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Synthetic Magnetism in Nonlinear Optics

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Spice workshop on Spin Textures: Magnetism meets Plasmonics July 2024



- Sum frequency generation as a two-level system
 - Rabi oscillations
 - Adiabatic frequency conversion
 - Adiabatic geometric phase
- Pauli equation for diffracting beams and the all-optical Stern-Gerlach effect in space.
- The all-optical Stern-Gerlach effect in the time domain.
- Spintronic effects and devices in optics:
 - Anderson localization of the signal and idler in disordered nonlinear media
 - Spin valve controlling the transmission and reflection of signal-idler in-phase beam with a pump beam
 - Spin waveguide guiding of signal-idler in-phase beam
 - Skyrmionic nonlinear photonic crystals and the topological Hall effect for light

Sum Frequency Generation – textbook problem



Signal and idler intensities vs. nonlinear coupling, for undepleted pump

Saleh and Teich, Fundamentals of Photonics



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Sum Frequency Generation – a 2-mode system

Sum Frequency Generation with strong (un-depleted) pump



Assuming slowly varying amplitudes, we obtain a two-coupledmode system:

$$-i\frac{dA_{1}}{dz} = K_{1}A_{3}e^{-i\Delta kz}$$

$$-i\frac{dA_{3}}{dz} = K_{3}A_{1}e^{i\Delta kz}$$

$$A_{i} \equiv \frac{A_{i}}{\sqrt{K_{i}}}$$

$$-i\frac{dA_{1}}{dz} = \kappa A_{3}e^{-i\Delta kz}$$

$$-i\frac{dA_{3}}{dz} = \kappa^{*}A_{1}e^{i\Delta kz}$$

R. W. Boyd, *Nonlinear Optics*, 2nd Edition (2003)

Examples of two-coupled-mode systems



Suchowski et al, Lasers and Photonics Reviews (2014)

One can utilize representation methods (e.g. Bloch sphere, eigenvalue diagram) and concepts (adiabatic following, geometric phase) developed in other disciplines for nonlinear optics.

Specifically, here we focus on **analogy with spin-1/2 in magnetic field.**

SFG representation with Bloch sphere

All the possible states can be represented using a sphere, whose coordinates are determined the complex amplitudes A_1 and A_3

 $U_{SFG} = A_1^* A_3 + A_3^* A_1$ Real coherence operator

 $V_{SFG} = i(A_1^*A_3 - A_3^*A_1)$ Imaginary coherence operator

 $W_{SFG} = |A_3|^2 - |A_1|^2$ "Excitation" operator



Torque or "magnetic field" vector

$$B = \sqrt{\kappa^2 + \Delta k^2}; \quad \vec{B} = [\operatorname{Re}(\kappa), \operatorname{Im}(\kappa), \Delta k]$$

System dynamics:

$$\frac{d\vec{\rho}_{SFG}}{dz} = \vec{B} \times \vec{\rho}_{SFG}$$

Constant phase mismatch ($\Delta k=0$)



Perfect phase-matching



 $\vec{B} = [\operatorname{Re}(\kappa), \operatorname{Im}(\kappa), 0]$

φ₁(0) **↓**

Upconverted



(b)

Upconverted

signal $\phi_3(z)$

Signal $\phi_1(z)$

SFG analogy with atomic two-level system



Parameter	Optical Bloch sphere	SFG sphere realization
Evolution parameter	time	z axis
Ground/excited state population	$ a_g ^2, a_e ^2$	$ A_1 ^2, A_3 ^2$
Energy difference	$\omega_0 = \omega_{fg}$	$n(\omega_2)\omega_2/c$
Detuning/phase mismatch	Δ	Δk
"Rabi" frequency	$\Omega_0 = \frac{1}{\hbar} \mu E_{in}$	$\kappa = \frac{4\pi w_1 w_3}{(k_1 k_3)^{1/2} c^2} \chi^{(2)} E_2$
Torque vector	$\Omega = (\operatorname{Re}\{\Omega_0\}, \operatorname{Im}\{\Omega_0\}, \Delta)$	$g = (\operatorname{Re}\{\kappa\}, \operatorname{Im}\{\kappa\}, \Delta k)$
State vector— ρ	$(\operatorname{Re}\{a_{f}^{*}a_{g}\}, \operatorname{Im}\{a_{f}^{*}a_{g}\}, a_{f} ^{2} - a_{g} ^{2})$	$(\operatorname{Re}\{A_3^*A_1\}, \operatorname{Im}\{A_3^*A_1\}, A_3 ^2 - A_1 ^2)$

How to structure nonlinear crystals?1D/2D E field polingLaser induced poling
(allows 3D modulation)



Poling periods: 3-40 µm (state of the art: 0.317 µm).

Crystal thickness: 1 mm (state of the art: 10 mm).

A. Arie and N. Voloch, Laser Photon. Rev. 4, 355 (2010)Y. Zhang, Y. Sheng, S. Zhu, M. Xiao, and W. Krolikowski, Optica 8, 372 (2021).

Examples of poled crystals by Efield poling (selective etching of the top surface)





Sharabi,..AA, PRL (2014) Broderick et al, PRL (2000) Examples of 3D lasers-poled crystals (Cerenkov nonlinear microscopy)



Zhang et al, Optica (2021)

Adiabatic frequency conversion



Far off Resonant |Δ| >> Ω₀
 Adiabatic changing of Δ<0 to Δ>0

In nonlinear optics:

1. Large phase mismatch $|\Delta k| >> \kappa$

2. Adiabatic changing of $\Delta k < 0$ to $\Delta k > 0$

(or vice versa)



- Efficient Frequency conversion
- Robust for parameter variation



Adiabatic frequency conversion on Bloch Sphere

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Adiabatic SFG



H. Suchowski, D. Oron, A. Arie and Y Silberberg, PRA 78, 063821 (2008)

Adiabatic SFG: broad spectral acceptance



Conversion efficiency vs. pump intensity

Adiabatic frequency conversion is efficient, broadband and robust. It is suitable for conversion of ultrafast pulses.



Conversion efficiency ~100%

Landau-Zener limit: $\eta_{LZ}(z \to \infty) = 1 - \exp\left(-2\pi \frac{\kappa^2}{d\Delta k/dz}\right)$ J. Moses, H. Suchowski, F. Kartner, Opt. Lett. (2012)

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Adiabatic following and M.C. Escher



Generalize coupled wave equation

So far we considered only variations in Δk , which correspond to changes in the torque vector from the north pole to the south pole. A general coupled wave equation:

$$i\frac{d}{d\tau}\begin{pmatrix}\tilde{A}_i\\\tilde{A}_s\end{pmatrix} = -\boldsymbol{\sigma}\cdot\boldsymbol{B}\begin{pmatrix}\tilde{A}_i\\\tilde{A}_s\end{pmatrix}$$

where $\boldsymbol{\sigma}$ represents the Pauli matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and the Magnetization, or torque vector, is given by

$$\boldsymbol{B} = \frac{1}{2}\kappa\sin\pi D\left(\hat{x}\cos\phi + \hat{y}\sin\phi\right) + \frac{\Delta k}{2}\hat{z}$$

D – duty cycle, ϕ - pump phase – poling phase, Δk – phase mismatch

Details:

Move to the *rotating frame* and define amplitudes:

$$A_{i} = \sqrt{k_{s}}\omega_{i}e^{-\frac{i}{2}(\Delta k_{0}z - \int q(z')dz')}\tilde{A}_{i} \qquad A_{s} = \sqrt{k_{i}}\omega_{s}e^{+\frac{i}{2}(\Delta k_{0}z - \int q(z')dz')}\tilde{A}_{s}$$

Dimensionless propagation coordinate: $\tau = \sqrt{k_i k_s} z$

Phase mismatch: $\Delta k(\tau) = \Delta k_0 - q(\tau)$; Nonlinear coupling: $\kappa(z) = \frac{4\sqrt{k_i k_s}}{n_i n_s} |d(z)A_p|$

Controlling the torque vector on the Bloch sphere

The torque vector, or magnetic-field equivalent vector:

$$\mathbf{B}(\tau) = B_0(\tau) \hat{\mathbf{B}}[\theta(\tau), \phi(\tau)],$$

$$B_0(\tau) = \frac{1}{2\sqrt{k_i k_s}} \sqrt{\kappa^2 \bar{d}^2(\tau) + \Delta k^2(\tau)},$$

$$\cos\theta(\tau) = \frac{\Delta k(\tau)}{\sqrt{\kappa^2 \bar{d}^2(\tau) + \Delta k^2(\tau)}},$$
$$\phi(\tau) = \phi_p - \phi_d(\tau).$$



Phase offset control ϕ



Duty cycle d (together with Δk) control vector size,

QPM Period – controls phase mismatch Δk

=> Z axis



Circular adiabatic rotation in SFG

Starting with eigenstate in the north pole, we can utilize the nonlinear modulation degrees of freedom to realize a closed rotation on the Bloch sphere

Circular Rotation $\gamma = -\pi(1 - \cos \Theta_0)$ Simulation results for LiNbO₃, $L = 2.5 \ cm$ and $I_p = 100 \ MW/cm^2$ with $\gamma = -\frac{\pi}{2}$





Aviv Karnieli



Karnieli and Arie, Opt. Exp. 26, 4920(2018)

Circular adiabatic rotation in SFG



Prepared by Inbar Hurvitz

Broadband mode converter: HG00->HG01



Paraxial SFG and the Pauli equation

So far we assumed that the interacting waves act as *plane waves*, with negligible diffraction.

Let's consider now the case of *paraxial diffraction*. The wave equation can be written as:

Assuming the long-wavelength pump approximation $k_p \ll k_i, k_s$: $\mathbf{M}^{-1} \rightarrow \frac{1}{m}, \ m = \frac{2\sqrt{k_i k_s}}{k_i + k_s} \cong 1$

This equation has a similar form to the Pauli equation, describing a spin ¹/₂ particle in a weak transverse magnetic field

$$i\hbar \frac{\partial}{\partial t} |\psi\rangle = \left(\frac{\boldsymbol{p}_T^2}{2m} - \mu \boldsymbol{\sigma} \cdot \boldsymbol{B}_T\right) |\psi\rangle$$

 μ is the magnetic moment and σ the Pauli matrix



The Stern-Gerlach experiment



Memorial

Plaque in

Frankfurt

For more details: B. Friedrich and D. Hershbach, Phys. Today 53-59, Dec. 2003.

The Stern-Gerlach effect in NLO



Transverse magnetization gradient:





Karnieli and Arie, *Phys. Rev. Lett.* **120**, 053901 (2018) O. Yesharim,...A. Arie, *Nature Photonics* **16**, 582 (2022)

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The all-optical Stern-Gerlach effect - experiment







Ofir Yesharim Aviv Karnieli

O. Yesharim,...A. Arie, Nature Photonics 16, 582 (2022)



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Stern-Gerlach effect for eigenstates

Input: idler only
$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Input: idler – signal, $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

Input: idler + signal,
$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Controlling the signal-idler relative phase





Measurement of the two eigenstates



Quantum optical Stern-Gerlach effect: two-photon input

Two different photons in idler and signal frequency: never detected at two outputs simultaneously and transformed into superposition!



Stern-Gerlach-HOM effect with distinguishable photons

Karnieli and Arie, *Optica* **5**, 1297 (2018) Also see: Kobayashi et al, *Nature Photonics* **10**, 441 (2016); Imani et al, *Opt. Lett.* **43**, 2760 (2018).

Dispersion-Diffraction analogy



Sum Frequency Generation: $\chi^{(2)}$ vs. $\chi^{(3)}$

Coupled wave equation for *quadratic* nonlinear material with undepleted pump:

$$\nabla_T^2 A_i + 2ik_i \frac{\partial A_i}{\partial z} = -\frac{4d_{eff}\omega_i^2}{c^2} A_p^* A_s e^{-i\Delta k(\mathbf{r})z}$$
$$\nabla_T^2 A_s + 2ik_s \frac{\partial A_s}{\partial z} = -\frac{4d_{eff}\omega_s^2}{c^2} A_p A_i e^{i\Delta k(\mathbf{r})z}$$

Coupled wave equation for *qubic* nonlinear material with two undepleted pump:

$$\frac{\partial^{2}}{\partial \tau^{2}} A_{i} - \frac{i2}{\beta_{2}} \frac{\partial A_{i}}{\partial z} = \frac{4}{\beta_{2}} \gamma A_{p1} A_{p2}^{*} A_{s} e^{-i\Delta kz}$$
$$\frac{\partial^{2}}{\partial \tau^{2}} A_{s} - \frac{i2}{\beta_{2}} \frac{\partial A_{s}}{\partial z} = \frac{4}{\beta_{2}} \gamma A_{p2} A_{p1}^{*} A_{i} e^{i\Delta kz}$$

Stern-Gerlach effect in the time domain

Using four-wave mixing in an optical fiber. Two undepeted pumps + signal and idler

- Two pump tones in a ramp envelope
- Couple light between signal and idler pulses
- Creates two eigenstates that • different in the optical phase
- The eigenstates have different speeds
- The spectrum of the eigenstates shifts in frequency domain to opposite directions



Gil



Eigenstates and mixed states Eigenstates Mixed states



Dashed white line – frequency shift owing to cross-phase modulation.

Experimental setup

- First EOMs modulate the pulses
- Second EOMs split the frequencies

EDFA

LO

Pulse

generator

EOM

Pulse generator

EOM

• Self-heterodyne detection

Microwave

generator

•

EOM

Microwave

generator

EOM

Pumps laser

diode

Signal and idler

fiber laser



Output spectra: Eigenstates and mixed statesEigenstatesMixed states







Frequency splitting of the eigenstates vs pump power

Low pump power, small splitting





Bashan,...AA, Opt. Exp. 32, 9589 (2024)

High pump power, large splitting

Application in classical and quantum communication

(Temporal) splitting of frequency-bin eigenstates in the Hadamard basis



Bashan,...AA, Opt. Exp. 32, 9589 (2024)

Optical Stern Gerlach – beyond nonlinear SFG

Splitting of polarization eigenstates using the electro-optic effect



Zhu et al, *Laser Photonics Rev.* **2024**, 2301030

Splitting of spatial modes using tilted planar waveguides



Li et al, *Laser Photonics Rev.* **2024**, 2301055

Spintronic effects and devices

• Anderson localization of the signal and idler in disordered nonlinear media (spin glass)

• **Spin valve** – controlling the transmission and reflection of signal-idler in-phase beam with a pump beam

• **Spin waveguide** – guiding of signal-idler in-phase beam

 Skyrmionic nonlinear photonic crystals and the topological Hall effect for light









Anderson localization



Conditions for Anderson Localization:

- 1. Time-invariant potential: $V(\mathbf{r}, t) = V(\mathbf{r})$
- 2. No interactions between the electrons (true for photos as bosons)
- 3. Strong disorder
- Wave phenomenon: can be observed in many waves systems e.g., optical waves.
- Most works so far studied Anderson localization of a *single* wave.
- Here we focus on transverse Anderson localization of *pseudo-spin* (two color

light waves $\begin{pmatrix} A_i \\ A_s \end{pmatrix}$ coupled by nonlinearity).

P. W. Anderson, *Phys. Rev.* 109, 1492–1505 (1958).
T. Schwartz et al, *Nature* 446, 52 (2007).



Spin glass in nonlinear media

Shani Izhak

The effective magnetization for phase matched process ($\Delta k = 0$) is $\mathbf{M}(\mathbf{r}) = \mathbf{M}_{\mathrm{T}}(\mathbf{r}) = \operatorname{Re} \kappa \, \hat{\mathbf{x}} + \operatorname{Im} \kappa \, \hat{\mathbf{y}}$ $= |\kappa(\mathbf{r})| \cos \varphi(\mathbf{r}) \, \hat{\mathbf{x}} + |\kappa(\mathbf{r})| \sin \varphi(\mathbf{r}) \, \hat{\mathbf{y}}$ If the poplinger coupling $\kappa(\mathbf{r}) \propto \kappa^{(2)} \, \mathbf{A}$ is constant we get an order

If the nonlinear coupling $\kappa(\mathbf{r}) \propto \chi^{(2)} A_p$ is constant we get an ordered ferromagnet, where all the spins are pointing at the same direction:

Ferromagnet



Spin - Glass





Psuedo-spin localization

Results for shaping the pump field





○ optical ferromagnet

$$I_s \propto \exp\left(-2\frac{\mathbf{r}_T^2}{\sigma^2}\right) \rightarrow \log(I_s) \propto -\mathbf{r}_T^2$$

Disordered synthetic magnetization:
 optical spin-glass

$$\overline{I_s} \propto \exp\left(-2\frac{|\mathbf{r}_T|}{\xi_{loc}}\right) \rightarrow \log(\overline{I_s}) \propto -|\mathbf{r}_T|$$



MO.0

Spintronic spin valve and its optical analogue

Our goal: To mimic the functionality of a spintronic spin valve device.



Characteristic	Spintronic Spin Valve	All-optical Spin Valve
Structure	Two ferromagnetic layers	Two synthetic magnetization layers
Operation modes	Parallel ("P"), open – high electric current, Anti-parallel ("AP"), closed – no current	 "P", open – full light transmission, "AP", closed – full light reflection
How to switch?	An external magnetic field Based on the Giant Magnetoresistance (GMR) effect	An external optical pump field

S. Izhak, .. AA, Opt. Lett. 49, 1025 (2024)

How the spin valve works?

The effective magnetization:



$$\phi_{NLPC}(\mathbf{r}) = \begin{cases} \pi, & x < 0\\ 0, & x > 0 \end{cases}, \quad \phi_p(\mathbf{r}_T) = const \to \mathbf{M}_{\mathbf{T}}(\mathbf{r}) = \begin{cases} -M_0 \, \hat{\mathbf{x}}, & x < 0\\ +M_0 \, \hat{\mathbf{x}}, & x > 0 \end{cases}, \quad M_0 \propto |\chi^{(2)}A_p|$$
The eigenstates: $\Psi_{\pm}(\mathbf{r}) = \langle \mathbf{r} | \Psi_{\pm} \rangle, \ |\Psi_{\pm} \rangle = |\psi(\mathbf{r})_{\pm} \rangle \otimes \frac{|\omega_i\rangle \pm |\omega_s\rangle}{\sqrt{2}} \xrightarrow{\Lambda_{b_i} \to \omega} \omega$

For $|\Psi_{-}\rangle$ we get *potential step*, whereas for $|\Psi_{+}\rangle$ we get *potential drop*



Therefore, by turning on the pump, we can get *total internal reflection for* $|\Psi_{-}\rangle$ No pump, $P_{pump} = 0 W$ With pump power, $P_{pump} = P_{opt}$



Spin waveguide

A waveguide only for spin-up, i.e. the in-phase superposition state,

$$|\Psi_{+}\rangle = \frac{|\omega_{i}\rangle + |\omega_{s}\rangle}{\sqrt{2}}$$

The nonlinear pattern:

$$d(x,z) = \begin{cases} 0.5, & |x| \le a, \\ 0, & else \end{cases}, \qquad \phi_{NLPC}(x) = 0 \ \forall x \end{cases}$$

Potential well for $|\Psi_+\rangle$

Potential barrier for $|\Psi_-\rangle$



Nonlinear crystal for guiding the spin-up superposition beam



Spin waveguide – experimental results

Microscope image of poled KTP crystal



Measured output intensity for $|\Psi_+\rangle$





Ofir Yesharim

Magnetic skyrmions and the topological Hall effect

Skyrmions – topologically robust chiral magnetic structures. The circular symmetric magnetization on a region of radius R is :

$$\widehat{\mathbf{M}}(\rho,\phi) = \sqrt{1 - m^2(\rho)} [\cos(n\phi + \eta)\,\widehat{\mathbf{x}} + \sin(n\phi + \eta)\,\widehat{\mathbf{y}}] + m(\rho)\widehat{\mathbf{z}}$$

Where *n* determines the *skyrmion number*:

$$S = [m(R) - m(0)] \times (n/2),$$

Skyrmions can be used to route spin currents via the *topological Hall effect*.



Tsymbal and Panagopoulos, *Nature Materials* **17**, 1054 (2018).

Nonlinear skyrmions & the topological Hall effect for light

Design the nonlinear structure and/or shape of pump beam to form synthetic magnetism in the form of a skyrmion.



A. Karnieli .. A. Arie, Nature Communications 12,1092 (2021)

Topological Hall Effect for Light Beams



A. Karnieli .. A. Arie, *Nature Communications* **12**,1092 (2021) ⁴⁶

Topological Hall Effect for different skyrmion numbers



- a, b skyrmion and anti-skyrmion for n=1
- d, e skyrmion and anti-skyrmion for n=4 (considered *unstable* in 'real' magnetic structures).
- c- deflection vs. skyrmion number
- f-idler and signal dependence on propagation.

Effects of domain shapes on deflection Cubic: $m(\rho) = 1 - 2(\rho/R)^3$ Linear: $m(\rho) = 1 - 2\rho/R$



Exponential: $m(\rho) = exp(-\rho/R)[1 - (\rho/R)(1 + e)]$

All-optical angular-momentum control of the topological Hall effect for light



A. Karnieli .. A. Arie, Nature Communications 12,1092 (2021)

Future directions (1)

Spin-1 dynamics using quasiperiodic crystal and two pumps





Karnieli and Arie, Optica 5, 1297 (2018)

Future directions (2)

Fully nonlinear interaction (pump can also be depleted): magnetization is perturbed by the pseudospin, emulating spin-transfer torque.



Porat and Arie, *JOSAB* (2013) Li,..AA, *PRA* (2020)

Ultrafast all-optical controlled multiplexers, de-multiplexers and routers.



Summary

- SFG with undepleted pump can be modeled as a **two-level system**.
- The nonlinear dynamics is analogous to that of **spin-1/2 in magnetic field**, where the signal-idler vector is the pseudo-spin and the nonlinear coupling emulates the magnetization.
- Enables realization of **new devices and effects that split, switch or guide signal-idler waves according to their relative phase**.











Thank You



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