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SPIN PHENOMENA INTERDISCIPLINARY CENTER

Topology by Dissipation in Systems of Free Bosons

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Papers & collaborators

❖ This talk:

PHYSICAL REVIEW LETTERS **127**, 245701 (2021)

Topology by Dissipation: Majorana Bosons in Metastable Quadratic Markovian Dynamics

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PHYSICAL REVIEW B **108**, 214312 (2023)

Topological zero modes and edge symmetries of metastable Markovian bosonic systems

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[2]*Department of Mathematics and Physics, SUNY Polytechnic Institute, 100 Seymour Road, Utica, New York 13502, USA*

❖ Related:

The interplay of finite and infinite size stability in quadratic bosonic Lindbladians

Mariam Ughrelidze, Vincent P. Flynn, Emilio Cobanera, and Lorenza Viola

E-print arXiv:2405.08873

Vincent Flynn

POSTER

Emilio Cobanera

Mariam Ughrelidze

Broad context

Challenge: To fully characterize the *roles / implications of topology* in quantum phases of matter and light – naturally occurring and engineered, in equilibrium and [far] beyond?...

- *Fundamental and practical significance* across quantum science – can topology
  - Provide an *organizing principle* for classifying quantum phases (beyond Landau's paradigm)?...
  - Mandate the existence of *protected edge/surface localized states* with exotic properties?...
  - Enable new types of *device applications* (quantum sensors, amplifiers, lasers)?...
  - Provide new paths to *robust* quantum information storage and processing?...

Qi & Zhang, RMP **83** 2011... Chiu *et al*, ibid **88** 2016 ... Ozawa *et al*, *ibid.* **91** 2019… Hurst & Flebus, JAP **132** 2022…

- **Stepping-stone:** Symmetry-protected topological (SPT) phases
  - Phases of *gapped, stable free-fermion* (mean-field) Hamiltonians
  - Complete classification ⇒ "tenfold way"
    - **Bulk-boundary correspondence:** Relation between bulk topological invariants ⇔ Emergence of topologically protected *boundary zero modes*...

TABLE VI. Symmetry classes that support topologically nontrivial line defects and their associated protected gapless modes.

Symmetry	Topological classes	1D gapless fermion modes
A	$\mathbb{Z}$	Chiral Dirac
D	$\mathbb{Z}$	Chiral Majorana
DIII	$\mathbb{Z}_2$	Helical Majorana
AII	$\mathbb{Z}_2$	Helical Dirac
C	$2\mathbb{Z}$	Chiral Dirac

Chiu *et al*, RMP **88** 2016

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Majorana fermions

- ✓ A pair of Hermitian operators that (approximately) *commute* with the Hamiltonian.
- ✓ *Exponentially localized* on opposite end of the chain.
- ✓ Normalized to satisfy the *Majorana algebra*, forming a 'split' Dirac fermion: $\{ \gamma_L , \gamma_R \} = 0,\ \gamma_L^2 = 1 = \gamma_R^2$
- ✓ Zero energy ⇒ Many-body *ground-state degeneracy*.

No-go for SPT phases of free bosons in equilibrium

- A closed system of free bosons in equilibrium is described (within MF) by a thermodynamically stable [bounded from below or above] *quadratic bosonic Hamiltonian* (QBH):

$$H = H^\dagger \equiv \frac{1}{2}\Phi^\dagger \mathbf{H}\Phi, \quad \Phi \equiv [a_1, a_1^\dagger, \ldots, a_N, a_N^\dagger]^T, \quad \mathbf{H} = \mathbf{H}^\dagger, \mathbf{H} > 0$$

Theorem 1 [No SPTs]: Let H_1 , H_2 denote gapped, thermodynamically stable QBHs sharing a group of symmetries. Then H_1 *can be adiabatically deformed* into H_2 w/o breaking any of the symmetries or closing the many-body gap.

Theorem 2 [No ZMs]: A gapped, thermodynamically stable QBH *cannot host edge states* in the gap surrounding zero energy under open boundary conditions.

Xu, Flynn, Alase, Cobanera, LV & Ortiz, PRB **102** 2020

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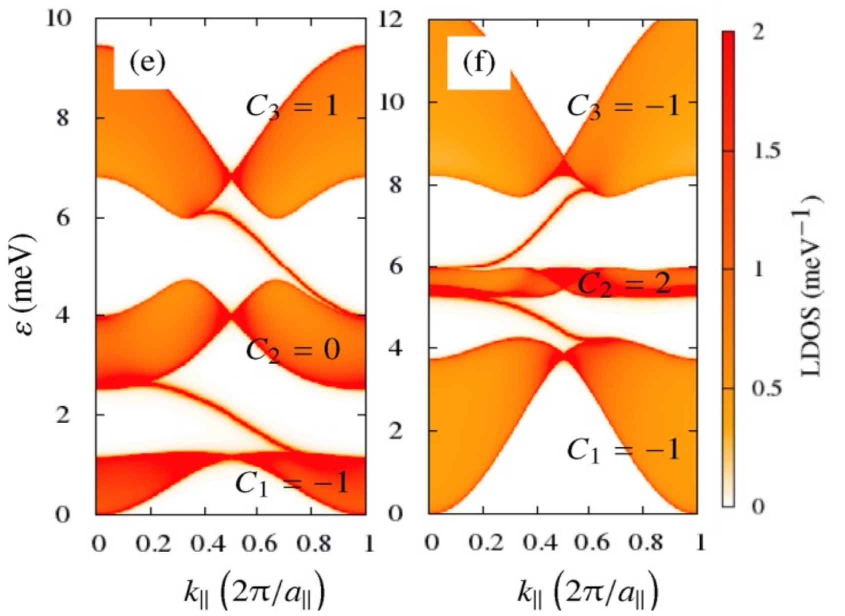


- Non-trivial manifestations of topology *can* emerge
  - Away from zero energy – *e.g.*, topological magnonics …

    Mook *et al*, PRB **90** 2014… Zhuo *et al*, Adv. Phys. Res. 2023…
  - By forgoing thermodynamical stability, at zero energy – but at the cost of intrinsic *dynamical instability*…

    Flynn, Cobanera & LV, NJP **22** 2020

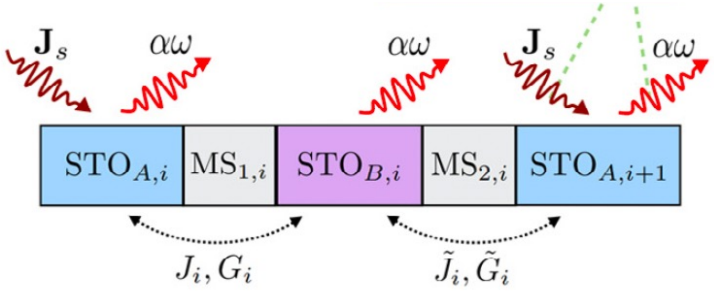
Topology of free bosons beyond equilibrium

❖ Topological phenomena can emerge away from equilibrium, in the context of *non-unitary, open quantum dynamics* – *e.g.*, metamaterials, light-matter hybrid/driven-dissipative quantum systems…

$$\dot{\rho} = -i\big(H_{\text{eff}}\rho - \rho H_{\text{eff}}^{\dagger}\big) \qquad \dot{\rho} = -i\big[H,\rho\big] + \sum_{\alpha}\Big(L_{\alpha}\rho L_{\alpha}^{\dagger} - \tfrac{1}{2}\{L_{\alpha}^{\dagger}L_{\alpha},\rho\}\Big), \quad H_{\text{eff}} \equiv H - \frac{i}{2}\sum_{\alpha} L_{\alpha}^{\dagger}L_{\alpha}$$

- Semiclassical / phenomenological ⇒ *Non-Hermitian effective Hamiltonians* (*e.g.*, via post-selection)
  - Exponentially enhanced quantum sensing… Topologically protected lasing modes…

    McDonald & Clerk, Nat. Comm. **11** 2020… Flebus *et al* PRB **102** 2020…



- Fully quantum ⇒ *Markovian dynamics*, via Lindblad ME or input-output formalism…
  - Dissipative, steady-state phase transitions…

    Kessler et al, PRA **86** 2012… Fitzpatrick *et al*, PRX **7** 2017…
  - Topological amplification in photonic arrays…

    Porras & Fernández-Lorenzo, PRL **122** 2019; Wanjura *et al*, Nat. Commun. **11** 2020…

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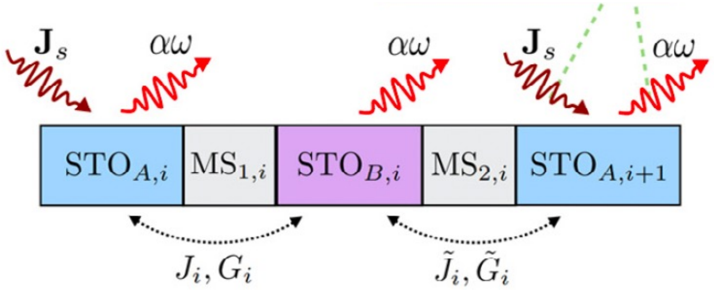

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Question: These topological bosonic matter and phenomena have very little in common with SPT fermionic physics from a *many-body standpoint*. Do *tight bosonic analogs* exist?...

Spoiler: Shift focus away from (many-body) ground states or even steady states…

Setting the stage: Quadratic bosonic Lindbladians

- Restrict to purely Gaussian (quadratic) Markovian dissipative dynamics:
  - *Heisenberg-picture* generator on N-mode bosonic Fock space

$$\partial_t A = \mathcal{L}^\star(A) = i[H,A] + \sum_\mu \left(L_\mu^\dagger A L_\mu - \frac{1}{2}\{L_\mu^\dagger L_\mu, A\} \right), \quad H = \frac{1}{2}\Phi^\dagger \mathbf{H} \Phi, \quad L_\mu = \sum_j \ell_{\mu,j}\Phi_j, \quad \ell_{\mu,j} \in \mathbb{C}$$

$$\Phi \equiv [a_1, a_1^\dagger, \dots, a_N, a_N^\dagger]^T, \quad [\Phi_j, \Phi_k^\dagger] = (\boldsymbol{\tau}_3)_{jk}, \quad \boldsymbol{\tau}_\ell \equiv \mathbb{1}_N \otimes \boldsymbol{\sigma}_\ell$$

  - Equations of motion of linear observables are determined by (*generally non-normal!*) dynamical matrix

$$\partial_t \Phi = -i\mathbf{G}\Phi, \quad \mathbf{G} \equiv \tau_3 \mathbf{H} - \tfrac{i}{2}\tau_3(\mathbf{M} - \tau_1 \mathbf{M}^T \tau_1) \qquad \mathbf{G}\mathbf{G}^\dagger \neq \mathbf{G}^\dagger \mathbf{G}$$

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- A QBL is dynamically stable if expectations of arbitrary physical observables in any state are *bounded for all times*.
  - Dynamical instability can be diagnosed from the rapidity spectrum, $\sigma(-i\boldsymbol{G})$

Spectral gap $\Delta_\mathcal{L} \equiv |\sup \mathrm{Re}[\sigma(\mathcal{L}) \setminus \{0\}]| \geq 0$

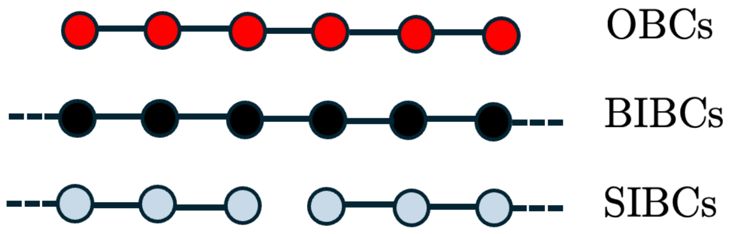
$\Delta_S \equiv \max \mathrm{Re}[\sigma(-i\boldsymbol{G})]$ **Stability gap**

$\Delta_S < 0$	$\Delta_S > 0$	$\Delta_S = 0$
❑ Dynamically stable	❑ Dynamically unstable	❑ Dynamically stable or unstable
❑ Unique steady state	❑ No steady states	❑ Many or no steady states

Prosen, NJP **10** 2008; Prosen & Seligman, JPA **43** 2010; Barthel & Zhang, J. Stat. Mech. **113101** 2022…

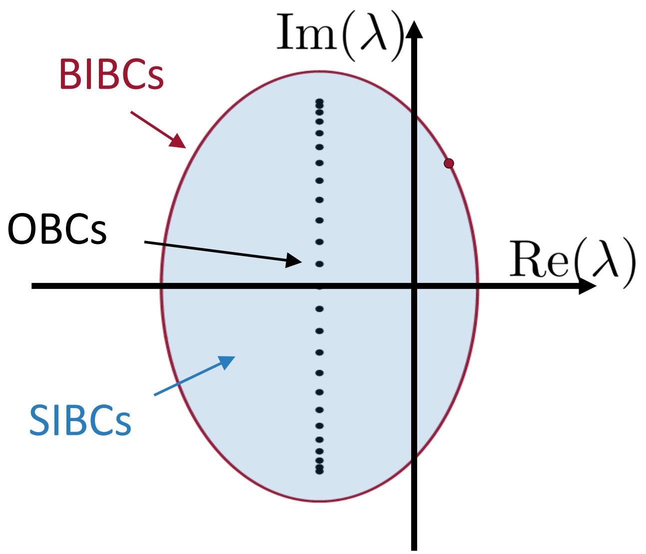
Dynamically metastable QBLs

- Focus on 1D QBLs that are 'translationally invariant up to BCs':
  - The dynamical matrix $\mathbf{G}$ is *block-Toeplitz* (*block-circulant*) under OBCs (BIBCs).
  - The rapidities for BIBCs form closed curves, *rapidity bands*, as $N \to \infty$.



- If $\mathbf{G}$ is highly non-normal, the finite-N spectrum need *not* converge to that of the (semi-)infinite-size limit, and the *stability gap can change discontinuously!*

$$\lim_{N\to\infty} \sigma(\mathbf{G}_N^{\mathrm{OBC}}) \neq \sigma(\mathbf{G}_\infty) \qquad \lim_{N\to\infty} \Delta_{S,N}^{\mathrm{OBC}} < 0 < \Delta_S^{\mathrm{SIBC}}$$



> The OBC chain is [type-I] *dynamically metastable* if it is dynamically *stable* for *arbitrary finite system size*, but dynamically *unstable* in the infinite-size limit.

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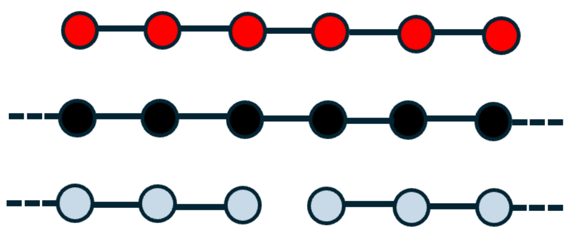

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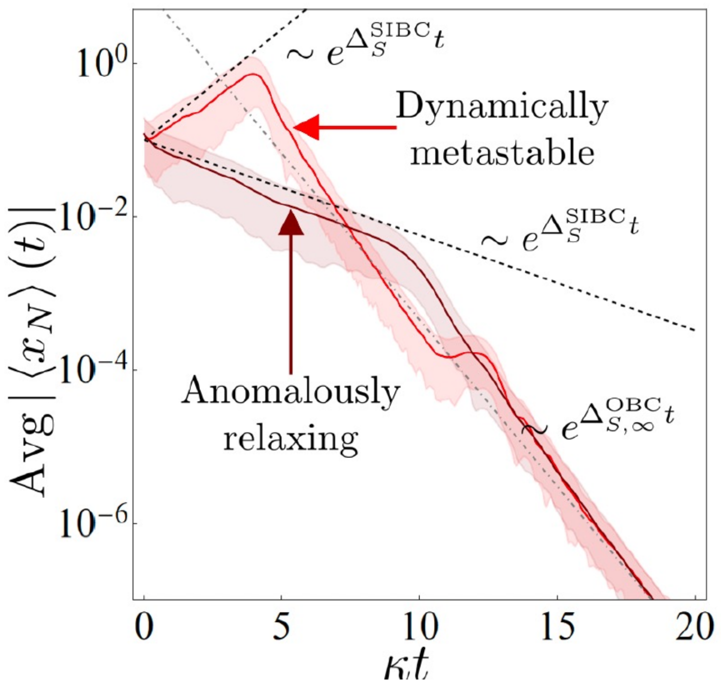
The OBC chain is [type-I] *dynamically metastable* if it is dynamically *stable* for *arbitrary finite system size*, but dynamically *unstable* in the infinite-size limit.

  - Proper mathematical tool to describe infinite-size limit: 'Pseudospectrum'

$$\sigma_\epsilon(\mathbf{X}) \equiv \{\lambda \in \mathbb{C} : \exists \vec{v},\ ||v|| = 1,\ ||(\mathbf{X} - \lambda \mathbb{1})\vec{v}|| < \epsilon\}$$

  - Dynamical metastability elicits distinctive 'anomalous' relaxation behavior: The infinite-size rapidities 'imprint' into the pseudorapidities of the finite OBC chain…



Refining the search for bosonic SPT physics

- **Goal:** Uncover 1D, bulk-translationally invariant QBLs that, under OBCs:
  - Are dynamically stable with a nonzero spectral gap – and thus, possess a unique steady state;
  - Host robust, edge-localized boundary zero modes (ZMs) mandated by a bulk topological invariant.

- There are two complications arising from non-unitarity and bosonic physics:
  - **(1)** *Exact* bosonic ZMs are *highly susceptible to instability* (even in closed systems) and require closing of the spectral gap…

Zero mode $\Leftrightarrow$ Zero rapidity $\Longrightarrow$ Zero spectral gap

Solution: The pseudospectrum. Non-normality allows for a system to have a nonzero spectral gap and an *approximate* zero rapidity (equivalently, a zero *pseudo*-eigenvalue) ⇒ The QBL is *necessarily dynamically metastable*…

$$0 \in \sigma_\epsilon(\mathbf{G}_N^{\mathrm{OBC}}), \quad \epsilon \text{ exponentially small in } N$$

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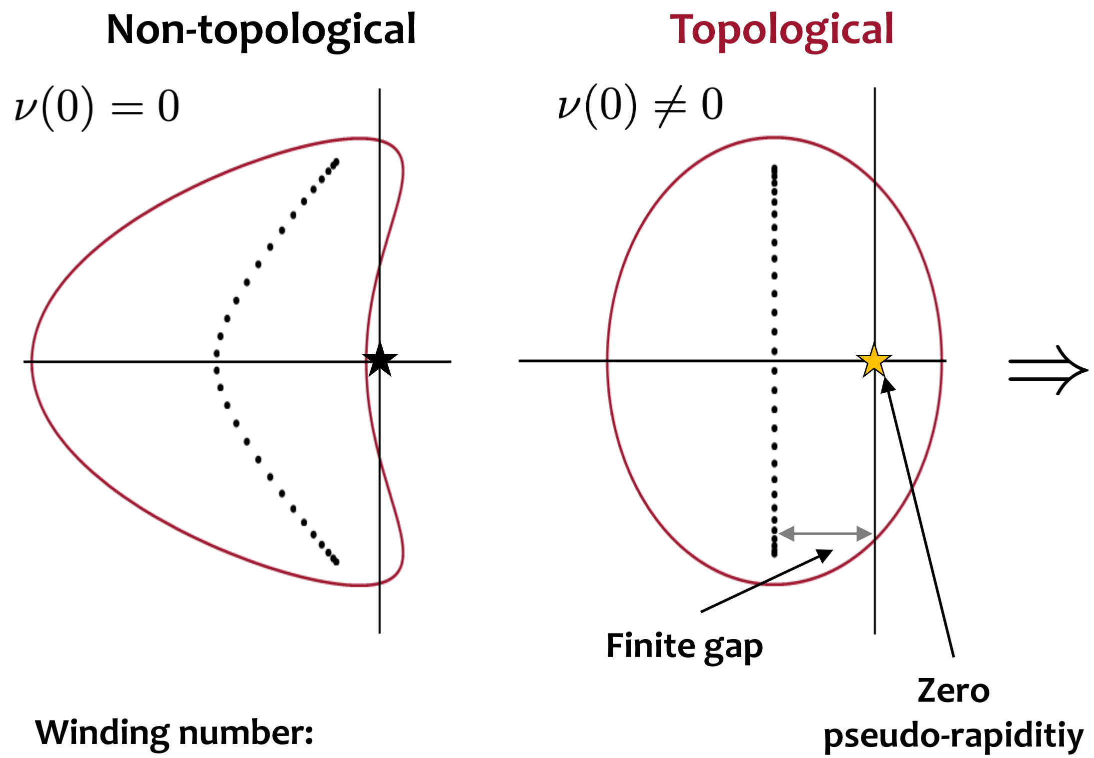
  - **(2)** For a Hamiltonian system, a ZM ($[H, \gamma] = 0$) is *both* a conserved quantity *and* a generator of a continuous symmetry, but Noether's theorem breaks down for open dynamics…

Solution: In searching for localized edge modes, allow *generically* for the possibility of a mode being *either* a ZM *or* a symmetry generator (SG), and not both…

Topological dynamical metastability and Majorana bosons

- A dynamically metastable QBL that i) is *point-gapped at zero* ($\lambda = 0$ does not belong to any rapidity band); and ii) has at least one rapidity band *winding around zero* is topologically dynamically metastable.
  - U(1) number symmetry may or may not be broken...



Winding number:

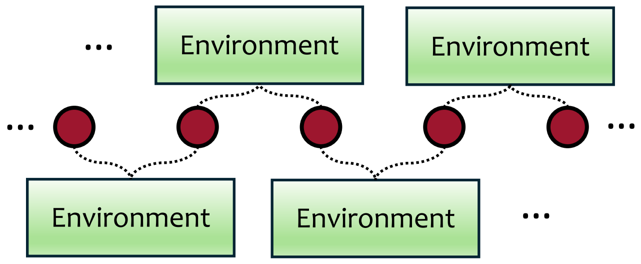
$$\nu(\lambda, \mathbf{x}(e^{ik})) \equiv \frac{1}{2\pi i}\int_{-\pi}^{\pi} dk \frac{d}{dk} \ln[\mathbf{x}(e^{ik}) - \lambda]$$

Majorana bosons (MBs) [Broken U(1)]

- A pair of edge-localized, *canonically conjugate* modes:
$$\gamma^{z\dagger} = \gamma^z,\ \gamma^{s\dagger} = \gamma^s,\ [\gamma^z, \gamma^s] \sim i$$
- One approximate ZM: $\mathcal{L}^\star(\gamma^z) \propto \mathcal{O}(e^{-N})$
- One approximate SG:
$$\mathcal{L}(e^{i\theta\gamma^s}\rho e^{-i\theta\gamma^s}) = e^{i\theta\gamma^s}\mathcal{L}(\rho)e^{-i\theta\gamma^s} + \mathcal{O}(e^{-N})$$
- Persisting in a transient regime that scales with N.
- Robust against disorder.
- SG generates a degenerate *quasi-steady-state* manifold:
$$\rho_\theta = e^{i\gamma^s\theta}\rho_{\rm ss}e^{-i\gamma^s\theta}, \quad \rho_\theta(t) \simeq \rho_\theta(0) + \mathcal{O}(e^{-N})$$

Example 1: Purely dissipative bosonic Majorana chain

- Simplest setting: Forgo a Hamiltonian completely ⇒ *Purely dissipative QBL*



$$\mathcal{L}_{\mathrm{PDC}} = \mathcal{D}_- + \mathcal{D}_+ + \mathcal{D}_{-,\mathrm{NN}} + \mathcal{D}_{+,\mathrm{NN}} + \mathcal{D}_{p,\mathrm{NN}}$$

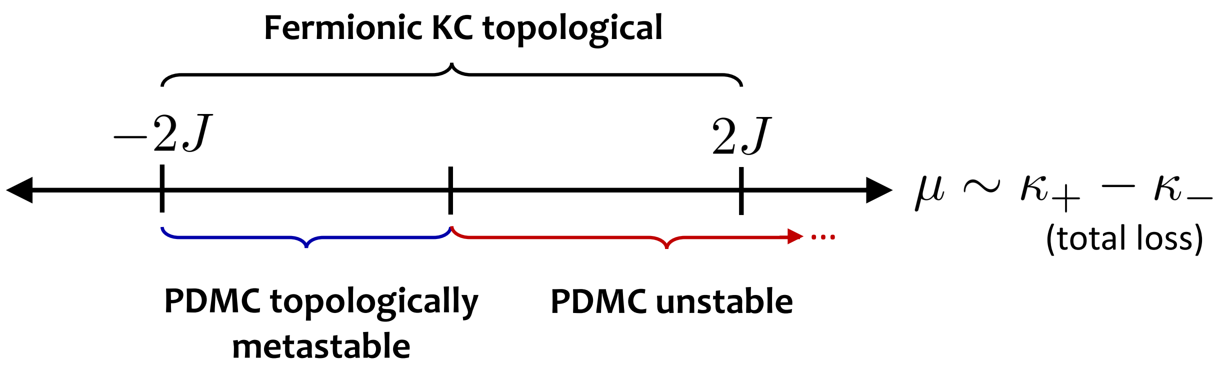
$\mathcal{D}_-$: Uniform on-site damping at rate κ_-

$\mathcal{D}_+$: Uniform on-site pumping at rate κ_+

$\mathcal{D}_{-,\mathrm{NN}}$: Uniform NN damping at rate J_-

$\mathcal{D}_{+,\mathrm{NN}}$: Uniform NN pumping at rate J_+

$\mathcal{D}_{p,\mathrm{NN}}$: NN dissipative 'pairing' of strength Δ



- 'Topologically equivalent' to the fermionic Kitaev chain!
- In purely dissipative settings, each MB is *both* a ZM *and* a SG: These MBs are 'non-split'.

$$\gamma_L = \sum_{j=1}^{N} \delta^{j-1} x_j \, , \;\; \gamma_R = \sum_{j=1}^{N} \delta^{N-j} p_j$$

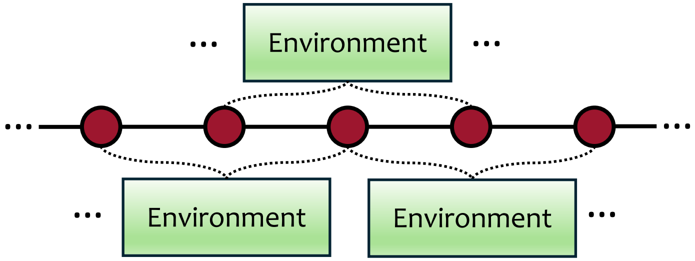
$$\delta = \frac{\kappa_- - \kappa_+}{2J} \;\; (J = \Delta)$$

Example 2: Dissipative bosonic Kitaev chain

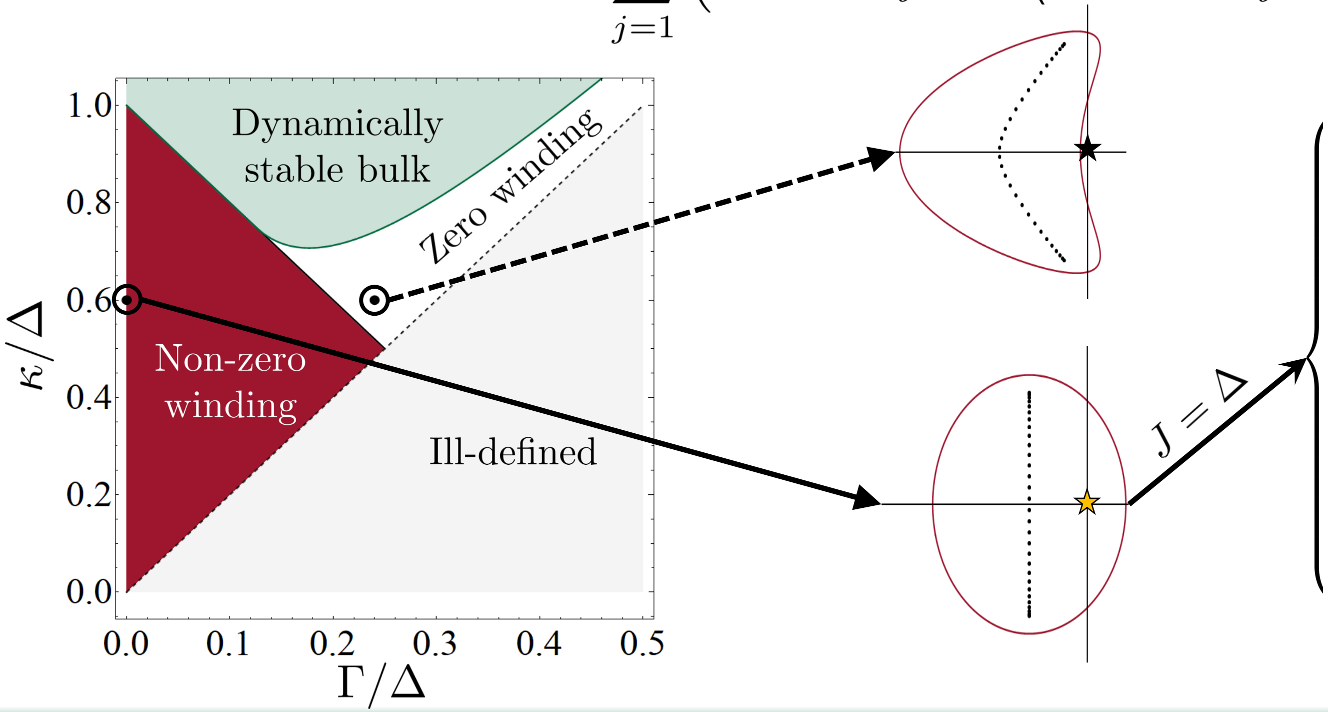
❖ Exploit Hamiltonian-dissipative interplay:

$$H_{\mathrm{BKC}} \equiv \frac{1}{2}\sum_{j=1}^{N}\Big(\overbrace{iJa_{j+1}^{\dagger}a_j}^{\text{Hopping}} + \overbrace{i\Delta a_{j+1}^{\dagger}a_j^{\dagger} + i\mu a_j^{\dagger 2}}^{\text{Two-photon 'drive'}} + \mathrm{H.c.}\Big),$$

McDonald *et al*, PRX **8** 2018; Flynn *et al*, NJP **22** 2020 [TH];
Slim *et al*, Nature **627** 2024; Busnaina *et al*, Nat. Communs. **15** 2024 [EXP]



$$\mathcal{L}^{\star}(A) = i[H_{\mathrm{BKC}}, A] + 2\sum_{j=1}^{N}\Big(\overbrace{\kappa\mathcal{D}^{\star}[a_j, a_j^{\dagger}]}^{\text{Local loss}} + \overbrace{\Gamma\left(\mathcal{D}^{\star}[a_{j+2}, a_j^{\dagger}] + \mathcal{D}^{\star}[a_j, a_{j+2}^{\dagger}]\right)}^{\text{NNN loss}}\Big)A, \quad \kappa \geq 2\Gamma \geq 0\ :$$



$$\gamma_L^z = \sum_{j=1}^{N}\delta_-^{j-1}, \qquad \gamma_R^s = \sum_{j=1}^{N}\delta_-^{N-j}p_j$$

$$\delta_{\pm} = -\frac{\mu \pm \kappa}{J}$$

$$\gamma_L^s = \sum_{j=1}^{N}\delta_+^{j-1}, \qquad \gamma_R^z = \sum_{j=1}^{N}\delta_+^{N-j}p_j$$

Observable signatures of topological metastability

- Topological metastability can be detected through steady state *two-time correlation functions* and *power spectra*:

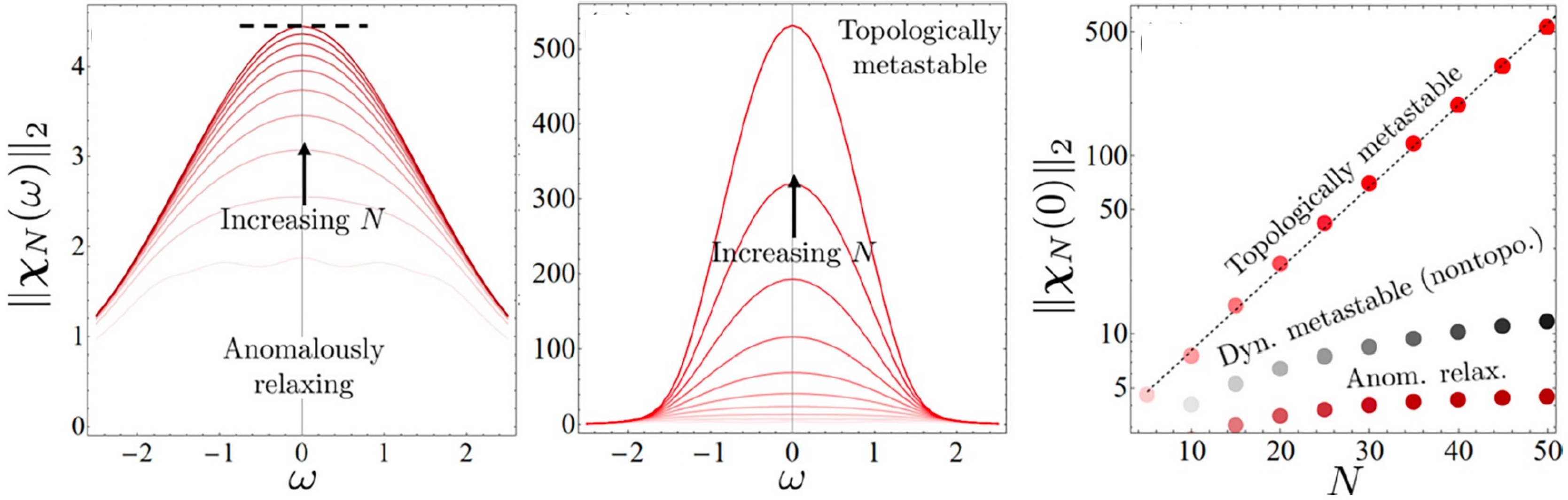
$$C_{\alpha,\beta}(t,\tau) = \langle \alpha(t+\tau)\beta(t)\rangle \xrightarrow[t\to\infty]{\text{QRT}} \text{tr}[\alpha(\tau)\beta(0)\rho_{\text{ss}}] = C^{\text{ss}}_{\alpha,\beta}(\tau), \quad \tau \geq 0 \qquad S^{\text{ss}}_{\alpha,\beta}(\omega) = \int_{-\infty}^{\infty} e^{i\omega\tau} C_{\alpha,\beta}(\tau)\, d\tau$$

  - **Linear observables: (1)** The power spectra are directly related to *susceptibility (response) matrix*

$$S^{\text{ss}}_{\alpha,\beta}(\omega) \sim \boldsymbol{\chi}(\omega), \quad \boldsymbol{\chi}(\omega) = -i(\mathbf{G} - \omega)^{-1}$$

(2) The *anti-symmetrized* correlation function becomes state-independent: $C^{\text{qu}}_{\alpha,\beta}(t,\tau) \propto \frac{1}{2}\langle[\alpha(t+\tau), \beta(t)]\rangle$

- **Prediction:** MBs elicits a *peak of the power spectrum at zero frequency*, which grows unbounded with system size.



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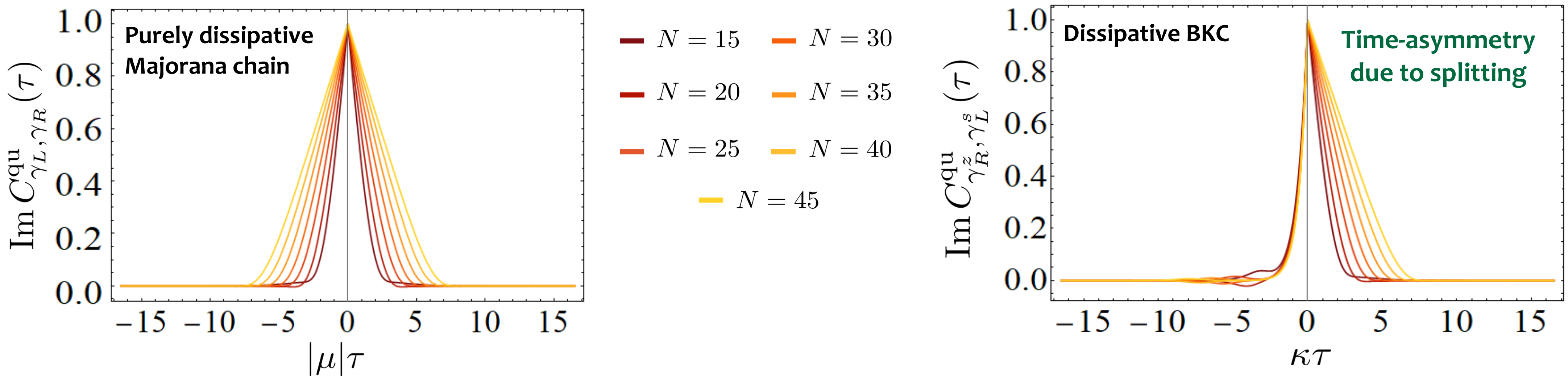
$$S^{\text{ss}}_{\alpha,\beta}(\omega) \sim \boldsymbol{\chi}(\omega), \quad \boldsymbol{\chi}(\omega) = -i(\mathbf{G}-\omega)^{-1}$$

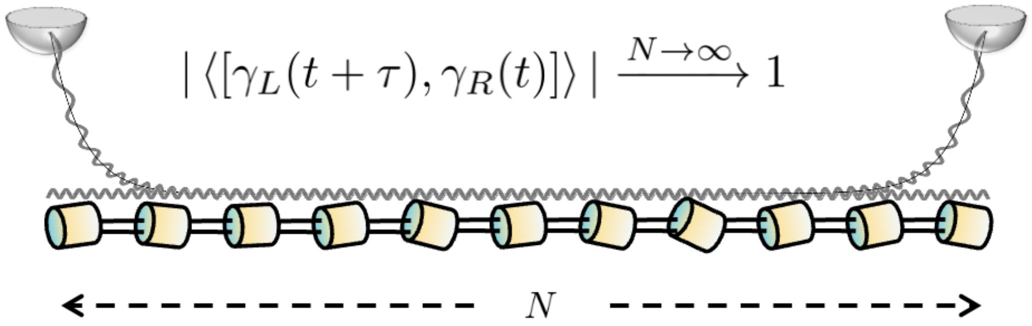
    (2) The *anti-symmetrized* correlation function becomes state-independent: $C^{\text{qu}}_{\alpha,\beta}(t,\tau) \propto \frac{1}{2}\langle[\alpha(t+\tau),\beta(t)]\rangle$

- **Prediction:** MBs have *long-lived quantum correlation functions*, despite macroscopic separation!



Conclusion & outlook

- Transient (dynamically) metastable regimes of QBLs point to a *new route for realizing genuine SPT phases* of free bosons in 1D.
- Topological metastability yields robust, *topologically-mandated edge zero modes*, symmetries, and quasi-steady states – over transient timescales that grow with system size.
- Topological metastability manifests in *distinctive signatures* in two-time quantum correlation functions and power spectra.



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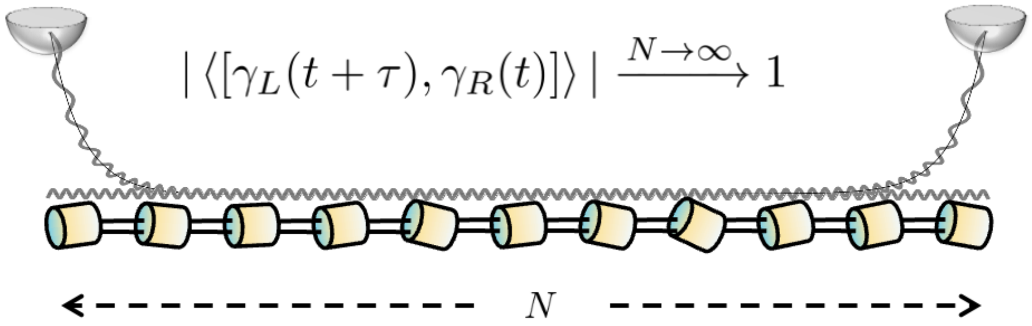

- **(Some) Outstanding questions:**
  - Response of topologically metastable QBLs to *weak interactions (nonlinearities)* – Stability of MBs?...
  - Detection of MBs in *experimental settings*: Cavity/cQED, magnonic realizations?…
  - Connections between topological metastability and genuine (dissipative) SPT phase transitions?...
  - Extensions to *higher dimensions* – Bosonic analogues to topological surface bands in 2D ($p + ip$)?…
  - Application to *quantum technologies*: Quantum sensors, amplifiers, continuous-variable quantum computation processing (relevance to GKP codes?)…

  ⋮

Postdoc position available!