Associative Memory and Spin Glasses with Confocal Cavity QED

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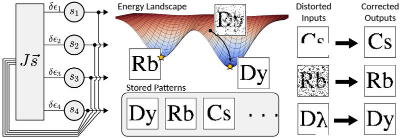
SUPA

Quantum Spinoptics, Ingleheim

Multimode cavity QED

- Spin glass/associative memory:
  - Energy $E = -\sum_{ij} J_{ij} s_i s_j$
  - Energy minimisation dynamics



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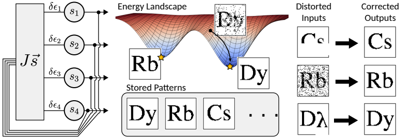

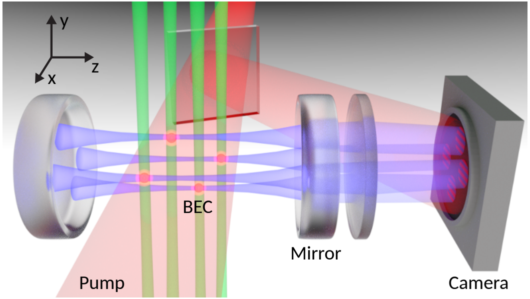

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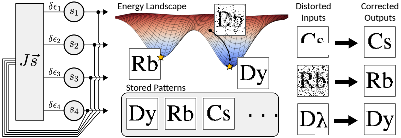

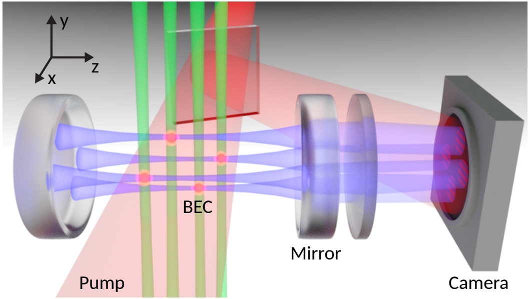

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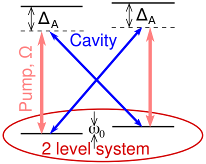

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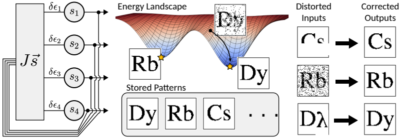

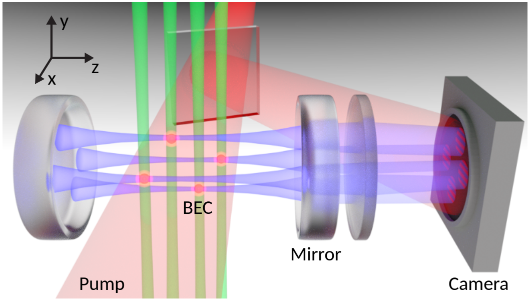

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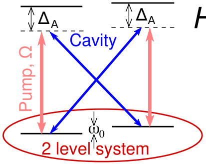

$$H_{\text{eff}} = \sum_\mu \overbrace{(\omega_\mu - \omega_P)}^{-\Delta_\mu} a_\mu^\dagger a_\mu + \sum_i \frac{\omega_0}{2} S_i^z + \underbrace{\frac{\Omega g}{\Delta_A}}_{g_{\text{eff}}} \sum_\mu \Xi_\mu(\mathbf{r}_n) S_n^x (a_\mu + a_\mu^\dagger)$$

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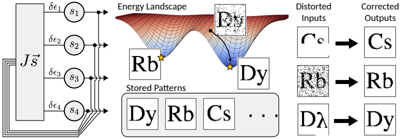

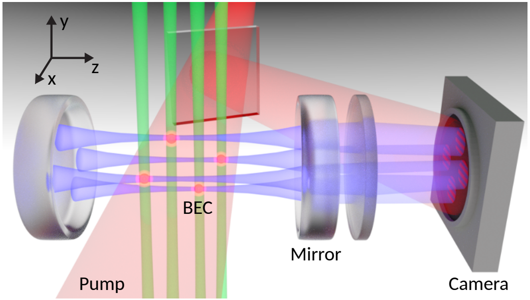

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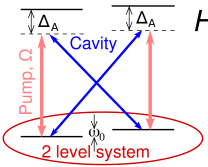

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   - Connectivity
   - Dynamics

3. Theory: Quantum dynamics near threshold

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$$J_{ij} = -J_0 \left[\beta \delta_{ij} + \cos\left(2 \frac{\mathbf{r}_i \cdot \mathbf{r}_j}{w_0^2} \right) \right]$$

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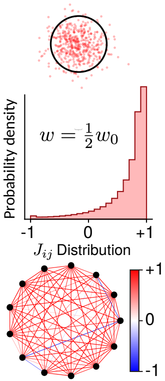

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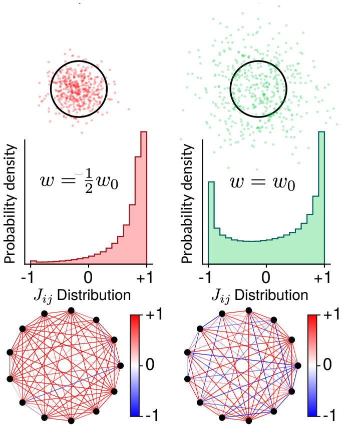

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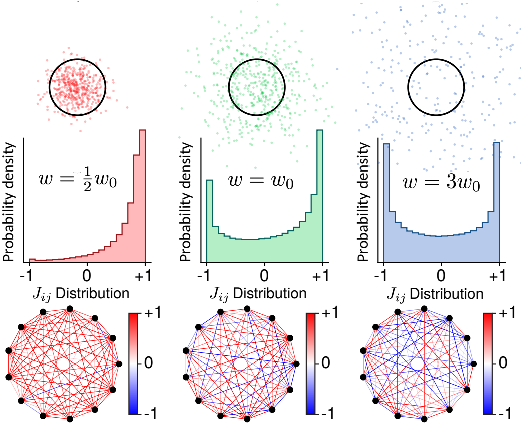

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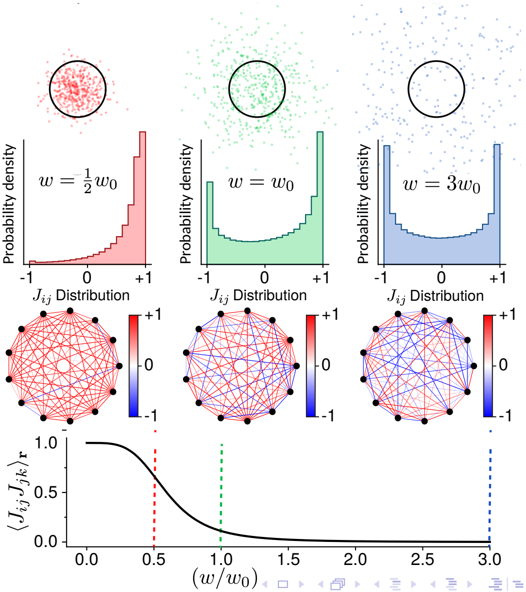
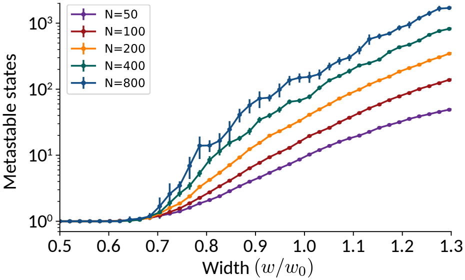


Associative memory behaviour



- Minima of $E = -\sum_{ij} J_{ij} s_i s_j$
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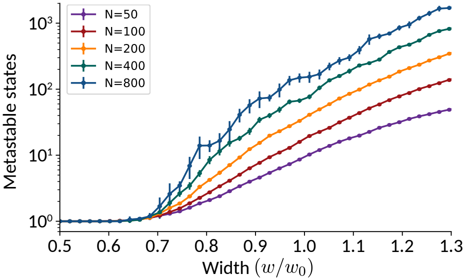

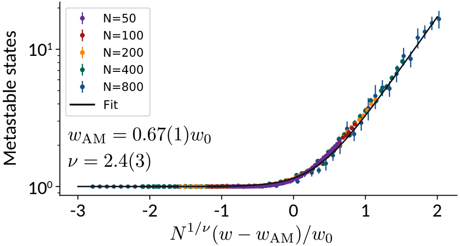

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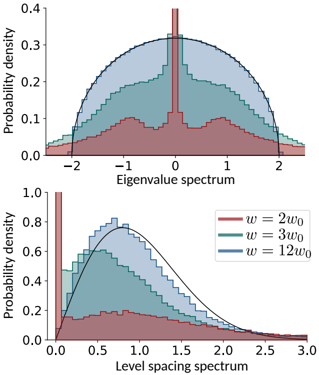

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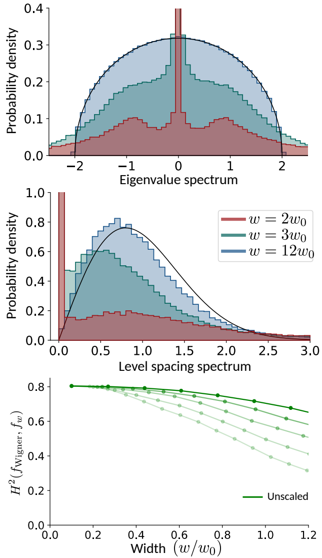

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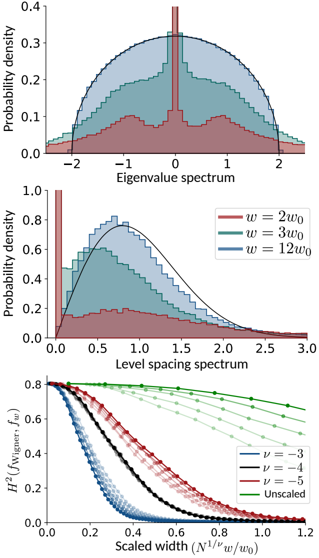

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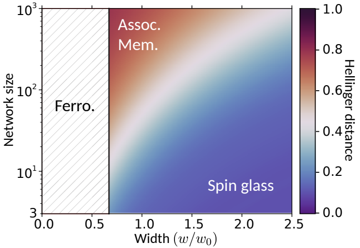

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$$H = -\Delta \sum_\mu a^\dagger_\mu a_\mu + g_{\rm eff} \sum_{\mu,i} \tilde{\Xi}_\mu(\mathbf{r}_i) S^x_i (a^\dagger_\mu + a_\mu) + \omega_0 \sum_i S^z_i$$

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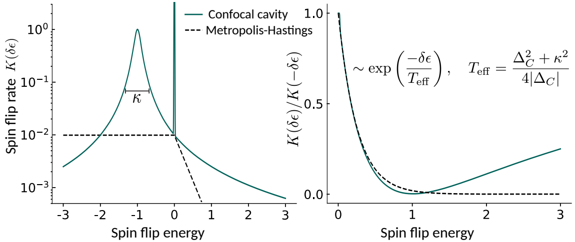

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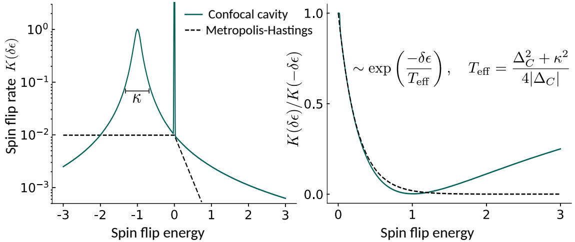

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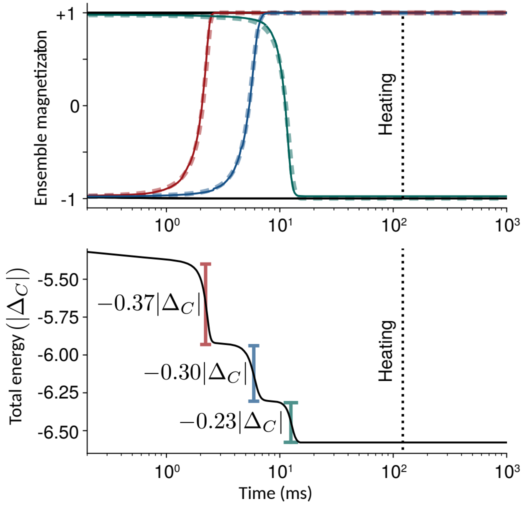

- Simulate via stochastic flips (solid)
- Fast (superradiant) flip, long wait.
  $\rightarrow$ Deterministic model (dashed)
- Solution:

$$S_i^x(t) = \mathrm{sgn}\left(\sum_j J_{ij} S_j^x\right) \tanh\left[\frac{t - t_i^0}{\tau_i^{\mathrm{flip}}}\right]$$

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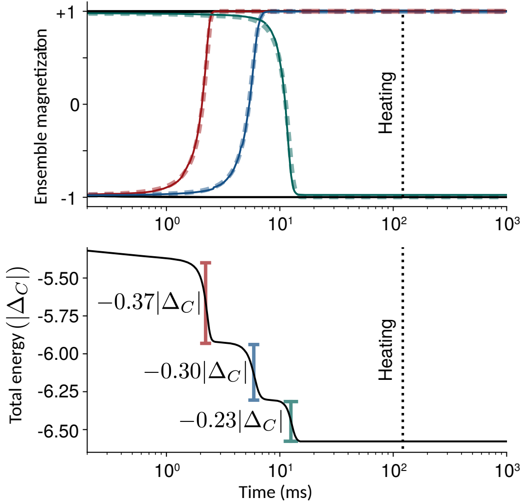

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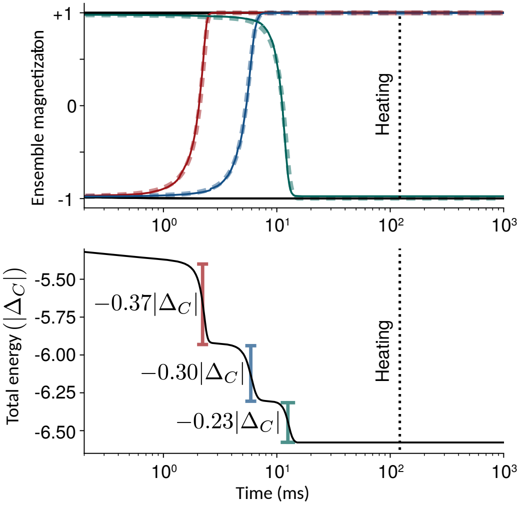

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- Solution:

$$S_i^x(t) = \mathrm{sgn}\left(\sum_j J_{ij} S_j^x\right) \tanh\left[\frac{t - t_i^0}{\tau_i^{\mathrm{flip}}}\right],$$

$$t_i^0 \propto -\mathrm{sgn}\left(\sum_j J_{ij} S_j^x S_i^x\right) \frac{S_i / \ln(S_i)}{|\delta\epsilon_i|}$$

- Smallest t_i^0 flips first —
  superradiant steepest descent!

Hebbian Connectivity + Dynamics: Basins of attraction

- Basin of attraction: Fixed point, distort, relax
- Illustrate for Hebbian $J_{ij} = \sum_{\mu=1}^{P} \xi_i^{(\mu)} \xi_j^{(\mu)}$
- Memories more robust with SD. Why?

- Robust memories in (SK) spin glass!

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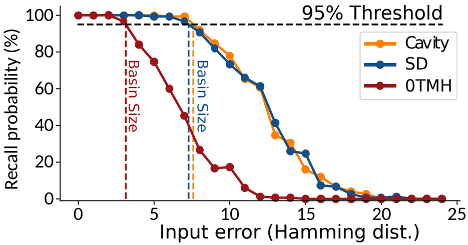

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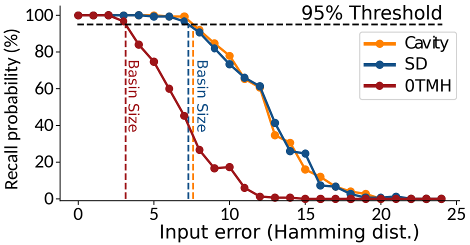

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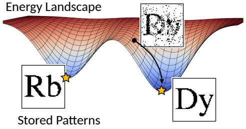

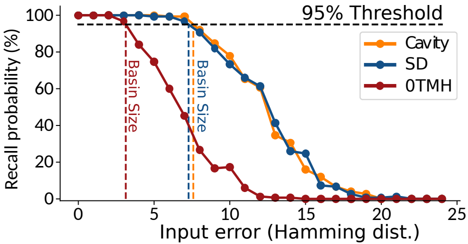

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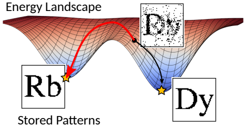

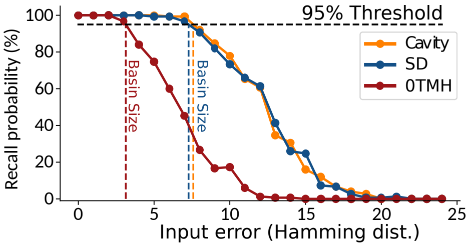

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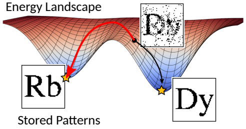

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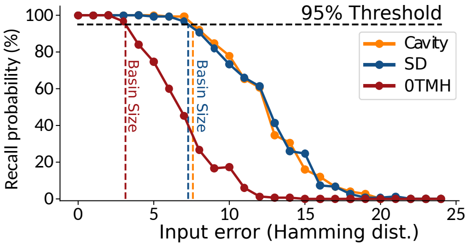

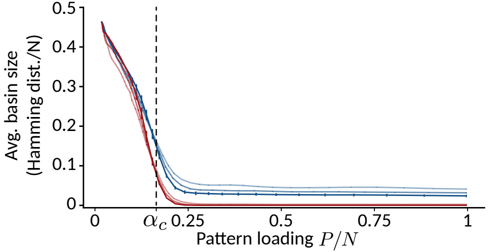

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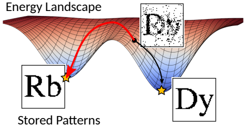

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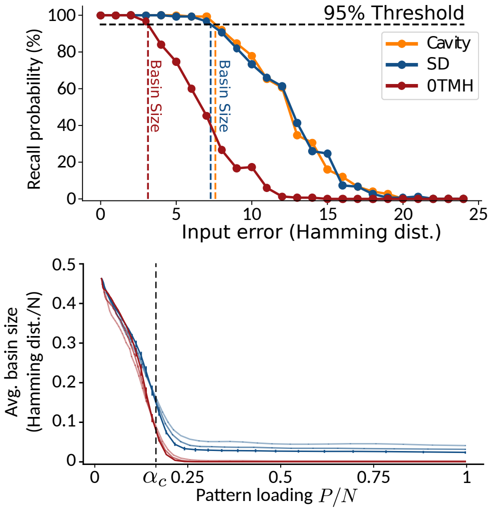

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Simulating quantum dynamics

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- Atom-only dynamics, after [Jäger *et al.* , PRL '22]: $\frac{d}{dt}\rho = -i[H, \rho] + \sum_k \mathcal{D}[C_k]$

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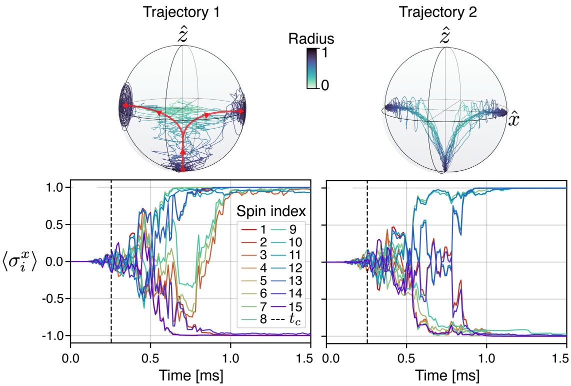
Quantum vs semiclassical

Quantum



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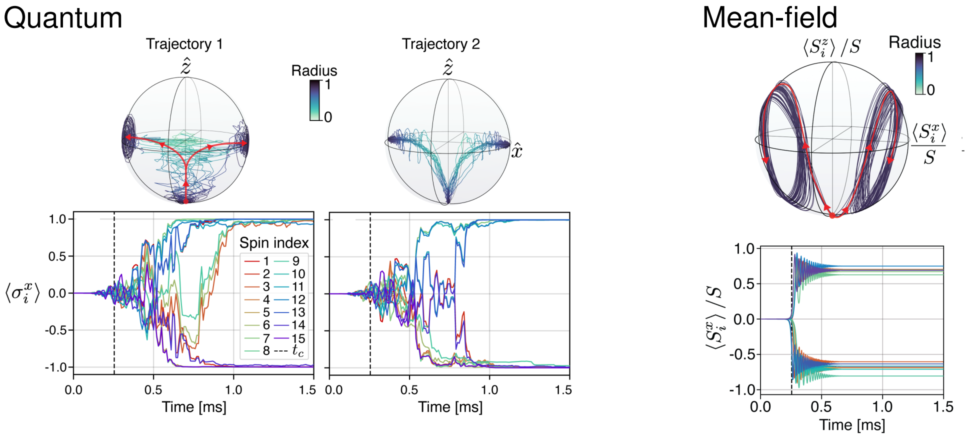

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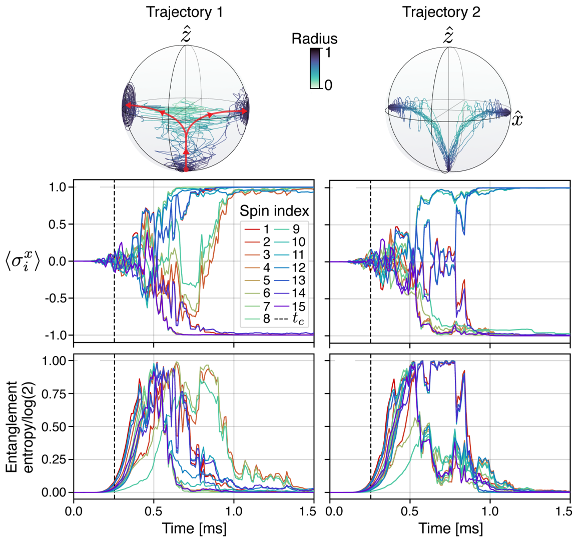
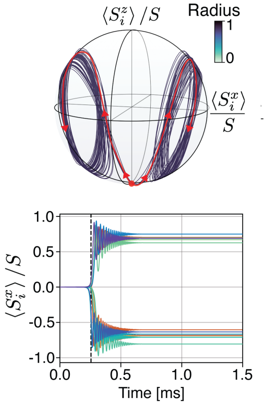


Mean-field



Trajectories and replicas

- Spin glass: replica overlap:

$$q_{\alpha\beta} = \sum_i \langle S_i^\alpha \rangle \langle S_i^\beta \rangle / N$$

- Trajectory overlap:

$$q_{\alpha\beta} = \sum_i \langle \Psi^\alpha | \sigma_i^x | \Psi^\alpha \rangle \langle \Psi^\beta | \sigma_i^x | \Psi^\beta \rangle / N$$

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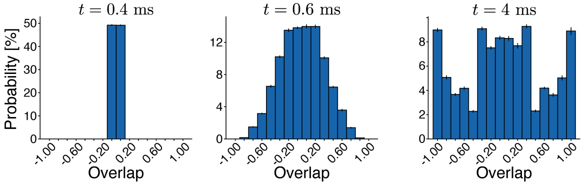

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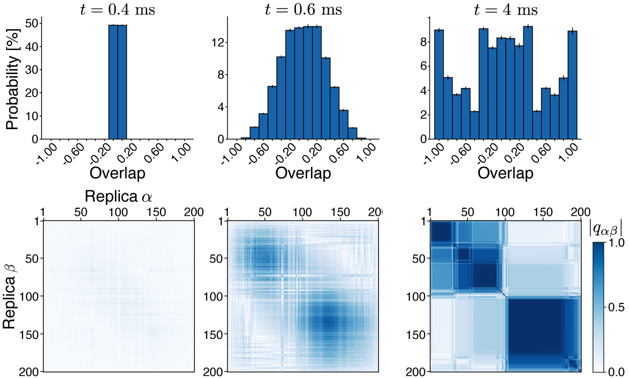

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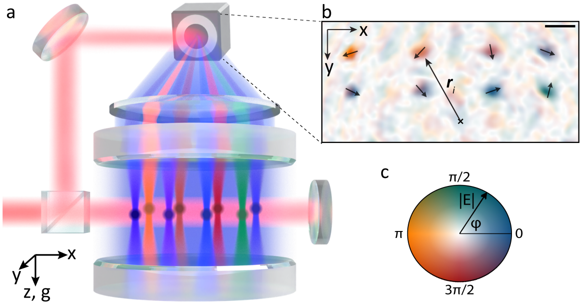

- More complex model

- Vector overlaps:
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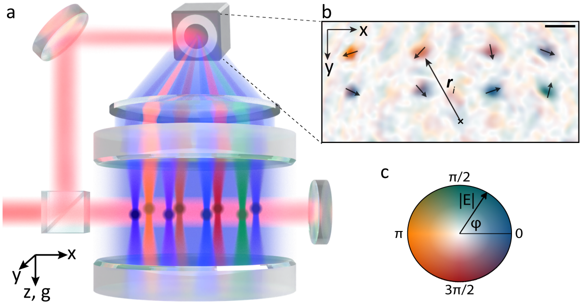

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  - Discrete symmetry
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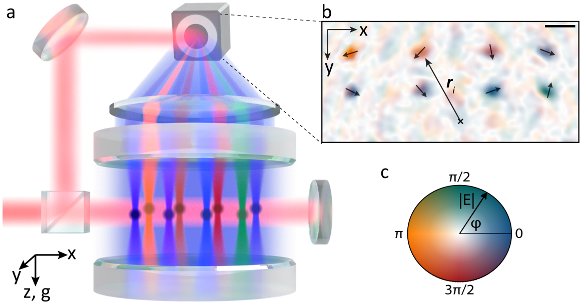

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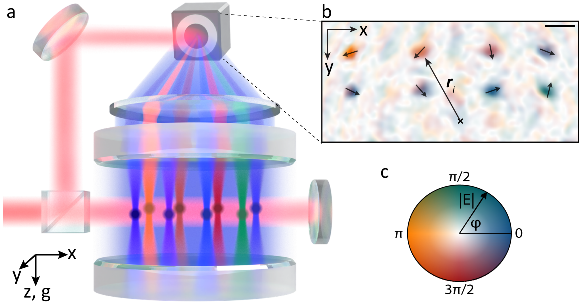

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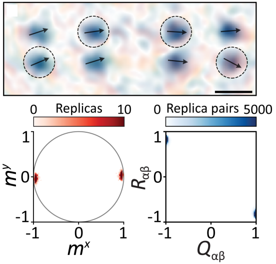
Results

Ferromagnetic J_{ij}

$J_{ij} > 0$



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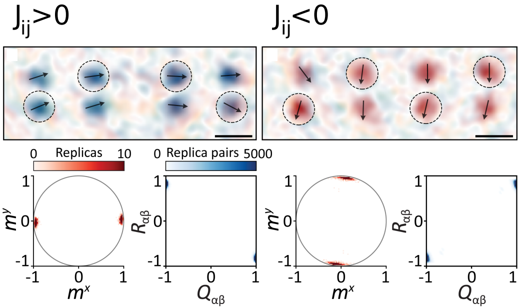

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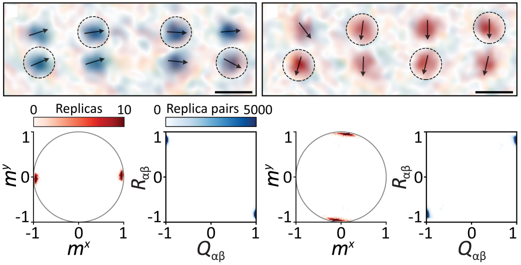

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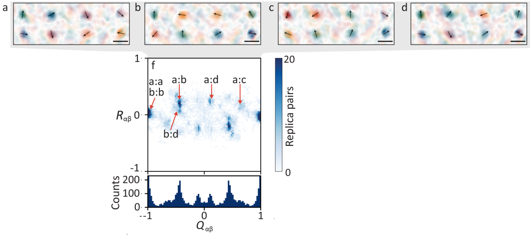

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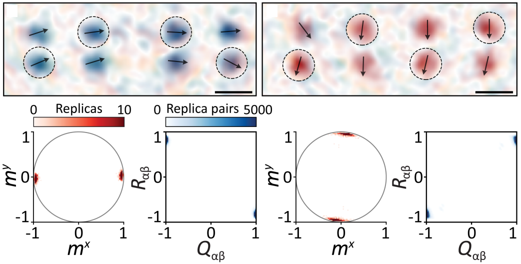

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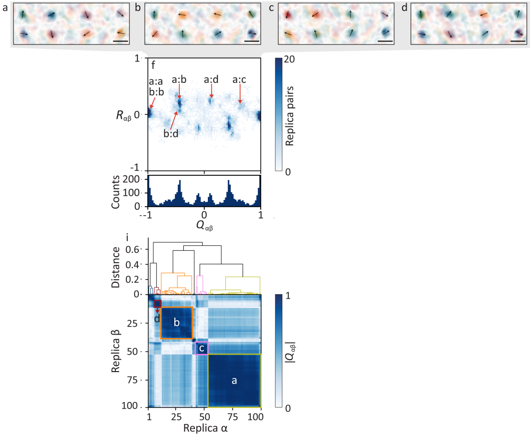

Directly measure RSB & ultrametricity

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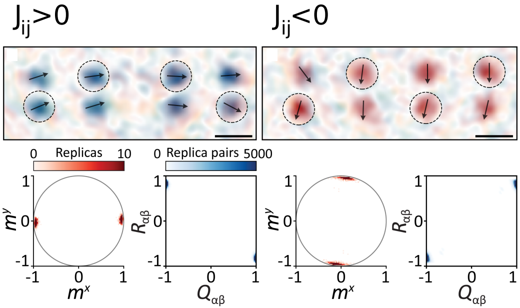

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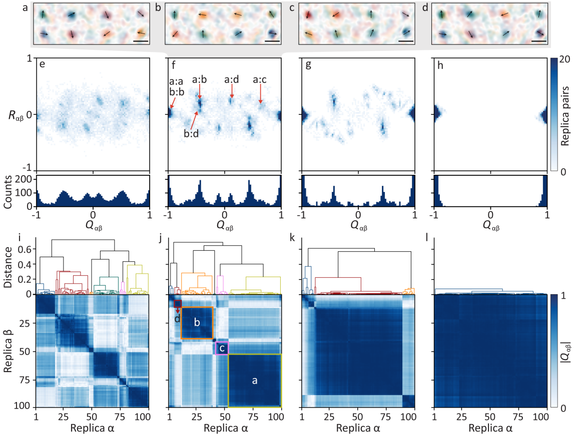

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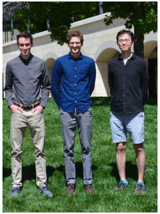
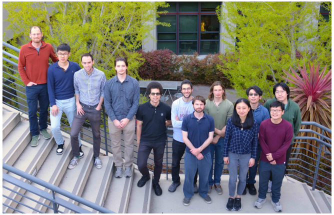
Acknowledgments

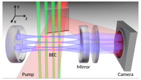
Lev Group:

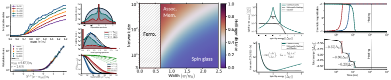
Sarang Gopalakrishnan (Princeton)
Surya Ganguli (Stanford)


NSF
STANFORD UNIVERSITY
GORDON AND BETTY MOORE FOUNDATION
the David & Lucile Packard FOUNDATION

Summary

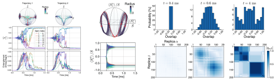
- Confocal CQED associative memory/spin glass. 

- Ferromagnet/Associative Memory/Spin Glass; superradiant steepest descent



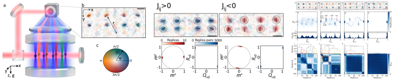
[Marsh, Guo, Kroeze, Gopalakrishnan, Ganguli, Keeling & Lev. PRX '21]

- Quantum behaviour near transition



[Marsh, Kroeze, Ganguli, Gopalakrishnan, Keeling & Lev. PRX '24]

- Experimental spin glass



[Kroeze, Marsh, Atri Shuller, Hunt, Gopalakrishnan, Keeling & Lev. arXiv:2311.04216]

5 Confocal cavities

6 More open system dynamics

7 Basins vs width

8 Training

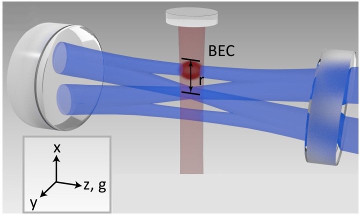
9 Replicas and trajectories

Confocal cavities

- Gaussian standing waves:

$$\Phi_\mu(\mathbf{r}) = \Xi_\mu(x,y)\cos\left[k_r\left(z + \frac{x^2+y^2}{z + z_R^2/z}\right) - \theta_\mu(z)\right]$$

- Gauss-Hermite modes $\Xi_\mu(x,y) = \xi_{l_\mu}(x)\xi_{m_\mu}(y)$



- Gouy phase $\theta_\mu(z) = \text{const.} + (l_\mu + m_\mu)\,\text{atan}(z/z_R)$

- Cavity Geometry:

- Tune degeneracy via L/R

Confocal cavities

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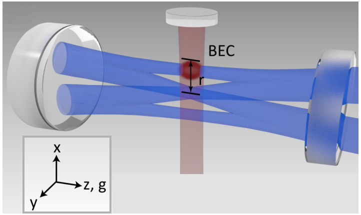

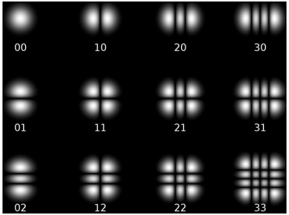

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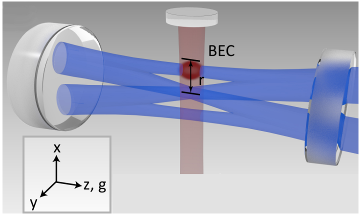

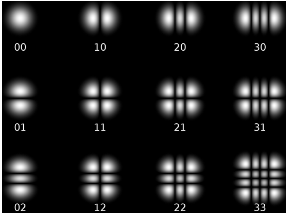

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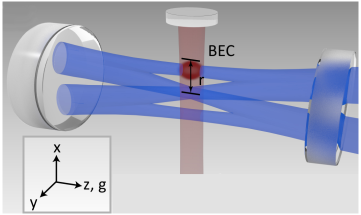

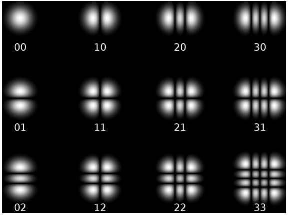

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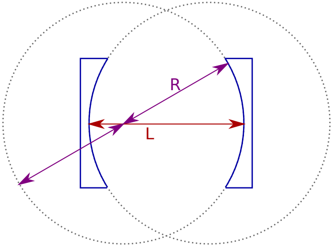

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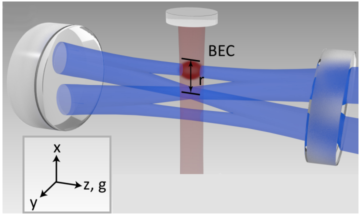

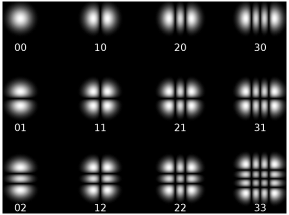

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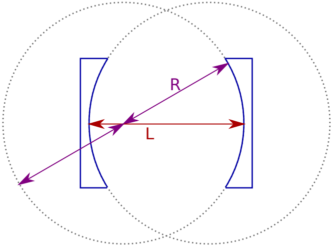

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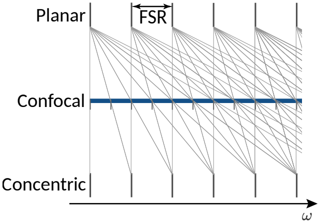

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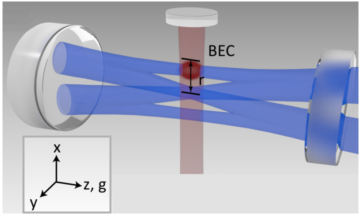

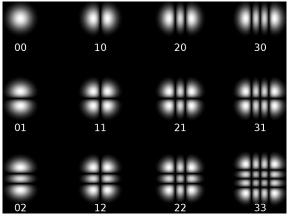

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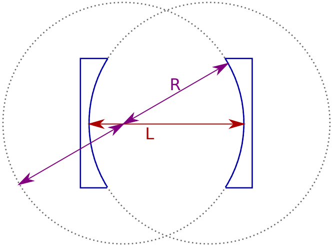

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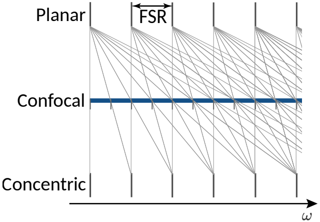

Confocal cavity QED Dynamics

- Open quantum system dynamics

$$\dot{\rho} = -i[H, \rho] + \kappa \sum_{\mu} \mathcal{L}[a_\mu]$$

$$H = -\Delta \sum_{\mu} a^\dagger_\mu a_\mu + g_{\text{eff}} \sum_{\mu,i} \tilde{\Xi}_\mu(\mathbf{r}_i) S^x_i (a^\dagger_\mu + a_\mu) + \omega_0 \sum_i S^z_i$$

- Simplify:
  - Far above SR transition
  - Atom-only equations
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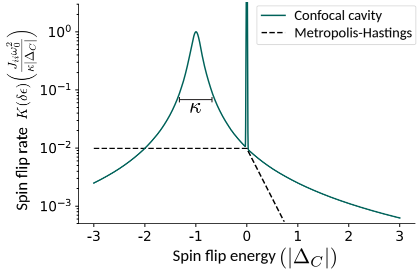

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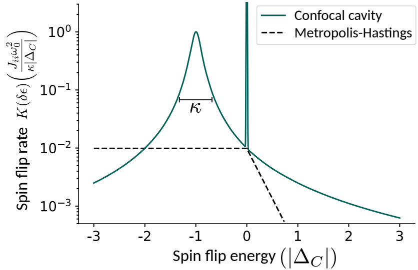

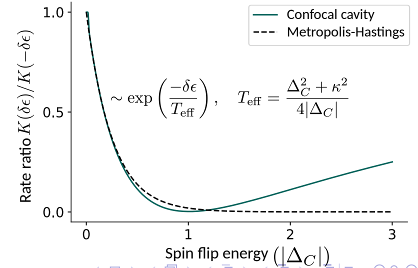

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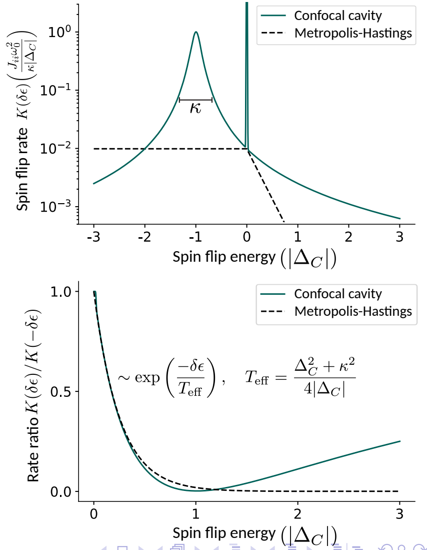

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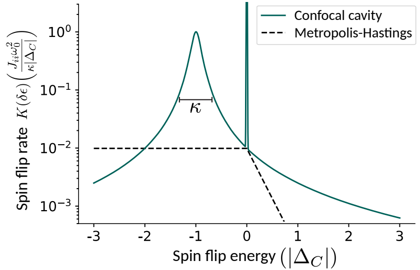

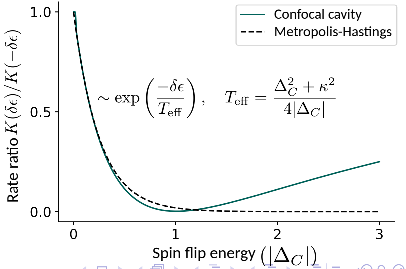

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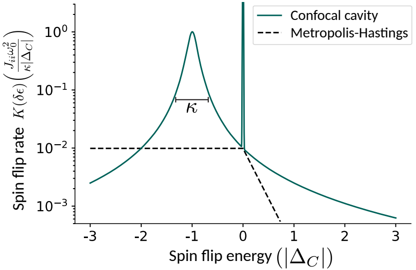
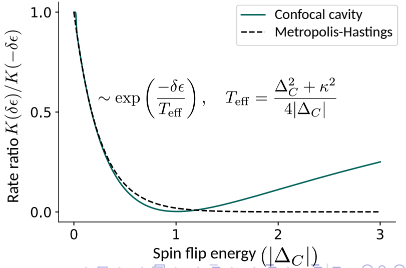
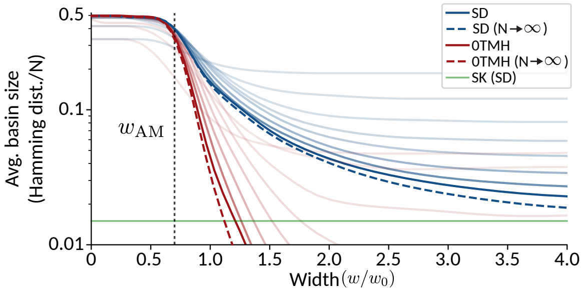




Confocal Connectivity + Dynamics: Basins of attraction

$$\text{Use } J_{ij}^{CCQED} \rightarrow \begin{cases} \text{FM} & w < w_{AM} \\ \text{AM} & w_{AM} < w < w_{SG} \\ \text{SG} & w_{SG} < w \end{cases}$$



How to train your cavity

- Confocal connectivity:
  $\mathbf{r}_i \rightarrow J_{ij} \rightarrow S_i^{*,\nu}$
  - Hard to invert
- Alternative: encoding.
  - Fixed points: $\{\mathbf{S}^*\}$
  - Desired patterns: $\{\mathbf{P}\}$
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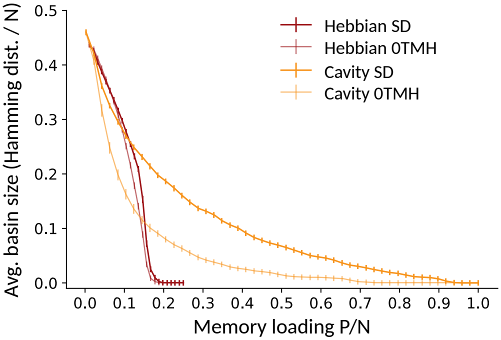

Replica and trajectory in general

$$P(q) = \frac{1}{N_{\text{traj.}}{}^2} \sum_{\alpha,\beta} \sum_{q'} \delta(q - q') \langle \psi^\alpha | \otimes \langle \psi^\beta | \mathcal{P}_{q'} | \psi^\beta \rangle \otimes | \psi^\alpha \rangle$$

where

$$\sum_{q'} q' \mathcal{P}_{q'} = \frac{1}{N_{\text{spins}}} \sum_i \sigma_i^x \otimes \sigma_i^x$$