Dynamics of Cavity QED Simulators of Frustrated Quantum Matter

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arXiv:2311.05682 arXiv:2312.11624





Spintronics



Landau-Lifshiftz-Gilbert equation

$$\frac{d\vec{S}}{dt} = -\gamma \left(\vec{S} \times \vec{H}_{\text{eff}} - \eta \vec{S} \times \frac{d\vec{S}}{dt}\right)$$

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Cavity QED

Collective interactions



 $H \sim (a + a^{\dagger}) \sum \sigma_i^x$

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Cavity QED

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 $H \sim (a + a^{\dagger}) \sum \sigma_i^x = (a + a^{\dagger}) S^x$ $\frac{S}{\hbar} \sim 10^5$

→ Mean field classical dynamics













Cavity QED

Collective interactions



 $H \sim (a + a^{\dagger}) \sum \sigma_i^x = (a + a^{\dagger}) S^x$

Small spins

Strong but non-collective interactions





Spin Liquids

Semeghini et al., Sci '21 / Chiocchetta et al., Nat Com '21



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Programmable arrays

Periwal et al., Nat '21/ Setharam et al., PRR&PRB '22



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Density waves (fermions)

Zwettler et al., '24



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Spin Glasses

Kroeze et al., arXiv:2311.04216 / Marsh et al., PRX '24/ Buchhold et al. PRA '13 / Strack & Sachdev, PRL '11

Simple example: Sherrington-Kirkpatrick model

Sherrington & Kirkpatrick '75



Conflicting couplings (FM or AFM ?)

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 $H = h \sum \sigma_i^z + \sum J_{ij} \sigma_i^x \sigma_j^x$

Conflicting couplings (FM or AFM ?)







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Conflicting couplings (FM or AFM ?)















Rugged free energy landscape



Unusual relaxation





Unusual relaxation

(Quasi) ergodicity at short times \checkmark Effective temperature T_1 of quick fluctuations





Unusual relaxation

(Quasi) ergodicity at short times \checkmark Effective temperature T_1 of quick fluctuations

Energy barriers obstruct more ``flips" \rightarrow Slow thermalization

Equilibration becomes slower with time (aging)

Effective temperature T_2 for fluctuations over longer timescales





Noble metals (Cu, Ag, Au) doped with transition metal ions (Fe, Mn, ...)

RKKY coupling of moments

 $J(R) \sim \frac{\cos(2k_f R + \theta_0)}{(k_f R)^3}$





Noble metals (Cu, Ag, Au) doped with transition metal ions (Fe, Mn, ...)

RKKY coupling of moments





From Mulder et al. '81



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Experimental realization

Experimental realization Confocal cavity QED platforms



Figure: Marsh et al., PRX '21

Experimental realization Confocal cavity QED platforms

Cavity mediated couplings:

$$H_{int} \propto -\sum_{ij} J_{ij} S_i^x S_j^x + \dots \qquad \begin{array}{c} \text{Guo et al., PRA '19} \\ \text{Marsh et al., PRX '21} \end{array}$$
$$= \underbrace{f_{ij}} \longrightarrow BEC$$
$$J_{ij}/J_0 = \delta(r_i - r_j) + \delta(r_i + r_j) + \frac{1}{\pi} \cos(\frac{2r_i \cdot r_j}{w_0^2})$$



Figure: Marsh et al., PRX '21

Experimental realization Confocal cavity QED platforms

Cavity mediated couplings:



Figure: Marsh et al., PRX '21

e + sign changing

Experimental realization Confocal cavity QED platforms Cavity mediated couplings:

For large spins ($S \sim 10^5$) \rightarrow direct imaging of spin configuration. Kroeze et al., arXiv:2311.



Figure: Kroeze et al., arXiv:2311.04216

$$H = \omega_c \sum_{\alpha=1}^{M} a_{\alpha}^{\dagger} a_{\alpha} + \frac{\Delta}{2} \sum_{i=1}^{N} S_{i\lambda}^z - \sum_{\alpha}^{M} \sum_{i}^{N} \sum_{\lambda}^{N_s} g_{\alpha,i} (a_{\alpha} + a_{\alpha}^{\dagger}) S_i^x$$
$$\mathscr{L}\rho = \kappa \sum_{\alpha=1}^{M} \mathscr{D}[a_{\alpha}]\rho \qquad \overline{g_{\alpha,i}} = 0 \quad \overline{g_{\alpha,i}g_{\beta,j}} = \delta_{\alpha\beta} \delta_{ij} g^2 / \sqrt{2S(N)}$$





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Adiabatic elimination:

$$H = \Delta \sum_{i} S_i^z - \frac{4\omega_c^{-1}}{2S(N+M)} \sum_{ij}^N \sum_{\alpha}^M g_{\alpha,i} g_{\alpha,j} S_i^x S_j^x$$





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patterns \approx # groundstates

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cavity modes (M)η # atomic clouds (N)

 $\eta < \eta_c \rightarrow$ Easy to choose the groundstate Spins retrieve one of the patterns Hopfield, PNAS '82 / Amit, Gutfreund & Sompolinsky, PRL '85 / Marsh et al., PRX '21

 $\eta > \eta_c \rightarrow$ Frustration \rightarrow **spin glass**

Gopalakrishnan, Lev & Goldbart, PRL '11 / Strack & Sachdev, PRL '11 Buchhold et al., PRA '13











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To solve for non-eq. dynamics

Disordered systems are very complicated, but averaging makes life easier

Write spins in terms of fermions σ^{lpha}



$$-i\epsilon_{\alpha\beta\gamma}\psi^{\beta}\psi^{\gamma} \qquad \{\psi^{\alpha},\psi^{\beta}\} = \delta_{\alpha\beta} \qquad (\alpha,\beta=x,y,z)$$

Use non-equilibrium field theory





Start from $| \rightarrow \rangle | \rightarrow \rangle \dots | \rightarrow \rangle$

Sudden change of coupling $g_i \rightarrow g_f$





Start from $| \rightarrow \rangle | \rightarrow \rangle \dots | \rightarrow \rangle$

Sudden change of coupling $g_i \rightarrow g_f$

Let's try different spin sizes



Long lasting prethermal plateau Quantum fluctuation accelerate relaxation





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Long lasting prethermal plateau









Long-time correlations

$$C(\tau, \tau - t) = \frac{1}{S^2} \overline{\langle S_i^x(\tau) S_i^x(\tau - t) \rangle}$$



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Edwards-Anderson order parameter for spin glass

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for spin glass

Quantum fluctuations melt spin glass

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Statistical correlations between similar samples

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Take two copies with the same profile of disorder.

How much do they look similar after evolution?

$$Q(t) = \frac{1}{S^2} \overline{\langle S_{i,A}^x(t) S_{i,B}^x(t) \rangle_c}$$



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Quantum fluctuation can completely melt the spin glass (without cavity loss)

Playing with the cavity frequency

Loss rate of the cavity modes $\rightarrow \kappa$

$$\left(\partial_t \rho\right)_{\text{loss}} = \kappa \sum_{\alpha} \left(2a_{\alpha} \rho a_{\alpha}^{\dagger} - \{a_{\alpha}^{\dagger} a_{\alpha}, \rho\}\right)$$

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Loss is good, but not too much!

Playing with the cavity frequency



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Summary

- clean/controlled dynamics of SG vs solid state experiments (access to certain macroscopic quantities only/short range interactions - RKKY)
- possibility to engineer tunable range interactions (from long to short) (open problems in theory of spin glasses)
- role of (controlled) dissipation
 (moderate cavity loss helps SG formation)
- tunable quantum fluctuations at the network nodes (un-thinkable in solid state)
- quantum-to-classical crossover in a many body system (in general: intractable/very few examples/ purely phenomenological)
- <u>broad applicability</u>: programmable interactions & dissipation in c-QED with tweezers, correlated emission in atomic arrays, Rydbergs in cavities.