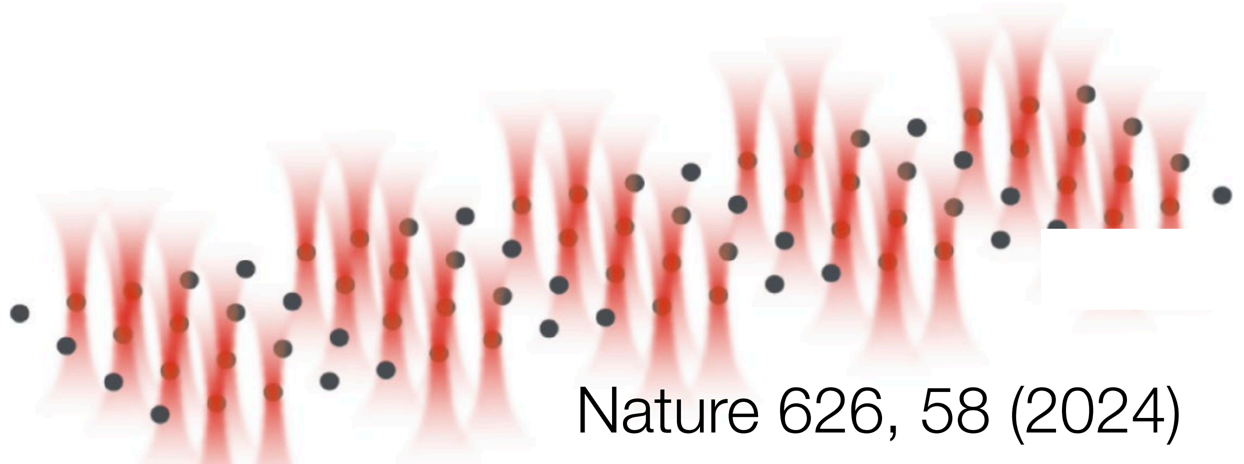
Quantum optics with spins: from single-excitation to many-body physics

Lecture 1

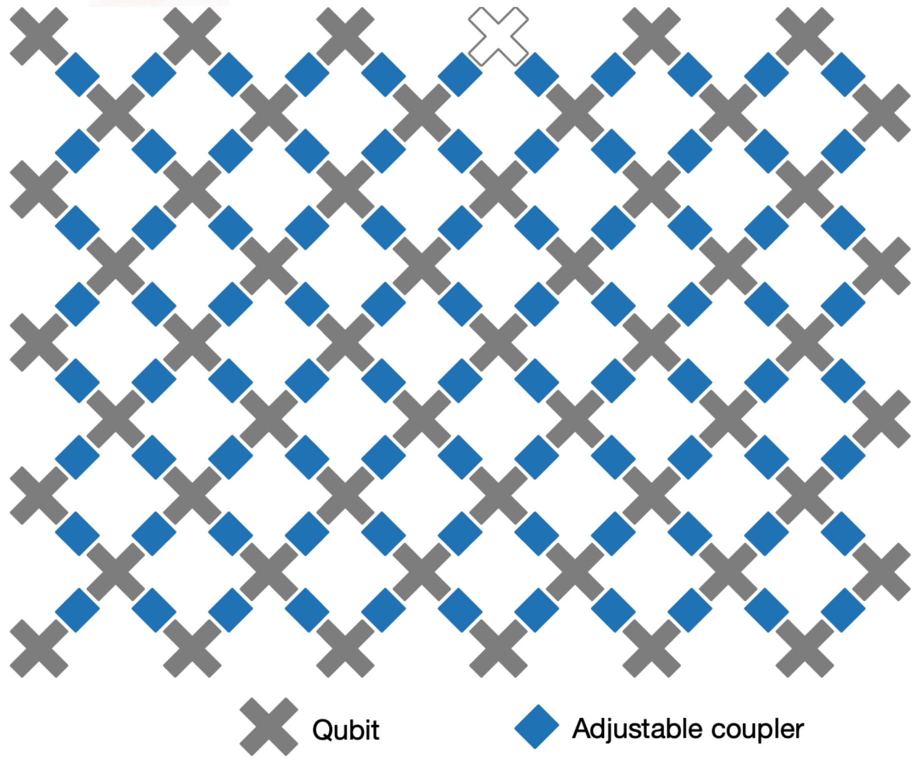
Ana Asenjo-Garcia
Columbia University

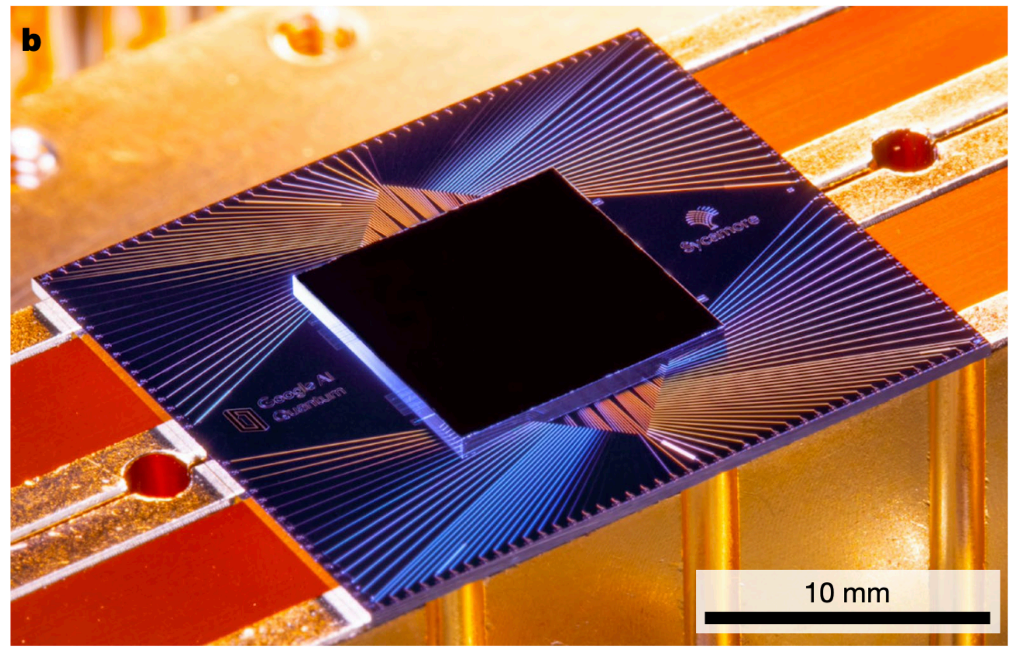
Current state in quantum science: diverse set of applications

Quantum computation



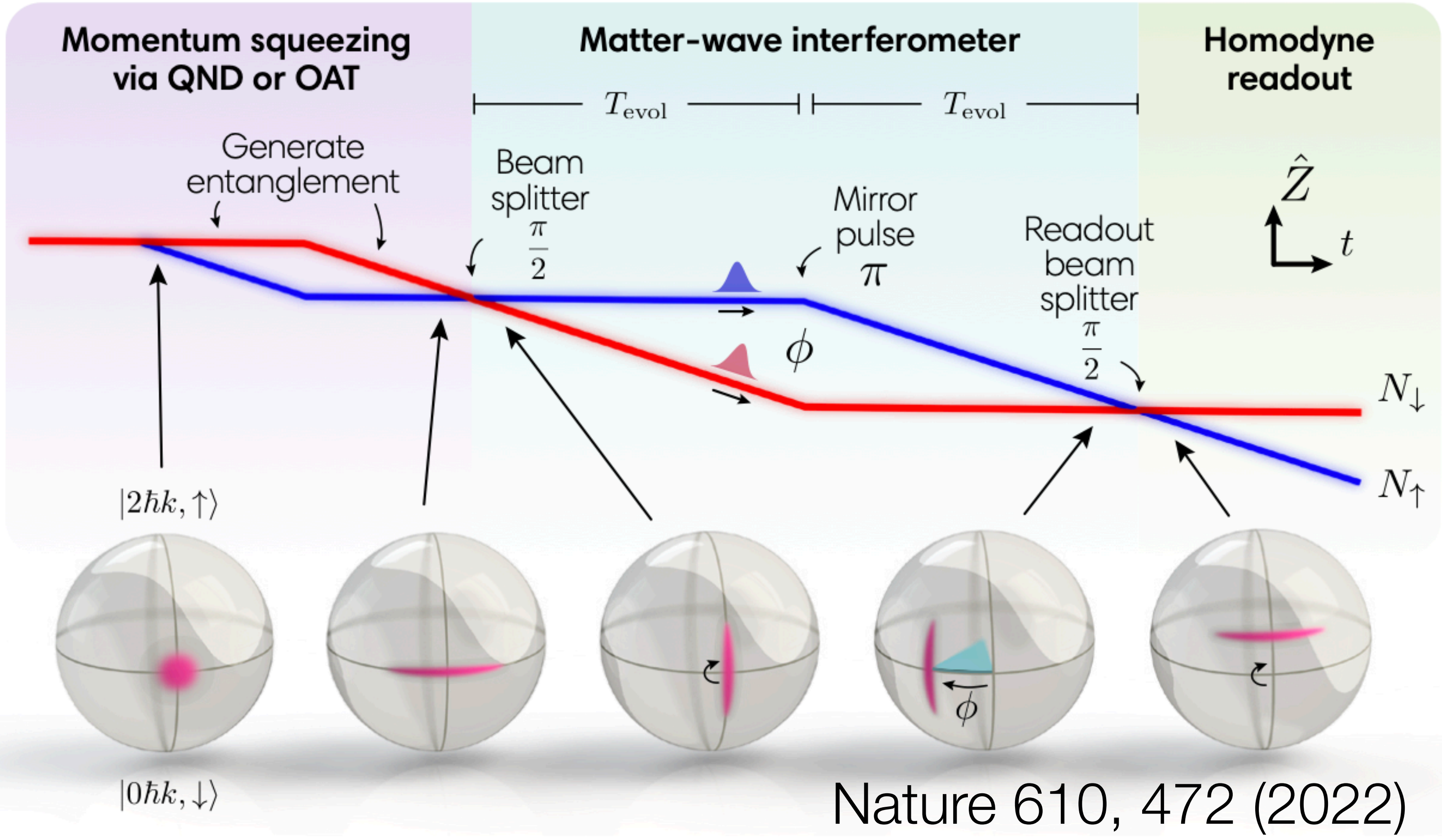
Nature 626, 58 (2024)



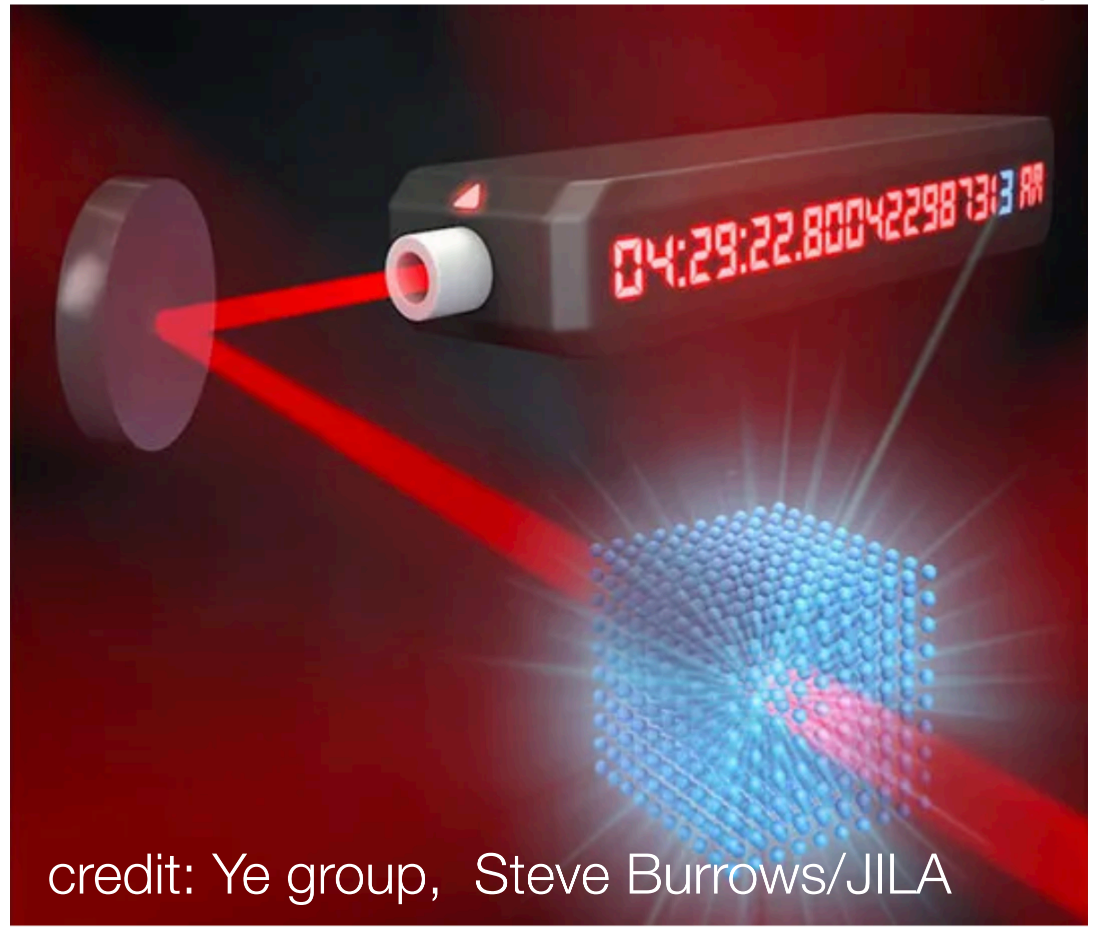


Nature 574, 505 (2019)

Precision measurement: atomic clocks, squeezing

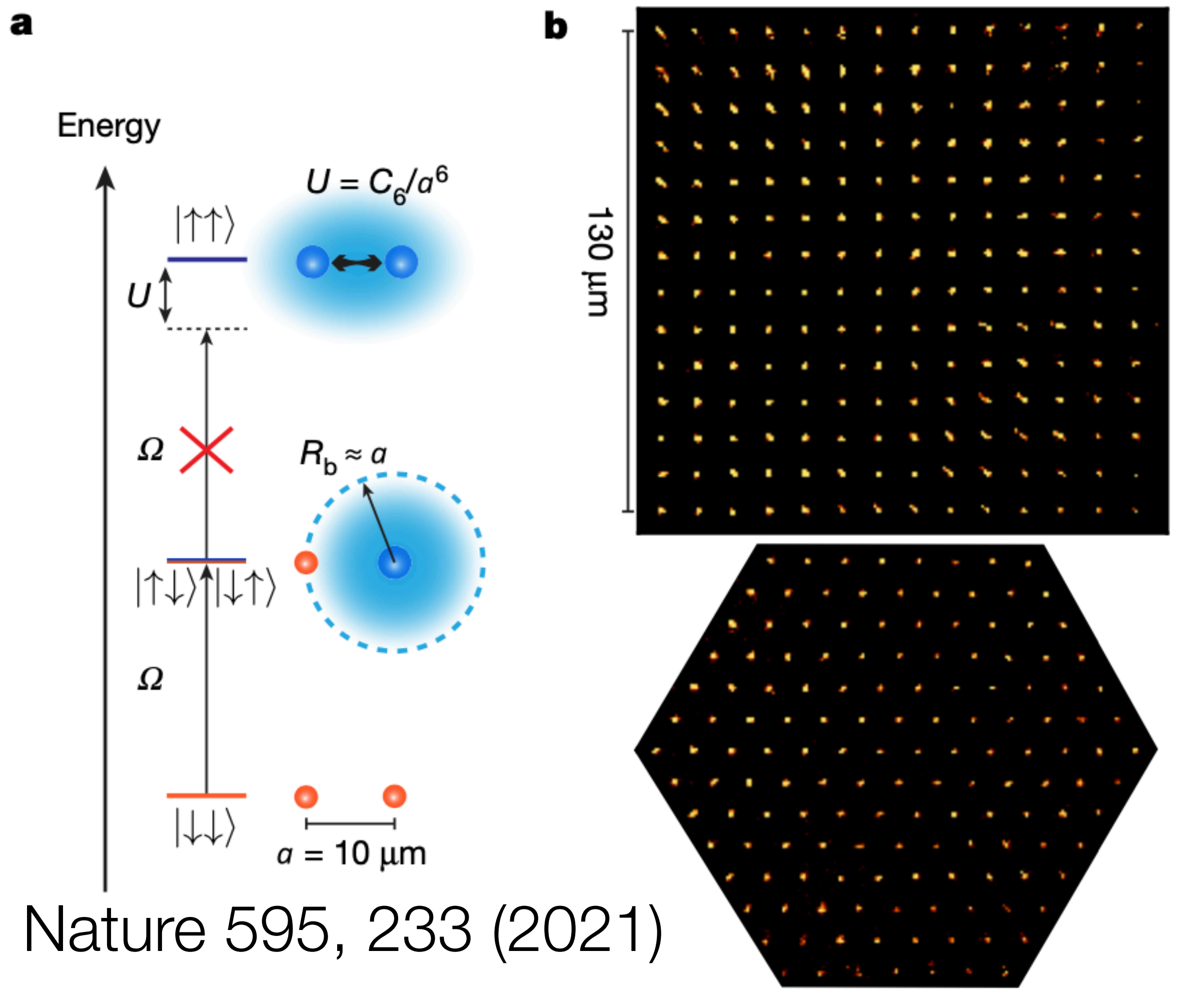


Nature 610, 472 (2022)



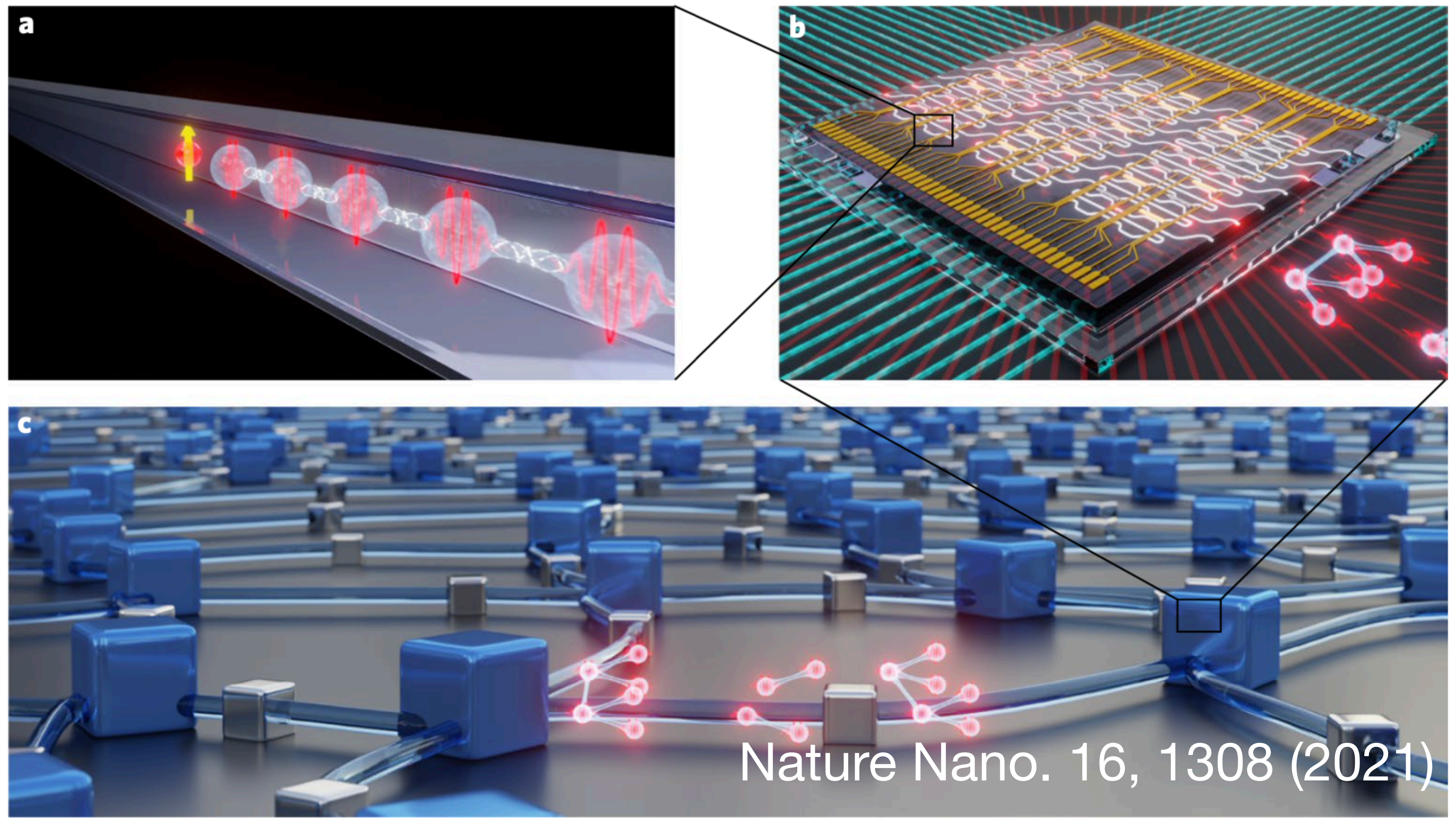
credit: Ye group, Steve Burrows/JILA

Quantum simulation



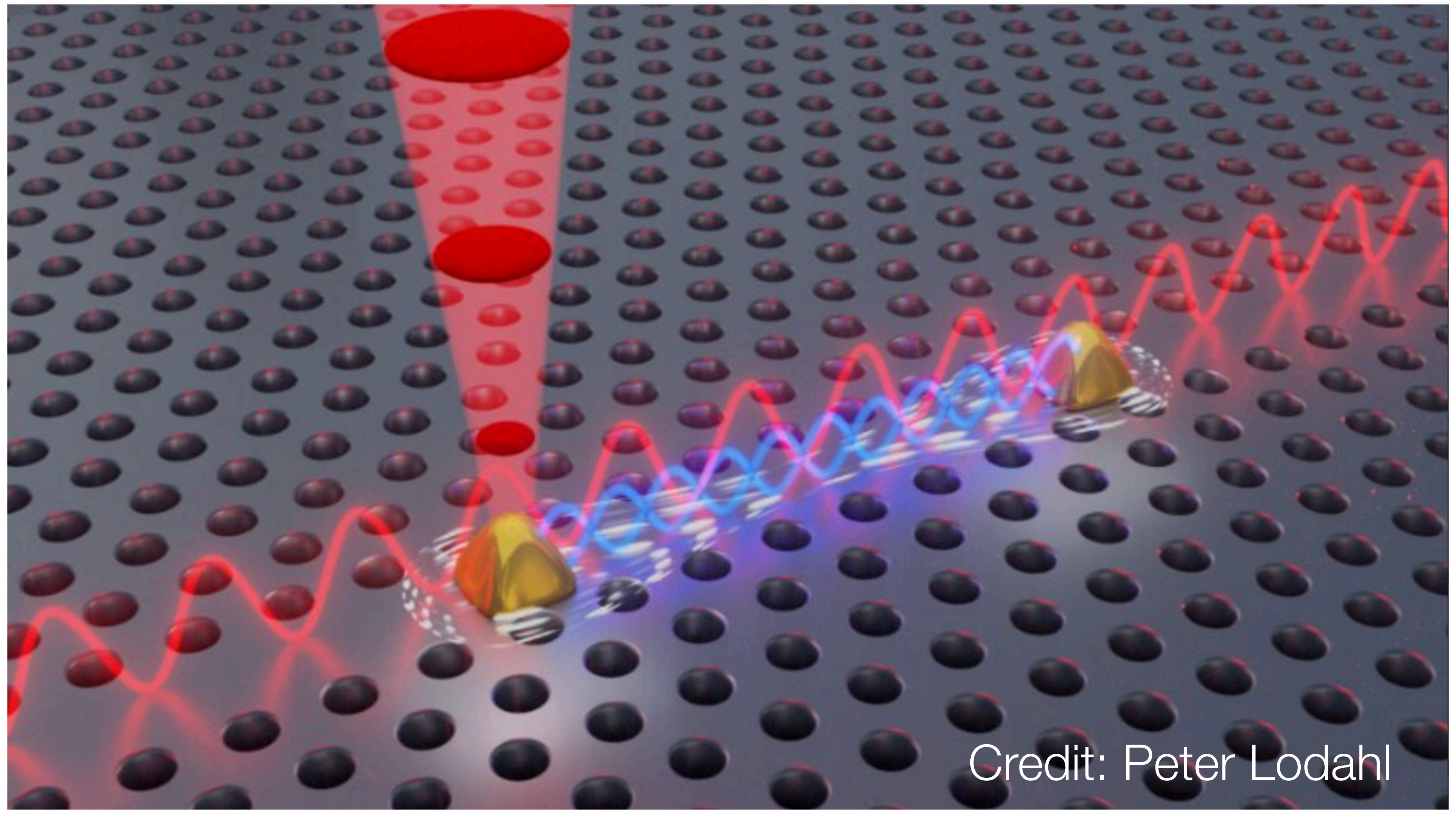
Nature 595, 233 (2021)

Quantum networks

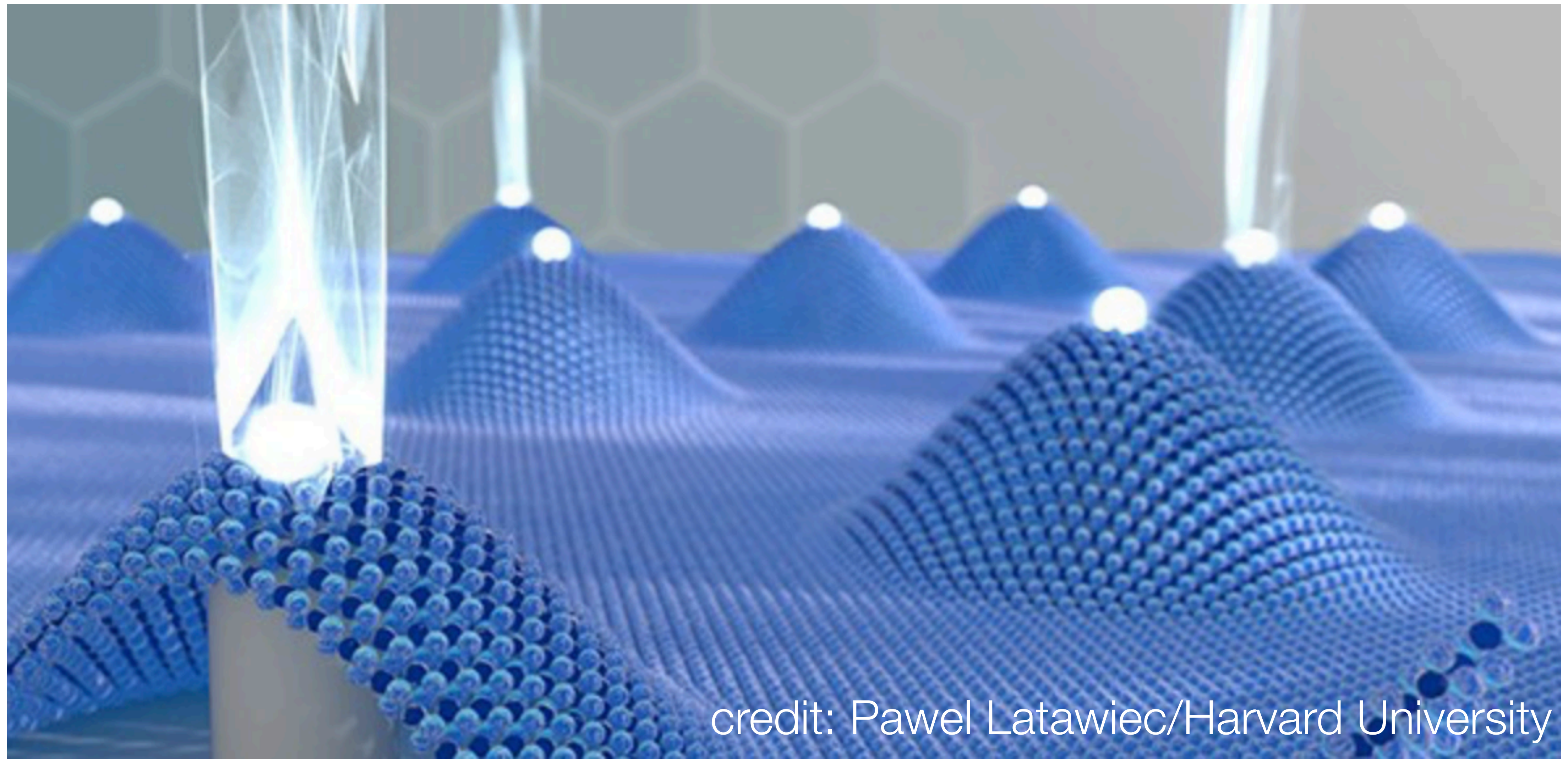


Nature Nano. 16, 1308 (2021)

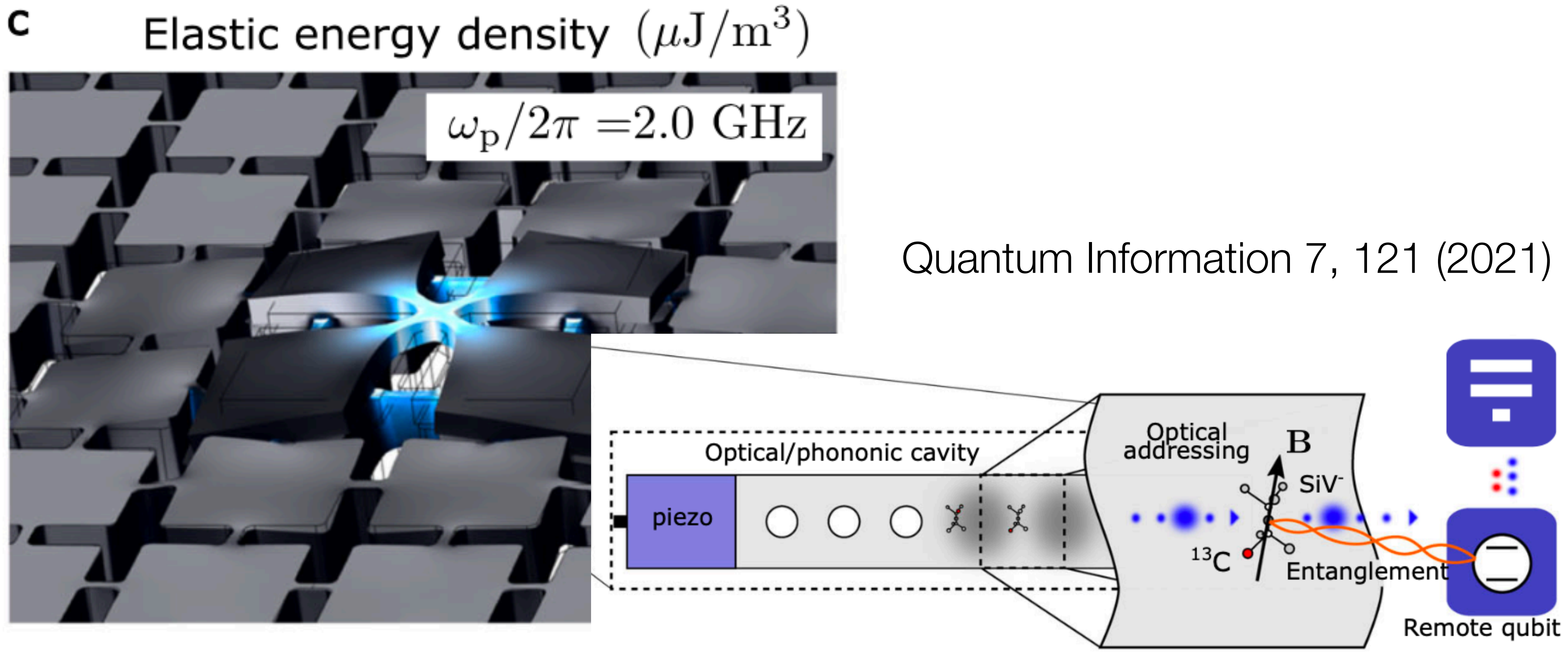
Diverse set of systems and interactions



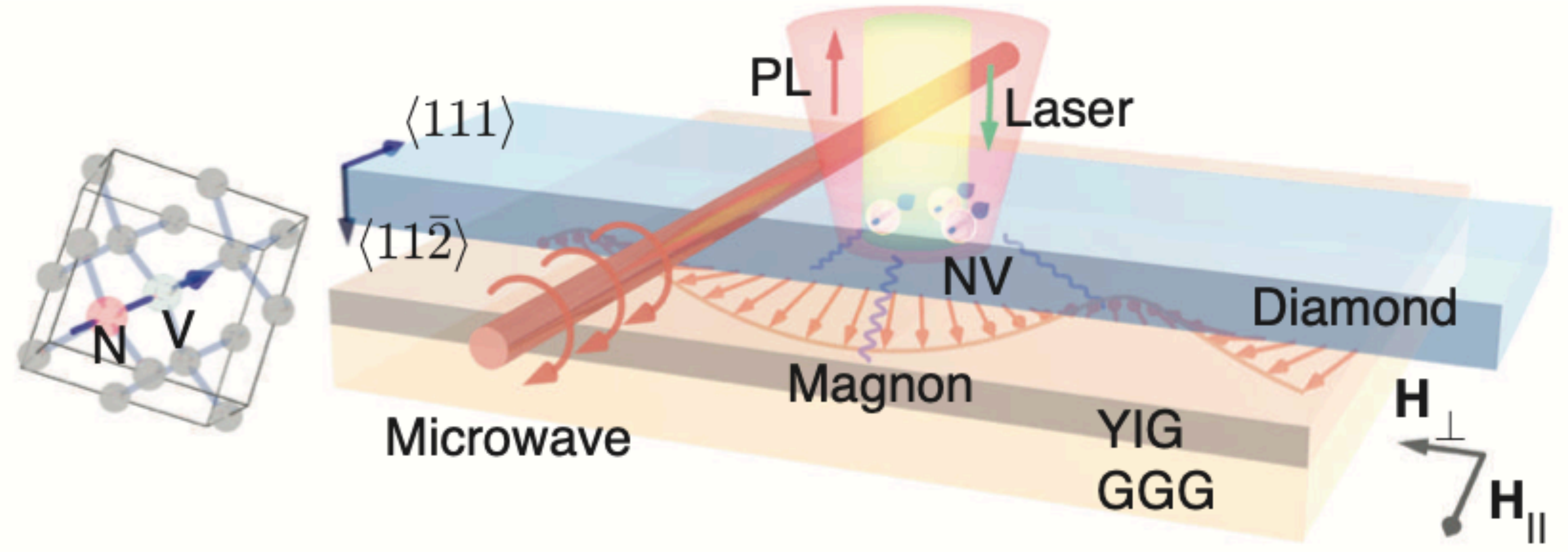
Credit: Peter Lodahl



credit: Pawel Latawiec/Harvard University



Quantum Information 7, 121 (2021)

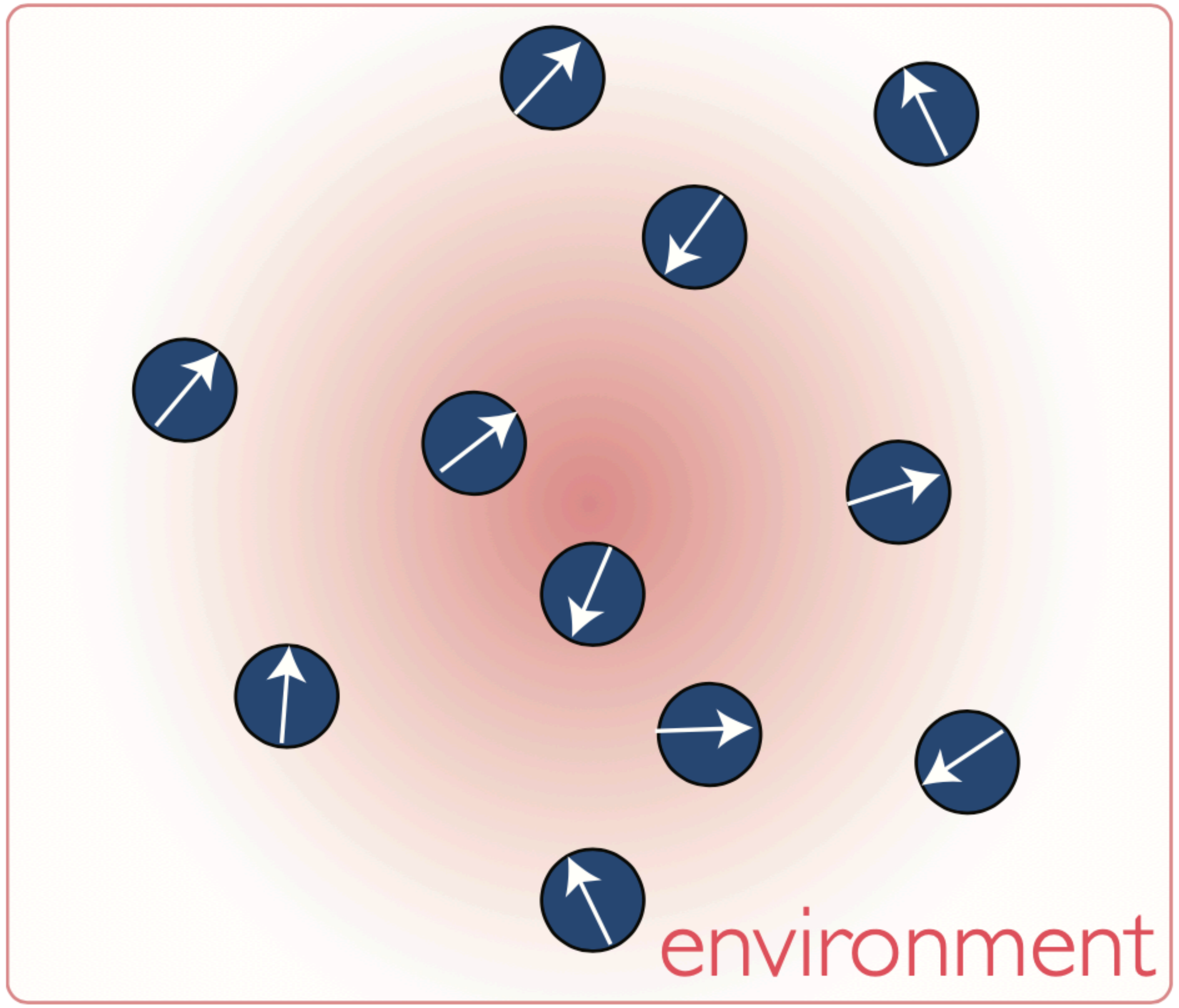


arXiv: 2308.11710 (2023)

Goal: to control LARGE quantum systems

Issue: they are all fundamentally open systems

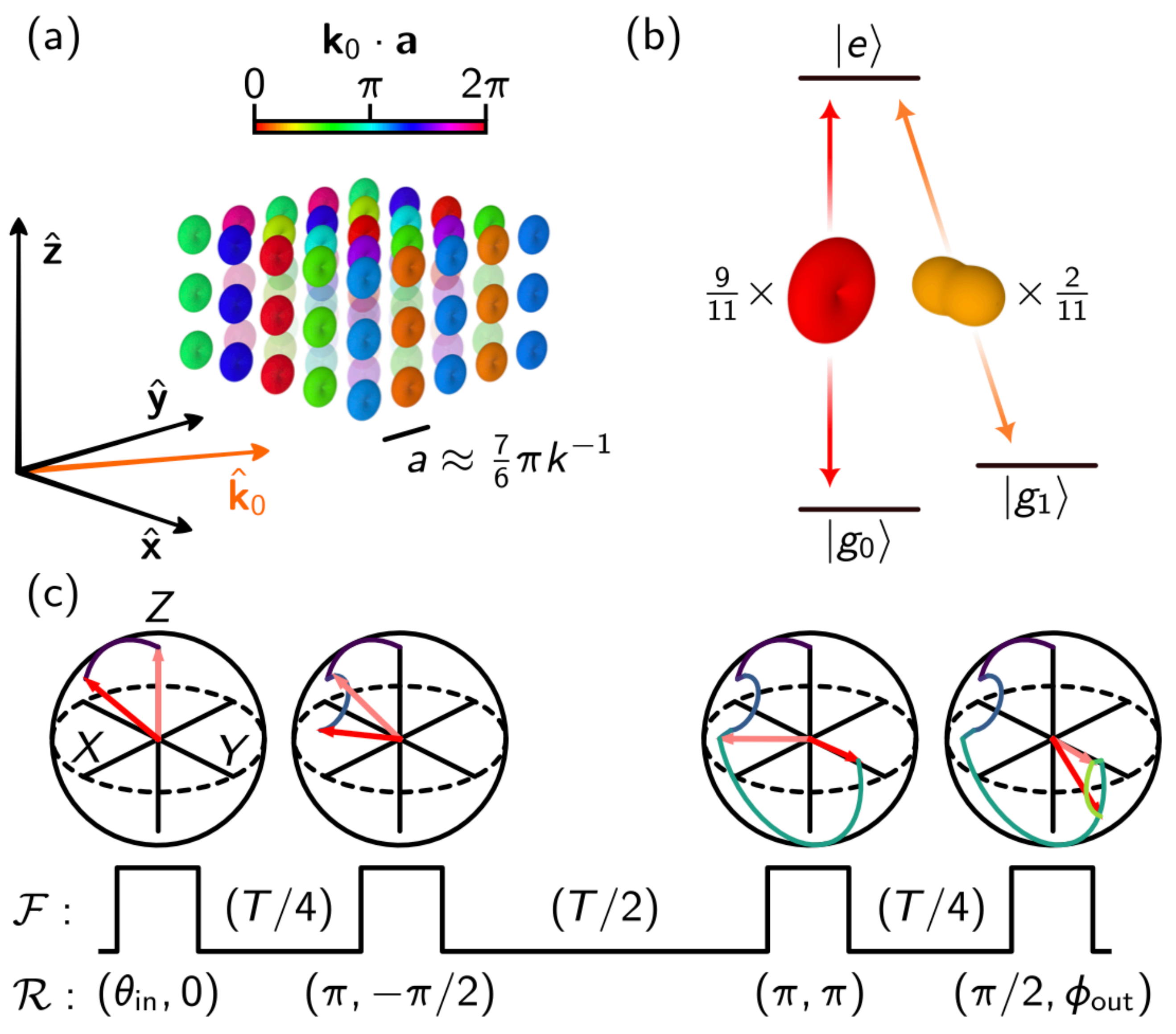
From a fundamental perspective, systems are always coupled to an environment



vacuum fluctuations cannot be switched off

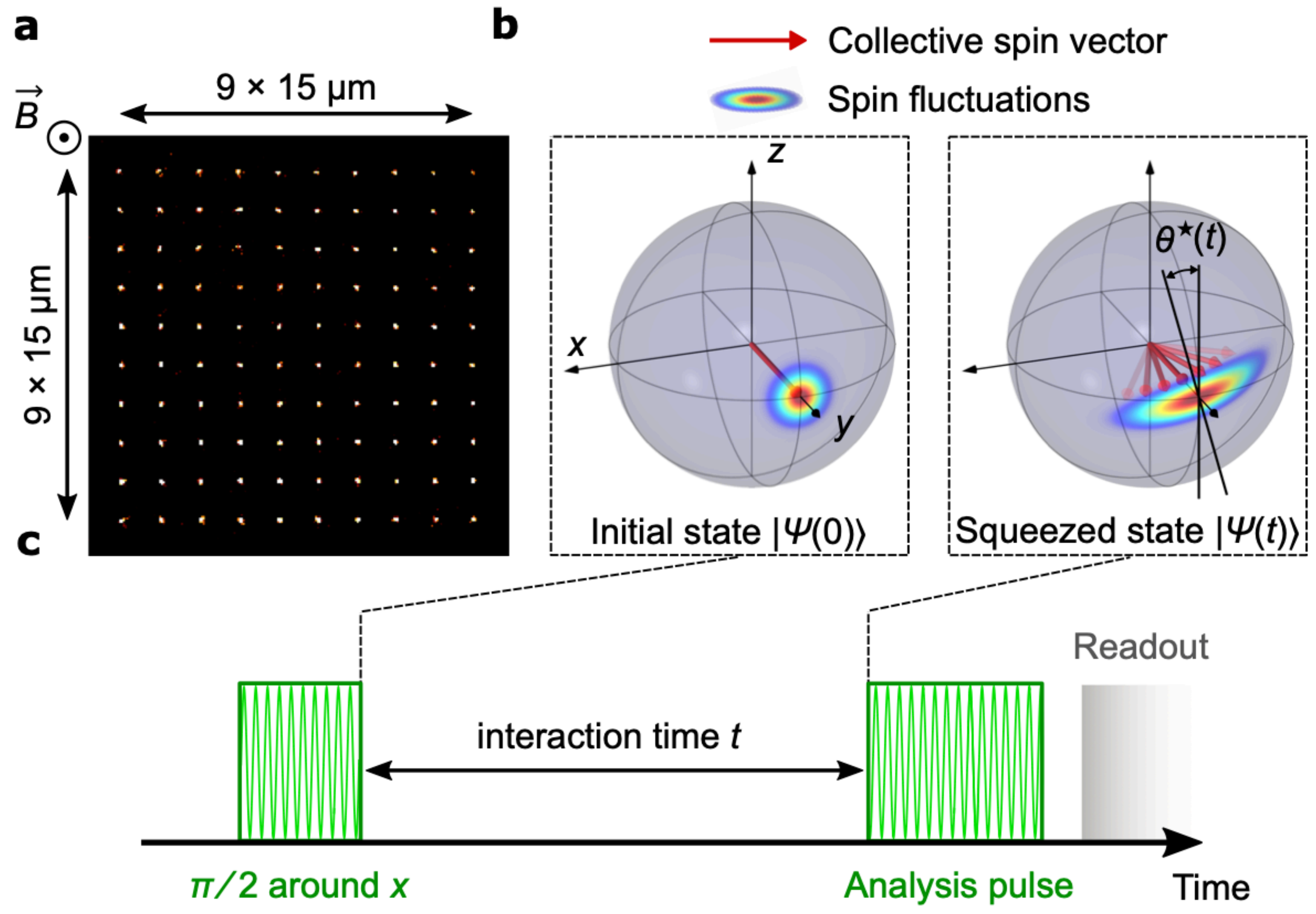
Vacuum fluctuations generate coherent dynamics, exploited for metrology, computing, and sensing applications

Cooperative Lamb shifts in a lattice



Hutson et al., Science 383, 384 (2024)

Spin squeezing in a Rydberg atom array



Bornet et al., Nature 621, 728 (2023)

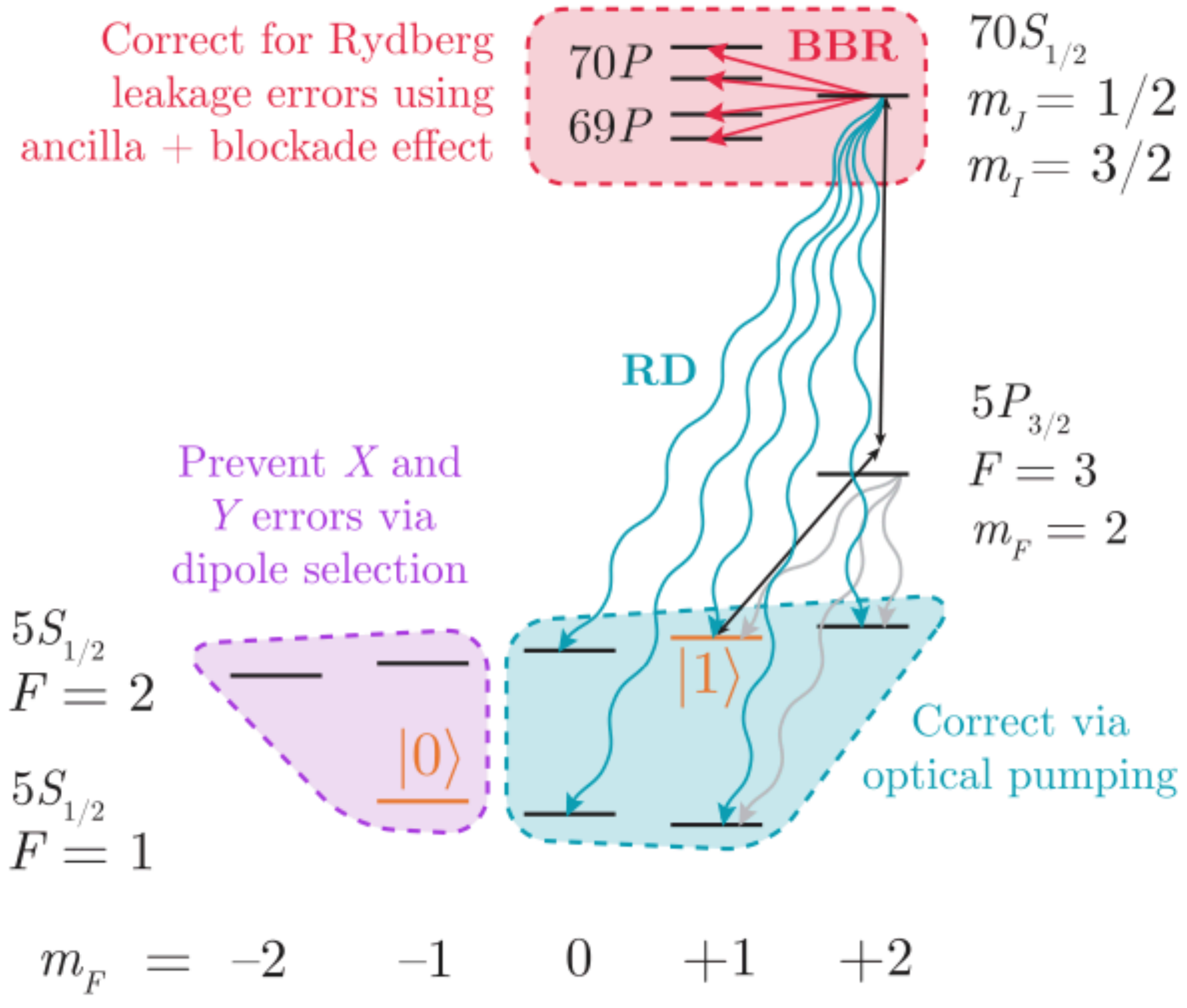
related work (same issue of Nature): Eckner et al., Franke et al.

However, they also mediate long-range dissipative interactions, leading to correlated decay

The role of correlated dissipation:

as a foe…

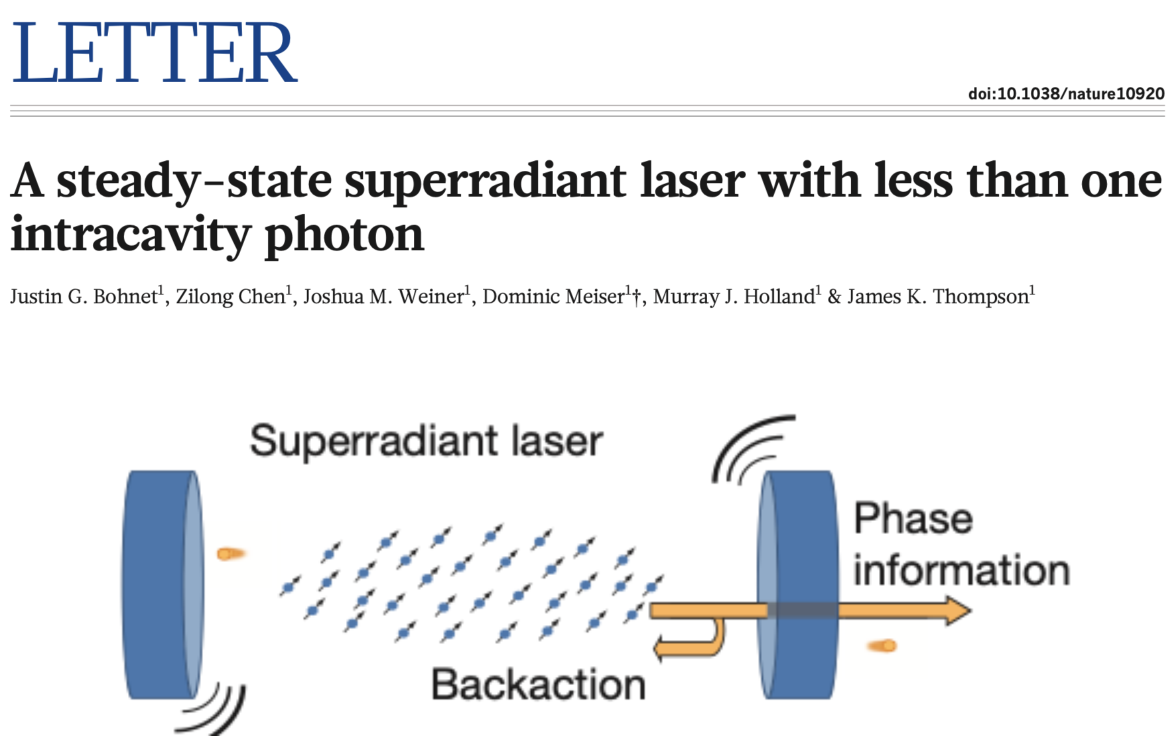
- reduced coherence times for metrology, quantum simulation…
- correlated errors for QC



Phys. Rev. X 12, 021049 (2022)

as a friend…

- To prepare dark (and potentially useful) states
- New light sources (particularly useful for metrology)



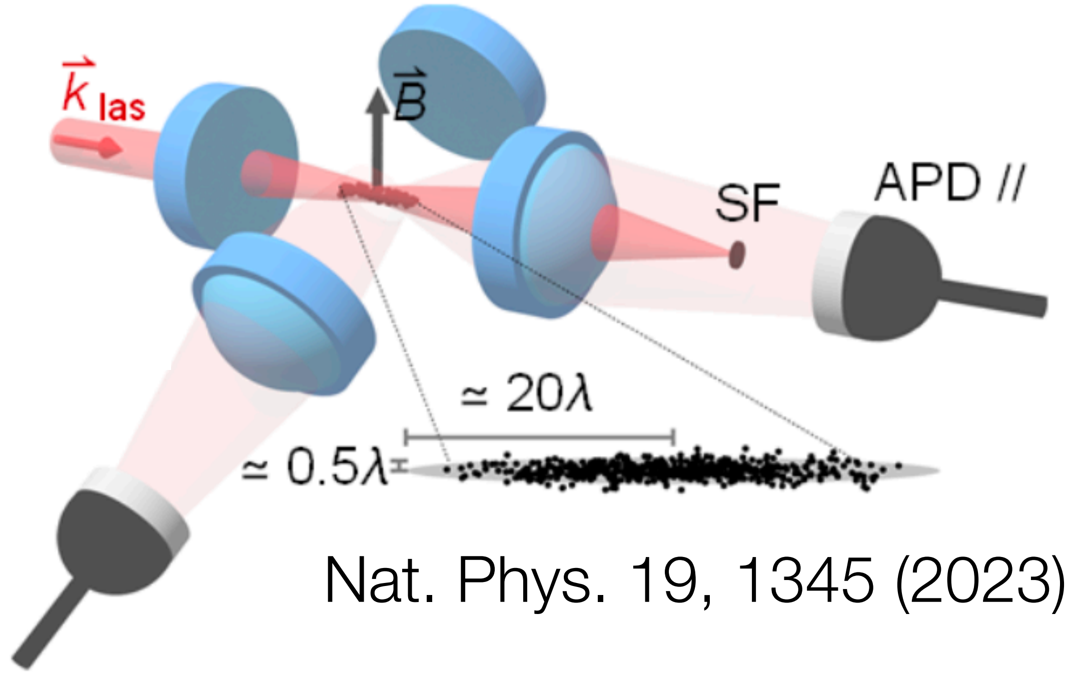
LETTER

doi:10.1038/nature10920

A steady-state superradiant laser with less than one intracavity photon

Justin G. Bohnet[1], Zilong Chen[1], Joshua M. Weiner[1], Dominic Meiser[1]†, Murray J. Holland[1] & James K. Thompson[1]

- Driven-dissipative out-of-equilibrium phase transitions

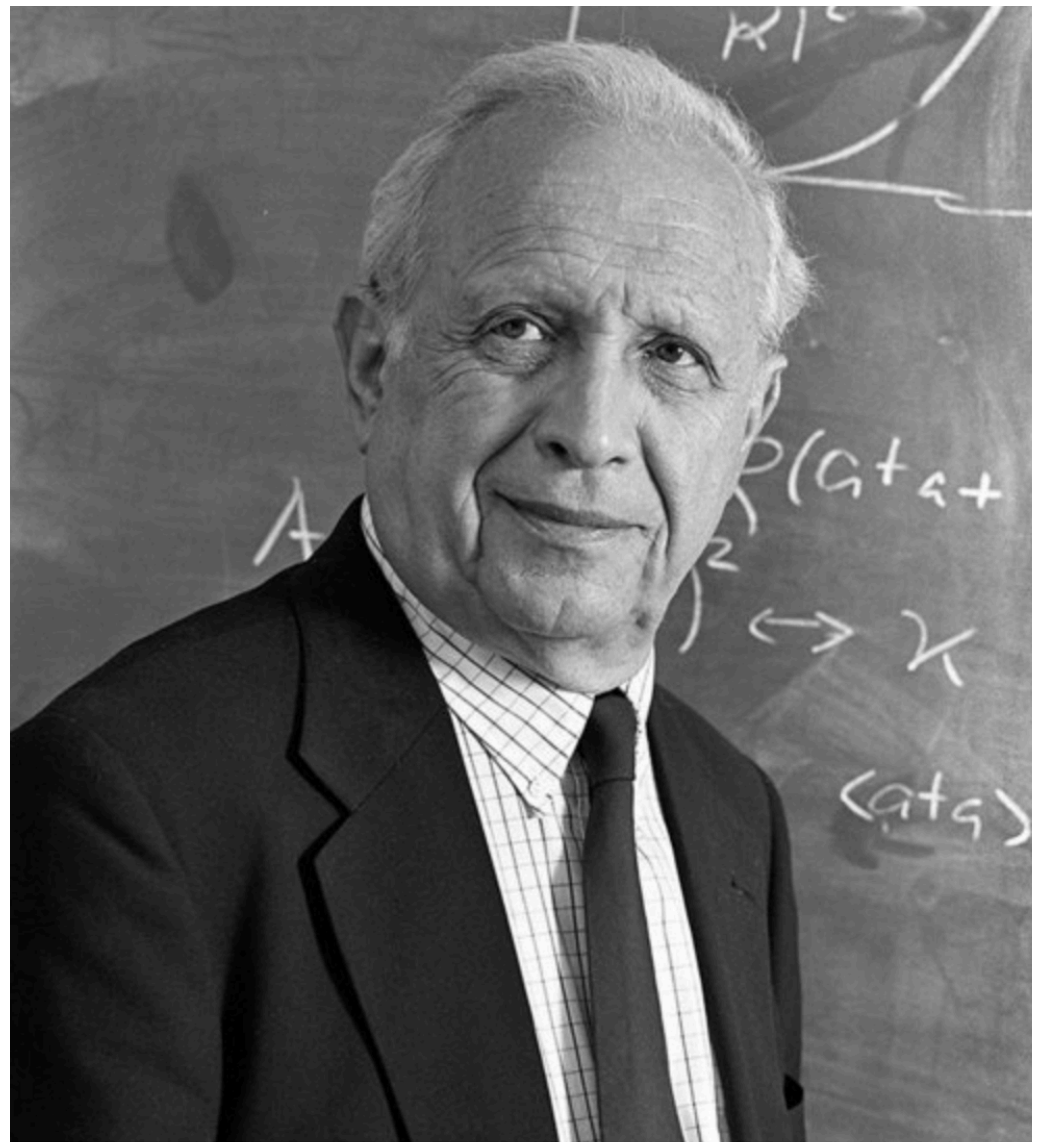


Nat. Phys. 19, 1345 (2023)

Many-body quantum optics: coherence by dissipation

Roy Glauber

Nobel prize "for […] the quantum theory of optical coherence"



Ilya Prigogine

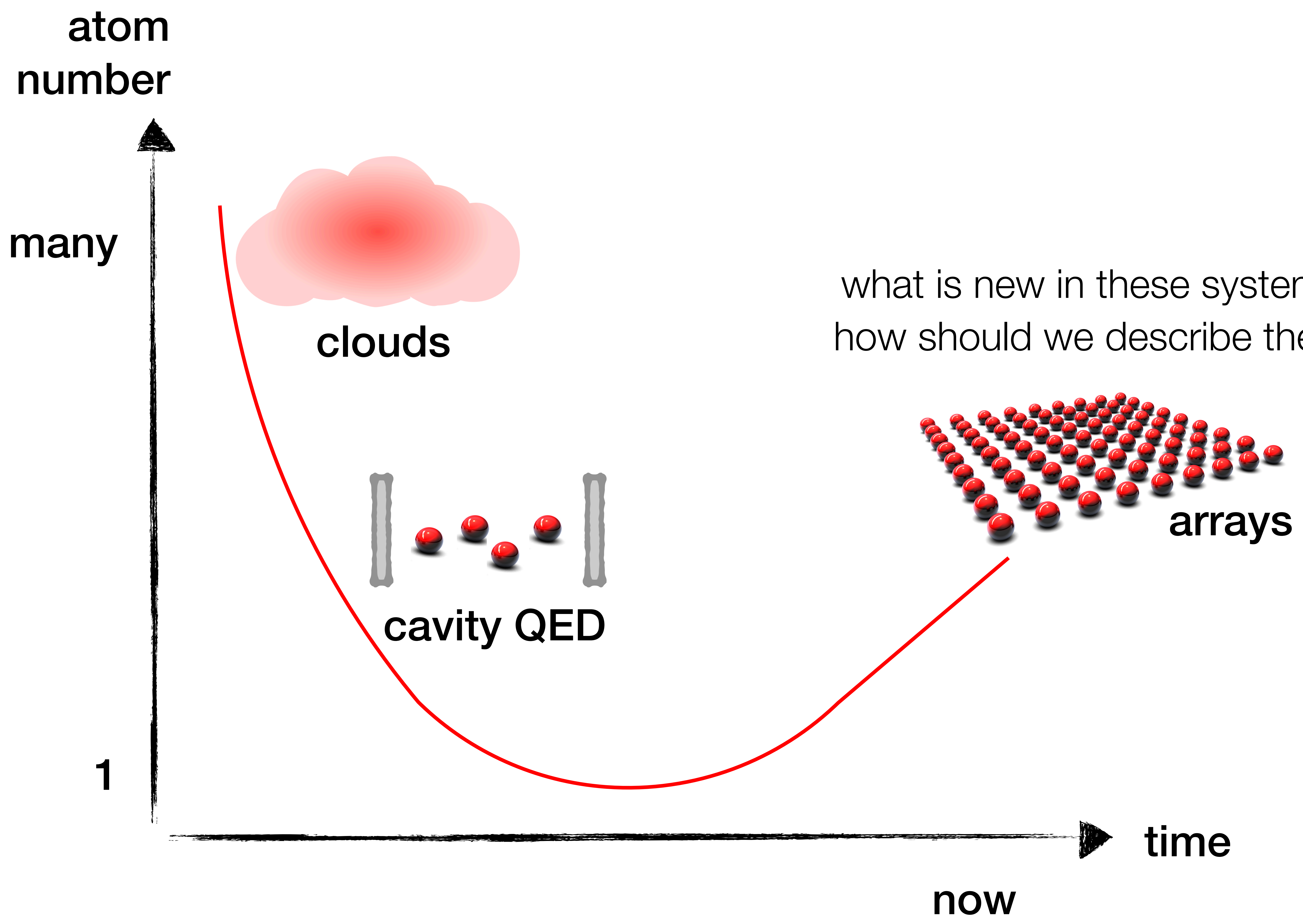
Nobel prize "for […] the theory of dissipative structures"

"[…] all of the delicate and ingenious techniques of optics are exercises in the constructive use of noise"

~dissipation

"Irreversibility […] leads to coherence, to effects that encompass billions and billions of particles"

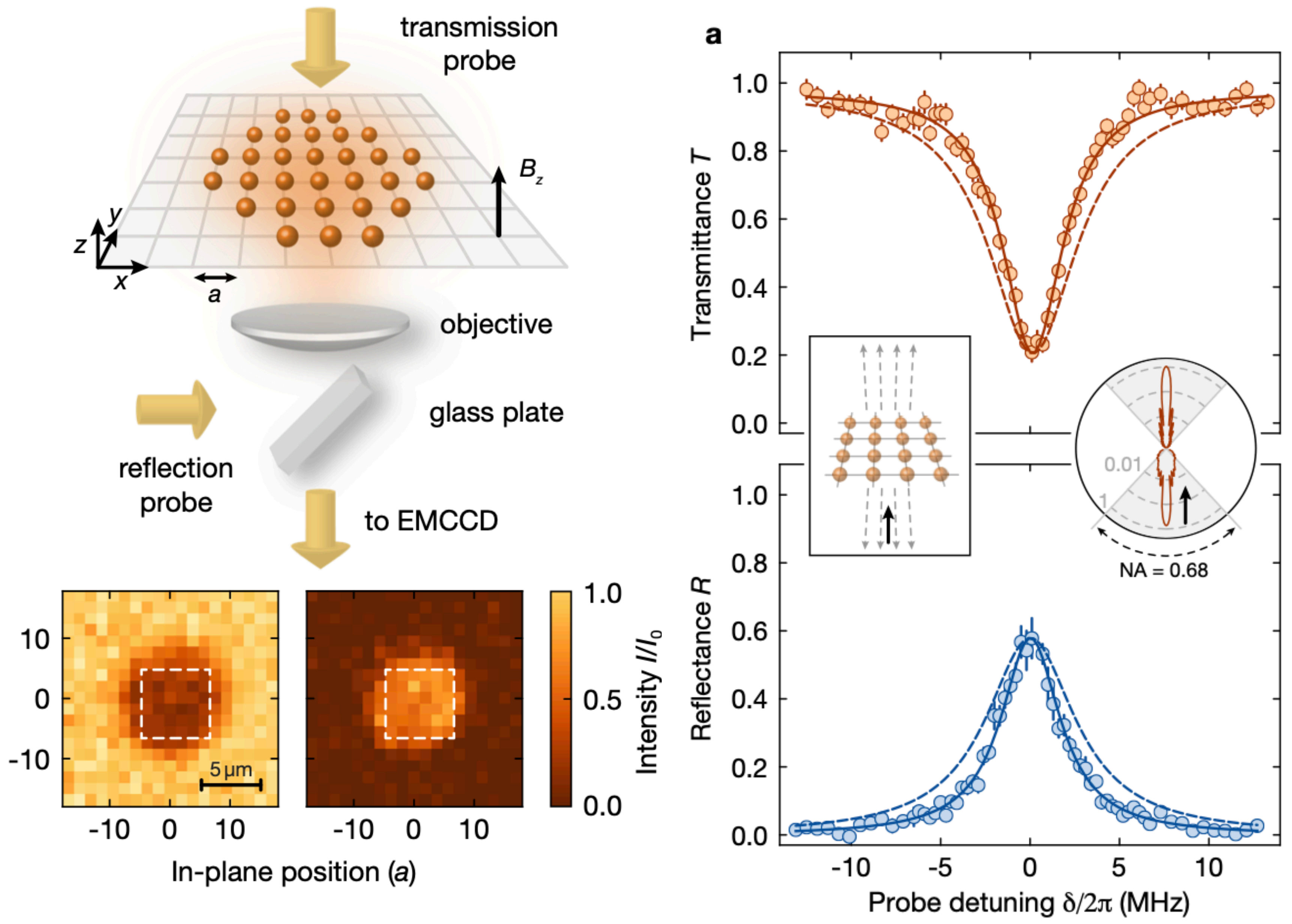
Ordered atomic arrays have appeared as a new platform in QO



what is new in these systems?
how should we describe them?

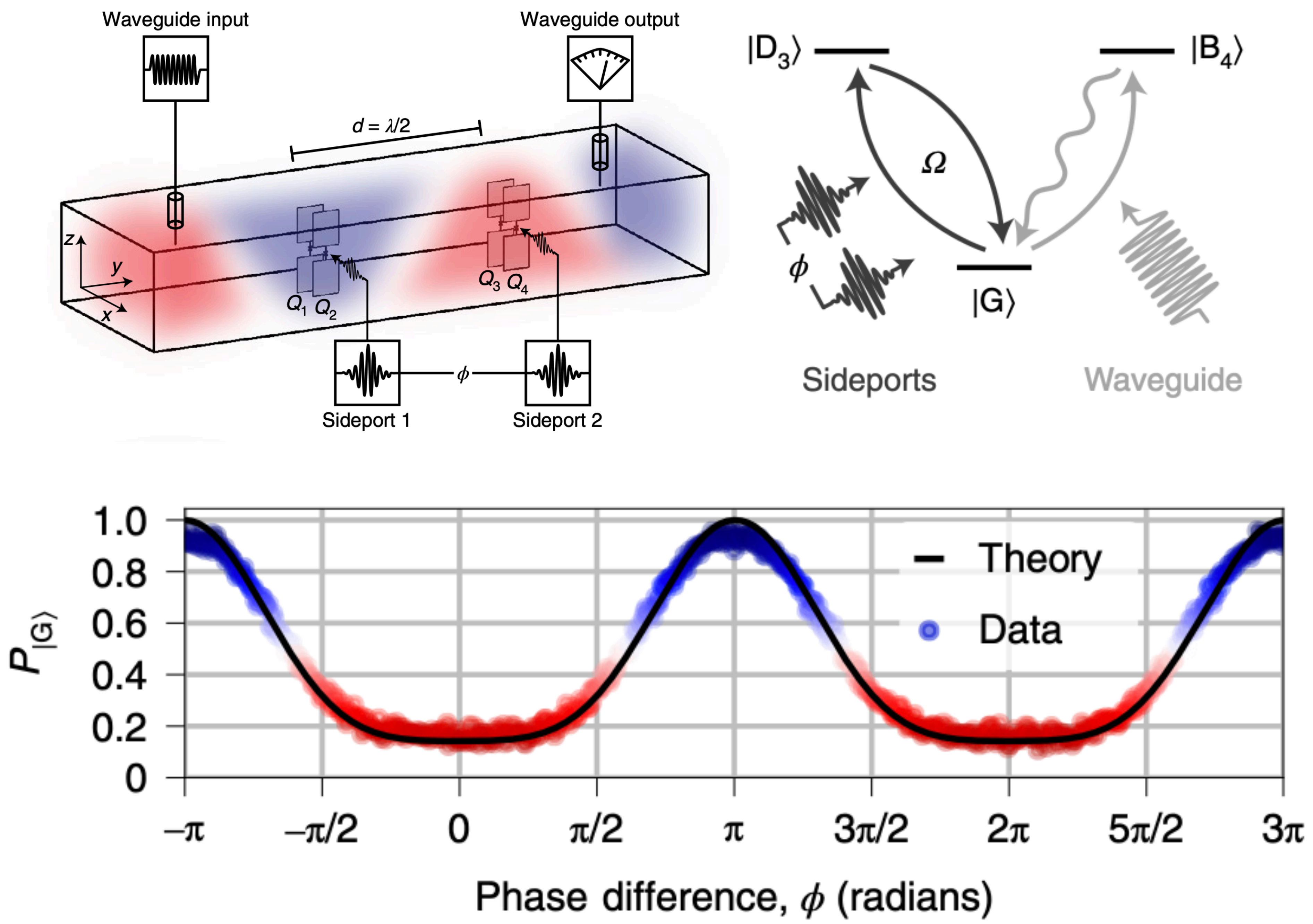
Recent experimental work on quantum optics with atomic arrays

2D atomic array as a perfect mirror



Rui et al., Nature 583, 369 (2020)

Coherent driving of multi-qubit dark state



M. Zanner et al., Nat. Phys. 18, 538 (2022)

+ recent preliminary results by the Greiner group on many-body superradiance in an Erbium lattice

Lecture plan

Lecture 1 - Spin model: spontaneous emission, theoretical description, approximations

Lecture 2 - Few excitation physics: super- and sub-radiance, selective radiance, applications

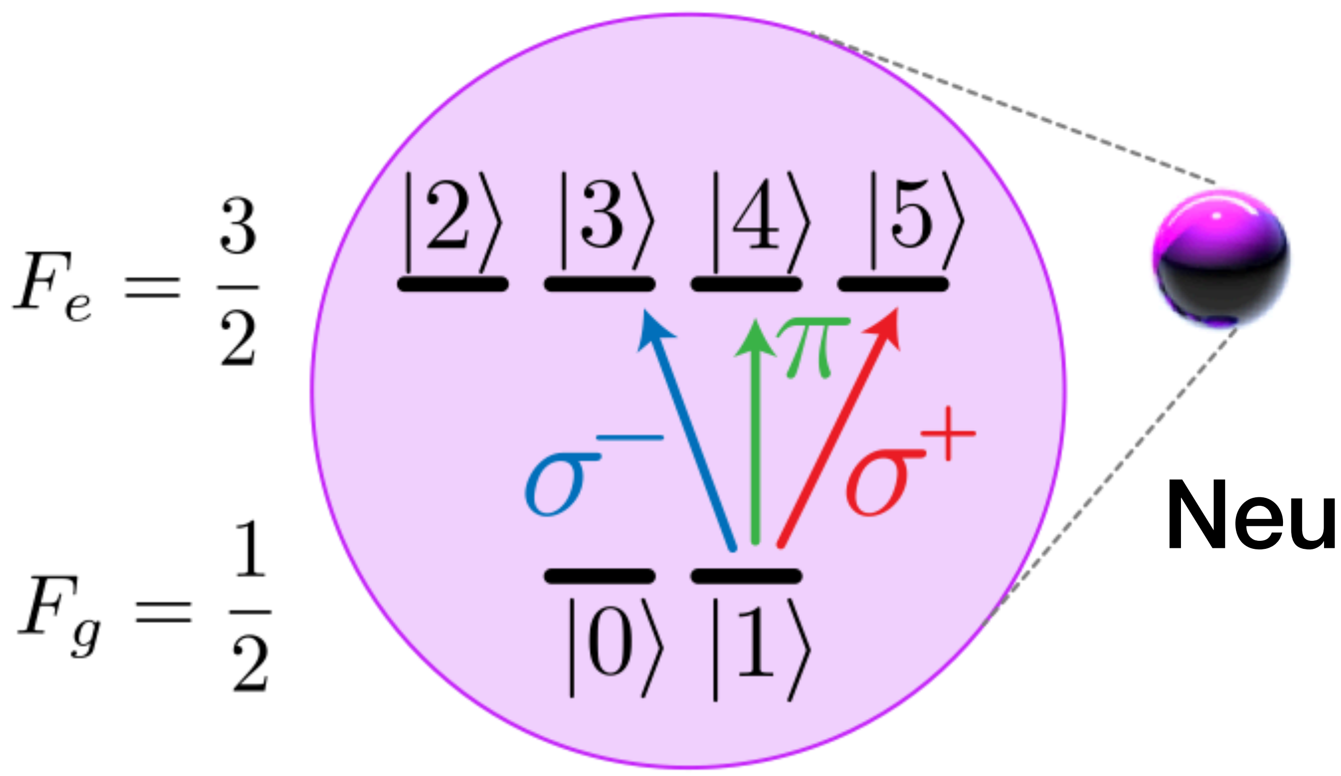
Lecture 3 - Many body physics: some adventures in the deep(er) Hilbert space

Lecture 1

Setting up the stage

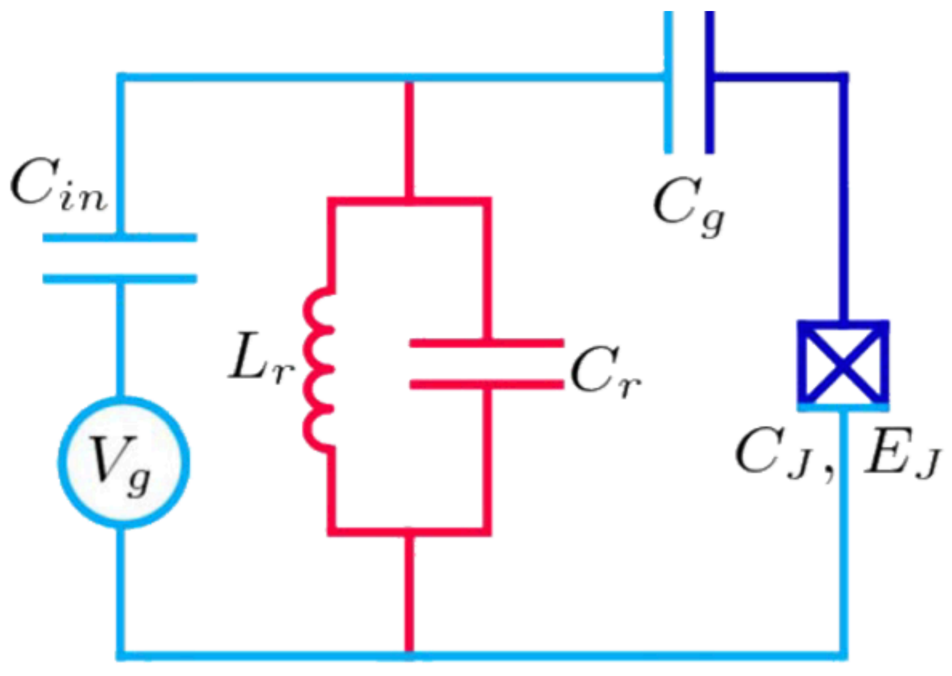
Spin model

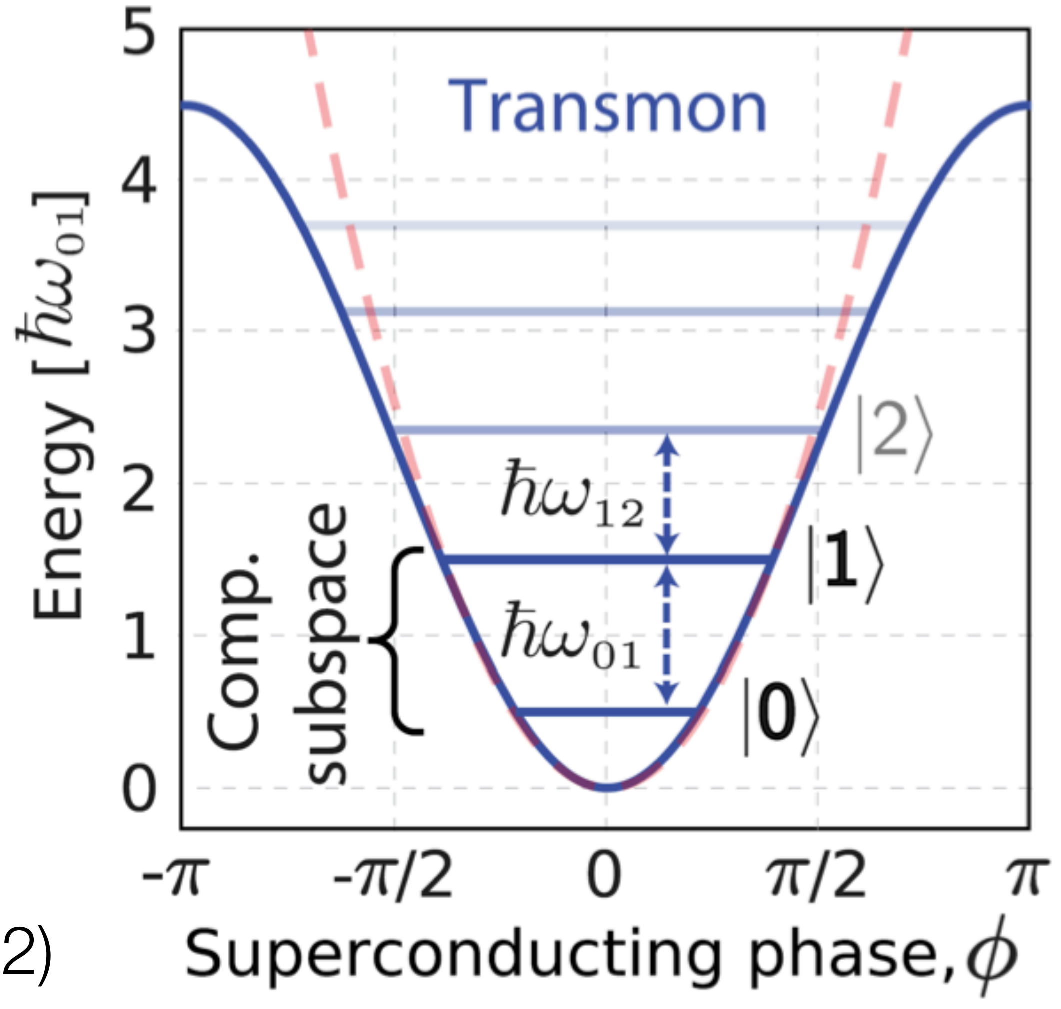
Examples of “two-level” systems/spins



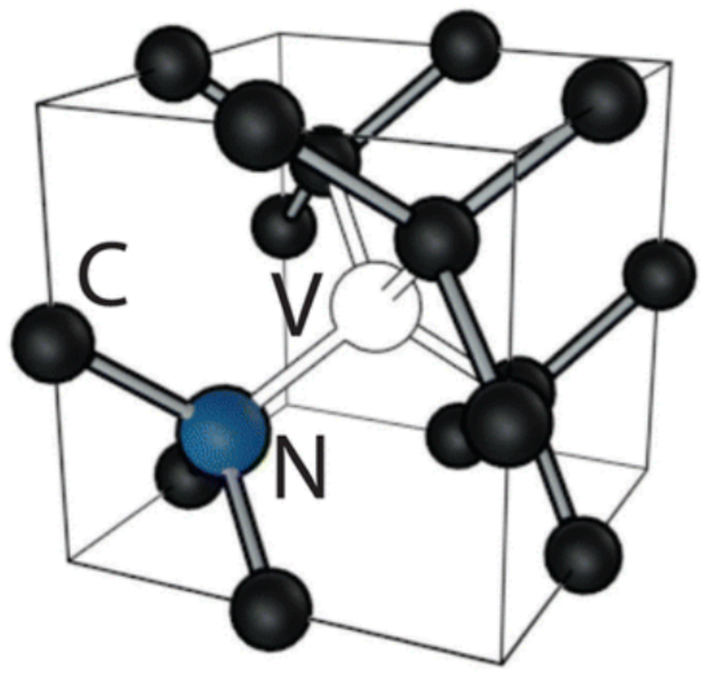
Neutral atoms

SC qubits (transmons)

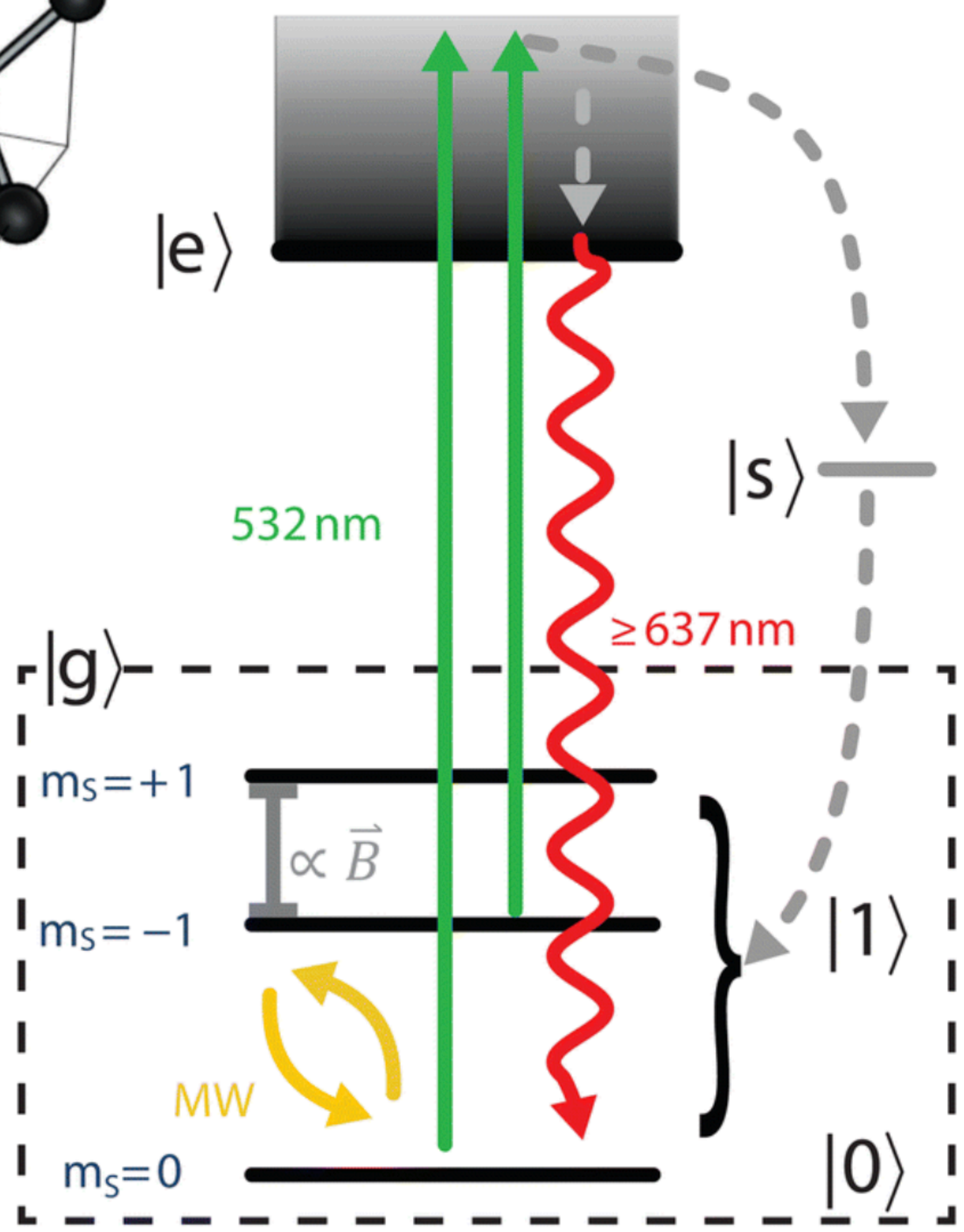




from J. Knoll thesis, ETH (2022)



NV centers in diamond

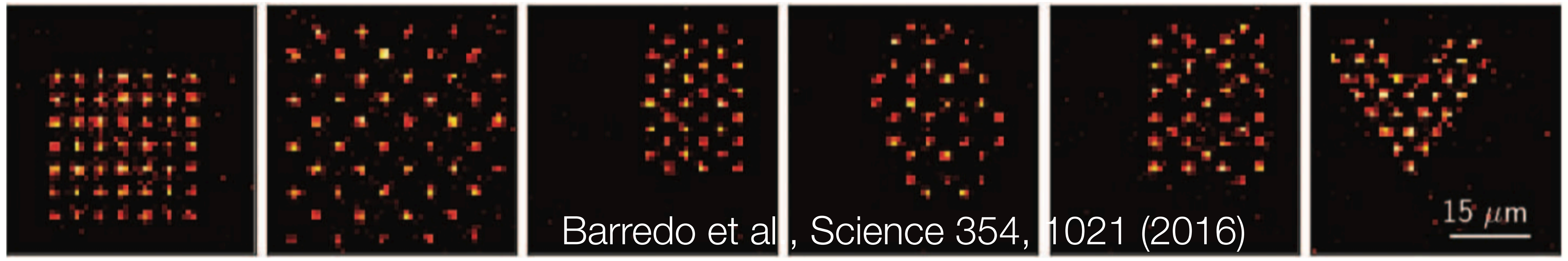


from J. Mater. Chem. C **10**, 13533 (2022)

Examples of environments/baths

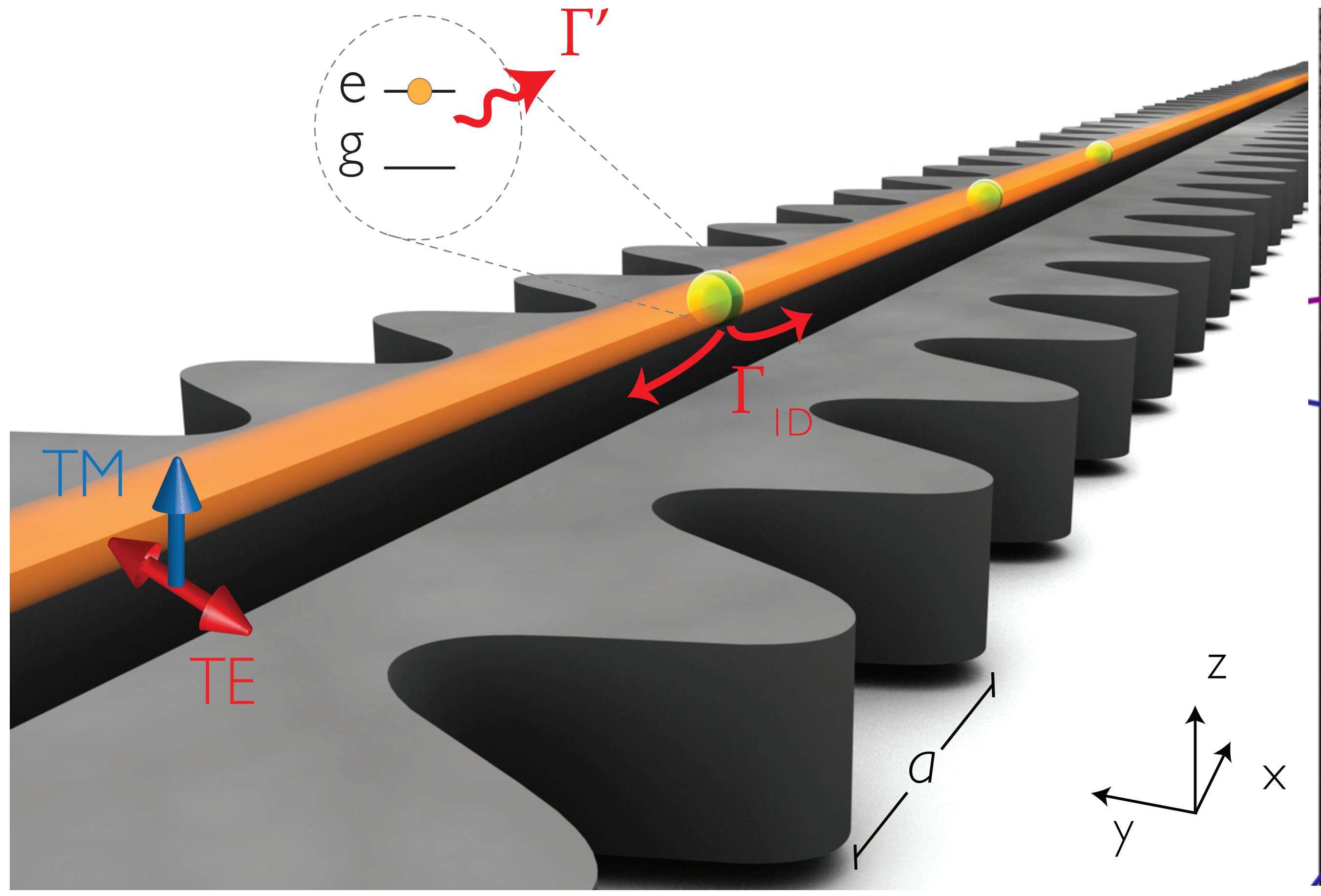
Electromagnetic vacuum (3D free space)

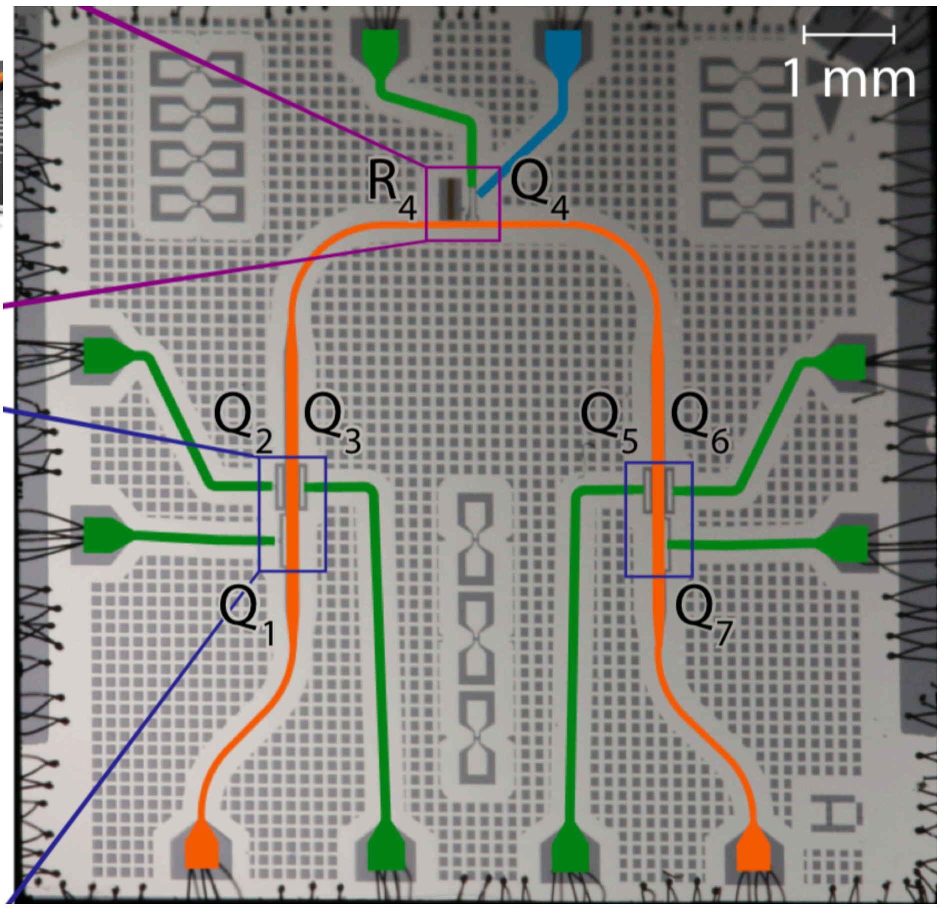
Endres et al., Science 354, 1024 (2016)



Barredo et al., Science 354, 1021 (2016)

Electromagnetic vacuum (engineered)

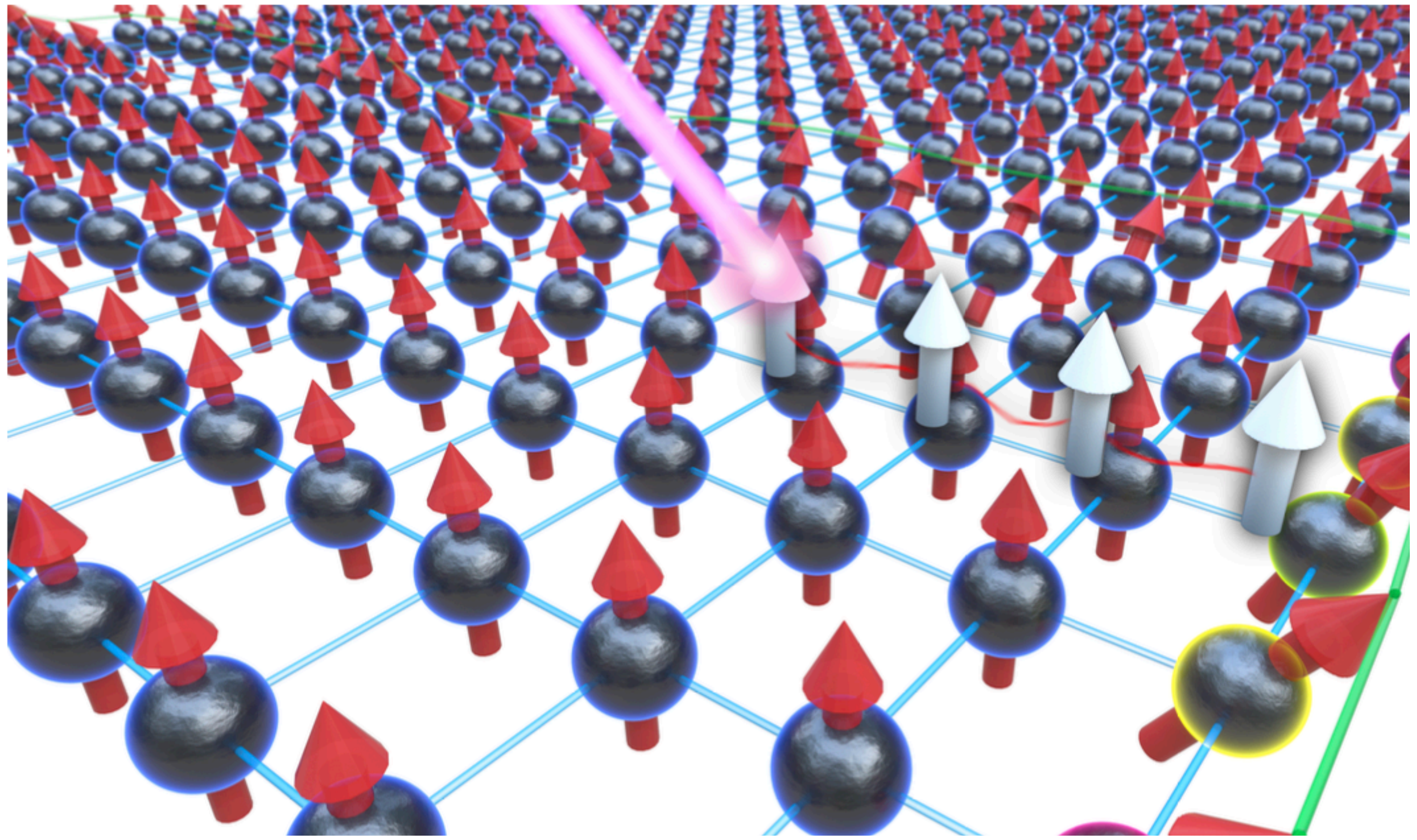




Hood et al., PNAS 113, 10507 (2016)

Mirhosseini et al., Nature 569, 692 (2019)

Magnetic bath or other collective excitation in a material

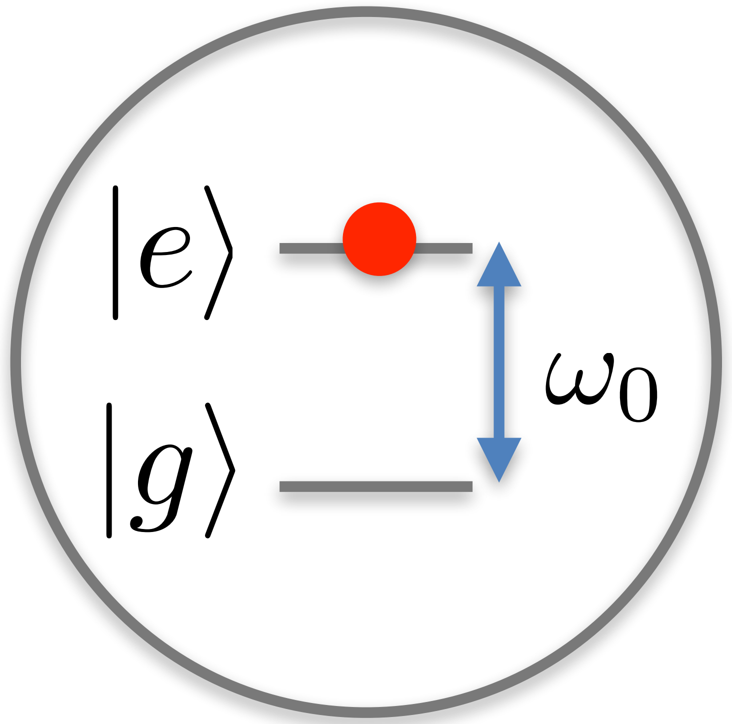


(potentially-interacting) magnon bath, Yttrium Iron Garnet (YIG) film

Li, Marino, Chang, Flebus, arXiv: 2309.08991 (2023)

Setting up the stage: a few basics

Minimal toy model: spins/qubits/“two-level atoms”



For optical transitions $\hbar\omega_0 \gg k_B T$ at room T (otherwise, cool bath)

no thermal excitation of excited state, we can take T=0

ground state is “trivial” $|g\, g \ldots g\rangle \equiv |g\rangle^{\otimes N}$

As system is coupled to the environment, dynamics is no longer of Hamiltonian

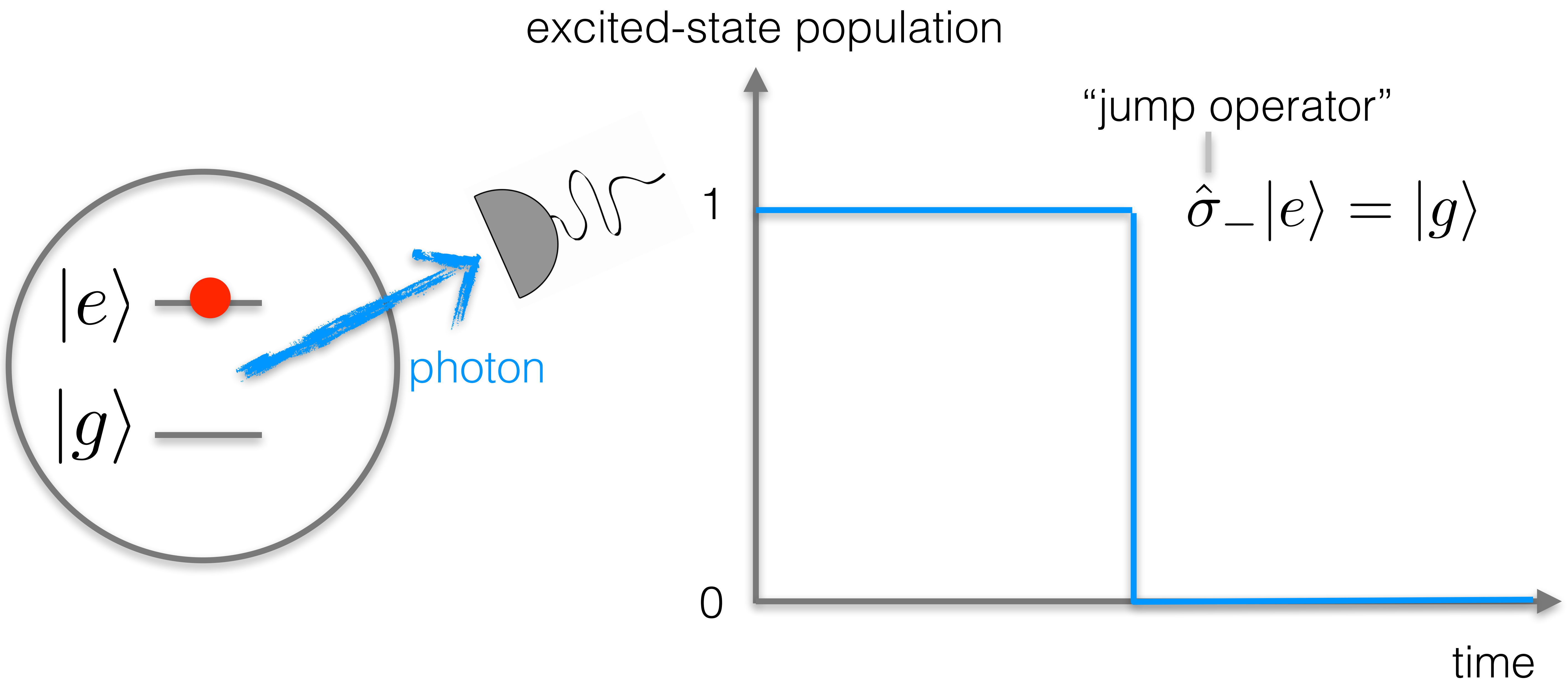
Pure state $|\psi\rangle$ ⟶ Density operator ρ

Schrödinger equation $i\hbar\, \partial_t |\psi\rangle = \mathcal{H}|\psi\rangle$ ⟶ Master equation $\dot{\rho} = \mathcal{L}[\rho]$

Phenomenology: spontaneous emission from a single atom

- Stochastic process of photon emission driven by vacuum fluctuations

- Photons are detected in random directions, with an average dipole pattern



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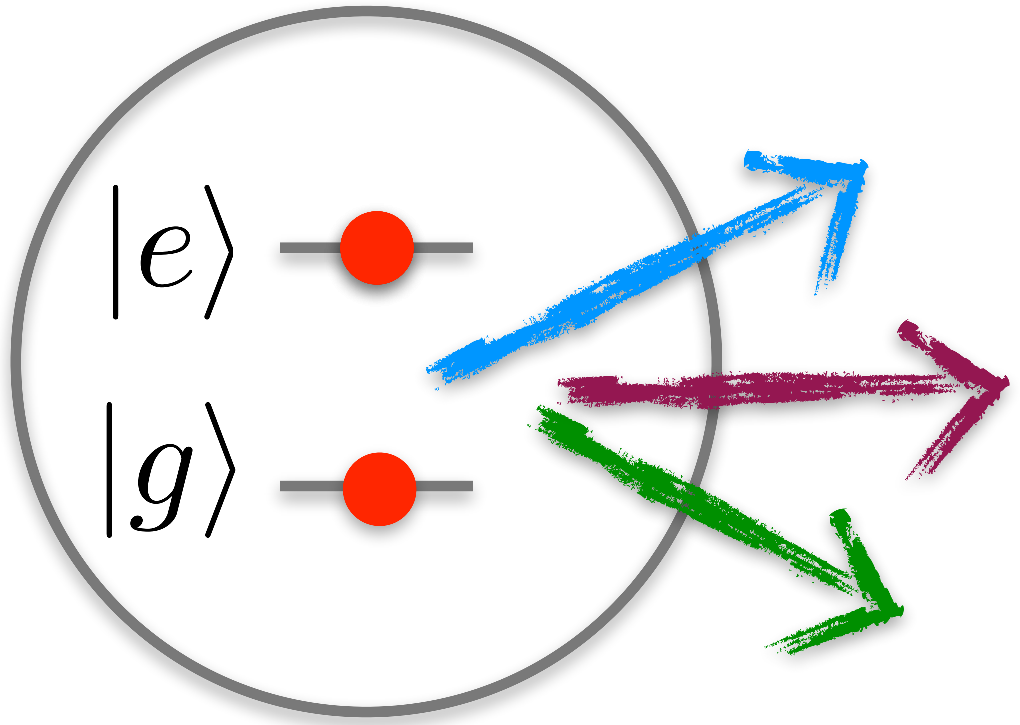

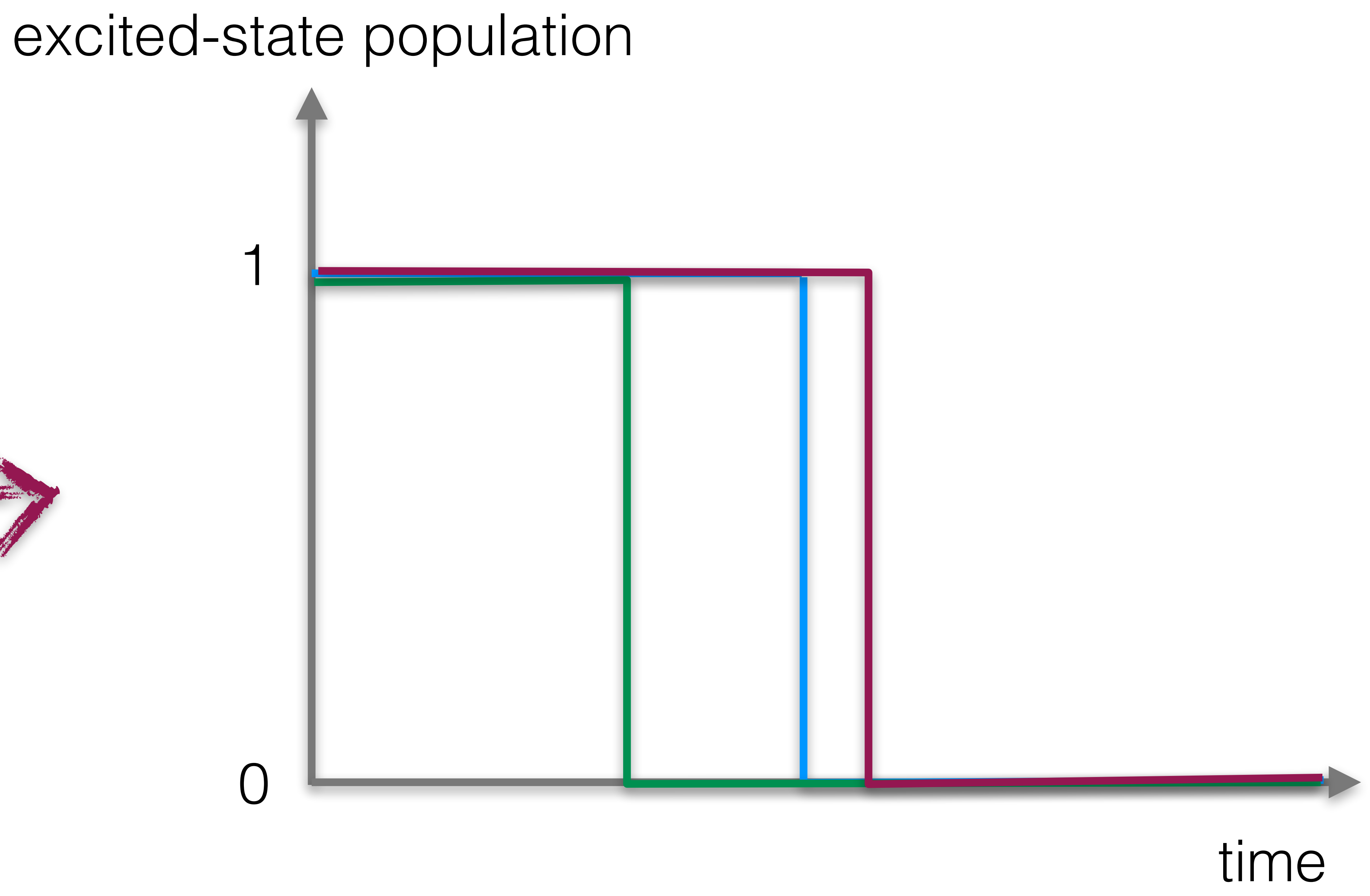

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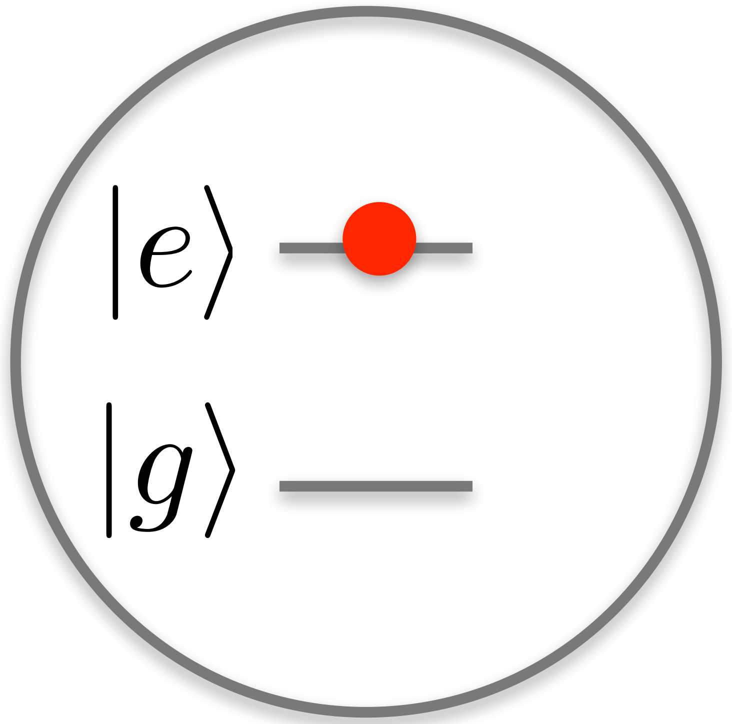
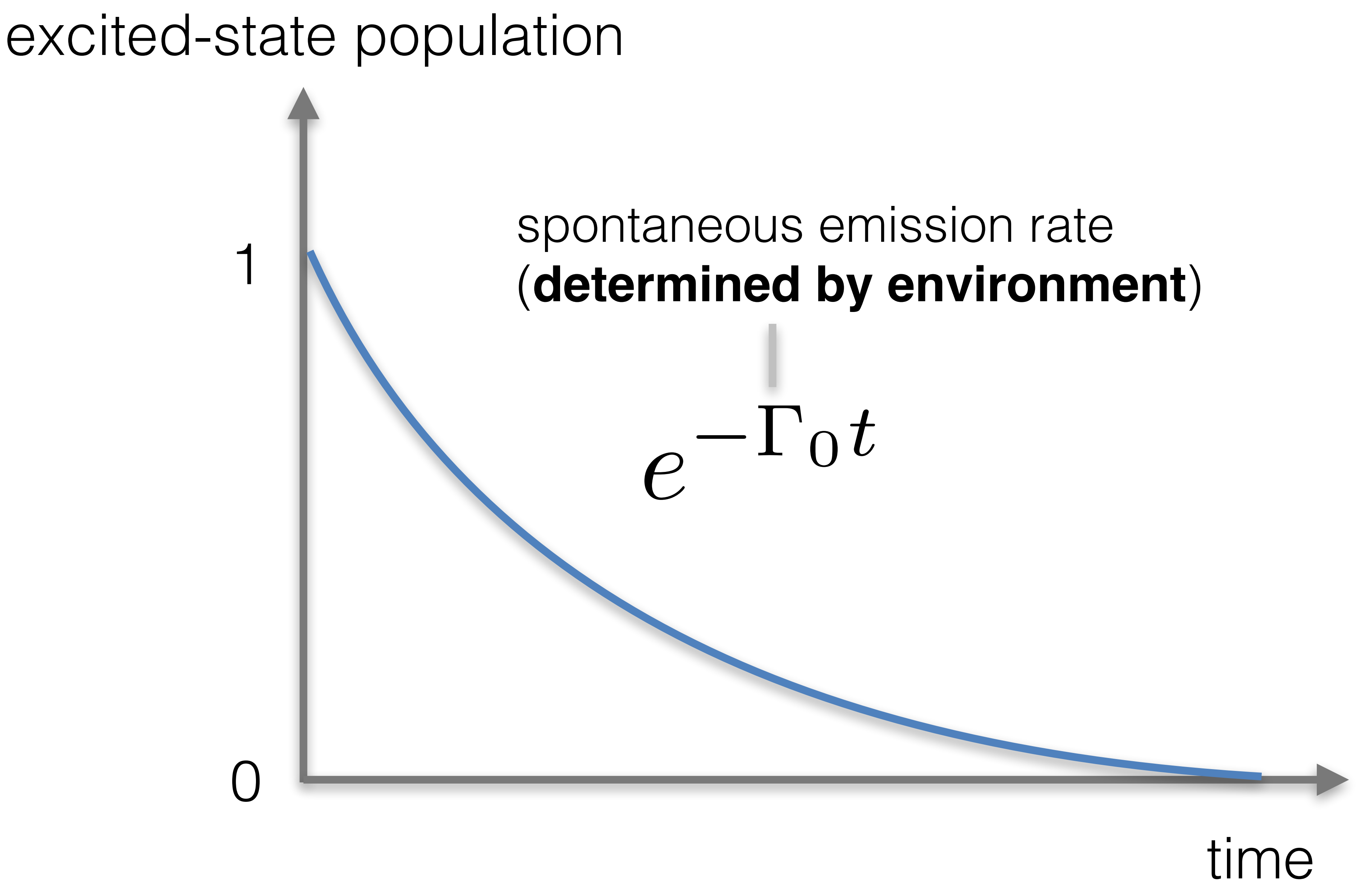
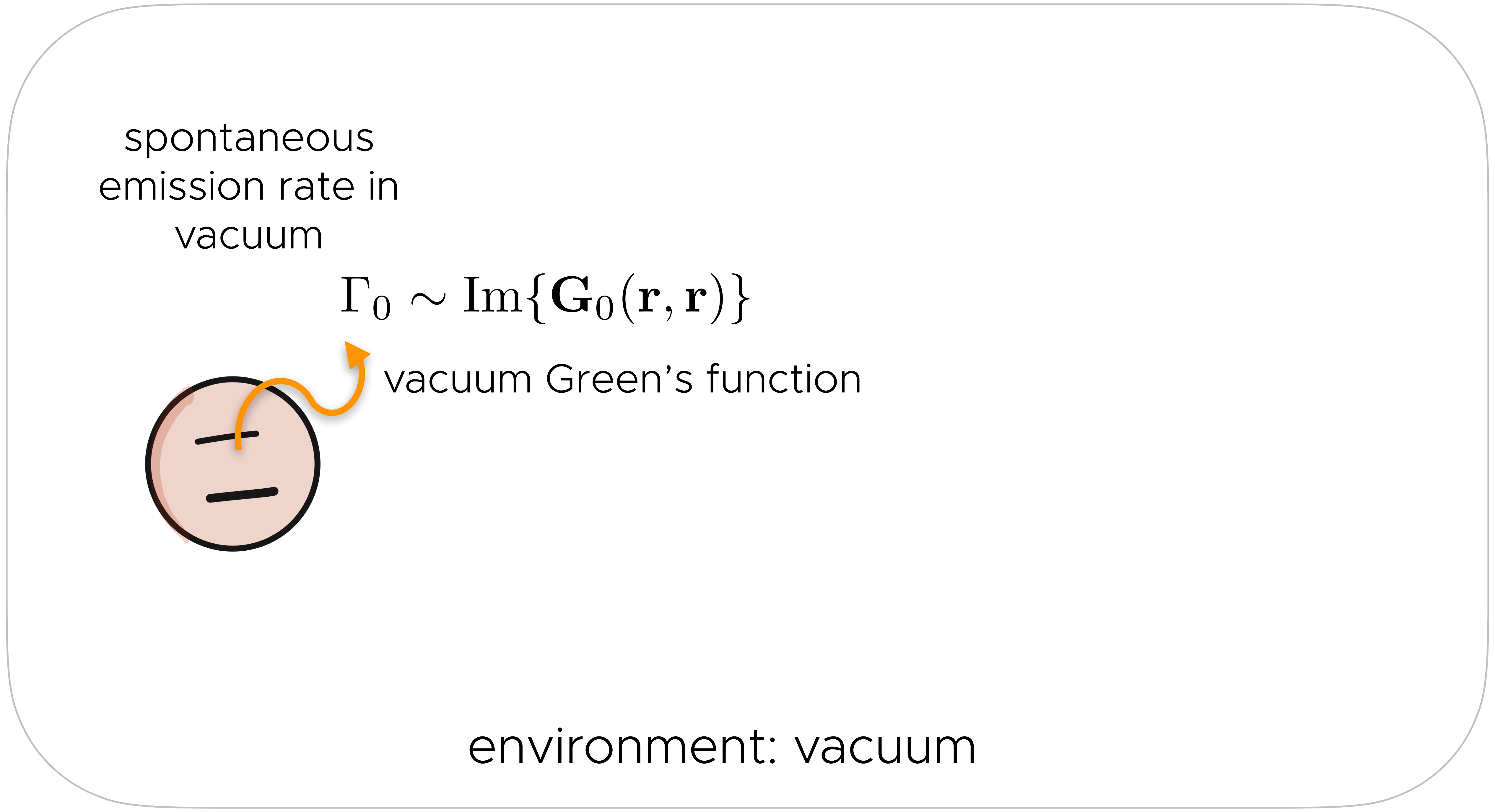




spontaneous emission rate
(**determined by environment**)

$$e^{-\Gamma_0 t}$$

Decay properties are dictated by the environment (Purcell)



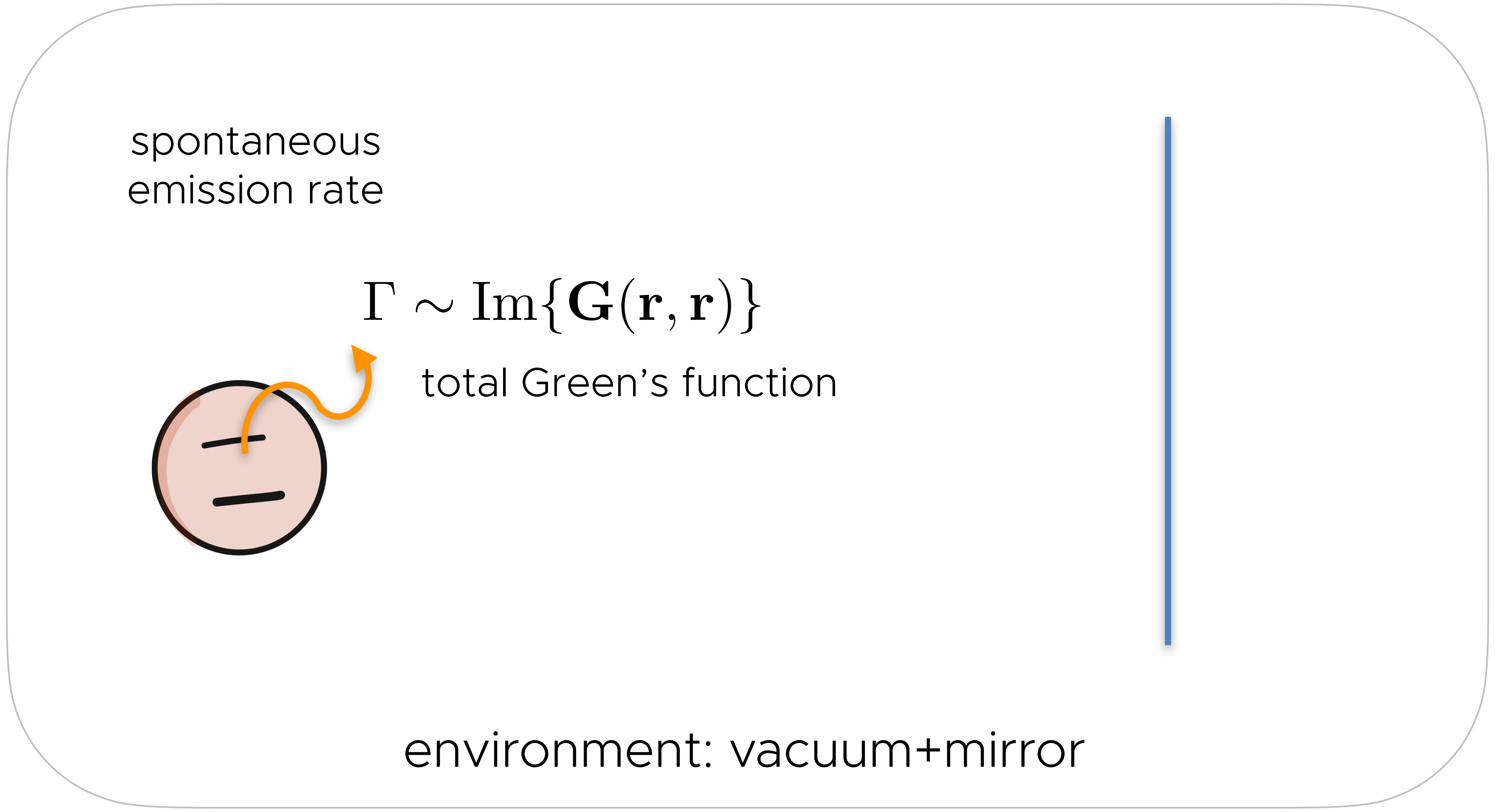
spontaneous
emission rate in
vacuum

$$\Gamma_0 \sim \mathrm{Im}\{\mathbf{G}_0(\mathbf{r},\mathbf{r})\}$$

vacuum Green's function

environment: vacuum

Decay properties are dictated by the environment (Purcell)



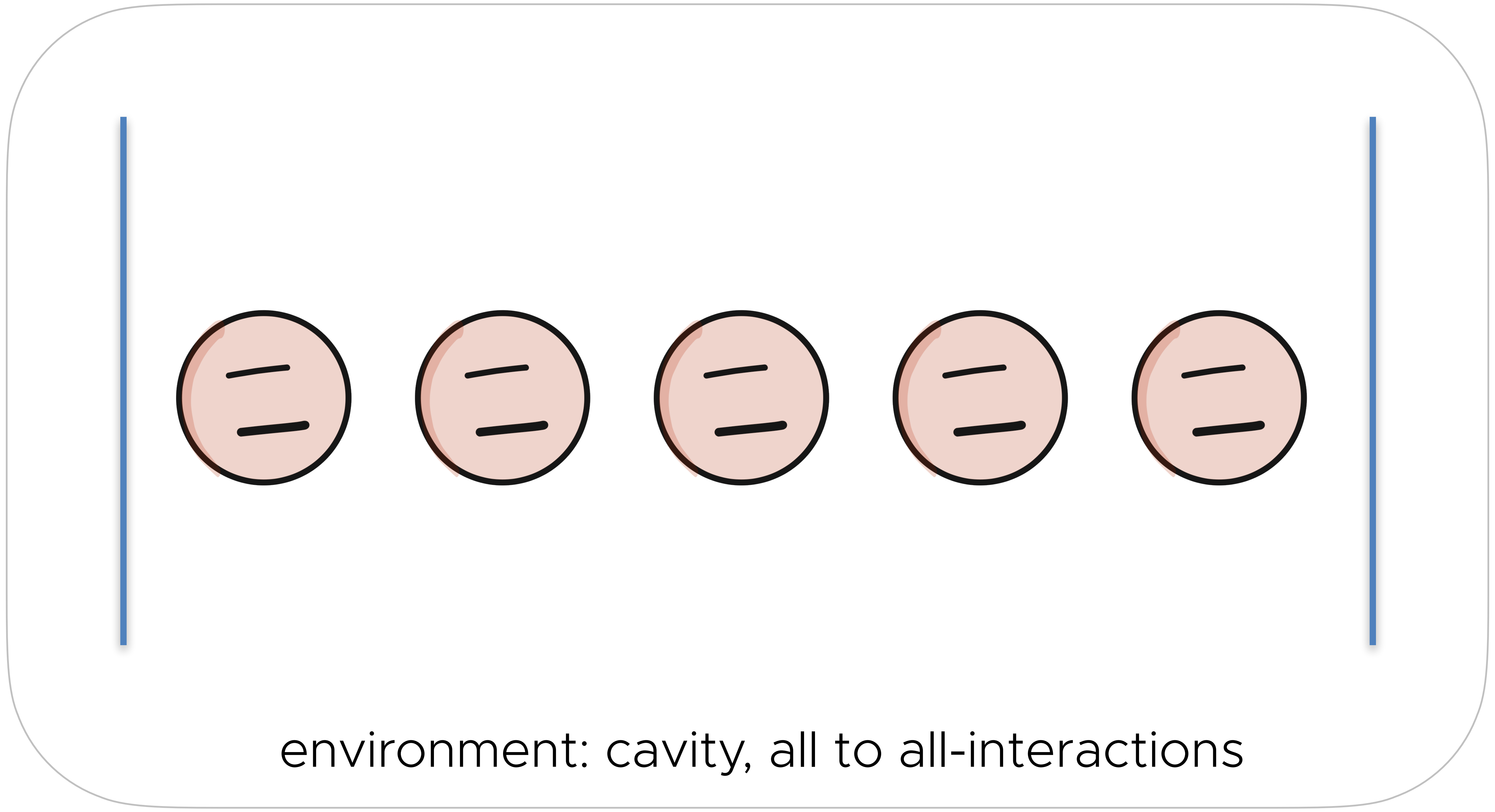
spontaneous
emission rate

$$\Gamma \sim \mathrm{Im}\{\mathbf{G}(\mathbf{r},\mathbf{r})\}$$

total Green's function

environment: vacuum+mirror

Atom-atom interactions are also dictated by the environment

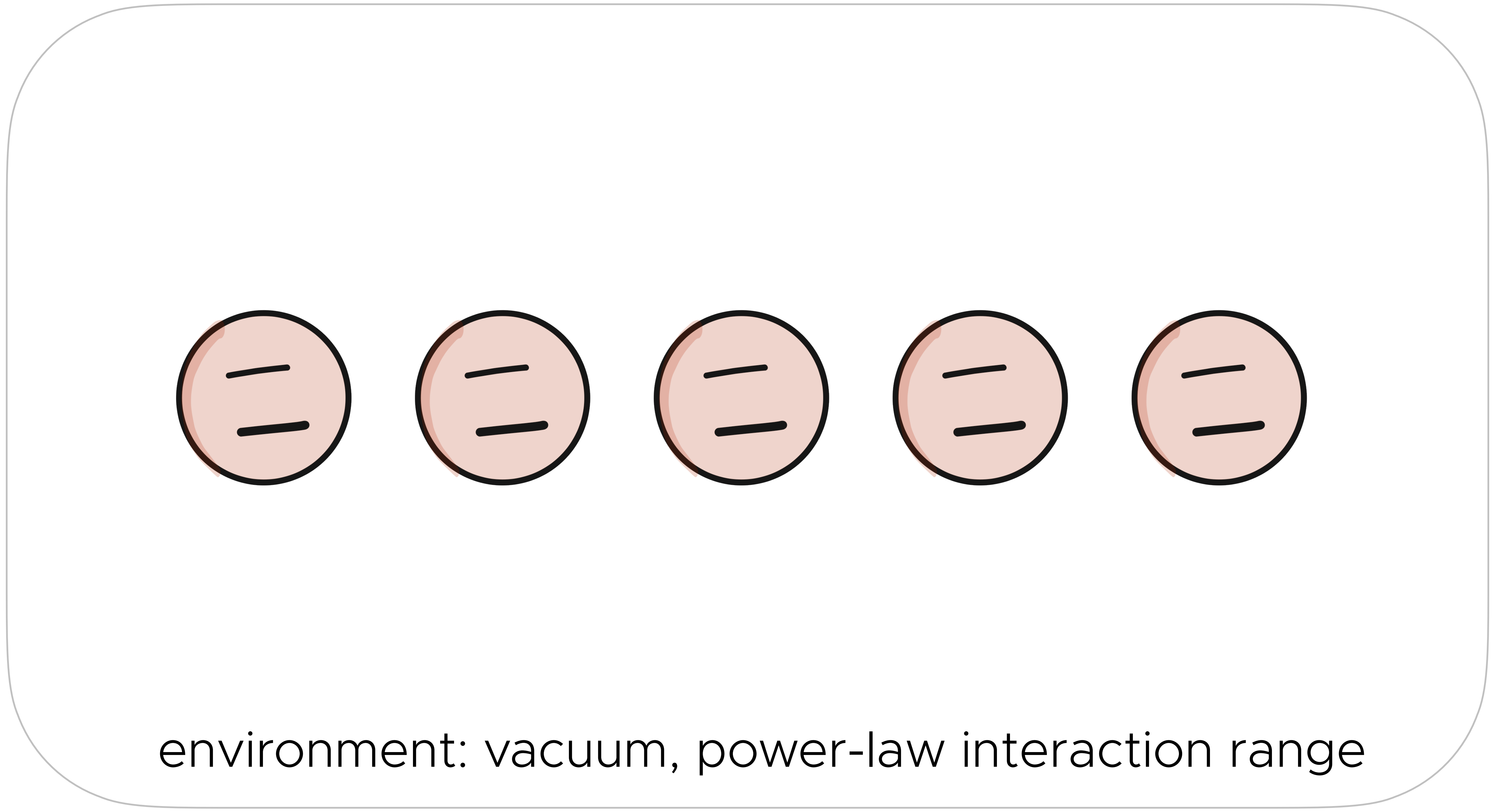


environment: cavity, all to all-interactions

“bad cavity” limit of cavity QED

permutational symmetry makes problem “easy”

Atom-atom interactions are also dictated by the environment



environment: vacuum, power-law interaction range

interactions depend on distance,
permutational symmetry is broken
interference in photon emission and absorption gives rise to non-trivial phenomena

Lecture 1

Setting up the stage

Spin model

Conventional derivation of the master equation

matter Hamiltonian

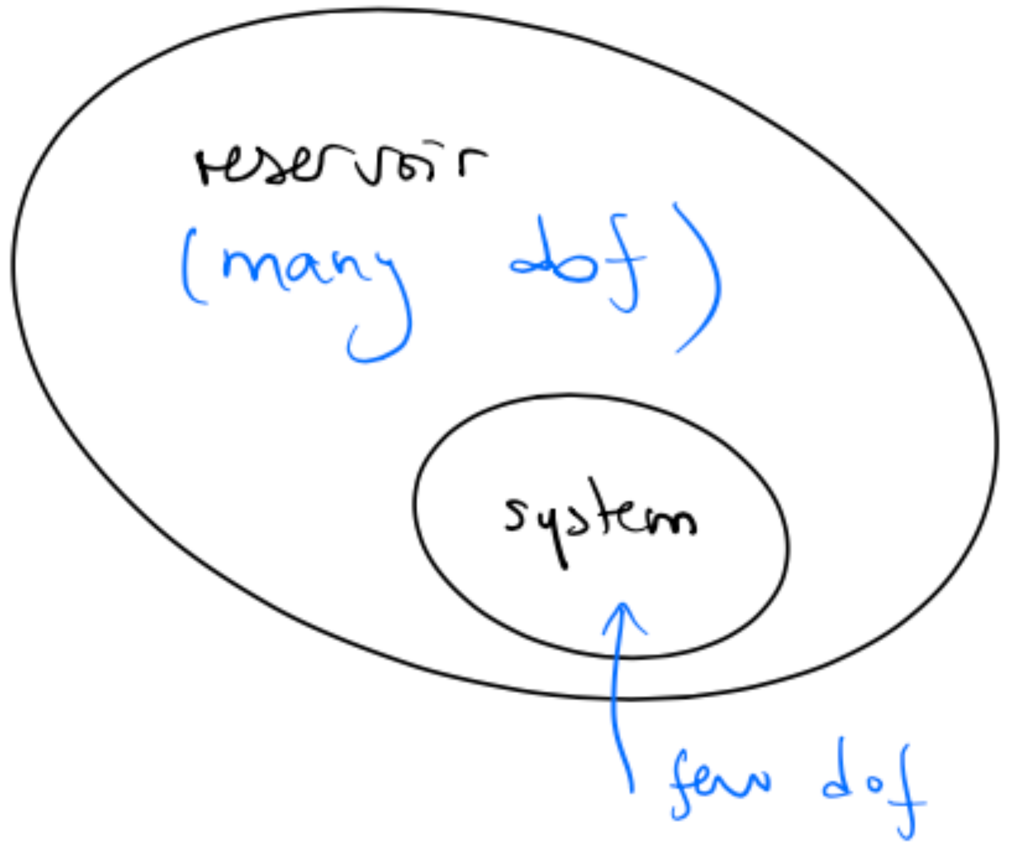
$$\mathcal{H} = \sum_k \hbar\omega_k a_k^\dagger a_k + \sum_j \hbar\omega_0 \sigma_{ee}^j + \sum_{k,j} \hbar g_{k,j}(a_k^\dagger + a_k)(\sigma_-^j + \sigma_+^j)$$

normal modes
of EM field (or bosonic excitations
that mediate interactions)

light-matter coupling
(field x dipole)

Problem scales poorly…. dimension of Hilbert space=2^Nxdimension Hilbert space of bath dof

Under Born-Markov approximation, we integrate out the photonic degrees of freedom:



$$\dot{\rho}_{SR} = -\frac{i}{\hbar}[\mathcal{H}, \rho_{SR}] \longrightarrow \dot{\rho} = \mathcal{L}[\rho]$$

$$\rho = \mathrm{Tr}_R\{\rho_{SR}\}$$

master equation

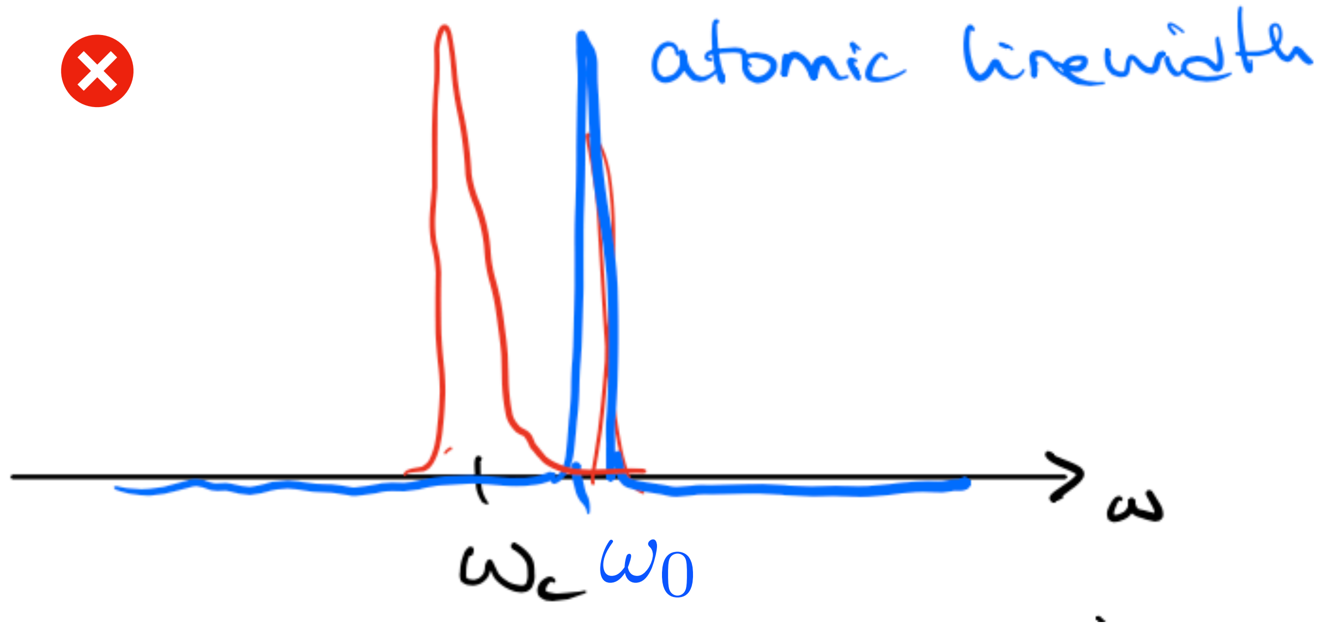
Lindblad superoperator

- only depends on spin degrees of freedom
- coefficients depend on “bath” correlators of the type $\langle a_k^\dagger a_k \rangle_R$

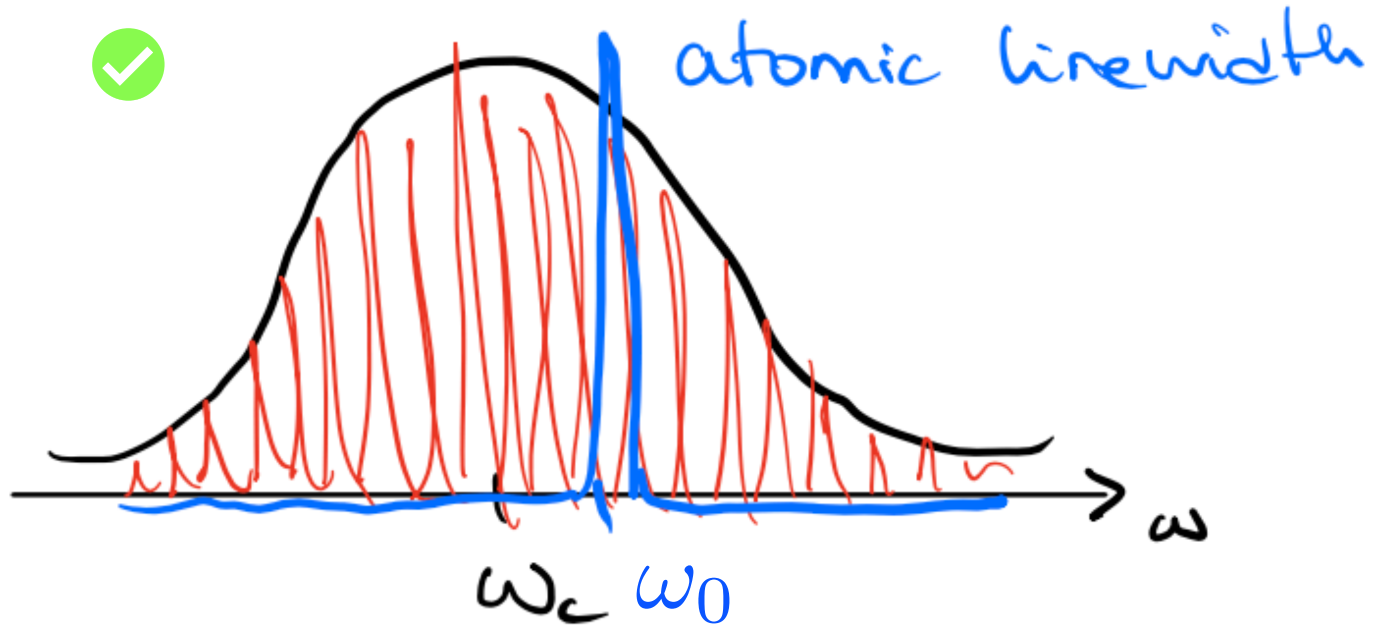
see for instance Howard Carmichael’s textbook

When is this valid?

- Weak coupling (perturbative): $g_k \ll \omega_0$
- Memory-less reservoir (akin to broadband). Consider for instance a cavity:

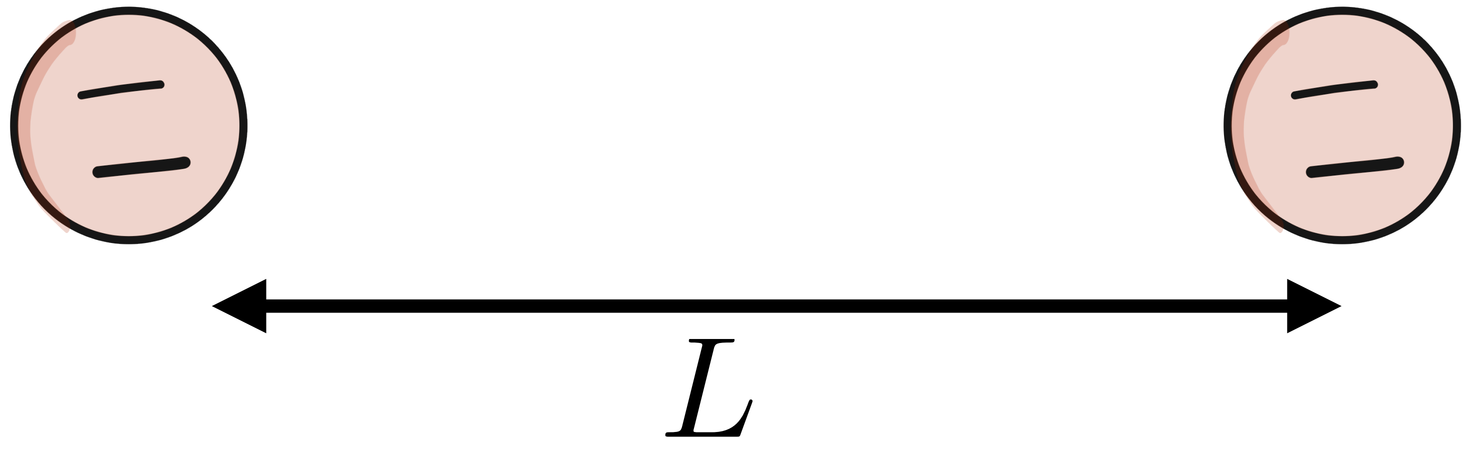


narrow, single mode, Jaynes-Cummings type



multimode, broad

- No retardation (resulting equation is time local)



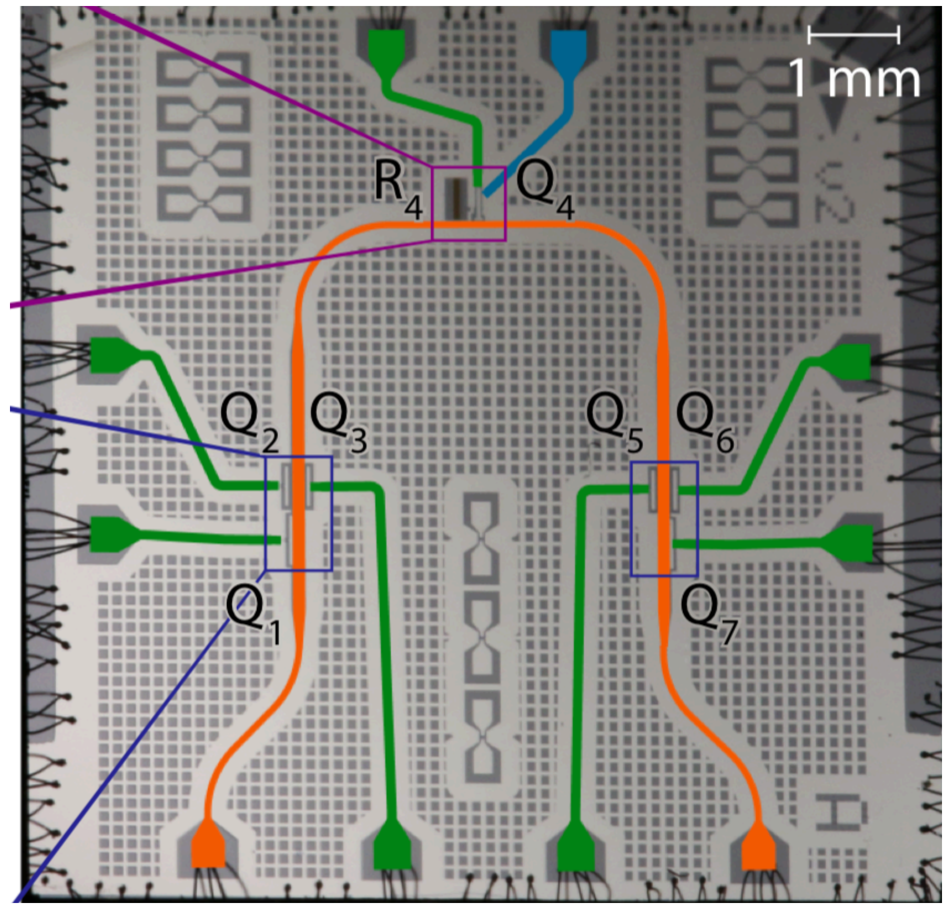
$$t_{\text{propagation}} \ll t_{\text{decay}}$$

$$\frac{\Gamma_0 L}{c} \ll 1$$

Alternatively, solve EM problem first, take care of QM later

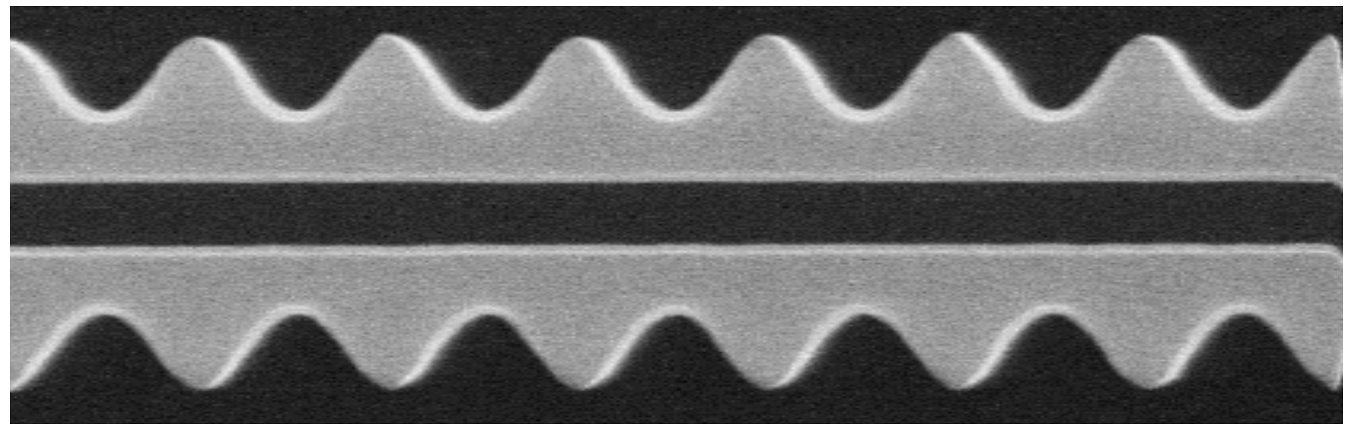
- Previous approach is fine if only a few modes are relevant or if dielectric environment is simple
- However, what happens in more complicated reservoirs?

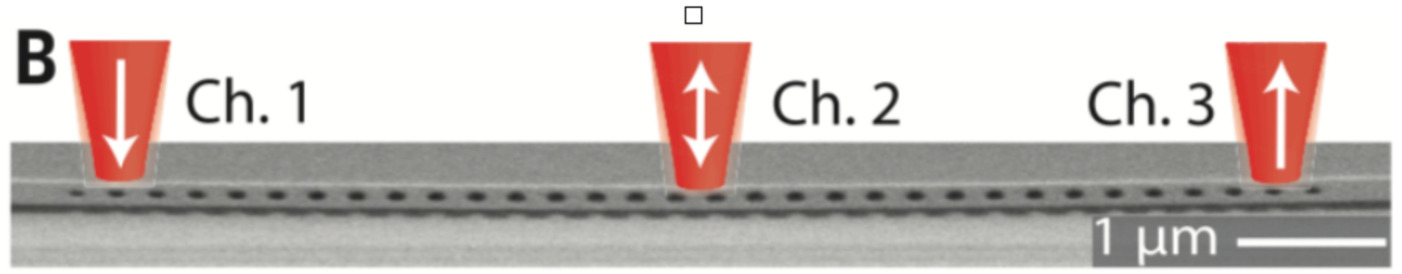
nanofiber
Laurat (2015)



SC waveguide,
Painter (2019)

photonic crystal, Kimble (2016)

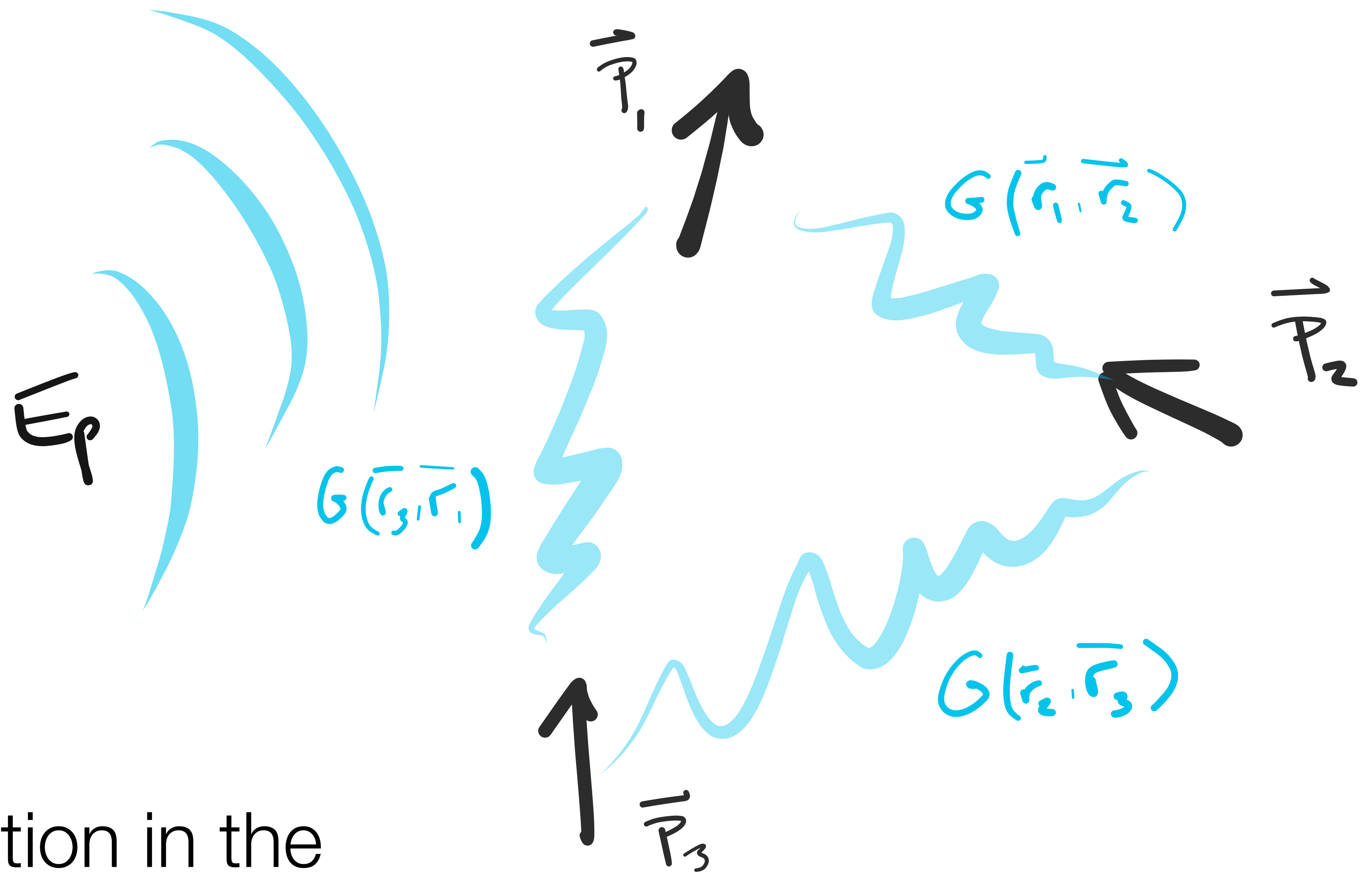




NV in diamond waveguide, Lukin (2016)

- Solution: quantization in terms of Green’s functions (propagator of EM field)

Propagator of the EM field: Green's function



Let's consider an ensemble of driven classical dipoles.

The field at dipole i is:

$$\mathbf{E}(\mathbf{r}_i,\omega) = \underbrace{\mathbf{E}_{\mathrm{p}}(\mathbf{r}_i,\omega)}_{\text{pump field}} + \mu_0\omega^2 \sum_{j=1}^{N} \underbrace{\overbrace{\mathbf{G}(\mathbf{r}_i,\mathbf{r}_j,\omega)}^{\text{EM Green's function}} \cdot \mathbf{p}(\mathbf{r}_j,\omega)}_{\text{scattered field}}$$

The Green's tensor is the fundamental solution of the wave equation in the medium (obtained from Maxwell's equations):

$$\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\mathbf{G}(\mathbf{r},\mathbf{r}',\omega) - \frac{\omega^2}{c^2}\underbrace{\epsilon(\mathbf{r},\omega)}_{\text{boundary conditions}}\mathbf{G}(\mathbf{r},\mathbf{r}',\omega) = \delta(\mathbf{r}-\mathbf{r}')\mathbb{1}$$

One can write the classical Hamiltonian as (hand wavy!):

$$\mathcal{H} \sim \sum_j \mathbf{p}_j \cdot \mathbf{E}(\mathbf{r}_j,\omega) = \mathcal{H}_{\text{pump}} + \sum_{i,j=1}^{N} \mathbf{p}_j \cdot \mathbf{G}(\mathbf{r}_j,\mathbf{r}_i,\omega)\cdot\mathbf{p}_i$$

Promoting fields to operators

Atoms couple to the EM field via their dipole moment. Since:

$$\mathbf{E}(\mathbf{r}_i,\omega) = \mathbf{E}_\mathrm{p}(\mathbf{r}_i,\omega) + \mu_0\omega^2 \sum_{j=1}^{N} \mathbf{G}(\mathbf{r}_i,\mathbf{r}_j,\omega)\cdot\mathbf{p}(\mathbf{r}_j,\omega)$$

Promoting fields to operators, we find an input-output equation (in time t for spins):

Markov approximation

$$\hat{\mathbf{E}}^{+}(\mathbf{r}) = \hat{\mathbf{E}}_\mathrm{p}^{+}(\mathbf{r}) + \mu_0\omega_0^2 \sum_{j=1}^{N} \mathbf{G}(\mathbf{r},\mathbf{r}_j,\omega_0)\cdot\wp\hat{\sigma}_-^{j}$$

positive frequency component of field operator

dipole matrix element

*classical and quantum fields propagate identically

Pretty tempting to do:

$$\mathcal{H}\sim\sum_{i,j=1}^{N} \mathbf{G}(\mathbf{r}_j,\mathbf{r}_i,\omega_0)\hat{\sigma}_+^{i}\hat{\sigma}_-^{j}$$

*not correct, missing parts of the evolution!

for rigorous procedure, see extensive work of Welsch, Buhmann et al., for instance Buhmann's "Dispersion forces"

Spin model in terms of Green's functions

Starting from spins+field Hamiltonian (in terms of Green's functions) and integrating out we find a Lindblad master equation for the spins:

$$\dot{\rho} = -\frac{i}{\hbar}\left[\mathcal{H}, \rho\right] + \sum_{i,j=1}^{N} \frac{\Gamma_{ij}}{2} \left(2\sigma_-^i \rho \sigma_+^j - \rho \sigma_+^j \sigma_-^i - \sigma_+^j \sigma_-^i \rho\right)$$

coherent term

dissipation

$$\mathcal{H} = \hbar\omega_0 \sum_{i=1}^{N} \hat{\sigma}_{ee}^i + \hbar \sum_{i,j=1}^{N} J^{ij}\, \sigma_+^i \sigma_-^j$$

with:

$$J^{ij} = -\frac{\mu_0 \omega_0^2}{\hbar} \wp^* \cdot \mathrm{Re}\,\mathbf{G}(\mathbf{r}_i, \mathbf{r}_j, \omega_0) \cdot \wp, \qquad \Gamma^{ij} = \frac{2\mu_0 \omega_0^2}{\hbar} \wp^* \cdot \mathrm{Im}\,\mathbf{G}(\mathbf{r}_i, \mathbf{r}_j, \omega_0) \cdot \wp,$$

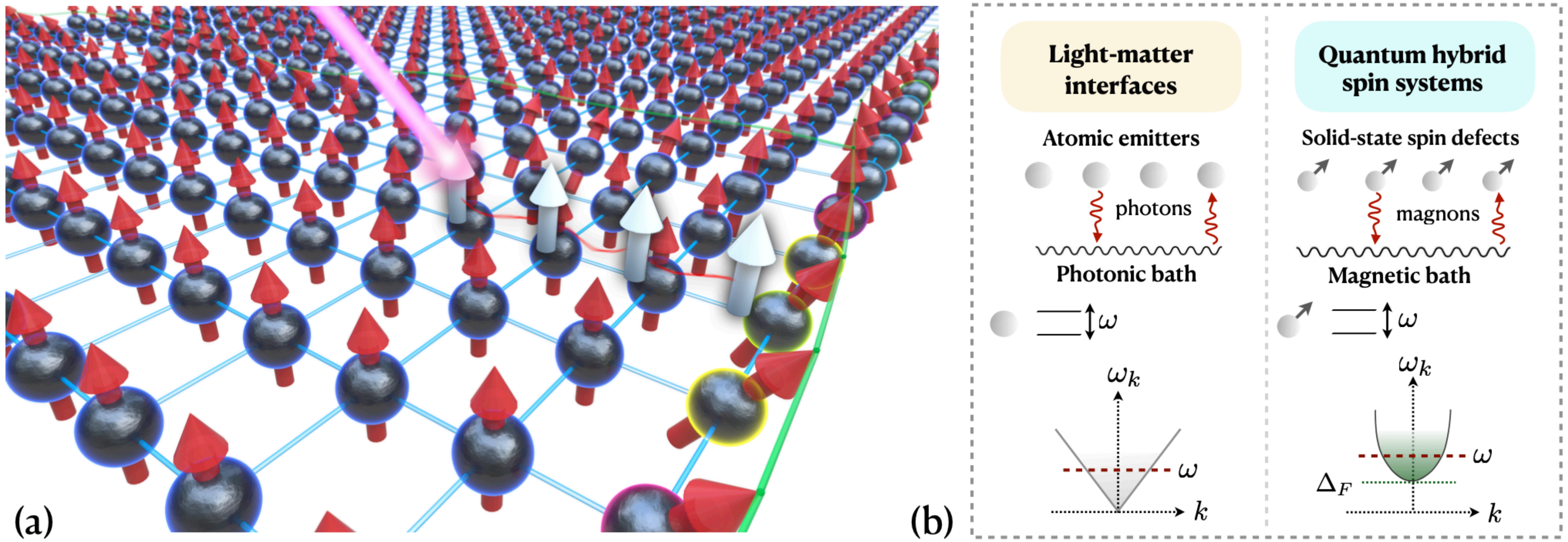
XY model, but (potentially) long-ranged and open

Green's function is analytical if there is enough symmetry… worst case scenario: numerical solver

Arbitrary complications can be incorporated, complex internal structure, non-reciprocal/chiral baths, inhomogeneous broadening, etc.

see AAG et al, PRX 7, 031024 (2017)

Similarly, for NV centers coupled to a ferromagnetic bath



Li, Marino, Chang, Flebus, arXiv: 2309.08991 (2023)

Green’s function can be obtained for other baths via 2-point correlation functions:

$$\frac{d\rho}{dt} = -i\left[\mathcal{H}_s + \sum_{\alpha\neq\beta} \underset{\substack{| \\ \frac{\pi}{2}\frac{\omega_{qi}-\Delta_F}{\Delta_0}Y_0\left(\frac{\rho_{\alpha\beta}}{\lambda}\right)}}{J_{\alpha\beta}}\sigma_\alpha^+\sigma_\beta^-,\ \rho\right] + \sum_{\alpha\beta} \underset{\substack{| \\ \pi\left[n_B(\omega_{qi})+1\right]\frac{\omega_{qi}-\Delta_F}{\Delta_0}J_0\left(\frac{\rho_{\alpha\beta}}{\lambda}\right)e^{-\frac{2d}{\lambda}}}}{\Gamma_{\alpha\beta}}\left(\sigma_\beta^+\rho\sigma_\alpha^- - \frac{1}{2}\{\sigma_\alpha^-\sigma_\beta^+,\rho\}\right) + \sum_{\alpha\beta}\tilde{\Gamma}_{\alpha\beta}\left(\sigma_\beta^-\rho\sigma_\alpha^+ - \frac{1}{2}\{\sigma_\alpha^+\sigma_\beta^-,\rho\}\right)$$

correlated absorption

Non-Hermitian Hamiltonian and quantum trajectories

$$\dot{\rho} = -\frac{i}{\hbar}\left[\mathcal{H},\rho\right] + \sum_{i,j=1}^{N} \frac{\Gamma_{ij}}{2}\left(2\sigma_-^i \rho \sigma_+^j - \rho\sigma_+^j\sigma_-^i - \sigma_+^j\sigma_-^i\rho\right)$$

$$\mathcal{H} = \hbar\omega_0 \sum_{i=1}^{N} \hat{\sigma}_{ee}^i + \hbar \sum_{i,j=1}^{N} J^{ij}\, \sigma_+^i \sigma_-^j$$

We can rewrite the master equation as:

$$\dot{\rho} = -\frac{i}{\hbar}\left(\mathcal{H}_{\text{eff}}\rho - \rho\mathcal{H}_{\text{eff}}^{\dagger}\right) + \sum_{i,j=1}^{N} \Gamma_{ij}\sigma_-^i \rho \sigma_+^j$$

system evolve due to acquisition of information
due to negative measurement

population recycling term (quantum jumps)

Non-Hermitian Hamiltonian:

$$\mathcal{H}_{\text{eff}} = \hbar \sum_{i,j=1}^{N}\left(J_{ij} - i\frac{\Gamma_{ij}}{2}\right)\sigma_+^i\sigma_-^j = -\mu_0\omega_0^2 \sum_{i,j=1}^{N} \wp^* \cdot \mathbf{G}(\mathbf{r}_i, \mathbf{r}_j, \omega_0) \cdot \wp \;\; \sigma_+^i \sigma_-^j$$

For a single atom:

$$\mathcal{H}_{\text{eff}} = -i\frac{\hbar\Gamma_{ii}}{2}\sigma_{ee}^i \longrightarrow \langle \sigma_{ee}^i(t)\rangle = e^{-\Gamma_{ii}t}$$

spontaneous emission rate=local imaginary part of Gf.
(+Lamb shift, here taken to zero)

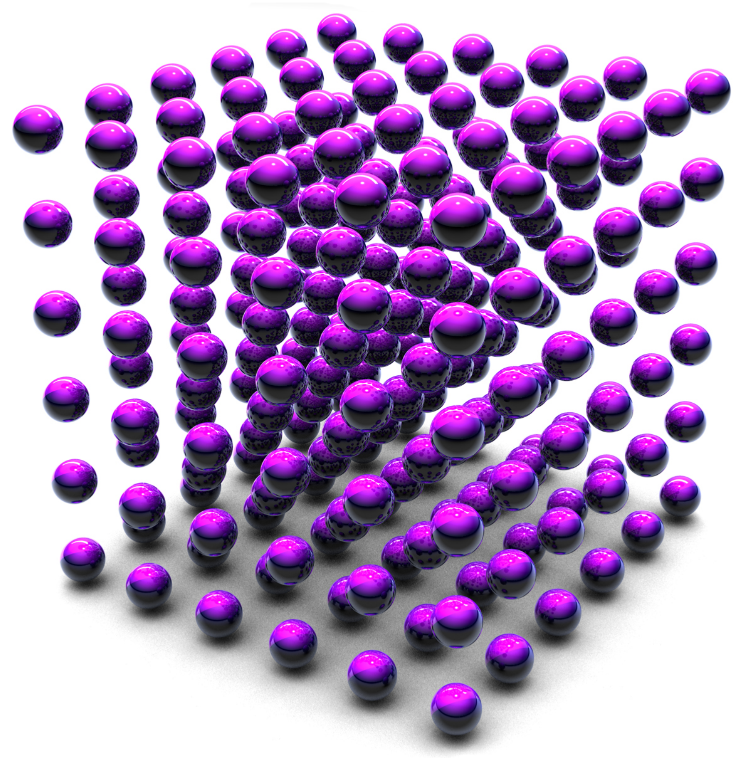
original work on quantum trajectories by Carmichael, and Dalibard, Castin, Molmer

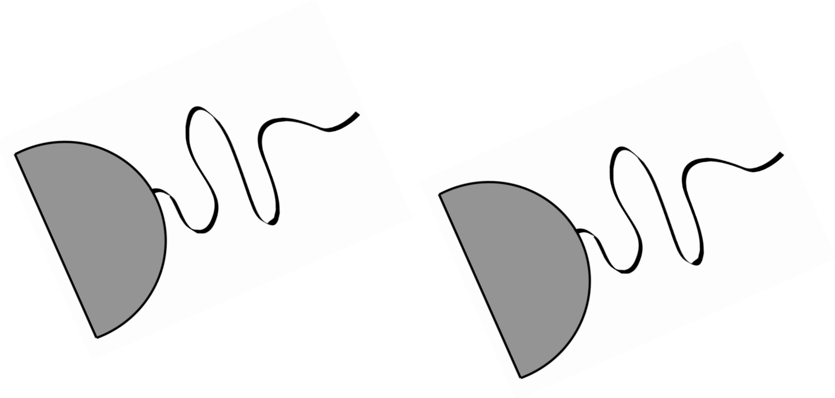
Recovering the field via input-output equations

Even though we have integrated out photons, we can recover field correlation functions via input-ouput equations

$$\hat{\mathbf{E}}^{+}(\mathbf{r}) = \hat{\mathbf{E}}_{\mathrm{p}}^{+}(\mathbf{r}) + \mu_0\omega_0^2 \sum_{j=1}^{N} \mathbf{G}(\mathbf{r},\mathbf{r}_j,\omega_0)\cdot\wp\sigma_-^j$$

Arbitrary order field correlation functions become correlation function of spin operators

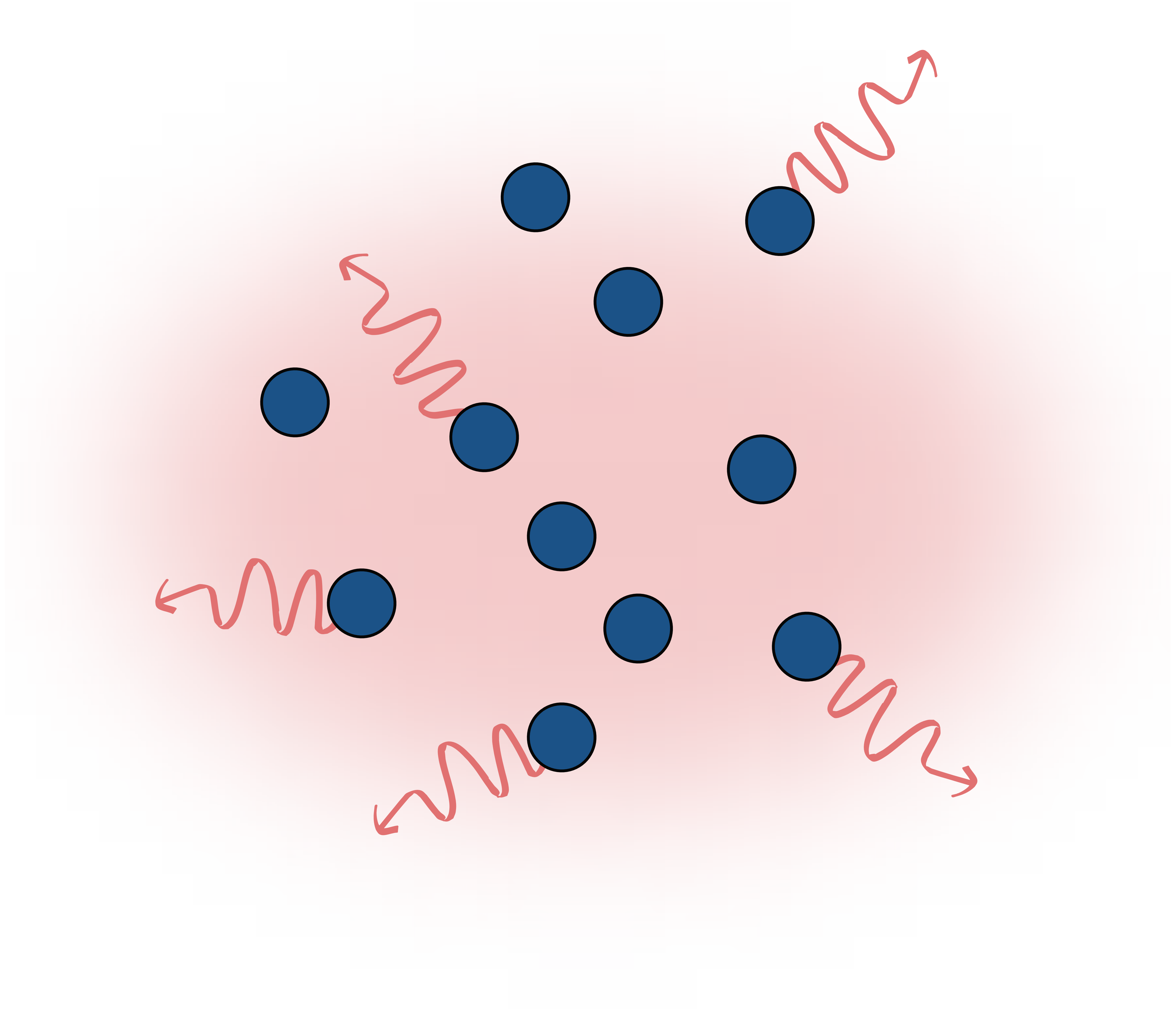




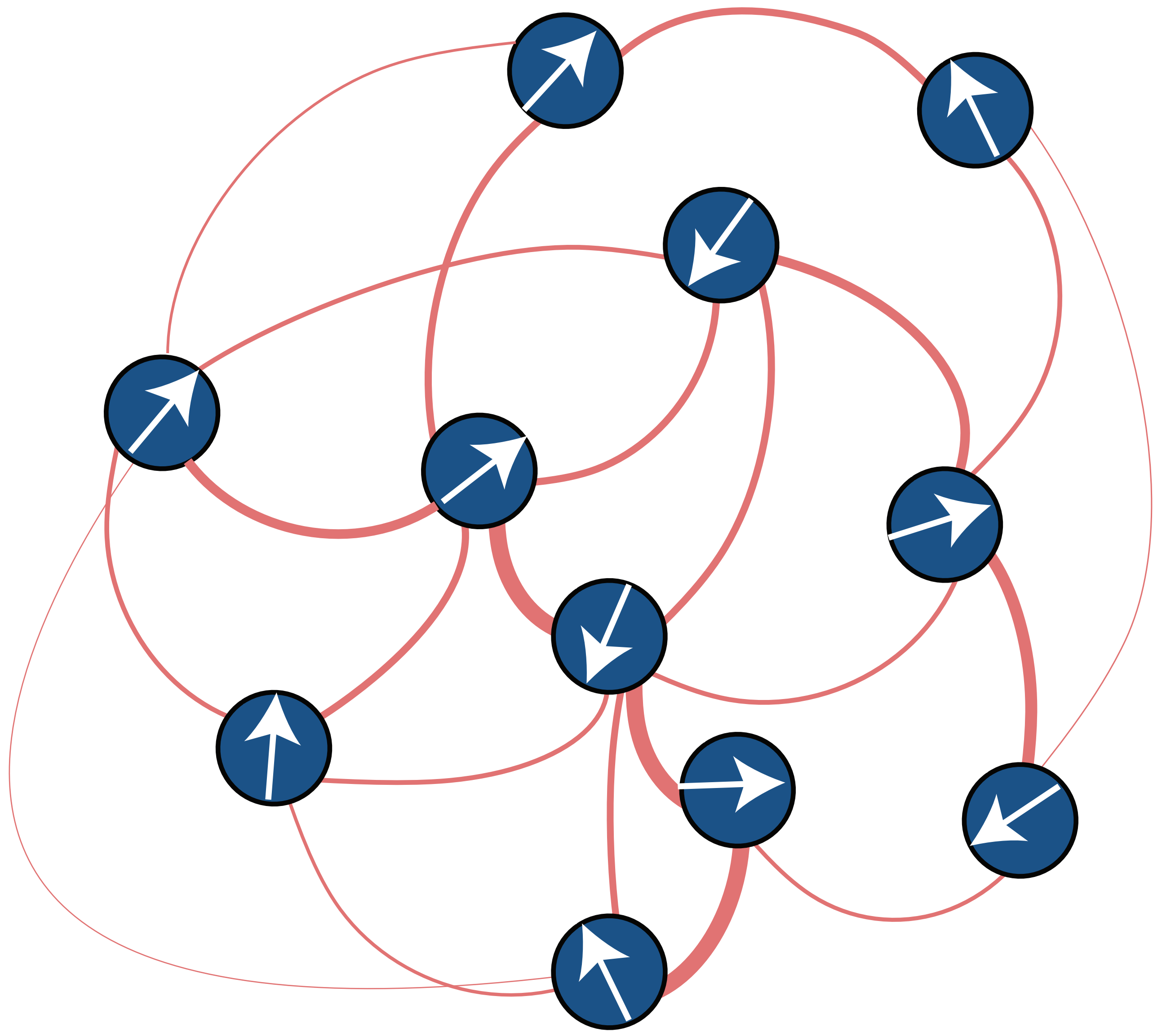
$$\langle \hat{E}^{-}(t)\hat{E}^{-}(t+\tau)\hat{E}^{+}(t+\tau)\hat{E}^{+}(t)\rangle$$

$$\sim \langle \sigma_+^p(t)\sigma_+^k(t+\tau)\sigma_-^i(t+\tau)\sigma_-^j(t)\rangle$$

In summary: light-matter interactions as a spin model



In summary: light-matter interactions as a spin model

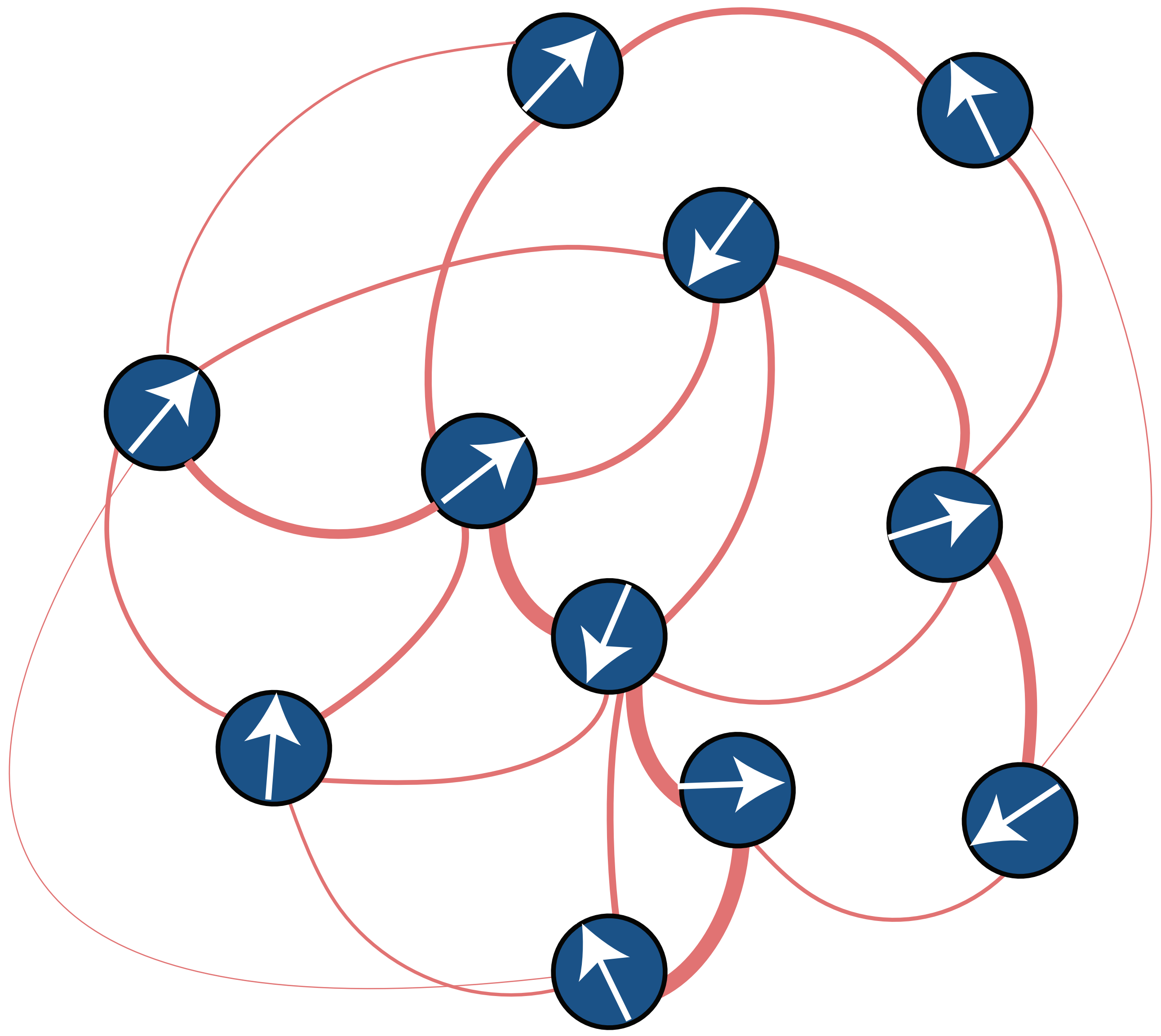


Described by a spin model with
dynamics governed
by a Lindblad master equation

$$\dot{\rho} = -\frac{i}{\hbar}\left[\sum_{i,j=1}^{N} J_{ij}\sigma_{+}^{i}\sigma_{-}^{j}, \rho\right] + \sum_{i,j=1}^{N}\frac{\Gamma_{ij}}{2}\left(2\sigma_{-}^{i}\rho\sigma_{+}^{j} - \rho\sigma_{+}^{j}\sigma_{-}^{i} - \sigma_{+}^{j}\sigma_{-}^{i}\rho\right)$$

conserves number of excitations

In summary: light-matter interactions as a spin model



Described by a spin model with
dynamics governed
by a Lindblad master equation

Eigenvalues
(collective transition rates) $\Gamma_\mu \geq 0$

$$\dot{\rho} = -\frac{i}{\hbar}\left[\sum_{i,j=1}^{N} J_{ij}\sigma_+^i\sigma_-^j, \rho\right] + \sum_{\mu=1}^{N}\frac{\Gamma_\mu}{2}\left(2\hat{c}_\mu\hat{\rho}\hat{c}_\mu^\dagger - \hat{\rho}\hat{c}_\mu^\dagger\hat{c}_\mu - \hat{c}_\mu^\dagger\hat{c}_\mu\hat{\rho}\right)$$

collective jump operator

conserves number of excitations

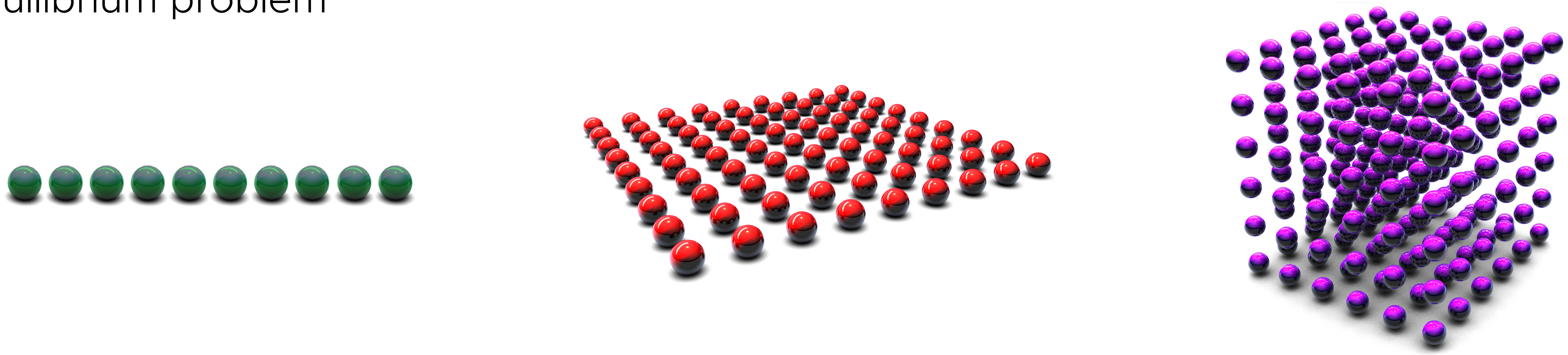
Take-home messages and some observations from Lecture 1

- Vacuum fluctuations give rise to interactions between spins

- Different boundary conditions/environments give rise to very different interactions
    - all-to-all, long-range, short range
    - dissipative vs coherent

- This dramatically impacts the physics, both single-particle (“linear optics”) and many-body

- Emergent behavior: for “close enough” spins, physics is that of a strongly-interacting out-of-equilibrium problem



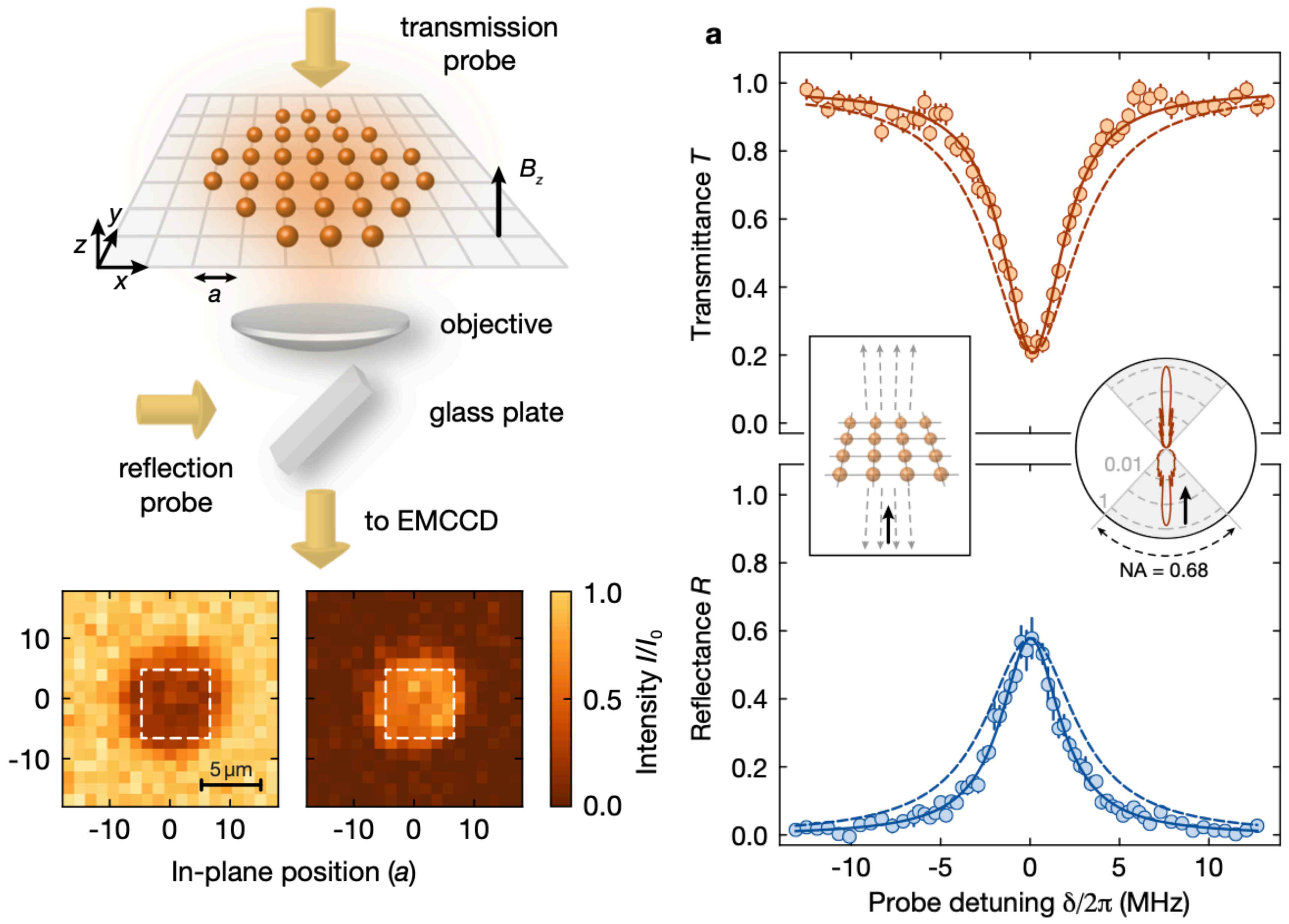
Quantum optics with spins: from single-excitation to many-body physics

Lecture 2

Ana Asenjo-Garcia
Columbia University

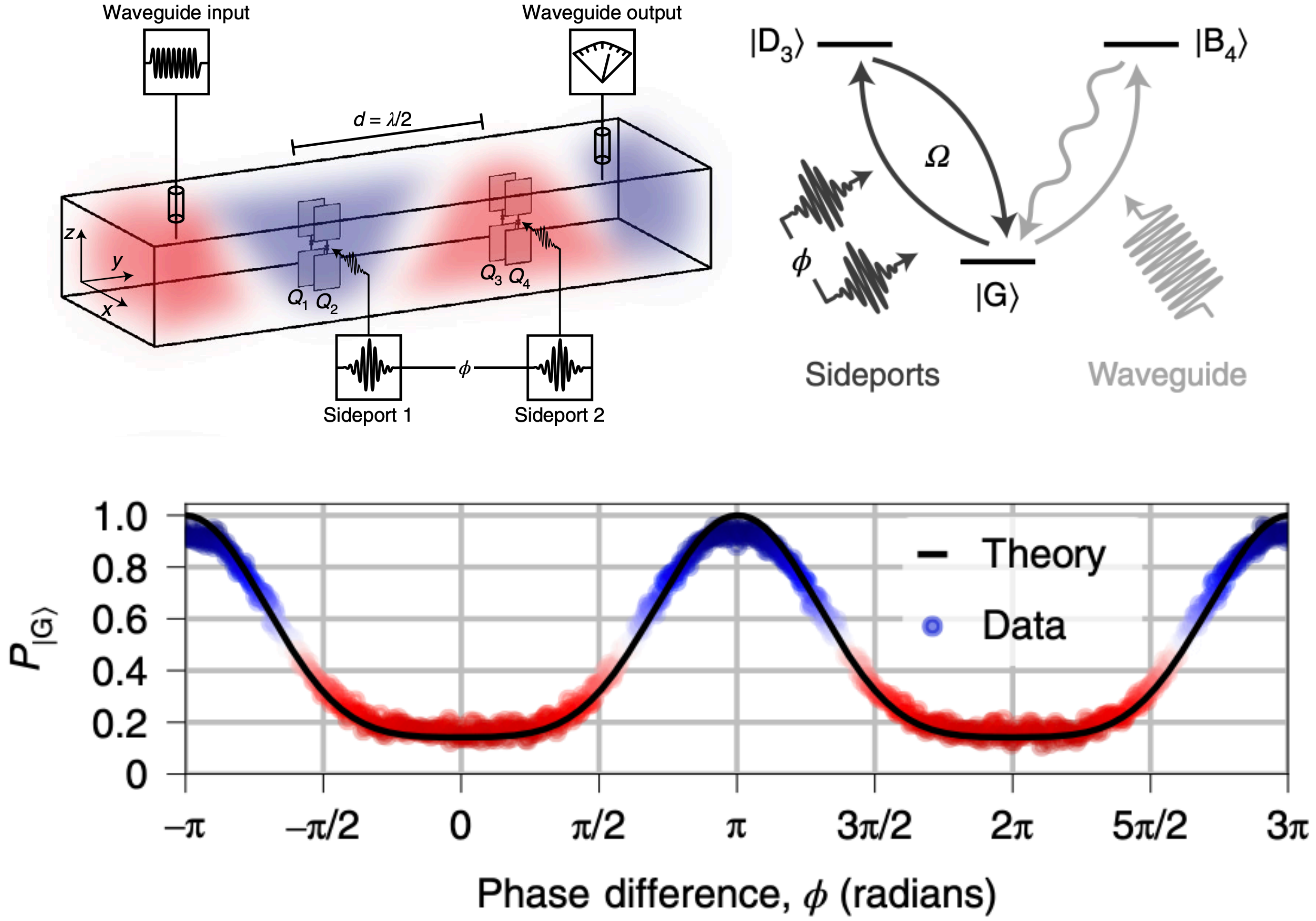
Recent experimental work on quantum optics with atomic arrays

2D atomic array as a perfect mirror



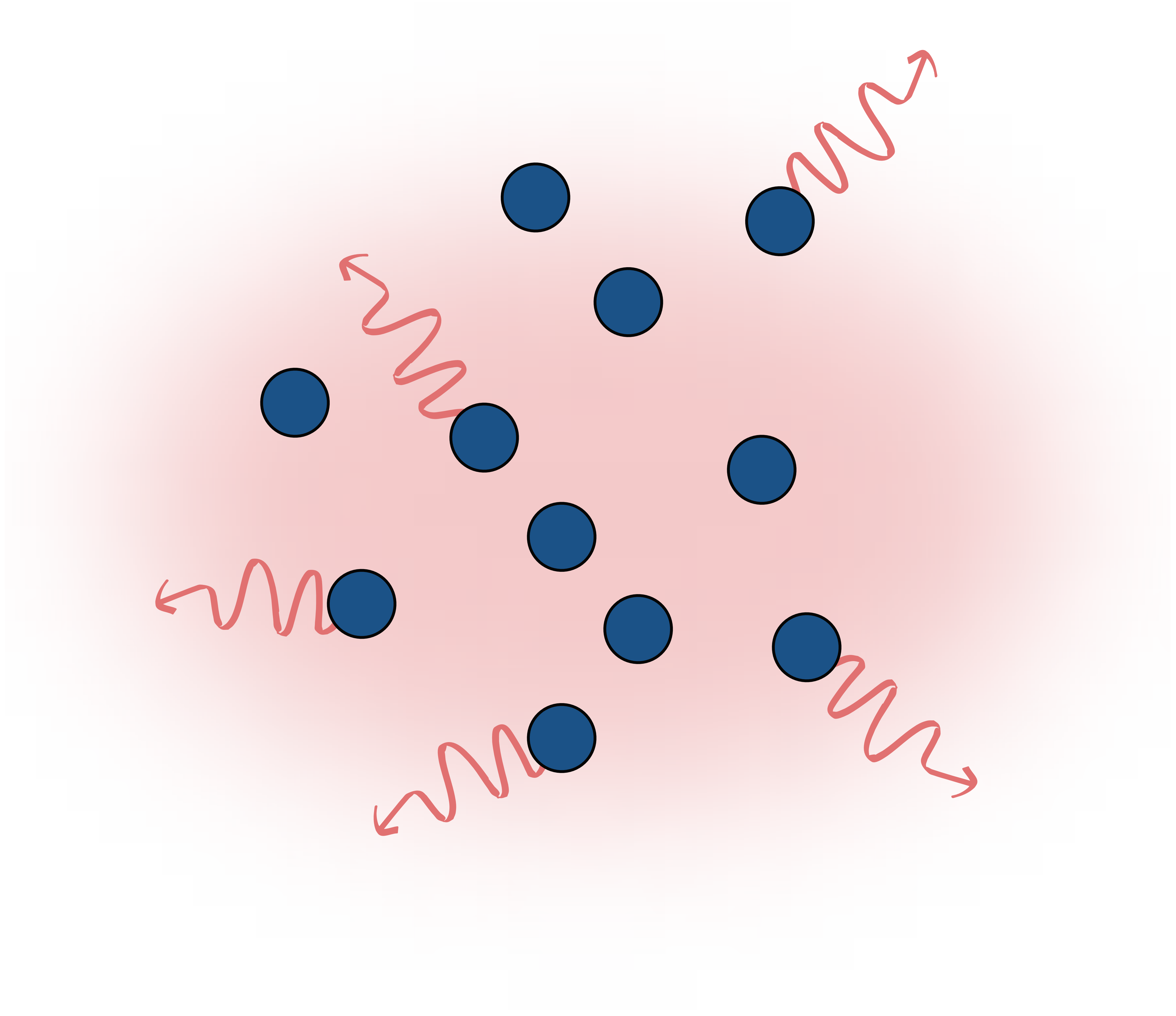
Rui et al., Nature 583, 369 (2020)

Coherent driving of multi-qubit dark state

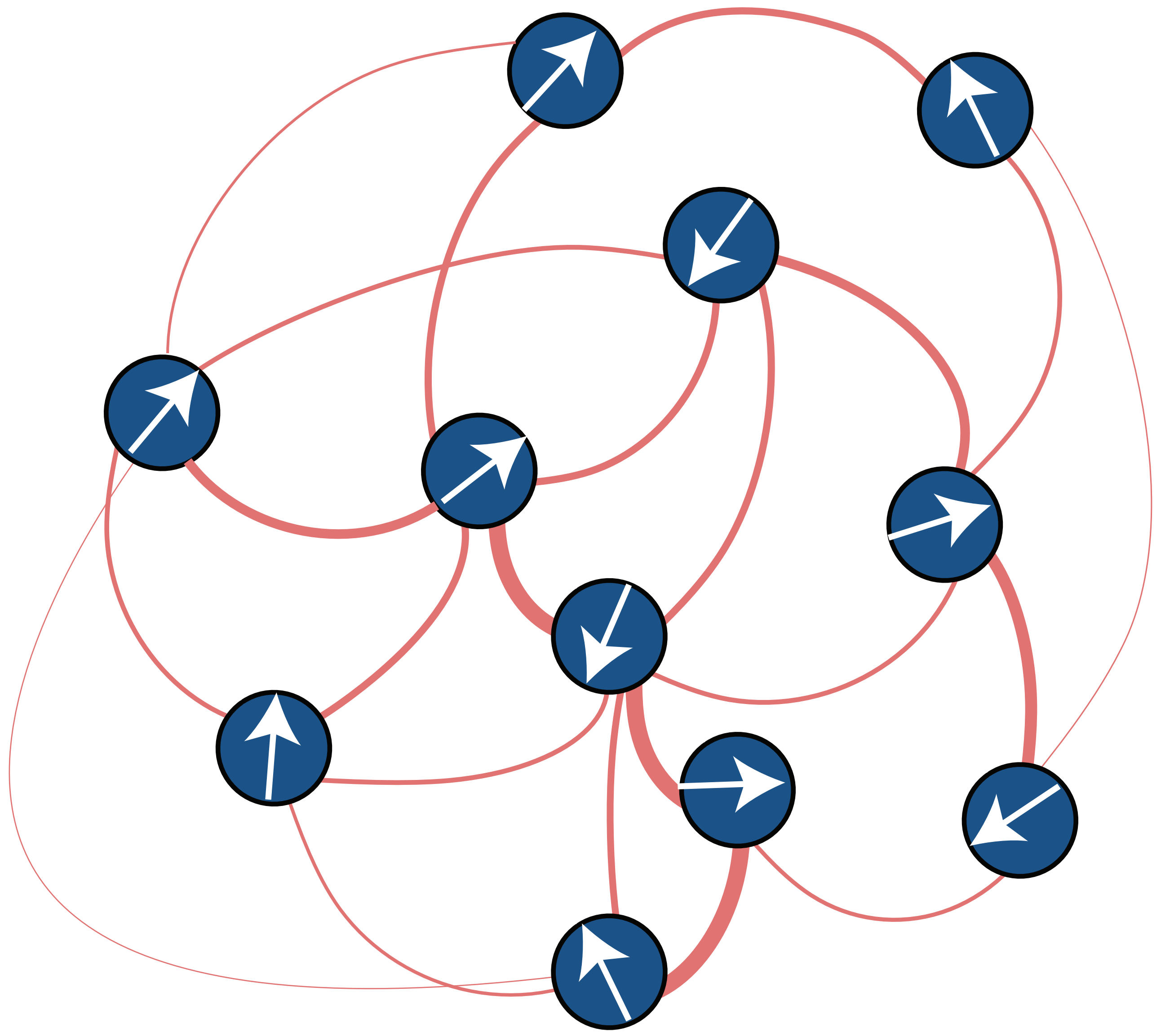


M. Zanner et al., Nat. Phys. 18, 538 (2022)

Recap from lecture 1: light-matter interactions as a spin model



Recap from lecture 1: light-matter interactions as a spin model

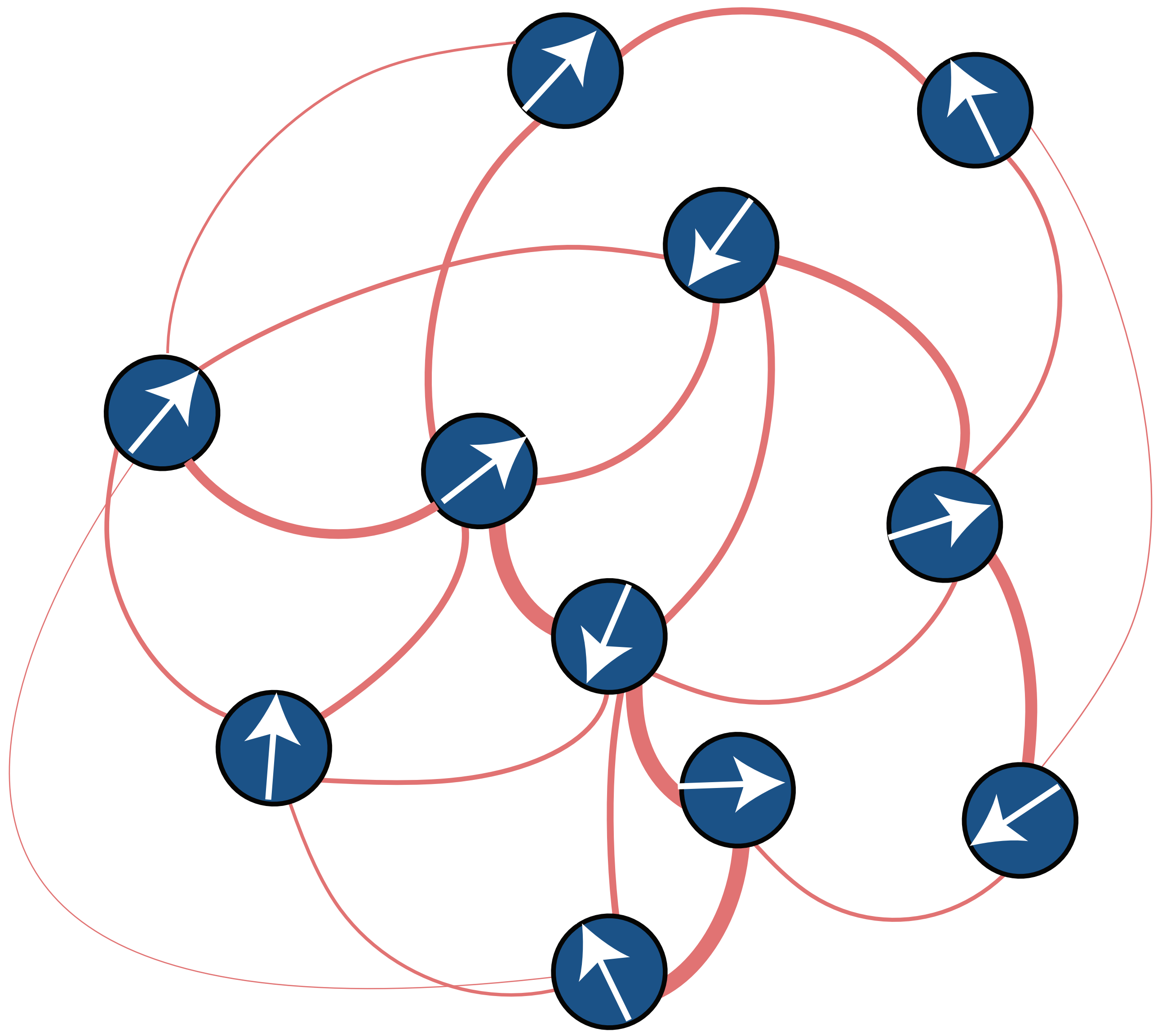


Described by a spin model with
dynamics governed
by a Lindblad master equation

$$\dot{\rho} = -\frac{i}{\hbar}\left[\sum_{i,j=1}^{N} J_{ij}\sigma_+^i\sigma_-^j, \rho\right] + \sum_{i,j=1}^{N}\frac{\Gamma_{ij}}{2}\left(2\sigma_-^i\rho\sigma_+^j - \rho\sigma_+^j\sigma_-^i - \sigma_+^j\sigma_-^i\rho\right)$$

conserves number of excitations

Recap from lecture 1: light-matter interactions as a spin model



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Eigenvalues
(collective transition rates) $\Gamma_\mu \geq 0$

$$\dot{\rho} = -\frac{i}{\hbar}\left[\sum_{i,j=1}^{N} J_{ij}\sigma_+^i\sigma_-^j, \rho\right] + \sum_{\mu=1}^{N}\frac{\Gamma_\mu}{2}\left(2\hat{c}_\mu\hat{\rho}\hat{c}_\mu^\dagger - \hat{\rho}\hat{c}_\mu^\dagger\hat{c}_\mu - \hat{c}_\mu^\dagger\hat{c}_\mu\hat{\rho}\right)$$

collective jump operator $\sim \sum_{i=1}^{N} \alpha_i \, \hat{\sigma}_-^i$

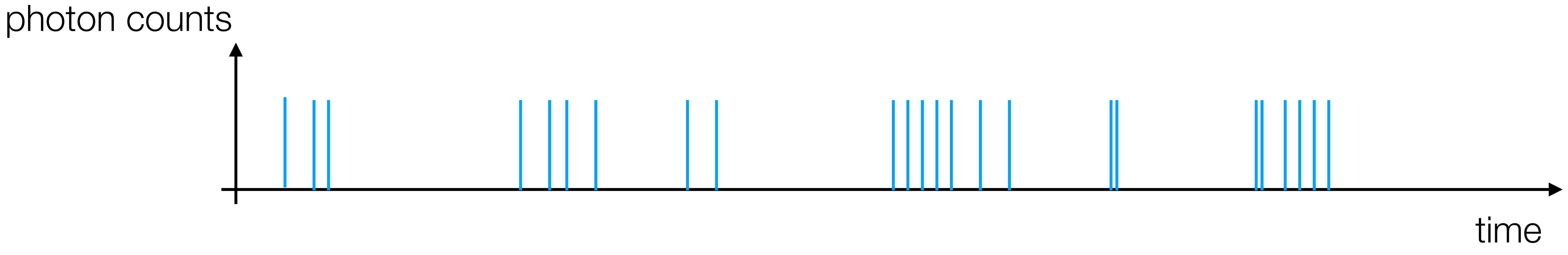
conserves number of excitations

Recap: non-Hermitian Hamiltonian and quantum trajectories

We can rewrite the master equation as:
$$\dot{\rho} = -\frac{i}{\hbar}\left(\mathcal{H}_{\text{eff}}\rho - \rho\mathcal{H}_{\text{eff}}^{\dagger}\right) + \sum_{i,j=1}^{N}\Gamma_{ij}\sigma_-^i\rho\sigma_+^j$$

system evolves due to acquisition of
information from negative measurement

population recycling term
(quantum jumps)



Quantum trajectories: evolution of **pure state** with non-Hermitian H and stochastic application of jumps

Non-Hermitian Hamiltonian:
$$\mathcal{H}_{\text{eff}} = \hbar\sum_{i,j=1}^{N}\left(J_{ij} - i\frac{\Gamma_{ij}}{2}\right)\sigma_+^i\sigma_-^j = -\mu_0\omega_0^2\sum_{i,j=1}^{N}\wp^*\cdot\mathbf{G}(\mathbf{r}_i,\mathbf{r}_j,\omega_0)\cdot\wp\;\sigma_+^i\sigma_-^j$$

Excitation conserving:
$$[\mathcal{H}_{\text{eff}}, \underset{\textstyle\sum_j \sigma_{ee}^j}{\underset{|}{n_{\text{exc}}}}] = 0$$

→ Excitation conserving (block-diagonal in excitation sector)

Lecture 2

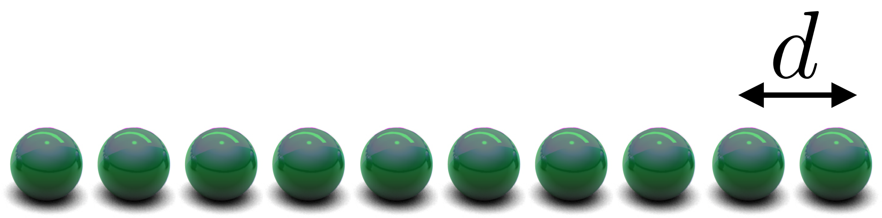
Atomic arrays in free space and waveguide QED

Few-excitation sector: super- and sub-radiance

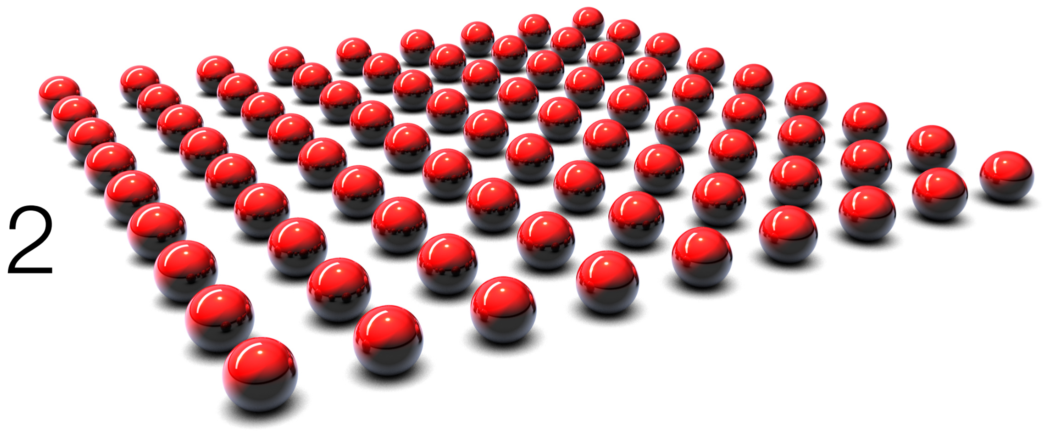
Selective radiance and applications

Green's function between spins in free space (or bulk medium)

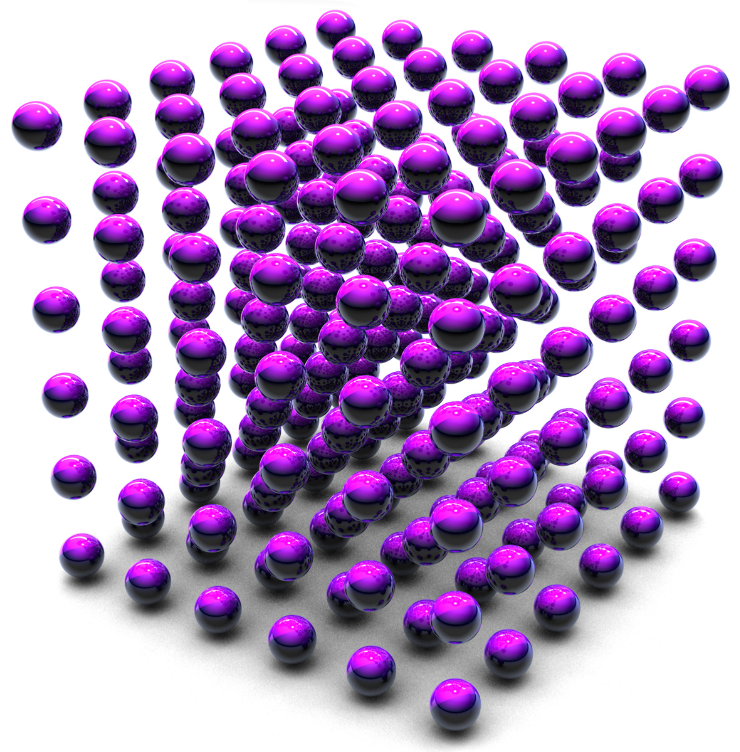
D=1



D=2



D=3



Dissipative matrix elements (in 3D vacuum):

$\omega_0^3|\wp|^2/\,3\pi\hbar\epsilon_0 c^3$

dipole matrix element

$$\Gamma_{ij} = \frac{6\pi\Gamma_0}{k_0}\hat{\wp}^* \cdot \operatorname{Im}\mathbf{G}(\mathbf{r}_i, \mathbf{r}_j, \omega_0) \cdot \hat{\wp}$$

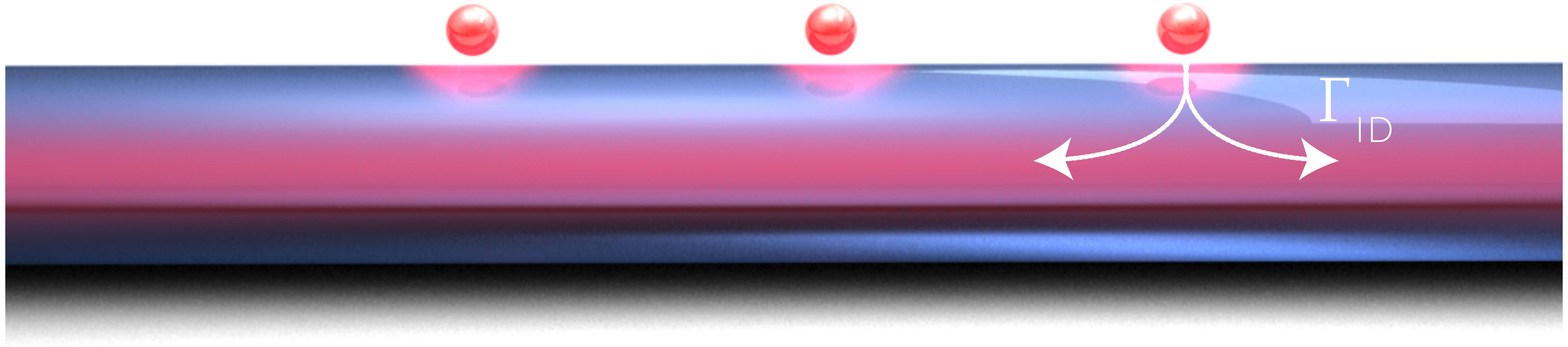
ω_0/c

$$\mathbf{G}(\mathbf{r}, \omega_0) \equiv \frac{e^{ik_0 r}}{4\pi k_0^2 r^3}\left[(k_0^2 r^2 + ik_0 r - 1)\mathbb{1} + (-k_0^2 r^2 - 3ik_0 r + 3)\frac{\mathbf{r}\otimes\mathbf{r}}{r^2}\right]$$

Single-spin decay rate: $\lim\limits_{i\to j}\Gamma_{ij} = \Gamma_0$

Lamb shift: $\lim\limits_{i\to j} J_{ij} = 0$ (technically diverges, yields resonance frequency renormalization)

Green’s function between spins in 1D vacuum (wQED)



$$\mathbf{G}(\mathbf{r}_i, \mathbf{r}_j, \omega_0) \sim -i\,\frac{\Gamma_{\mathrm{1D}}}{2} e^{ik_{\mathrm{1D}}|z_i - z_j|}$$

Single-spin decay rate: $\lim_{i\to j} \Gamma_{ij} = \Gamma_{\mathrm{1D}}$

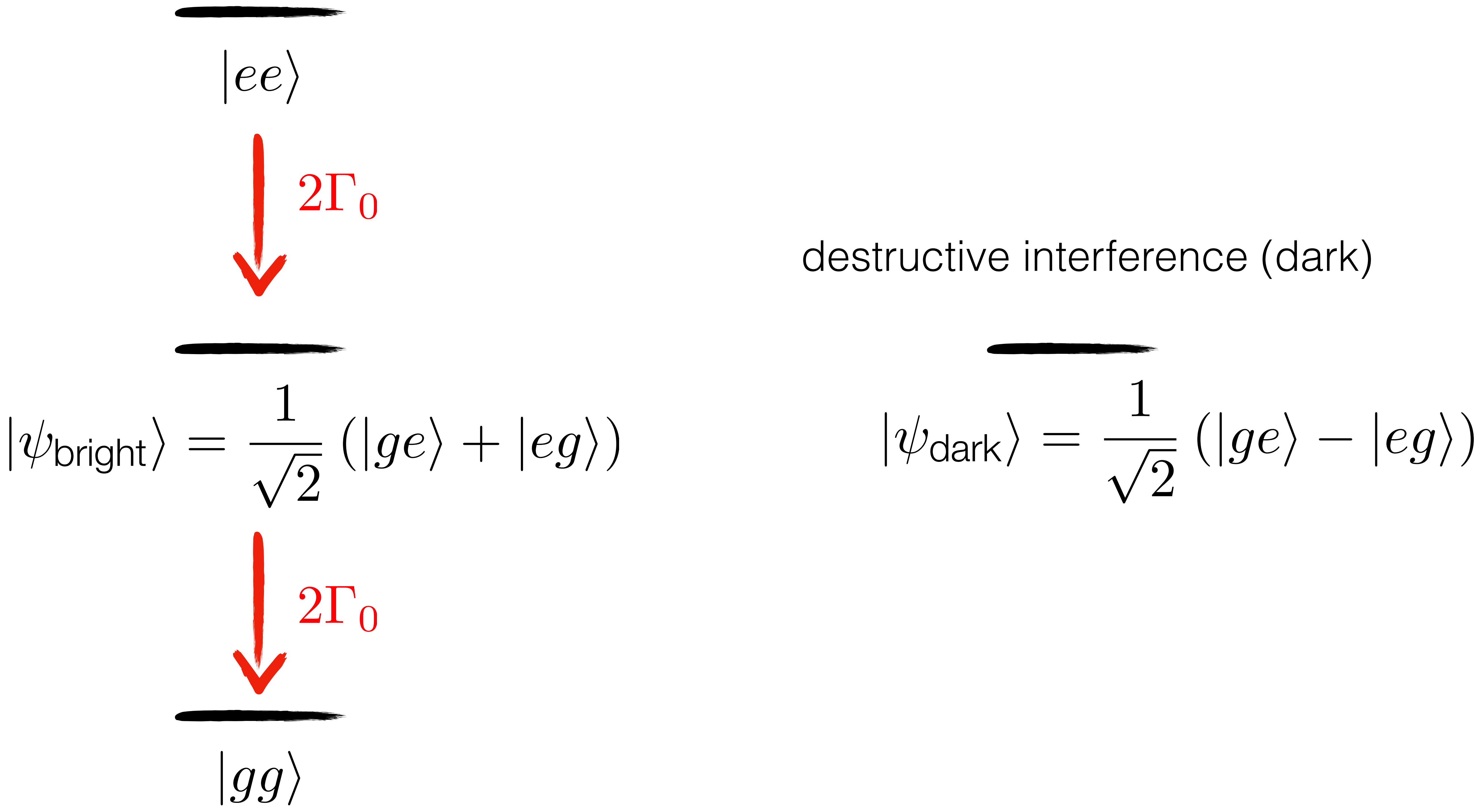
Lamb shift: $\lim_{i\to j} J_{ij} = 0$

Field does not decay, only relative distance matters (accumulated phase)

Despite looking different, free space and wQED are similar. Non-negligible H, several collective jump operators

Simplest limit (of both situations): d ⟶ 0

non-Hermitian H becomes quite trivial: $$\mathcal{H}_{eff} = -i\frac{\hbar\Gamma_0}{2}\sum_{i,j=1}^{N}\sigma_+^i\sigma_-^j$$



2 spins, two excitations:

$|ee\rangle$

destructive interference (dark)

2 spins, single excitation:

$$|\psi_{\text{bright}}\rangle = \frac{1}{\sqrt{2}}(|ge\rangle + |eg\rangle)$$

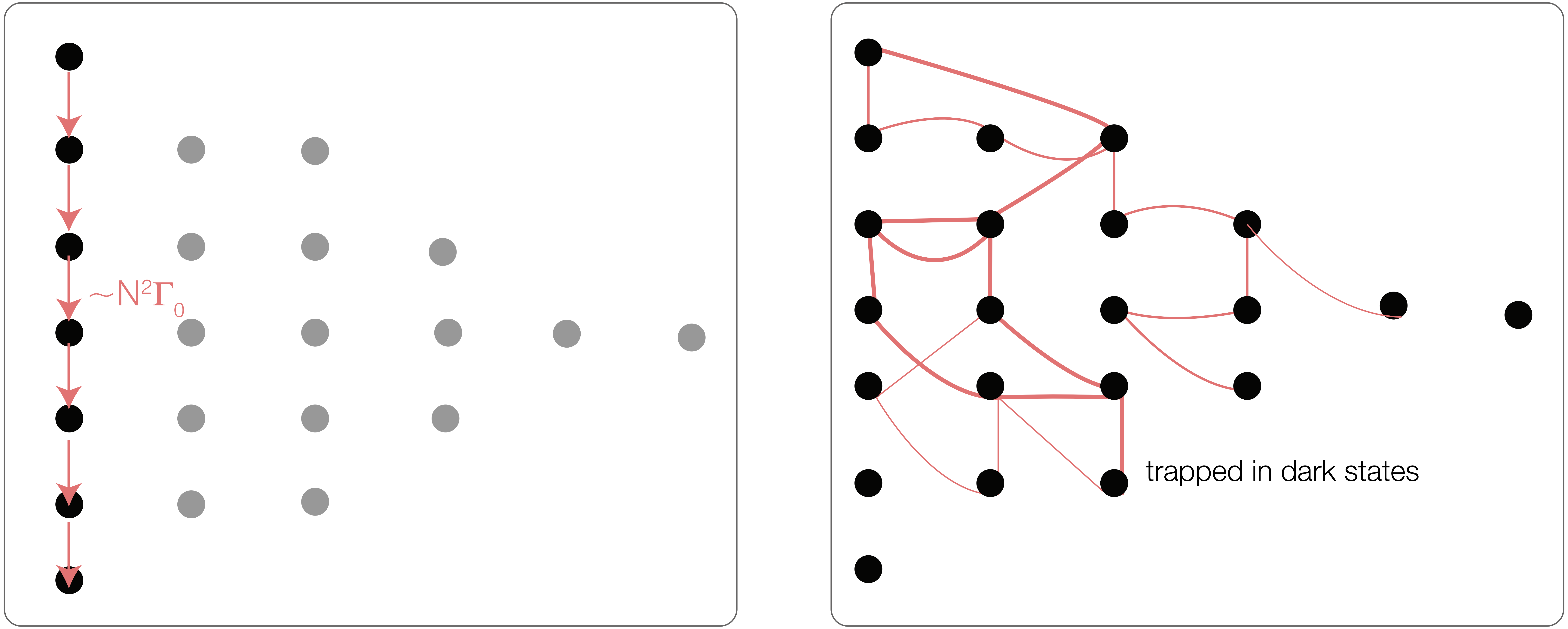
$$|\psi_{\text{dark}}\rangle = \frac{1}{\sqrt{2}}(|ge\rangle - |eg\rangle)$$

2 spins, zero excitations:

$|gg\rangle$

Generalizing to arbitrary number of excitations

schematics of possible trajectories in Hilbert space

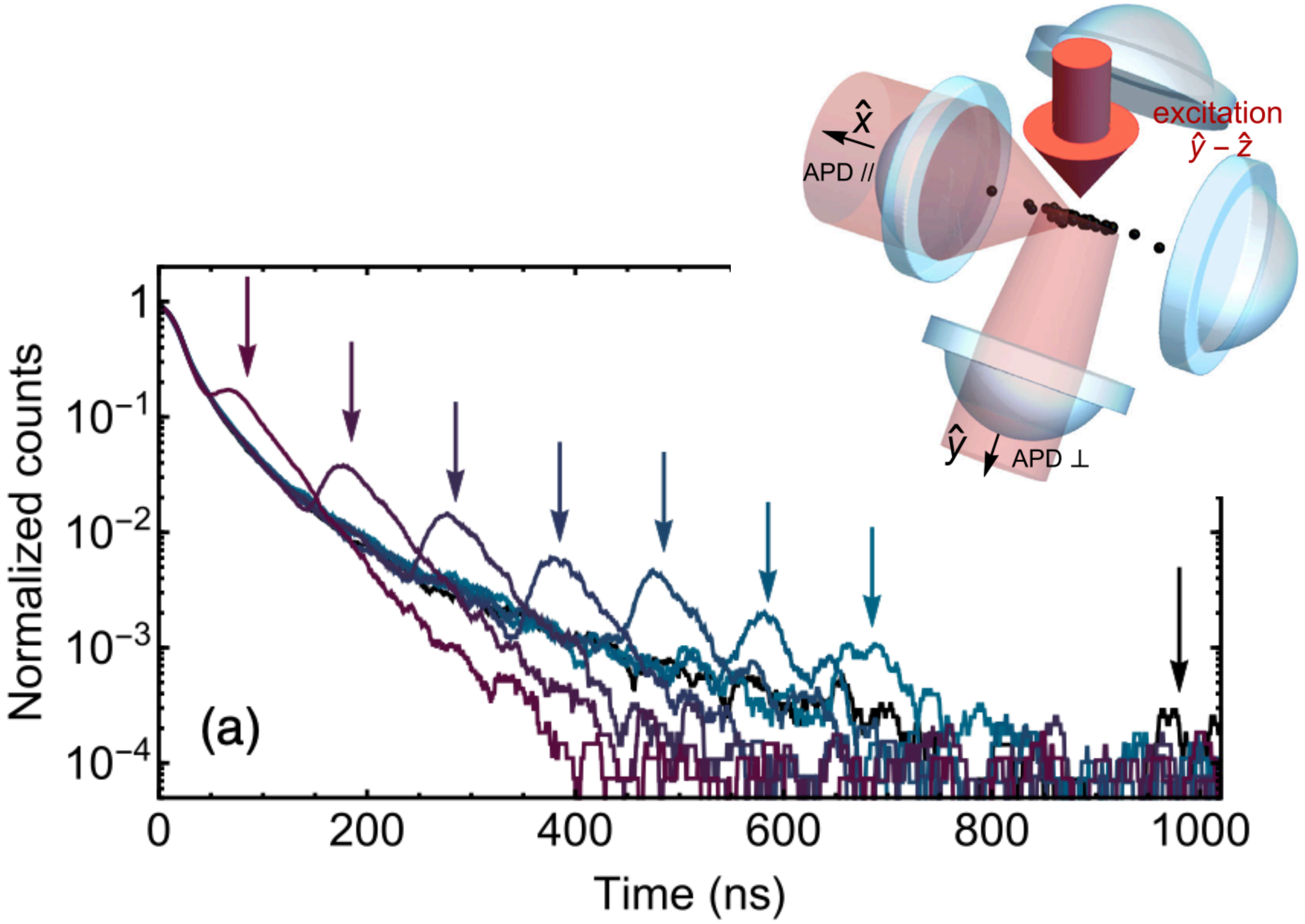


with permutational symmetry

in extended systems

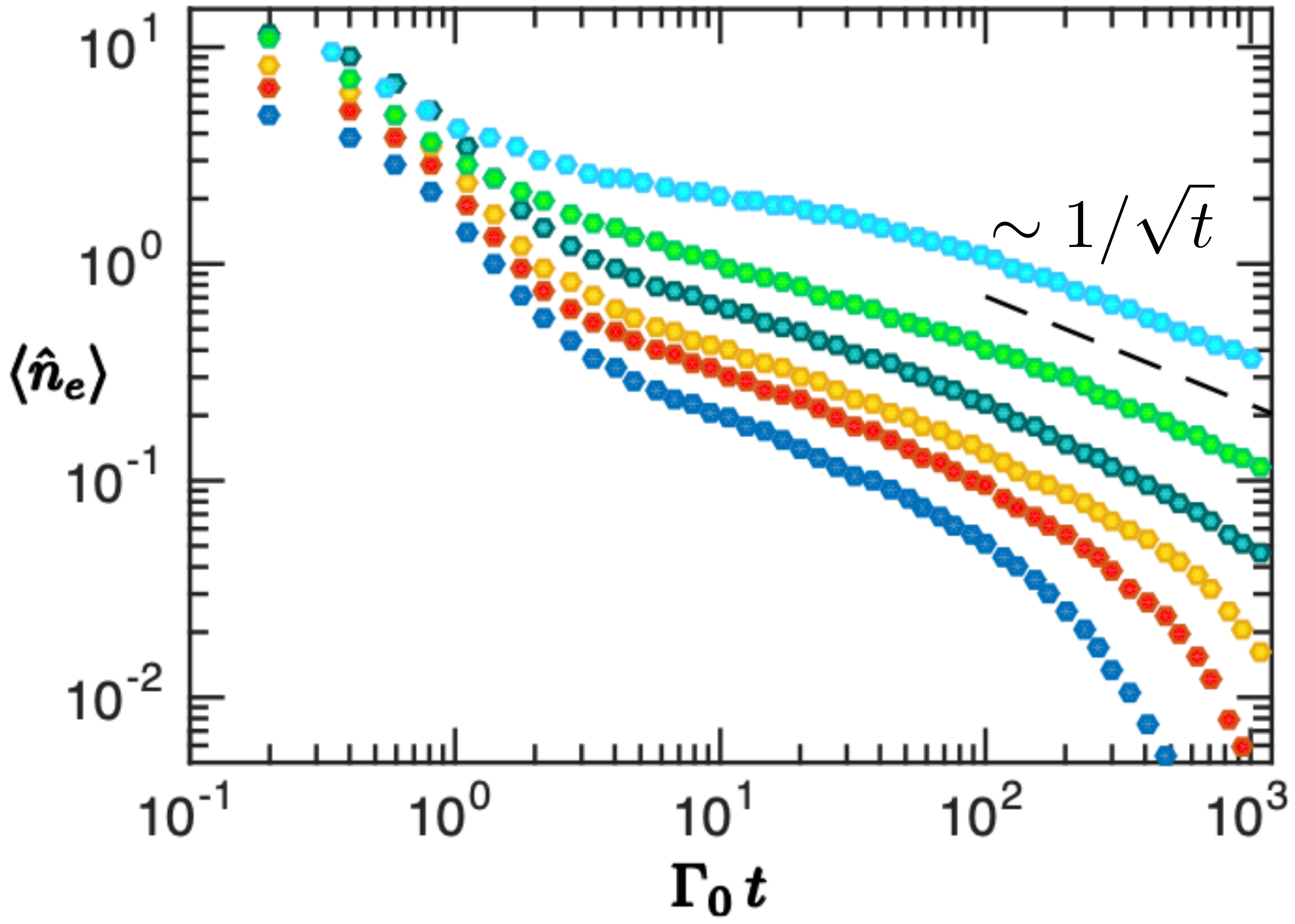
Accessing dark states via decay in extended atomic ensembles

Storage and release of subradiant excitations in a cloud



Ferioli et al., PRX 11, 021031 (2021)

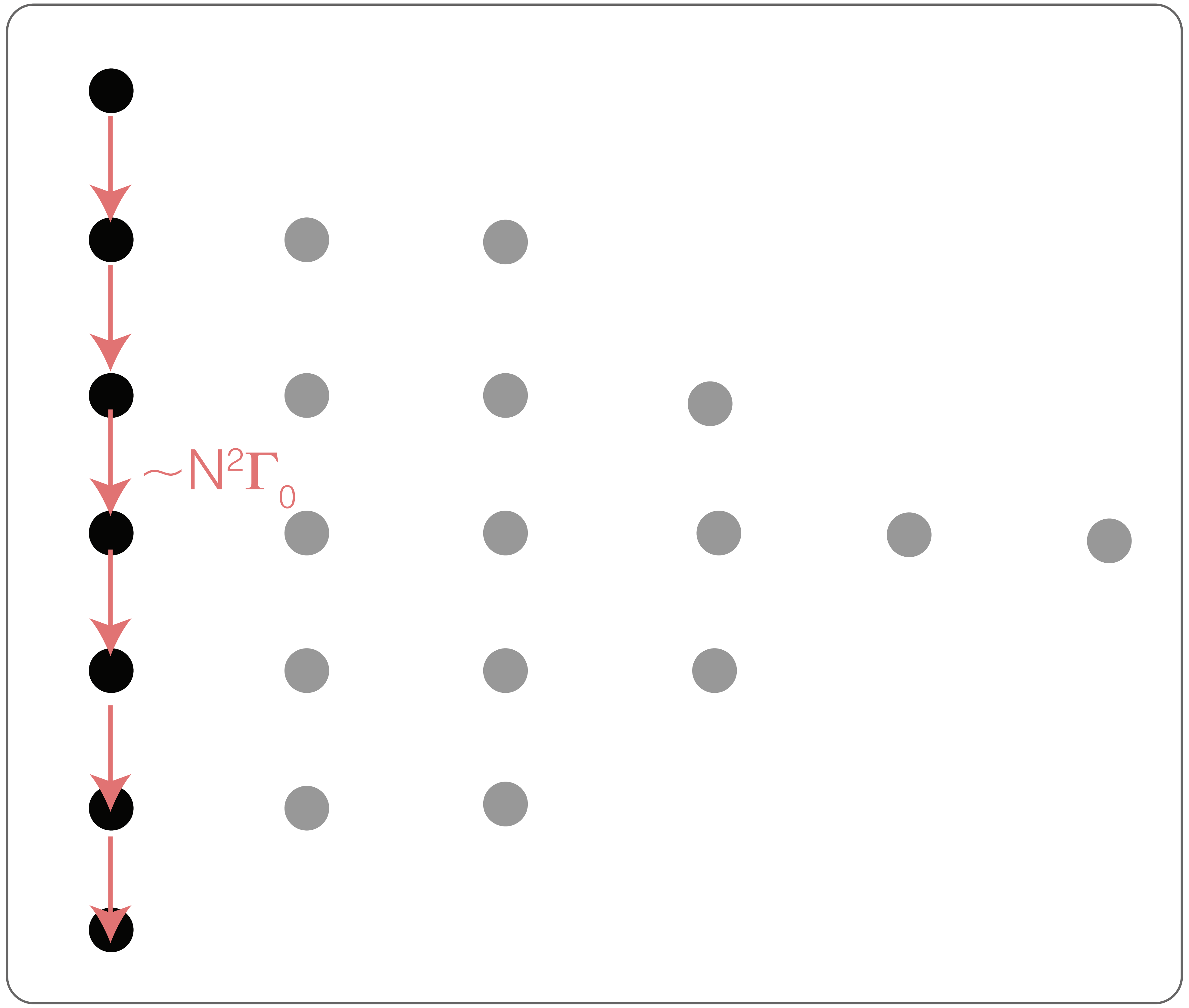
Power-law behavior



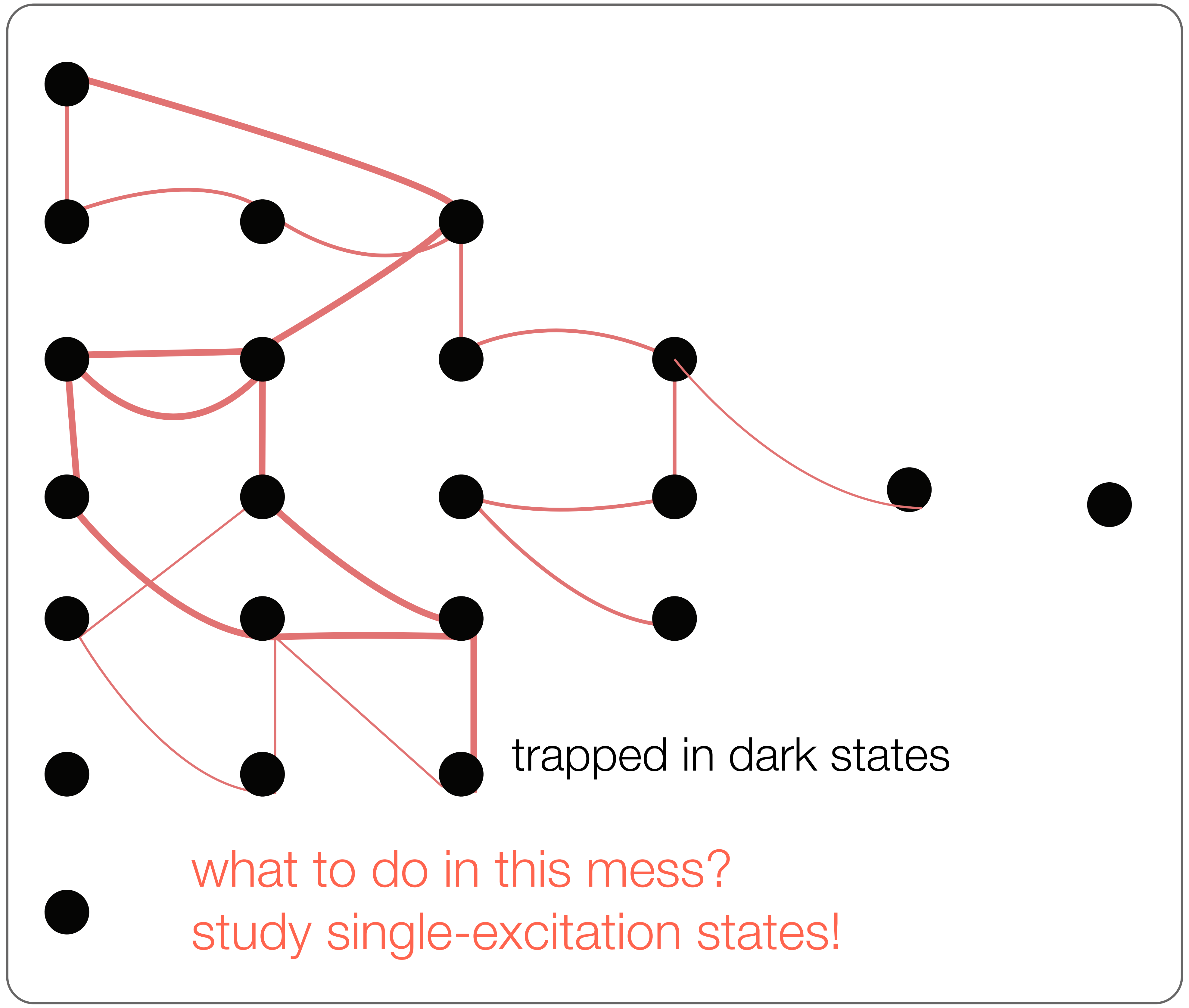
Henriet et al., PRA 99, 023802 (2019)

Generalizing to arbitrary number of excitations

schematics of possible trajectories in Hilbert space



with permutational symmetry



what to do in this mess?
study single-excitation states!

in extended systems

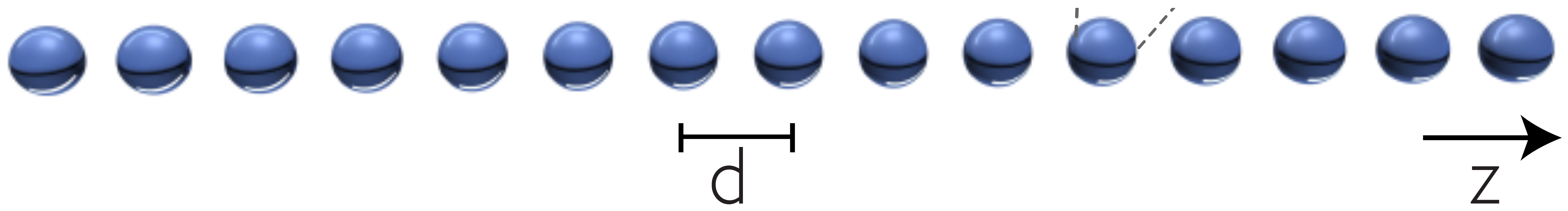
Lecture 2

Atomic arrays in free space and waveguide QED

Few-excitation sector: super- and sub-radiance

Selective radiance and applications

Collective modes of an infinite 1D chain (single excitation sector)



Non Hermitian Hamiltonian: $\mathcal{H}_{\text{eff}} = -\mu_0 \omega_0^2 \sum_{i,j=1}^{N} \wp^* \cdot \mathbf{G}(\mathbf{r}_i, \mathbf{r}_j, \omega_0) \cdot \wp \; \hat{\sigma}_+^i \hat{\sigma}_-^j$

For an infinite chain, we diagonalize the Hamiltonian in momentum space via spin waves:

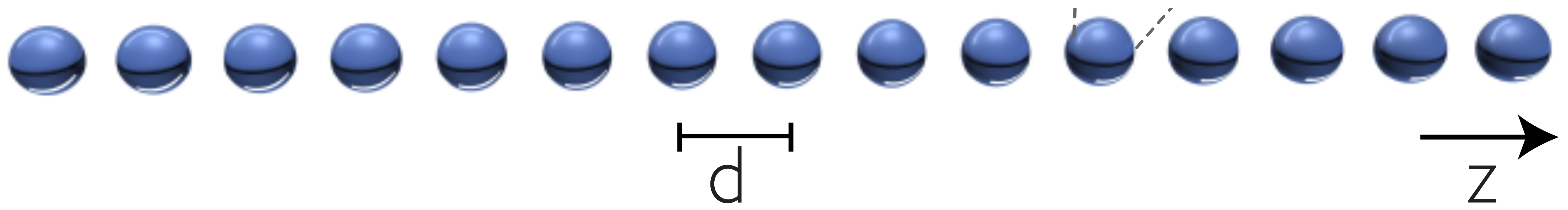
Creation operator: $S_{\mathbf{k}}^\dagger = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} e^{i\mathbf{k}\cdot\mathbf{r}_j} \sigma_+^j$ such that:

$$\mathcal{H}_{\text{eff}} S_{\mathbf{k}}^\dagger |g\rangle^{\otimes N} = \hbar(J_{\mathbf{k}} - i\Gamma_{\mathbf{k}}/2)\; S_{\mathbf{k}}^\dagger |g\rangle^{\otimes N}$$

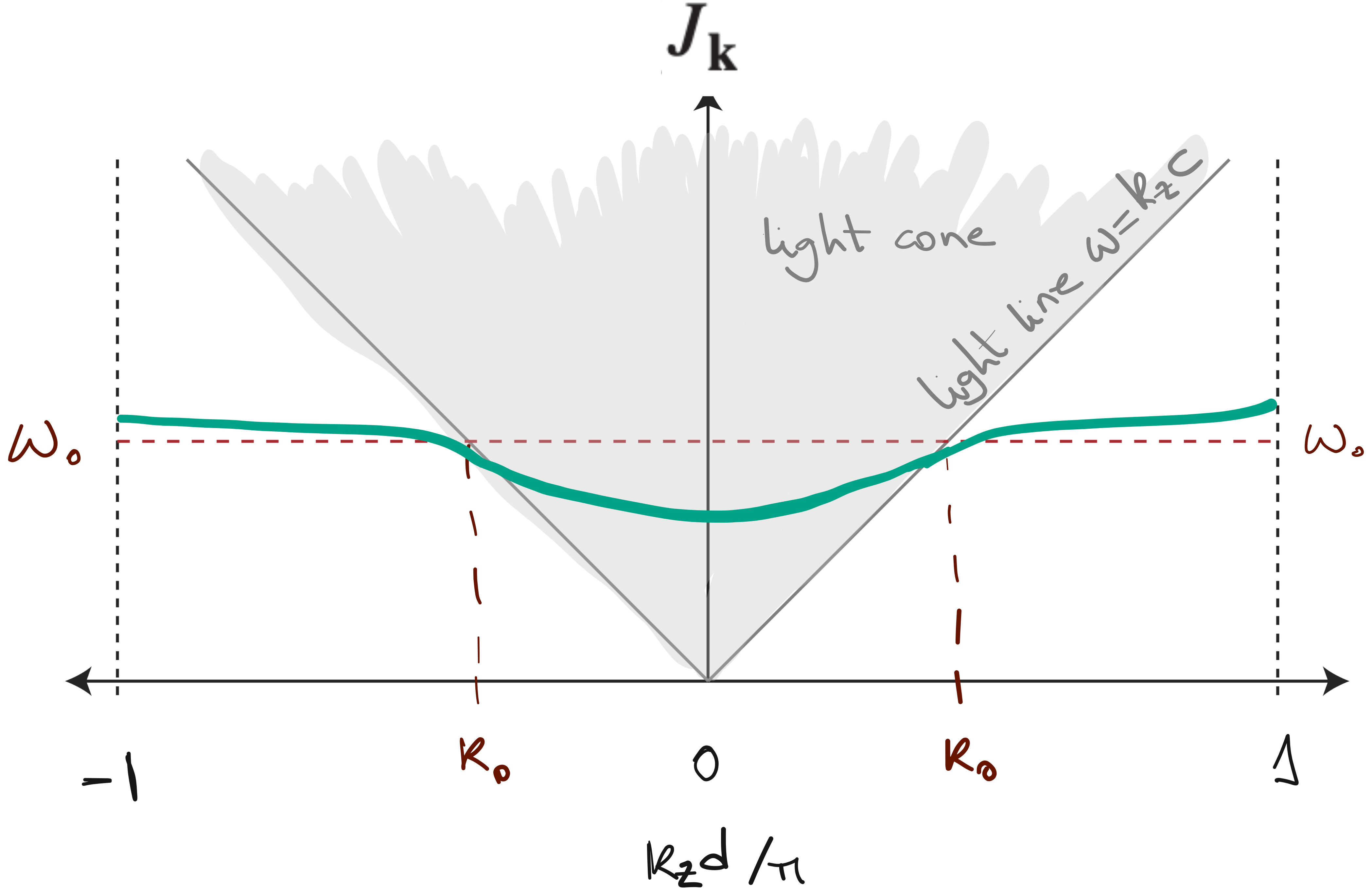
$$-\frac{3\pi}{k_0} \hat{\wp}^* \cdot \tilde{\mathbf{G}}_0(\mathbf{k}) \cdot \hat{\wp}$$

dispersion relation
and decay rate of spin wave

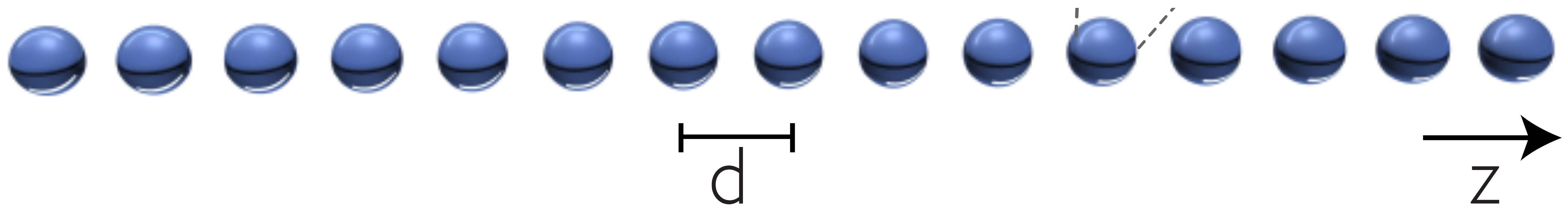
Collective modes of an infinite 1D chain (single excitation sector)



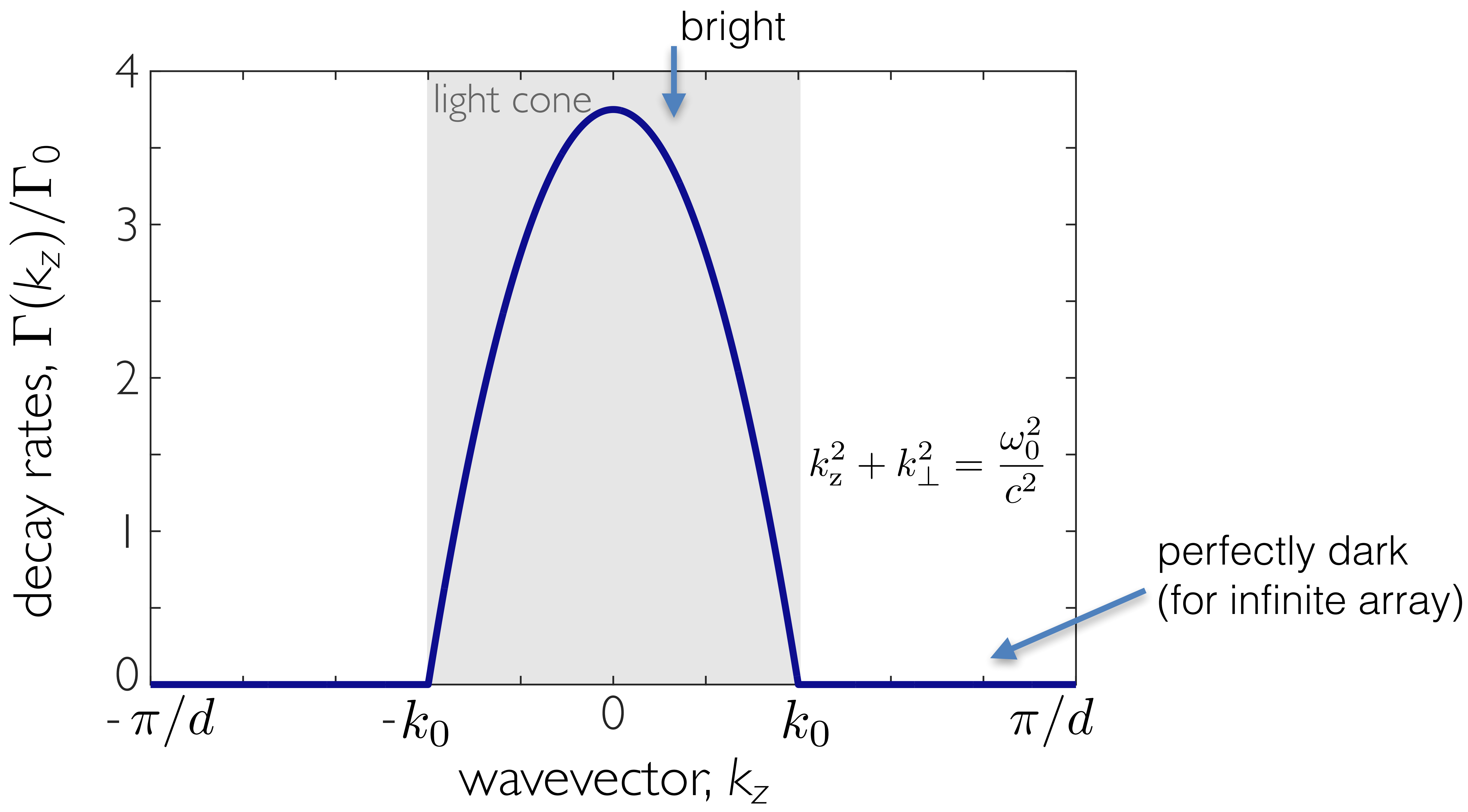
Dispersion relation of spin wave:



Collective modes of an infinite 1D chain (single excitation sector)



For $d<\lambda_0/2$ dark states emerge:

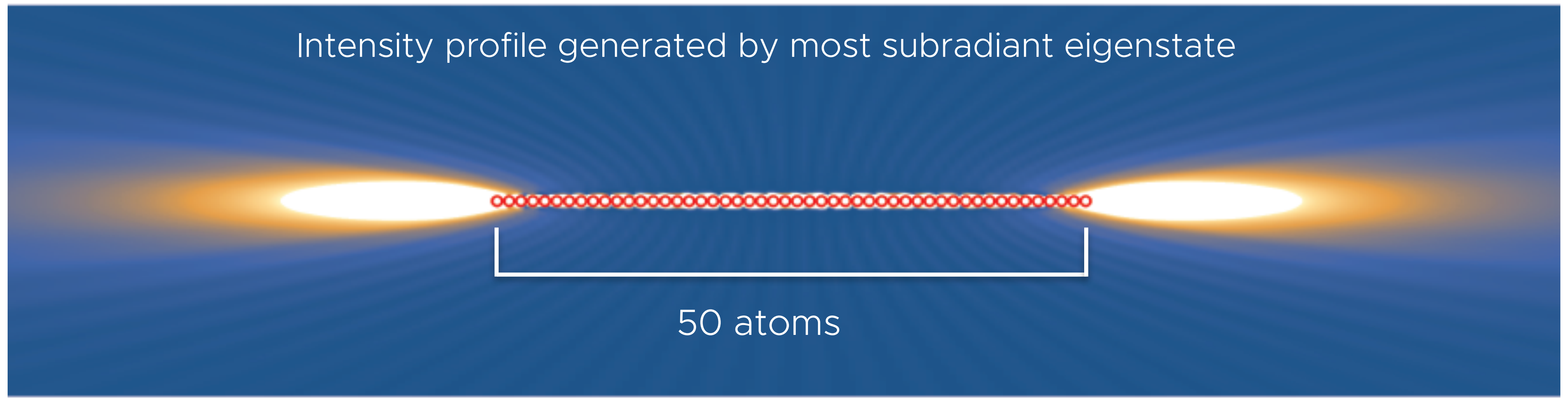


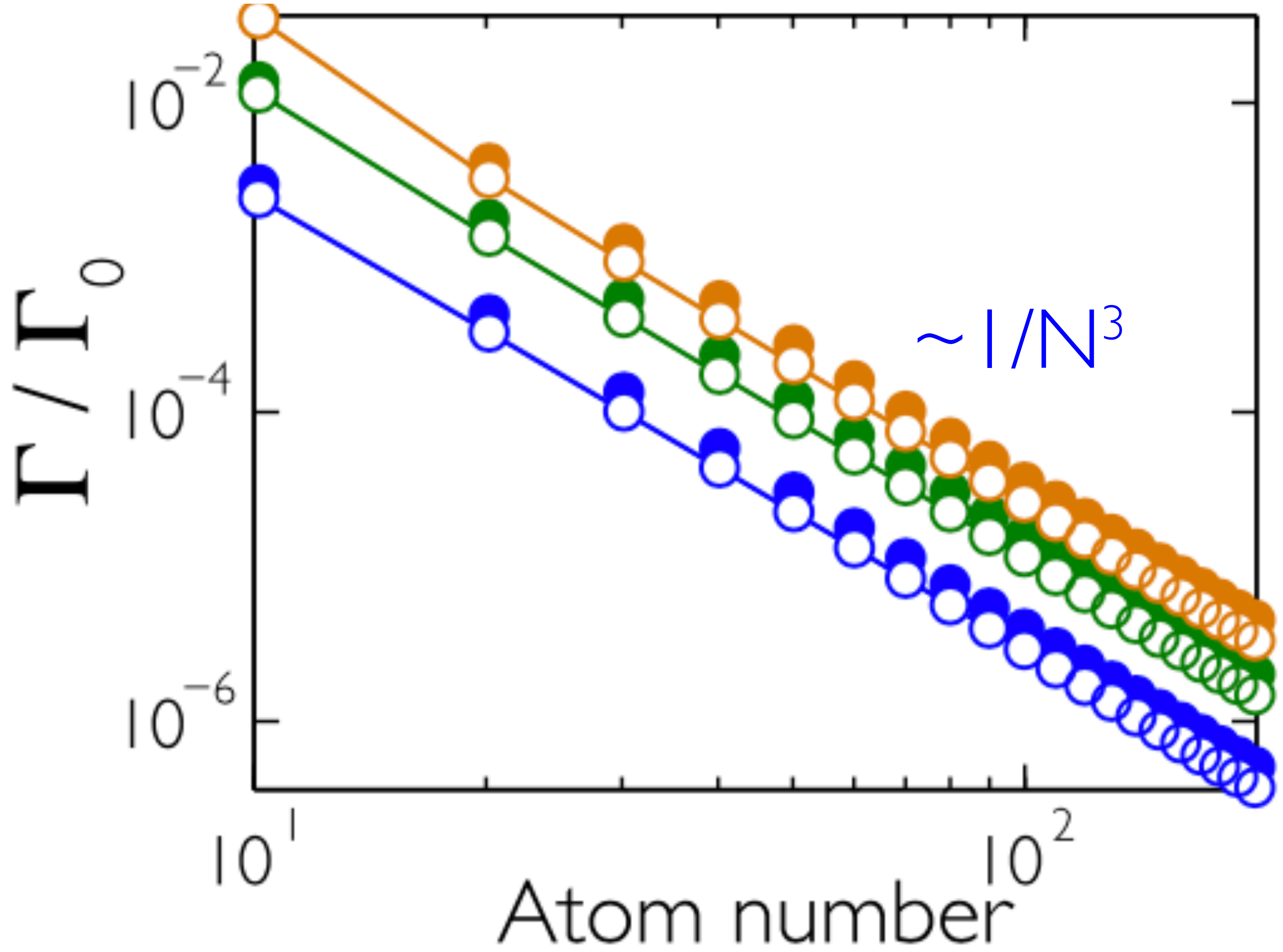
bright

$$k_z^2 + k_\perp^2 = \frac{\omega_0^2}{c^2}$$

perfectly dark
(for infinite array)

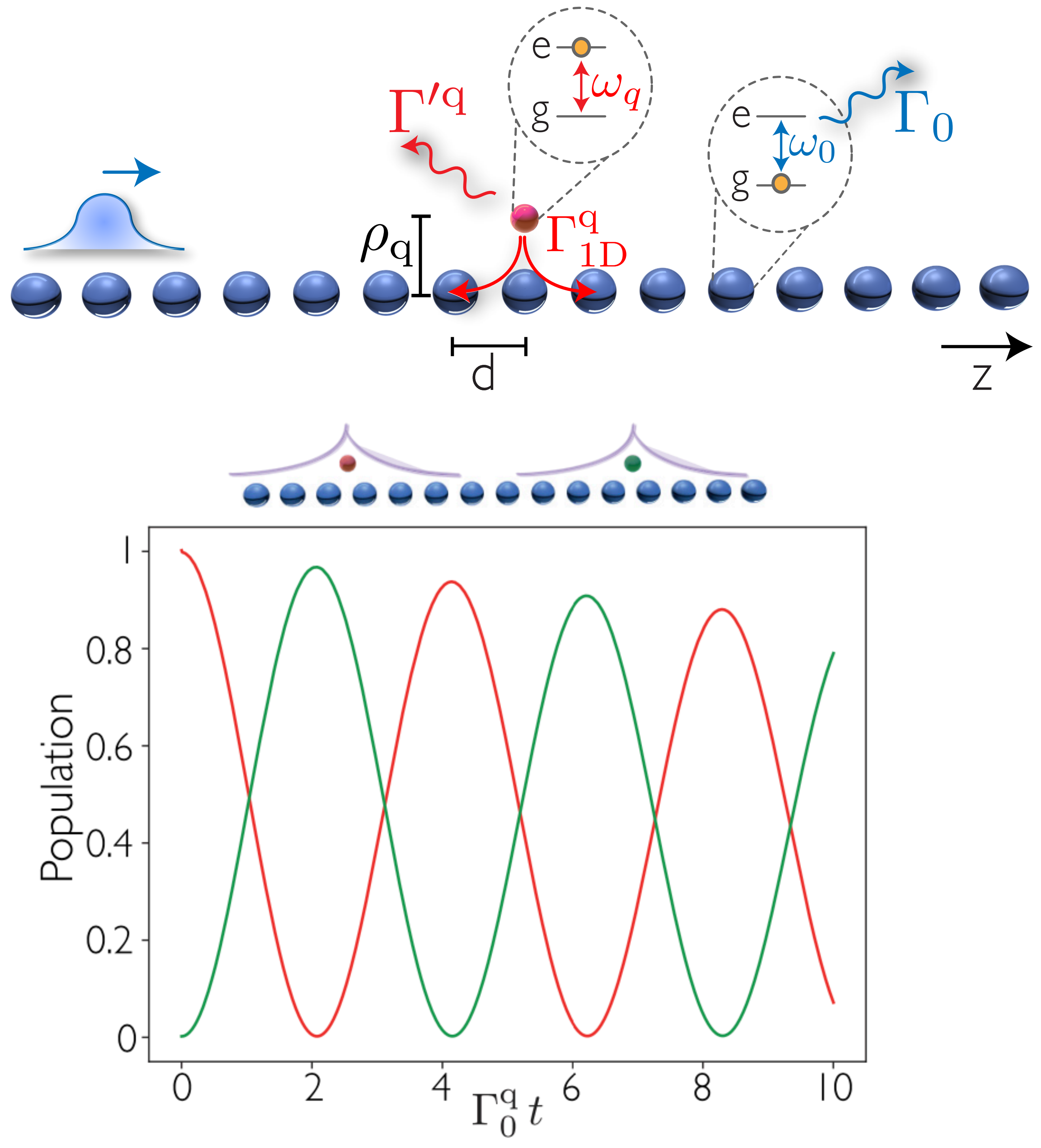
Scaling of dark states for a finite chain





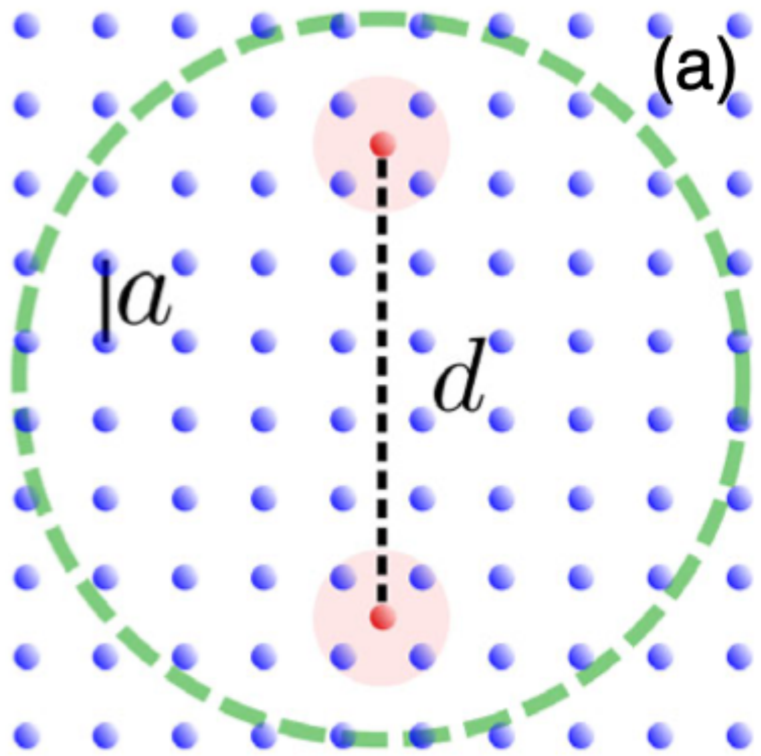
“Atomic-waveguide” QED

array as a bath:

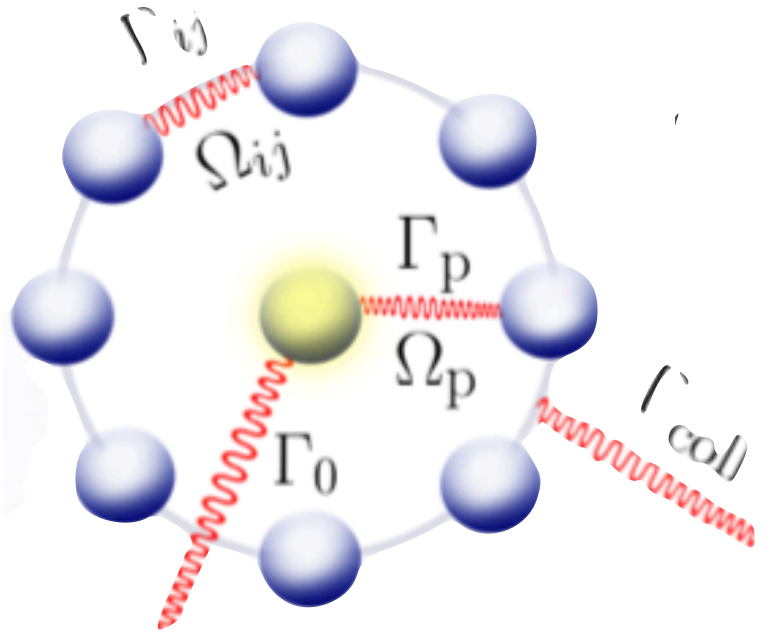


Masson & AAG, PRR 2, 043213 (2020)

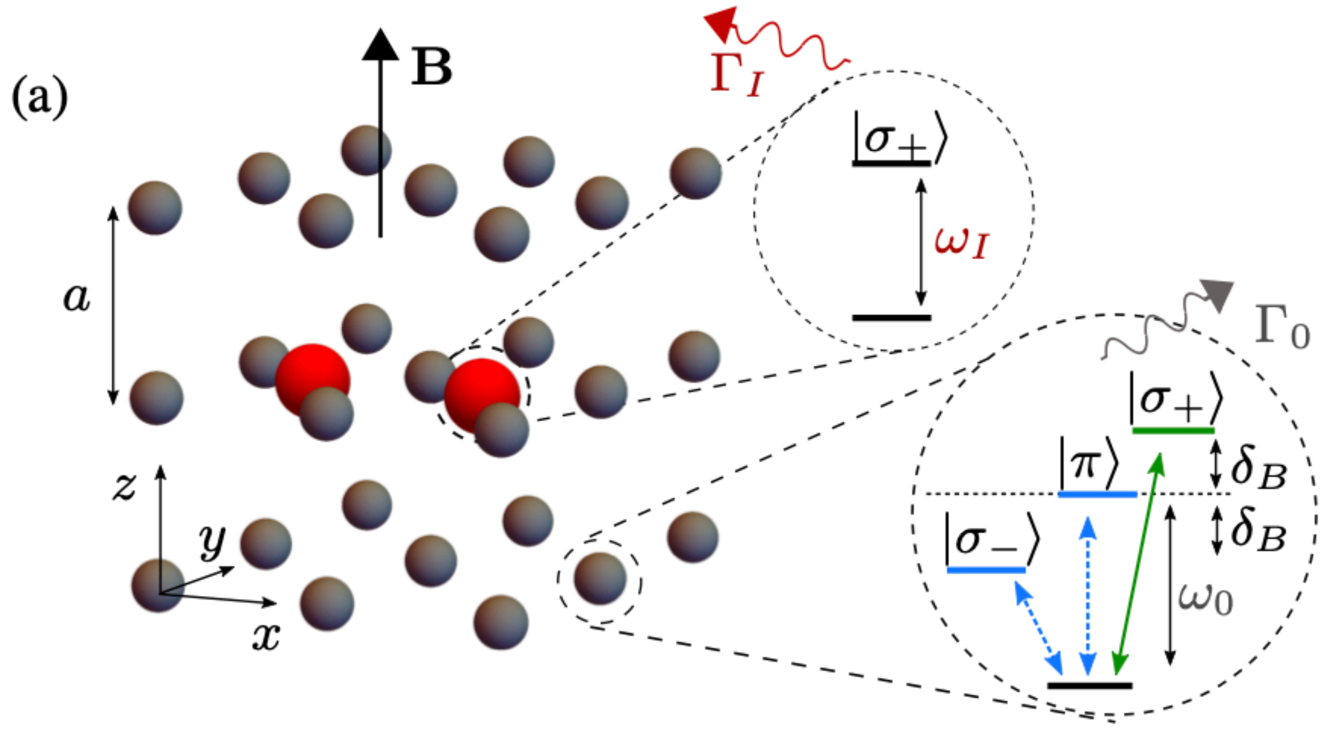
2D: Patti et al.,
PRL 126, 223602 (2021)



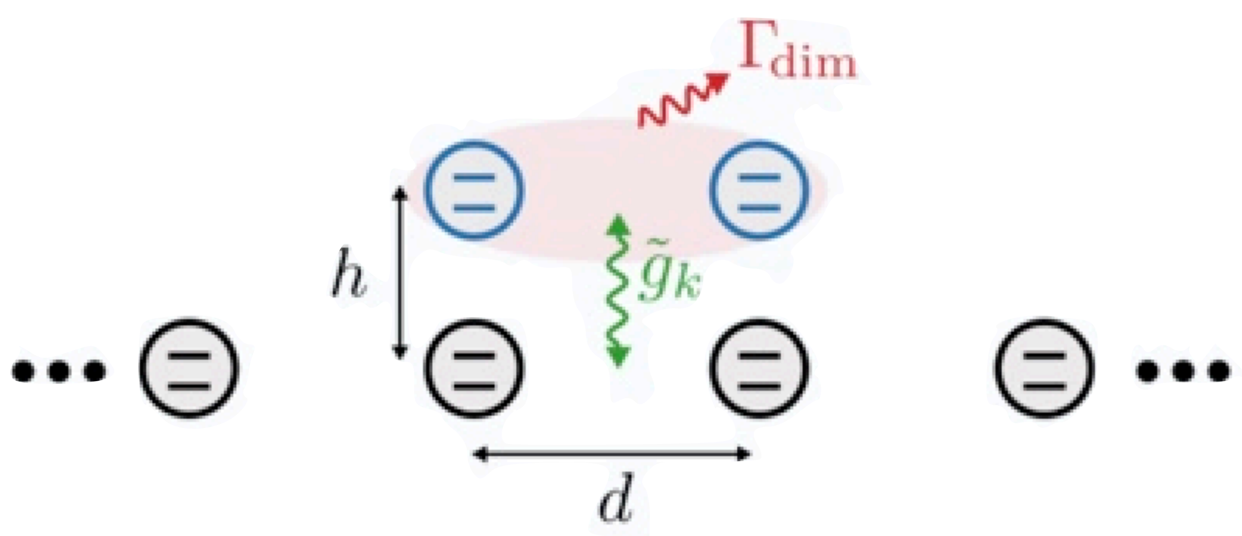
Ring: Holzinger et al.,
PRL 124, 253603 (2020)



3D: Brechtelsbauer and Malz,
Phys. Rev. A 104, 013701 (2021)

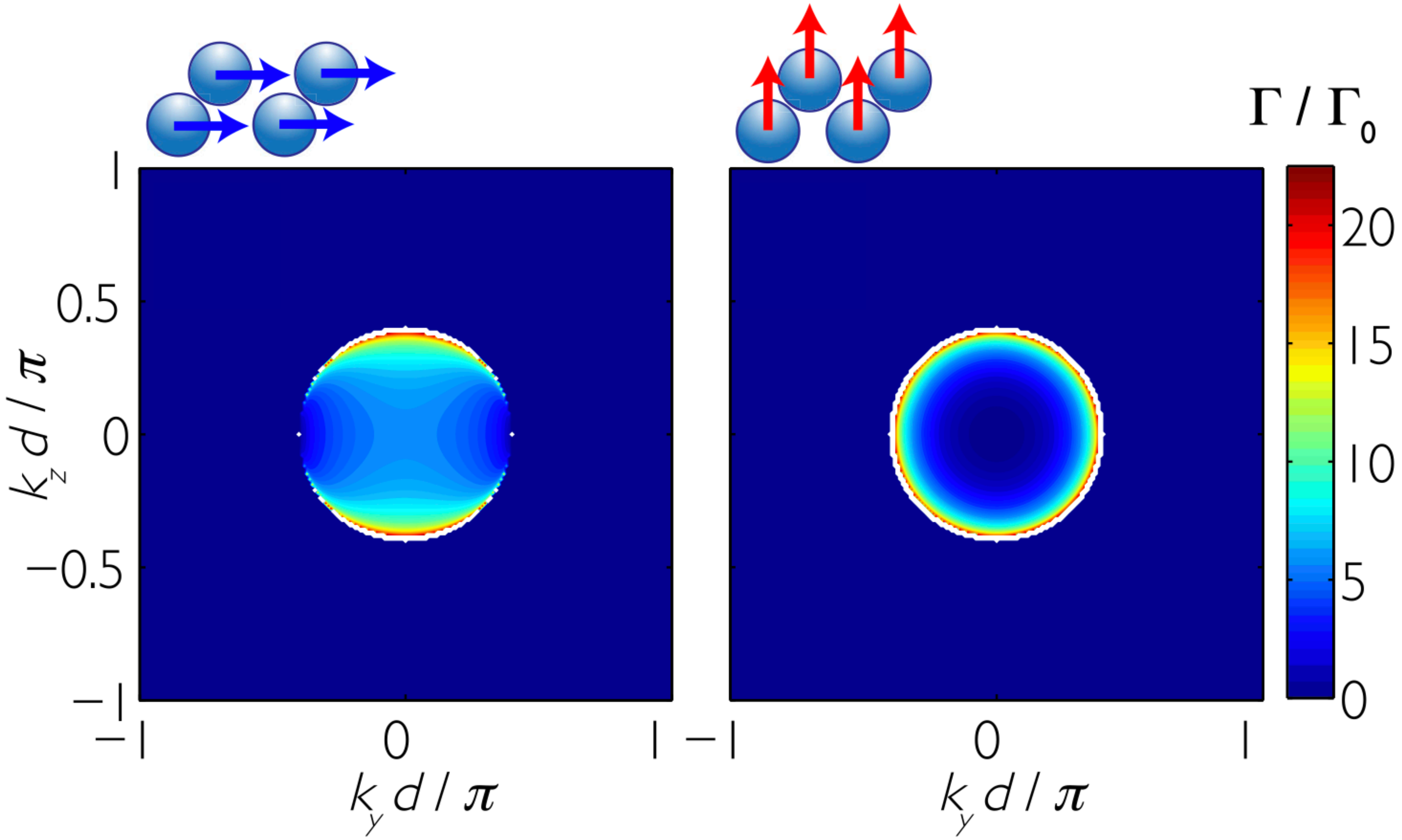


1D+dimer: Castell-Graells et al.,
Phys. Rev. A 104, 063707 (2021)



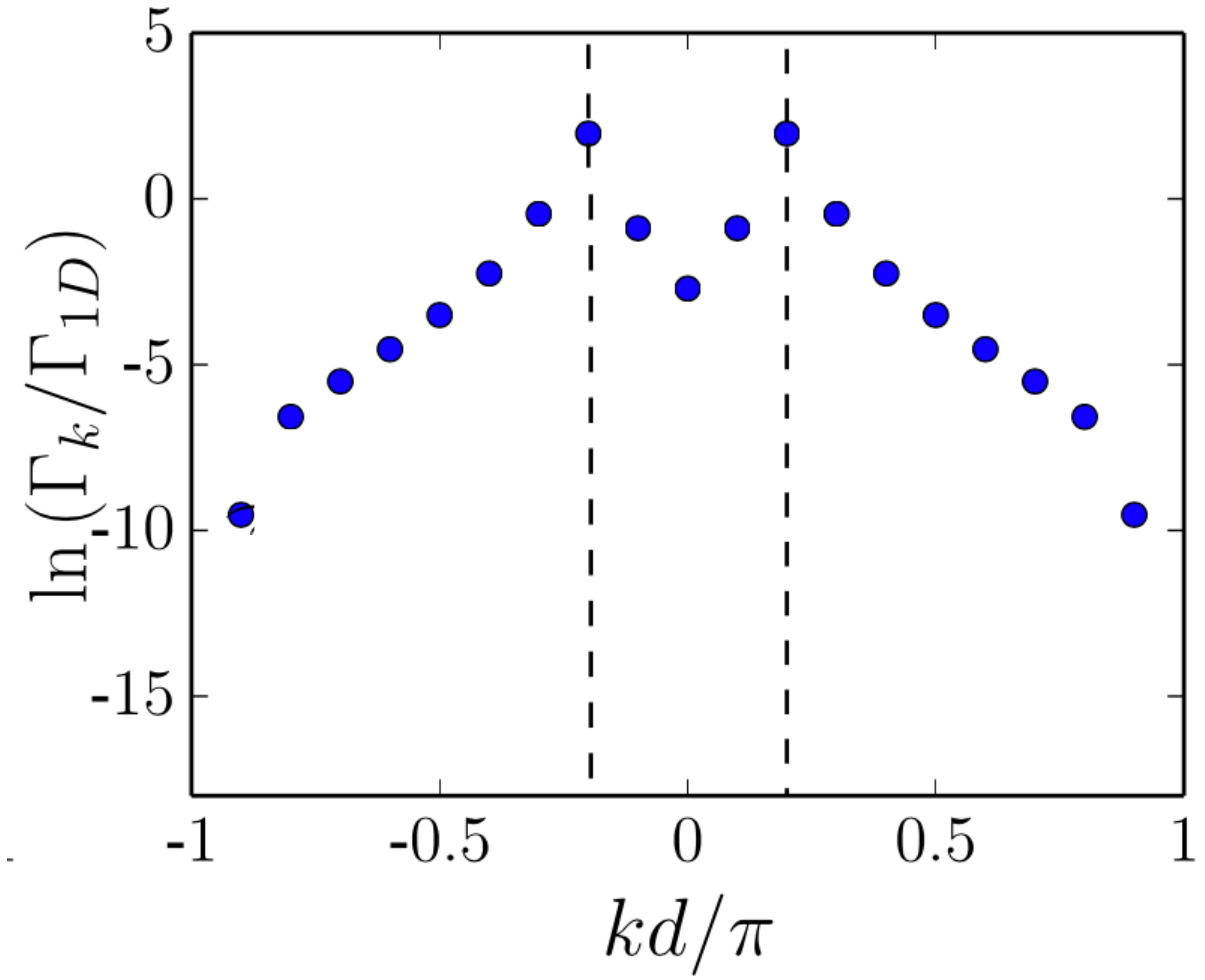
Decay of rates of spin waves in 2D arrays and in wQED

Brillouin zone of 2D arrays,
showing divergences at the light line



AAG et al., PRX **7**, 031024 (2017)

Brillouin zone of 1D arrays,
coupled to 1D waveguide



Albrecht et al., NJP 21, 025003 (2019)

Two-excitation states in chains: revealing their spin nature

Consider subradiant excitation (with decay 1/N^3)

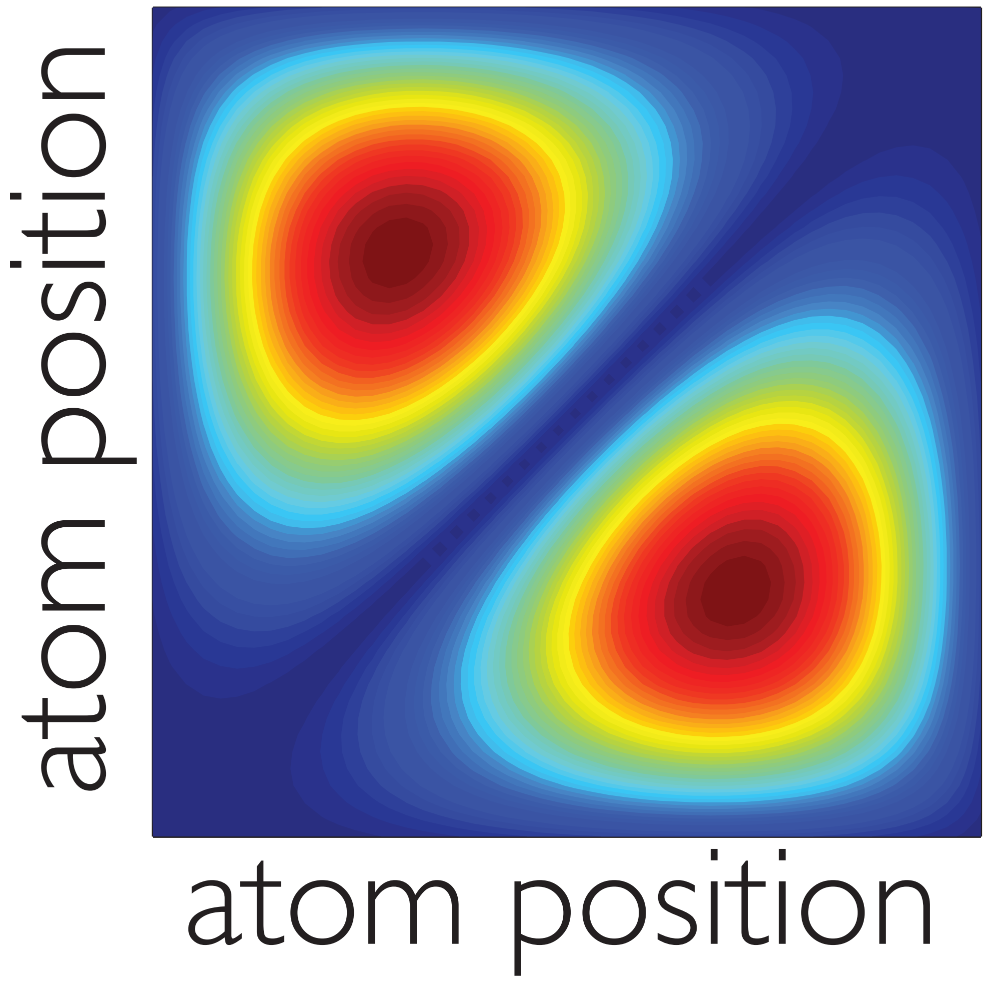
$$|\psi\rangle \propto S^\dagger |g\rangle^{\otimes N} = \sum_{i=1}^{N} c_i |e_i\rangle$$

Most subradiant eigenstate of H$_{\text{eff}}$ in the 2-ex sector is “fermonic”. Ansatz:

$$|F_{1,2}\rangle \propto \sum_{i<j} (c_{1,i}c_{2,j} - c_{1,j}c_{2,i}) |e_i, e_j\rangle$$

$$= \sum_{i,j=1}^{N} c_{ij} |e_i e_j\rangle$$

$|c_{ij}|$



Pauli exclusion principle in space

AAG et al., PRX **7**, 031024 (2017)

Two-excitation states in chains: revealing their spin nature

Consider subradiant excitation (with decay 1/N^3)

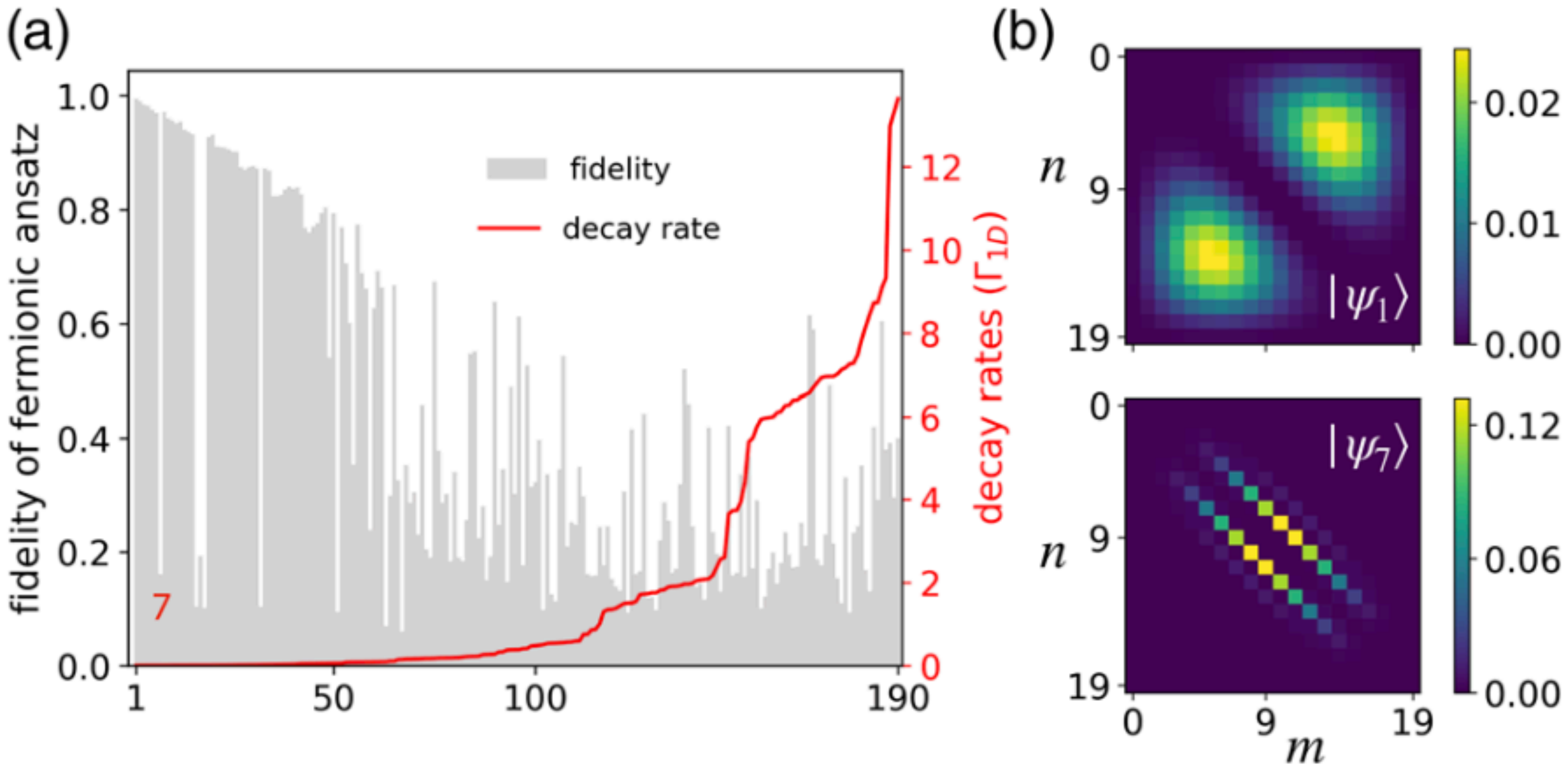
$$|\psi\rangle \propto S^\dagger |g\rangle^{\otimes N} = \sum_{i=1}^{N} c_i |e_i\rangle$$

Most subradiant eigenstate of H$_{\text{eff}}$ in the 2-ex sector is "fermonic". Ansatz:

$$|F_{1,2}\rangle \propto \textstyle\sum_{i<j}(c_{1,i}c_{2,j} - c_{1,j}c_{2,i})|e_i, e_j\rangle$$

$$= \sum_{i,j=1}^{N} c_{ij} |e_i e_j\rangle$$

2-excitation "zoology"



Zhang & Molmer, PRL 122, 203605 (2019)

AAG et al., PRX **7**, 031024 (2017)

Lecture 2

Atomic arrays in free space and waveguide QED

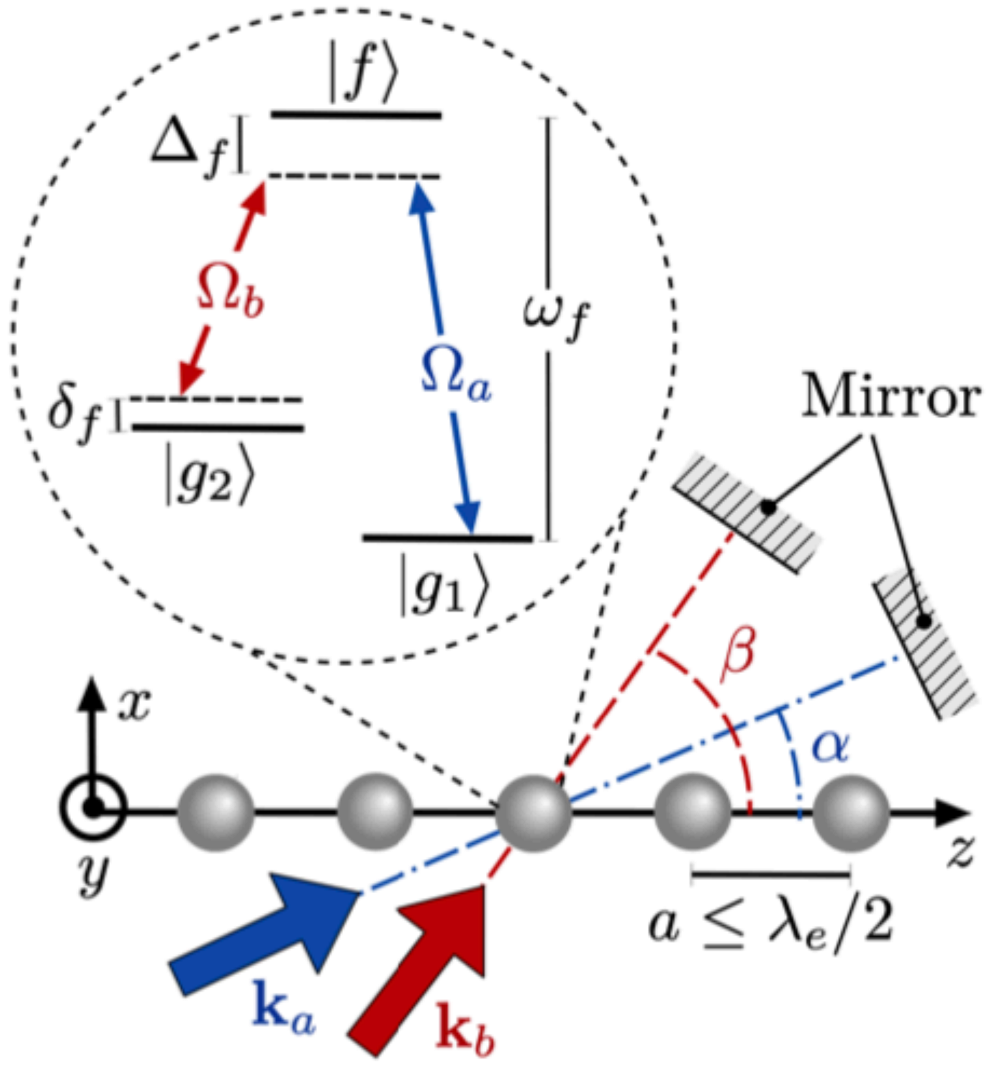
Few-excitation sector: super- and sub-radiance

Selective radiance and applications

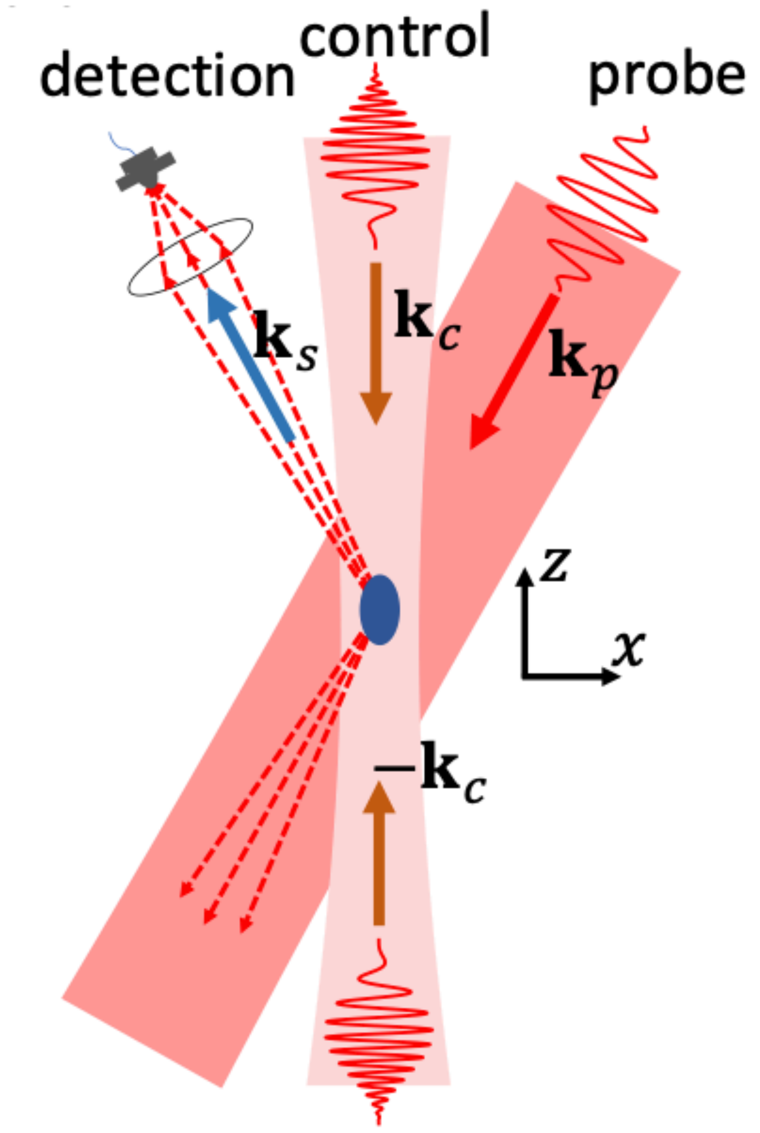
Problem: how to access dark states and exploit their properties?

States are dark, do not couple directly to photons in the bath they are dark to. Options:

Two drive fields to overcome momentum mismatch

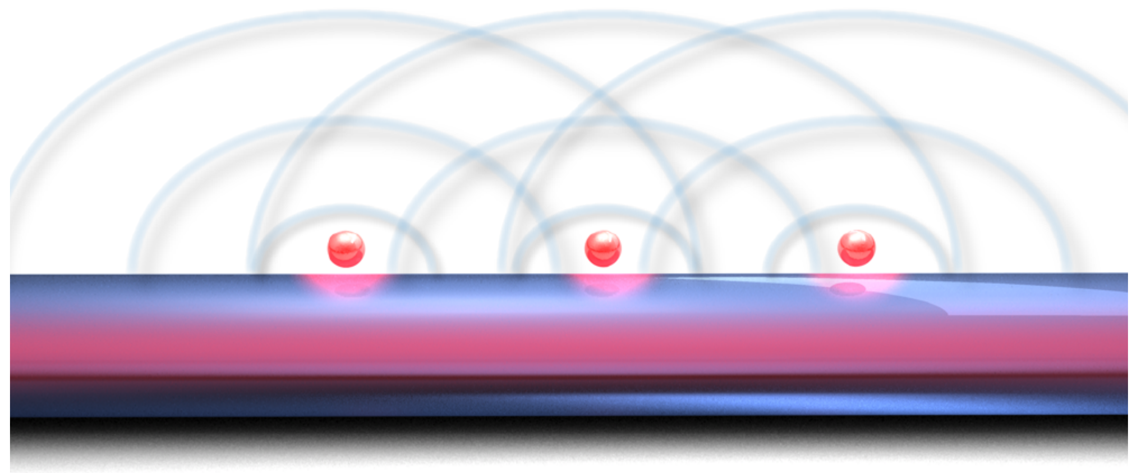


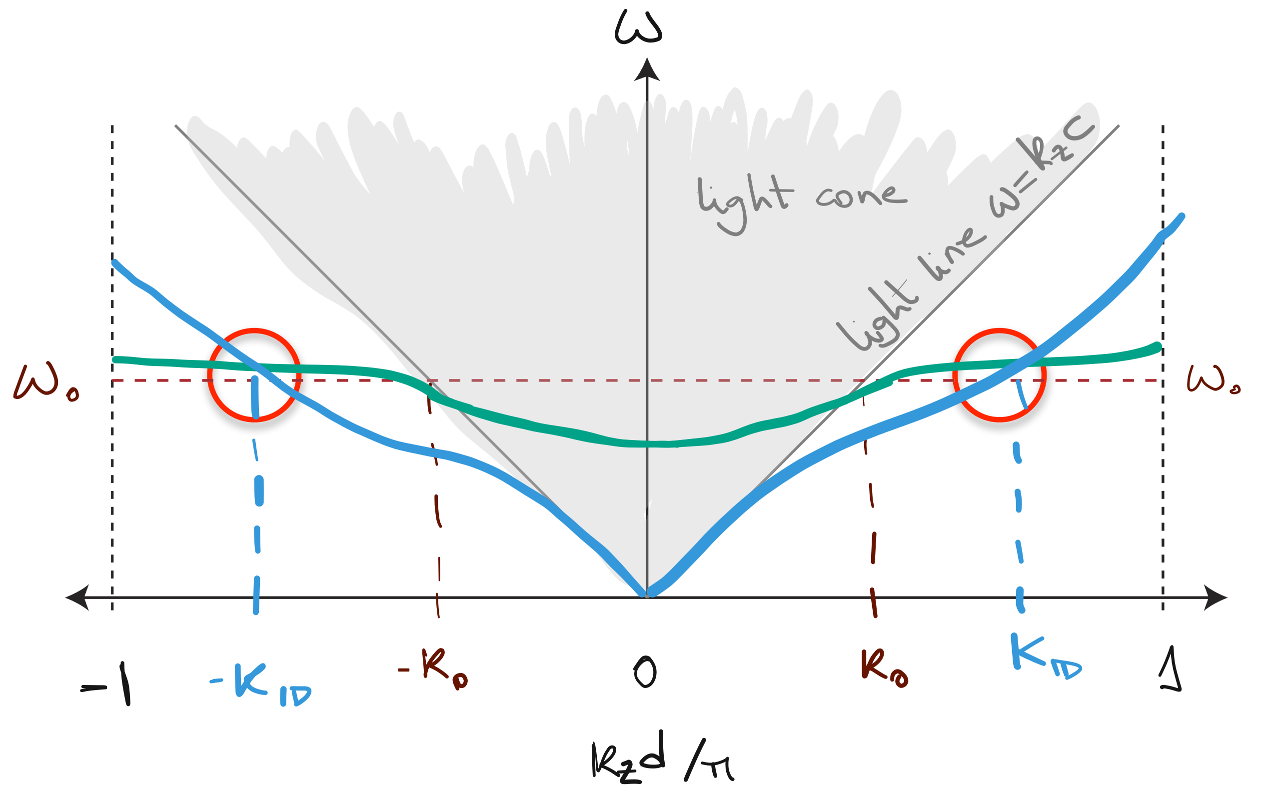
PRA 104, 033718 (2021)



PRL 125, 213602 (2020)

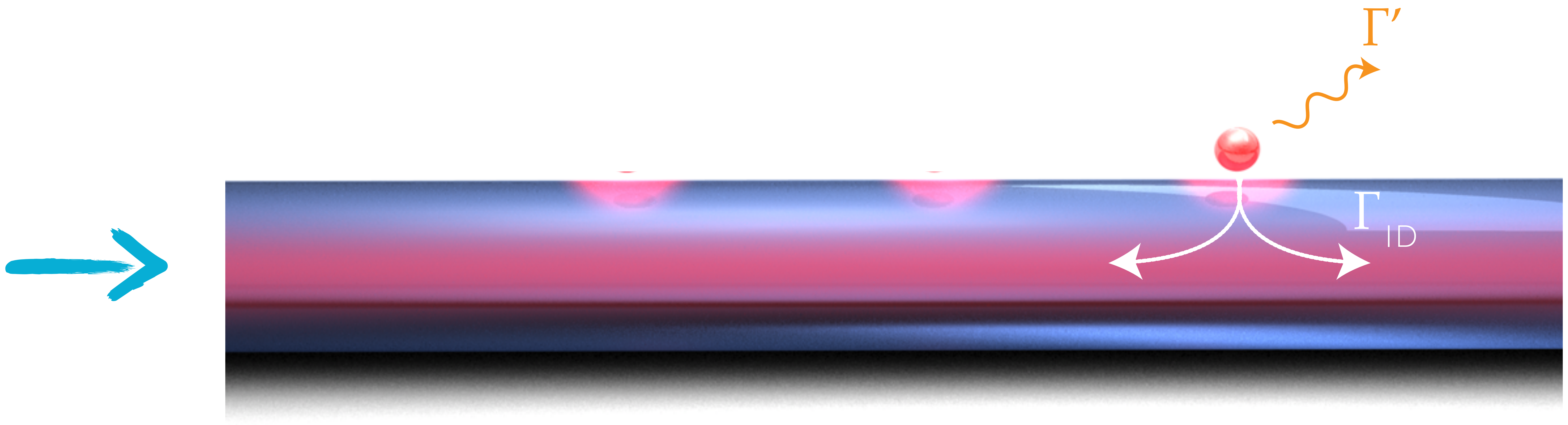
Add additional bath, "selective radiance"





Optical depth: figure of merit for performance of protocols

Recall Susanne Yelin's talk on the "perfect" mirror. Let's do a trivial example with a waveguide:



From input-output plus steady state equation for coherence operator (from H_{eff}) the reflectance on resonance is:

$$R_{\text{indep}} = 1 - 4/D \quad \text{with:} \quad D = 2\Gamma_{1D}/\Gamma'$$

Optical depth: figure of merit for performance of protocols

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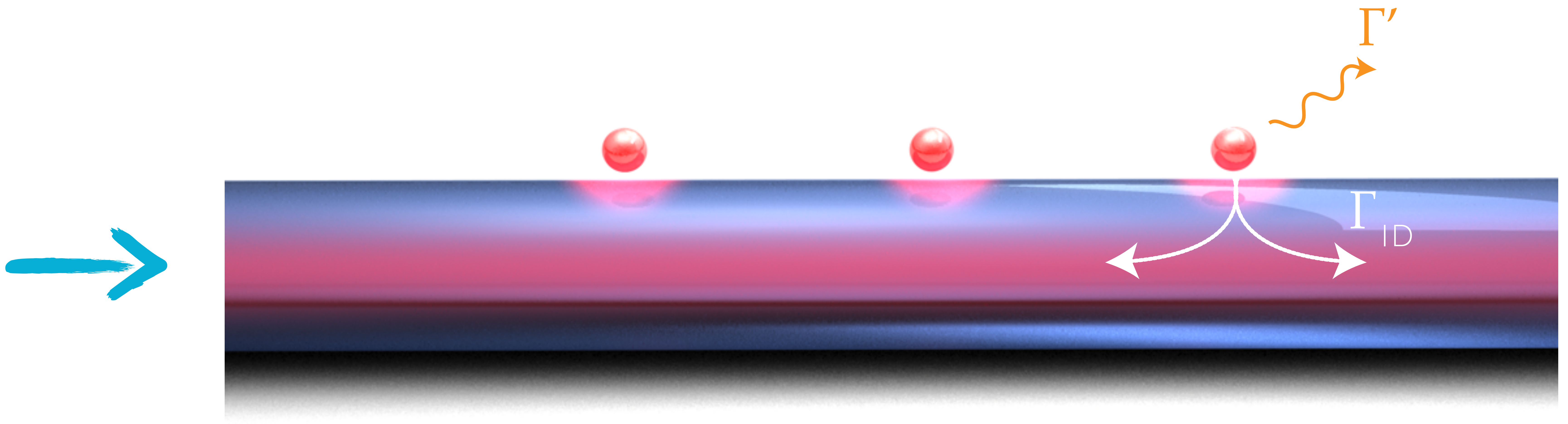


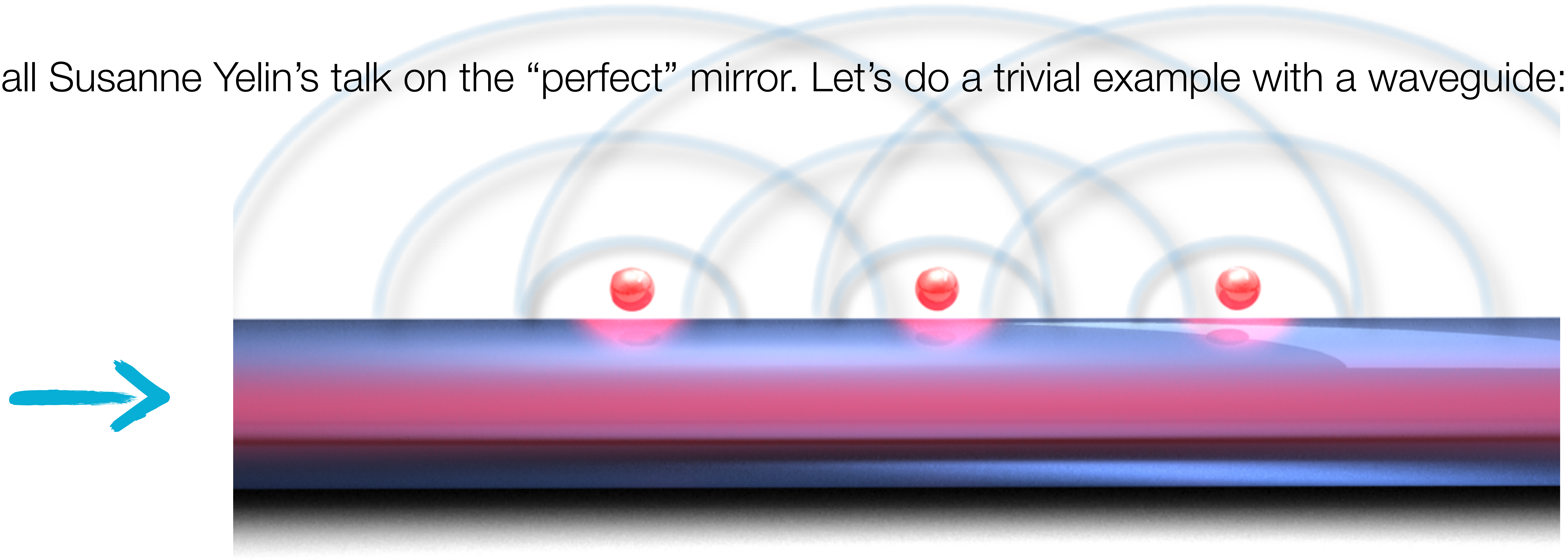
From input-output plus steady state equation for coherence operator (from H_{eff}) the reflectance on resonance is:

$$R_{\text{indep}} = 1 - 4/D \quad \text{with:} \quad D = 2N\Gamma_{1D}/\Gamma'$$

Fidelity of many quantum protocols (gates, memories, etc) determined by optical depth

Optical depth: figure of merit for performance of protocols

Recall Susanne Yelin's talk on the "perfect" mirror. Let's do a trivial example with a waveguide:

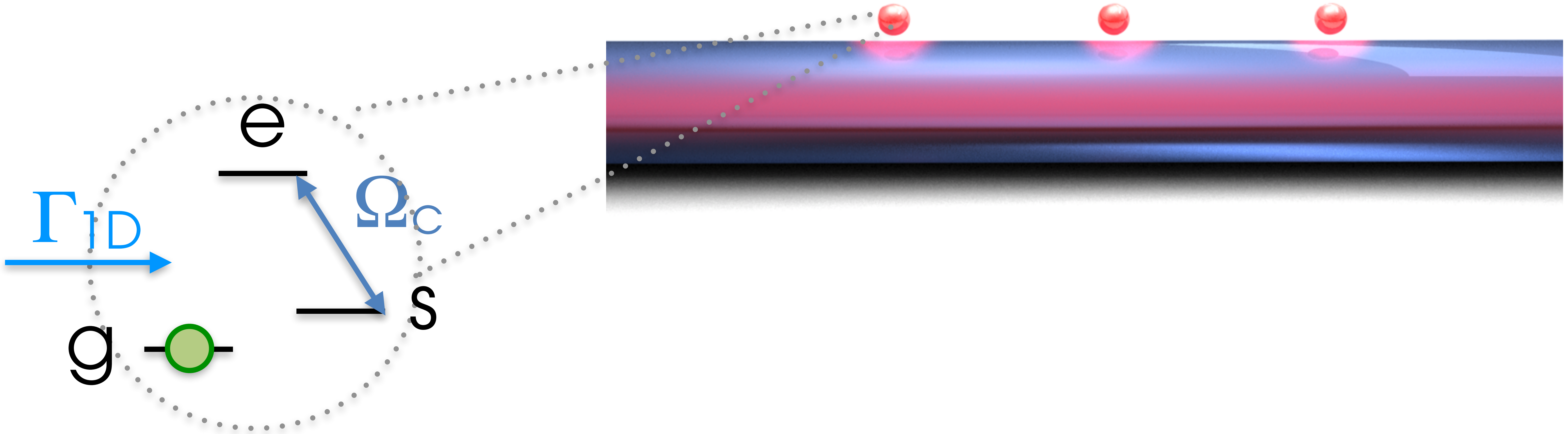


Key assumption: emission to free space is uncorrelated. But if we harness selective radiance:

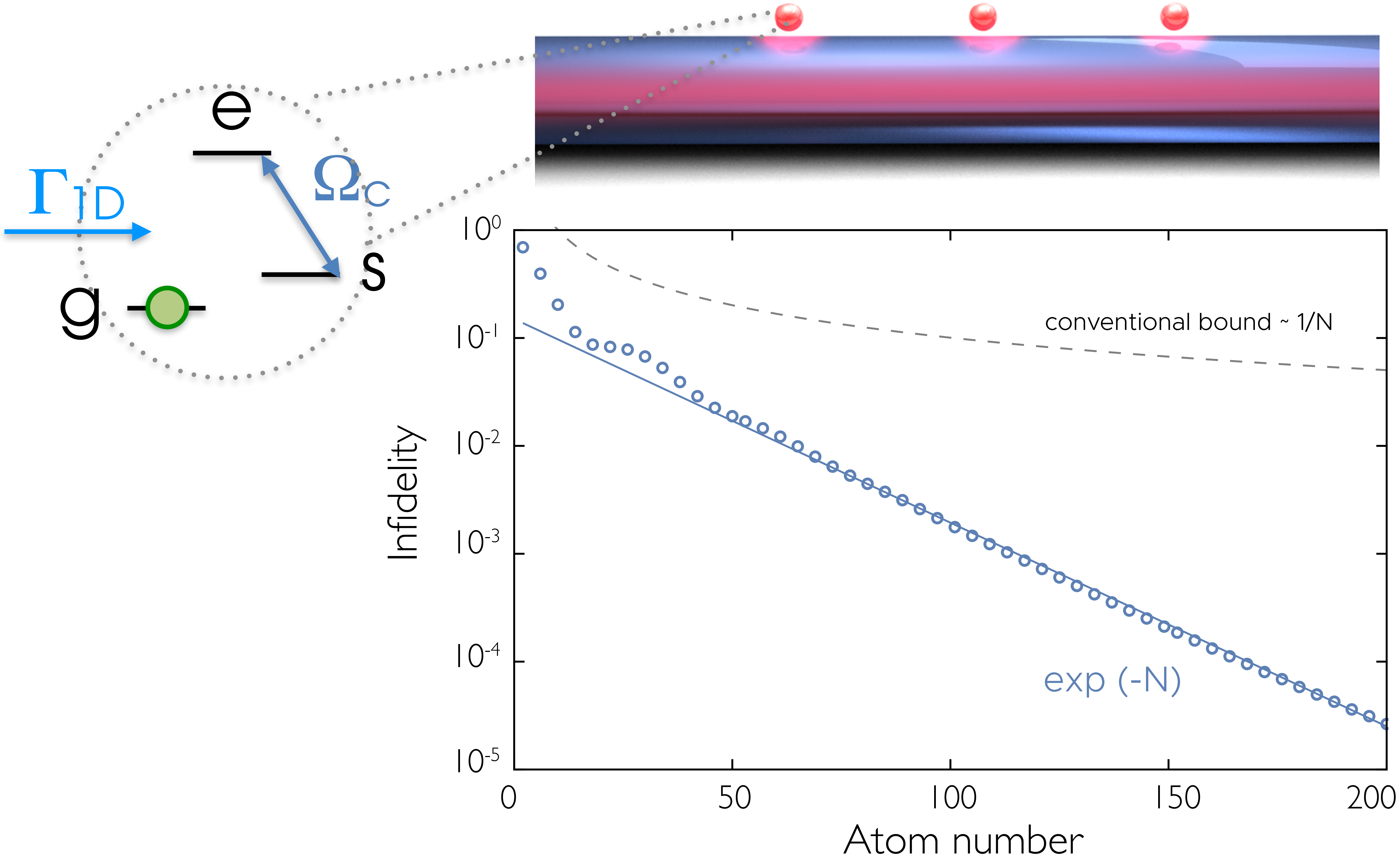
$$D = 2N\Gamma_{1\text{D}}/\Gamma'(N)$$

Fidelity of many quantum protocols (gates, memories, etc) can be improved beyond conventional bounds!

Example: exponential improvement of a quantum memory

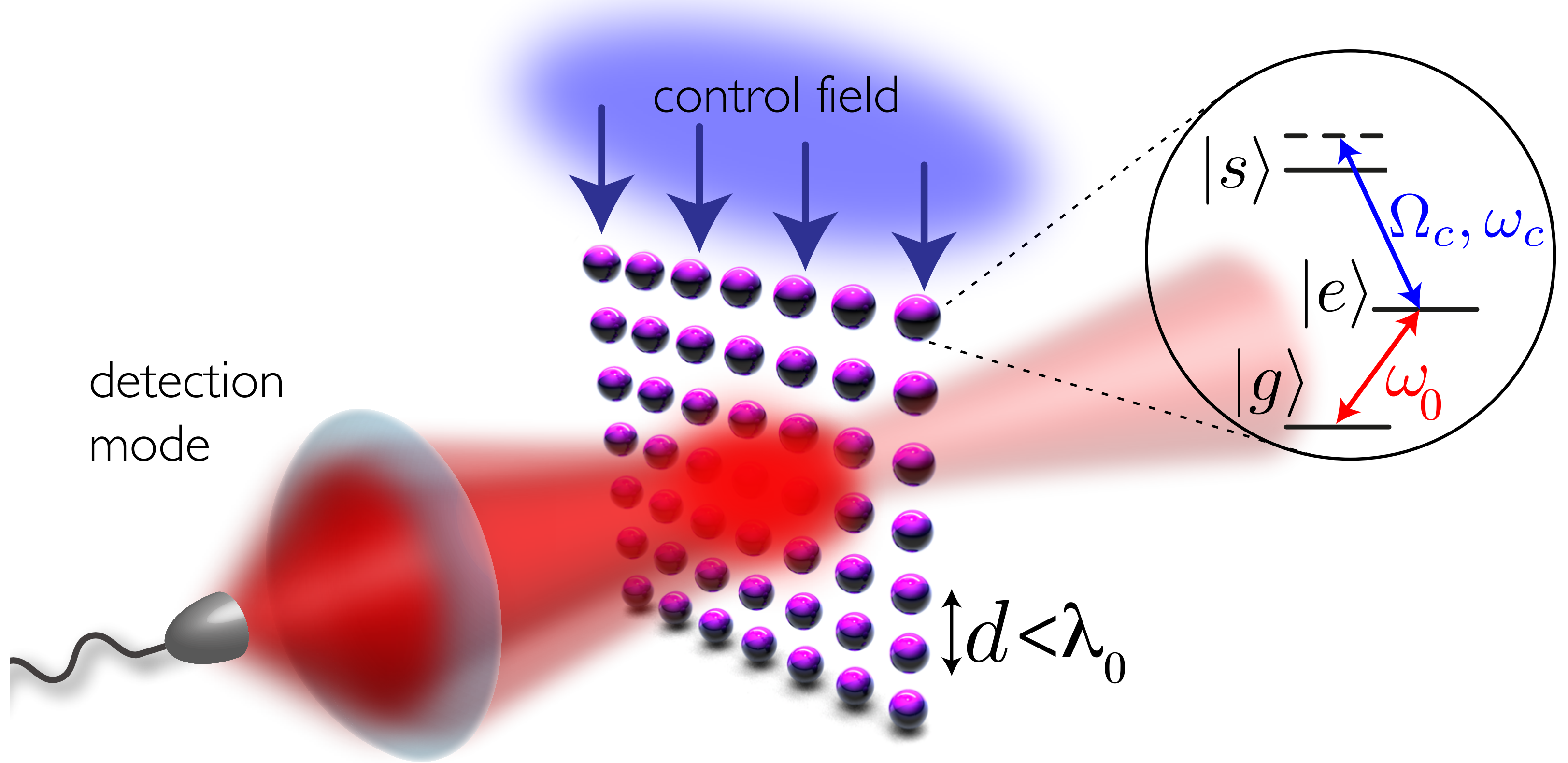


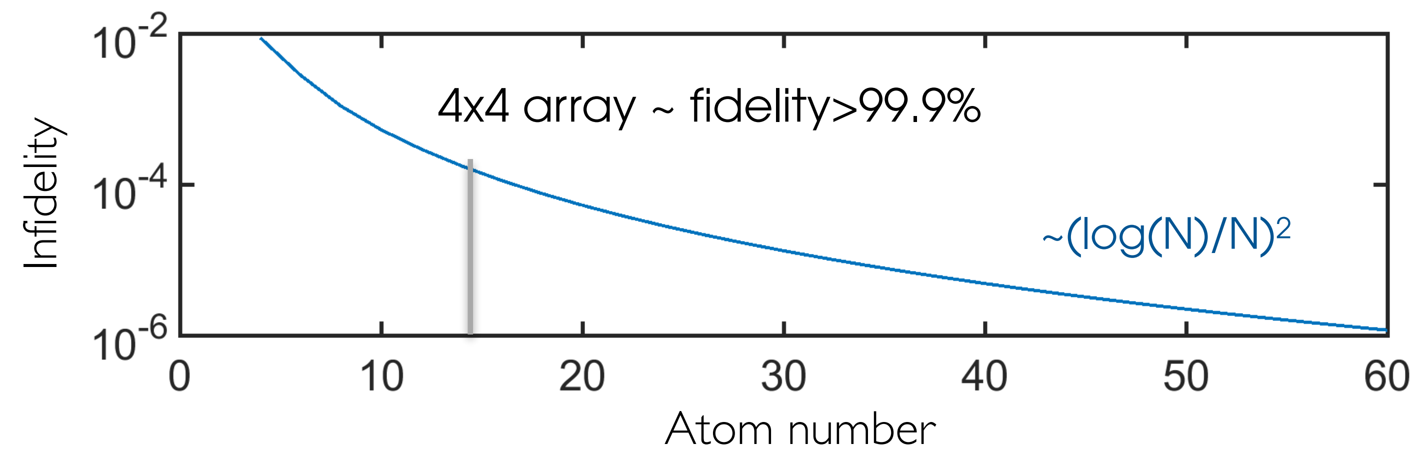
Example: exponential improvement of a quantum memory



Examples of improved light-matter interfaces and applications

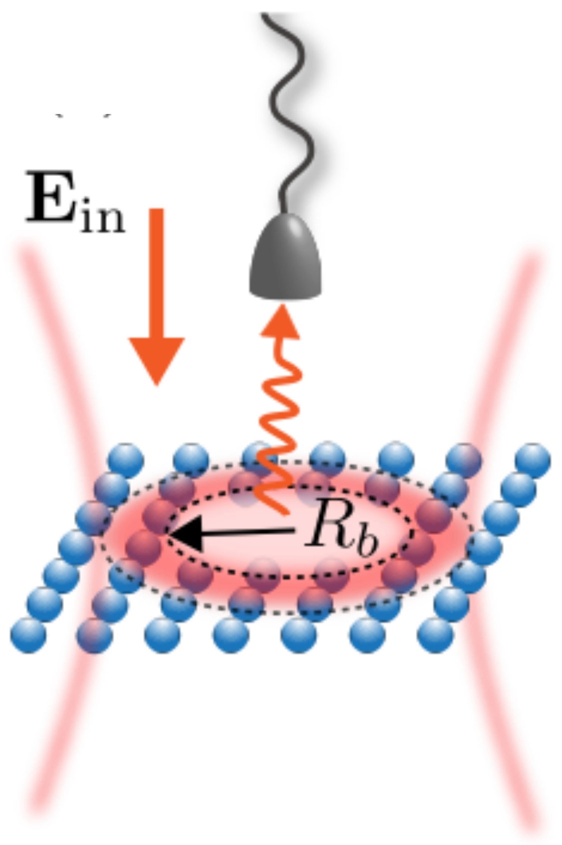
Free-space memories

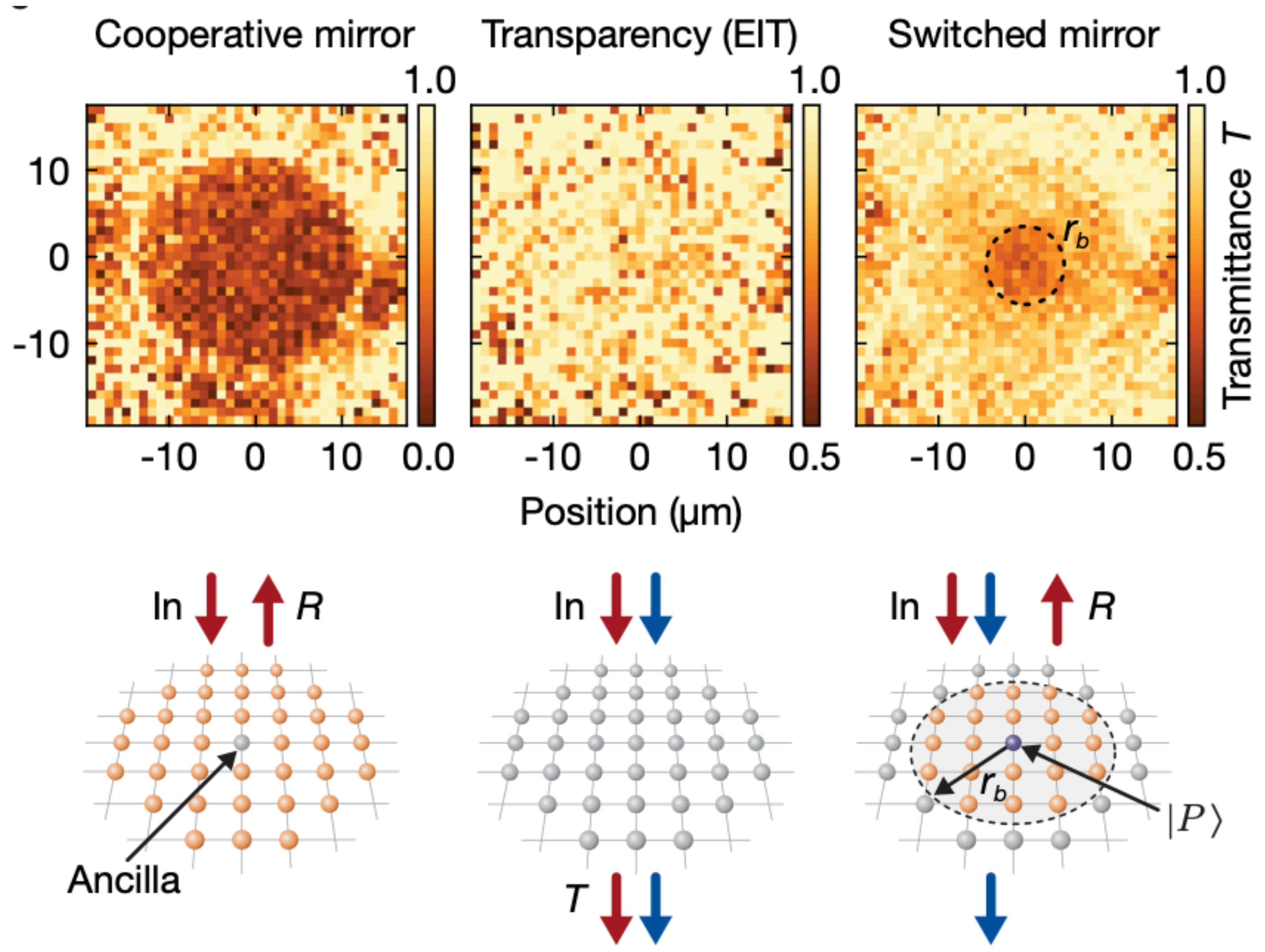




NJP 21, 025003 (2019)

Quantum non-linear optics: Photon-photon gates





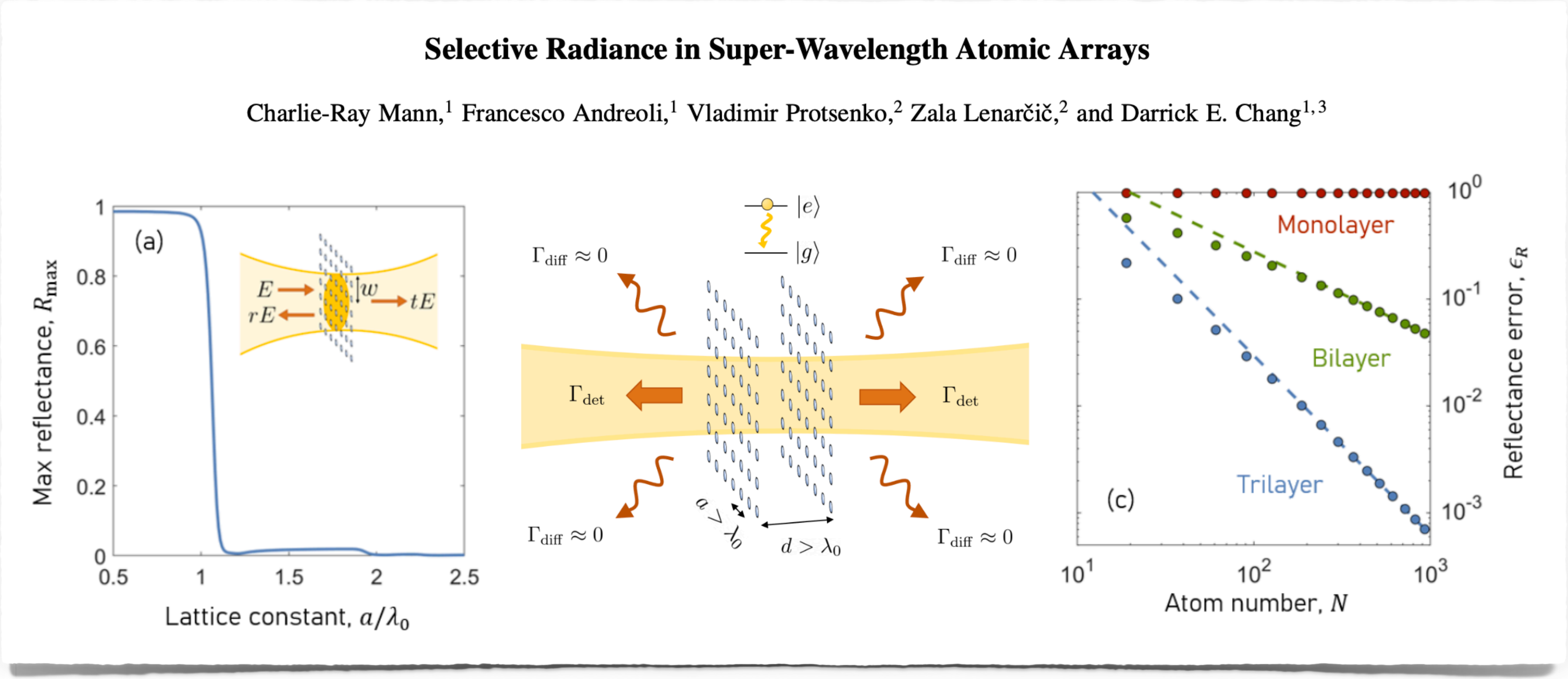
PRL **127**, 263602 (20 ,

Nat. Phys. 16, 676 (2020).

PRA 104, 033718 (2021)

Nat. Phys. 19, 714 (2023)

Overcoming subwavelength-lattice-constant requirement



Selective Radiance in Super-Wavelength Atomic Arrays

Charlie-Ray Mann,[1] Francesco Andreoli,[1] Vladimir Protsenko,[2] Zala Lenarčič,[2] and Darrick E. Chang[1,3]

Take-home messages and some observations from Lecture 2

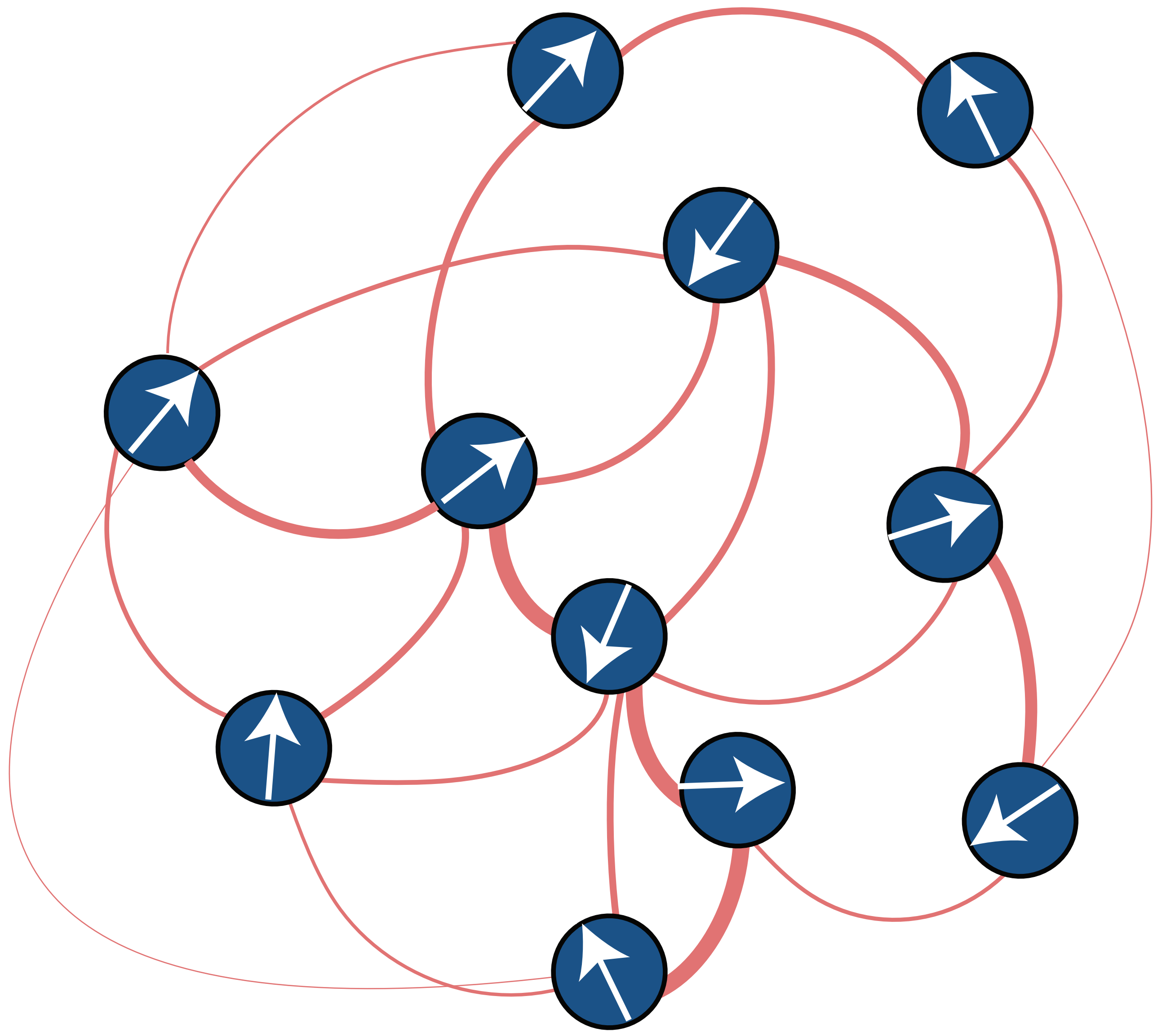
- Subradiant states are protected from decay
- Selective radiance gives rise to improved light-matter interfaces
- However:
    - dark states are not very robust
    - how to access them or harness selective radiance is not exactly trivial
    - how to harness multi-excitation dark states is quite unknown

Quantum optics with spins: from single-excitation to many-body physics

Lecture 3

Ana Asenjo-Garcia
Columbia University

Recap from lecture 1: light-matter interactions as a spin model

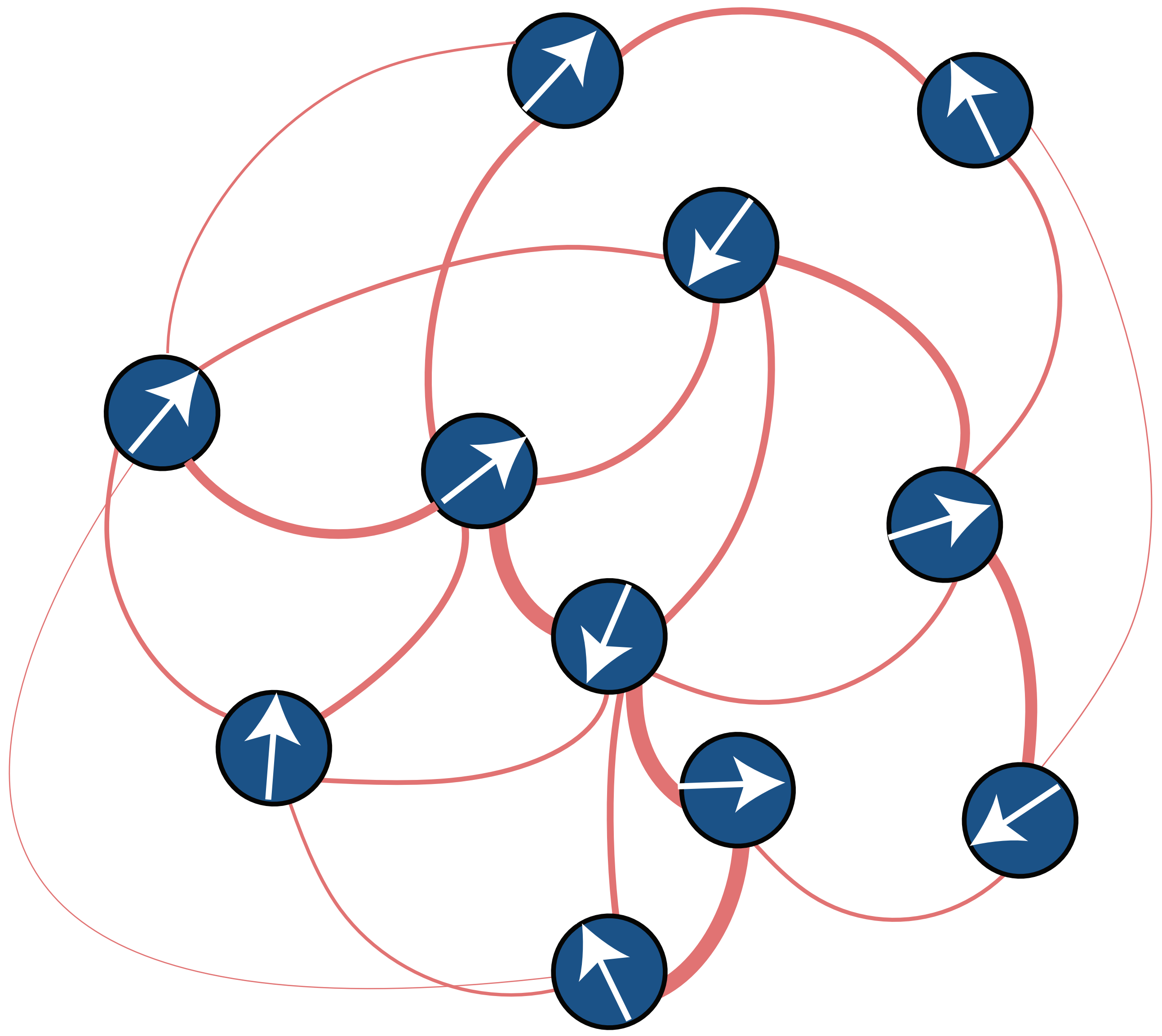


Described by a spin model with
dynamics governed
by a Lindblad master equation

$$\dot{\rho} = -\frac{i}{\hbar}\left[\underbrace{\sum_{i,j=1}^{N} J_{ij}\sigma_+^i\sigma_-^j}_{}, \rho\right] + \sum_{i,j=1}^{N}\frac{\Gamma_{ij}}{2}\left(2\sigma_-^i\rho\sigma_+^j - \rho\sigma_+^j\sigma_-^i - \sigma_+^j\sigma_-^i\rho\right)$$

conserves number of excitations

Recap from lecture 1: light-matter interactions as a spin model



Described by a spin model with dynamics governed by a Lindblad master equation

Eigenvalues
(collective transition rates) $\Gamma_\mu \geq 0$

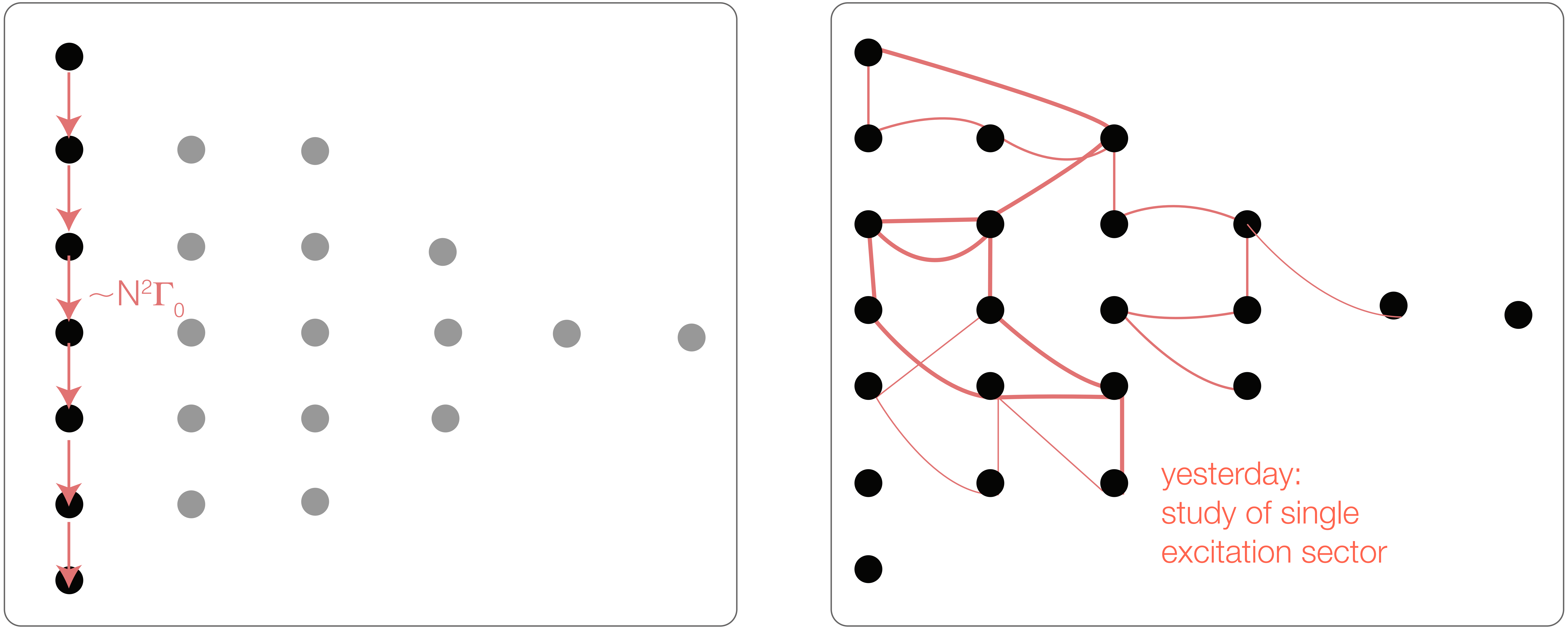
$$\dot{\rho} = -\frac{i}{\hbar}\left[\sum_{i,j=1}^{N} J_{ij}\sigma_+^i\sigma_-^j, \rho\right] + \sum_{\mu=1}^{N}\frac{\Gamma_\mu}{2}\left(2\hat{c}_\mu\hat{\rho}\hat{c}_\mu^\dagger - \hat{\rho}\hat{c}_\mu^\dagger\hat{c}_\mu - \hat{c}_\mu^\dagger\hat{c}_\mu\hat{\rho}\right)$$

conserves number of excitations

collective jump operator $\sim \sum_{i=1}^{N}\alpha_i\,\hat{\sigma}_-^i$

Recap from lecture 2: evolution in Hilbert space

schematics of possible trajectories in Hilbert space

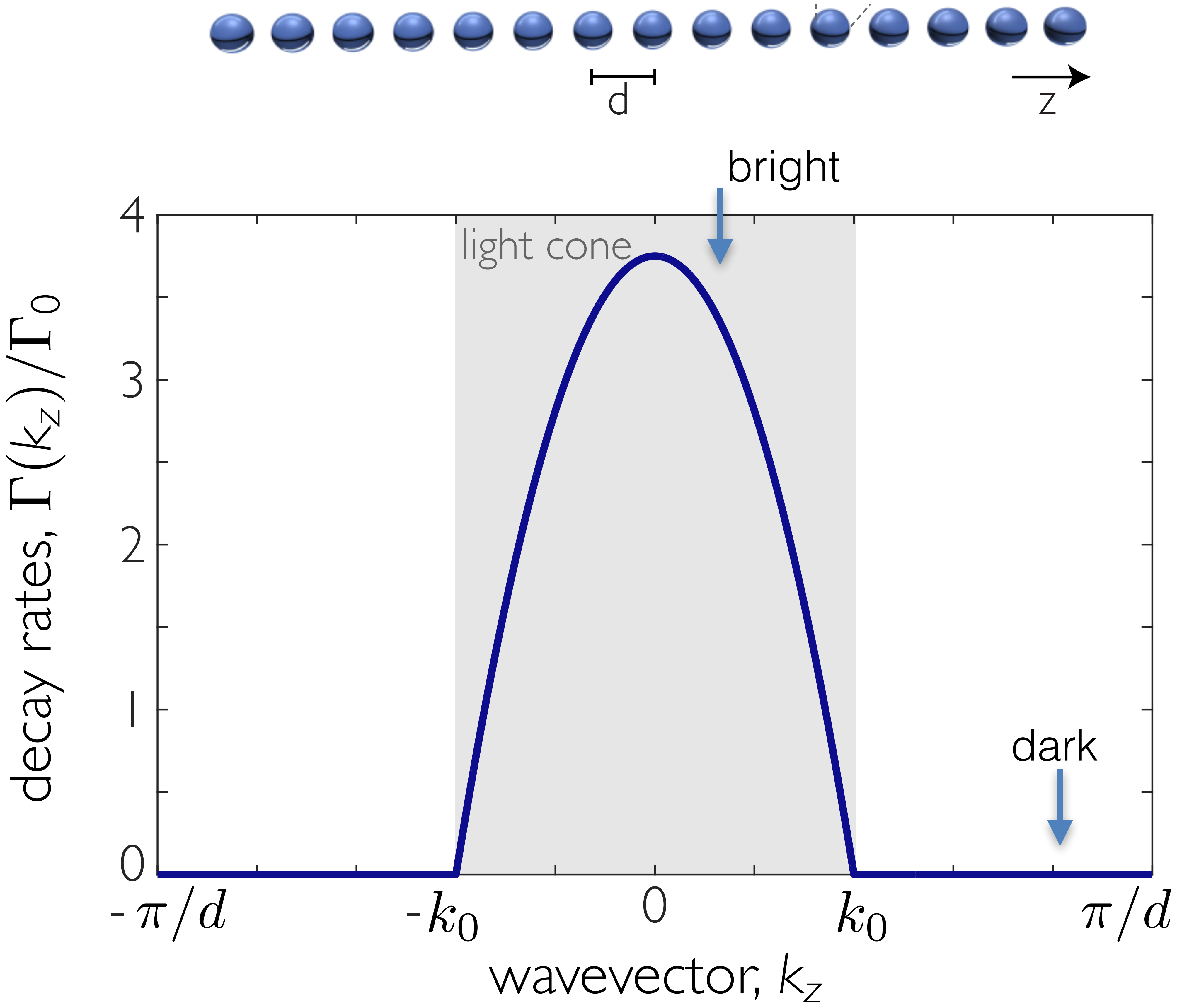


yesterday:
study of single
excitation sector

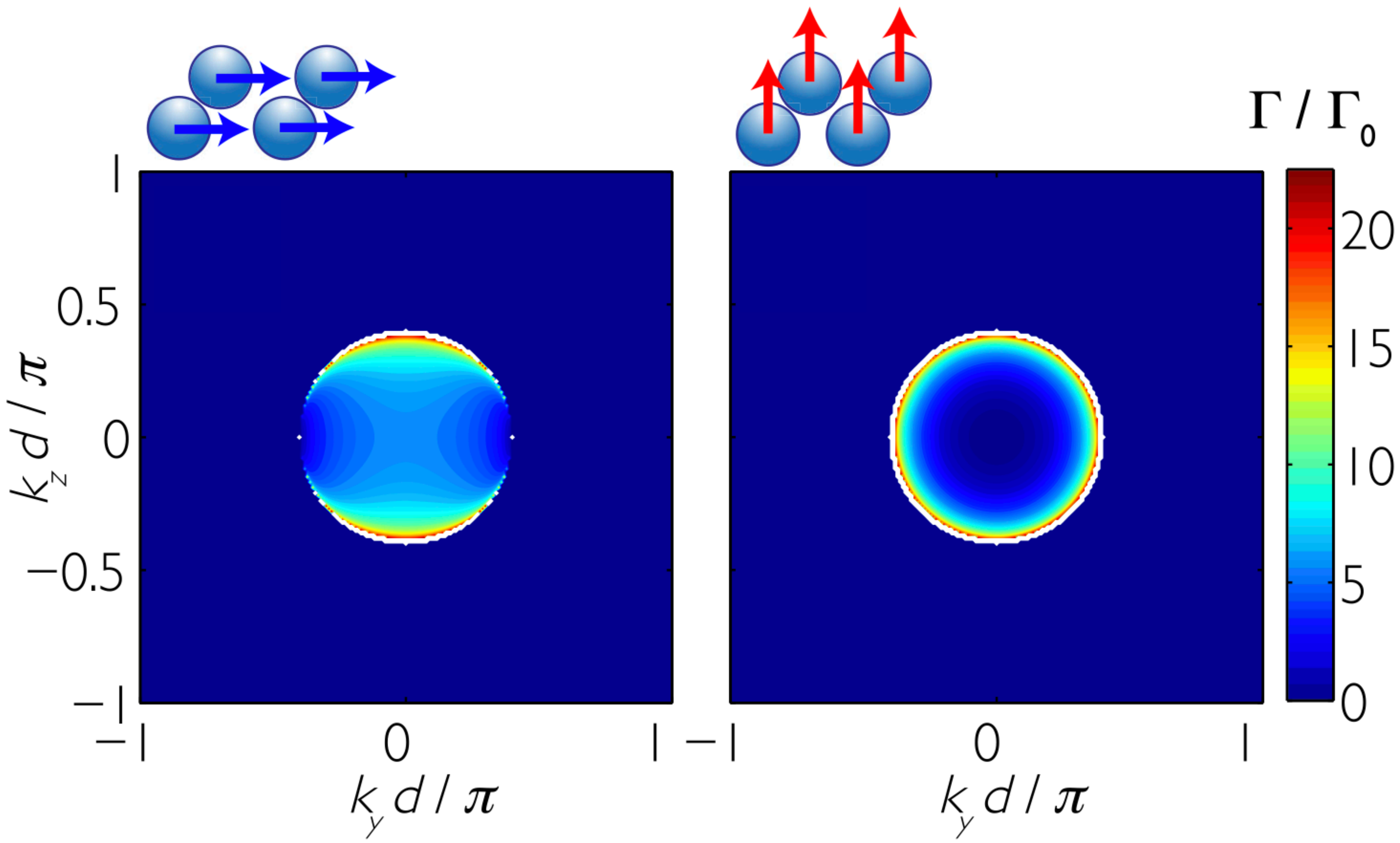
with permutational symmetry

in extended systems

Recap from lecture 2: Emergence of bright and dark states



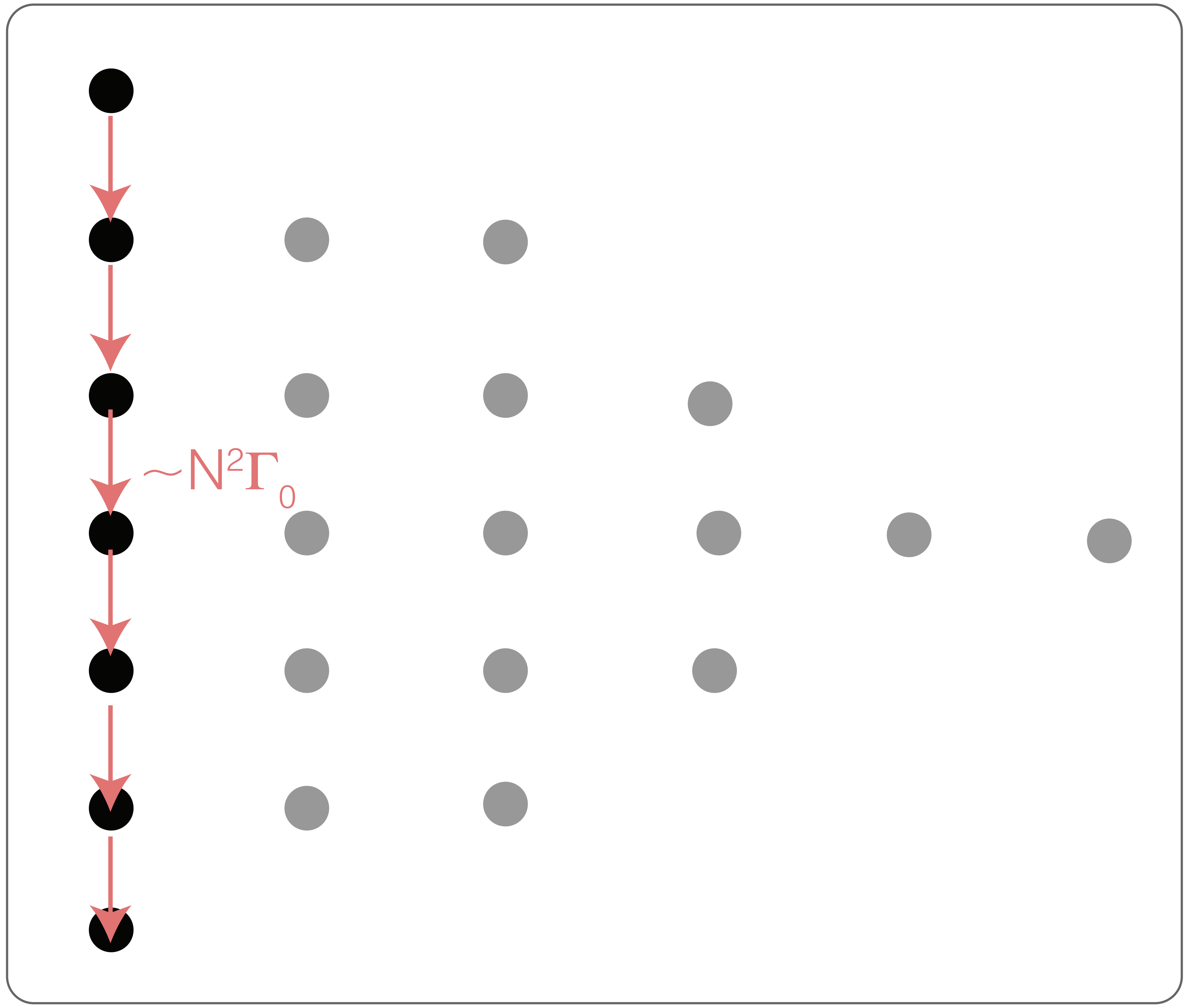
Brillouin zone of 2D arrays,
showing divergences at the light line



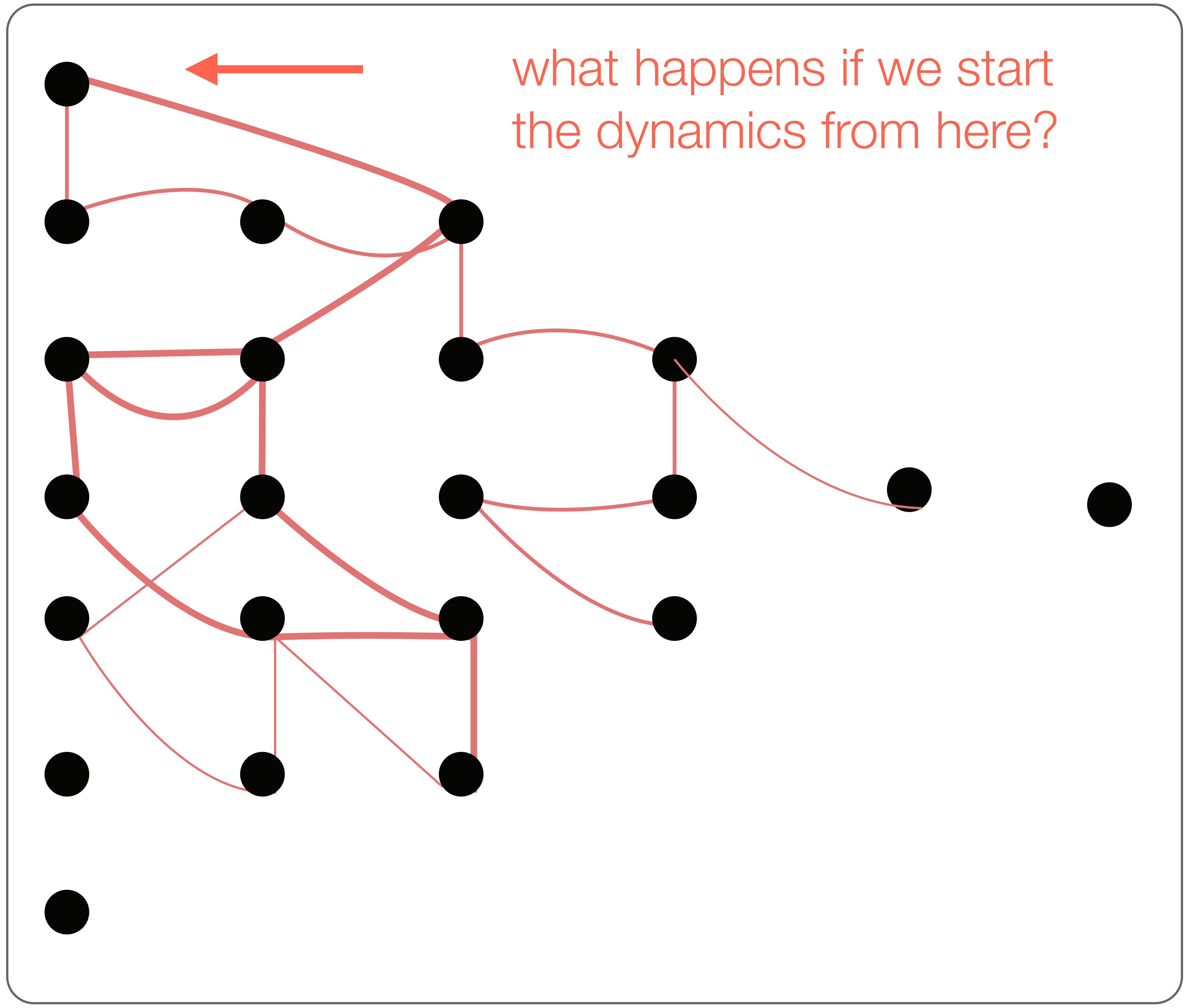
AAG et al., PRX **7**, 031024 (2017)

Alternatively: explore “few hole” sectors

schematics of possible trajectories in Hilbert space



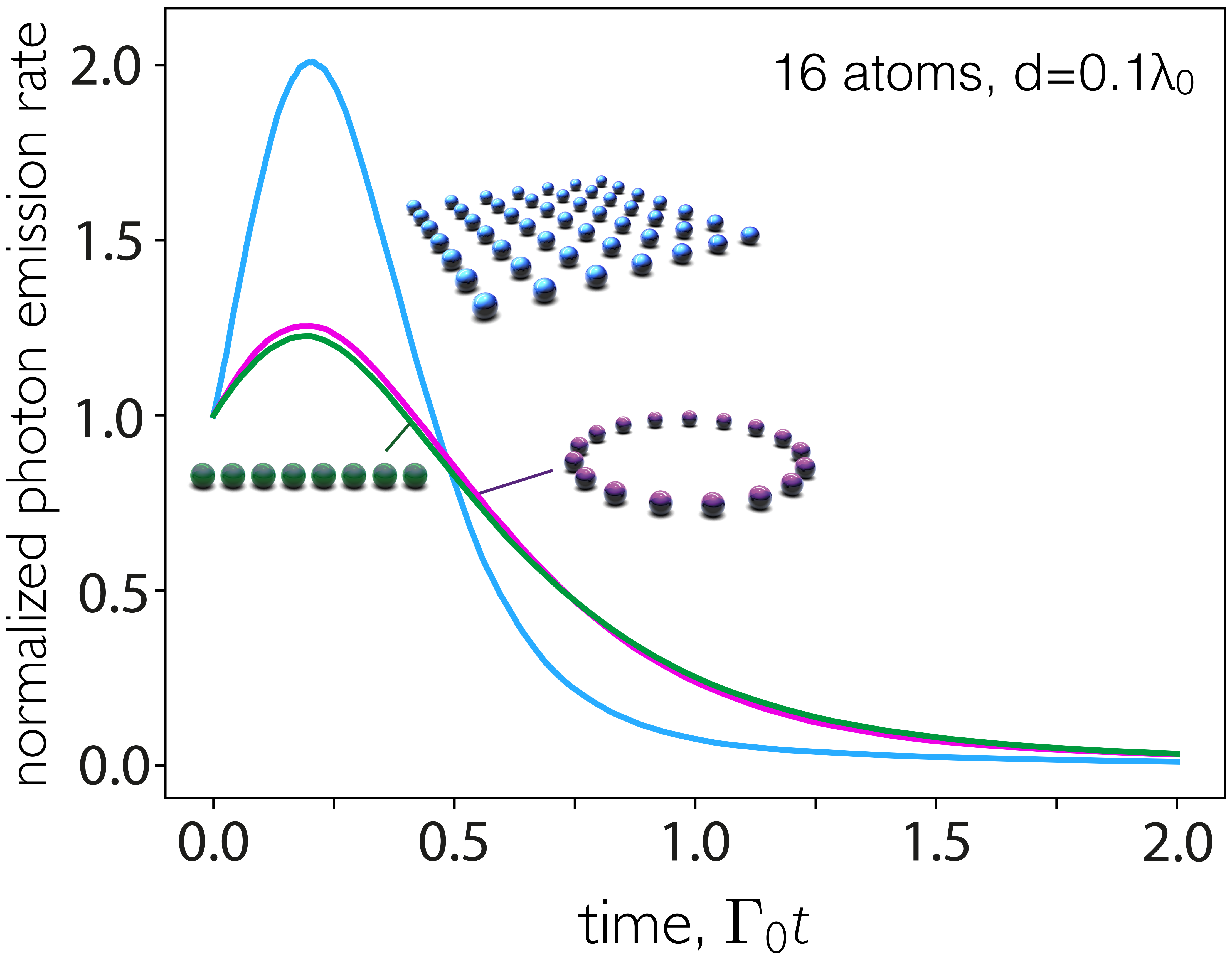
with permutational symmetry



what happens if we start the dynamics from here?

in extended systems

Many-body superradiance in extended systems (L>>λ_0)



Masson & AAG, Nat. Comm. 13, 2285 (2022)

- Coherence develops by dissipation (akin to transient synchronization)
- Computing exact dynamics only possible for few spins
- Studying what happens at short times we find a minimal condition for a burst:

$$\text{Variance}\left(\frac{\{\Gamma_\nu\}}{\Gamma_0}\right) > 1$$

notion of magnetic avalanches in molecular nano magnets (Sarachik, Chudnovsky & Garanin)

Understanding the dynamics of open many-body systems remains a theoretical challenge

+ many-body

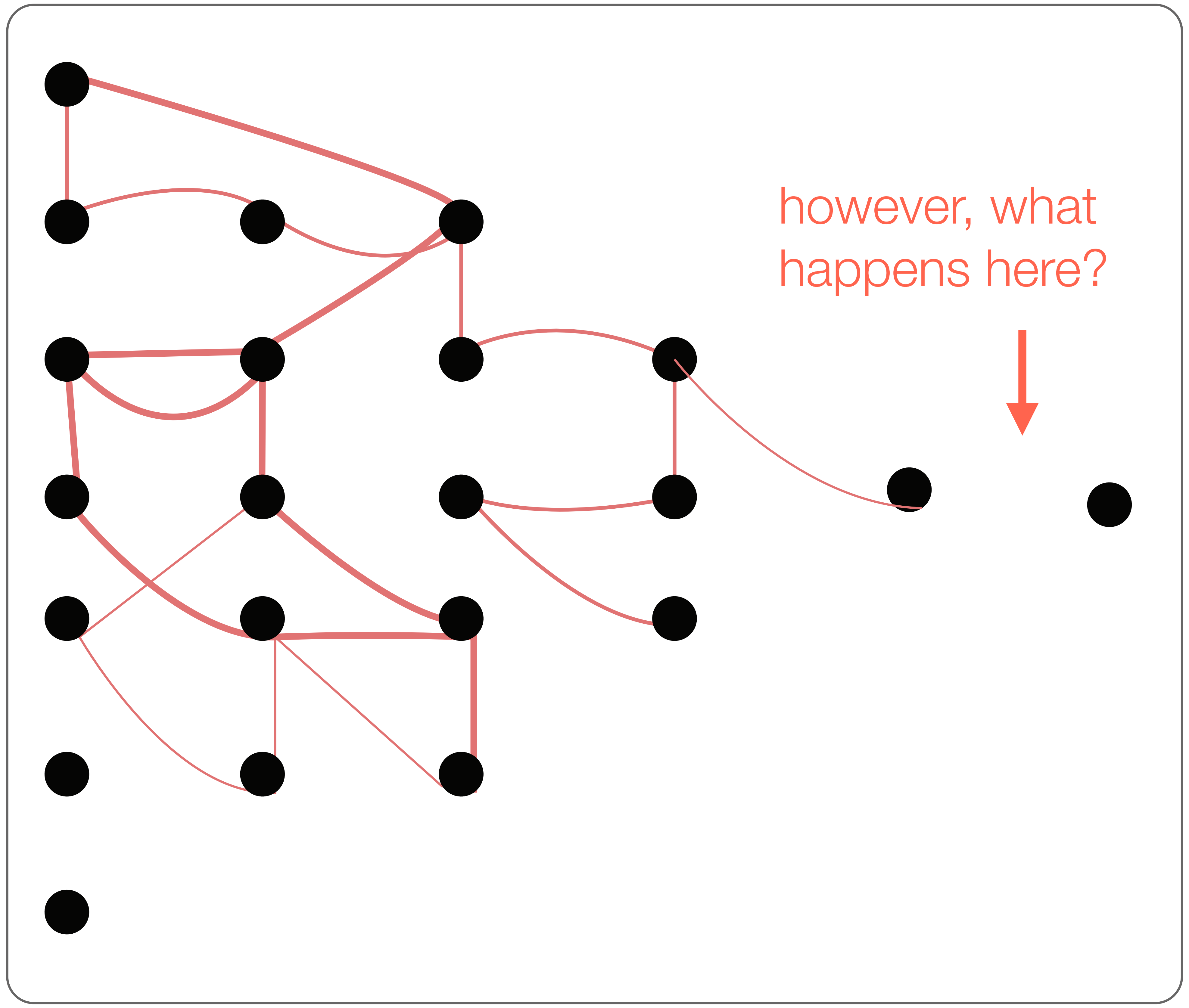
+ long-range interactions

+ open, out of equilibrium

+ dynamics

~ hard

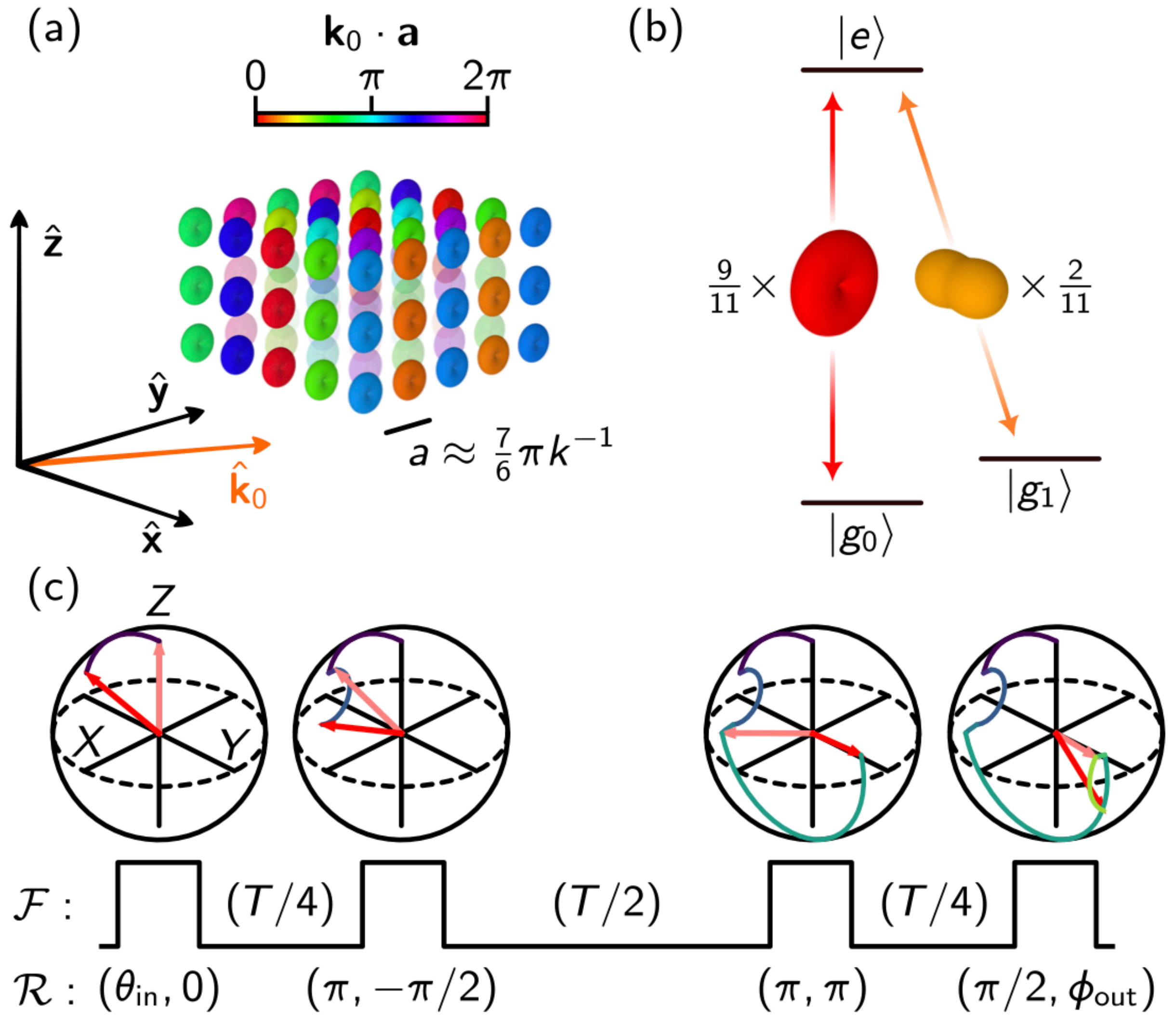
Approximation methods have been developed (cumulants, phase space methods, etc) but are of limited applicability



in extended systems

All quantum systems are open. We need to understand correlated decay

- reduced coherence times for metrology, quantum simulation…



Hutson et al., Science 383, 384 (2024)

- correlated errors for QC

so many types of qubits, baths, applications!

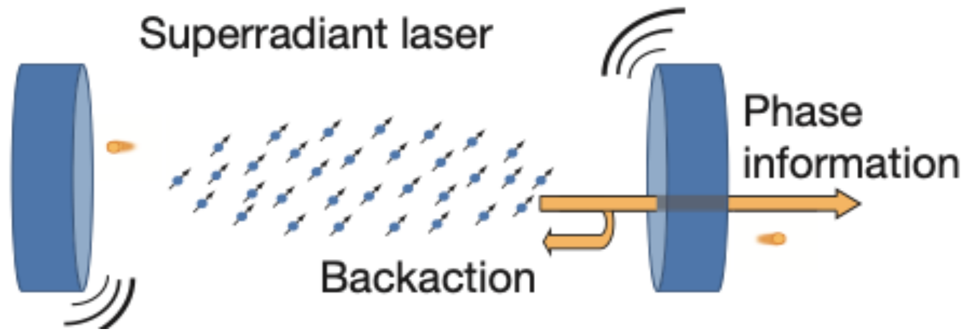
- New light sources (particularly useful for metrology)

LETTER

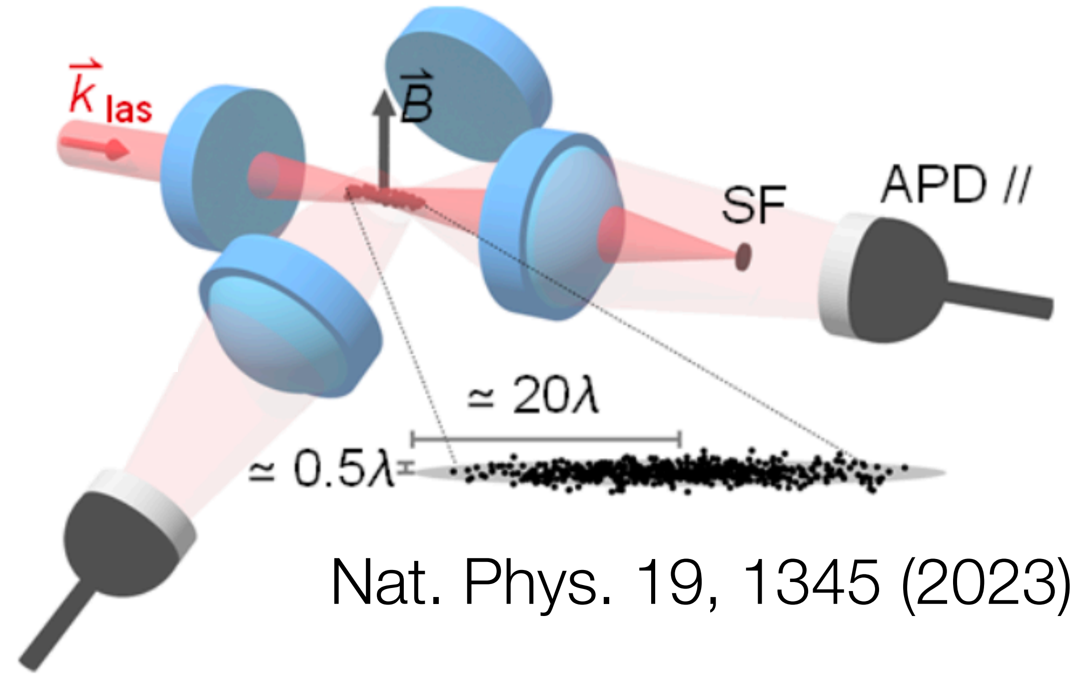
doi:10.1038/nature10920

A steady-state superradiant laser with less than one intracavity photon

Justin G. Bohnet[1], Zilong Chen[1], Joshua M. Weiner[1], Dominic Meiser[1]†, Murray J. Holland[1] & James K. Thompson[1]



- Out-of-equilibrium (driven-dissipative) phases



Nat. Phys. 19, 1345 (2023)

properties of phases and thresholds depend on N

Today's lecture:
what is the maximal decay rate of a quantum system?

Goal: to find scaling laws for correlated decay with system size (N=number of particles)

Hypothesis: the environment and arrangement of particles will affect the scaling

Result: simple, universal scaling laws that can be obtained for a broad class of (Markovian) systems… "easily"

D. Mok, A. Poddar, E. Sierra, C. Rusconi, J. Preskill, AAG, arXiv:2406.00722 (2024)

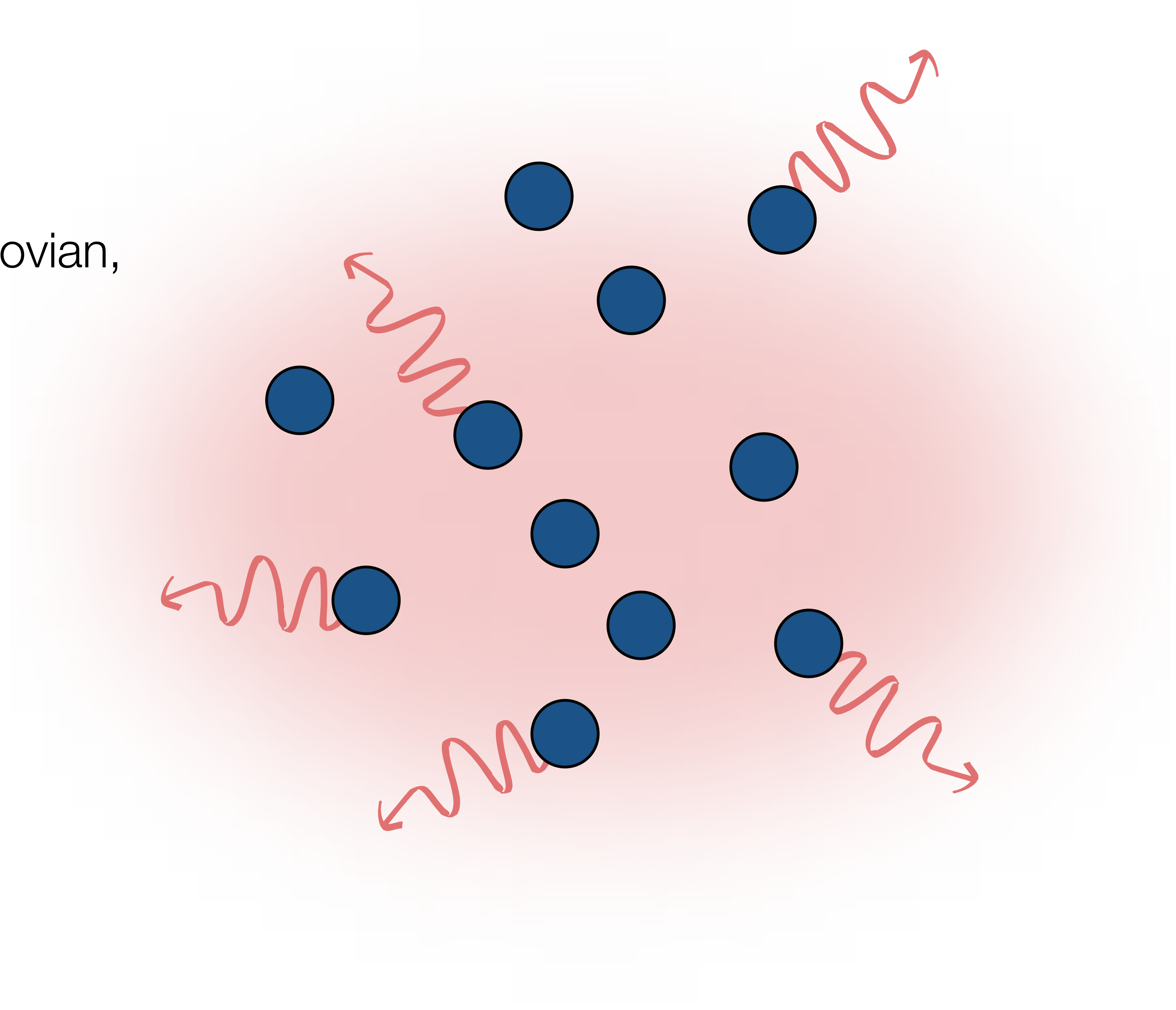
Generic systems: mapping to a ground-state problem

Scaling laws for atom arrays in free space

“Bonus”: other results, interesting problems

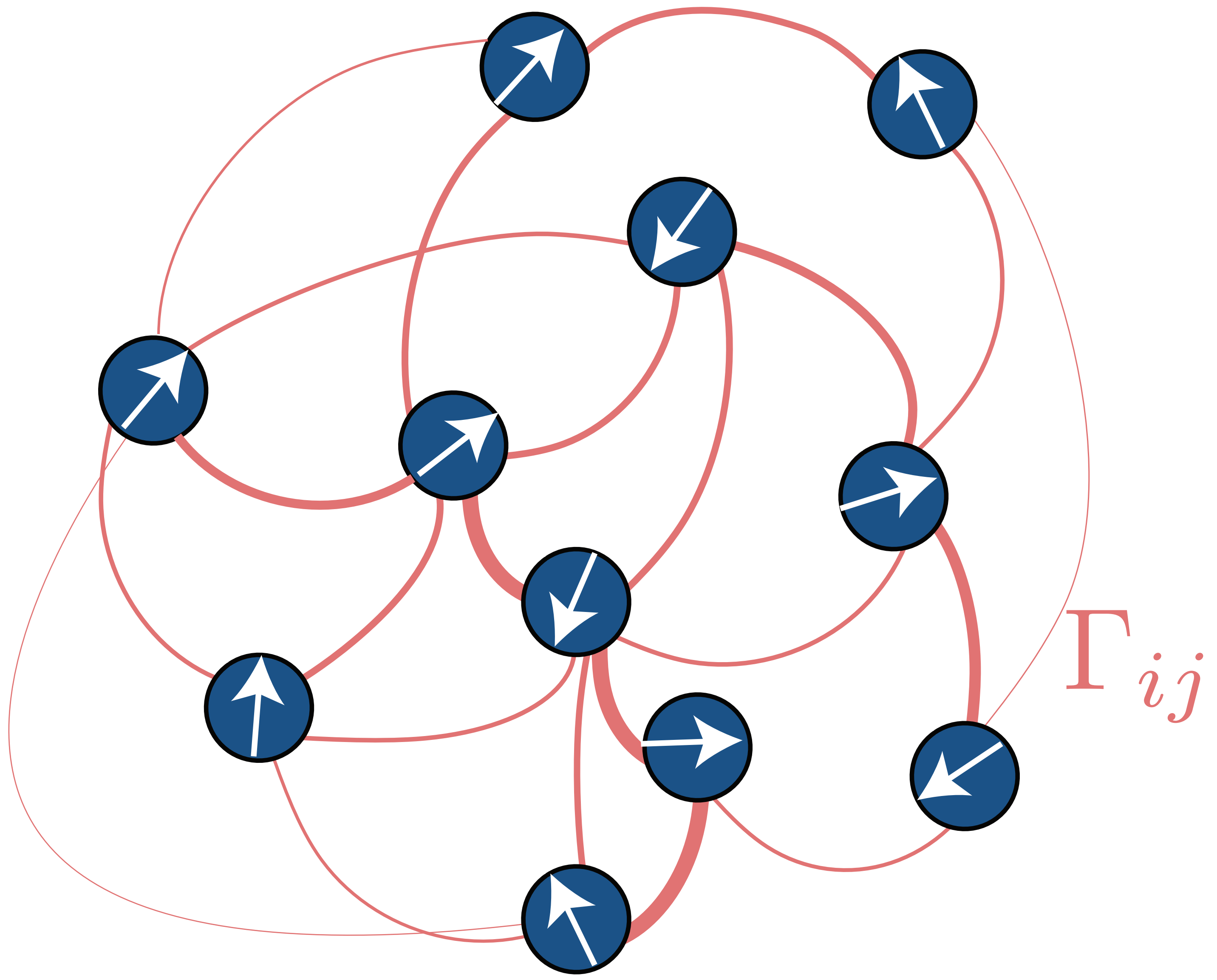
What is the maximum decay rate of a quantum system?

Let’s consider a generic, Markovian, many-body dissipative system



What is the maximum decay rate of a quantum system?

Let’s consider a generic, Markovian, many-body dissipative system



Described by a
spin model with dynamics governed by a Lindblad master equation

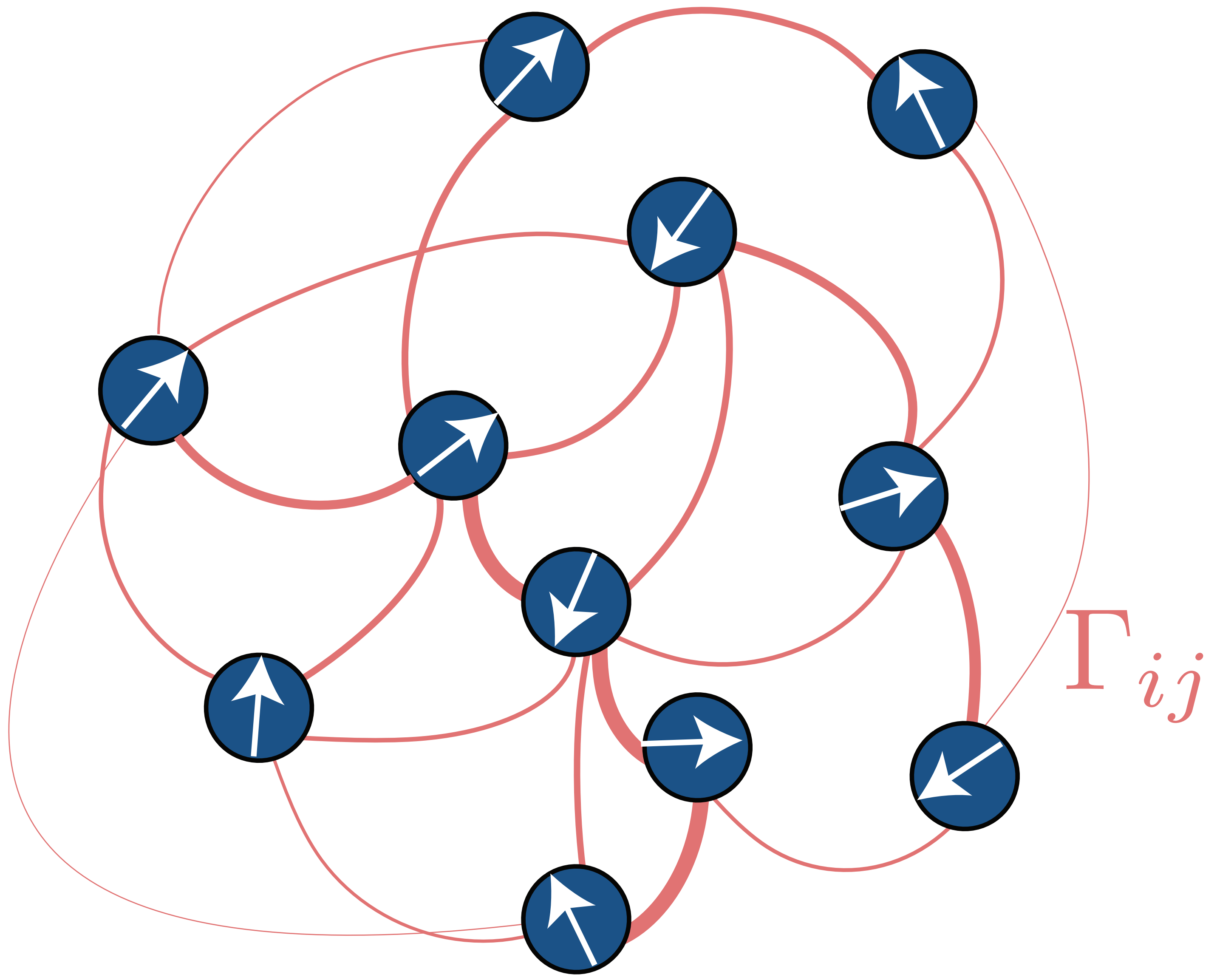
NxN matrix $\mathbf{\Gamma}$ is positive semidefinite to ensure evolution as a CPTP map

$$\dot{\hat{\rho}} = -\frac{\mathrm{i}}{\hbar}[\mathcal{H}, \hat{\rho}] + \sum_{i,j=1}^{N} \frac{\Gamma_{ij}}{2} \left(2\hat{\sigma}_-^i \hat{\rho} \hat{\sigma}_+^j - \hat{\rho}\hat{\sigma}_+^j \hat{\sigma}_-^i - \hat{\sigma}_+^j \hat{\sigma}_-^i \hat{\rho}\right)$$

conserves number of excitations

What is the maximum decay rate of a quantum system?

Let’s consider a generic, Markovian, many-body dissipative system



Described by a spin model with dynamics governed by a Lindblad master equation

Eigenvalues
(collective transition rates) $\Gamma_\mu \geq 0$ (largest one is Γ_{max})

$$\dot{\hat{\rho}} = -\frac{\mathrm{i}}{\hbar}[\mathcal{H}, \hat{\rho}] + \sum_{\mu=1}^{N} \frac{\Gamma_\mu}{2} \left(2\hat{c}_\mu \hat{\rho} \hat{c}_\mu^\dagger - \hat{\rho} \hat{c}_\mu^\dagger \hat{c}_\mu - \hat{c}_\mu^\dagger \hat{c}_\mu \hat{\rho}\right)$$

conserves number of excitations

What is the maximum decay rate of a quantum system?

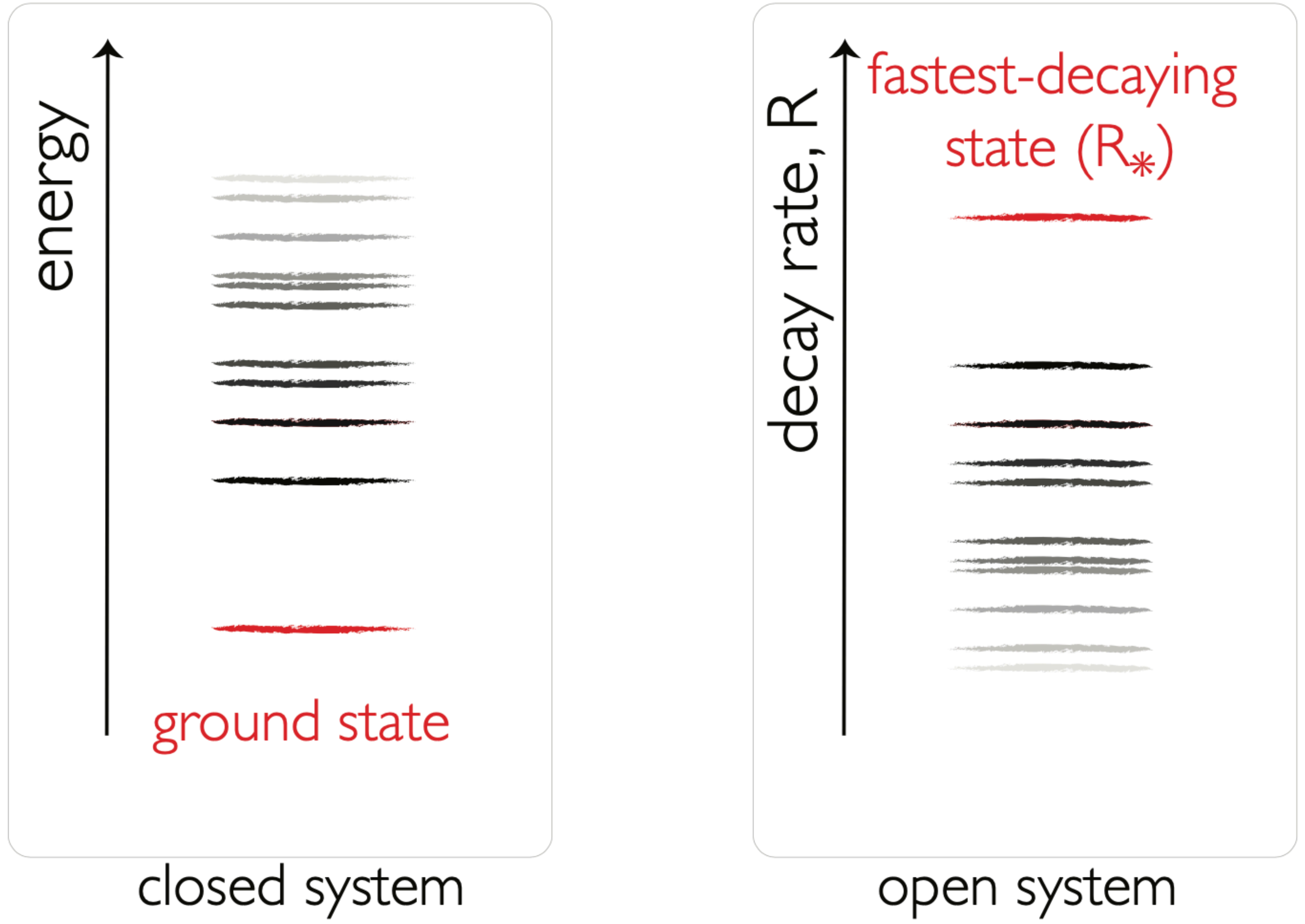
Instantaneous decay rate of any state is

“auxiliary/dissipative” Hamiltonian

$$R = -\frac{d}{dt}\left\langle \sum_{i=1}^{N} \hat{\sigma}_{ee}^{i} \right\rangle = \left\langle \sum_{i,j=1}^{N} \Gamma_{ij}\, \hat{\sigma}_{+}^{i} \hat{\sigma}_{-}^{j} \right\rangle \equiv \langle H_\Gamma \rangle$$

total excited state population

Finding the maximum decay rate R_* is equivalent to finding the ground state energy of $-H_\Gamma$



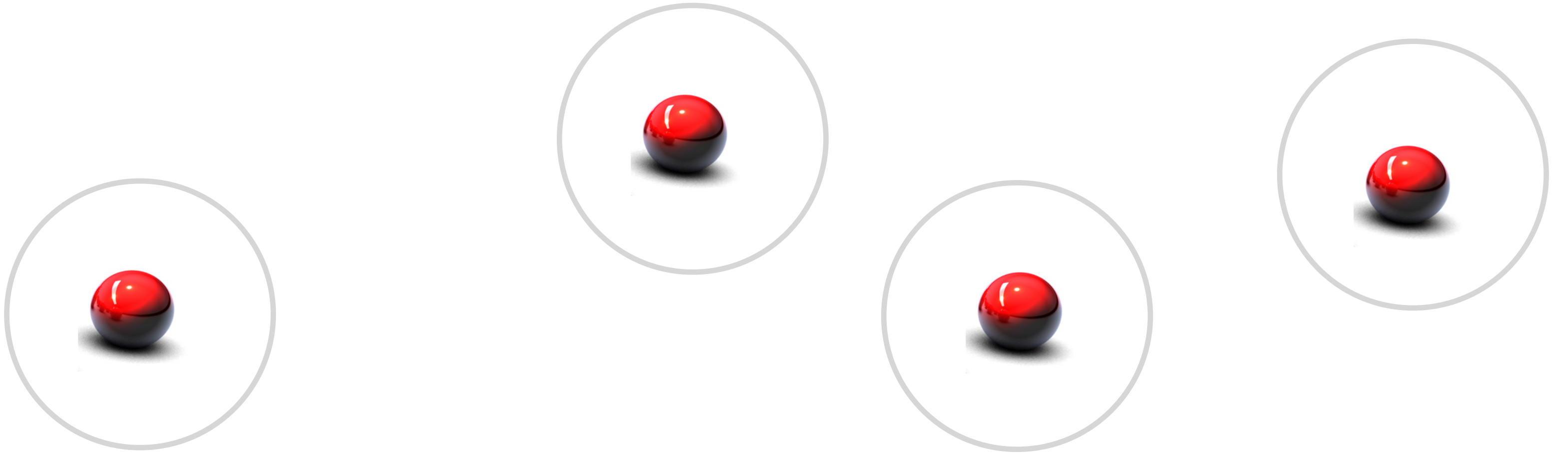
Mok, AAG, Sum, Kwek, Phys. Rev. Lett. 130, 213605 (2023)

What is the maximum decay rate of a quantum system?

Two trivial cases/limits:

Independent (non-interacting) atoms

$$\Gamma_{ij} = \Gamma_0 \, \delta_{ij}$$



$$R_* = N\Gamma_0$$

Atoms all at the same point (very small volume)

$$\Gamma_{ij} = \Gamma_0 \; \forall \; i,j$$

$$R_* = N(N+2)\Gamma_0/4$$

(Dicke state at half excitation)

How do the environment, interaction range, and dimensionality (encoded in $\mathbf{\Gamma}$) change R_*?

Mapping to a ground state problem

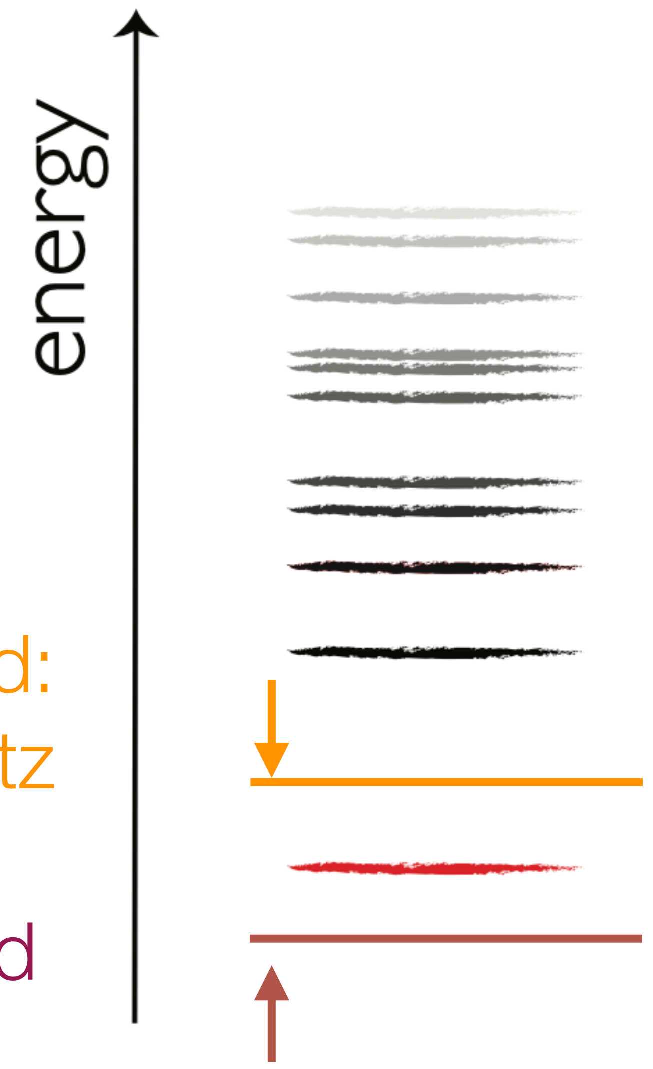
Exact diagonalization of $H_\Gamma = \sum_{i,j=1}^{N} \Gamma_{ij}\, \hat{\sigma}_+^i \hat{\sigma}_-^j$ is unfeasible for more than a few atoms

- Finding the ground state of a 2-local Hamiltonian is QMA complete

J. Kempe, A. Kitaev, O. Regev, SIAM 35, 1070 (2006)

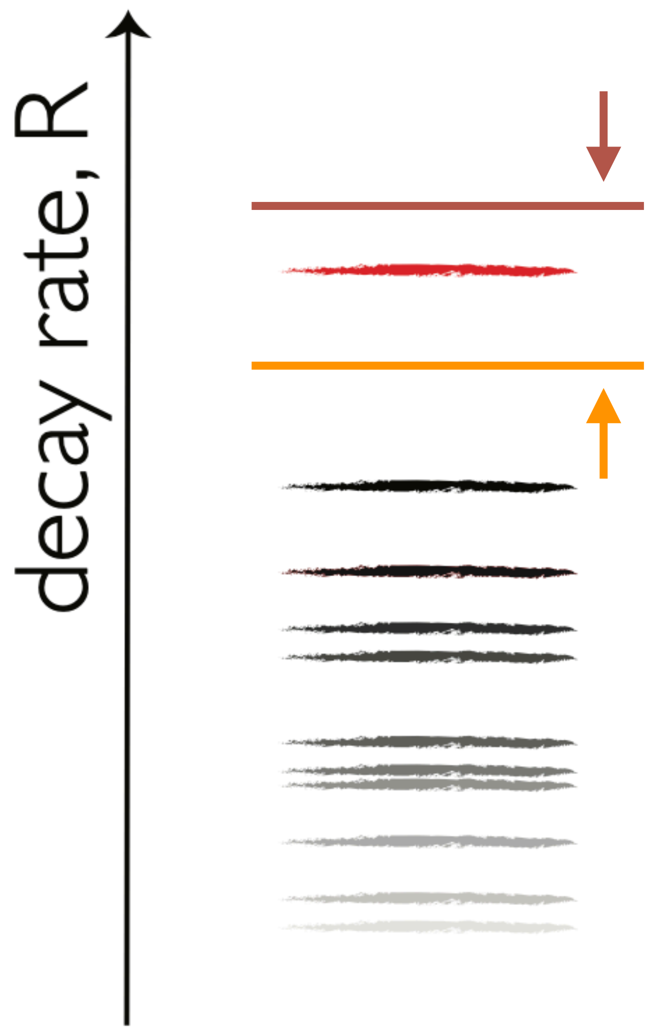
- For atoms in free space, this is an XY model with long-range interactions with oscillating sign

Instead, we set bounds:



upper bound:
variational ansatz

lower bound

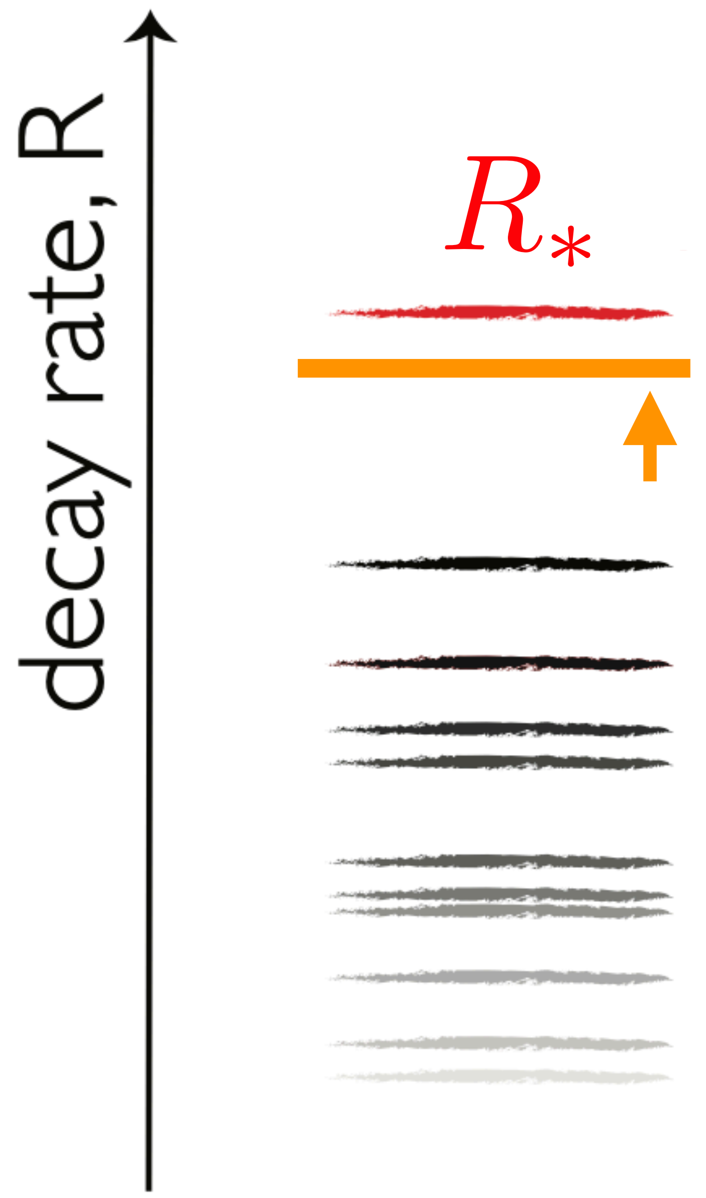


upper bound:
quantum approximation theory

lower bound:
variational ansatz

If tight, we find scaling laws

Lower bound: variational ansatz



Product state ansatz: $|\psi\rangle = \left(\cos\frac{\theta}{2}|g\rangle + \sin\frac{\theta}{2}|e\rangle\right)^{\otimes N}$

sum of dissipative interactions

$$\begin{cases} R_* \geq \max_\psi \langle\psi|H_\Gamma|\psi\rangle = \dfrac{(N\Gamma_0 + S)^2}{4S} & \text{if} \quad S = \sum_{i\neq j}\Gamma_{ij} \geq N\Gamma_0 \\ R_* \geq N\Gamma_0 & \text{otherwise} \end{cases}$$

For translationally invariant systems (with delocalized decay) a (better) variational ansatz yields:

$$R_* \geq \frac{N\Gamma_{\max}}{4}$$

largest eigenvalue of $\mathbf{\Gamma}$

The upper bound: quantum approximation algorithms

Let’s rewrite:

$$\hat{H}_\Gamma \;=\; \underbrace{\hat{H}_{\mathbf{diag}}}_{\Gamma_0 \sum_{i=1}^{N} \hat{\sigma}_i^+ \hat{\sigma}_i^-} + \underbrace{\hat{H}_{\mathrm{XY}}}_{\frac{1}{4} \sum_{i \neq j}^{N} \Gamma_{ij} \left(\hat{\sigma}_i^x \hat{\sigma}_j^x + \hat{\sigma}_i^y \hat{\sigma}_j^y\right)}$$

By the triangle inequality:

norm of diagonal Hamiltonian

$$R_* = \left\|\hat{H}_\Gamma\right\| \leq N\Gamma_0 + \left\|\hat{H}_{\mathrm{XY}}\right\|$$

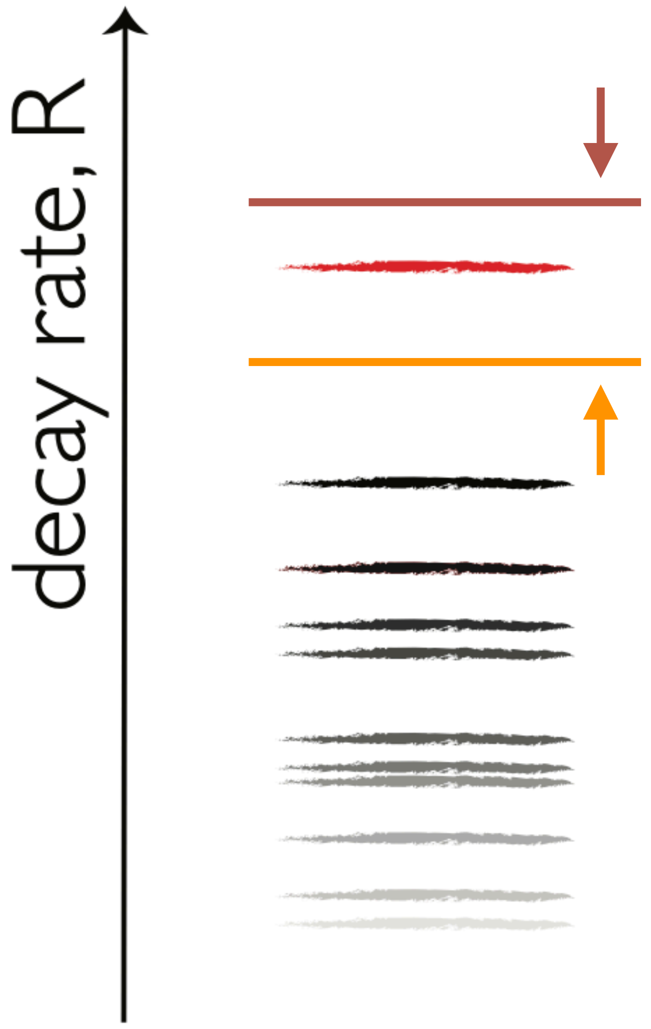
The XY Hamiltonian is traceless and 2-local. Its largest energy can be “well” approximated by a product state(*)

$$\left\|\hat{H}_{\mathrm{XY}}\right\| \leq 6R_{\mathrm{prod}}(\hat{H}_{\mathrm{XY}}) \;\leq\; \cdots \;\leq \frac{3N}{2}(\Gamma_{\max} - \Gamma_0)$$

best product-state approximation

(*) Bravyi et al. J Math Phys 60, 032203 (2019)

Putting everything together: scaling laws



upper bound,
quantum approximation theory: $R_* \leq \frac{N}{2}\left(3\Gamma_{\mathrm{max}} - \Gamma_0\right)$

lower bound,
variational ansatz: $R_* \geq \frac{(N\Gamma_0 + S)^2}{4S} \xrightarrow[\text{decay}]{\text{for delocalized}} R_* \geq \frac{N}{4}\Gamma_{\mathrm{max}}$

For delocalized decay, and assuming Γ_{max} scales with N, the bounds are tight:

$$R_* \sim N\Gamma_{\mathrm{max}}$$

“universal” scaling law

Universal scaling law

$$R_* \sim N\Gamma_{\mathrm{max}}$$

- For non-interacting particles, $\Gamma_{\mathrm{max}} \sim 1$

  and the maximal decay (per particle) does not grow with system size

- In a cavity, the N^2 scaling arises because of the all-to-all interactions, as $\Gamma_{\mathrm{max}} \sim N$

- Scaling is not related to entanglement:

  a product state captures it, even though generically the true state is entangled

Intuition: Hamiltonian is 2-local and the reduced two-particle state is similar
(~quantum deFinetti theorem)

C. M. Caves, C. A. Fuchs, and R. Schack, J. Math. Phys. 43, 4537 (2002)

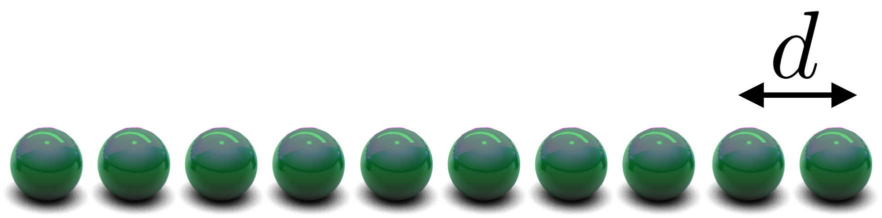
Generic systems: mapping to a ground-state problem

Scaling laws for atom arrays in free space

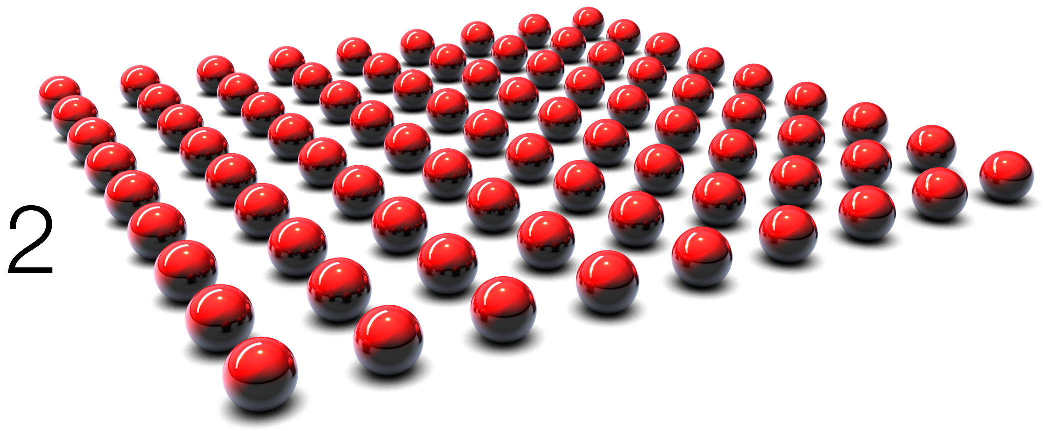
“Bonus”: other results, interesting problems

Dissipative interactions of ordered atomic lattices in free space

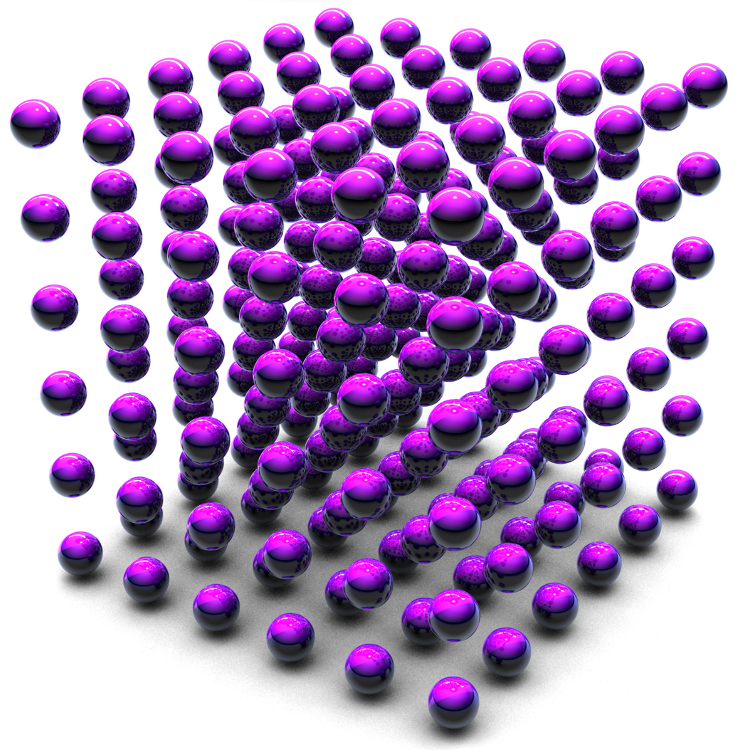
D=1



D=2



D=3



Dissipative matrix elements (in 3D vacuum):

dipole matrix element

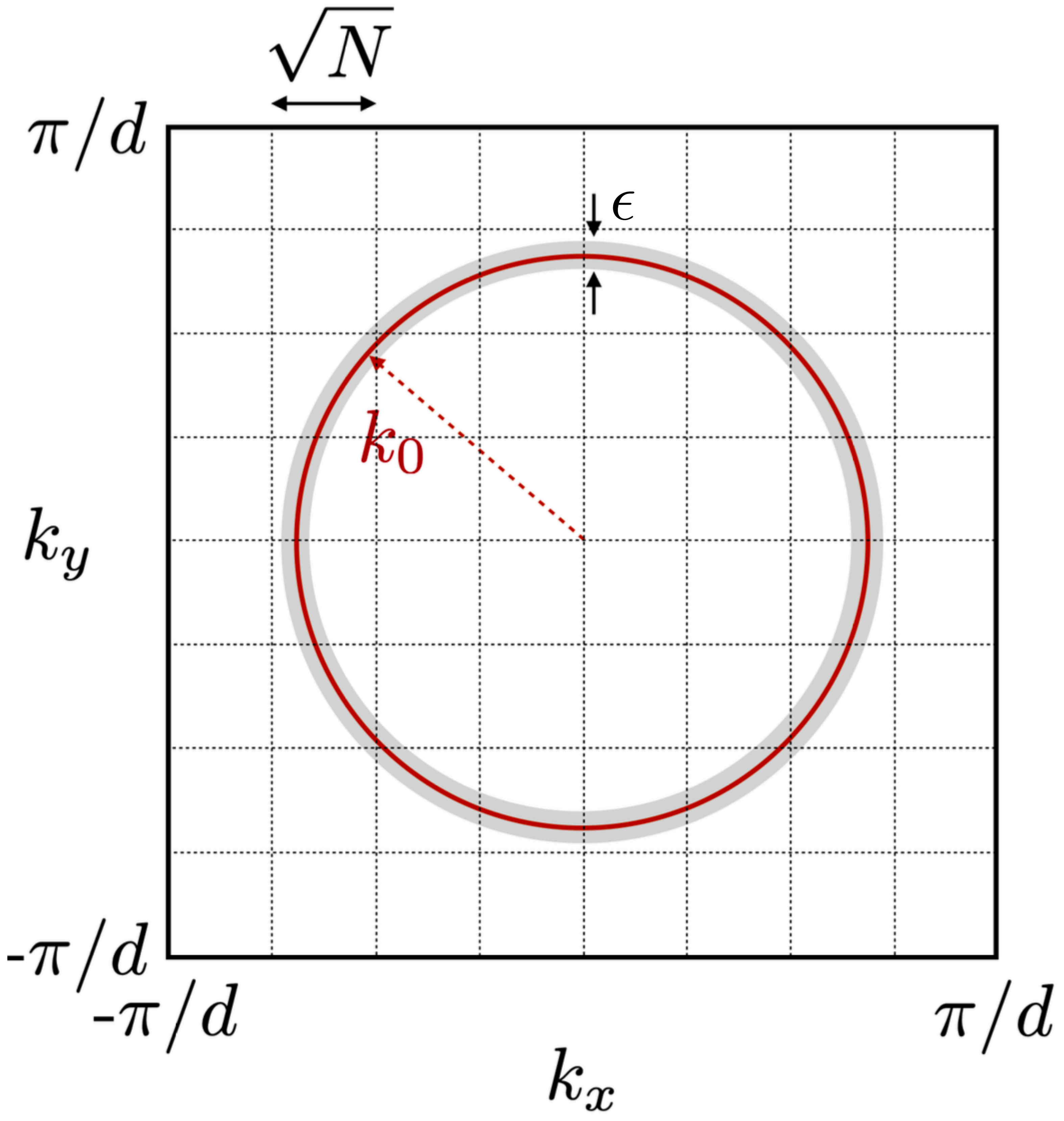
$$\Gamma_{ij} = \frac{6\pi\Gamma_0}{k_0}\hat{\wp}^* \cdot \mathrm{Im}\,\mathbf{G}(\mathbf{r}_i, \mathbf{r}_j, \omega_0) \cdot \hat{\wp}$$

ω_0/c

$$\mathbf{G}(\mathbf{r}, \omega_0) \equiv \frac{e^{ik_0 r}}{4\pi k_0^2 r^3}\Big[(k_0^2 r^2 + ik_0 r - 1)\mathbb{1} + (-k_0^2 r^2 - 3ik_0 r + 3)\frac{\mathbf{r}\otimes\mathbf{r}}{r^2}\Big]$$

How does the largest eigenvalue, Γ_{max}, scale with N for different D?

Finding the scalings analytically



Brillouin zone of an infinite 2D array, showing divergences at the light line

Dissipative matrix can be diagonalized in k-space.
For 2D arrays

$\epsilon = 2\pi/\sqrt{N}d$

$$\frac{\Gamma^{\perp}_{2\mathrm{D}}(\mathbf{k})}{\Gamma_0} = \frac{3\pi}{k_0^3 d^2}\frac{k^2}{\sqrt{k_0^2-k^2}} \xrightarrow{k\to k_0-\epsilon} \frac{3\pi}{k_0 d^2}\frac{1}{\sqrt{2k_0\epsilon}} \sim N^{1/4}$$

In summary (following the same idea):

$$\Gamma^{(1\mathrm{D})}_{\mathrm{max}}/\Gamma_0 \sim 1$$

$$\Gamma^{(2\mathrm{D})}_{\mathrm{max}}/\Gamma_0 \sim N^{1/4}$$

$$\Gamma^{(3\mathrm{D})}_{\mathrm{max}}/\Gamma_0 \sim N^{1/3}$$

Universal scaling laws for ordered atomic arrays

In summary, in the large N limit

$$\frac{R_*^{(\mathrm{D})}}{\Gamma_0} \sim N^{\frac{3}{2}-\frac{1}{2\mathrm{D}}}$$

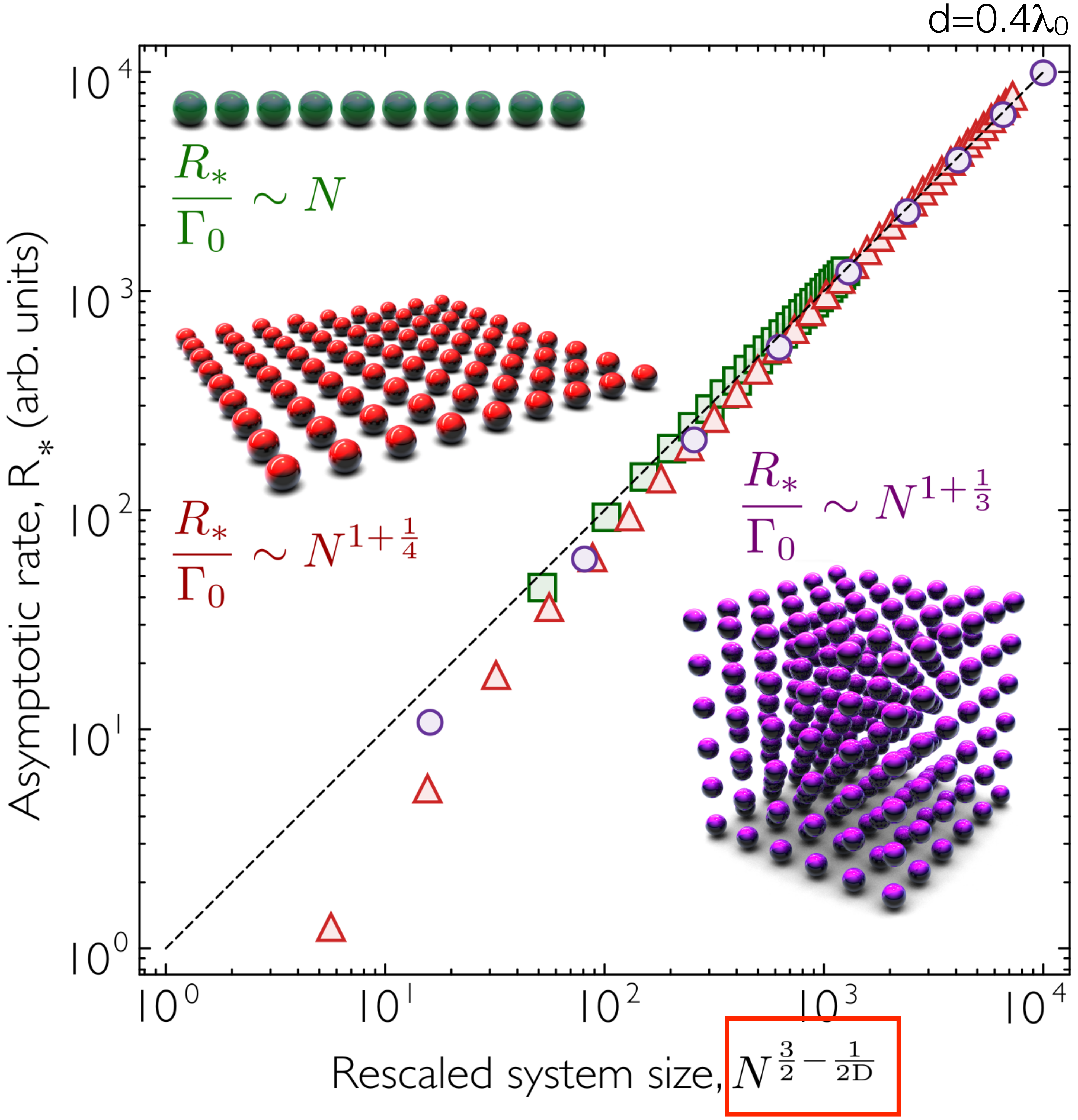
- 1D arrays behave as non-interacting atoms

- Scaling is independent of lattice geometry, polarization, etc

- Robust to “reasonable” disorder

Framing the problem as a semidefinite program

The problem of finding a lower bound (ground state of the classical XY model) can be mapped into a semidefinite program*, which can be solved efficiently.

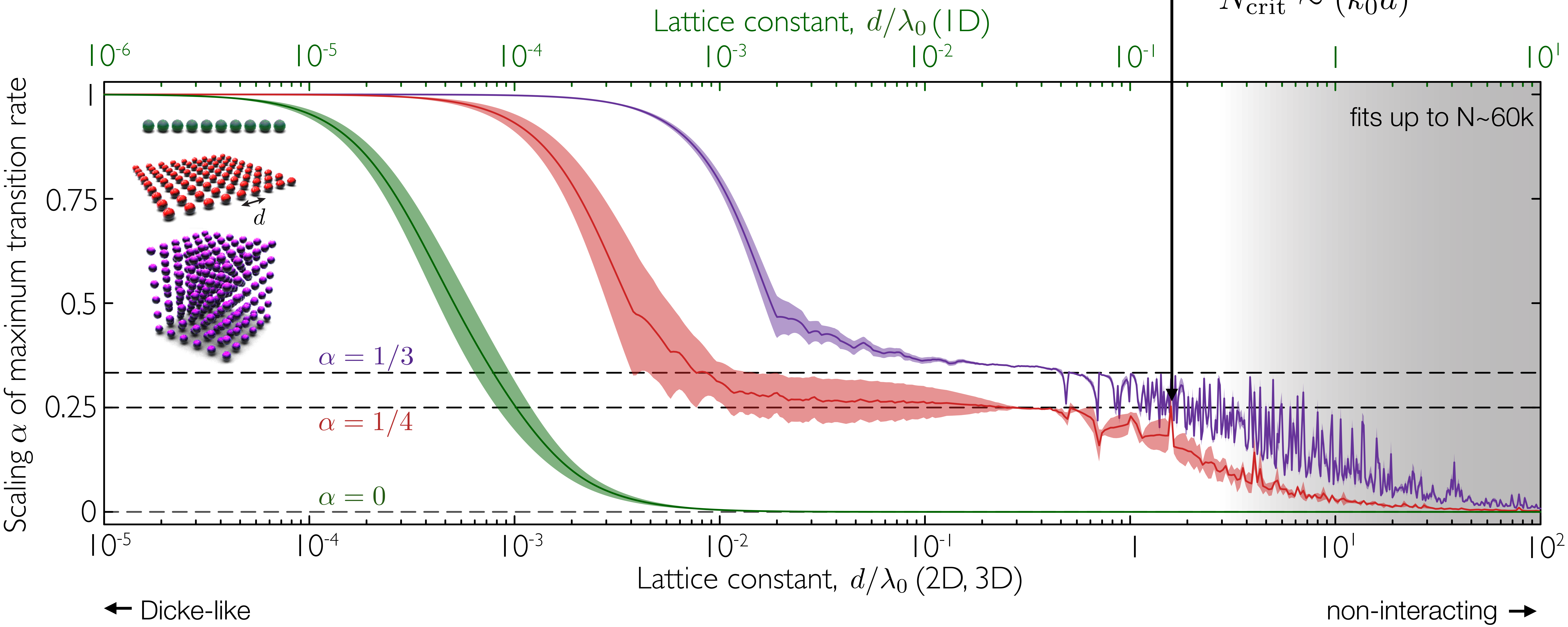


*see for instance Wang et al. arXiv:2310.05844 for a comprehensive discussion on SDPs

Finite-size effects: fits of largest eigenvalue $\Gamma_{\max}/\Gamma_0 \sim N^{\alpha}$

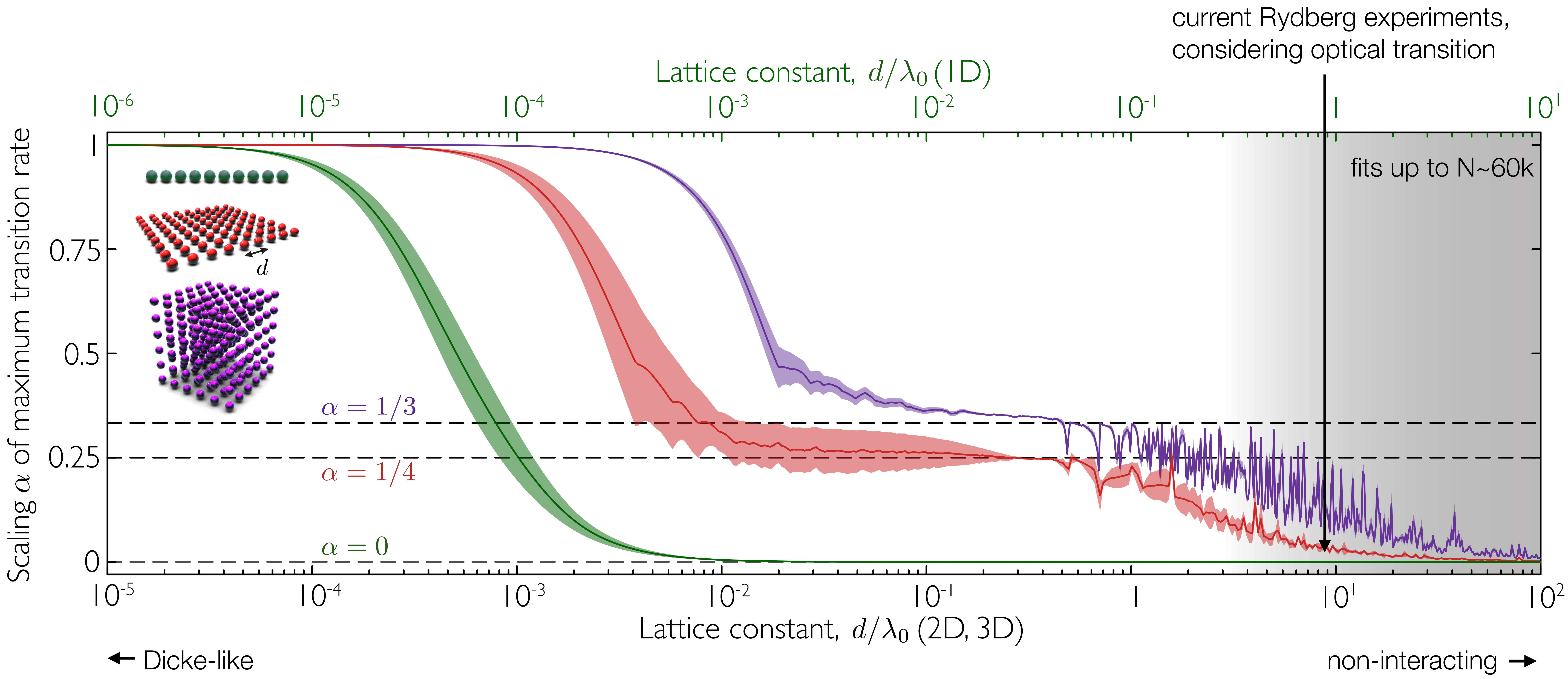
crossover from "non-interacting" to "collective"

$N_{\text{crit}} \sim (k_0 d)^6$



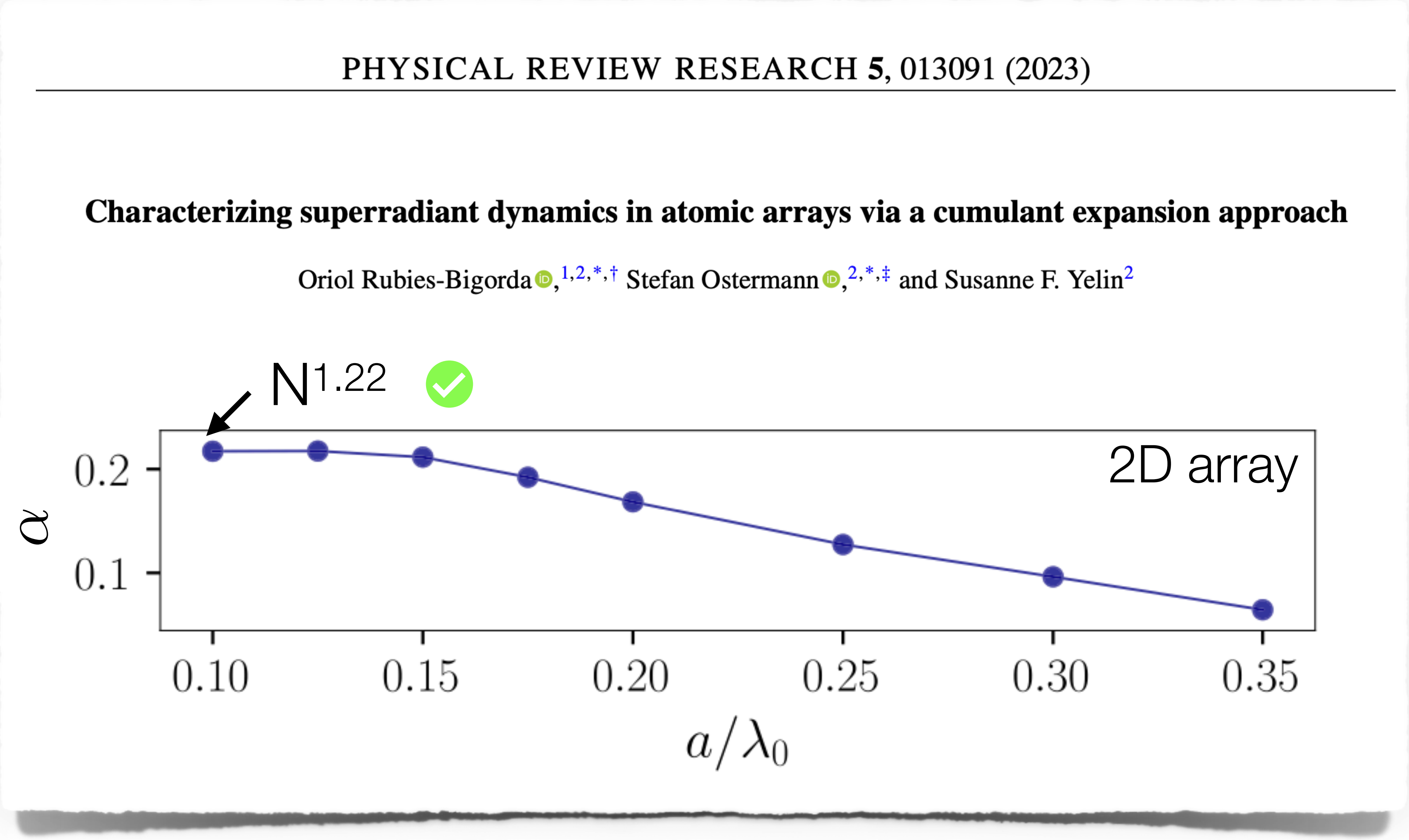
Universality: similar plots are obtained for other polarizations and lattice geometries

Finite-size effects: fits of largest eigenvalue $\Gamma_{\max}/\Gamma_0 \sim N^{\alpha}$



Universality: similar plots are obtained for other polarizations and lattice geometries

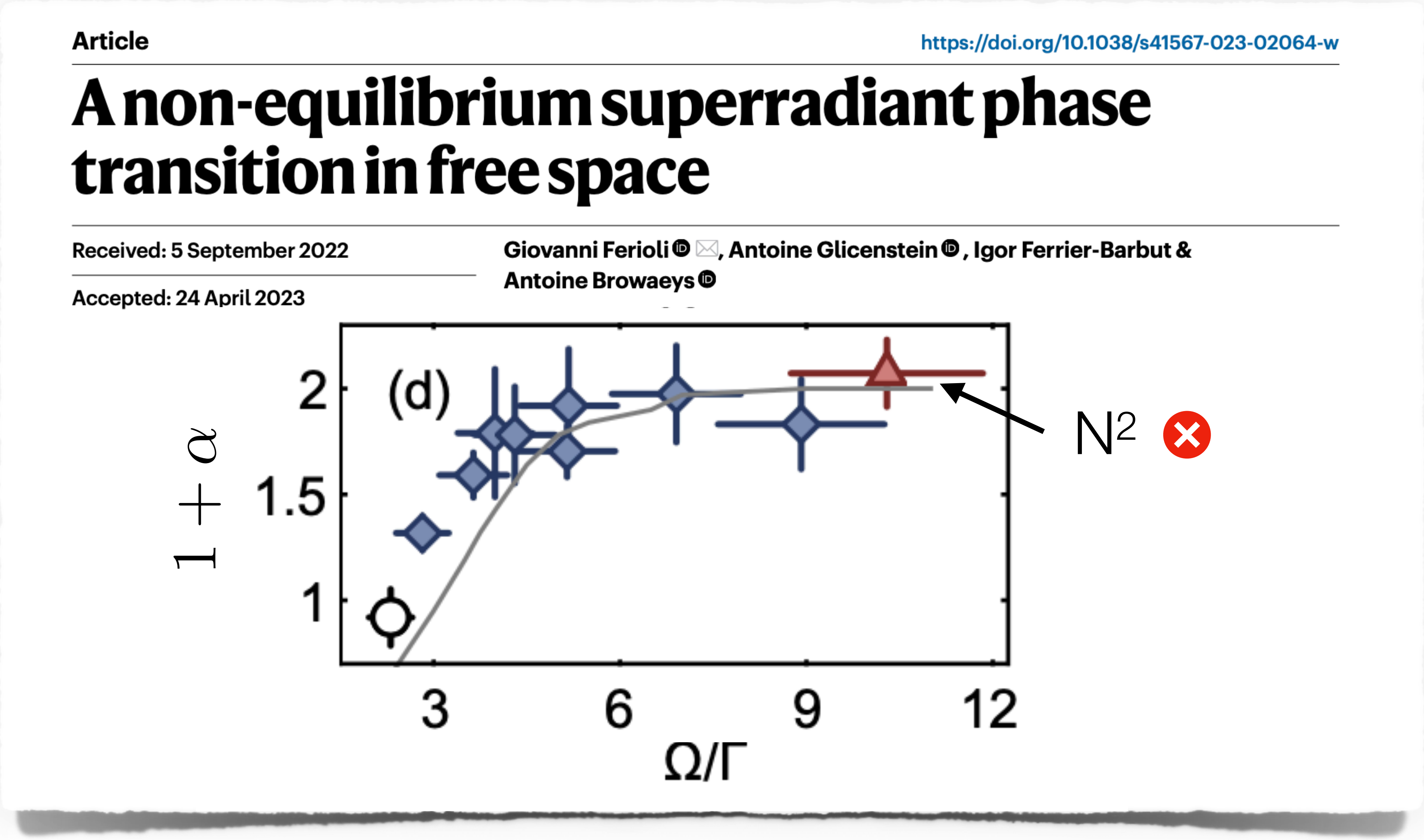
These scaling laws serve as upper bounds for superradiant emission



PHYSICAL REVIEW RESEARCH **5**, 013091 (2023)

Characterizing superradiant dynamics in atomic arrays via a cumulant expansion approach

Oriol Rubies-Bigorda,[1,2,*,†] Stefan Ostermann,[2,*,‡] and Susanne F. Yelin[2]



Article

https://doi.org/10.1038/s41567-023-02064-w

A non-equilibrium superradiant phase transition in free space

Received: 5 September 2022

Accepted: 24 April 2023

Giovanni Ferioli, Antoine Glicenstein, Igor Ferrier-Barbut & Antoine Browaeys

The scaling is obtained for the total decay rate, which is identical to the integrated photon emission rate

Bounds can be violated directionally. But we can obtain new bounds for directional emission

Outlook and open questions

<u>The scaling laws set an upper bound to the decay rate of any state</u>, in particular those achieved dynamically. What are the implications/next questions?

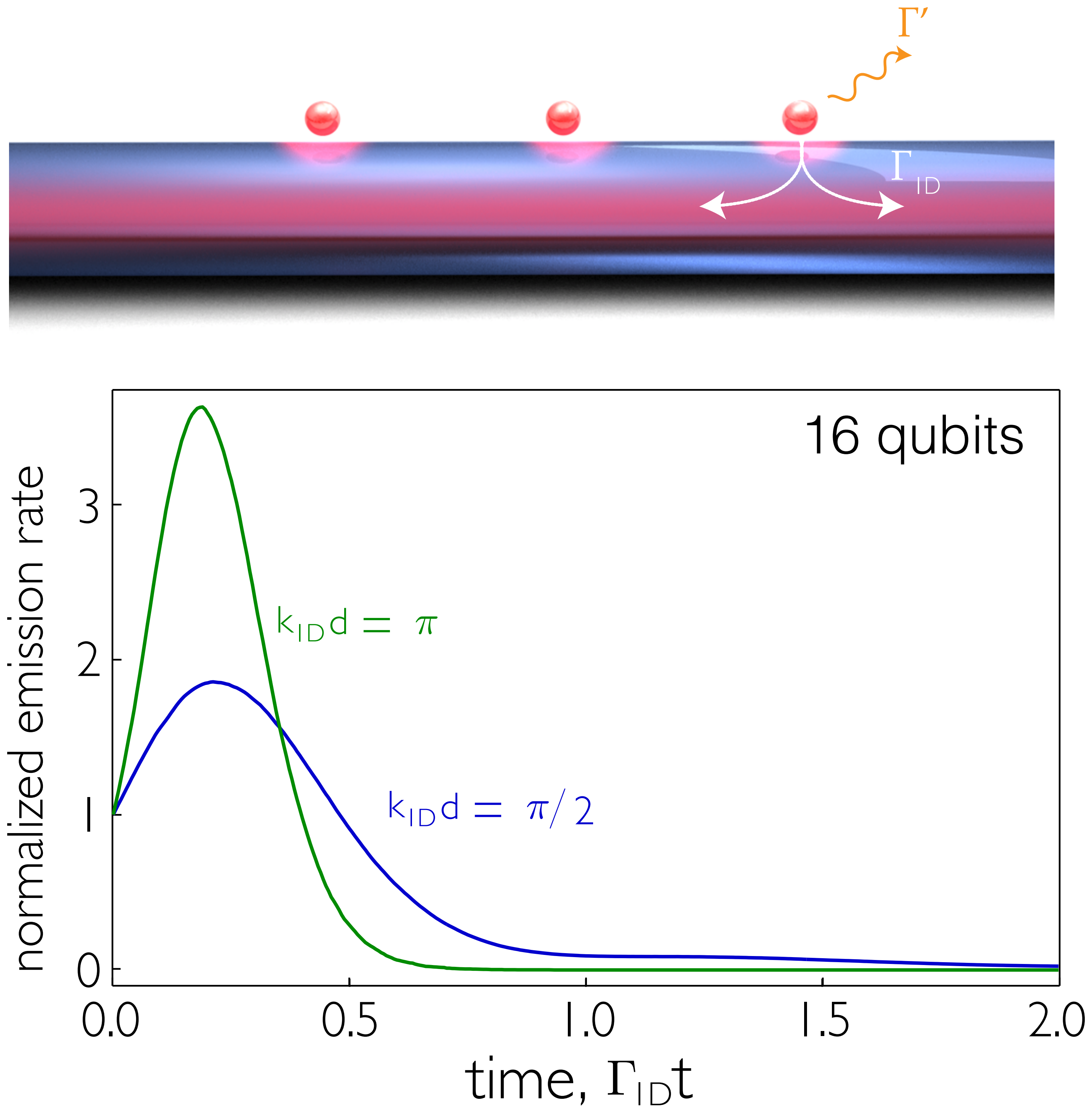
- Error correction: in 2D and above the error scales with system size
- Decay of “typical” (Haar-random) states is not superlinear
- Limit of the coherence time in metrology experiments (clocks, spin-squeezing)
- Fully disordered systems, connection with random matrix theory
- Use of correlated decay to probe material’s properties/phase transitions

Generic systems: mapping to a ground-state problem

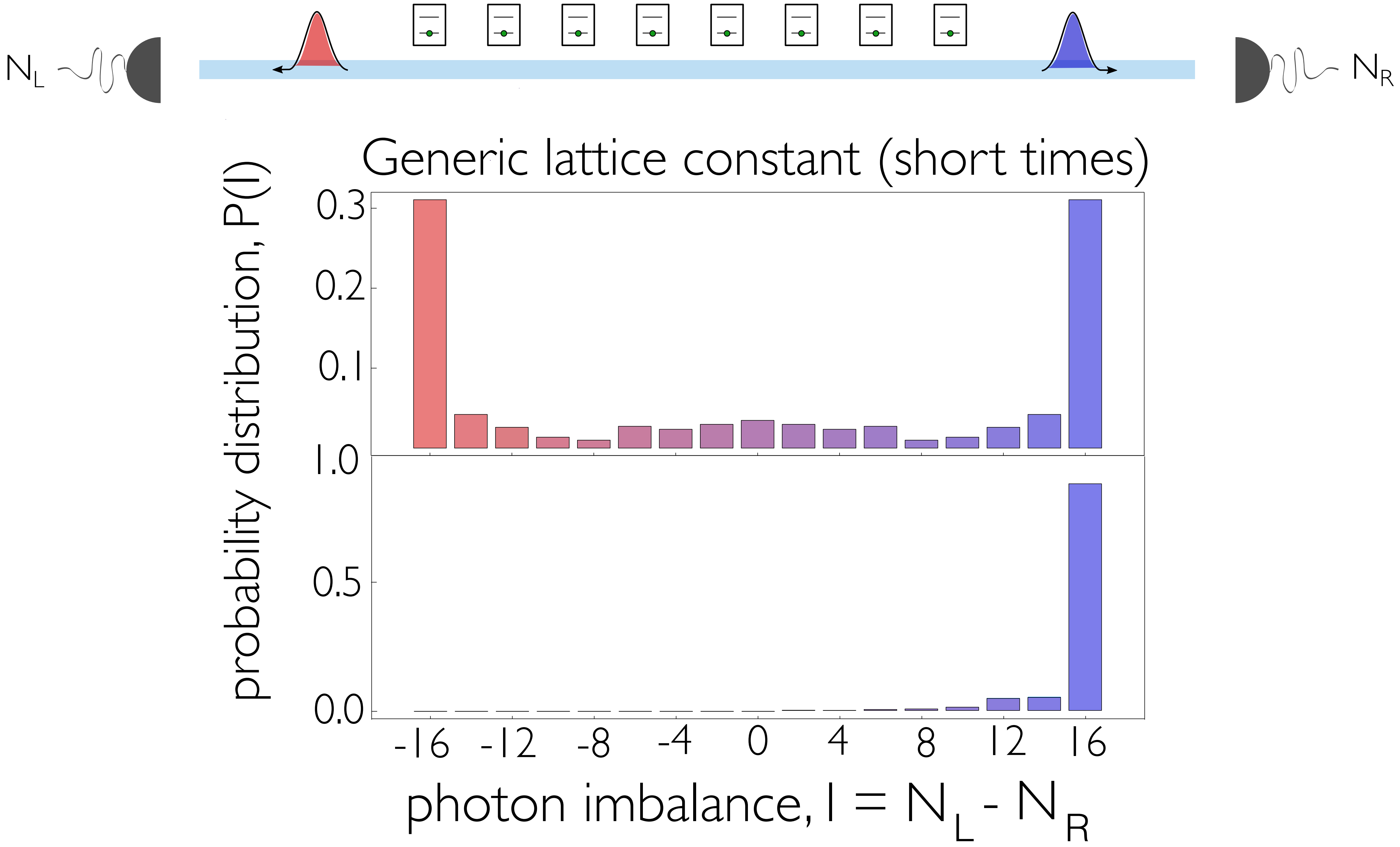
Scaling laws for atom arrays in free space

“Bonus”: other results, interesting problems

“Dynamical” symmetry breaking due to many-body superradiance



Only 2 collective jump operators: left and right

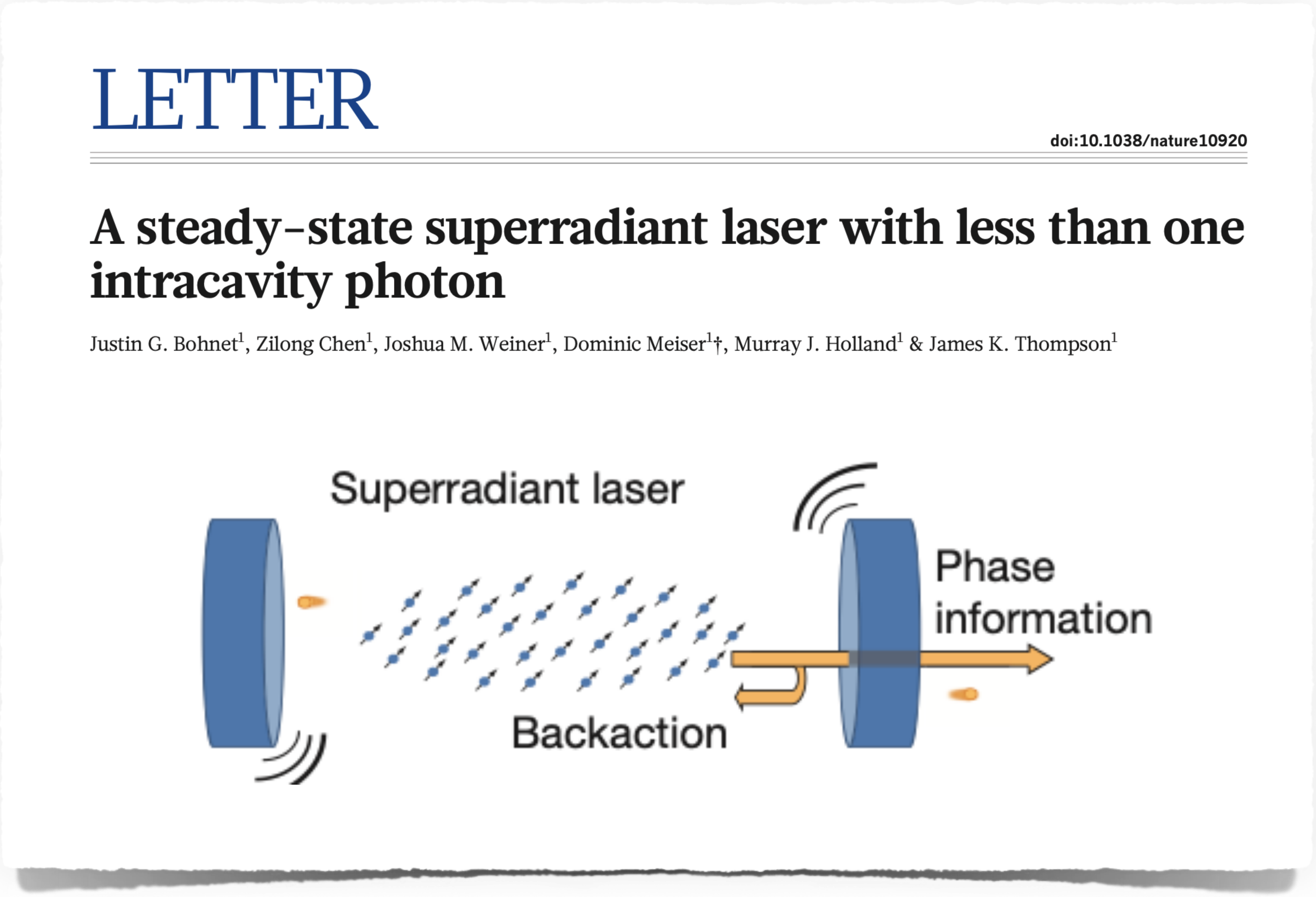


Spontaneous (mirror) symmetry breaking gets dynamically amplified

Emergent chirality

Cardenas-Lopez, Masson, Zager, AAG, PRL 131, 033605 (2023)

Can we stabilize this order in the steady state? Superradiant lasing



LETTER

doi:10.1038/nature10920

A steady-state superradiant laser with less than one intracavity photon

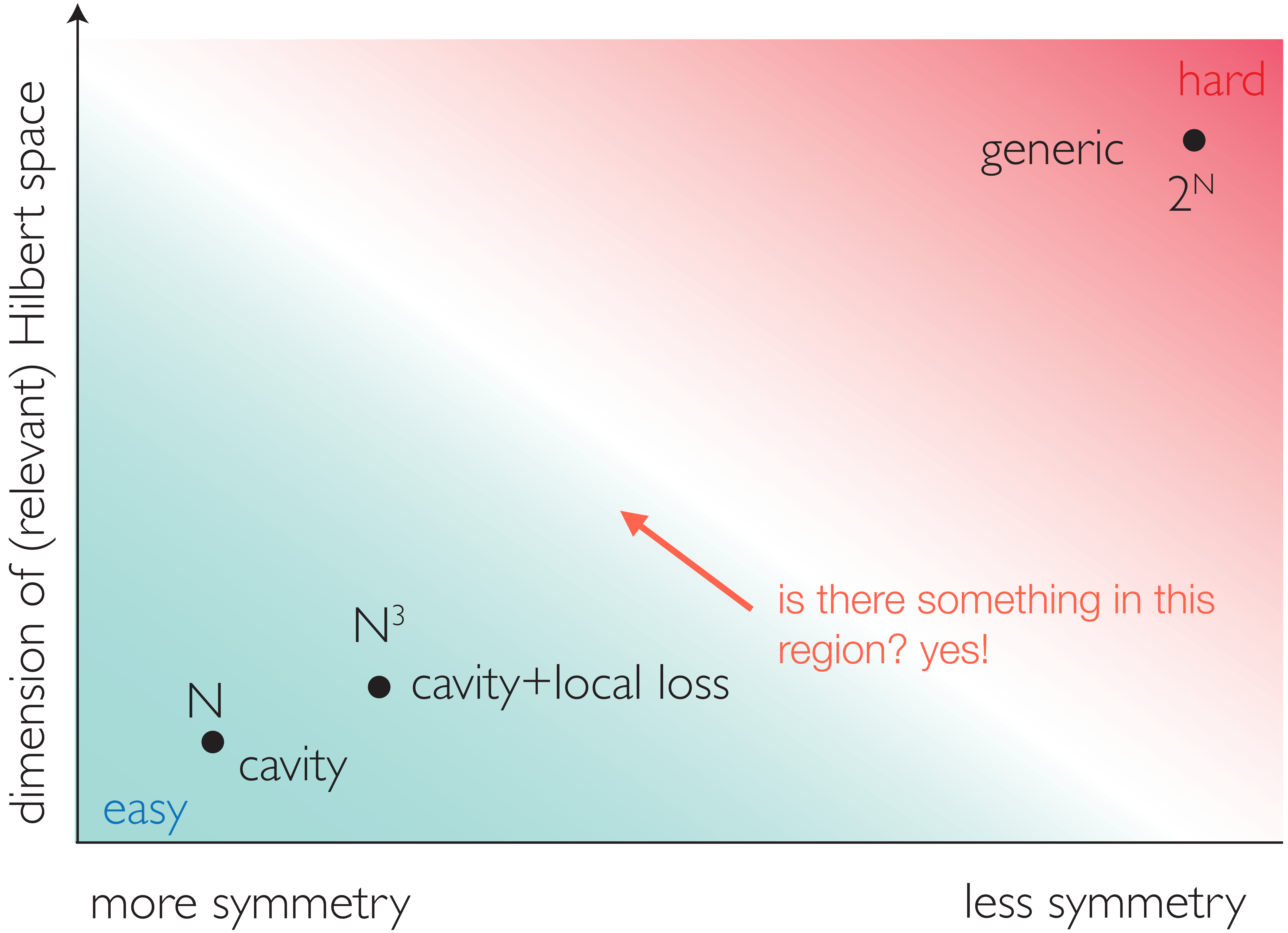
Justin G. Bohnet[1], Zilong Chen[1], Joshua M. Weiner[1], Dominic Meiser[1]†, Murray J. Holland[1] & James K. Thompson[1]

Can we have coherent light ($g^{(2)}=1$) and line narrowing without a cavity?

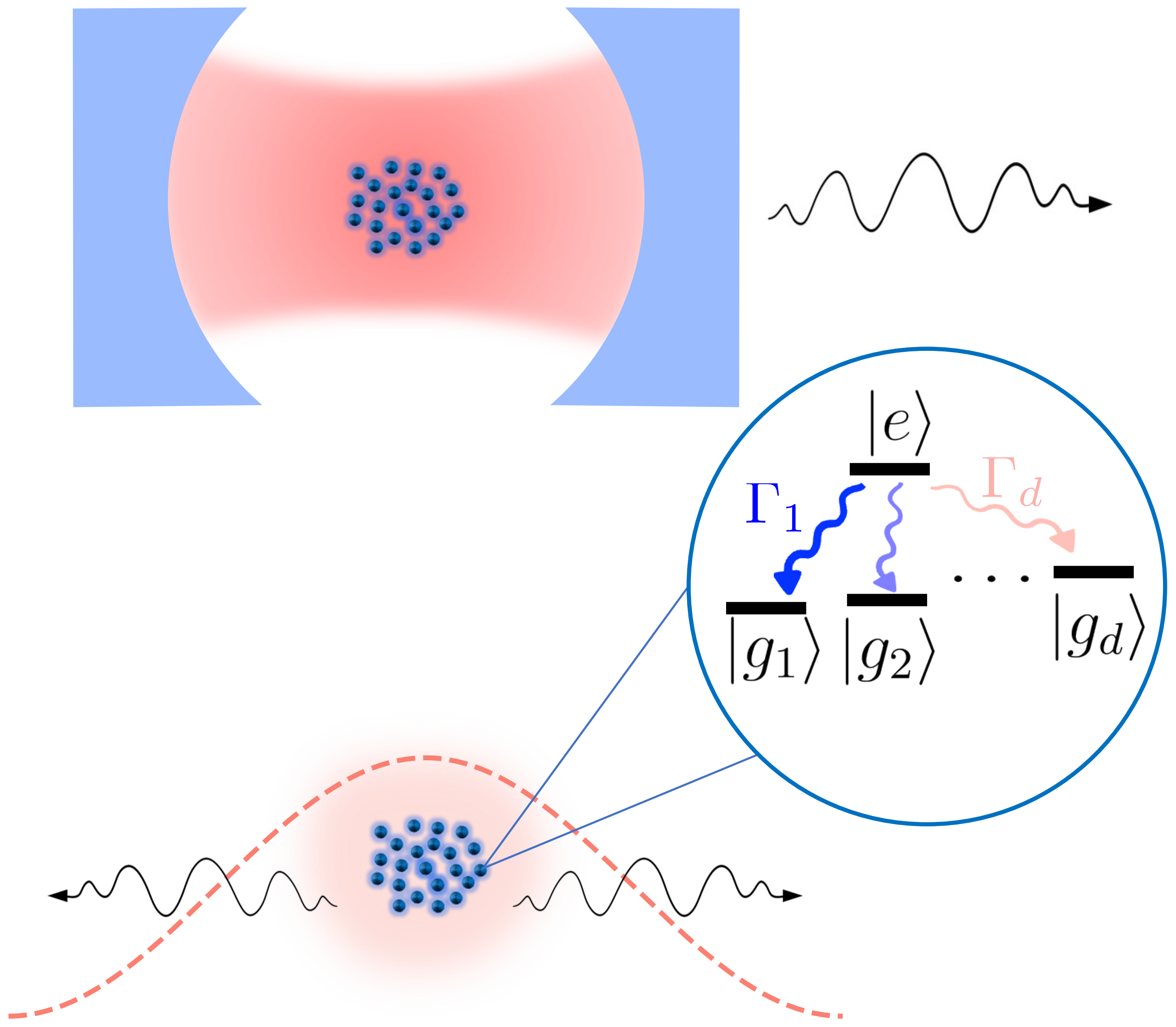
Is there a lasing phase transition?

What does order mean? Is there directionality?

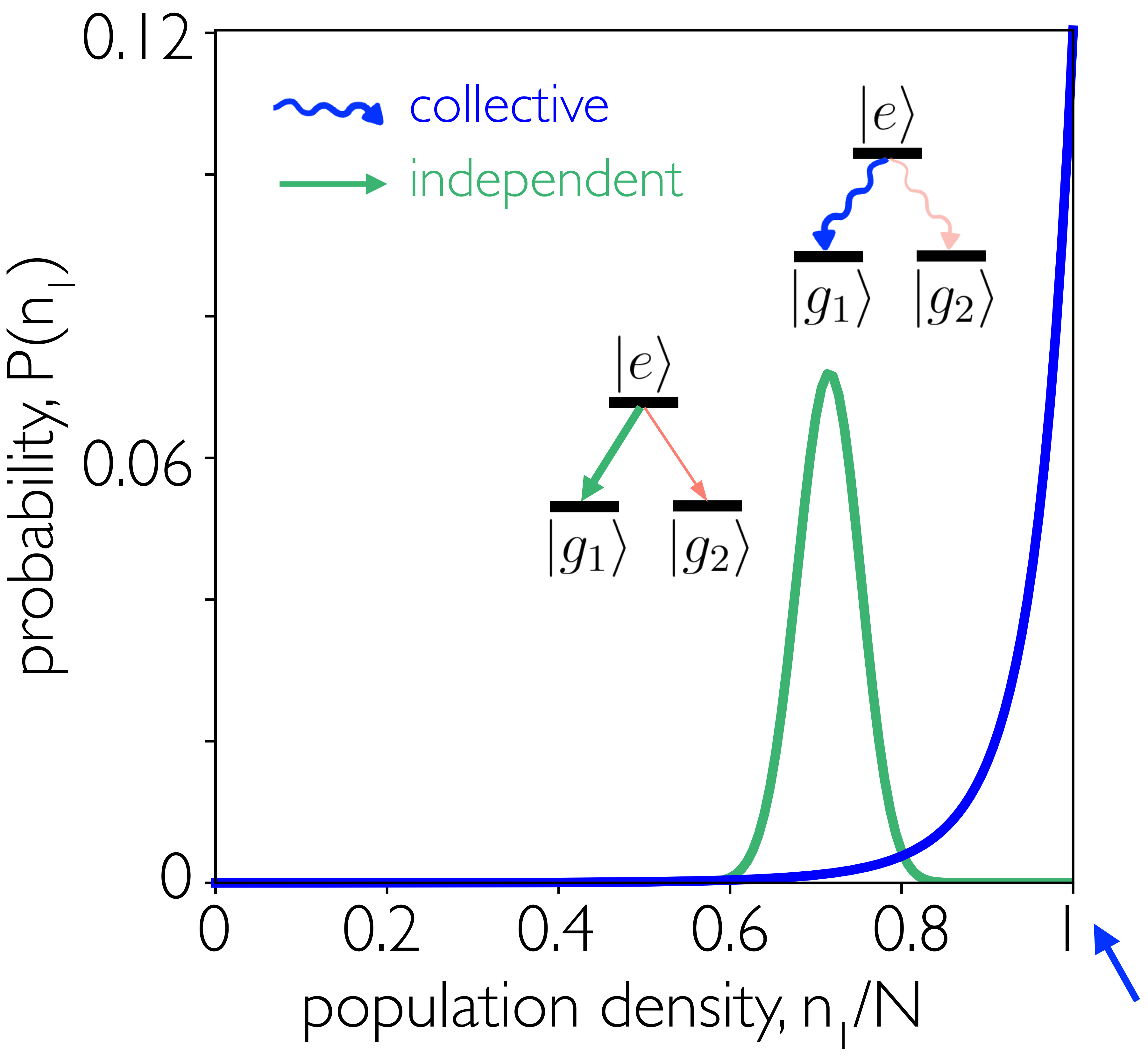
Exact dynamics: harnessing partial permutation symmetry



Collective transition quenching due to correlated decay



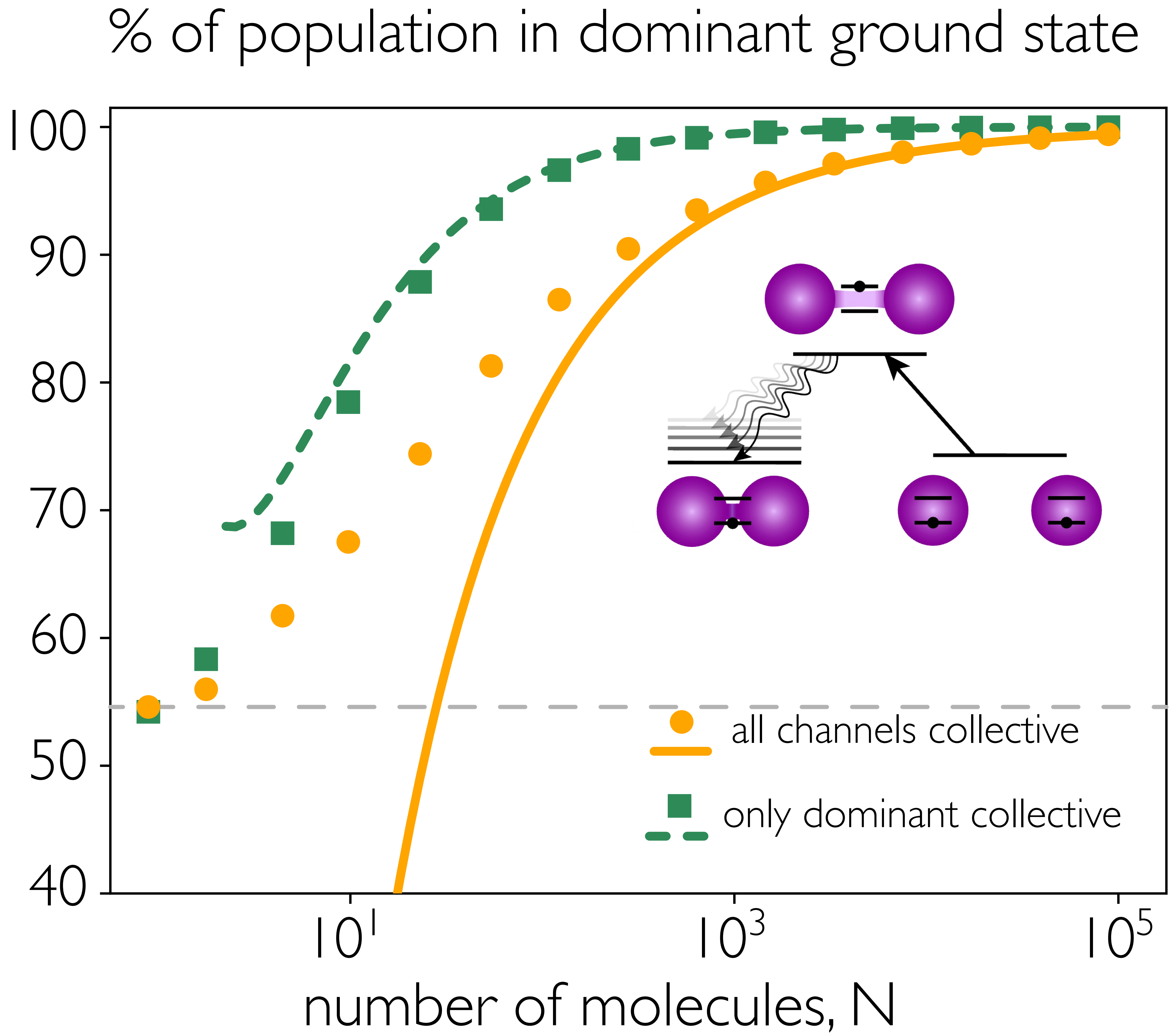
multiple competing channels
that decay superrradiantly



mean $\sim 1 - \sum_{\mu=2}^{d} \frac{1}{N^{1-\Gamma_\mu/\Gamma_1}}$

joint work with Dariel Mok, Stuart Masson, Dan Stamper-Kurn, Tanya Zelevinsky… soon to appear on arXiv
related work by Rey, Robicheaux

Collective decay acts as a “catalyst” for photochemistry



calculation for Sr dimer
1-color photoassociation

calculations for decay to
4 rovibrational states

joint work with Dariel Mok, Stuart Masson, Dan Stamper-Kurn, Tanya Zelevinsky… soon to appear on arXiv

Some final take-home messages

- We know almost nothing about the many-body regime
- Some interesting open problems:
  - new out-of-equilibrium phases in the presence of competing decay channels (is there order, what does order mean in this context?)
  - applications: mirrorless lasing, collective transition quenching
  - stabilization of many-body dark states
  - spin squeezing/metrology in the presence of dissipation, etc.
- Yarovslav's question: What to diagonalize? Answer: needs to be informed by the physics!
- Great time to work on these topics (new theory results, experiments)

Acknowledgments

Eric Sierra Avishi Poddar Cosimo Rusconi Silvia Cardenas-Lopez

@caltech:

Dariel Mok

John Preskill

D. Mok, A. Poddar, E. Sierra, C. Rusconi, J. Preskill, AAG, arXiv:2406.00722 (2024)

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