

Associative Memory and Spin Glasses with Confocal Cavity QED

Jonathan Keeling



University of
St Andrews

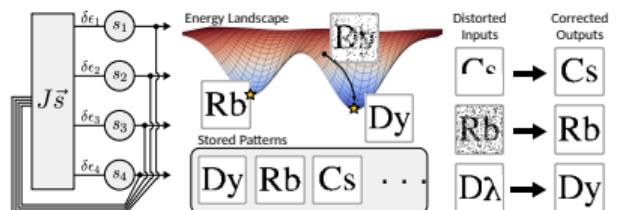
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Quantum Spinoptics, Ingleheim

Multimode cavity QED

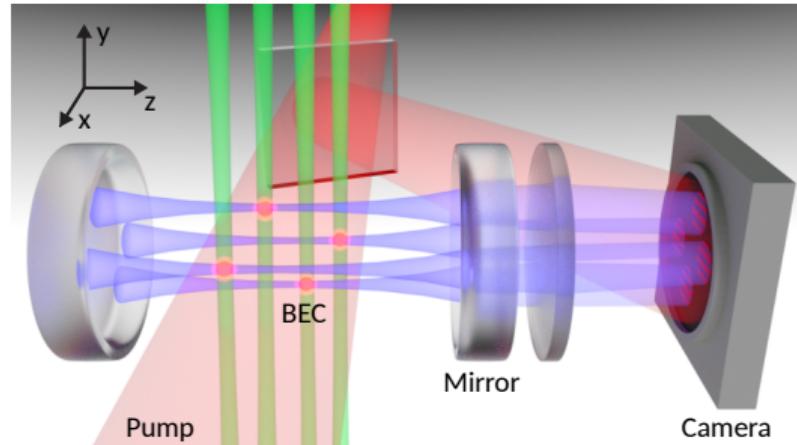
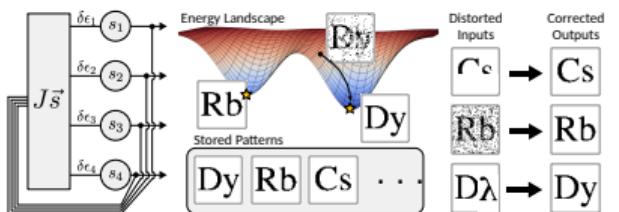
- Spin glass/associative memory:
 - ▶ Energy $E = - \sum_{ij} J_{ij} s_i s_j$
 - ▶ Energy minimisation dynamics



- Confocal cavity QED:
 - ▶ Atom (clumps) \rightarrow spins
 - ▶ Cavity $\rightarrow J_i \propto \sum_s \Xi_s(\vec{r}) \Xi_s(\vec{r})$
 - ▶ Open quantum system dynamics
 - ▶ Superadiabatic phase

Multimode cavity QED

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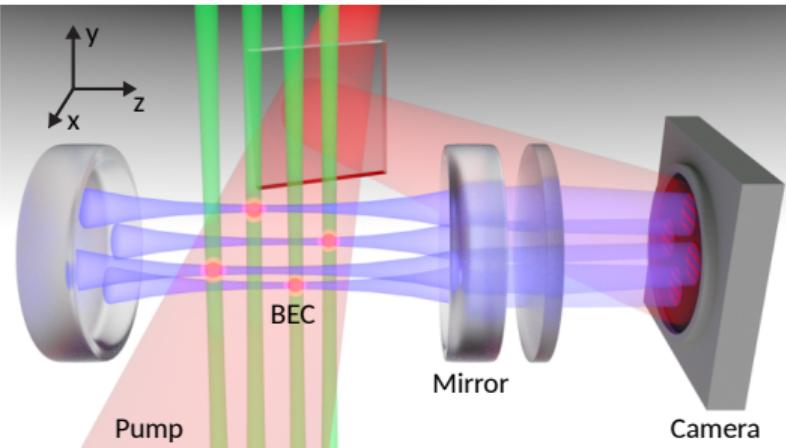
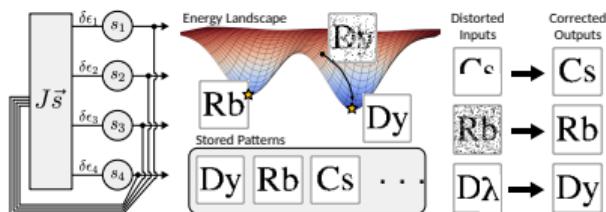
– Cavity → $J_{ij} \propto \sum_n \Xi_n(i) \Xi_n(j)$

– Open quantum system dynamics

– Superadiabatic phase

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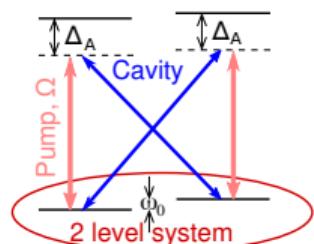
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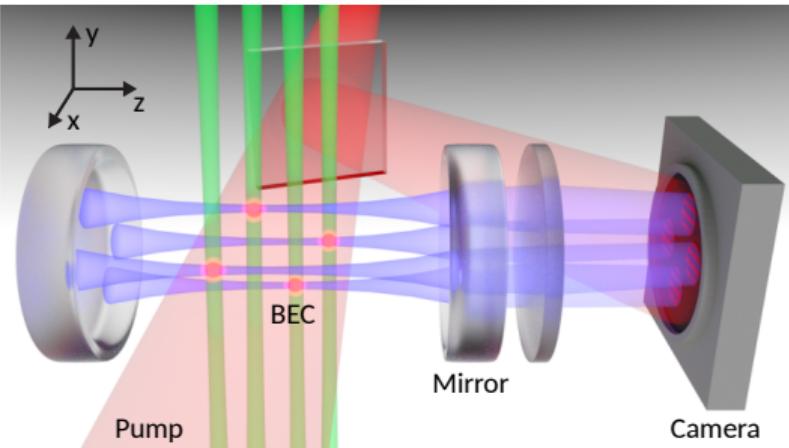
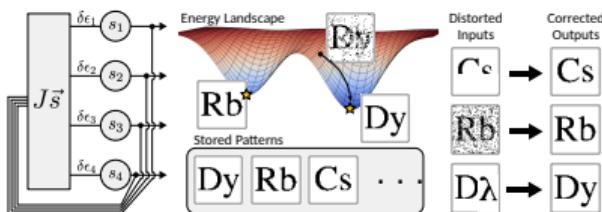
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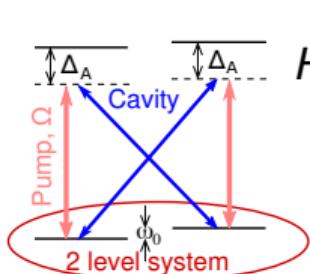
Confocal cavity QED

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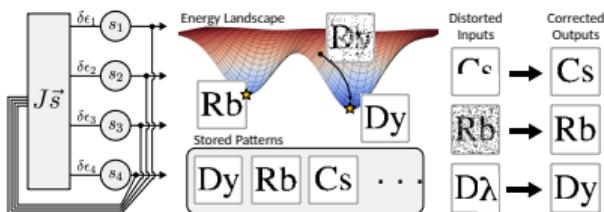
→ Superadiabatic phase



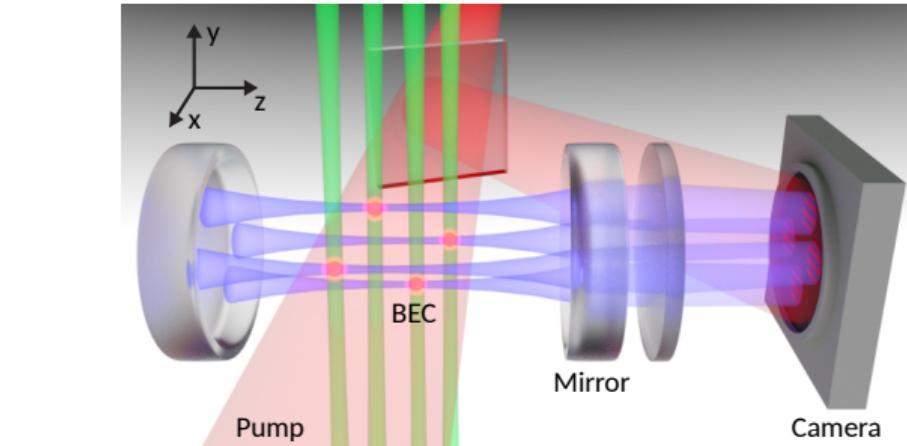
$$H_{\text{eff}} = \sum_{\mu} \overbrace{(\omega_{\mu} - \omega_P)}^{-\Delta_{\mu}} \hat{a}_{\mu}^{\dagger} \hat{a}_{\mu} + \sum_i \frac{\omega_0}{2} \mathbf{S}_i^z + \frac{\Omega g}{\Delta_A} \sum_{\mu} \Xi_{\mu}(\mathbf{r}_n) \mathbf{S}_n^x (\hat{a}_{\mu} + \hat{a}_{\mu}^{\dagger})$$

Multimode cavity QED

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- Confocal cavity QED:
 - ▶ Atom (clumps) \rightarrow spins
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 - ▶ Open quantum system dynamics
 - ▶ Superradiant phase



A schematic diagram of a 2-level system coupled to a cavity. A red oval labeled '2 level system' contains a dipole moment d_0 . Two red arrows labeled 'Pump, C' point towards the system from opposite directions. Above the system, a blue double-headed arrow indicates a frequency difference Δ_A . A blue arrow labeled 'Cavity' points upwards from the system. The entire setup is labeled 'Cavity'.

$$H_{\text{eff}} = \sum_{\mu} \overbrace{(\omega_{\mu} - \omega_P)}^{-\Delta_{\mu}} \hat{a}_{\mu}^{\dagger} \hat{a}_{\mu} + \sum_i \frac{\omega_0}{2} \mathbf{S}_i^z + \frac{\Omega g}{\Delta_A} \sum_{\mu} \Xi_{\mu}(\mathbf{r}_n) \mathbf{S}_n^x (\hat{a}_{\mu} + \hat{a}_{\mu}^{\dagger})$$

g_{eff} Navigation icons

1 Introduction: Multimode CCQED & spin models

2 Theory: classical dynamics above threshold

- Connectivity
- Dynamics

3 Theory: Quantum dynamics near threshold

4 Experiment: Directly observing replica symmetry breaking

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Confocal cavity QED Connectivity

- Cavity QED:

$$J_{ij} = - \sum_{\mu} \left(\frac{g_{\text{eff}}^2 \Delta_{\mu}}{\Delta_{\mu}^2 + \kappa^2} \right) \tilde{\Xi}_{\mu}(\mathbf{r}_i) \tilde{\Xi}_{\mu}(\mathbf{r}_j)$$

- Confocal limit

$$J_{ij} = -\omega_0 \left[\beta \delta_{ij} + \cos\left(2\frac{\mathbf{r}_i - \mathbf{r}_j}{w_0}\right) \right]$$

- Random (Gaussian) positions, width w

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- Random (Gaussian) positions, width w

• $w \ll w_0$, Frustration — memory?

• $w \gg w_0$, Un-correlated.

$$\langle \delta_{ij} \rangle = 1/(1 + 8(w/w_0)^2)$$

Confocal cavity QED Connectivity

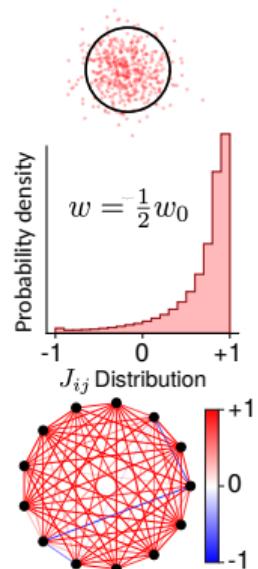
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- Random (Gaussian) positions, width w
 - ▶ $w \ll w_0$, Ferromagnet



What does this mean for memory?
Is it a ferromagnet?
 $J_{ij} = 1/(1 + \exp(M_{ij}))$

Confocal cavity QED Connectivity

- Cavity QED:

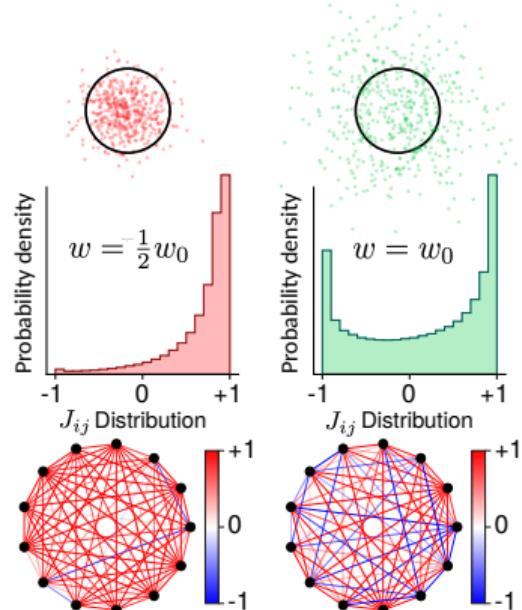
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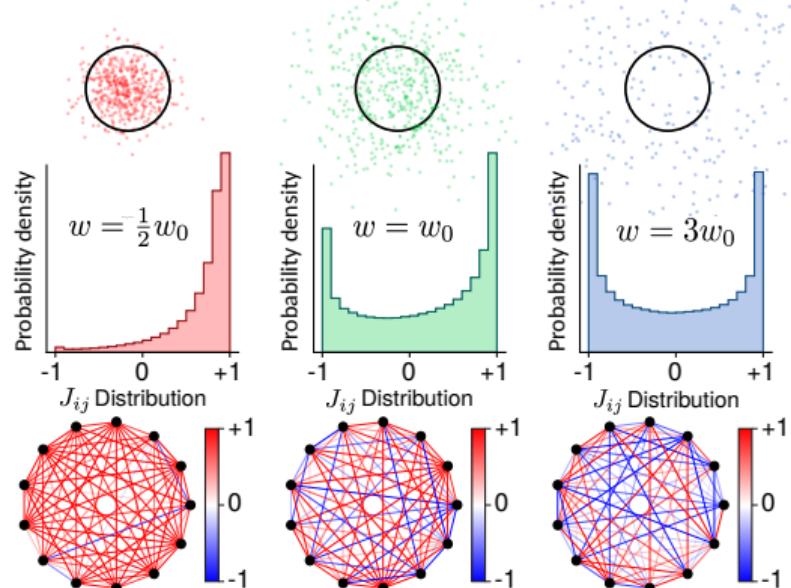
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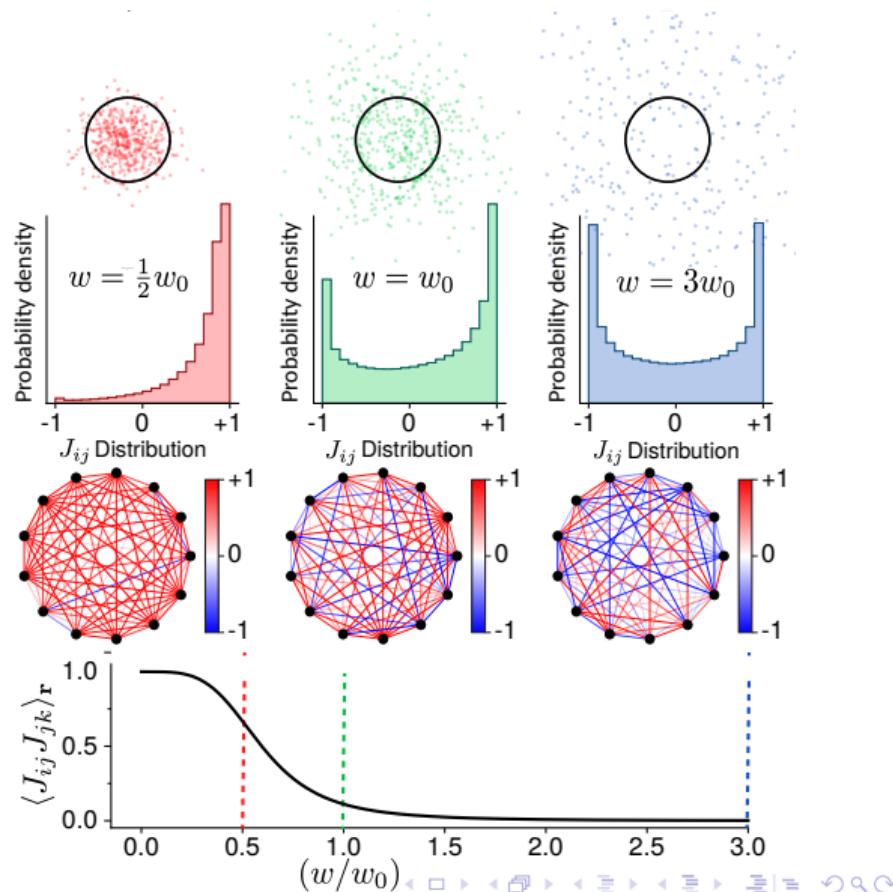
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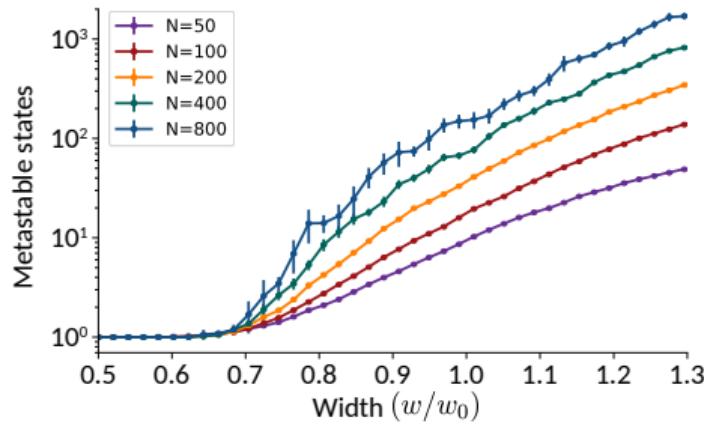
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- Random (Gaussian) positions, width w

- ▶ $w \ll w_0$, Ferromagnet
- ▶ $w \gtrsim w_0$, Frustration — memory?
- ▶ $w \gg w_0$, Uncorrelated.
 $\langle J_{ij} J_{jk} \rangle = 1/[1 + 8(w/w_0)^4]$



Associative memory behaviour

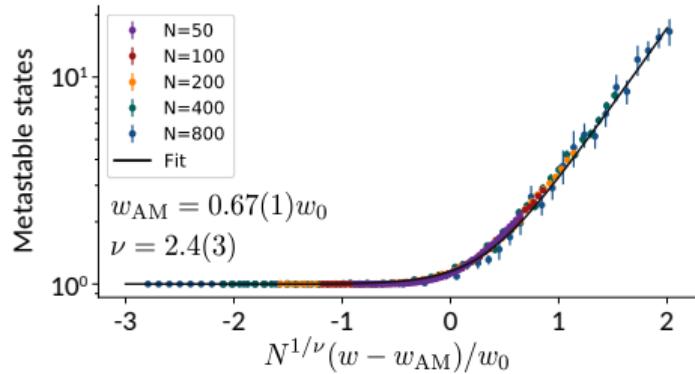
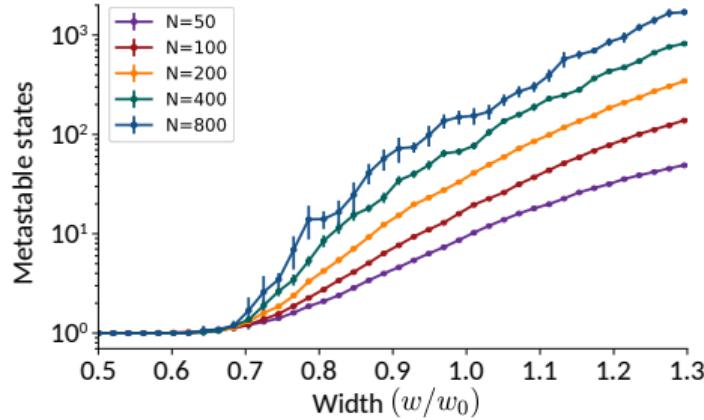


- Minima of $E = - \sum_{ij} J_{ij} s_i s_j$
- **Associative memory** – # minima $\gg 1$

Near transition

$$\# \sim \sqrt{1 + A \exp(BN^2/(w - w_m)/w_0)}$$

Associative memory behaviour



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Spin glass transition?

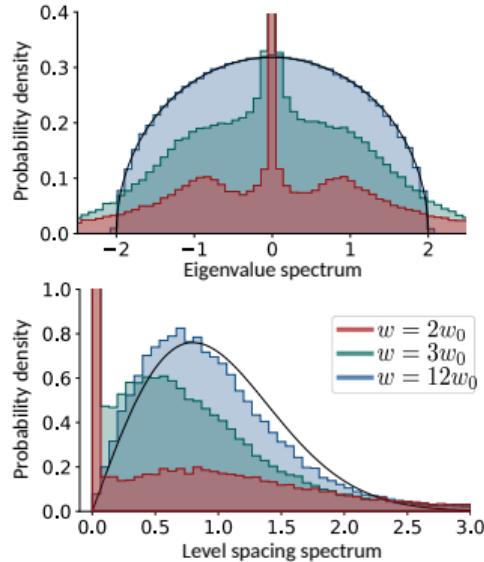
- Spin glass has $\# \sim e^N$ vs $\# \sim e^{N^{0.4}}$

— Sherrington-Kirkpatrick model
 $J_{ij} \sim K$ Gaussian iid.

Spin glass transition?

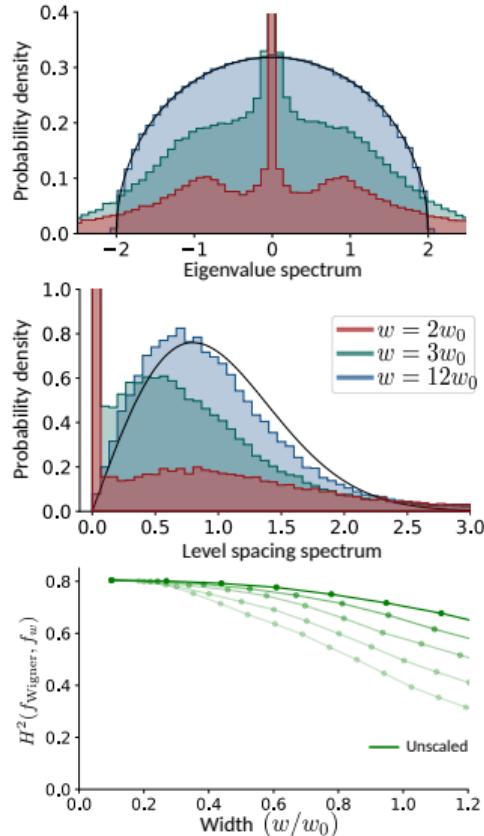
- Spin glass has $\# \sim e^N$ vs $\# \sim e^{N^{0.4}}$
- Sherrington-Kirkpatrick model:
 J_{ij}^{SK} Gaussian iid.
 - Compare J_{ij}^{SK} to random matrix
 - Measure difference, $\int dx (\sqrt{J_{ii}} - \sqrt{J_{jj}})^2$
 - $J_{ij}^{SK} \sim J^K$ if $(w/w_0)^2 > N$

Spin glass transition?



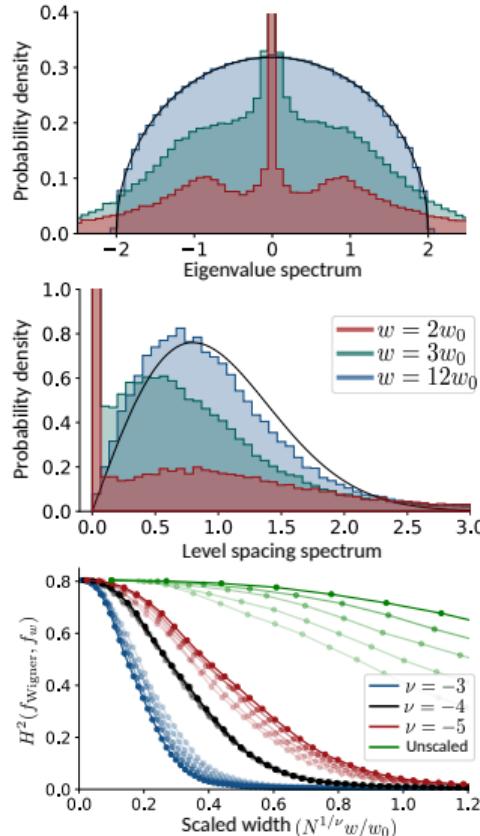
- Spin glass has $\# \sim e^N$ vs $\# \sim e^{N^{0.4}}$
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Spin glass transition?



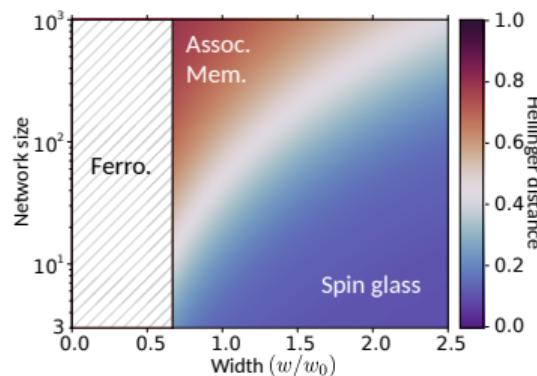
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 - ▶ Compare J_{ij}^{CCQED} to random matrix
 - ▶ Measure difference, $\int dx (\sqrt{f_w} - \sqrt{f_{\text{Wigner}}})^2$
 - ▶ $J_{ij}^{\text{CCQED}} \sim J_{ij}^{SK}$ if $(w/w_0)^4 \gg N$

Summary:



1 Introduction: Multimode CCQED & spin models

2 Theory: classical dynamics above threshold

- Connectivity
- Dynamics

3 Theory: Quantum dynamics near threshold

4 Experiment: Directly observing replica symmetry breaking

Confocal cavity QED Dynamics

- Open quantum system dynamics

$$\frac{d}{dt}\rho = -i[H, \rho] + \sum_{\mu} \mathcal{D}[\sqrt{\kappa}a_{\mu}]$$

$$H = -\Delta \sum_{\mu} a_{\mu}^{\dagger} a_{\mu} + g_{\text{eff}} \sum_{\mu, i} \tilde{\Xi}_{\mu}(\mathbf{r}_i) S_i^x (a_{\mu}^{\dagger} + a_{\mu}) + \omega_0 \sum_i S_i^z$$

• Simplify

• Function of spin-flip energy

→ Atom-only equations

• $T_{\text{ex}} \gtrsim J_z$, so use $S_y \gg 1$

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- Simplify:

- ▶ Atom-only equations
 - ▶ Treats perturbatively
 - ▶ Independent Boson model
 - ▶ Effective EQM

$$\frac{d}{dt}\rho = \sum_j K(\rho_j) D(S_j^+)$$

$$+ K(\rho_j^*) D(S_j^-) + \dots$$

→ Function of spin-flip energy

→ $T_{\text{ex}} \gtrsim J$, so use $\delta \gg J$

Confocal cavity QED Dynamics

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$$\frac{d\rho}{dt} = \sum_j K(j) \rho(S_j^z)$$

$$+ K(j) \rho(S_j^z) + \dots$$

Function of spin-flip energy

$\rightarrow T_{\text{ex}} \gtrsim J$, so use $\delta j \gg 1$

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$$\begin{aligned} \frac{d}{dt}\rho &= \sum_i K_i(\delta\epsilon_i^+) \mathcal{L}[S_i^{x+}] \\ &\quad + K_i(\delta\epsilon_i^-) \mathcal{L}[S_i^{x-}] + \dots \end{aligned}$$

Confocal cavity QED Dynamics

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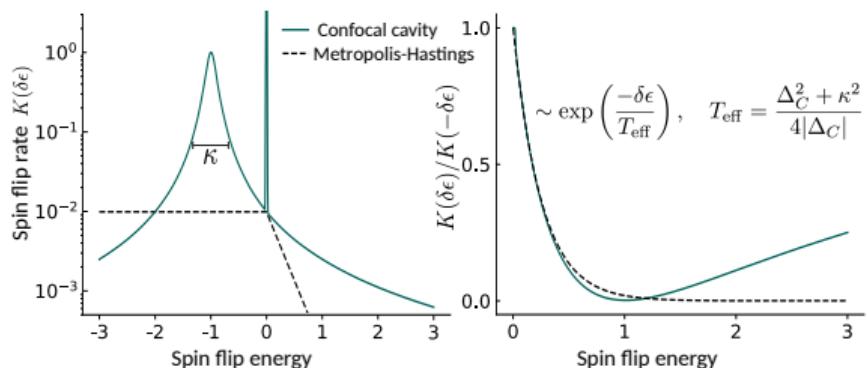
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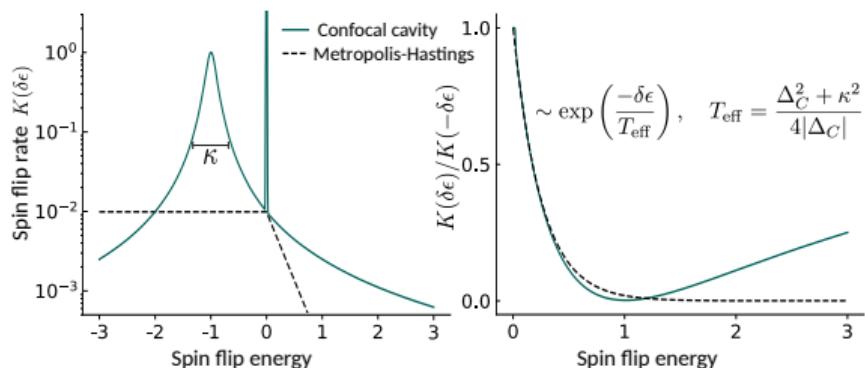
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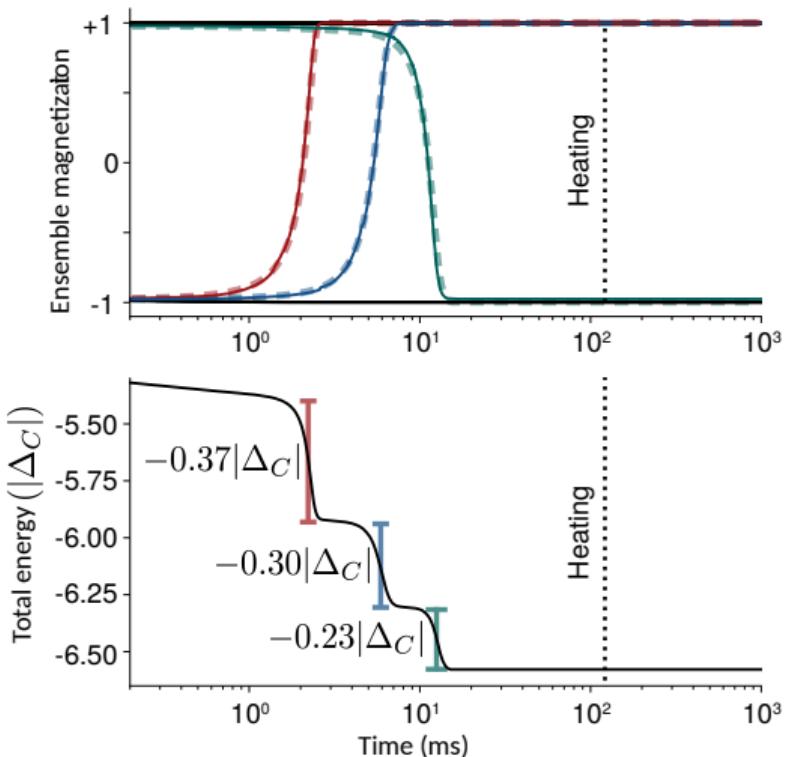
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- $T_{\text{eff}} \gtrsim J_{ij}$ so use $S_i \gg 1$

Effective dynamics of clumps



- Simulate via stochastic flips (solid)
- Fast (superradiant) flip, long wait.
→ Deterministic model (dashed)

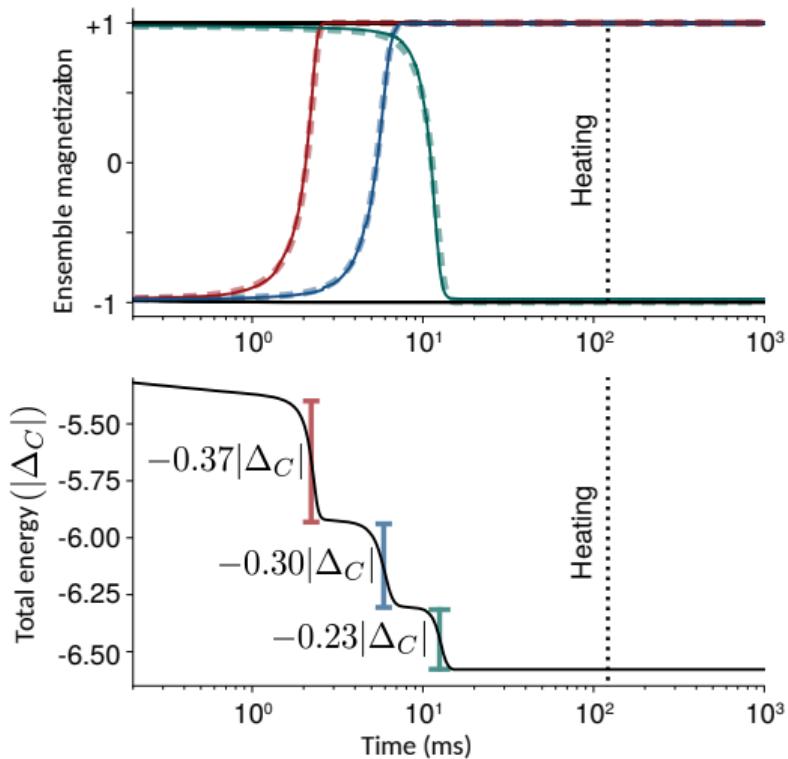
Solution:

$$S(t) = \text{sgn}(\sum_j S_j) \tanh(\beta E_j)$$

$$\dot{E}_j \propto -\text{sgn}(S_j) S_j \frac{\partial}{\partial E_j} \ln S_j$$

Smallest \dot{E} flips first →
superradiant steplike decay!

Effective dynamics of clumps

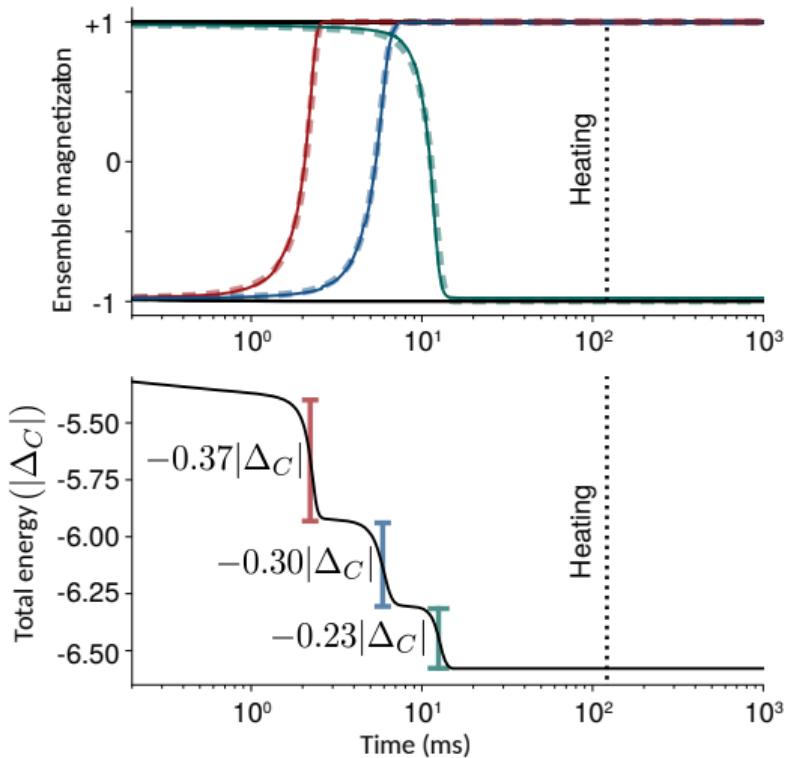


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$$S_i^x(t) = \text{sgn} \left(\sum_j J_{ij} S_j^x \right) \tanh \left[\frac{t - t_i^0}{\tau_i^{\text{flip}}} \right],$$
$$t_i^0 \propto -\text{sgn} \left(\sum_j J_{ij} S_j^x S_i^x \right) \frac{S_i / \ln(S_i)}{|\delta \epsilon_i|}$$

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Hebbian Connectivity + Dynamics: Basins of attraction

- Basin of attraction: Fixed point, distort, relax

- Illustrate for Hebbian $J_{ij} = \sum_{\sigma, \sigma'} G_i^\sigma G_j^{\sigma'}$

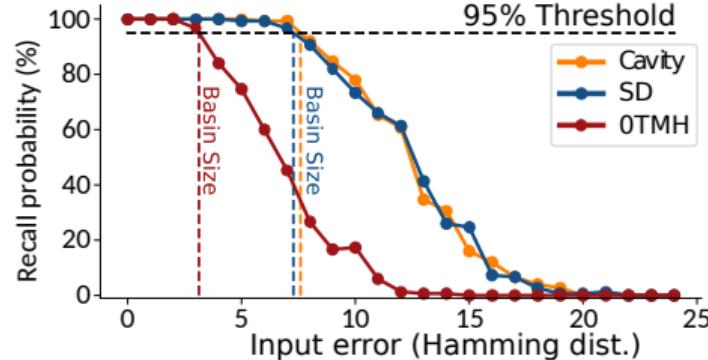
- Memories more robust with SD. Why?

- Robust memories in (SK) spin glass!

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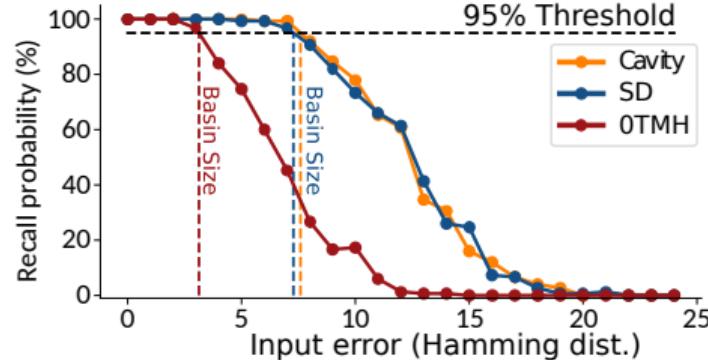
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→ SD → deterministic

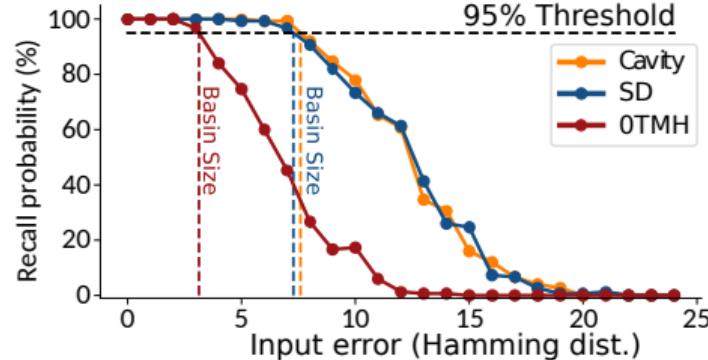
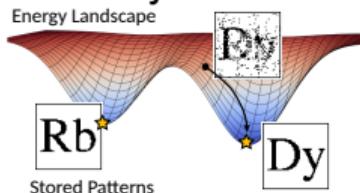
→ OTMH → stochastic

→ Robust memories in (SK) spin glass!



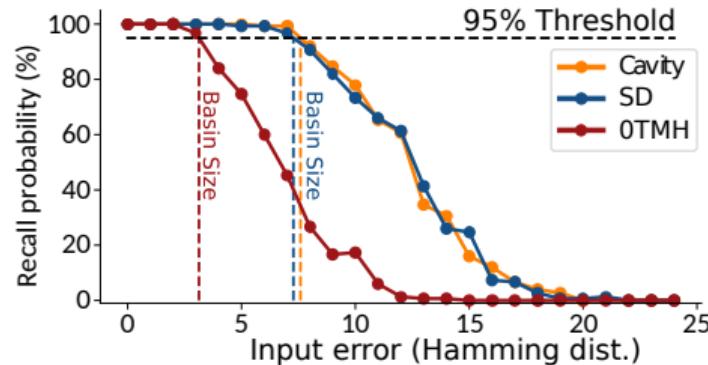
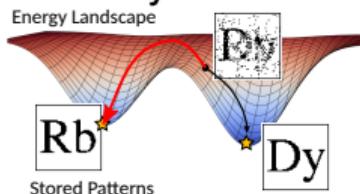
Hebbian Connectivity + Dynamics: Basins of attraction

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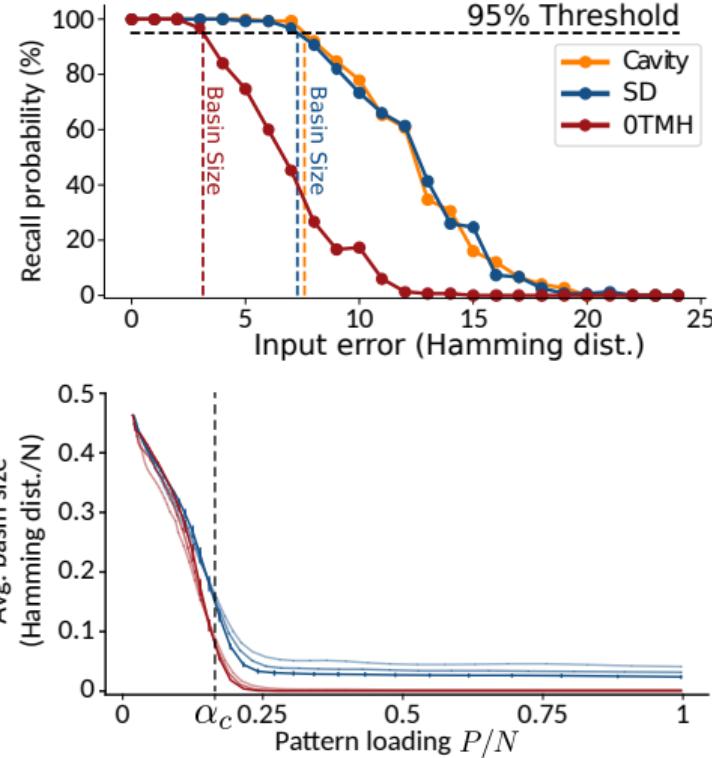
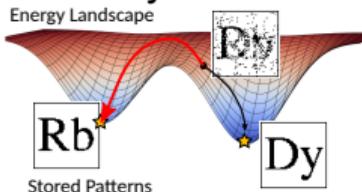
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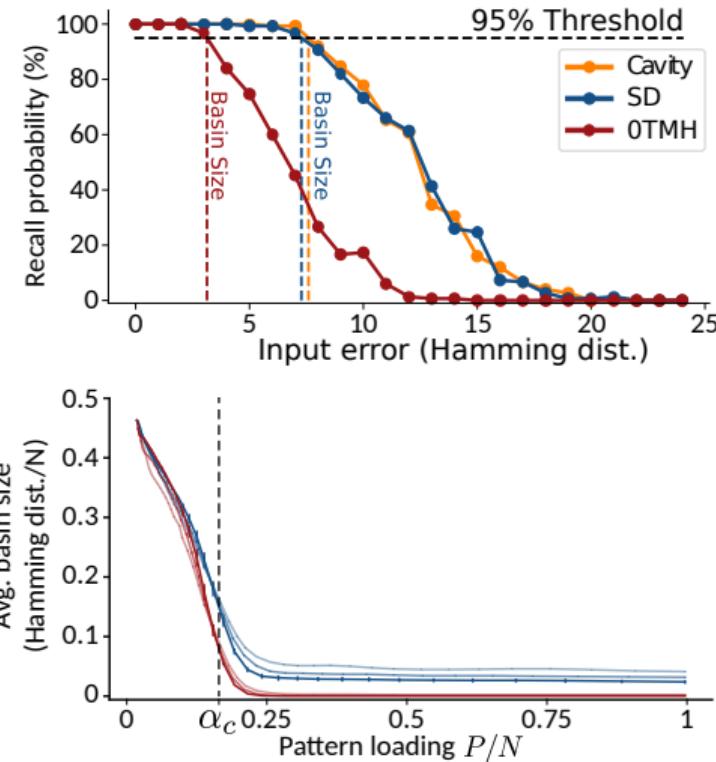
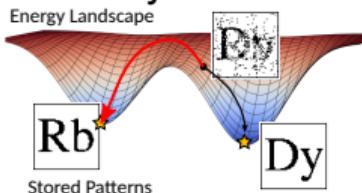
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- Dynamics

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4 Experiment: Directly observing replica symmetry breaking

Simulating quantum dynamics

- Difficult problem!

Atom-only dynamics, after Jäger et al., PRL '22; $\frac{d}{dt}\rho = -i[H, \rho] + \sum_k D[G_k]$

Quantum trajectories $|\psi\rangle$ —homodyne detection:

$N=15$ spin 1/2 possible.

Glassy J_p , ramp through threshold.

Compare:

Quantum trajectories

Mean-field: $|\Psi\rangle = \prod_i |\psi_i\rangle$

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Simulating quantum dynamics

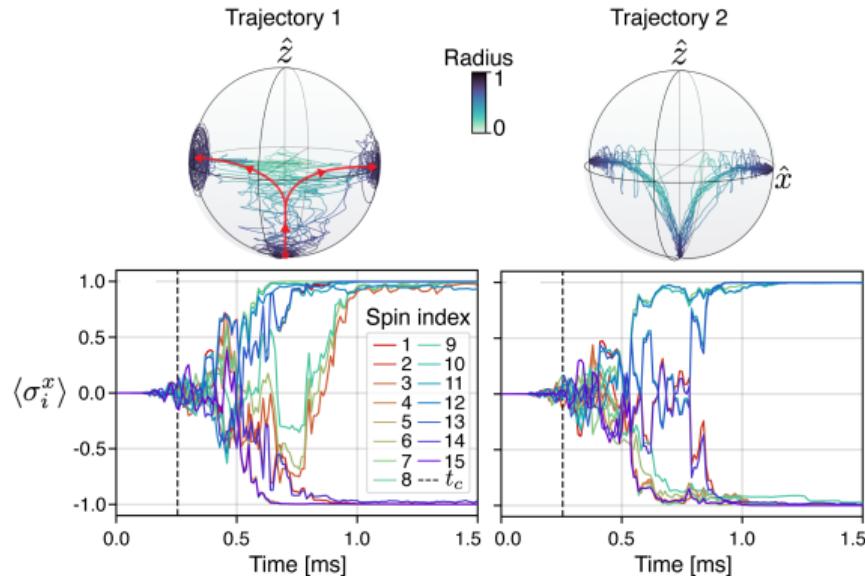
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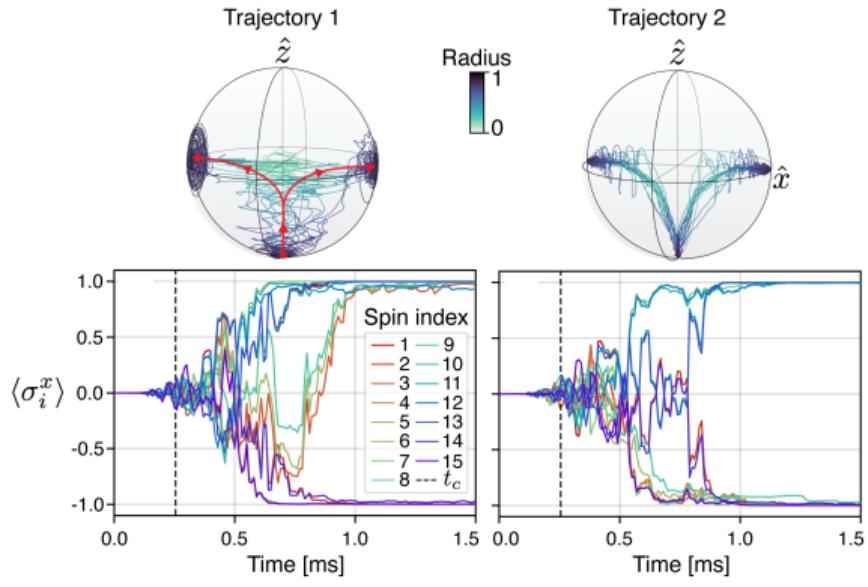
Quantum vs semiclassical

Quantum

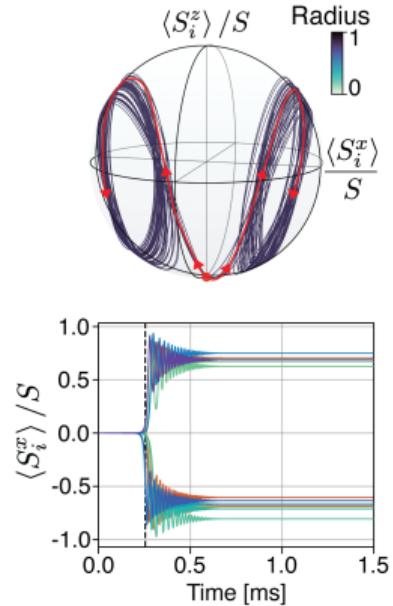


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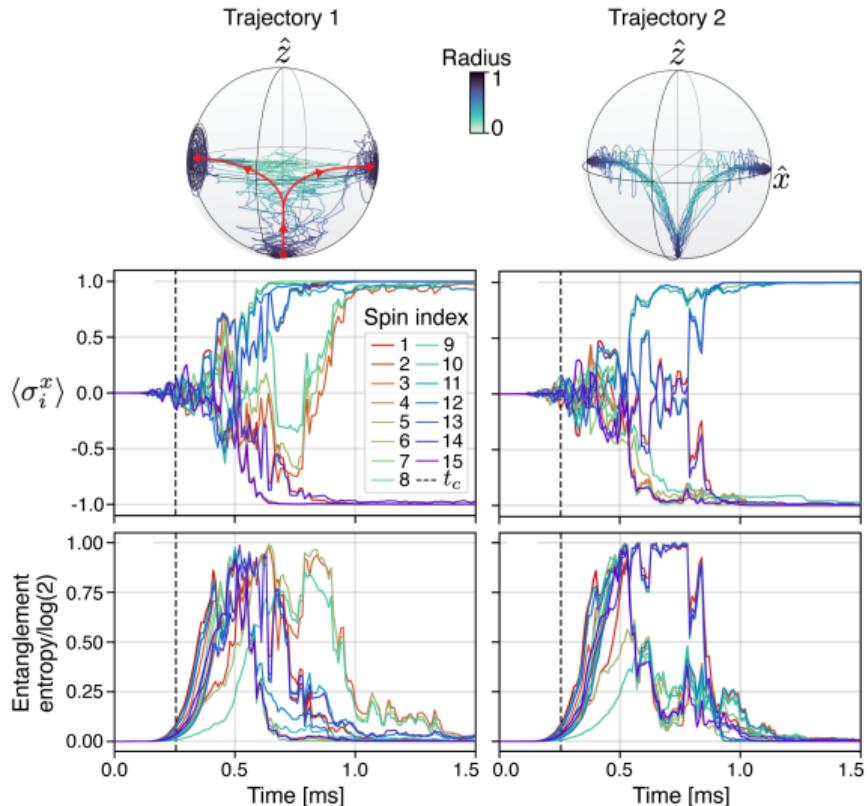


Mean-field

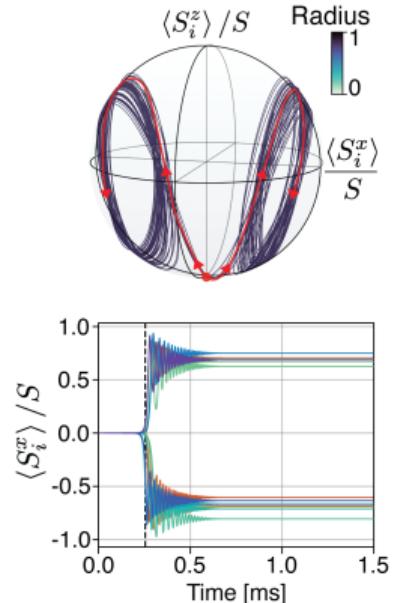


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Trajectories and replicas

- Spin glass: replica overlap:

$$q_{\alpha\beta} = \sum_i \langle S_i^\alpha \rangle \langle S_i^\beta \rangle / N$$

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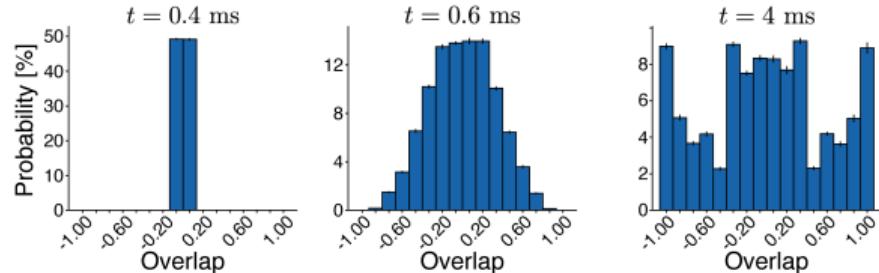
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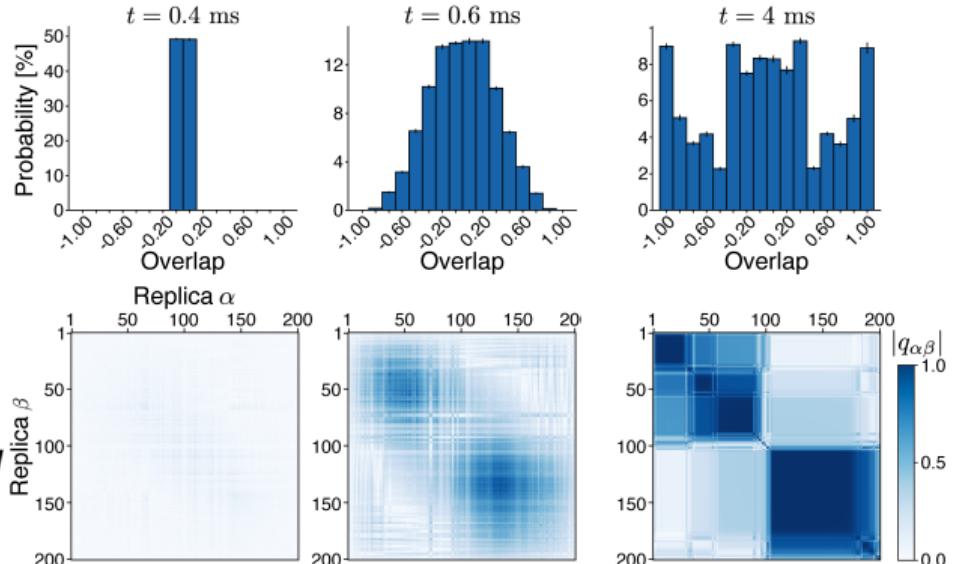
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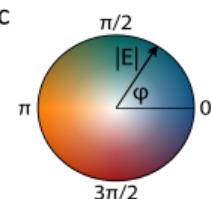
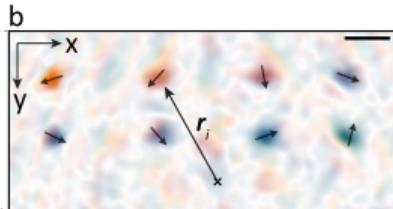
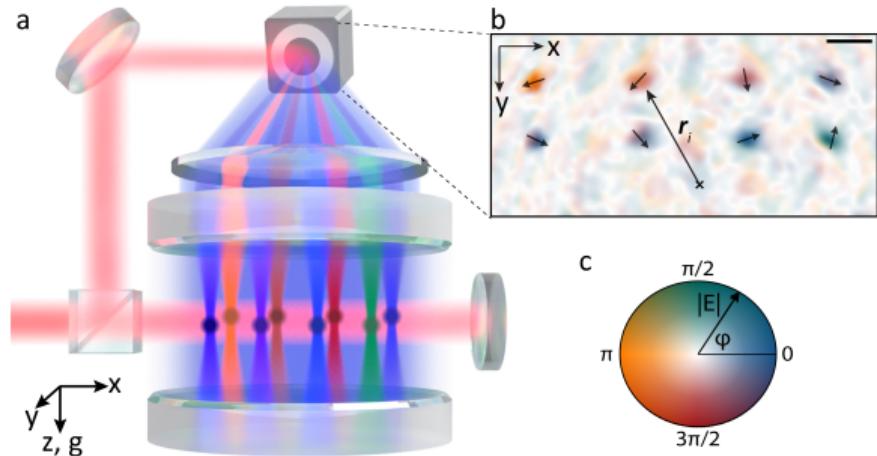
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Experimental realisation

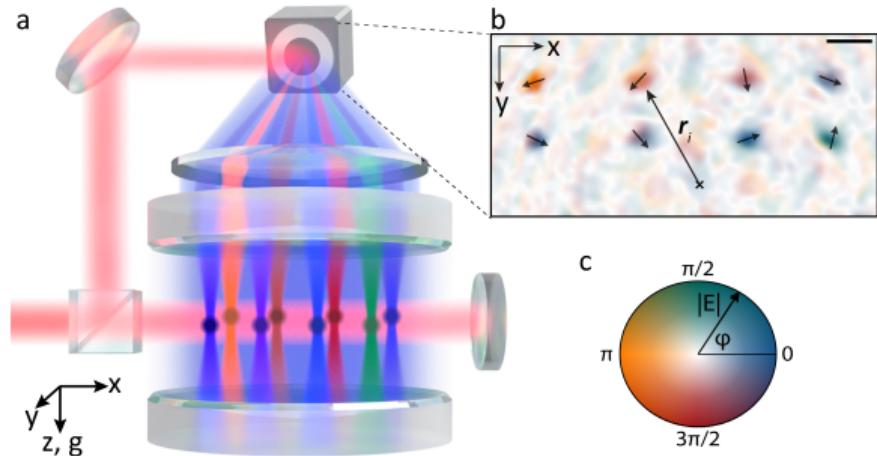
→ More complex model



→ Vector overlaps,
 $Q_{ab} = q_{ab}^2 + q_{ba}^2, R_{ab} = q_{ab}^2 - q_{ba}^2$

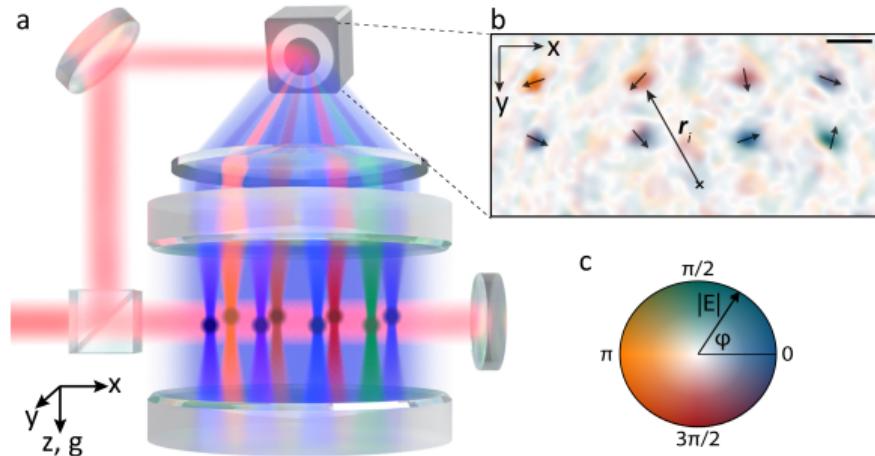
Experimental realisation

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 - Vector spin-density-wave phase



- Discrete symmetry
- k_f finite clump size effect
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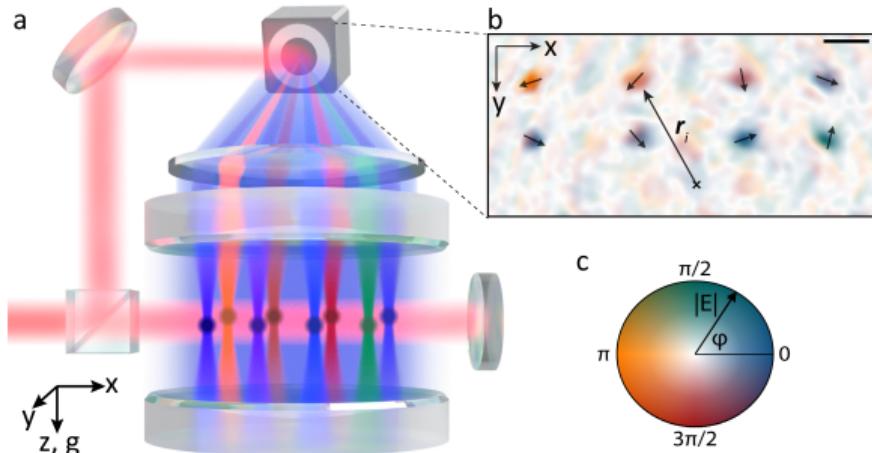
$$E_{\text{int}} = - \sum_{i,j=1}^n [J_{ij} (S_i^x S_j^x - S_i^y S_j^y) + K_{ij} (S_i^x S_j^y + S_i^y S_j^x)]$$

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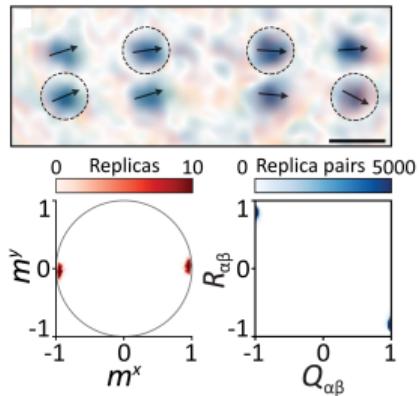
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Results

Ferromagnetic J_{ij}

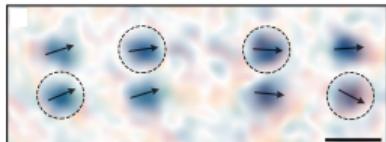
$$J_{ij} > 0$$



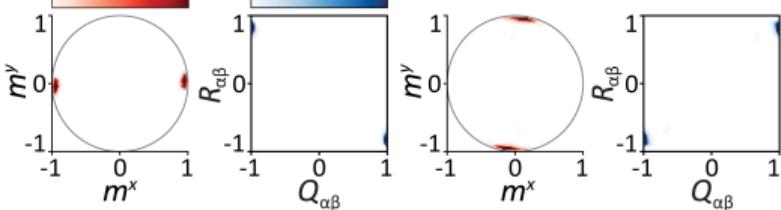
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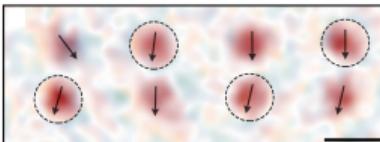
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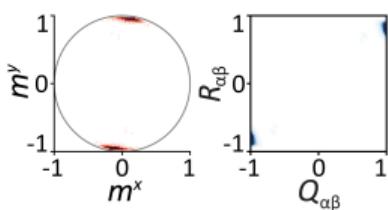
0 Replicas 10



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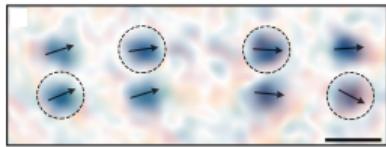
0 Replica pairs 5000



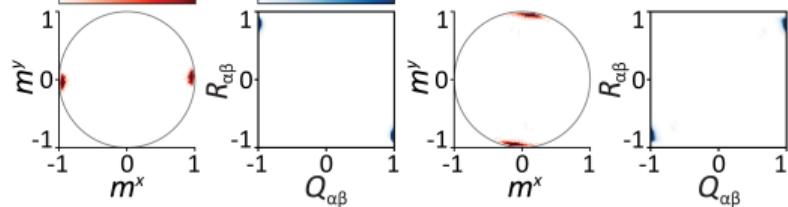
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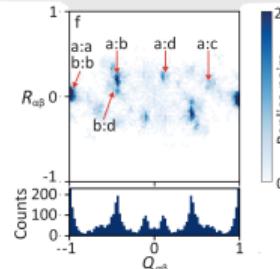
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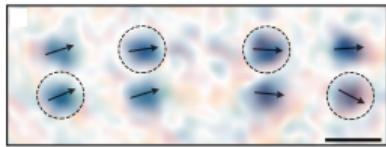
Spin glass J_{ij}



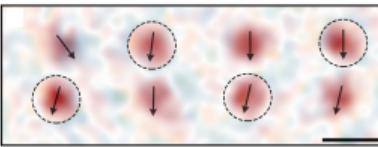
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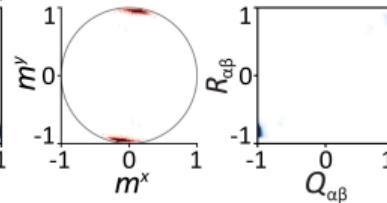
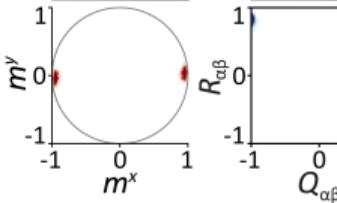
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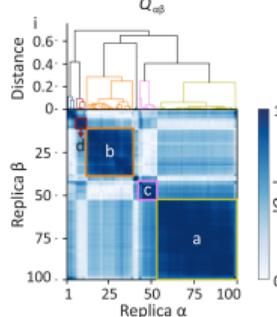
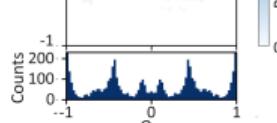
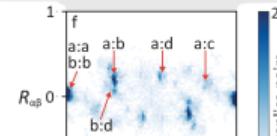
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0 Replicas 10 0 Replica pairs 5000



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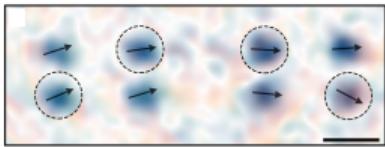


Directly measure RSB & ultrametricity

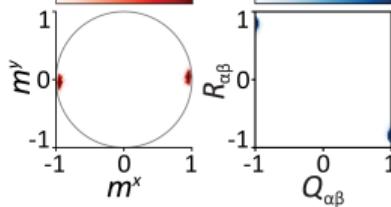
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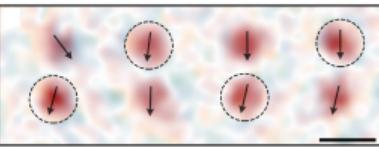
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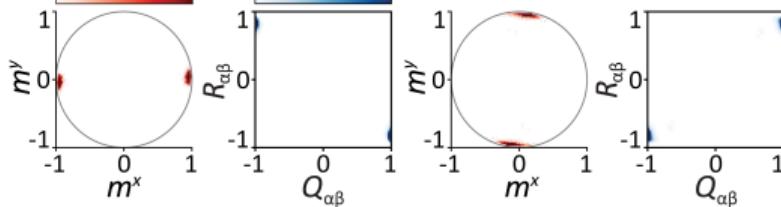
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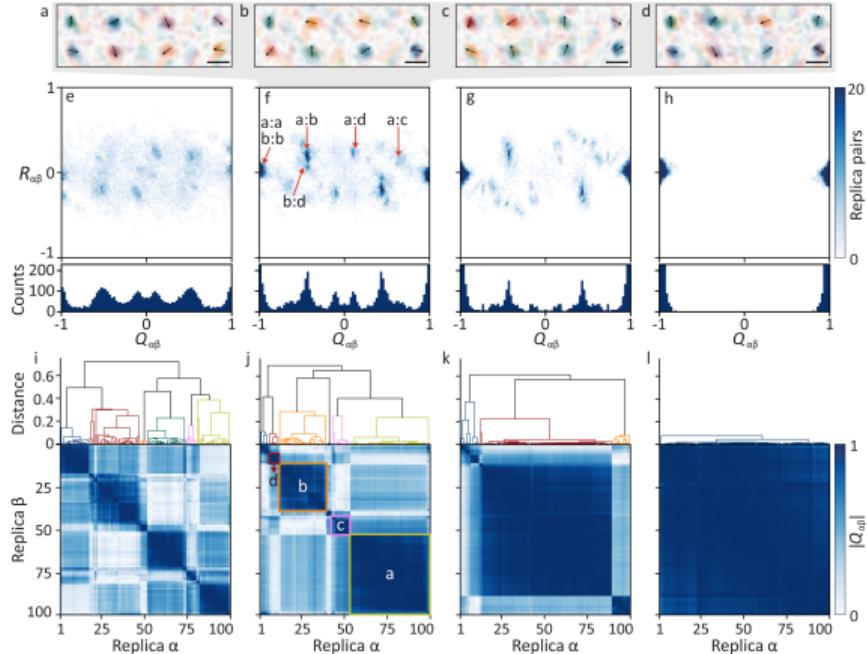
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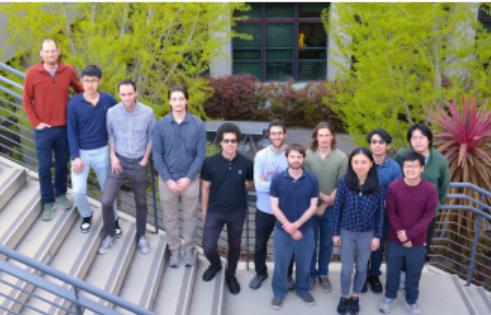
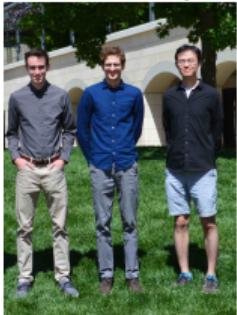
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Acknowledgments

Lev Group:

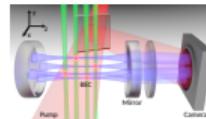


Sarang Gopalakrishnan (Princeton)
Surya Ganguli (Stanford)

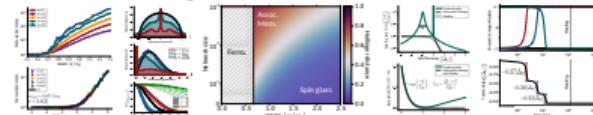


Summary

- Confocal CQED associative memory/spin glass.

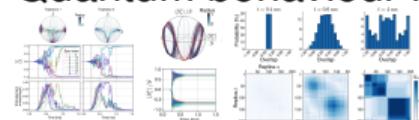


- Ferromagnet/Associative Memory/Spin Glass; superradiant steepest descent



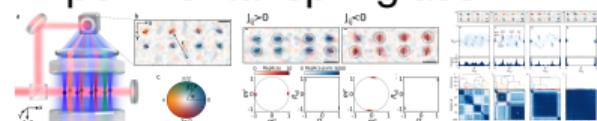
[Marsh, Guo, Kroeze, Gopalakrishnan, Ganguli, Keeling & Lev. PRX '21]

- Quantum behaviour near transition



[Marsh, Kroeze, Ganguli, Gopalakrishnan, Keeling & Lev. PRX '24]

- Experimental spin glass



[Kroeze, Marsh, Atri Shuller, Hunt, Gopalakrishnan, Keeling & Lev. arXiv:2311.04216]

5 Confocal cavities

6 More open system dynamics

7 Basins vs width

8 Training

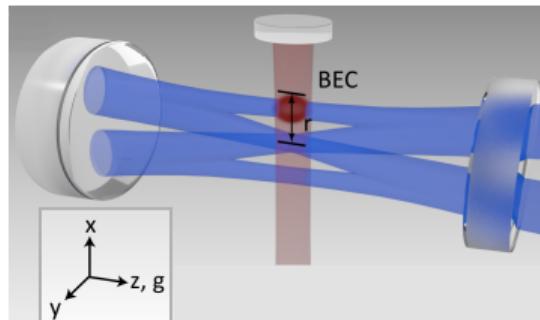
9 Replicas and trajectories

Confocal cavities

- Gaussian standing waves:

$$\Phi_\mu(\mathbf{r}) = \Xi_\mu(x, y) \cos \left[k_r \left(z + \frac{x^2 + y^2}{z + z_R^2/z} \right) - \theta_\mu(z) \right]$$

Gauss-Hermite modes $\Xi_\mu(x, y) = a_\mu(x) E_{m_\mu}(y)$



Gouy phase $\theta_\mu(z) = \text{const.} + (l_\mu + m_\mu) \arctan(z/z_R)$

Cavity Geometry:

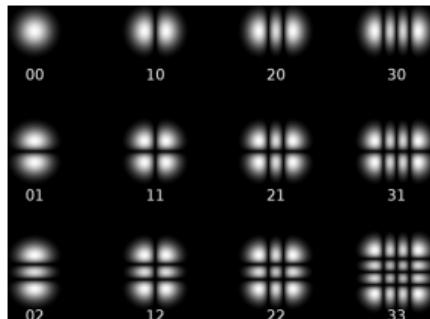
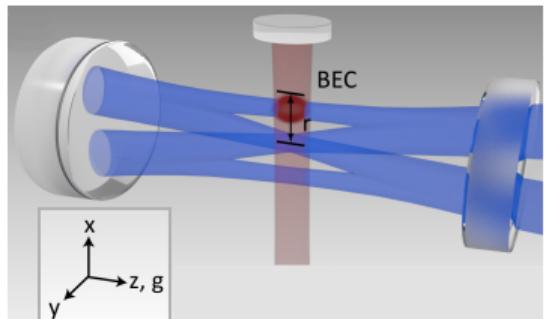
Tune degeneracy via L/R

Confocal cavities

- Gaussian standing waves:

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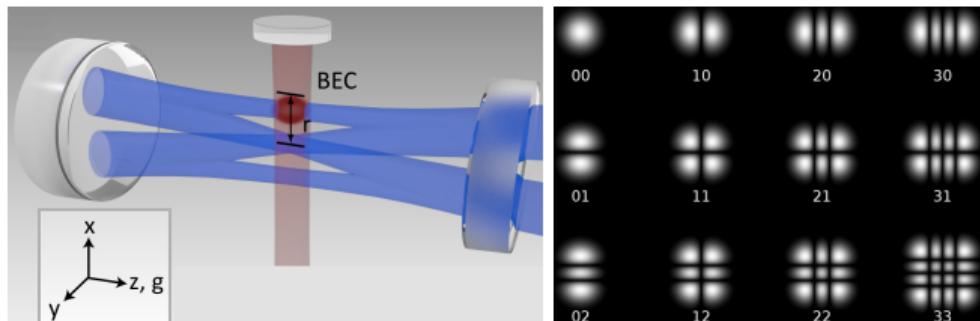
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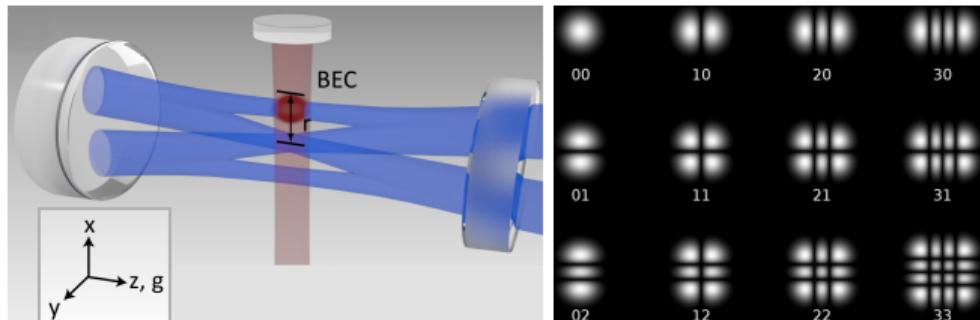
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 - ▶ Fix frequency — remove z nodes

Confocal cavities

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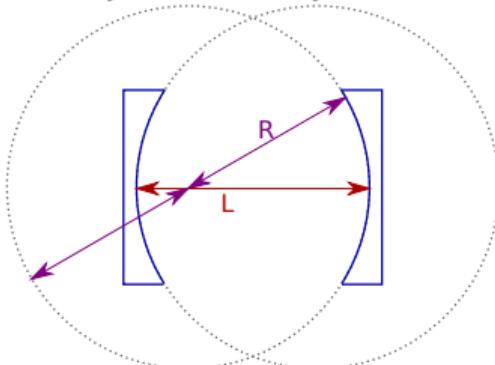
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- Cavity Geometry:



- Tune degeneracy via L/R

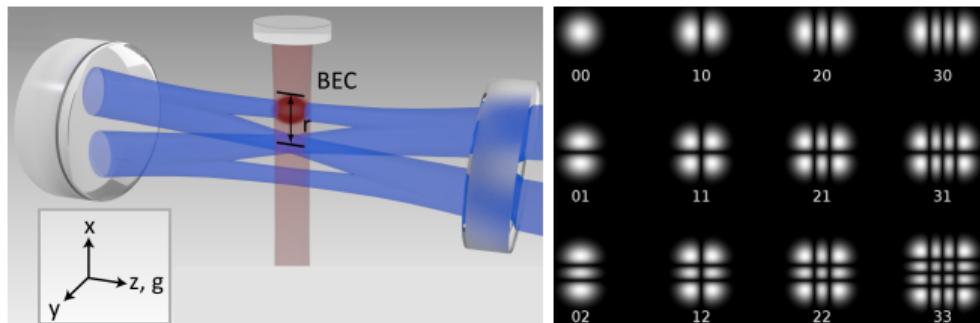
Jonathan Keeling

Confocal cavities

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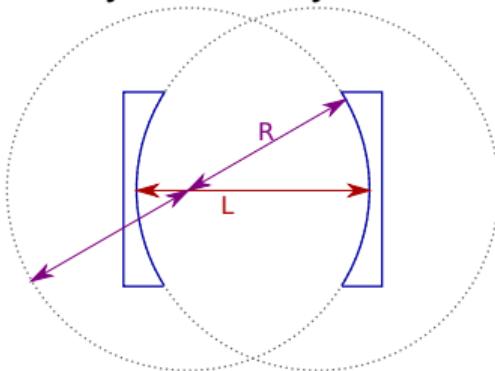
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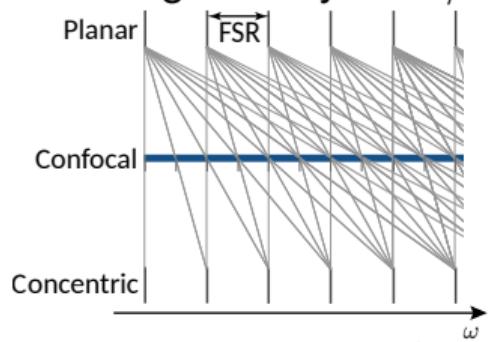


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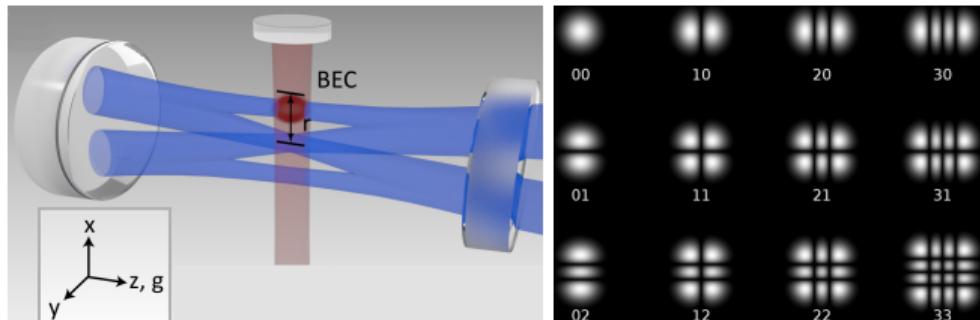


Confocal cavities

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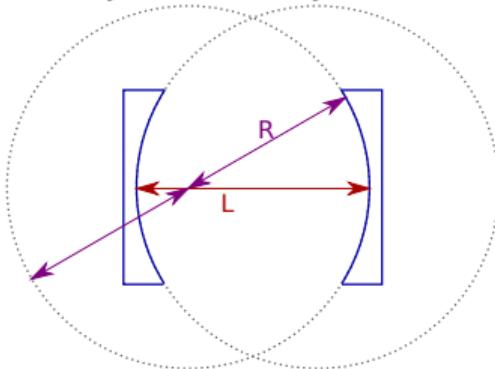
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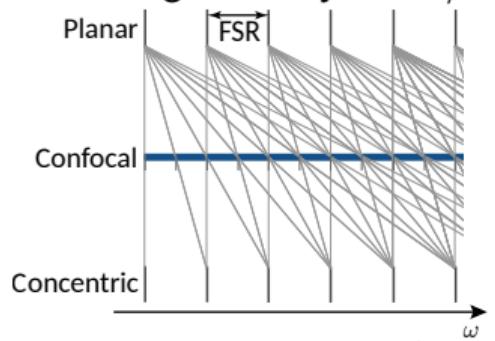


- Gouy phase $\theta_\mu(z) = \text{const.} + (l_\mu + m_\mu) \text{atan}(z/z_R)$
 - Fix frequency — remove z nodes
 - Odd/Even resonances — odd/even $l_\mu + m_\mu$.

- Cavity Geometry:



- Tune degeneracy via L/R



Confocal cavity QED Dynamics

- Open quantum system dynamics

$$\dot{\rho} = -i[H, \rho] + \kappa \sum_{\mu} \mathcal{L}[a_{\mu}]$$

$$H = -\Delta \sum_{\mu} a_{\mu}^{\dagger} a_{\mu} + g_{\text{eff}} \sum_{\mu, i} \tilde{\Xi}_{\mu}(\mathbf{r}_i) S_i^x (a_{\mu}^{\dagger} + a_{\mu}) + \omega_0 \sum_i S_i^z$$

• Simplify:

- Far above SR transition
- Atom-only equations
- Semiclassical S^z states

• Define $\tilde{\Xi}_{\mu}(t) S_i^z$ — classical S^z states

- Treat H perturbatively
- Diagonalise by

$$U = \exp \left[\sum_{\mu, i} S_i^z \left(\frac{\tilde{\Xi}_{\mu}(t)}{\Delta + i\kappa} - h.c. \right) \right]$$

[See also Fiorelli *et al.* PRL '20]

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- Simplify:

- ▶ Far above SR transition
- ▶ Atom-only equations
- ▶ Semiclassical S^x states

• $\tilde{\Xi}_{\mu}(\mathbf{r}_i) \sim \delta(r_i)$ — classical S^x states
• Treat H perturbatively
• Diagonalise by

$$U = \exp \left[\sum_{\mu} S_{\mu}^x \left(\frac{e^{i \vec{S}_{\mu} \cdot \vec{r}_i}}{\Delta + i\kappa} - \text{h.c.} \right) \right]$$

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- Simplify:
 - ▶ Far above SR transition
 - ▶ Atom-only equations
 - ▶ Semiclassical S^x states
- Noise $\sum_i \chi_i(t) S_i^x$ — classical S_i^x states
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 - ▶ Far above SR transition
 - ▶ Atom-only equations
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- Treat H_1 perturbatively
- Diagonalise by

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Stochastic and deterministic dynamics

- Effective EOM:

$$\dot{\rho} = \dots + \sum_i K_i(\delta\epsilon_i^+) \mathcal{L}[S_i^{x+}] + K_i(\delta\epsilon_i^-) \mathcal{L}[S_i^{x-}]$$

• $K_i \propto \omega_0^2$, function of:

$$\delta\epsilon_i^\pm = E((S_{j,i}^x, S_j^z \rightarrow S_j^z \pm 1)) - E(S_j^z)$$

- Differs from OTMH, $K(\delta\epsilon) = \Theta(-\delta\epsilon)$
- Detailed balance → effective temperature
- Problem: $T_{\text{eff}} \gg J_b$

Stochastic and deterministic dynamics

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Stochastic and deterministic dynamics

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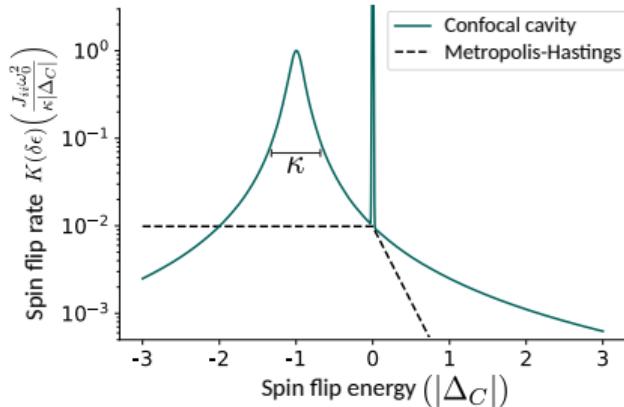
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• Detailed balance at circuit temperature

• Problem: $T_{\text{ext}} \gg J_C$



Stochastic and deterministic dynamics

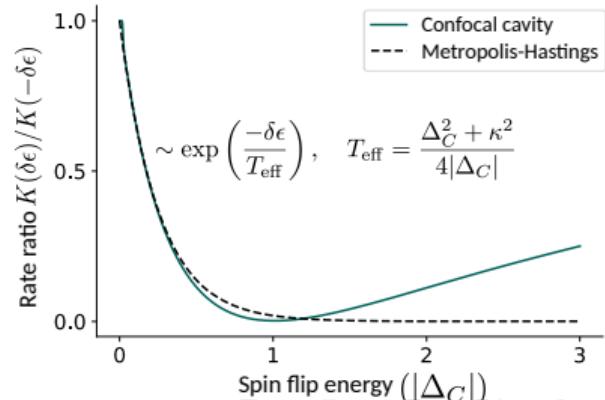
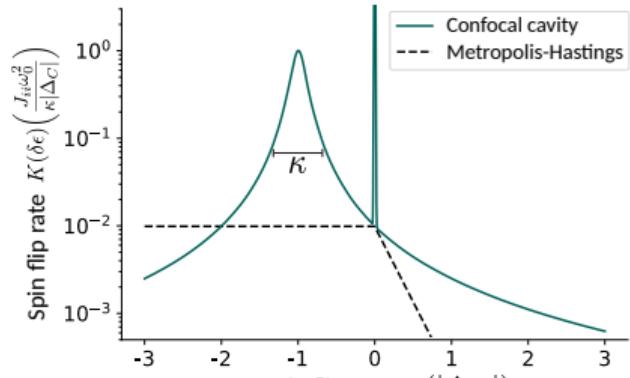
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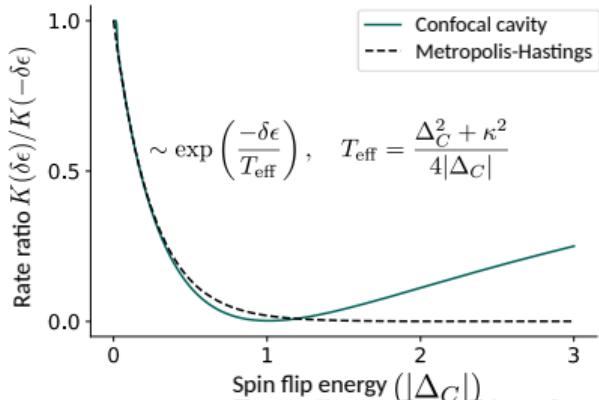
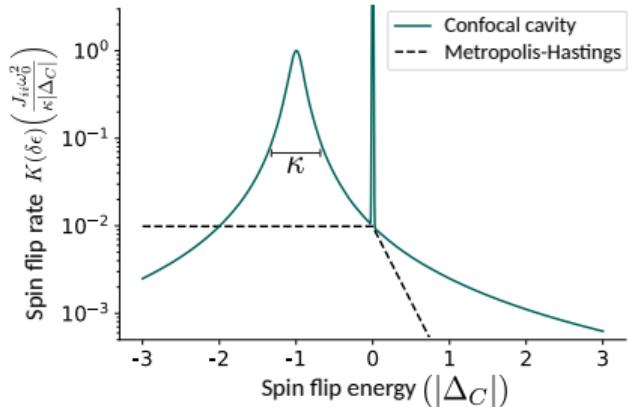
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- Problem: $T_{\text{eff}} \gg J_{ij}$

↳ Solution: $\sigma_j > 1$, $\sigma_i \sim \sum_j \sigma_j S_{ij}^{-1}$
↳ Further benefit — superadiant spin-flip dynamics



Stochastic and deterministic dynamics

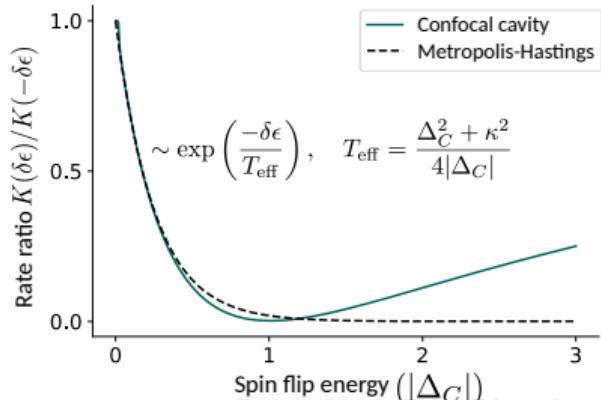
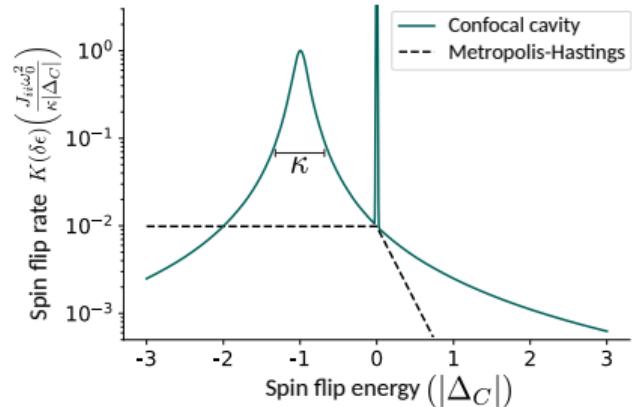
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Stochastic and deterministic dynamics

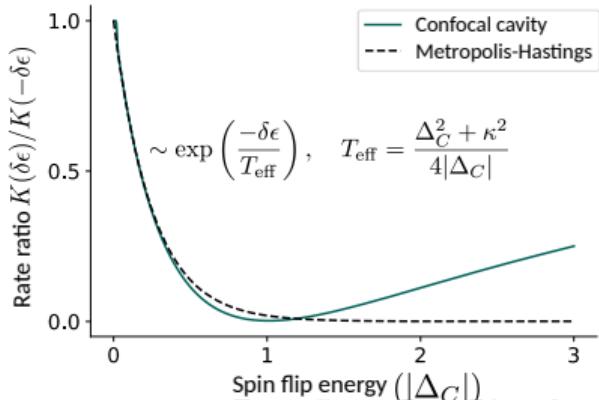
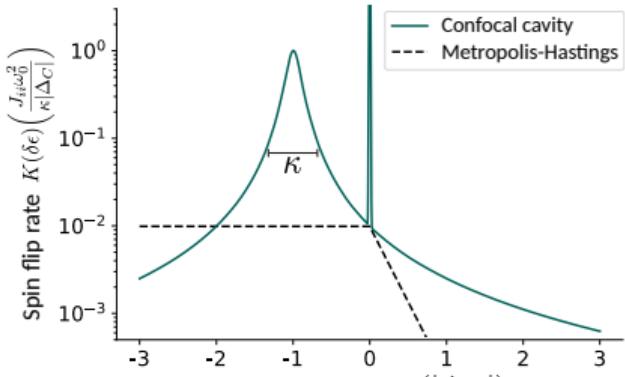
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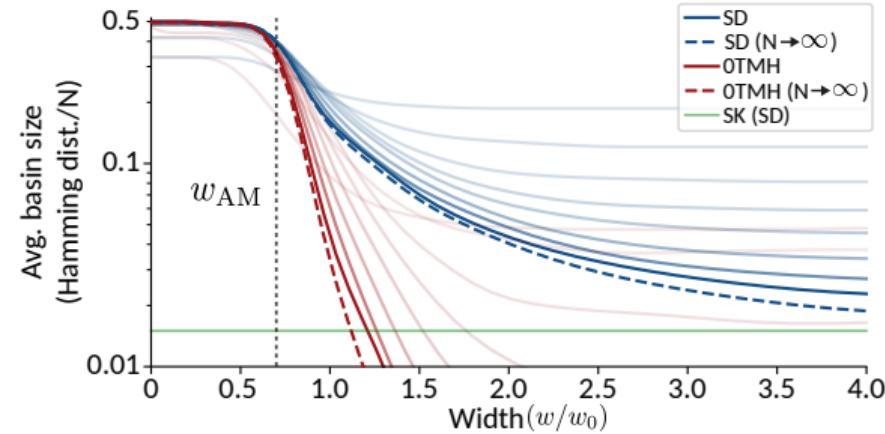
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 - Solution, $S_i \gg 1$, $\delta\epsilon \sim \sum_j J_{ij} S_j + \dots$
 - Further benefit — superradiant spin-flip dynamics



Confocal Connectivity + Dynamics: Basins of attraction

Use J_{ij}^{CCQED} \rightarrow $\begin{cases} \text{FM} & w < w_{AM} \\ \text{AM} & w_{AM} < w < w_{SG} \\ \text{SG} & w_{SG} < w \end{cases}$



How to train your cavity

- Confocal connectivity:

$$\mathbf{r}_i \rightarrow J_{ij} \rightarrow S_i^{*,\nu}$$

local states

- Alternative encoding

- Fixed points: $\{\mathbf{S}^*\}$

- Desired patterns: $\{\mathbf{P}\}$

- Define M, M^ν to minimise

- Recall:

- Encode $\mathbf{P}^{\text{target}} \rightarrow \mathbf{S}^{\text{init}} = \text{sgn}(\mathbf{M} \cdot \mathbf{P}^{\text{target}})$

- Evolve $\mathbf{S}^{\text{init}} \rightarrow \mathbf{S}^{\text{final}}$

- Decode $\mathbf{P}^{\text{final}} = \text{sgn}(\mathbf{M}^\nu \cdot \mathbf{S}^{\text{final}})$

How to train your cavity

- Confocal connectivity:
 $\mathbf{r}_i \rightarrow J_{ij} \rightarrow S_i^{*,\nu}$
 - ▶ Hard to invert
- Alternative encoding
 - ▶ Fixed points: (S^*)
 - ▶ Desired patterns: (P)
 - ▶ Define M, M^ν to minimise

• Recall:

- ▶ Encode $P^{\text{target}} \rightarrow S^{\text{out}} = \text{sgn}(M \cdot P^{\text{target}})$
- ▶ Evolve $S^{\text{out}} \rightarrow S^{\text{final}}$
- ▶ Decode $S^{\text{final}} \rightarrow \text{sgn}(M^\nu \cdot S^{\text{final}})$

How to train your cavity

- Confocal connectivity:
 $\mathbf{r}_i \rightarrow J_{ij} \rightarrow S_i^{*,\nu}$
 - ▶ Hard to invert
- Alternative: encoding.
 - ▶ Fixed points: $\{\mathbf{S}^*\}$
 - ▶ Desired patterns: $\{\mathbf{P}\}$
 - ▶ Define $\mathbf{M}, \mathbf{M}^{\text{inv}}$ to minimise

• Recall:

Encode $\mathbf{P}^{\text{out}} \rightarrow \mathbf{S}^{\text{out}} = \text{sgn}(\mathbf{M} \cdot \mathbf{P}^{\text{out}})$
Evolve $\mathbf{S}^{\text{out}} \rightarrow \mathbf{S}^{\text{in}}$
Decode $\mathbf{P}^{\text{in}} \rightarrow \text{sgn}(\mathbf{M}^{\text{inv}} \cdot \mathbf{S}^{\text{in}})$.

$$\sum_{p=1}^P |\mathbf{M} \cdot \mathbf{P}^p - \mathbf{S}^{*,p}|^2 + \lambda ||\mathbf{M}||^2,$$

$$\sum_{p=1}^P |\mathbf{M}^{\text{inv}} \cdot \mathbf{S}^{*,p} - \mathbf{P}^p|^2 + \lambda ||\mathbf{M}^{\text{inv}}||^2$$

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- Recall:

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- ▶ Evolve $\mathbf{S}^{\text{init}} \rightarrow \mathbf{S}^{\text{final}}$
- ▶ Decode $\mathbf{P}^{\text{recall}} = \text{sgn}(\mathbf{M}^{\text{inv}} \cdot \mathbf{S}^{\text{final}})$.

How to train your cavity

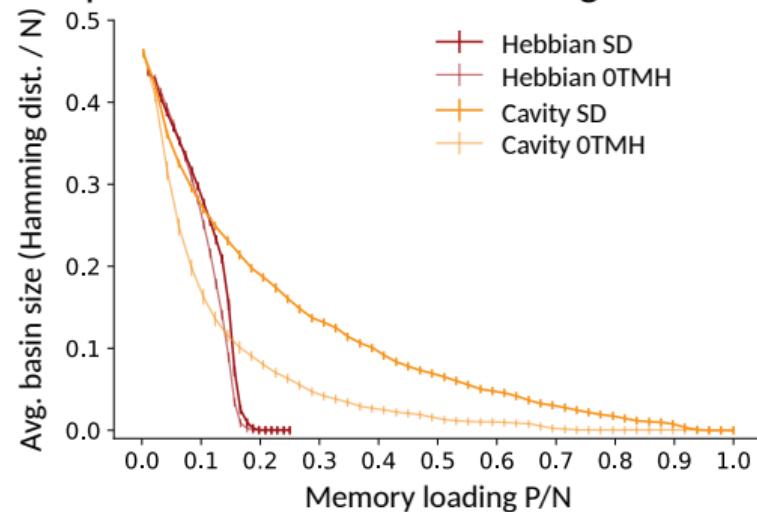
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$$\sum_{p=1}^P |\mathbf{M} \cdot \mathbf{P}^p - \mathbf{S}^{*,p}|^2 + \lambda \|\mathbf{M}\|^2,$$

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 - ▶ Decode $\mathbf{P}^{\text{recall}} = \text{sgn}(\mathbf{M}^{\text{inv}} \cdot \mathbf{S}^{\text{final}})$.

Outperforms Hebbian learning.



Replica and trajectory in general

$$P(q) = \frac{1}{N_{\text{traj.}^2}} \sum_{\alpha, \beta} \sum_{q'} \delta(q - q') \langle \psi^\alpha | \otimes \langle \psi^\beta | \mathcal{P}_{q'} | \psi^\beta \rangle \otimes | \psi^\alpha \rangle$$

where

$$\sum_{q'} q' \mathcal{P}_{q'} = \frac{1}{N_{\text{spins}}} \sum_i \sigma_i^x \otimes \sigma_i^x$$