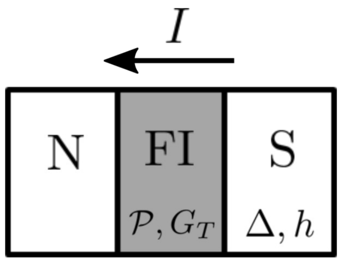
$$I_{\uparrow,\downarrow} = \frac{G_{\uparrow,\downarrow}}{e}\int d\epsilon\; N_{\uparrow,\downarrow}(\epsilon)[f_N(\epsilon - eV_{dc}) - f_S(\epsilon)]$$

Heat transport

$$\dot{Q}_{\uparrow,\downarrow} = \frac{G_{\uparrow,\downarrow}}{e}\int d\epsilon\; (\epsilon - eV_{dc})\, N_{\uparrow,\downarrow}(\epsilon)[f_N(\epsilon - eV_{dc}) - f_S(\epsilon)]$$

$$f_i = \left[1 + e^{\frac{\epsilon}{k_B T_i}}\right]^{-1}$$

DC transport in N/FI/S tunnel junctions



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$$f_i = \left[1 + e^{\frac{\epsilon}{k_B T_i}}\right]^{-1}$$

Total currents

$$I = I_\uparrow + I_\downarrow = \frac{G_T}{e}\int d\epsilon\; [N_+(\epsilon) + P\, N_-(\epsilon)][f_N(\epsilon - eV_{dc}) - f_S(\epsilon)]$$

$$\dot{Q} = \dot{Q}_\uparrow + \dot{Q}_\downarrow = \frac{G_T}{e}\int d\epsilon\; (\epsilon - eV_{dc})\, [N_+(\epsilon) + P\, N_-(\epsilon)][f_N(\epsilon - eV_{dc}) - f_S(\epsilon)]$$

$$N_\pm(\epsilon) = \frac{1}{2}[N_\uparrow(\epsilon) \pm N_\downarrow(\epsilon)]$$

$$P = (G_\uparrow - G_\downarrow)/(G_\uparrow + G_\downarrow) \in [-1,1]$$

Thermoelectric effects

Expansion for small V_{dc} and $\Delta T = T_S - T_N$

$$\begin{pmatrix} I \\ \dot{Q} \end{pmatrix} = \begin{pmatrix} G & \alpha \\ \alpha & G_{th}T \end{pmatrix} \begin{pmatrix} V_{dc} \\ -\Delta T/T \end{pmatrix}$$

$$G = G_T \int d\epsilon \, \frac{N_+(\epsilon)}{4\, k_B T \cosh^2 \frac{\epsilon}{2\, k_B T}}$$

$$G_{th} = \frac{G_T}{e^2} \int d\epsilon \, \frac{\epsilon^2 N_+(\epsilon)}{4\, (k_B T)^2 \cosh^2 \frac{\epsilon}{2\, k_B T}}$$

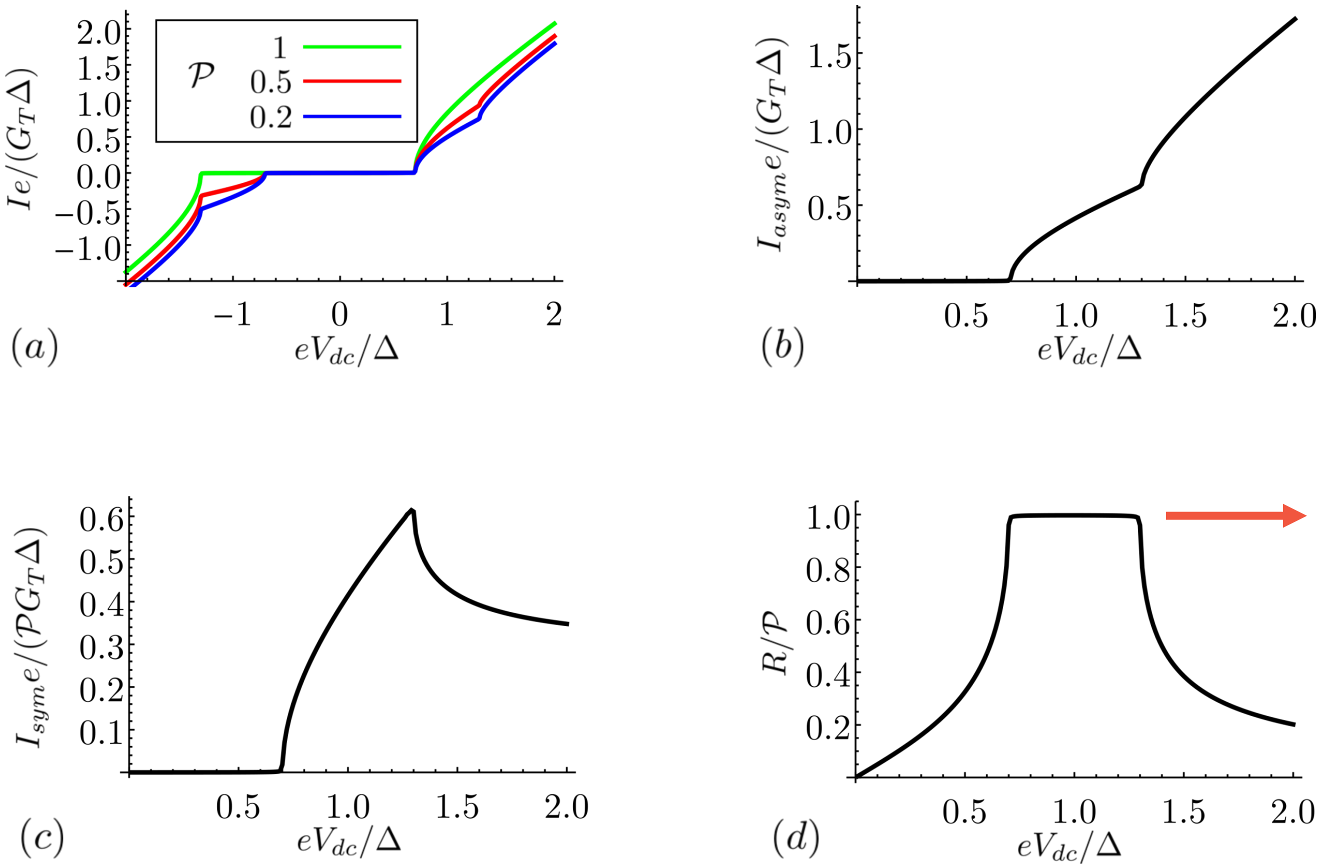
$$\alpha = \frac{G_T}{e} P \int d\epsilon \, \frac{N_-(\epsilon)}{4\, k_B T \cosh^2 \frac{\epsilon}{2\, k_B T}}$$

Ozaeta et al, PRL 112 (2014)

DC current at (near-) zero temperature

$$I = \frac{G_T}{e} \int d\epsilon \, [N_+(\epsilon) + P\, N_-(\epsilon)][f_N(\epsilon - eV_{dc}) - f_S(\epsilon)]$$

$$\frac{\Gamma}{\Delta} \gg \frac{\Delta}{k_B T} e^{-\Delta/k_B T}$$



$R_{max} = P$

$I(V) \neq -I(-V) \Rightarrow$ Rectification

$$I_{asym}(V) = \frac{|I(V_{dc}) - I(-V_{dc})|}{2}$$

$$I_{sym}(V) = \frac{|I(V_{dc}) + I(-V_{dc})|}{2}$$

Rectification ratio

$$R = I_{asym}/I_{sym}$$

DC current at finite temperature

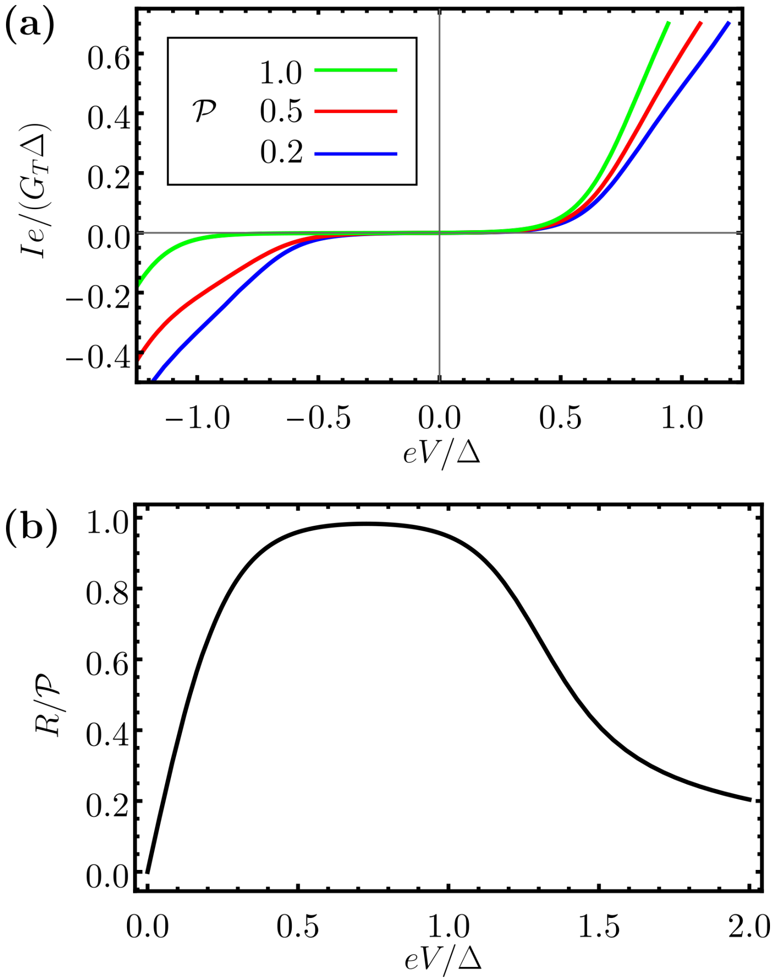
$$\frac{\Gamma}{\Delta} \ll \frac{\Delta}{k_B T} e^{-\Delta/k_B T} \qquad eV_{dc} < \Delta - h$$

$$I = I_s(e^{eV_{dc}/k_B T} - 1) + I_s(P-1)\left(\cosh\frac{eV}{k_B T} - 1\right)$$

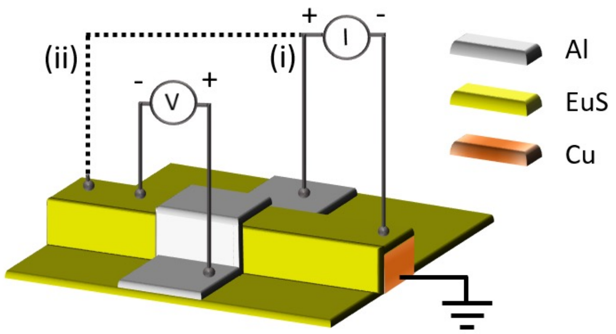
Ideal Schockley contribution

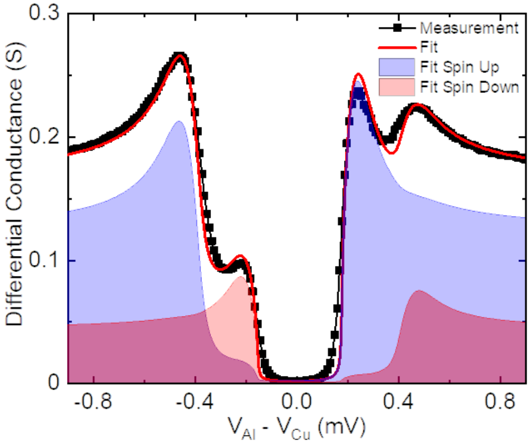
Non-ideality at $P \neq 1$

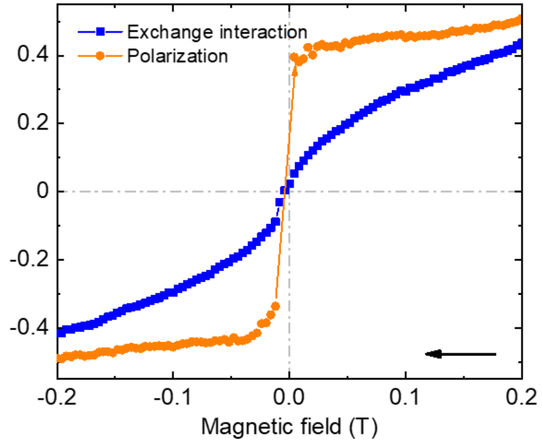
$$R = \frac{I_{sym}}{I_{asym}} = P \tanh\frac{eV_{dc}}{2\, k_B T}$$



Experimental results in Cu/Al/EuS

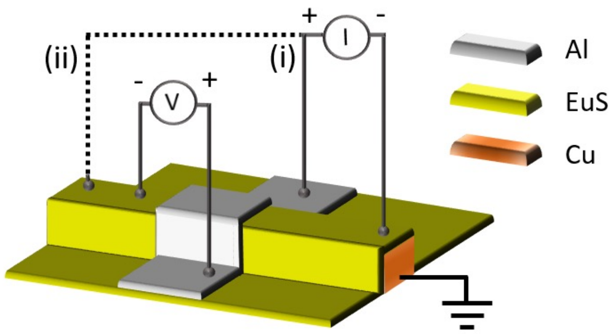


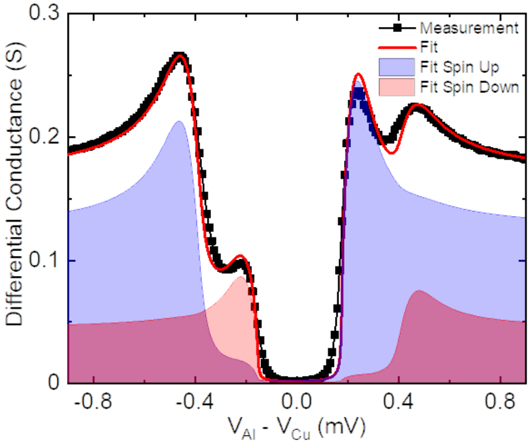


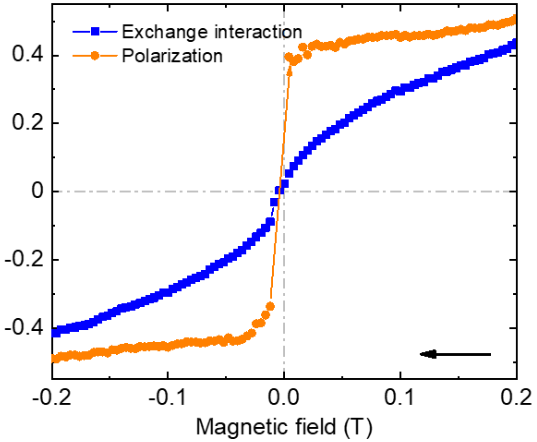


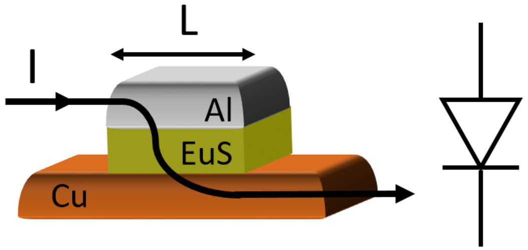
Strambini et al, Nat. Commun. 13 (2022)

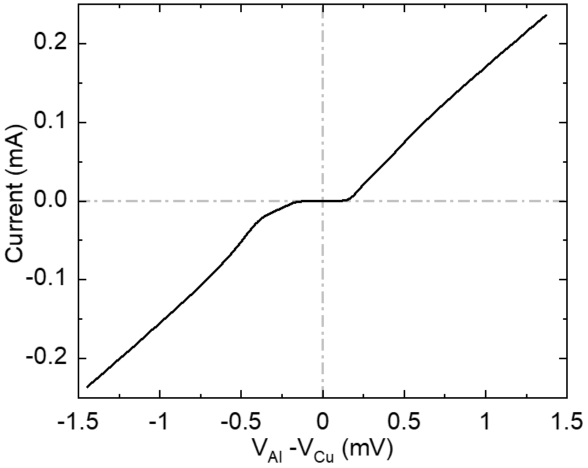
Experimental results in Cu/Al/EuS

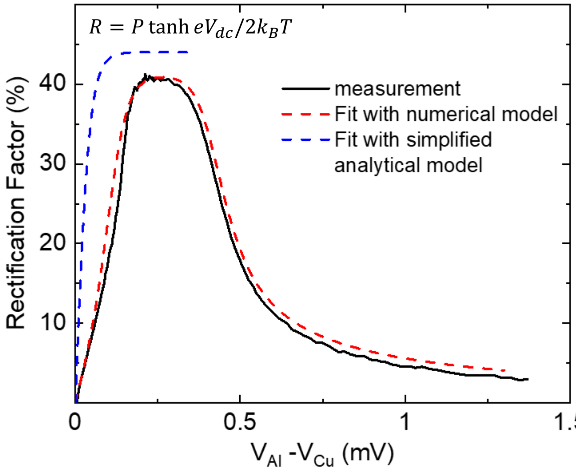






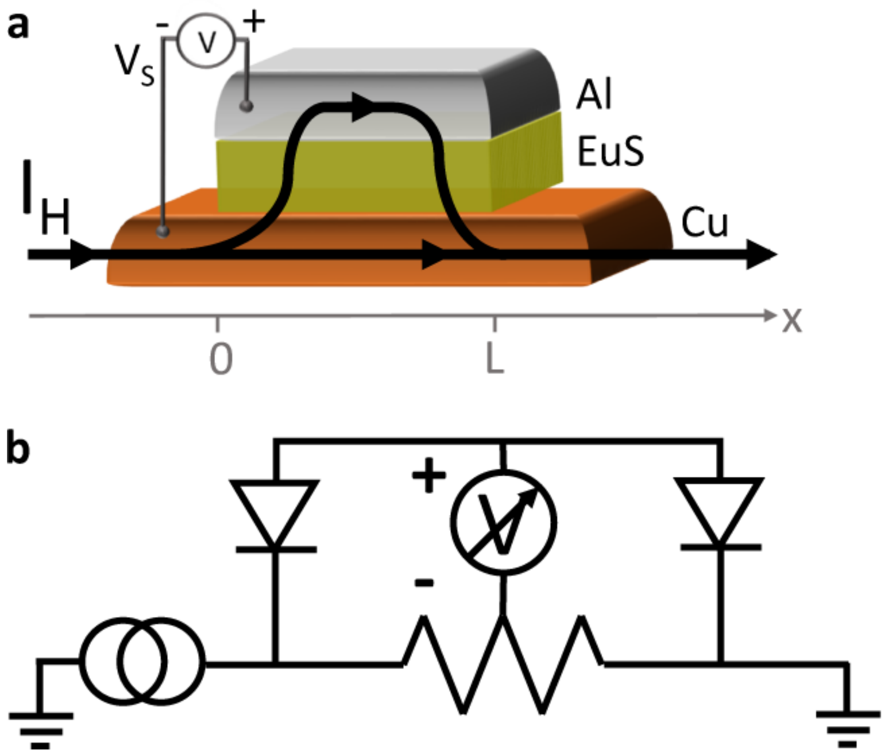


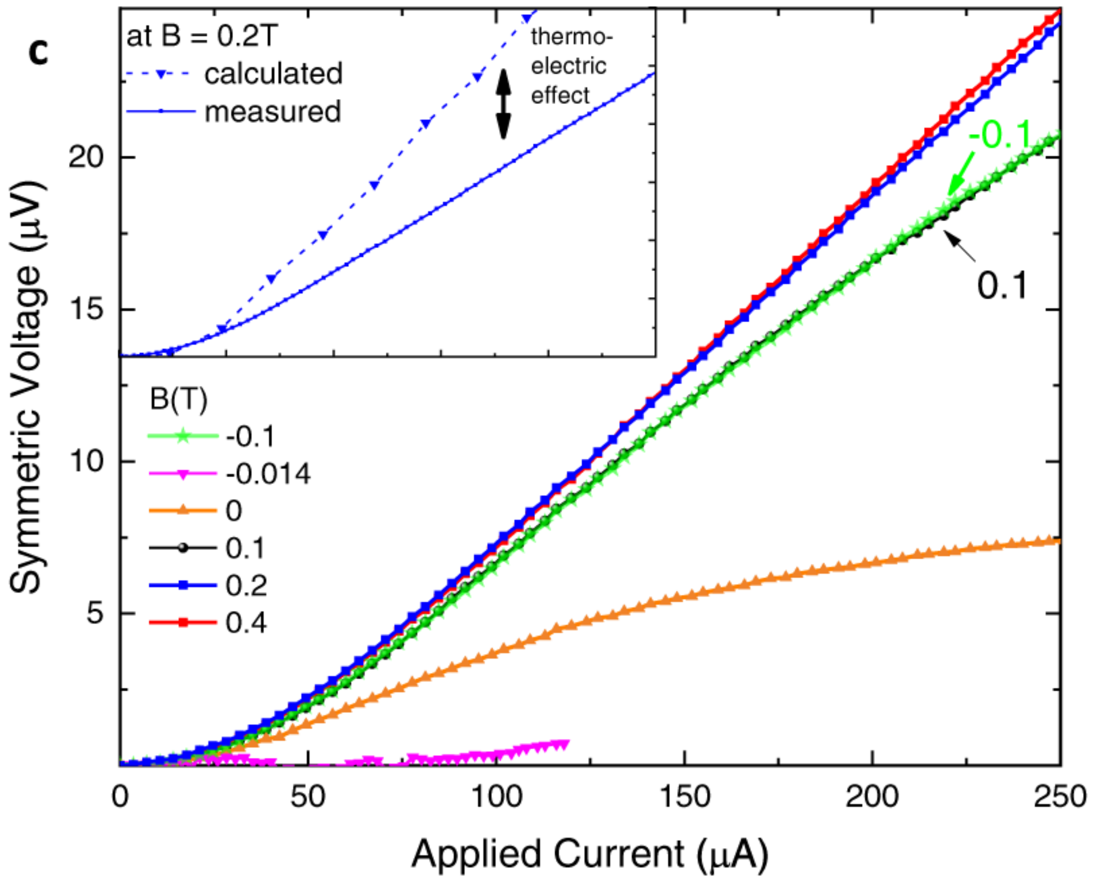




Strambini et al, Nat. Commun. 13 (2022)

Transversal rectifier





Strambini et al, Nat. Commun. 13 (2022)

AC current in N/FI/S junctions

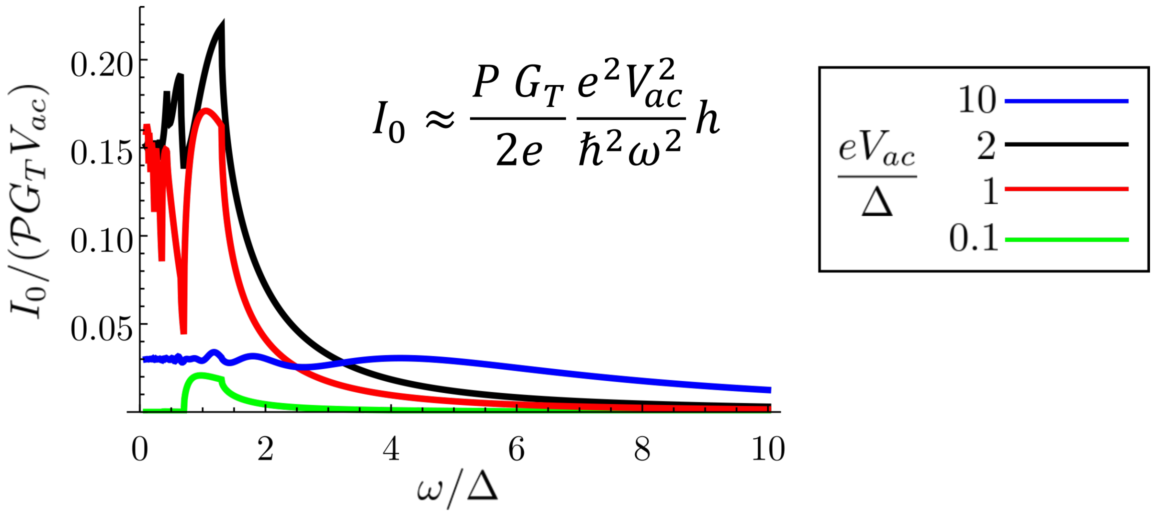
$$V(t) = V_{ac}\cos(\omega t)$$

$$I(t) = I_0 + \sum_{m=1}^{\infty}(I_m^c \cos m\omega t + I_m^s \sin m\omega t)$$

$$I_0 = \frac{P\,G_T}{e}\sum_{n=0}^{\infty} J_n^2\left(\frac{e\,V_{ac}}{\hbar\omega}\right)\int d\epsilon\, N_-(\epsilon)[f_N(\epsilon + n\hbar\omega) - f_S(\epsilon)]$$

at $\hbar\omega \gg \Delta, k_B T, h, eV_{ac}$

$$I_0 \approx \frac{P\,G_T}{2e}\frac{e^2V_{ac}^2}{\hbar^2\omega^2}h$$



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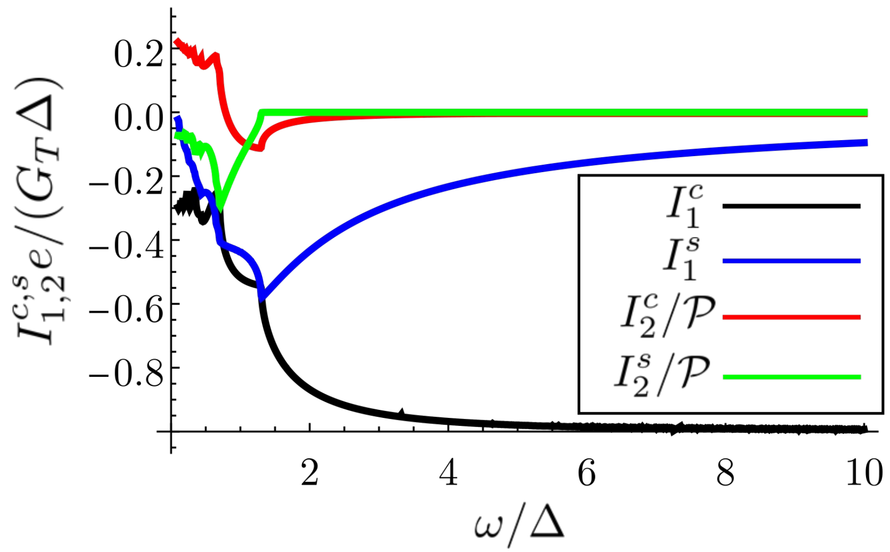
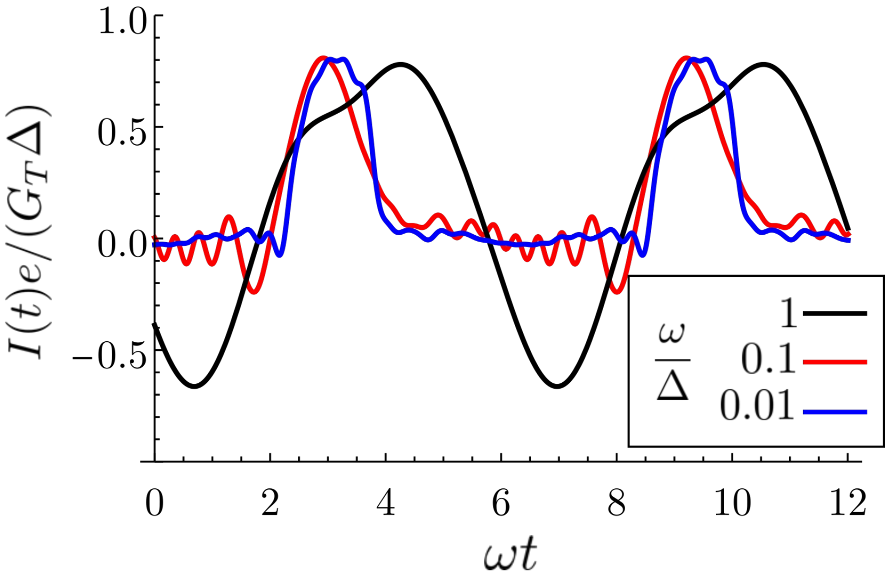
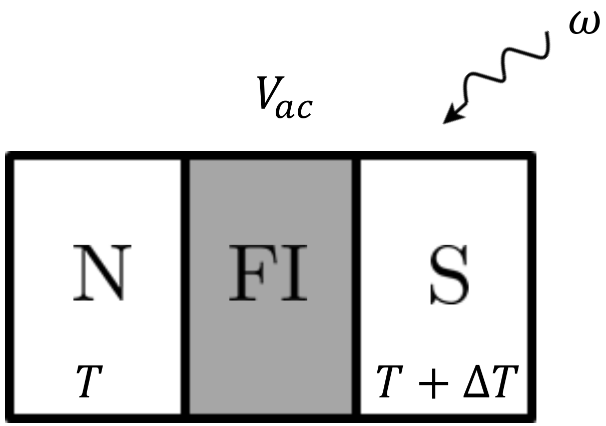


Half-wave rectifier



Application in radiation detectors



$$I = -\alpha \frac{\Delta T}{T} + \frac{P\ G_T}{2\ e} \frac{e^2 V_{ac}^2}{\hbar^2 \omega^2} h$$

- Detected radiaton is directly converted into current – self biased detector
- Works at cryogenic temperatues
- Thermoelectricity and rectification compete if $T_s > T_N$ ($\Delta T > 0$), since $\mathrm{sign}\ (\alpha) = \mathrm{sign}\ (P\ h)$
- Heat balance modelling needed to determine ΔT

Ilic et al, PR Applied 17 (2022)
Heikkilä et al, PR Applied 10 (2018)

Detector yield

- Estimate number of generated electrons per photon n at $\hbar\omega \gg \Delta$
- Incoming power $P_\gamma = \hbar\omega/\delta t$

Thermoelectric effect

$$G_{th}\Delta T \propto P_\gamma$$

$$I_{th} = \frac{\alpha\Delta T}{T}$$

$$n_{th} = \frac{I_{th}\,\delta t}{e} \propto \frac{\alpha\,\hbar\omega}{e\,TG_{th}} \approx \frac{P\ \ \hbar\,\omega}{\Delta - h} > 1$$

Detector yield

- Estimate number of generated electrons per photon n at $\hbar\omega > \Delta$
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Thermoelectric effect

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Rectification effect

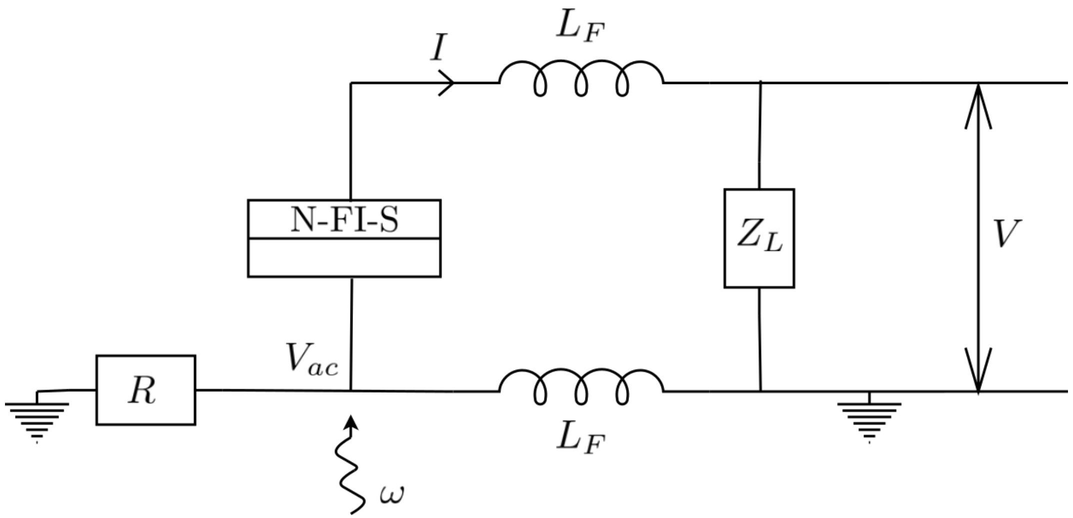
$$\frac{V_{ac}^2}{2\,\tilde{R}} \propto P_\gamma \qquad \tilde{R}\ = R/(1 + R\ G_T)$$

$$I_{rc} \approx \frac{P\ G_T}{2e}\frac{e^2 V_{ac}^2}{\omega^2}h$$

$$n_{rc} = \frac{I_{rc}\,\delta t}{e} \propto \frac{P\ \mathrm{h}}{\hbar\omega}\tilde{R}G_T < 1$$

- Conclusion: thermoelectric effect contributes more to detection at low temperatures

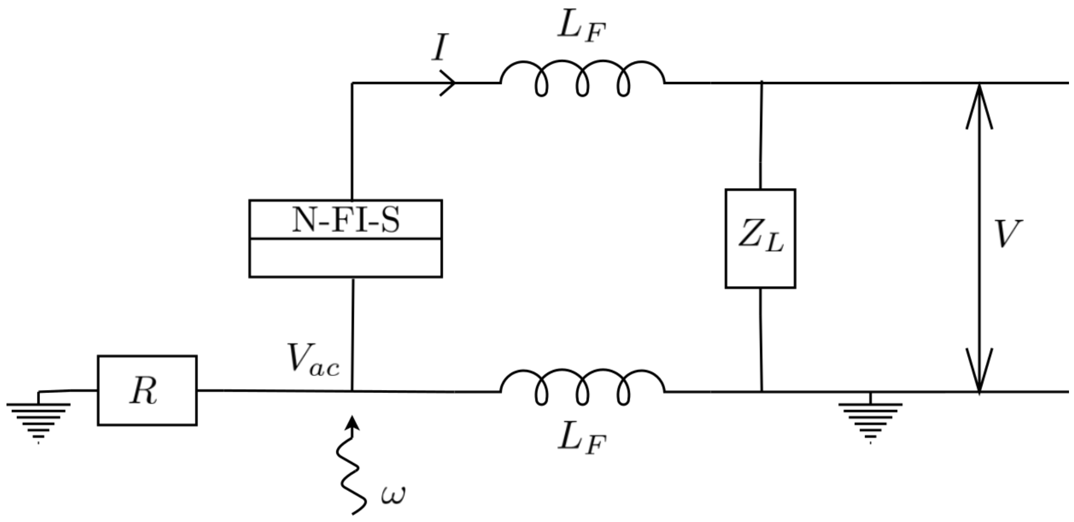
Radiation detector



$$NEP = \frac{\sqrt{\langle \delta V^2 \rangle}}{V} P_\gamma$$

Ilic et al, PR Applied 17 (2022)
Heikkilä et al, PR Applied 10 (2018)

Radiation detector



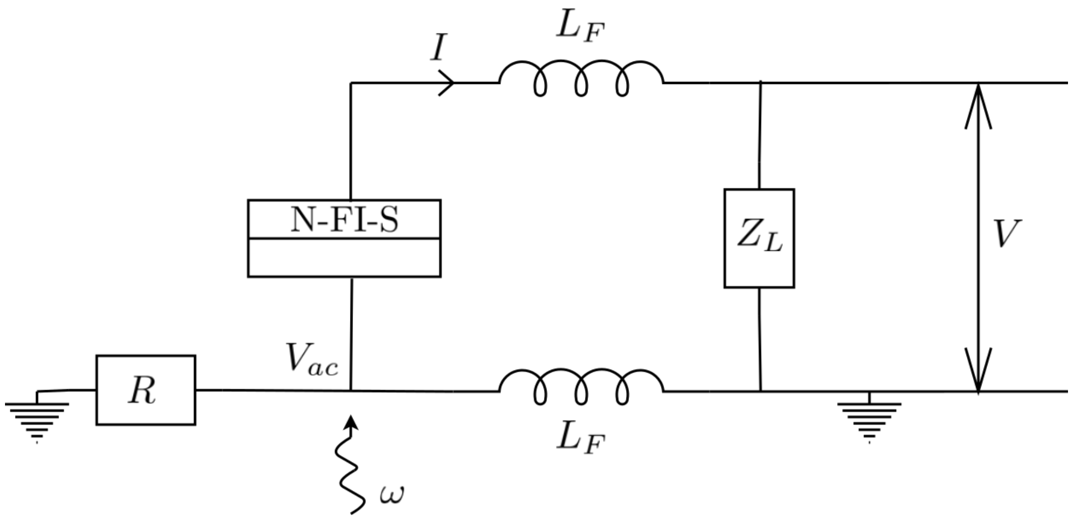
$$NEP = \frac{\sqrt{\langle \delta V^2 \rangle}}{V} P_\gamma$$

Current

$$I = \alpha \frac{\Delta T}{T} - GV - \frac{P\, G_T}{2\, e} \frac{e^2 V_{ac}^2}{\hbar^2 \omega^2} h$$

Ilic et al, PR Applied 17 (2022)
Heikkilä et al, PR Applied 10 (2018)

Radiation detector



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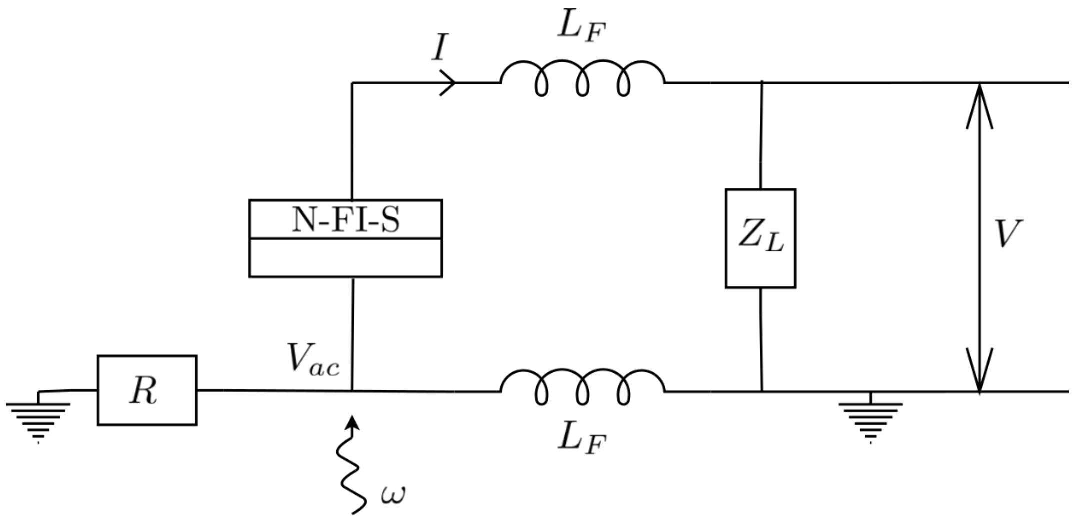
Heat balance

$$P_\gamma = (\mathrm{G_{th}} + \mathrm{G_{ph}})\Delta T - \alpha V + \frac{V_{ac}^2}{4} G_T$$

$$G_{ph} = \Sigma \Omega T^4 \Phi(\Delta, h, T)$$

Ilic et al, PR Applied 17 (2022)
Heikkilä et al, PR Applied 10 (2018)

Radiation detector



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Noise

$$(\mathrm{G_{th}} + \mathrm{G_{ph}})\, \delta T = \delta \dot{Q}_{ph} + \delta \dot{Q} + \alpha \delta V$$

$$(G + Z_L^{-1})\delta V = \delta I + \alpha \delta T / T$$

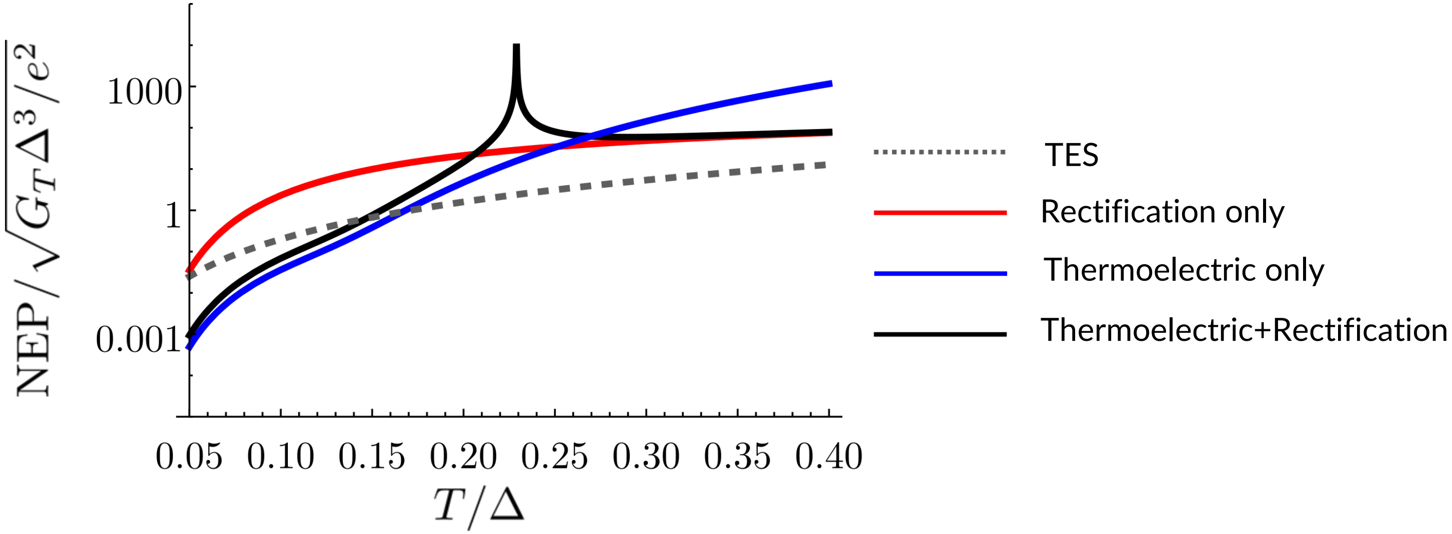
$$\langle \delta I^2 \rangle = 4\, k_B T\, G$$

$$\langle \delta \dot{Q}^2 \rangle = 4\, k_B^2 T^2 G_{th}$$

$$\langle \delta I \delta \dot{Q} \rangle = -4\, k_B T \alpha$$

Ilic et al, PR Applied 17 (2022)
Heikkilä et al, PR Applied 10 (2018)

Radiation detector



Ilic et al, PR Applied 17 (2022)
Heikkilä et al, PR Applied 10 (2018)

Summary

- Combination of spin-splitting and spin-filtering in F/S structures breaks electron-hole symmetry
- This enables thermoelectric effects and current rectification
- Applications:
  - Diode for cryogenic electronics
  - Radiation detector
- Review article: Geng et al, Supercond. Sci. Tech. 36 (2023)

We are looking for postdocs!

https://r.jyu.fi/CondMat