RPTU Rheinland-Pfälzische Technische Universität Kaiserslautern Landau

OPTIMAS

SFB/TR 185

Dissipative Dicke time crystals: an atoms' point of view

Simon B. Jäger

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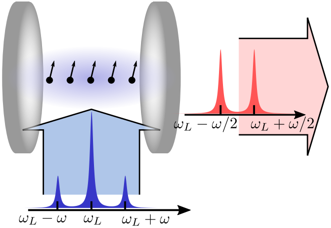
18.06.2024

Time-periodically driven atom-cavity setups



Cavity photons mediate time-periodic interactions between the atoms

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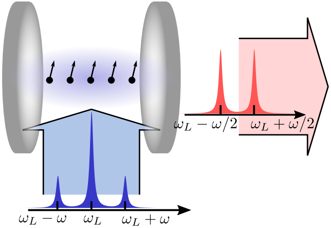

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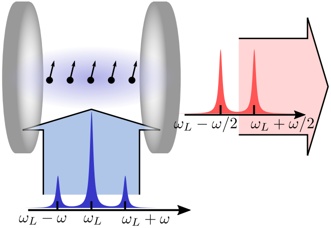

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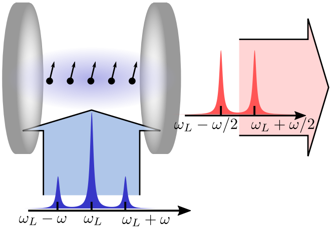
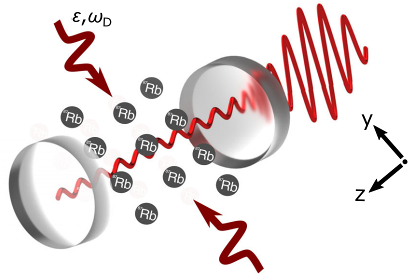




H. Keßler et al., PRL **127**, 043602 (2021).

Cavity photons mediate time-periodic interactions between the atoms

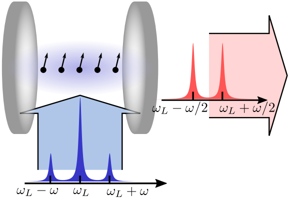
→ resonant creation of coherent spatio-temporal pattern

→ dissipation of cavity photons through mirrors stabilizes pattern

Generation of new phases of matter stabilized by dissipation

→ dissipative time crystals

Effective theoretical description



We are interested in the behavior of the atoms!

Can we eliminate the field degrees of fredom?

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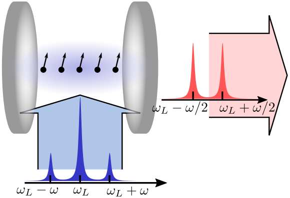

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Gain:

- more efficient description of the dynamics
- analytical results and predictions

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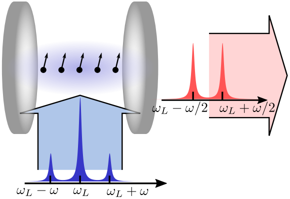

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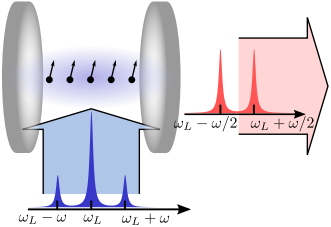
Challenge: correct descriptions of interactions + dissipation!

Periodically driven Dissipative Dicke model



Single diss. cavity mode coupled to collective spin

$$\frac{\partial \hat{\rho}}{\partial t} = -i\left[\hat{H}, \hat{\rho}\right] - \kappa(\hat{a}^\dagger \hat{a}\hat{\rho} + \hat{\rho}\hat{a}^\dagger \hat{a} - 2\hat{a}\hat{\rho}\hat{a}^\dagger)$$

$$\hat{H} = \omega_0 \hat{Z} + \omega_c \hat{a}^\dagger \hat{a} + \frac{2g(t)}{\sqrt{N}}(\hat{a} + \hat{a}^\dagger)\hat{X}$$

$$\hat{X} = \frac{1}{2}\sum_j \hat{\sigma}_j^x,\ \hat{Y} = \frac{1}{2}\sum_j \hat{\sigma}_j^y,\ \hat{Z} = \frac{1}{2}\sum_j \hat{\sigma}_j^z$$

$$g(t) = g_0 + g_1 \cos(\omega t)$$

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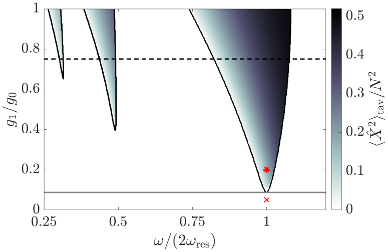

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Exhibits time crystalline phase for parameteric driving

R. Chitra and O. Zilberberg, Phys. Rev. A 92, 023815 (2015).
Z. Gong, R. Hamazaki, and M. Ueda, Phys. Rev. Lett. **120**, 040404 (2018).

- parameteric resonances $\frac{n\omega}{2} = \omega_{\rm res} = \omega_0\sqrt{1 - \frac{4g_0^2\omega_c}{\omega_0[\omega_c^2+\kappa^2]}}$

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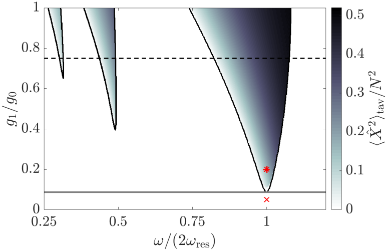

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- signaled by (i) $\langle[\hat{X}]^2\rangle \propto N^2$ and (ii) $\langle \hat{X}(t+t_0)\hat{X}(t_0)\rangle$ is $4\pi/\omega$ periodic in t

$\rightarrow$ Toy model for dissipative time crystals

We want to eliminate the cavity!

S. B. Jäger et al., Phys. Rev. Lett **129**, 063601 (2022).

Derivation of the master equation

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Step 1: Move into frame which decouples cavity and spin $\|\hat{\alpha}\| \ll 1$

$$\tilde{\rho} = \hat{D}^{\dagger}\hat{\rho}\hat{D}, \; \hat{D} = \exp\left[\hat{\alpha}^{\dagger}\hat{a} - \hat{a}^{\dagger}\hat{\alpha}\right]$$

and find operator $\hat{\alpha}$ that minimizes coupling between cavity and spin

$$\rightarrow \frac{\partial\hat{\alpha}}{\partial t} = -i[\omega_0\hat{Z}, \hat{\alpha}] - (i\omega_c + \kappa)\hat{\alpha} - i\frac{2g(t)}{\sqrt{N}}\hat{X}$$

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Step 2: Project onto the vacuum in the displaced picture $\hat{\rho}_{\mathrm{spin}} = \langle \mathrm{vac}|\tilde{\rho}|\mathrm{vac}\rangle$

$$\frac{\partial\hat{\rho}_{\mathrm{spin}}}{\partial t} = -i\left[\hat{H}_{\mathrm{eff}},\hat{\rho}_{\mathrm{spin}}\right] - \kappa(\hat{\alpha}^{\dagger}\hat{\alpha}\hat{\rho}_{\mathrm{spin}} + \hat{\rho}_{\mathrm{spin}}\hat{\alpha}^{\dagger}\hat{\alpha} - 2\hat{\alpha}\hat{\rho}_{\mathrm{spin}}\hat{\alpha}^{\dagger})$$

$$\hat{H}_{\mathrm{eff}} = \omega_0\hat{Z} + \frac{g}{\sqrt{N}}(\hat{\alpha}^{\dagger}\hat{X} + \hat{X}\hat{\alpha})$$

Master equation with cavity-mediated Interactions and Dissipation

When is it possible to eliminate the field?

What are the limitations?

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Valid if there is a timescale separation

$$\underbrace{\kappa^{-1}, \omega_c^{-1}}_{\text{timescale of cavity}} \ll \underbrace{\omega_0^{-1}, \omega^{-1}, g^{-1}}_{\text{timescale of atom and drive}}$$

→ **Cavity adiabatically follows atomic degrees**

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Solution for $\hat{\alpha}$:

$$\hat{\alpha} = \frac{\alpha_+(t)}{\sqrt{N}} \hat{S}^+ + \frac{\alpha_-(t)}{\sqrt{N}} \hat{S}^- \qquad \alpha_\pm \approx -\underbrace{\frac{g(t)}{\omega_c - i\kappa}}_{\omega_c^{-1} g} - i \underbrace{\frac{\dot{g}(t)}{[\omega_c - i\kappa]^2}}_{\omega_c^{-2} g\omega} \pm \underbrace{\frac{\omega_0 g(t)}{[\omega_c - i\kappa]^2}}_{\omega_c^{-2} g\omega_0}$$

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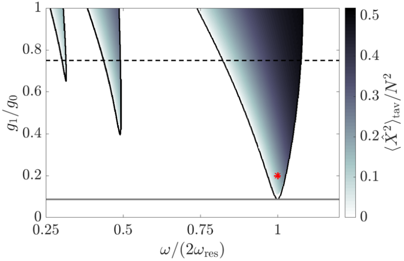
Comparison of atom-cavity and atom-only



Comparison in time-crystalline phase

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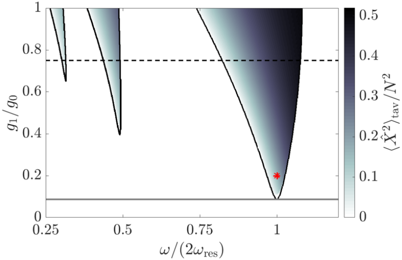

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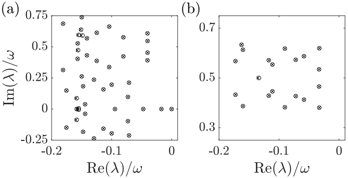

Comparison of quantum dynamics ($N \leq 20$)

Floquet spectrum $\lambda\hat{\varrho} = \left[\mathcal{L} - \frac{\partial}{\partial t}\right]\hat{\varrho}$

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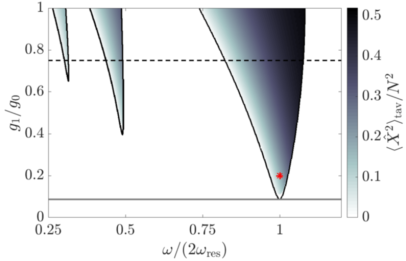

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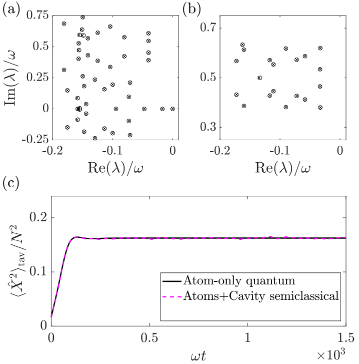

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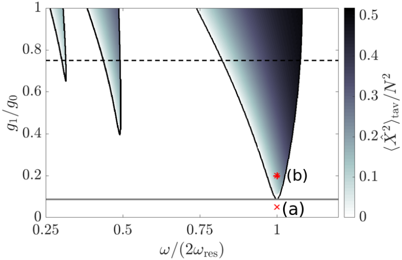
Comparison to semiclassical dyn. ($N \sim 100$)

Excellent agreement!

Spectral features of the time crystalline phase



We are now able to study spectral features for large N

Time-crystal breaks discrete time-translational symmetry

$\rightarrow$ Closing gap at $\lambda = i\frac{\omega}{2} + \gamma$ (subharmonic response)

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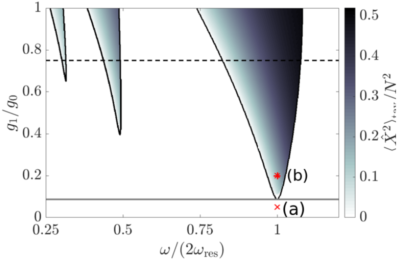

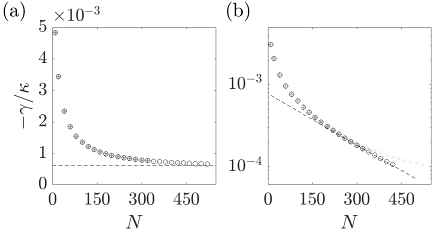

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(a) λ remains gapped. (b) λ closes exponentially in N

First description of this effect with only atomic degrees of freedom

Atom-only mean field theory

Mean-field description of dynamics

$$\hat{X} = \hat{b}_\uparrow^\dagger \hat{b}_\downarrow + \hat{b}_\downarrow^\dagger \hat{b}_\uparrow,\ \hat{Y} = i(\hat{b}_\uparrow^\dagger \hat{b}_\downarrow - \hat{b}_\downarrow^\dagger \hat{b}_\uparrow),\ \hat{Z} = \hat{b}_\uparrow^\dagger \hat{b}_\uparrow - \hat{b}_\downarrow^\dagger \hat{b}_\downarrow\ ,\ \varphi_s = \langle \hat{b}_s \rangle$$

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Time-dependent interactions V_0

and dissipation $V_1 \propto \frac{1}{(\omega-\omega_0)^2+\kappa^2} - \frac{1}{(\omega+\omega_0)^2+\kappa^2}$ (Cooling)

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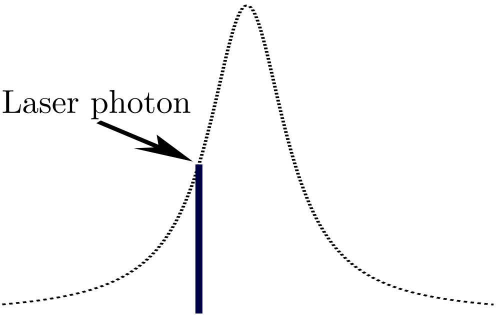

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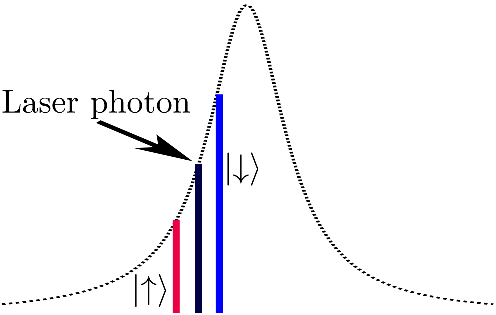

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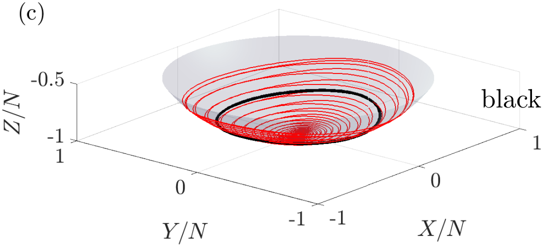

black $\rightarrow V_1 \neq 0$, red $\rightarrow V_1 = 0$

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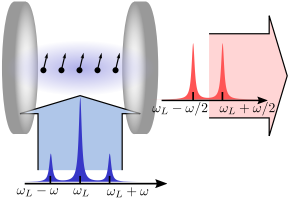
Conclusion



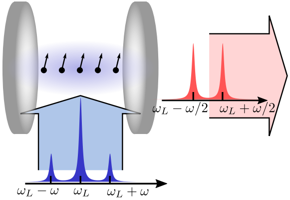
- Effective master equation for spins coupled by bosons

  $\rightarrow$ Description of mediated interactions + dissipation

  $\rightarrow$ More efficient description

  S. B. Jäger et al., Phys. Rev. Lett **129**, 063601 (2022).

Conclusion



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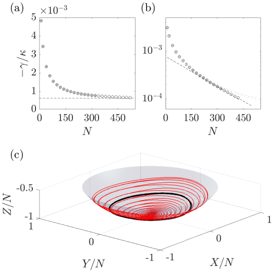
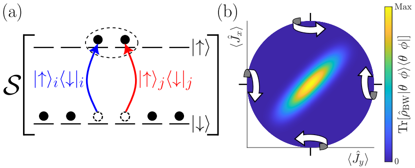


- Description for time-periodic dissipative Dicke model
  - → Description of exponentially closing gap
  - → New insights into stabilization mechanism

  S. B. Jäger et al., arXiv:2310.00046 (2023).



Squeezing → J. T. Reilly, S. B. Jäger, J. D. Wilson, J. Cooper, S. Eggert, M. J. Holland, arXiv:2310.07694

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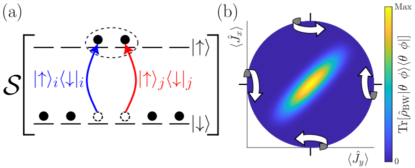
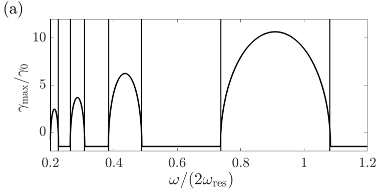
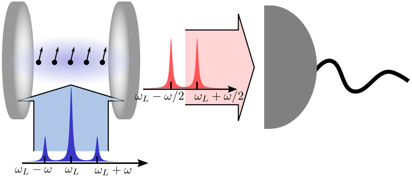
Thank you for your attention!



Squeezing $\rightarrow$ J. T. Reilly, S. B. Jäger, J. D. Wilson, J. Cooper, S. Eggert, M. J. Holland, arXiv:2310.07694

Comparison of Floquet theory to full dynamics





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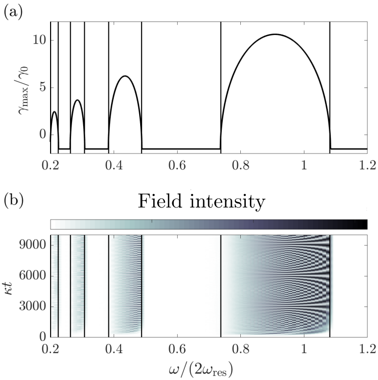
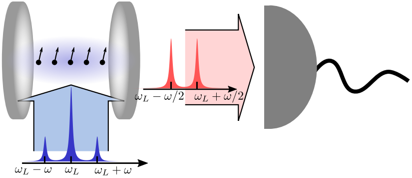
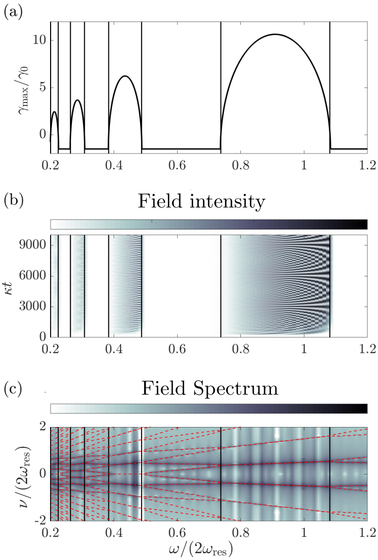
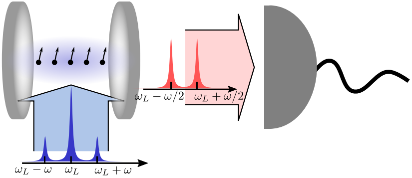




- If $\gamma > 0$ we expect large field
- Floquet theory predicts threshold

Comparison of Floquet theory to full dynamics





- If $\gamma > 0$ we expect large field
- Floquet theory predicts threshold
- Floquet theory predicts frequency