SPIN TEXTURES:
MAGNETISM MEETS PLASMONICS

Maria
Azhar

Pascal
Dreher

Karin
Everschor-Sitte

Frank
Meyer zu Heringdorf

SFB/TRR 270
HoMMage

SFB
1242

CENIDE
CENTER
FOR
NANO
INTEGRATION
DUISBURG
ESSEN

SPICE

Topological magnetic defects

23/07/2024 Spin textures: magnetism meets plasmonics

Dr Maria Azhar

University of Duisburg-Essen

Outline

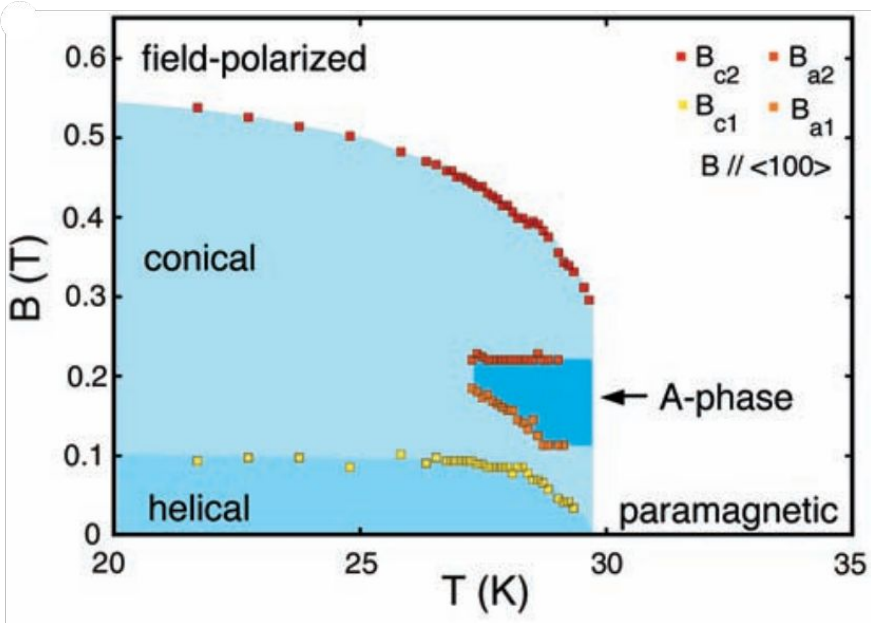
1. Introduction
2. Magnetic interactions
3. Topology and homotopy groups of spheres
4. Screw dislocations in chiral magnets

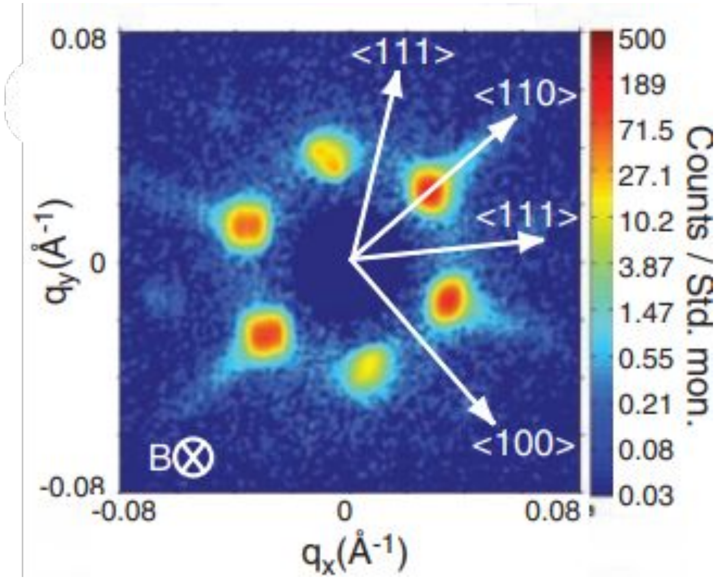
Introduction

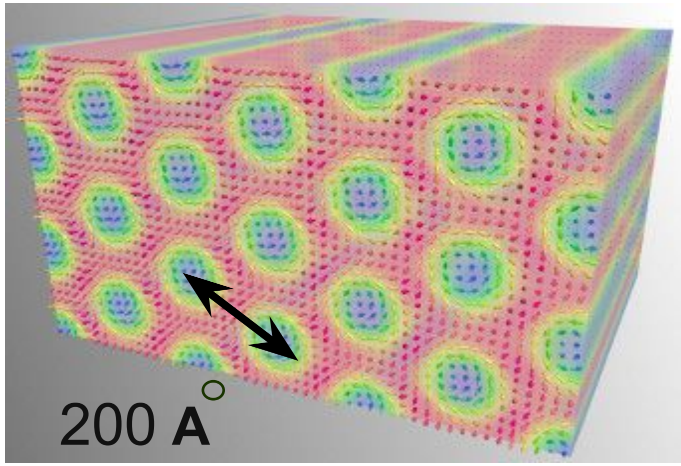
Skyrmion Lattice in a Chiral Magnet

S. Mühlbauer,[1,2] B. Binz,[3] F. Jonietz,[1] C. Pfleiderer,[1*] A. Rosch,[3]
A. Neubauer,[1] R. Georgii,[1,2] P. Böni[1]

Science 323, 915-919 (2009)



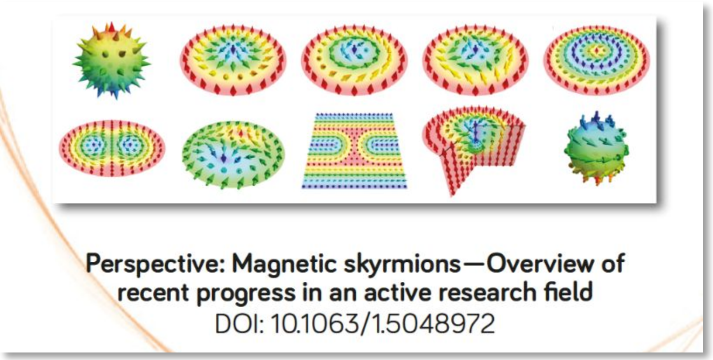


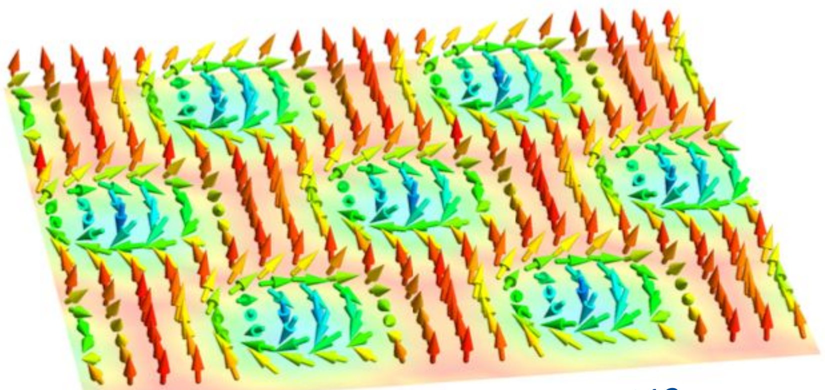


earlier theoretical work:
Bogdanov and Yablonskii,
Sov. Phys. JETP 1989

Introduction

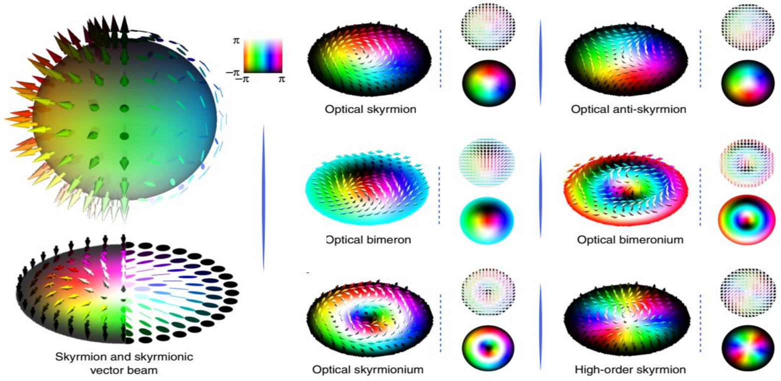
Magnetism



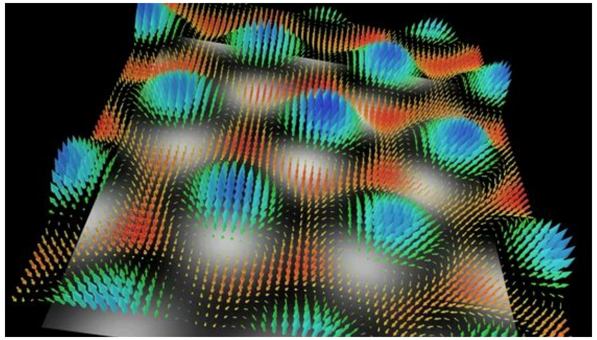


Everschor-Sitte, PhD Thesis 2012

Electromagnetic / Optical



He et al. Light: Science & Applications 11 (2022)



Davis, Janoschka, Dreher, Frank,
Meyer zu Heringdorf, Giessen, **Science 368, (2020)**

- Lots of similarities between magnetic topological excitations and electromagnetic ones
- But also key differences!

Interesting Questions:

- What is the role of topology in which quantity?
- What are measurable consequences?

Introduction

1920s Emmy Noether

- Symmetries lead to conservation laws

1920s Lev Landau

- Classify phases based on symmetry breaking

1970s Kenneth G. Wilson

- Phase transitions could be categorized into universality classes

1980s David J. Thouless, F. Duncan M. Haldane and J. Michael Kosterlitz

- **Topological** phase transitions

Introduction

1820 Hans Christian Oersted

- An **electric** current can deflect a **magnetic** compass needle

1820 André-Marie Ampère

- All **magnetic** phenomena are due to **electric** charges in motion

1831 Michael Faraday

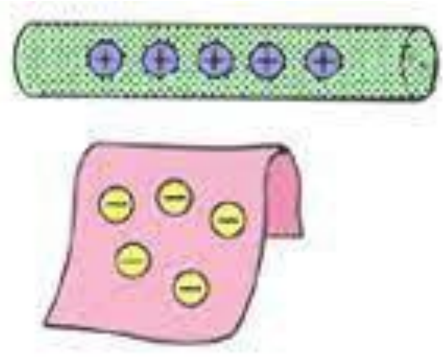
- All **magnetic** phenomena are due to **electric** charges in motion

1860s James Clerk Maxwell

- Unified theory of **electromagnetism**

1895 Hendrik Lorentz

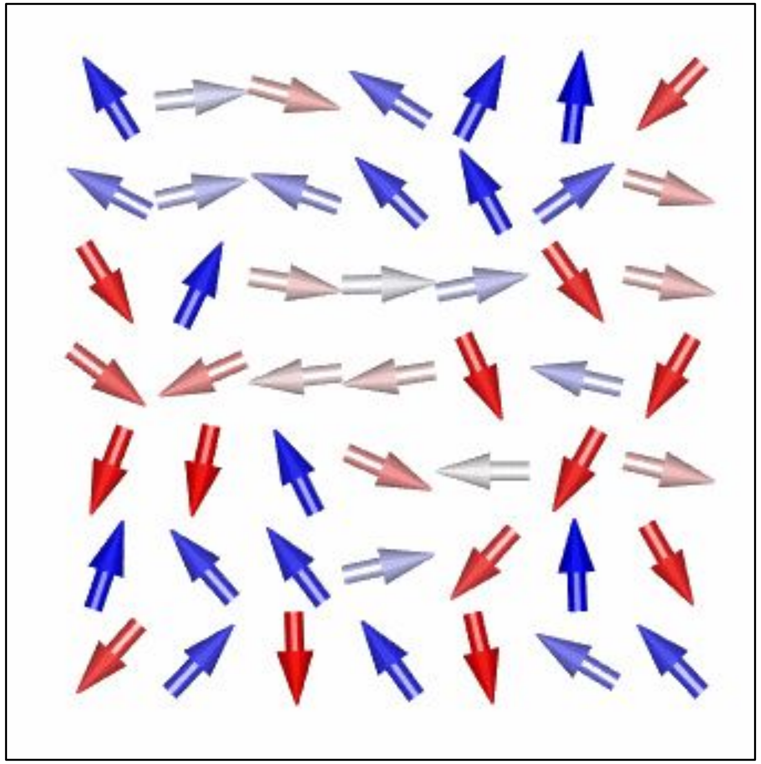
- Lorentz force law, which describes the **electromagnetic** force exerted on a charged particles



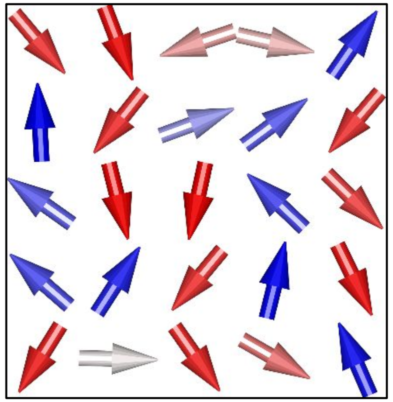
$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0}$$
$$\nabla \cdot \boldsymbol{B} = 0$$
$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$
$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \mu_0 \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}$$
$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$

Magnetic interactions

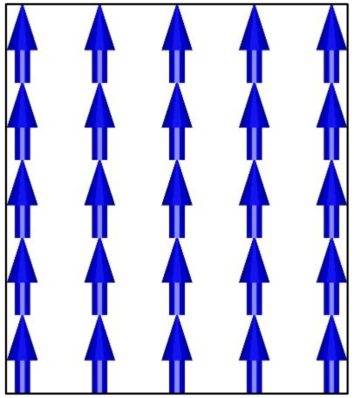
Magnetic interactions



•$H = 0$



H



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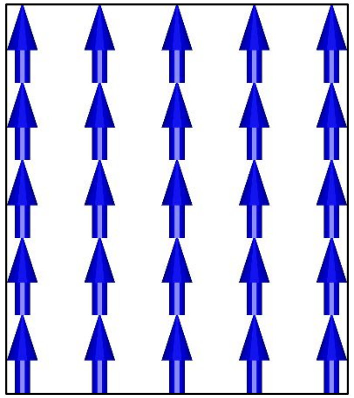
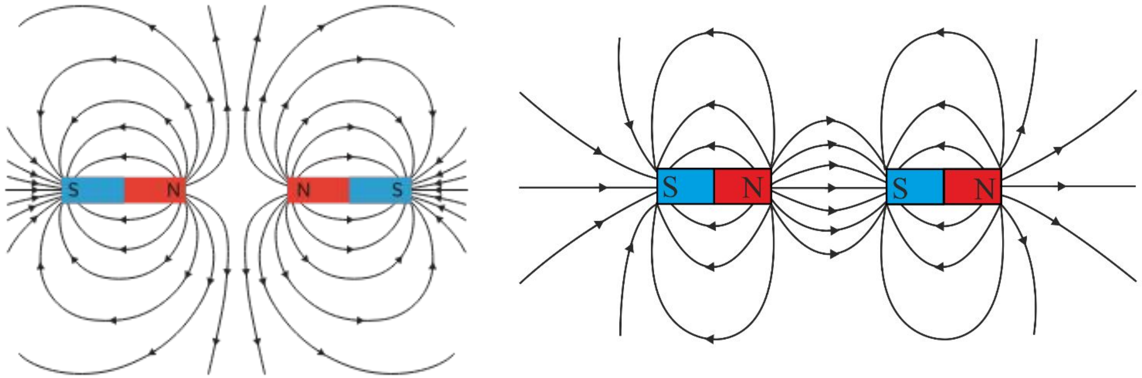


?

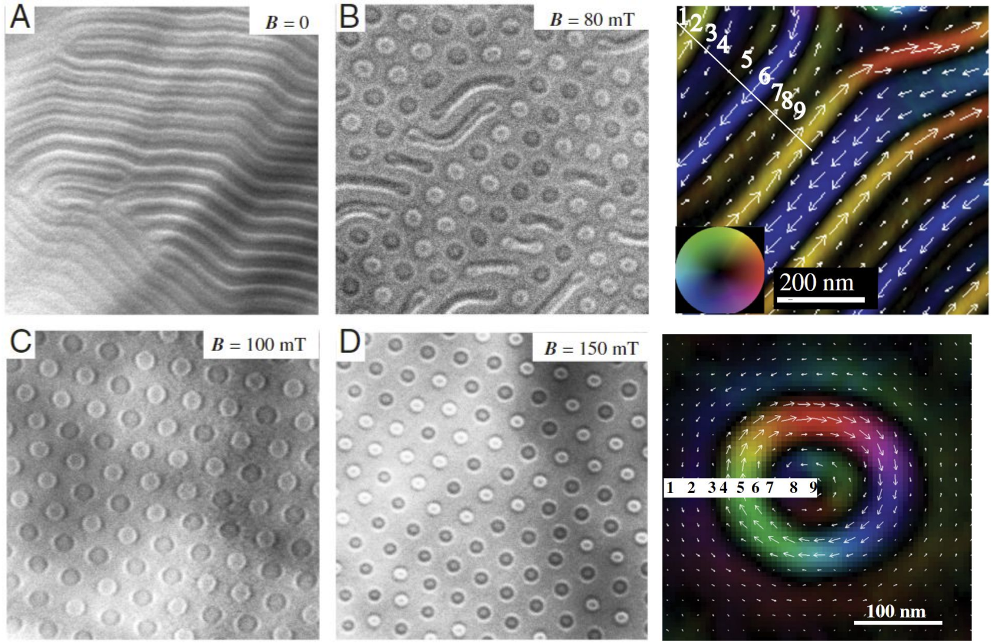
Magnetic interactions

Magneto dipolar interactions

$$E \propto \sum_{ij} \frac{1}{|\boldsymbol{r}_{ij}|^3}\left(\boldsymbol{S}_i \cdot \boldsymbol{S}_j - 3\frac{(\boldsymbol{S}_i \cdot \boldsymbol{r}_{ij})(\boldsymbol{S}_j \cdot \boldsymbol{r}_{ij})}{|\boldsymbol{r}_{ij}|^2}\right)$$



Stripe domains, bubbles, bubble array



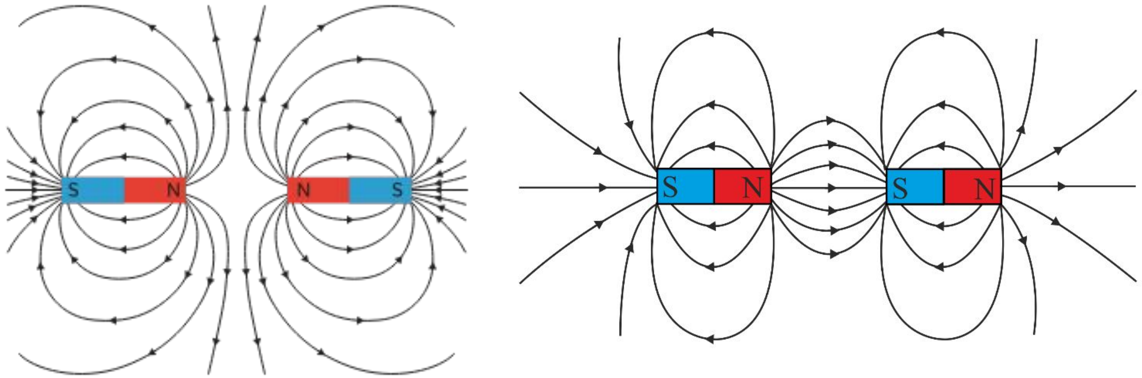
X.Yu, M. Mostovoy, Y. Tokunaga, W. Zhang, K. Kimoto, Y. Matsui, Y. Kaneko, N. Nagaosa, and Y. Tokura, PNAS, **109**, no. 23, 8856–8860 (2012)

Magnetic interactions

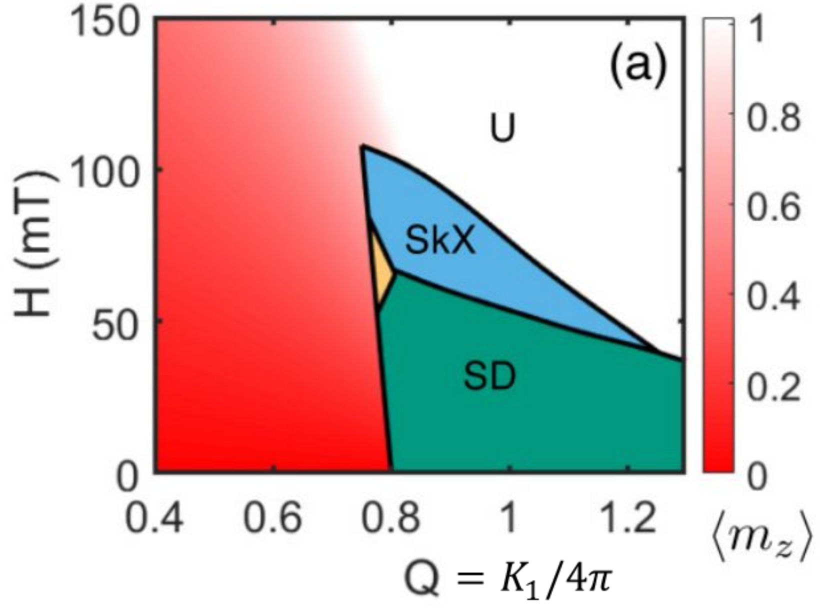
TWIST
Topological Whirls In SpinTronics

Magneto dipolar interactions

$$E \propto \sum_{ij} \frac{1}{|\boldsymbol{r}_{ij}|^3}\left(\boldsymbol{S}_i \cdot \boldsymbol{S}_j - 3\frac{(\boldsymbol{S}_i \cdot \boldsymbol{r}_{ij})(\boldsymbol{S}_j \cdot \boldsymbol{r}_{ij})}{|\boldsymbol{r}_{ij}|^2}\right)$$



Phase diagram for a thin film



P. Zhang, A. Das, E. Barts, M. Azhar, L. Si, K. Held, M. Mostovoy, and T. Banerjee
Phys. Rev. Research 2, 032026(R) (2020)

- Maxwell's equations

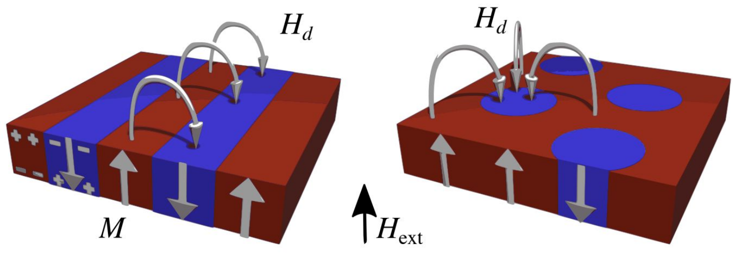
$$\nabla \cdot \boldsymbol{B} = \nabla \cdot (\mu_0 \boldsymbol{H} + \boldsymbol{M}) = 0$$

Definition of the stray field

$$\Rightarrow \nabla \cdot (\mu_0 \boldsymbol{H}_d) = -\nabla \cdot \boldsymbol{M}$$

Energy stored in the stray field is

$$E = \frac{1}{2}\mu_0 \int_{all\ space} H_d^2\, dV = -\frac{M_s}{2} \int_{sample} \boldsymbol{H}_d \cdot \boldsymbol{m}\, dV$$



PhD thesis Evgenii Barts (2023)

Define volume and surface charges

$$\boxed{\lambda_V = -\boldsymbol{\nabla} \cdot \boldsymbol{m}} \qquad \boxed{\sigma_S = \boldsymbol{m} \cdot \hat{n}}$$

Stray field energy:

$$E = \frac{1}{2} M_s \left[\int \lambda_V\, \Phi_d dV + \int \sigma_S\, \Phi_d\, dA \right]$$

energetically favorable to minimise volume charge λ_V and surface charge σ_S

Where the potential Φ_d is given by, $\boldsymbol{H}_d = -\nabla \Phi_d$

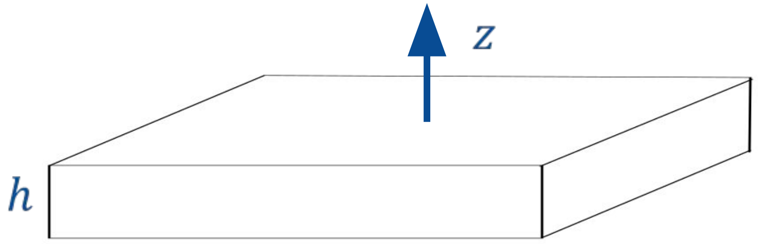
$$\Phi_d(\boldsymbol{r}) = \frac{M_s}{4\pi\mu_0} \left[\int \frac{\lambda_V(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|} dV + \int \frac{\sigma_S(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|} dA \right]$$

Magnetic interactions

Magneto-dipolar interactions for a thin film (magnetisation does not vary along film normal)

$$\varepsilon_{dd} = \frac{E_{dd}}{hS} = 2\pi \sum_{\boldsymbol{q}} \left[f(qh)|m_z(\boldsymbol{q})|^2 + (1 - f(qh))\left|\hat{\boldsymbol{q}} \cdot \boldsymbol{m}_{||}(\boldsymbol{q})\right|^2 \right]$$

$$\approx 2\pi {m_z}^2 + 2\pi \frac{qh}{2} \sum_{\boldsymbol{q}} \left[-|m_z(\boldsymbol{q})|^2 + \left|\hat{\boldsymbol{q}} \cdot \boldsymbol{m}_{||}(\boldsymbol{q})\right|^2 \right] + \cdots$$

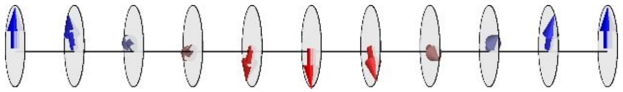


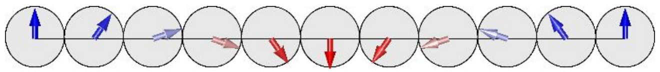
$$f(x) = \frac{1 - e^{-x}}{x}$$

$$\approx 1 - \frac{x}{2} + \frac{x^2}{6} + \cdots$$

second-order anisotropy
(shape anisotropy)

anisotropy for a spiral ordering
favours Bloch (helical) spirals

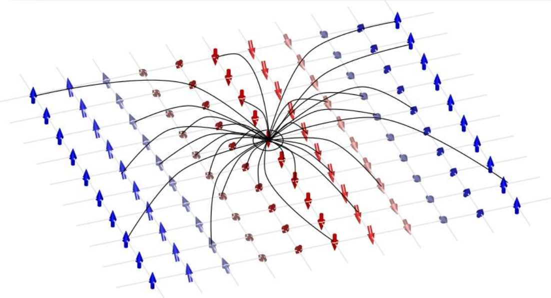




Néel (cycloidal)
spirals not favored

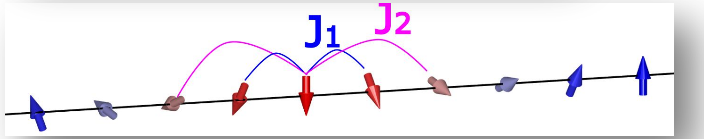
Magnetic interactions

1. Magnetic dipole-dipole interactions



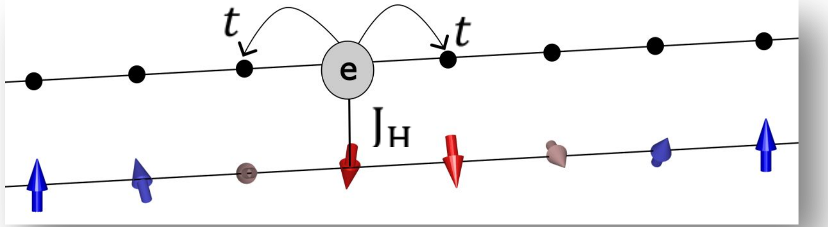
$$E \propto \sum_{ij} \frac{1}{|\boldsymbol{r}_{ij}|^3}\left(\boldsymbol{S}_i \cdot \boldsymbol{S}_j - 3\frac{(\boldsymbol{S}_i \cdot \boldsymbol{r}_{ij})(\boldsymbol{S}_j \cdot \boldsymbol{r}_{ij})}{|\boldsymbol{r}_{ij}|^2}\right)$$

2. Competing exchange (frustrated exchange interactions)



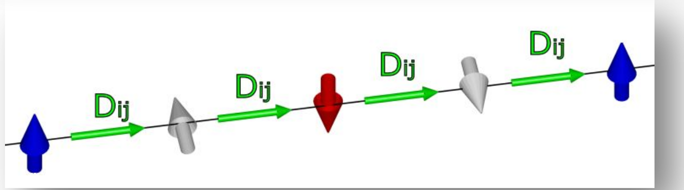
$$E = -\sum_{ij} J_{ij}\boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

3. Magnetism of conduction electrons (Kondo lattice model or Double Exchange model)



$$H = -t\sum_{ij}\psi_i^\dagger\psi_j - \mathrm{J_H}\sum_{i}\psi_i^\dagger\boldsymbol{\sigma}\psi_i \cdot \boldsymbol{S}_i$$

4. Dzyaloshinskii-Moriya interaction in chiral magnets



$$E = \sum_{ij}\boldsymbol{D}_{ij} \cdot \boldsymbol{S}_i \times \boldsymbol{S}_j$$

Magnetic interactions

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Topological Whirls In SpinTronics

	Discrete model	Continuum model *
Magnetic dipole-dipole interactions		
Heisenberg exchange or competing exchange		
Magnetism of conduction electrons		
Dzyaloshinskii-Moriya interaction		

* A ferromagnet is described by a magnetization vector $\boldsymbol{M} = M_s\boldsymbol{m}$ where $\boldsymbol{m}(\boldsymbol{x}, t)$ has a fixed length, $|\boldsymbol{m}(\boldsymbol{x}, t)|$ =1, i.e., $\boldsymbol{m} \in \mathbb{S}^2$

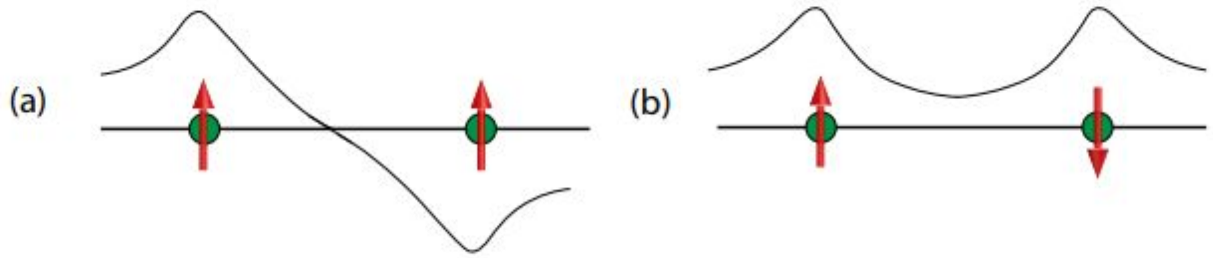
Magnetic interactions

- $$\mathcal{H} = -\sum_{ij\in N} J_{ij}\boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

  In general, magnetic interactions in solids are usually generated as an indirect manifestation of the stronger Coulomb interaction

$$\mathcal{H} = -\sum_{ij}\sum_{\sigma} a^{\dagger}_{i\sigma} t_{ij} a_{j\sigma} + \sum_{ii'jj'}\sum_{\sigma\sigma'} U_{ii'jj'} a^{\dagger}_{i\sigma} a^{\dagger}_{i'\sigma'} a_{j'\sigma'} a_{j\sigma} \xrightarrow{U_{ii'jj'}} -2\sum_{ij\in N} J_{ij}\boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Induced effective ferromagnetic exchange between electrons on *different* sites



Magnetic interactions

$$\mathcal{H} = -\sum_{ij\in N} J_{ij}\boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

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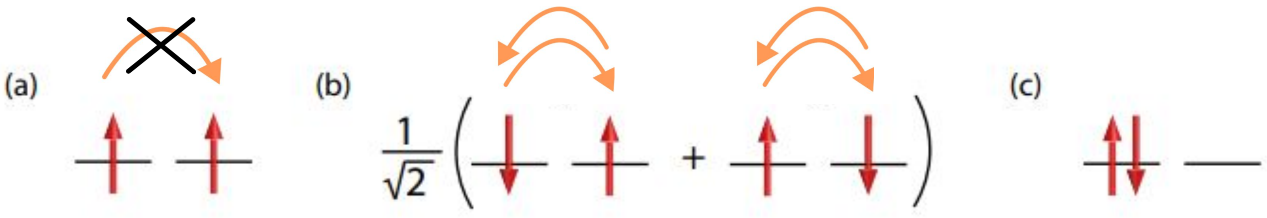


$E = 0$ $\qquad$ $E = -\dfrac{4t^2}{U}$ $\qquad$ $E = U$

effective antiferromagnetic exchange due to *on-site* Coulomb repulsion

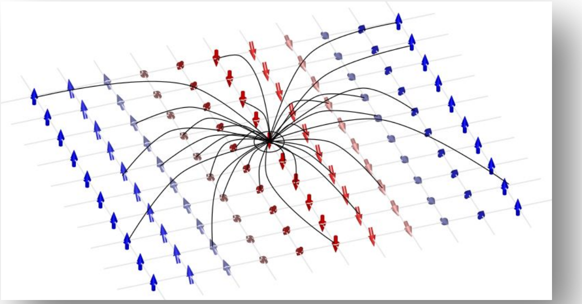
$$\mathcal{H} = const. + \frac{2t^2}{U}\sum_{ij} S_i \cdot S_j$$

Magnetic interactions

TWIST
Topological Whirls In SpinTronics

1. Magnetic dipole-dipole interactions

$$E_{dd} = \sum_{ij} \frac{\boldsymbol{S}_i \cdot \boldsymbol{S}_j - 3(\boldsymbol{S}_i \cdot \boldsymbol{r}_{ij})(\boldsymbol{S}_j \cdot \boldsymbol{r}_{ij})/|\boldsymbol{r}_{ij}|^2}{|\boldsymbol{r}_{ij}|^3}$$



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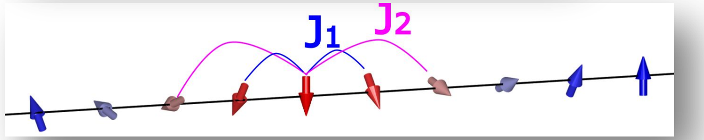

3. Magnetism of conduction electrons (Kondo lattice model or Double Exchange model)

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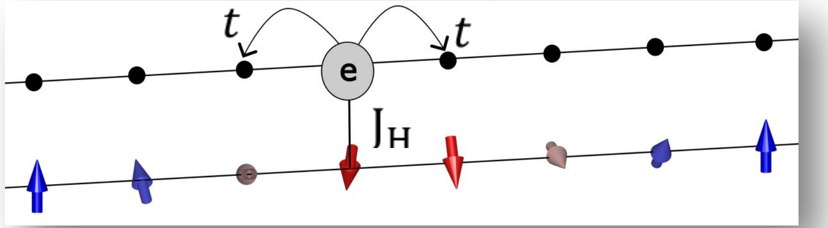
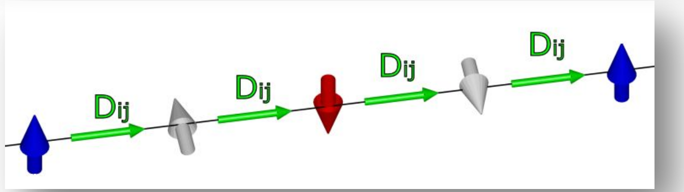


4. Dzyaloshinskii-Moriya interaction in chiral magnets

$$E = \sum_{ij} \boldsymbol{D}_{ij} \cdot \boldsymbol{S}_i \times \boldsymbol{S}_j$$



UDE

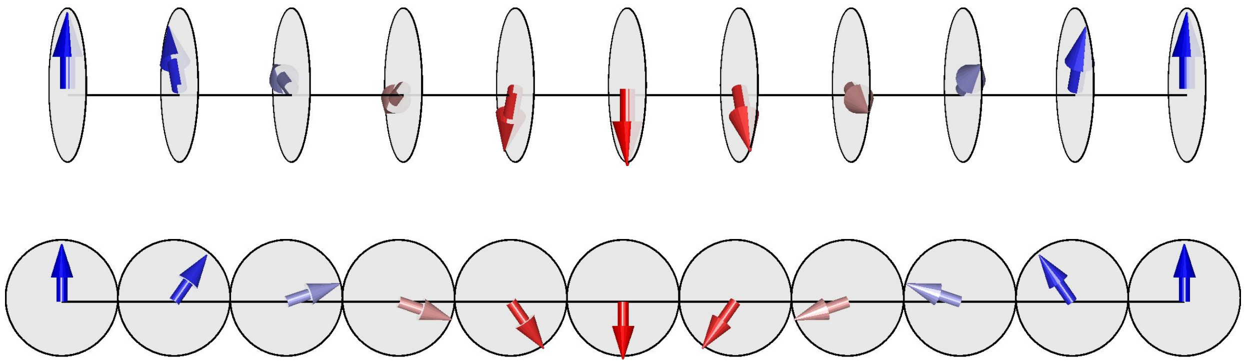
Competing exchange interactions

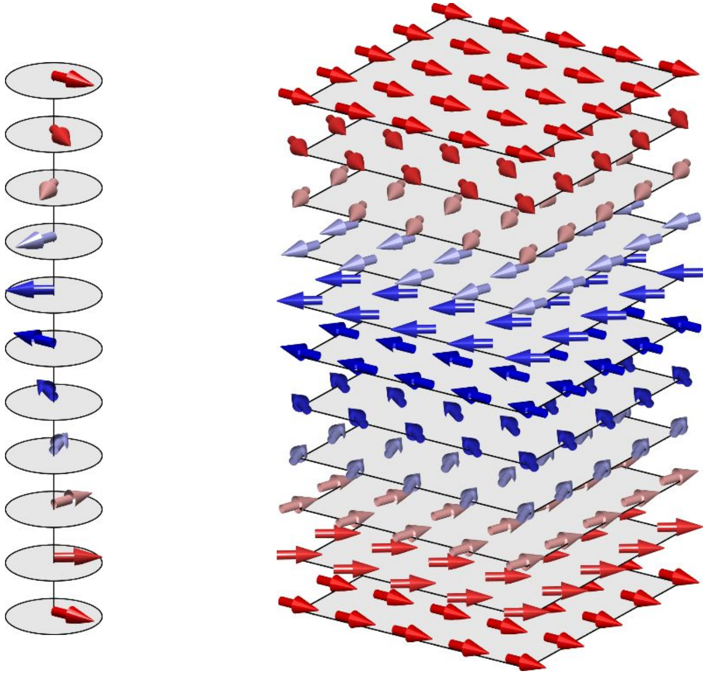
Topological Whirls In SpinTronics

$$\mathcal{H} = -J_1 \sum_{<ij>} S_i \cdot S_j + J_2 \sum_{\ll ij \gg} S_i \cdot S_j$$

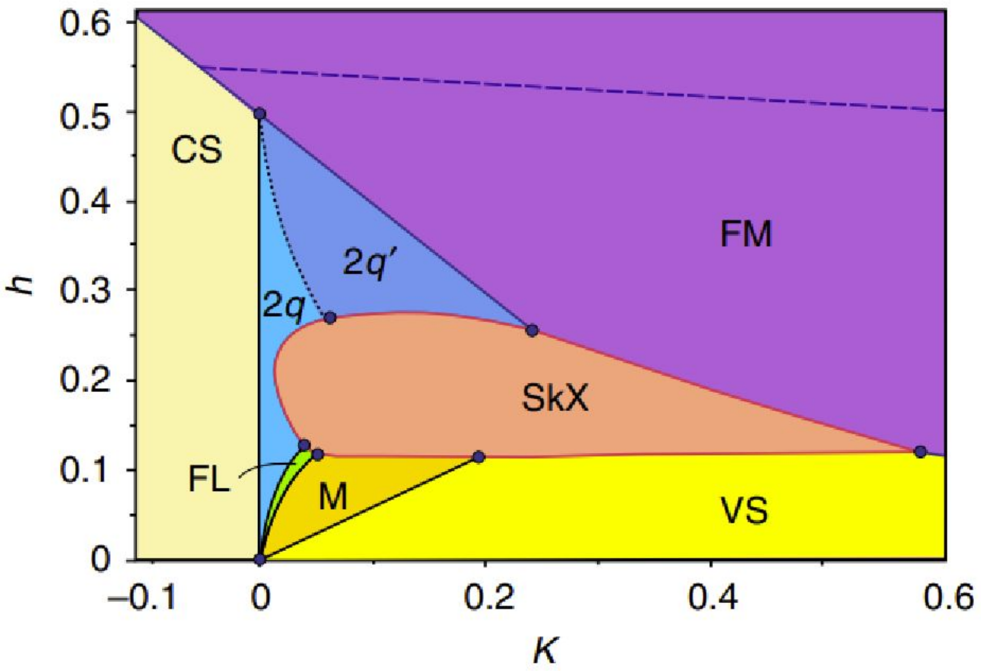
for a single-q spiral ansatz, $\boldsymbol{S}(\boldsymbol{r}) = (\cos \boldsymbol{q}\cdot\boldsymbol{r}, \sin \boldsymbol{q}\cdot\boldsymbol{r}, 0)$

the wavevector is given by $q = \cos^{-1}\frac{J_1}{4J_2}$

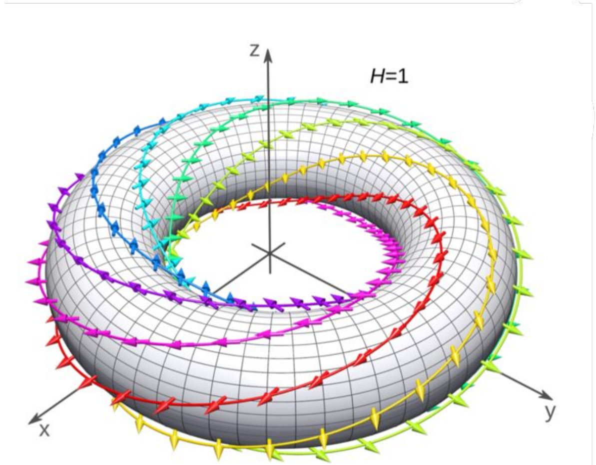


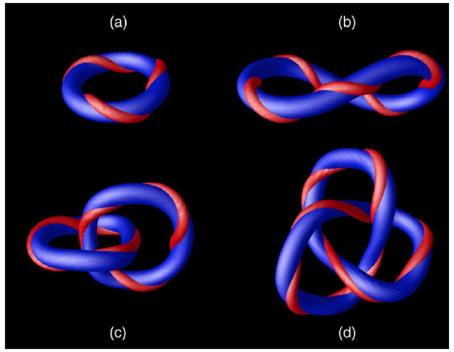


Magnetic interactions

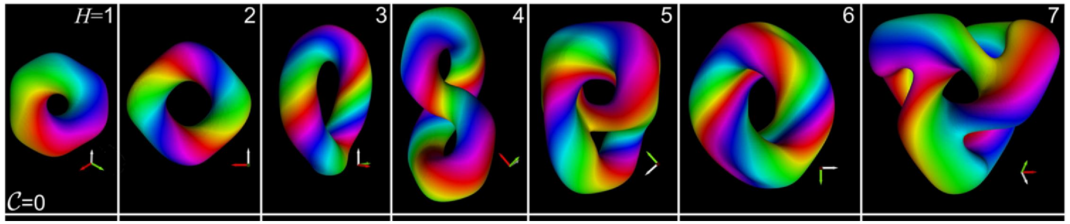


Leonov, A., Mostovoy, M. Multiply periodic states and isolated skyrmions in an anisotropic frustrated magnet. *Nat Commun* **6**, 8275 (2015)





Sutcliffe, PRL 118, 247203 (2017)

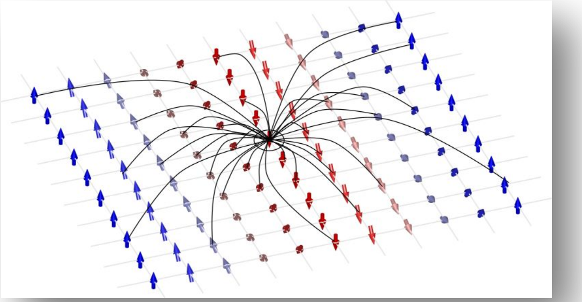


Filipp N. Rybakov, Nikolai S. Kiselev, Aleksandr B. Borisov, Lukas Döring, Christof Melcher, Stefan Blügel; Magnetic hopfions in solids. *APL Mater.* 1 November 2022; 10 (11): 111113

Magnetic interactions

1. Magnetic dipole-dipole interactions

$$E_{dd} = \sum_{ij} \frac{\boldsymbol{S}_i \cdot \boldsymbol{S}_j - 3(\boldsymbol{S}_i \cdot \boldsymbol{r}_{ij})(\boldsymbol{S}_j \cdot \boldsymbol{r}_{ij})/|\boldsymbol{r}_{ij}|^2}{|\boldsymbol{r}_{ij}|^3}$$



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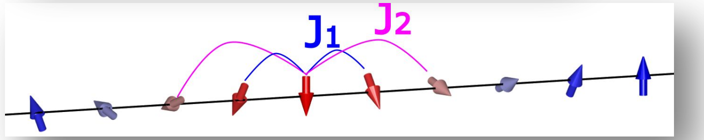

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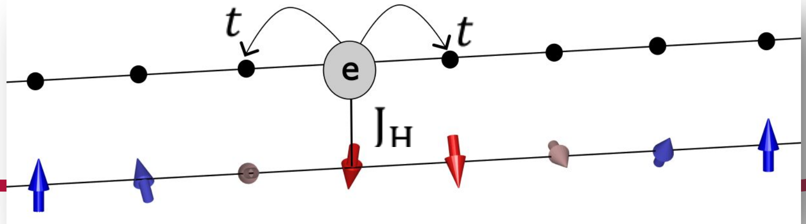

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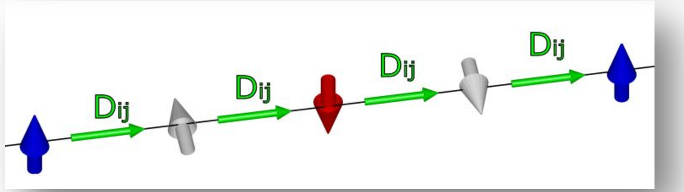
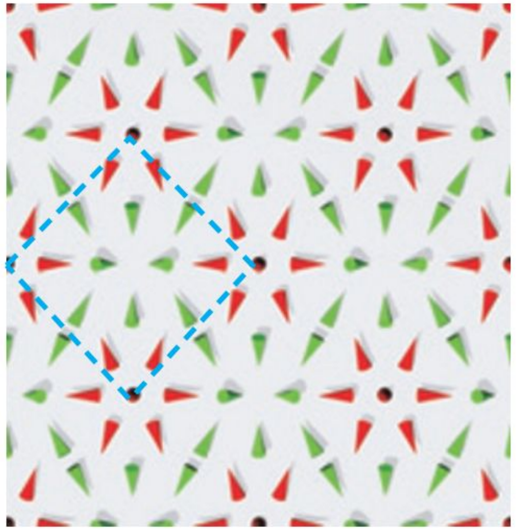


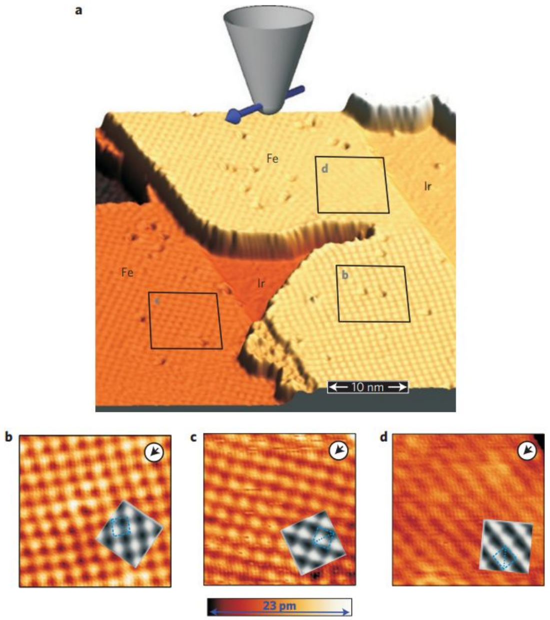
TWIST
Topological Whirls In SpinTronics

Spontaneous atomic-scale magnetic skyrmion lattice in two dimensions

Stefan Heinze, Kirsten von Bergmann, Matthias Menzel, Jens Brede, André Kubetzka, Roland Wiesendanger, Gustav Bihlmayer & Stefan Blügel

Nature Physics 7, 713–718 (2011)





$$H = -\sum_{ij} J_{ij}\boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

$$-\sum_{ijkl} K_{ijkl}[(\boldsymbol{S}_i \cdot \boldsymbol{S}_j)(\boldsymbol{S}_k \cdot \boldsymbol{S}_l) + (\boldsymbol{S}_i \cdot \boldsymbol{S}_l)(\boldsymbol{S}_j \cdot \boldsymbol{S}_k) - (\boldsymbol{S}_i \cdot \boldsymbol{S}_k)(\boldsymbol{S}_j \cdot \boldsymbol{S}_l)]$$

$$-\sum_{ij} \boldsymbol{D}_{ij} \cdot \boldsymbol{S}_i \times \boldsymbol{S}_j$$

Magnetism of conduction electrons

TWIST
Topological Whirls In SpinTronics

$$H = -\sum_{ij} t_{ij}\psi_i^\dagger\psi_j - \mathrm{J_H}\sum_i \psi_i^\dagger \boldsymbol{\sigma}\psi_i \cdot \boldsymbol{S}_i$$

Perturbation theory for gradients of magnetisation

$$H = H^{(0)} + H^{(2)} + H^{(4)} + \cdots$$

$H^{(0)}$ energy of ferromagnetic state

$H^{(2)}$ $\propto q^2$

$H^{(4)}$ $\propto q^4$

"Incommensurate Spiral Order from Double-Exchange Interactions"
Maria Azhar and Maxim Mostovoy
Phys. Rev. Lett. 118, 027203 (2017)

Perturbation theory for $\mathrm{J_H}$

$$H = H^{(0)} + H^{(2)} + H^{(4)} + \cdots$$

$H^{(0)}$ bare electrons

$H^{(2)} = -\mathrm{J_H^2}\sum_{\boldsymbol{q}} \chi_{\boldsymbol{q}}^0 S_{\boldsymbol{q}} \cdot S_{-\boldsymbol{q}}$ RKKY interaction

$H^{(4)} \propto \mathrm{J_H^4}$ multiple-spin interactions
4-spin interactions,
biquadratic exchange

Satoru Hayami and Yukitoshi Motome
"Topological spin crystals by itinerant frustration"
J. Phys.: Condens. Matter **33** 443001 (2021)

Magnetism of conduction electrons

TWIST
Topological Whirls In SpinTronics

- $$H = -\sum_{ij} t_{ij}\psi_i^\dagger\psi_j - \mathrm{J_H}\sum_i \psi_i^\dagger \boldsymbol{\sigma}\psi_i \cdot \boldsymbol{S}_i$$

Transform to co-rotating frame,

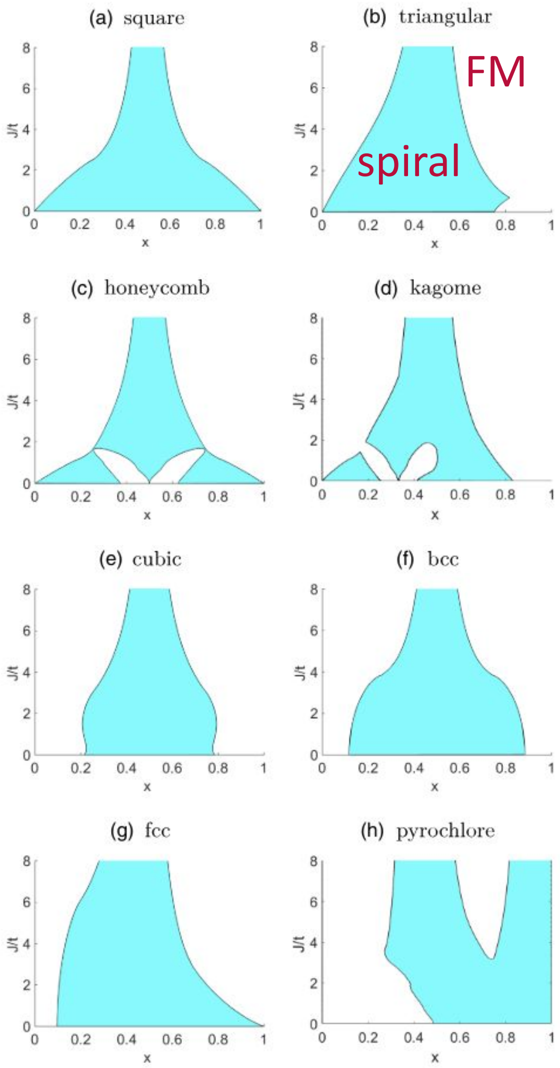
$$\psi_i = e^{-i\theta_i\sigma_x/2}\psi_i'$$

Expand the first term in powers of q,

$$T \approx T^{(0)} + T^{(1)} + T^{(2)}$$

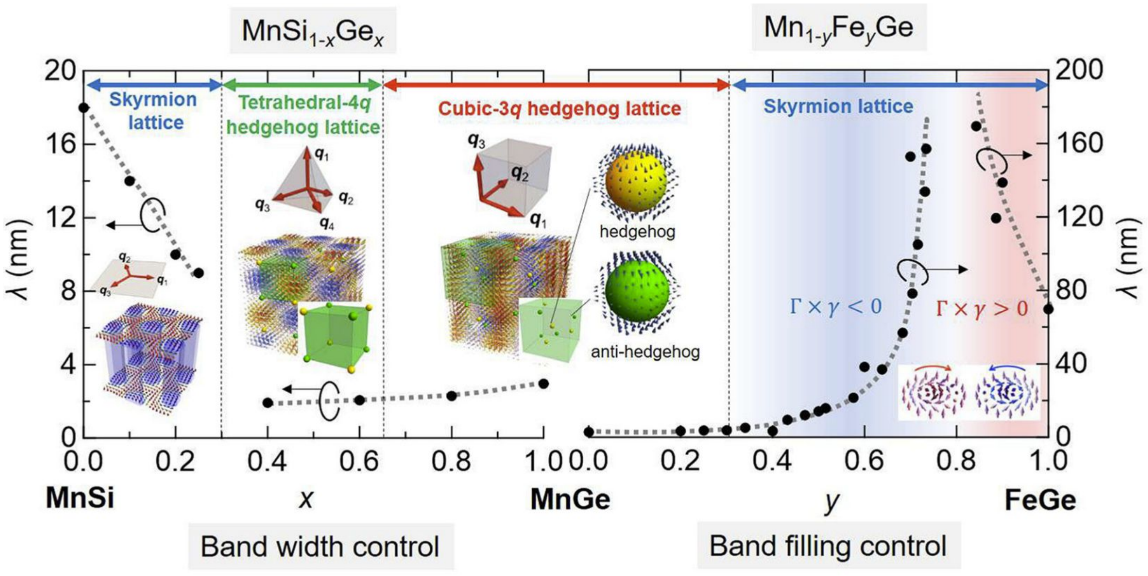
$$E_{spiral} - E_{FM} = -\sum_\nu \frac{\left|\langle\nu|T^{(1)}|0\rangle\right|^2}{E_\nu - E_0} + \langle 0|T^{(2)}|0\rangle$$

Spiral magnetic ordering in all lattices in 2d and 3d.

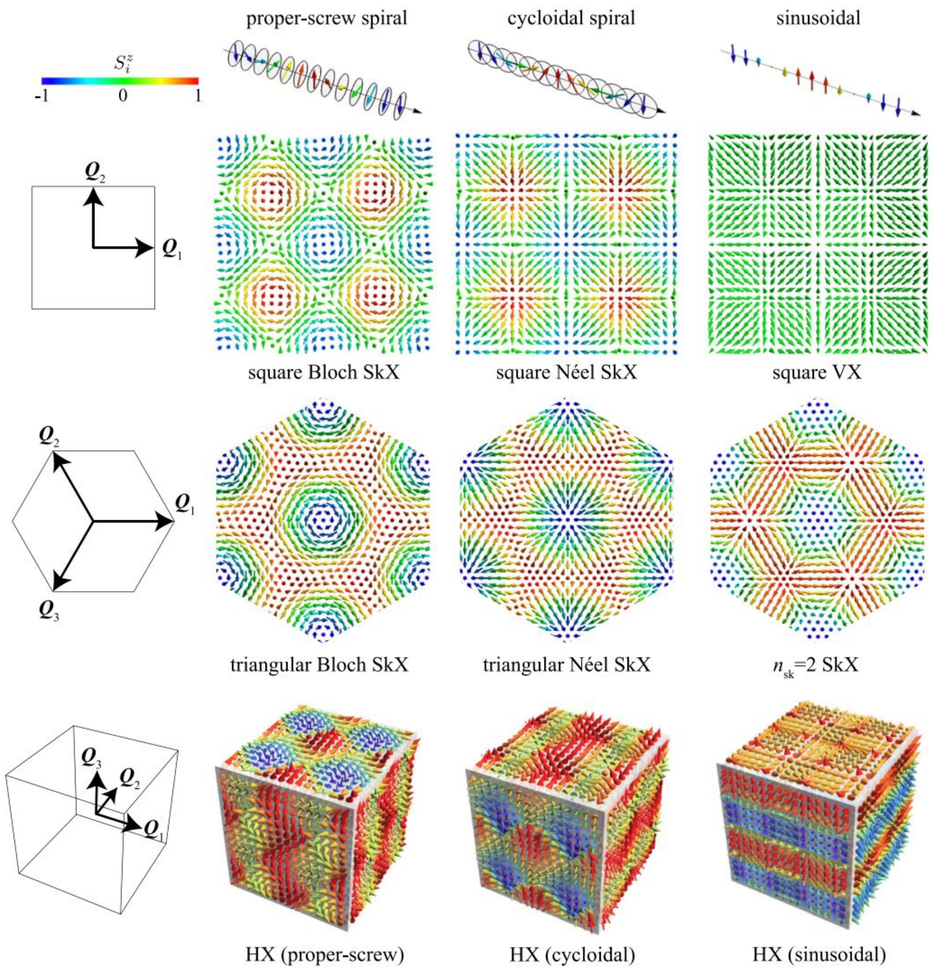


Incommensurate Spiral Order from Double-Exchange Interactions
Maria Azhar and Maxim Mostovoy
Phys. Rev. Lett. **118**, 027203 (2017)

Magnetism of conduction electrons



Y. Fujishiro, N. Kanazawa and Y. Tokura
“Engineering skyrmions and emergent monopoles in topological spin crystals”
Appl. Phys. Lett. **116**, 090501 (2020)



Satoru Hayami and Yukitoshi Motome
“Topological spin crystals by itinerant frustration”
J. Phys.: Condens. Matter **33** 443001 (2021)

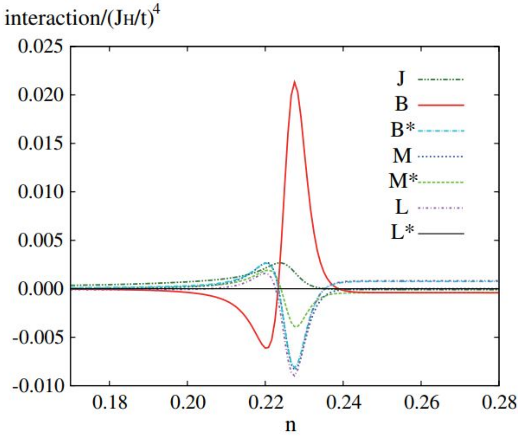
Magnetism of conduction electrons

TWIST
Topological Whirls In SpinTronics

- The Hamiltonian,

$$H = -\sum_{ij} t_{ij}\psi_i^\dagger\psi_j - \mathrm{J_H}\sum_i \psi_i^\dagger \boldsymbol{\sigma}\psi_i \cdot \boldsymbol{S}_i$$

is expanded in powers of J_H to give,



Y. Akagi, M. Udagawa, and Y. Motome
Phys. Rev. Lett. 108, 096401 (2012)

$$H^{(2)} = -\mathrm{J_H^2}\sum_{\boldsymbol{q}} \chi_{\boldsymbol{q}}^0 S_{\boldsymbol{q}} \cdot S_{-\boldsymbol{q}}$$

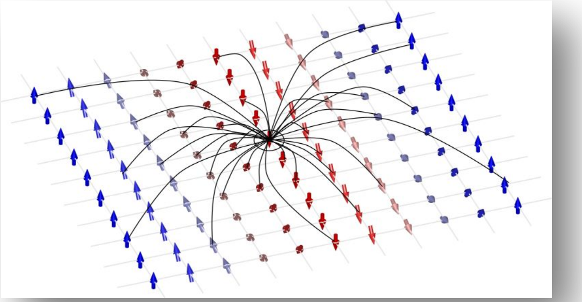
$$H^{(4)} = J_H^4 \sum_{1\le i<j\le 4} [J\boldsymbol{S}_i\cdot\boldsymbol{S}_j + B(\boldsymbol{S}_i\cdot\boldsymbol{S}_j)^2 + B^*(\boldsymbol{S}_i\times\boldsymbol{S}_j)^2] + \sum_{i\ne j\ne k}[M(\boldsymbol{S}_i\cdot\boldsymbol{S}_j)(\boldsymbol{S}_i\cdot\boldsymbol{S}_k) + M^*(\boldsymbol{S}_i\times\boldsymbol{S}_j)\cdot(\boldsymbol{S}_i\times\boldsymbol{S}_k)]$$
$$+ \sum_{i\ne j\ne k\ne l}[L(\boldsymbol{S}_i\cdot\boldsymbol{S}_j)(\boldsymbol{S}_k\cdot\boldsymbol{S}_l) + L^*(\boldsymbol{S}_i\times\boldsymbol{S}_j)\cdot(\boldsymbol{S}_k\times\boldsymbol{S}_l),$$

UDE

Magnetic interactions

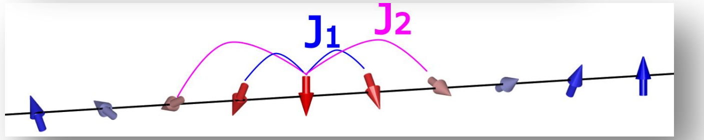
1. Magnetic dipole-dipole interactions

$$E_{dd} = \sum_{ij} \frac{\boldsymbol{S}_i \cdot \boldsymbol{S}_j - 3(\boldsymbol{S}_i \cdot \boldsymbol{r}_{ij})(\boldsymbol{S}_j \cdot \boldsymbol{r}_{ij})/|\boldsymbol{r}_{ij}|^2}{|\boldsymbol{r}_{ij}|^3}$$



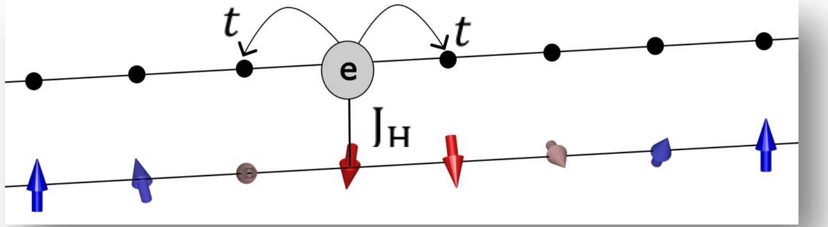
2. Competing exchange (frustrated exchange interactions)

$$E = -\sum_{ij} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



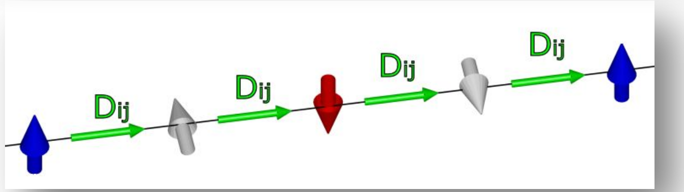
3. Magnetism of conduction electrons (Kondo lattice model or Double Exchange model)

$$H = -t \sum_{ij} \psi_i^\dagger \psi_j - \mathrm{J_H} \sum_i \psi_i^\dagger \boldsymbol{\sigma} \psi_i \cdot \boldsymbol{S}_i$$



4. Dzyaloshinskii-Moriya interaction in chiral magnets

$$E = \sum_{ij} \boldsymbol{D}_{ij} \cdot \boldsymbol{S}_i \times \boldsymbol{S}_j$$



Dzyaloshinskii-Moriya interaction

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Topological Whirls In SpinTronics

- most general form

$$\mathcal{F}_D = D_{ijk} m_i \partial_j m_k$$

antisymmetric part $D_{ijk} - D_{kji}$

contributes in bulk

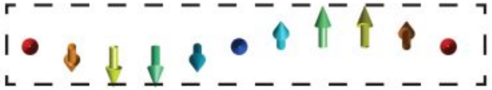
Symmetric part contributes at the boundaries

New Boundary-Driven Twist States in Systems with Broken Spatial Inversion Symmetry
Kjetil M. D. Hals and Karin Everschor-Sitte
Phys. Rev. Lett. 119, 127203 (2017)

Dzyaloshinskii-Moriya interaction

Topological Whirls In SpinTronics

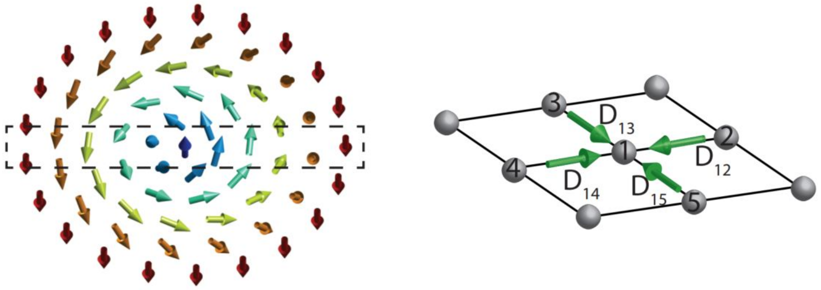
- On a lattice: $E_{DMI} = \boldsymbol{D}_{12} \cdot (\boldsymbol{S}_1 \times \boldsymbol{S}_2)$



Bloch-type DMI

Point groups: T, O, C_n, D_n

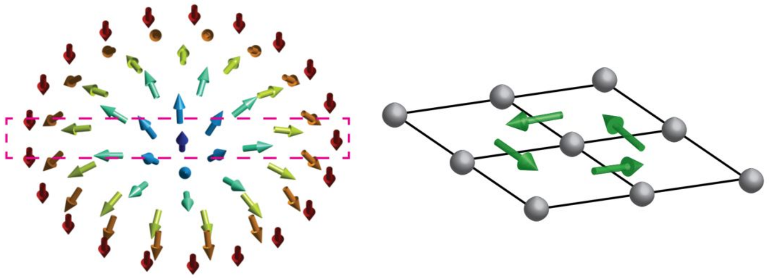
Lifshitz invariants: $\mathcal{L}_{yz}^{x} + \mathcal{L}_{zx}^{y} + (\mathcal{L}_{xy}^{z})$

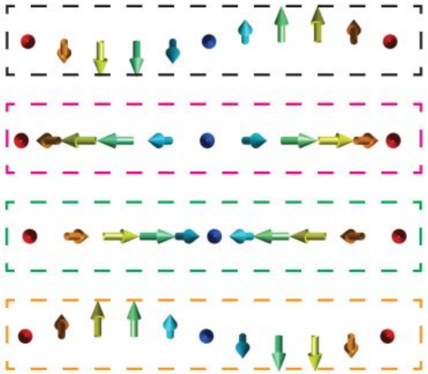


Neel-type DMI

Point groups: C_n, C_{nv}

Lifshitz invariants: $\mathcal{L}_{xz}^{x} + \mathcal{L}_{yz}^{y}$

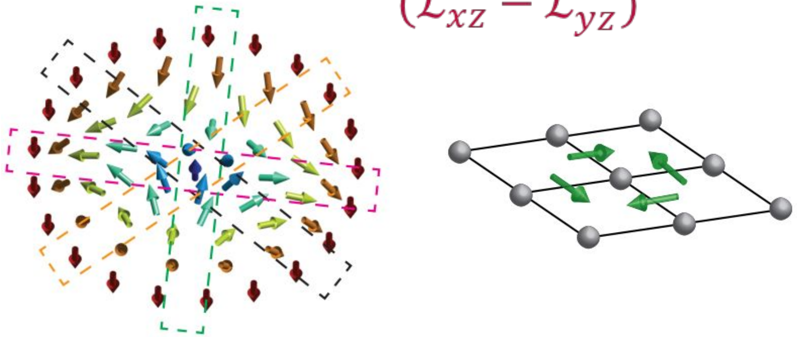




Antiskyrmion-type DMI

Point groups: D_{2d}, S_4

Lifshitz invariants: $\mathcal{L}_{yz}^{x} - \mathcal{L}_{zx}^{y} + (\mathcal{L}_{xz}^{x} - \mathcal{L}_{yz}^{y})$



Dzyaloshinskii-Moriya interaction

Lifshitz invariants $\mathcal{L}_{\alpha\beta}^{\gamma}$ (DMI energy terms) allowed by the symmetries of various point groups

$$\mathcal{L}_{\alpha\beta}^{\gamma} = m_\alpha \frac{\partial m_\beta}{\partial x_\gamma} - m_\beta \frac{\partial m_\alpha}{\partial x_\gamma}$$

Point group	Planar	Axial

Bloch-type DMI

Neel-type DMI

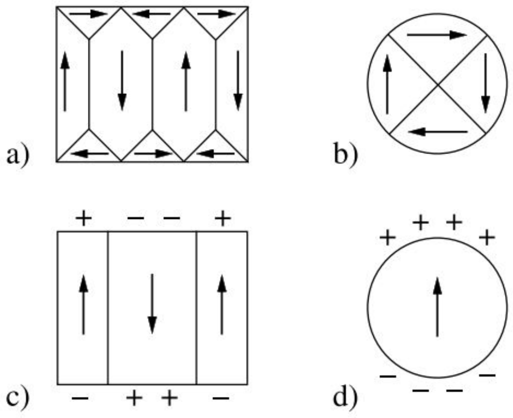
antiskyrmions

Dzyaloshinskii-Moriya interaction

Other spin-orbit coupling (soc) effects

Magnetocrystalline anisotropy

$$\varepsilon_{anis.} = K_{ij}^{(2)} m_i m_j + K_{ijkl}^{(4)} m_i m_j m_k m_l + K_{ijklrs}^{(6)} m_i m_j m_k m_l m_r m_s$$



flux closure patterns

open domain structures

Hubert, Schäfer
"Magnetic Domains" (1998)

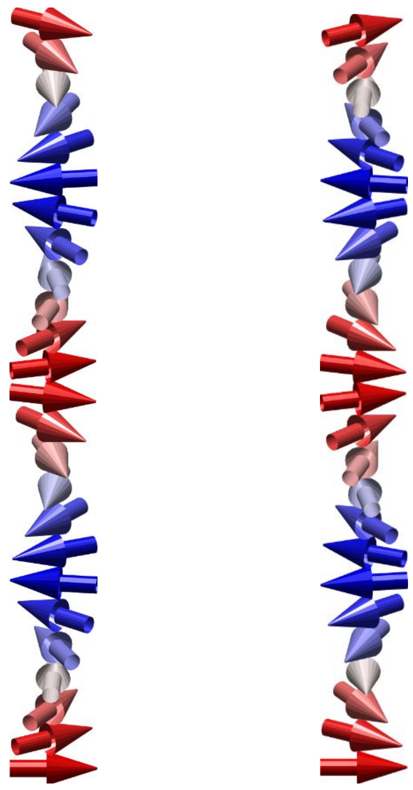
Dzyaloshinskii-Moriya interaction

- DM interaction in cubic chiral magnets, $\mathcal{L}^{x}_{yz} + \mathcal{L}^{y}_{zx} + \mathcal{L}^{z}_{xy}$

$$\mathcal{E} = A(\boldsymbol{\nabla n})^2 + D\boldsymbol{n} \cdot \boldsymbol{\nabla} \times \boldsymbol{n}$$

Exchange energy

DM energy



Favored for D>0 Favored for D<0

helical spiral wavelength

$$\lambda_h = \frac{2\pi}{q} = 4\pi \frac{A}{D}$$

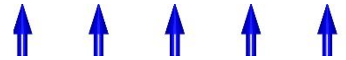
Dzyaloshinskii-Moriya interaction

TWIST
Topological Whirls In SpinTronics

- DM interaction in cubic chiral magnets, $\mathcal{L}_{yz}^{x} + \mathcal{L}_{zx}^{y} + \mathcal{L}_{xy}^{z}$

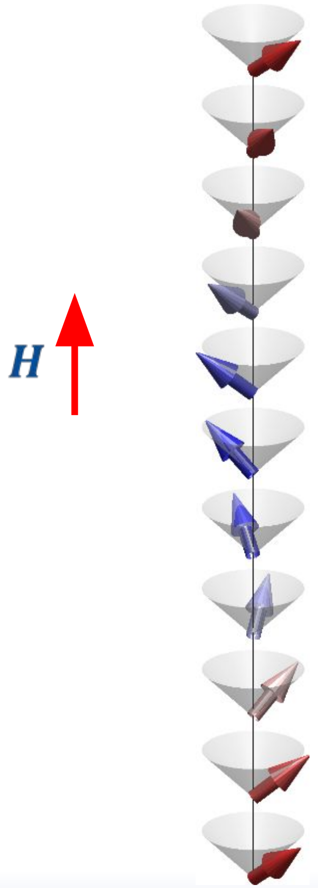
$$\mathcal{E} = A(\boldsymbol{\nabla n})^2 + D\boldsymbol{n} \cdot \boldsymbol{\nabla} \times \boldsymbol{n} - \mu_0 \boldsymbol{H} \cdot \boldsymbol{m}$$

Exchange energy



DM energy

Zeeman energy



helical spiral wavelength

$$\lambda_h = \frac{2\pi}{q} = 4\pi \frac{A}{D}$$

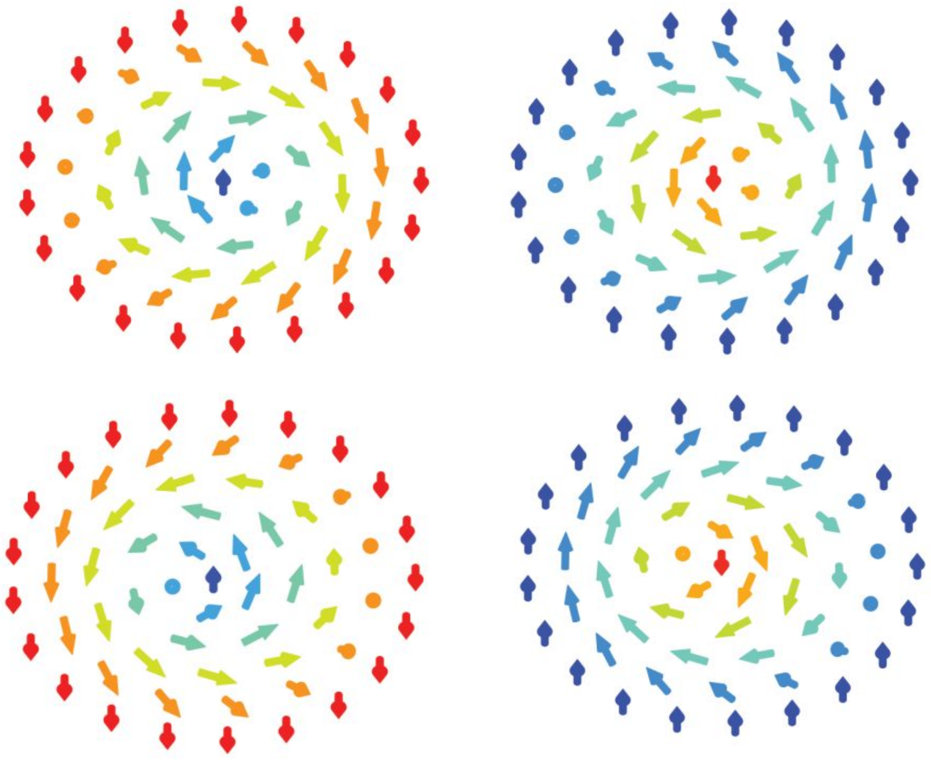
Dzyaloshinskii-Moriya interaction

- DM interaction in cubic chiral magnets, $\mathcal{L}_{yz}^{x} + \mathcal{L}_{zx}^{y} + \mathcal{L}_{xy}^{z}$

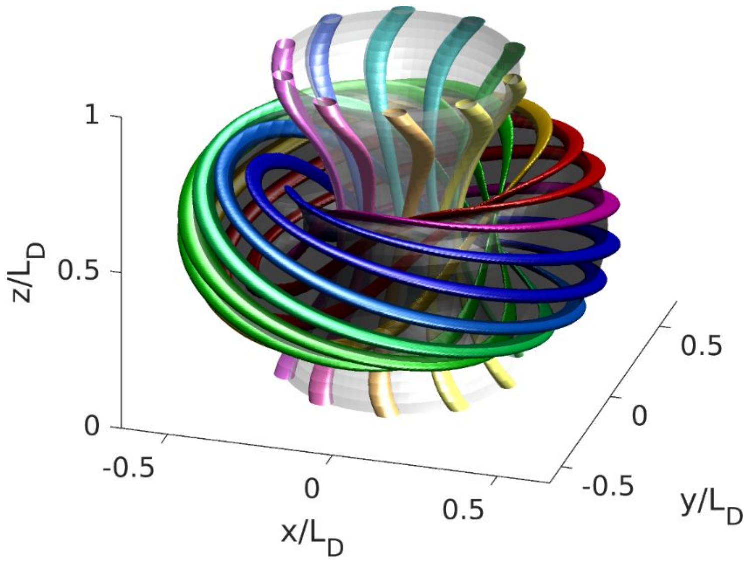
$$\mathcal{E} = A(\boldsymbol{\nabla n})^2 + D\boldsymbol{n} \cdot \boldsymbol{\nabla} \times \boldsymbol{n}$$

Favored for D>0

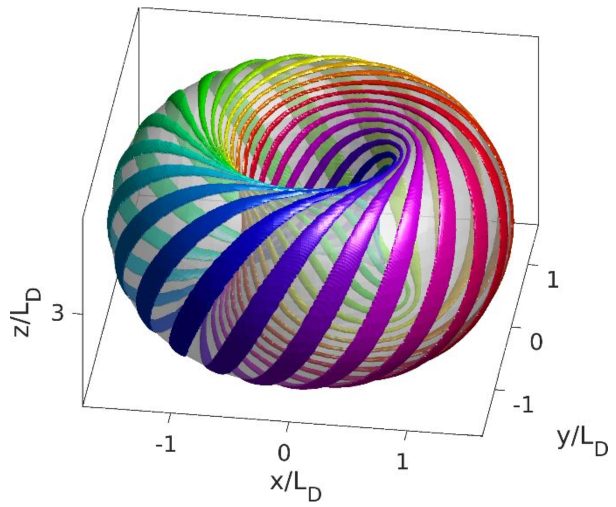
Favored for D<0



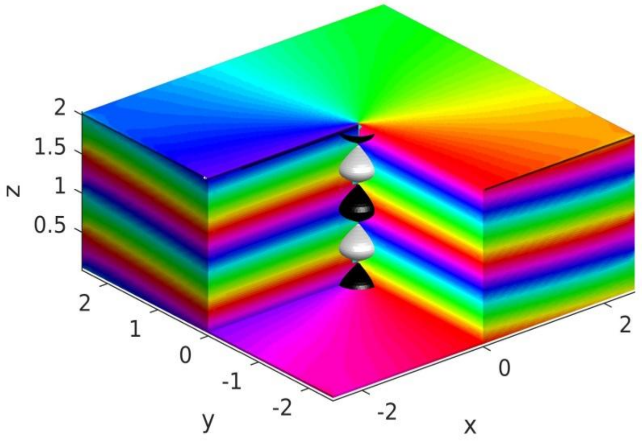
Topological magnetic textures



Screw dislocation,
Azhar et al, PRL 128, 157204 (2022)

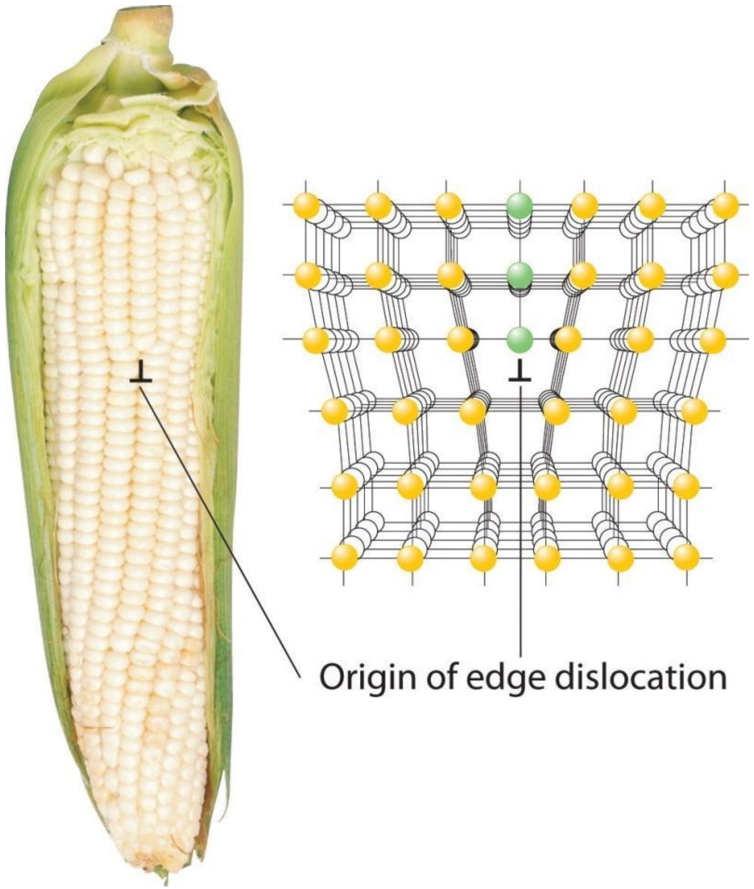


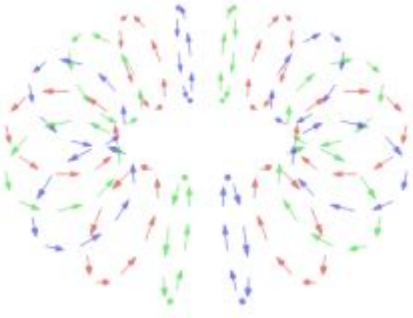
Hopfion

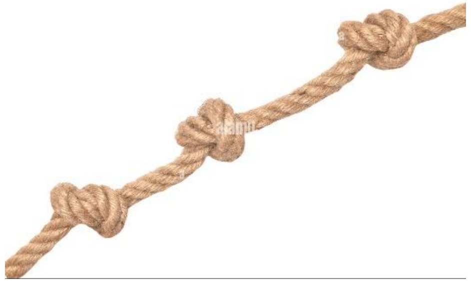


Screw dislocation

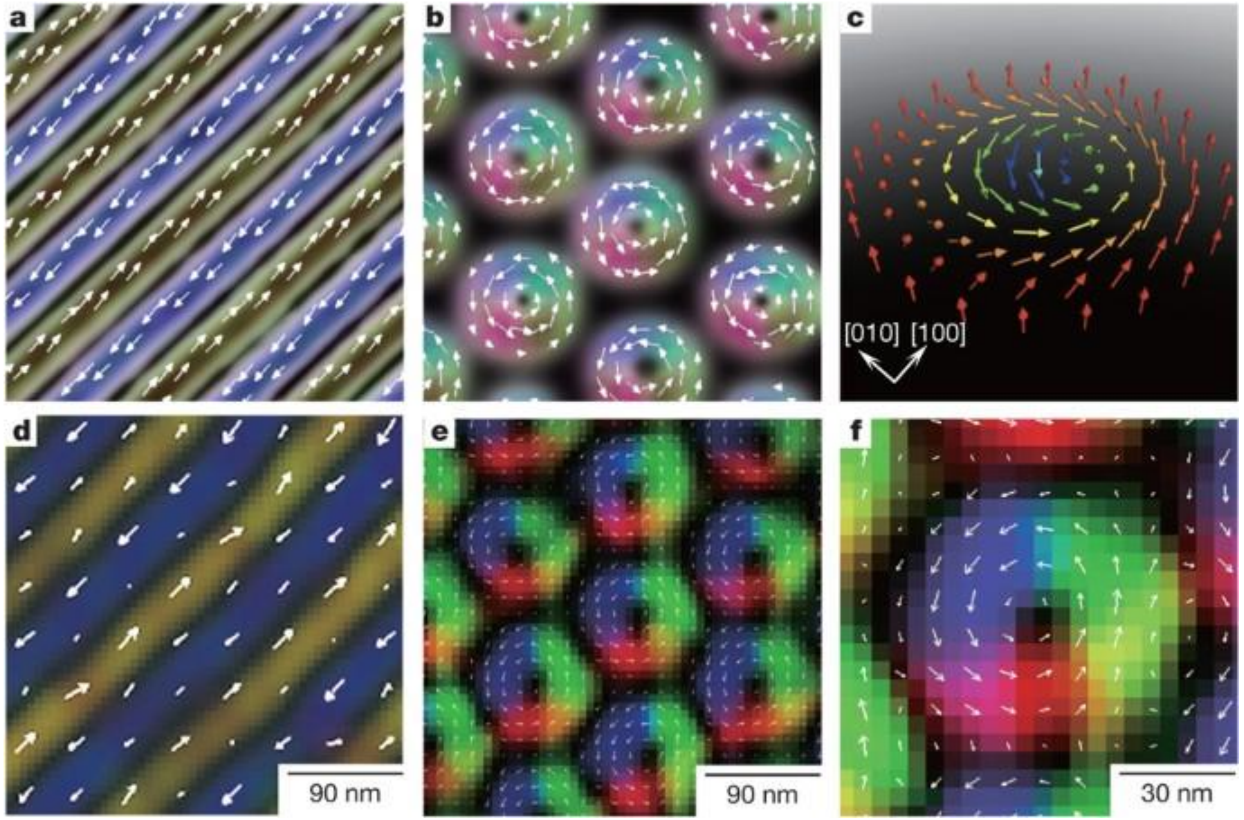
Topology





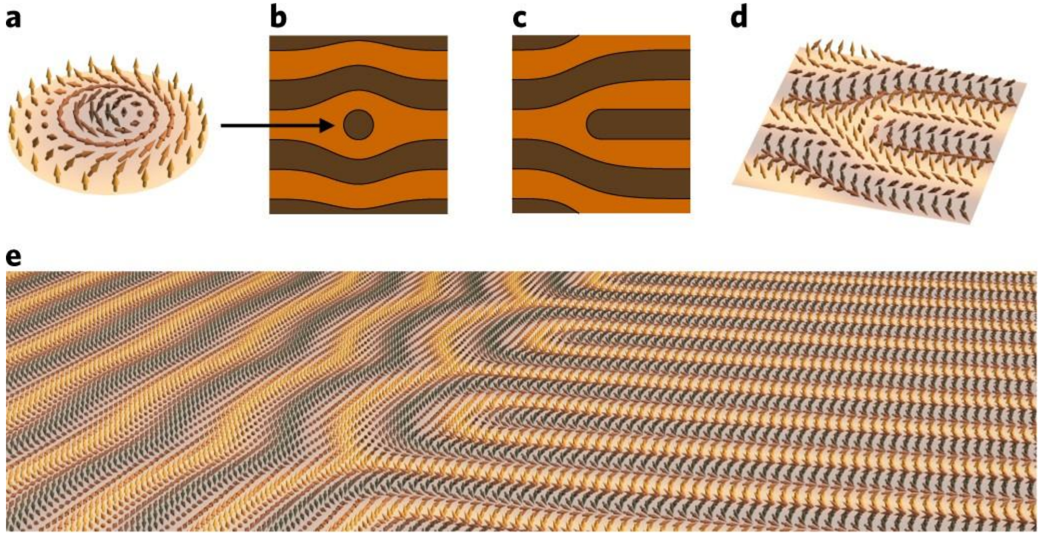


Topology in magnetism

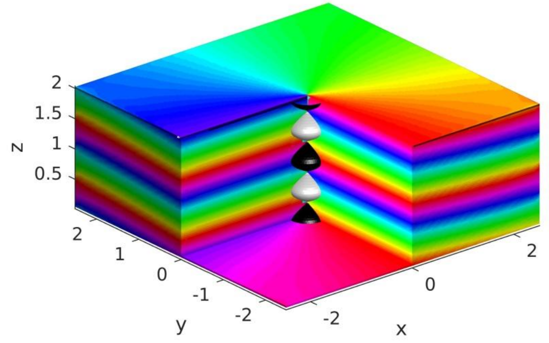


Real-space observation of a two-dimensional skyrmion crystal Nature 465, 901–904 (2010)

X. Z. Yu[1,2], Y. Onose[2,3], N. Kanazawa[3], J. H. Park[4], J. H. Han[4], Y. Matsui[1], N. Nagaosa[3,5] & Y. Tokura[2,3,5]



Schoenherr, P., Müller, J., Köhler, L. *et al.* Topological domain walls in helimagnets.
Nature Phys **14**, 465–468 (2018)



Maria Azhar, Volodymyr P. Kravchuk, and Markus Garst. Screw Dislocations in Chiral Magnets.
Phys. Rev. Lett. 128, 157204 (2022)

Topology in soft matter

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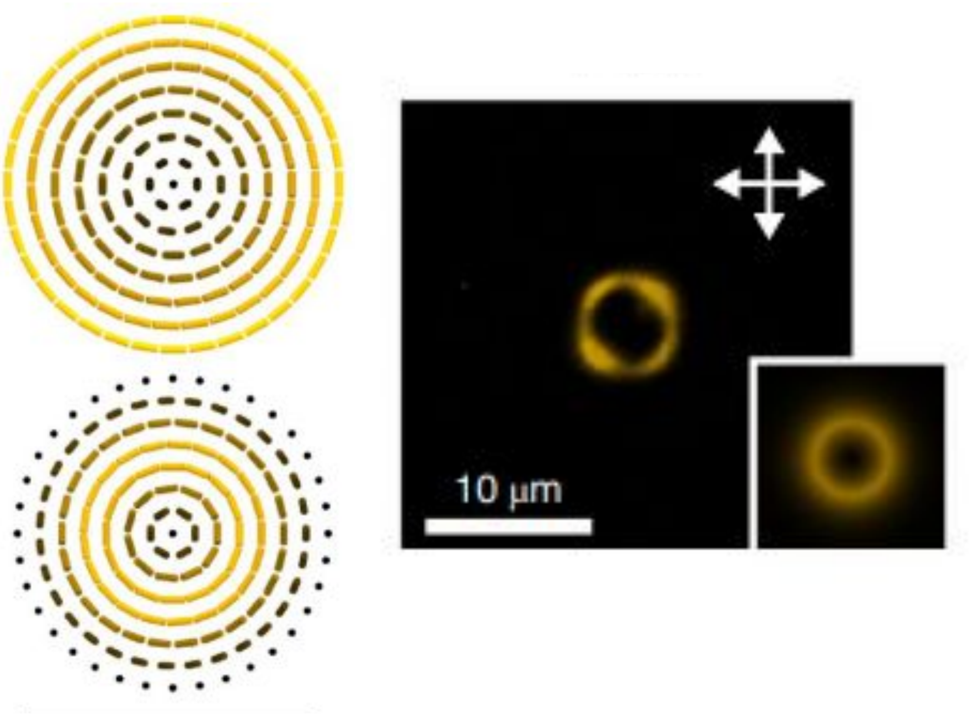
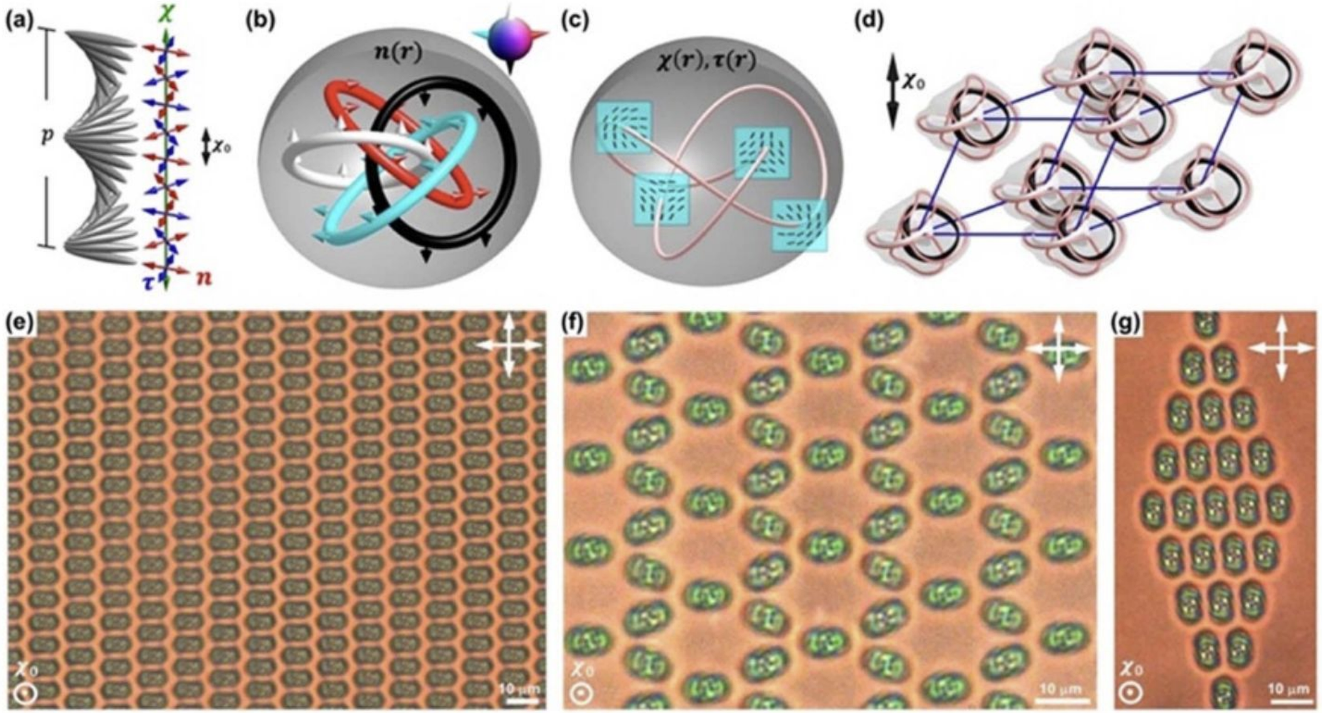
Topological Whirls In SpinTronics



Two-dimensional skyrmion bags in liquid crystals and ferromagnets
D. Foster et al, Nature Physics, **15**, 655–659 (2019)



Self-assembled crystals of heliknotons.
J-S B. Tai et al, Science, **365**, 1449-1453 (2019).

Topology in ferroelectrics

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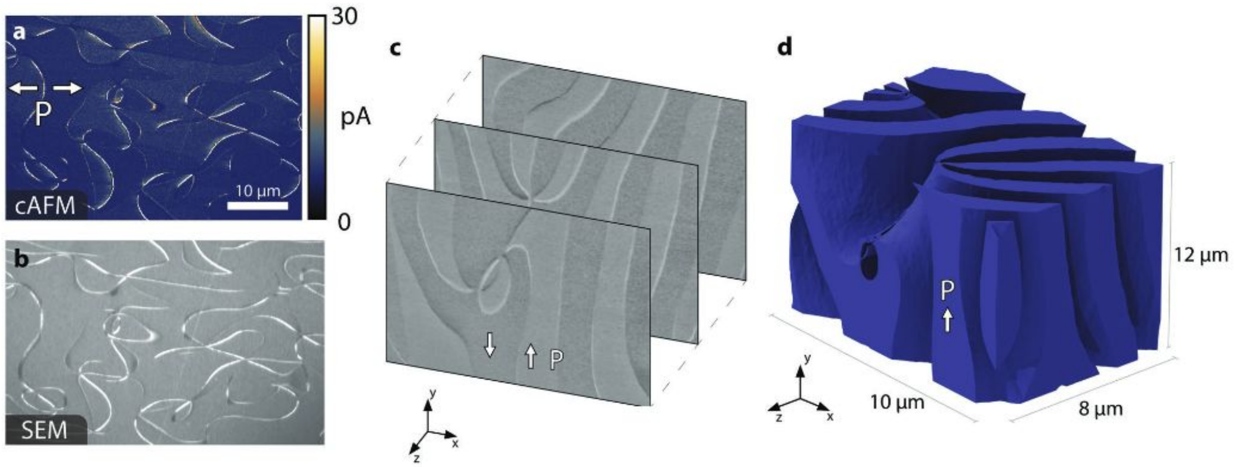
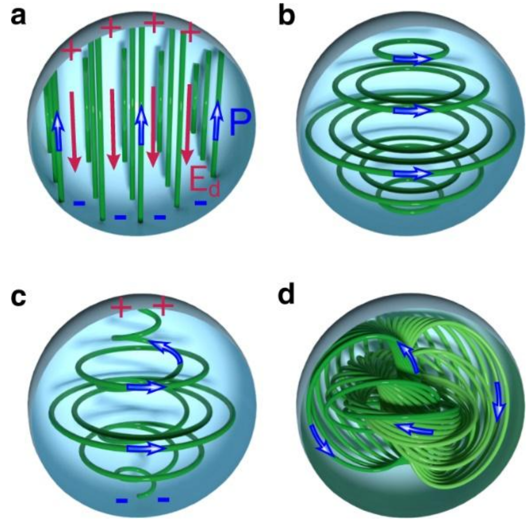
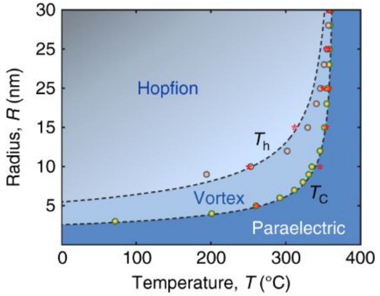
Topological Whirls In SpinTronics

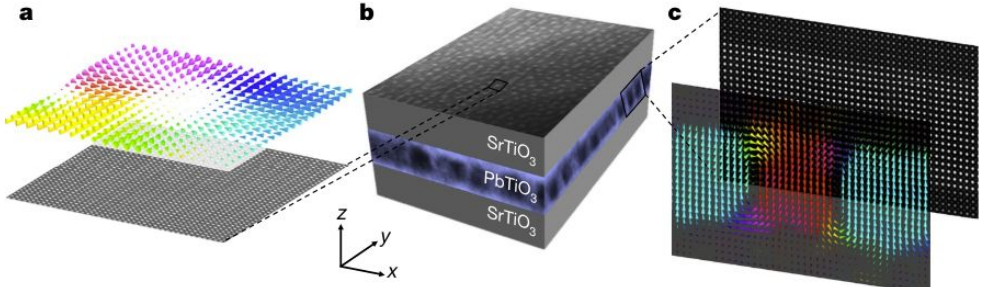


The Third Dimension of Ferroelectric Domain Walls
Erik D. Roede *et al*,
Advanced Materials (2022)





Hopfions emerge in ferroelectrics
Luk'yanchuk, I., Tikhonov, Y., Razumnaya, A. *et*
Nat Commun **11**, 2433 (2020)



Das, S., Tang, Y.L., Hong, Z. *et al.* Observation of room-temperature polar skyrmions. *Nature* **568**, 368–372 (2019)

Topology and homotopy groups of spheres

Homotopy groups of spheres

- Homotopy groups of spheres

Dimension of target space →

Dimension of order parameter ↓

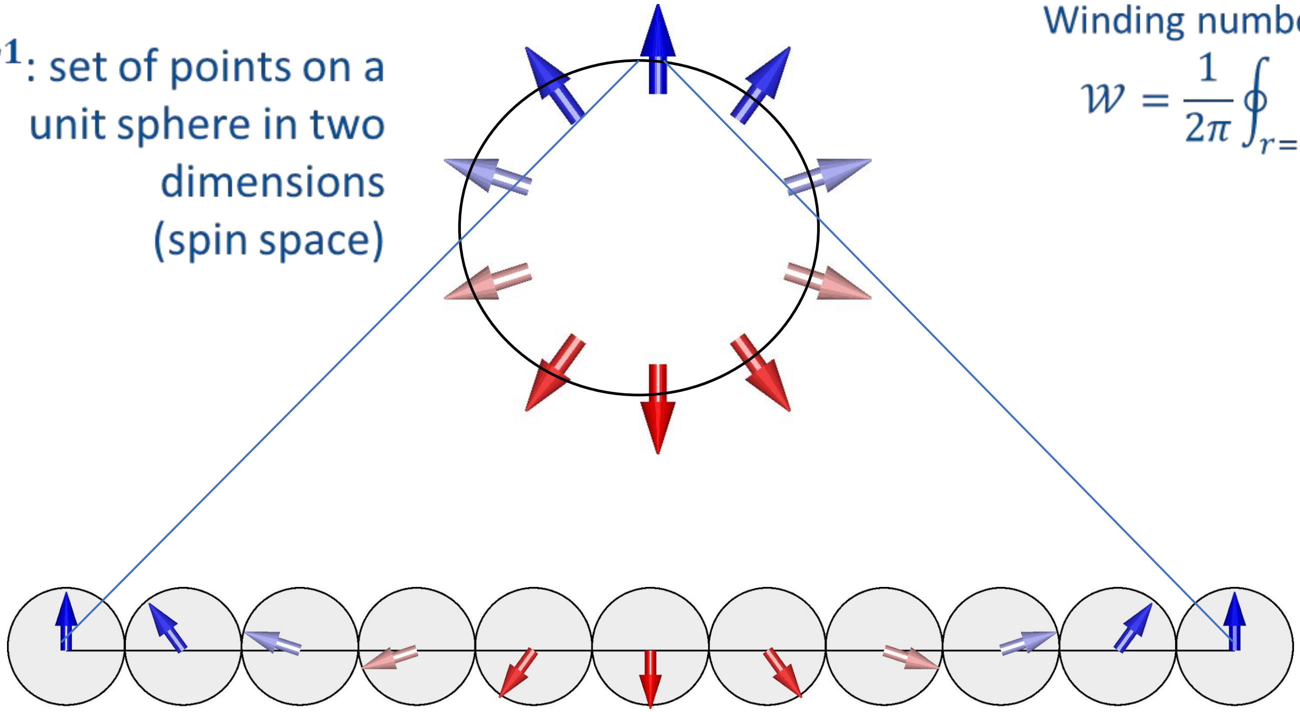
	π_1	π_2	π_3	π_4	π_5	π_6	π_7	π_8	π_9	π_{10}	π_{11}	π_{12}	π_{13}	π_{14}	π_{15}
S^1	Z	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S^2	0	Z	Z	Z_2	Z_2	Z_{12}	Z_2	Z_2	Z_3	Z_{15}	Z_2	Z_2^2	$Z_{12}{\times}Z_2$	$Z_{84}{\times}Z_2^2$	Z_2^2
S^3	0	0	Z	Z_2	Z_2	Z_{12}	Z_2	Z_2	Z_3	Z_{15}	Z_2	Z_2^2	$Z_{12}{\times}Z_2$	$Z_{84}{\times}Z_2^2$	Z_2^2
S^4	0	0	0	Z	Z_2	Z_2	$Z{\times}Z_{12}$	Z_2^2	Z_2^2	$Z_{24}{\times}Z_3$	Z_{15}	Z_2	Z_2^3	$Z_{120}{\times}Z_{12}{\times}Z_2$	$Z_{84}{\times}Z_2^5$
S^5	0	0	0	0	Z	Z_2	Z_2	Z_{24}	Z_2	Z_2	Z_2	Z_{30}	Z_2	Z_2^3	$Z_{72}{\times}Z_2$
S^6	0	0	0	0	0	Z	Z_2	Z_2	Z_{24}	0	Z	Z_2	Z_{60}	$Z_{24}{\times}Z_2$	Z_2^3
S^7	0	0	0	0	0	0	Z	Z_2	Z_2	Z_{24}	0	0	Z_2	Z_{120}	Z_2^3
S^8	0	0	0	0	0	0	0	Z	Z_2	Z_2	Z_{24}	0	0	Z_2	$Z{\times}Z_{120}$

Homotopy groups of spheres

$$\pi_1(S^1) = \mathbb{Z}$$

Coplanar domain wall

S^1: set of points on a unit sphere in two dimensions (spin space)

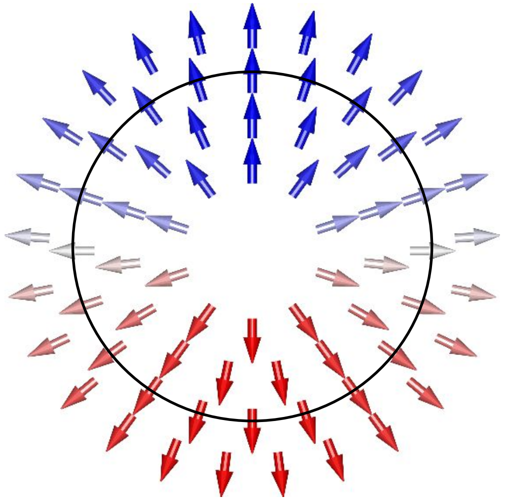


one-dimensional base space (physical space)

Winding number

$$\mathcal{W} = \frac{1}{2\pi} \oint_{r=\infty} dr \, \frac{d\,\phi}{dr}$$

Coplanar vortex



one-dimensional base space along circular contours

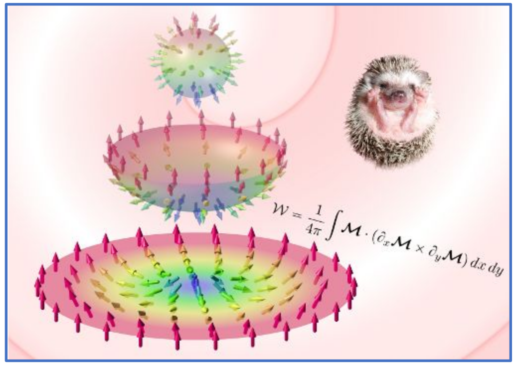
Homotopy groups of spheres

homotopy group

$\pi_2(S^2) = \mathbb{Z}$

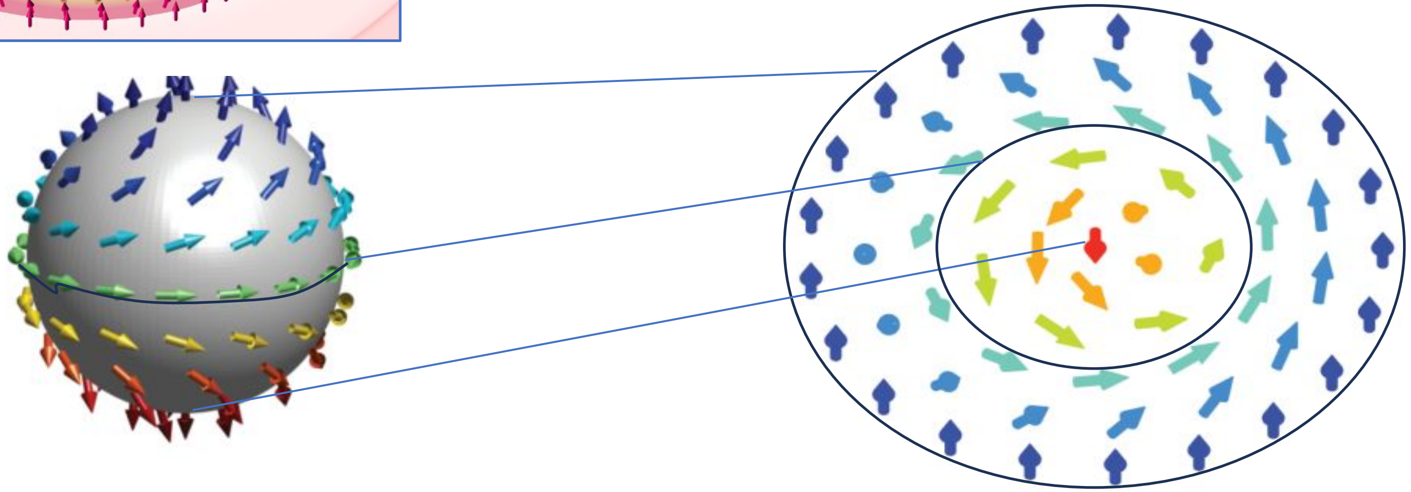
$$\mathcal{W} = \frac{1}{4\pi}\int d^2x\, \hat{\boldsymbol{m}} \cdot (\partial_x \hat{\boldsymbol{m}} \times \partial_y \hat{\boldsymbol{m}}) = \frac{1}{4\pi}\int d\Omega = integer$$

From a hedgehog to a skyrmion by Karin Everschor-Sitte and Matthias Sitte



mapping of points from a unit sphere in 3d to a 2d plane

S^2: set of points on a unit sphere in three dimensions (target space)



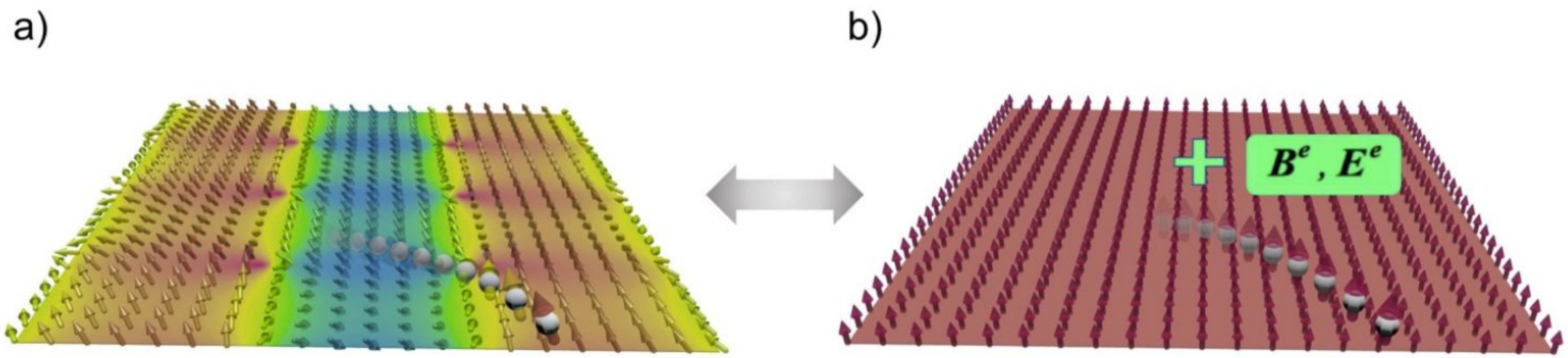
two-dimensional base space (physical space)

Example: Measuring topology via topological Hall effect

TWIST
Topological Whirls In SpinTronics

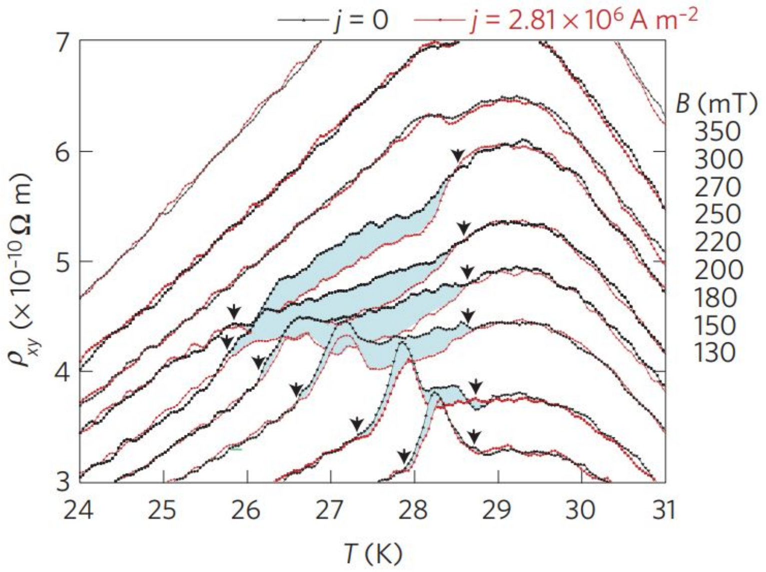
Emergent Electrodynamics

The 2020 Skyrmionics roadmap
J. Phys. D: Appl. Phys. 53 (2020)



- $$B_i^e = \frac{\hbar}{2e}\epsilon_{ijk}\hat{\boldsymbol{m}} \cdot \left(\partial_j \hat{\boldsymbol{m}} \times \partial_k \hat{\boldsymbol{m}}\right)$$
$$E_i^e = \frac{\hbar}{e}\hat{\boldsymbol{m}} \cdot \left(\partial_i \hat{\boldsymbol{m}} \times \partial_t \hat{\boldsymbol{m}}\right)$$

Emergent magnetic "flux" per skyrmion is quantized $\int eB^e \cdot d\sigma = 4\pi\hbar\mathcal{W}$



T.Schulz et al,
Nature Physics **8**, 301–304 (2012)

UDE

Example: Measuring topology via topological Hall effect

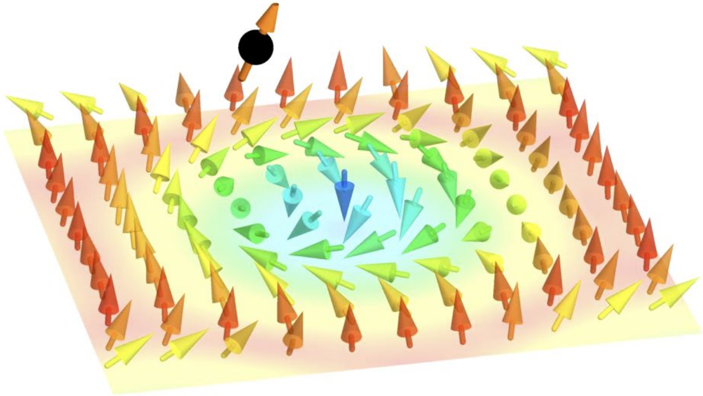
Magnetism

Neubauer, et al., PRL, 2009

Topological contribution to the Hall effect

If electron moves across a topological magnetic texture

$$i\hbar\partial_t\psi = \left[\frac{p^2}{2\mathrm{m}} + J\,\sigma\cdot\hat{M}(r,t)\right]\psi$$



Electromagnetic / Optical

nature COMMUNICATIONS

ARTICLE

https://doi.org/10.1038/s41467-021-21250-z OPEN

Emulating spin transport with nonlinear optics, from high-order skyrmions to the topological Hall effect

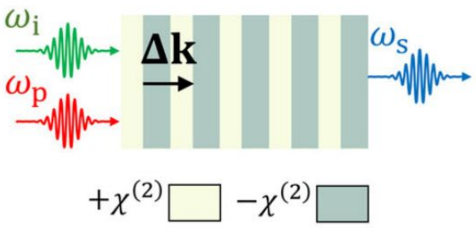
Aviv Karnieli[1], Shai Tsesses[2], Guy Bartal[2] & Ady Arie[1]

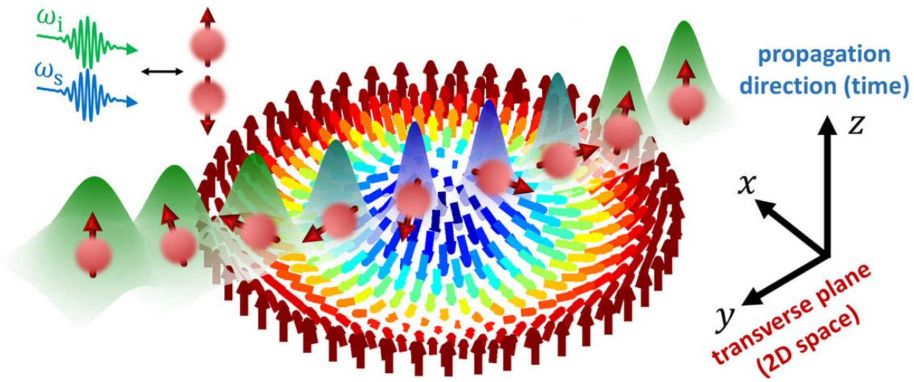
"optical system exhibiting the topological Hall effect"

$$i\frac{\partial}{\partial Z}\begin{pmatrix}E_\mathrm{i}\\ E_\mathrm{s}\end{pmatrix} = \left[\frac{\mathbf{p}_\mathrm{T}^2}{2\bar{k}} - \begin{pmatrix}0 & \kappa^* e^{-i\Phi}\\ \kappa e^{i\Phi} & 0\end{pmatrix}\right]\begin{pmatrix}E_\mathrm{i}\\ E_\mathrm{s}\end{pmatrix}$$

$$i\frac{\partial}{\partial Z}\Psi = \left[\frac{(\mathbf{p}_\mathrm{T} - \mathcal{A})^2}{2\bar{k}} - \boldsymbol{\sigma}\cdot\mathbf{M}\right]\Psi$$

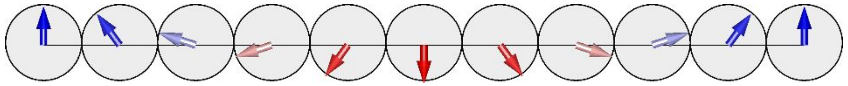
nonlinear photonic crystal



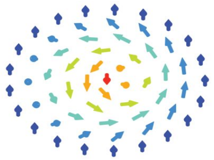


"In this article, we propose an optical system **emulating any 2D spin transport phenomena** with unprecedented controllability, by employing three-wave mixing in 3D **nonlinear photonic crystals**."

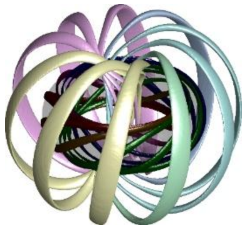
Homotopy groups of spheres



coplanar DW or two-dimensional vortex



skyrmion



Hopfion,
Hopf fibration $S^3 \to S^2$

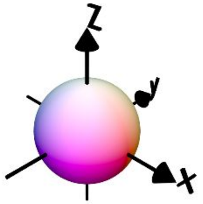
3D skyrmion (Shankar skyrmion)

Dimension of order parameter (target space)

	Z	0	0
	0	Z	Z
	0	0	Z
	0	0	0

Dimension of base space

Magnetic hopfion



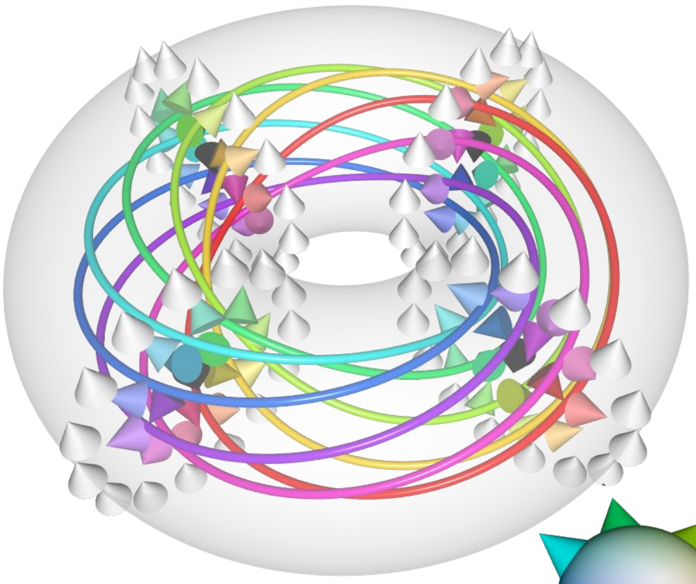
Hopf index: linking number of preimages of $\boldsymbol{m}(\boldsymbol{r}),\ \mathbb{R}^3 \to S^2$

$$\pi_3(S^2) = \mathbb{Z}$$

$$H = -\frac{1}{(8\pi)^2}\int_{\mathbb{R}^3} \mathrm{d}^3 r\, \boldsymbol{F}\cdot\boldsymbol{A}$$

$$F_i = \epsilon_{ijk}\boldsymbol{m}\cdot(\partial_j \boldsymbol{m}\times\partial_k \boldsymbol{m}),$$

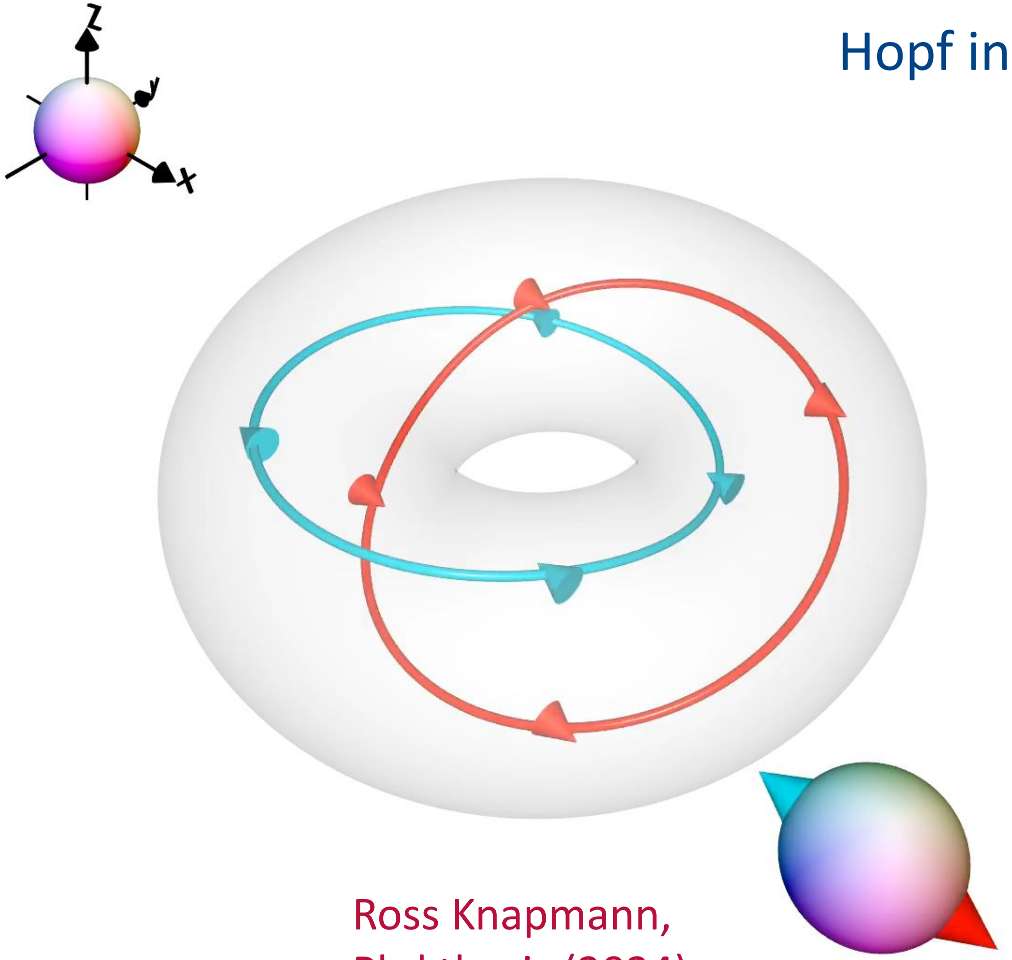
$$\boldsymbol{F} = \boldsymbol{\nabla}\times\boldsymbol{A}$$



Ross Knapmann,
Phd thesis (2024)

The preimages of $\boldsymbol{m}$ link, an integer number of times.

Magnetic hopfion



Ross Knapmann,
Phd thesis (2024)

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$$\boldsymbol{F} = \boldsymbol{\nabla} \times \boldsymbol{A}$$

The preimages of $\boldsymbol{m}$ link, an integer number of times.

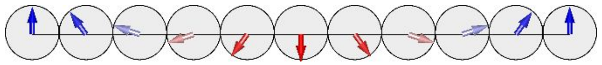
Homotopy groups of spheres

TWIST
Topological Whirls In SpinTronics

$\pi_1(S^1) = \mathbb{Z}$

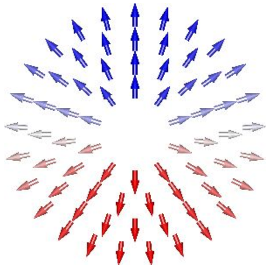
$$\mathcal{W} = \frac{1}{2\pi}\oint_{r=\infty} dr\, \frac{d\,\phi}{dr}$$

Coplanar domain wall



- Mapping of points at infinity (1d) to one point in order parameter space

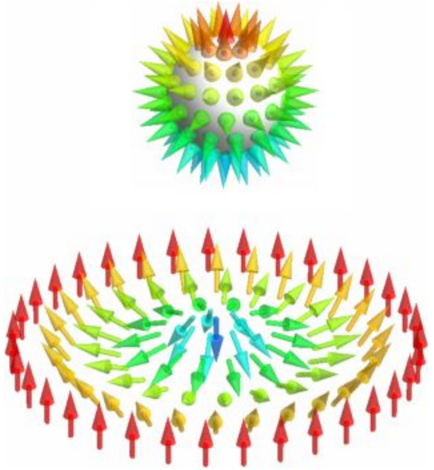
Vortex



- Winding far away from vortex core
- Mapping of points at infinity (1d) to one point in order parameter space
- Singular vortex core

$\pi_2(S^2) = \mathbb{Z}$

$$\mathcal{W} = \frac{1}{4\pi}\int d^2x\, \hat{\boldsymbol{m}} \cdot (\partial_x \hat{\boldsymbol{m}} \times \partial_y \hat{\boldsymbol{m}})$$

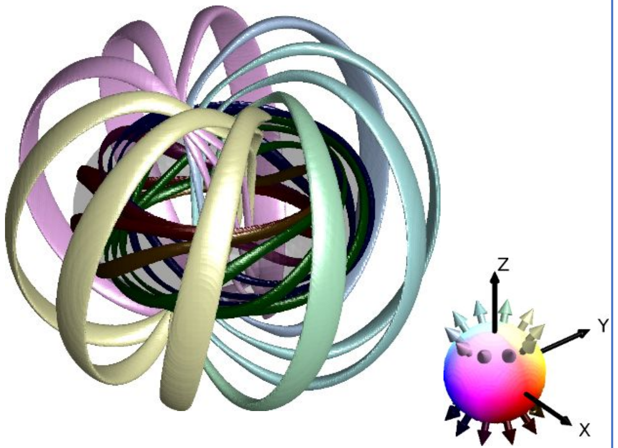


Skyrmion

- Trivial at spatial infinity
- Topologically quantized only as long as order parameter finite

$\pi_3(S^2) = \mathbb{Z}$

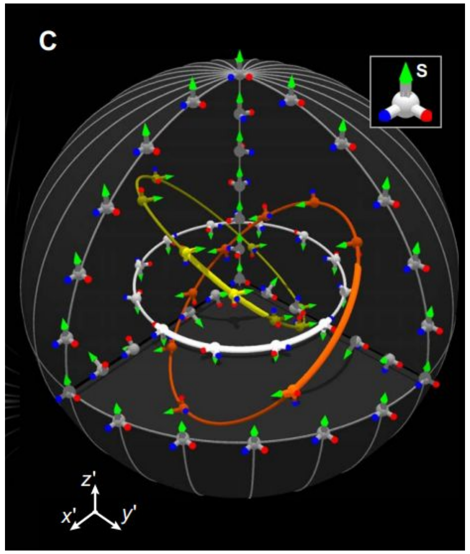
$$H = -\frac{1}{(8\,\pi)^2}\int_{\mathbb{R}^3} d^3r\, \boldsymbol{F} \cdot \boldsymbol{A}$$



Hopfion

- Trivial at spatial infinity
- $\mathbb{R}^3 \to S^3 \to S^2$

$\pi_3(S^3) = \mathbb{Z}$

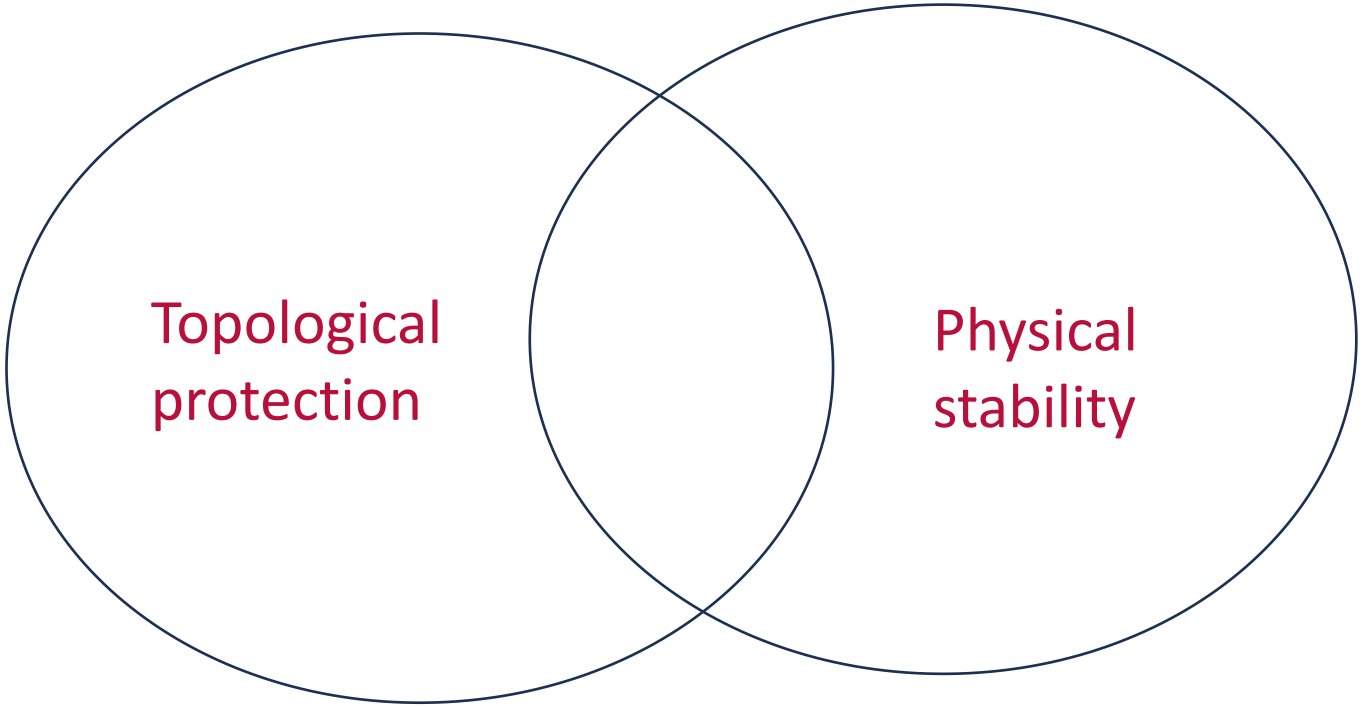


Sci. Adv. **4**, eaao3820 (2018)

3D Skyrmion

- SO(3) or SU(2) order parameter required
- Topologically quantized

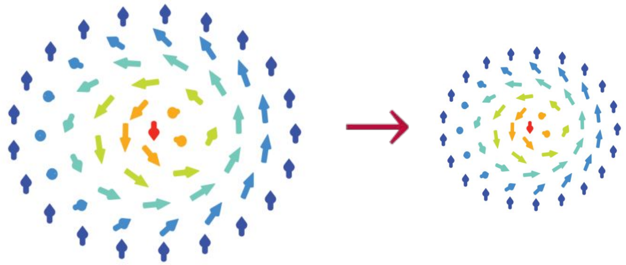
Homotopy groups of spheres



Derrick-Hobart theorem

TWIST
Topological Whirls In SpinTronics

Scaling $\boldsymbol{m}_0(\boldsymbol{r}) \to \boldsymbol{m}_0(\boldsymbol{r}/\lambda)$ in d dimensions yields conditions for the solution $\boldsymbol{m}_0(\boldsymbol{r})$ that minimises energy.



Consider the Hamiltonian,

$$\mathcal{H} = \mathcal{H}_{ex} + \mathcal{H}_{DMI} + \mathcal{H}_{an} + \mathcal{H}_{z}$$

Let $\boldsymbol{m}_0(\boldsymbol{r})$ be a finite-energy equilibrium solution.

Applying the scaling transformation $\boldsymbol{m}_0(\boldsymbol{r}) \to \boldsymbol{m}_0(\boldsymbol{r}/\lambda)$ we obtain

$$\mathcal{H} = \lambda^{d-2}\mathcal{H}_{ex} + \lambda^{d-1}\mathcal{H}_{DMI} + \lambda^{d}(\mathcal{H}_{an} + \mathcal{H}_{z})$$

For the equilibrium solution $\partial_\lambda \mathcal{H}|_{\lambda=1} = 0$ and $\partial_\lambda^2 \mathcal{H}|_{\lambda=1} > 0$.

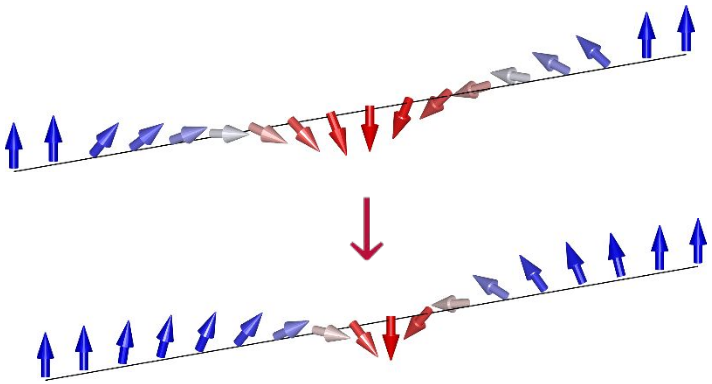
The virial relation is,

$$\boxed{(d-2)\mathcal{H}_{ex} + (d-1)\mathcal{H}_{DMI} + d(\mathcal{H}_{an} + \mathcal{H}_{z}) = 0}$$

Derrick-Hobart theorem

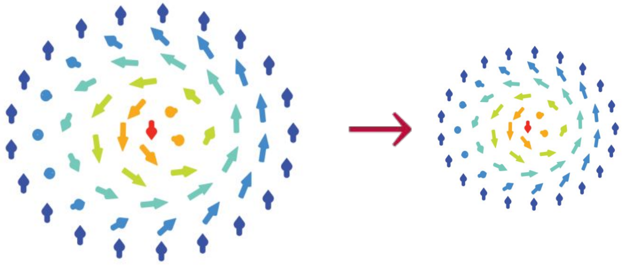
Scaling $\boldsymbol{m}_0(\boldsymbol{r}) \rightarrow \boldsymbol{m}_0(\boldsymbol{r}/\lambda)$ in d dimensions yields conditions for the solution $\boldsymbol{m}_0(\boldsymbol{r})$ that minimises energy.

$d = 1$



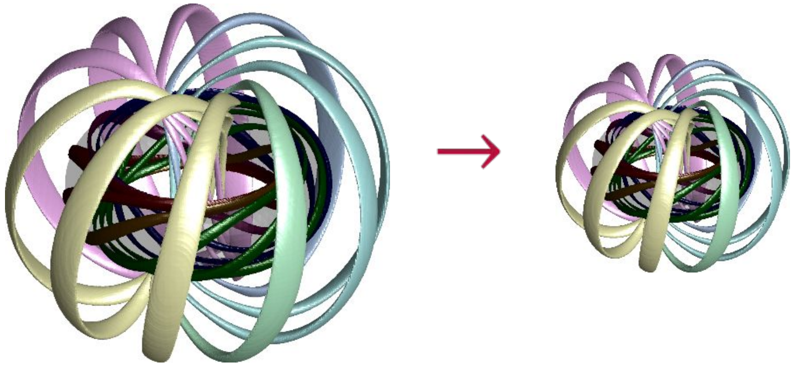
$$-\mathcal{H}_{ex} + (\mathcal{H}_{an} + \mathcal{H}_z) = 0$$

$d = 2$



$$\mathcal{H}_{DMI} + 2(\mathcal{H}_{an} + \mathcal{H}_z) = 0$$

$d = 3$



$$\mathcal{H}_{ex} + 2\mathcal{H}_{DMI} + 3(\mathcal{H}_{an} + \mathcal{H}_z) = 0$$

Homotopy groups of spheres

Beyond the $\boldsymbol{\pi_2(S^2)} = \mathbb{Z}$ or $\boldsymbol{\pi_3(S^2)} = \mathbb{Z}$ paradigm?

Under homogeneous boundary conditions e.g., $\boldsymbol{m}(\boldsymbol{r} \to \infty) = (0,0,1)$, the 3D physical space can be compactified to S^3.

Other boundary conditions allow other types of compactifications of R^3 such as

$$\boldsymbol{S}: S^2 \times T^1 \to S^2$$

$$\boldsymbol{S}: T^3 \to S^2$$

Homotopy groups of spheres

TWIST
Topological Whirls In SpinTronics

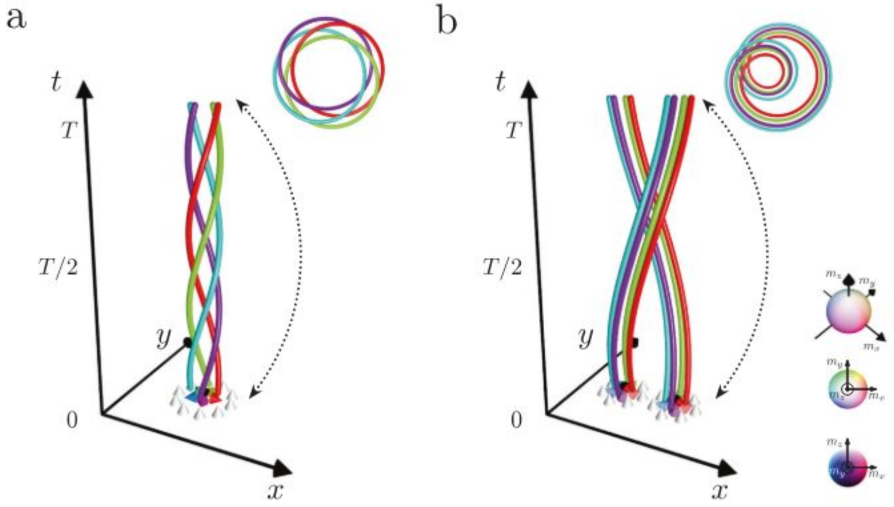
$S^2 \times T^1$ base space

https://doi.org/10.1038/s42005-024-01628-3

Spacetime magnetic hopfions from internal excitations and braiding of skyrmions

Check for updates

Ross Knapman[1,2], Timon Tausendpfund[2], Sebastián A. Díaz[1,3] & Karin Everschor-Sitte[1]

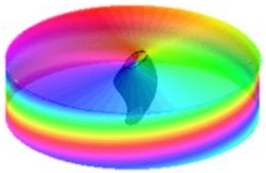


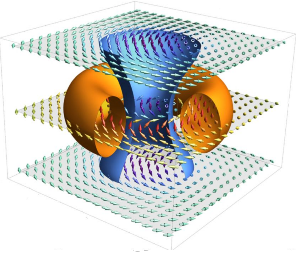
PHYSICAL REVIEW LETTERS 128, 157204 (2022)

Screw Dislocations in Chiral Magnets

Maria Azhar,[1] Volodymyr P. Kravchuk,[1,2] and Markus Garst[1,3]
[1]Institut für Theoretische Festkörperphysik, Karlsruhe Institute of Technology, 76131 Karlsruhe, Germany
[2]Bogolyubov Institute for Theoretical Physics of National Academy of Sciences of Ukraine, 03143 Kyiv, Ukrai
[3]Institute for Quantum Materials and Technology, Karlsruhe Institute of Technology, 76131 Karlsruhe, Germa

(Received 22 September 2021; revised 23 December 2021; accepted 28 February 2022; published 12 April 2





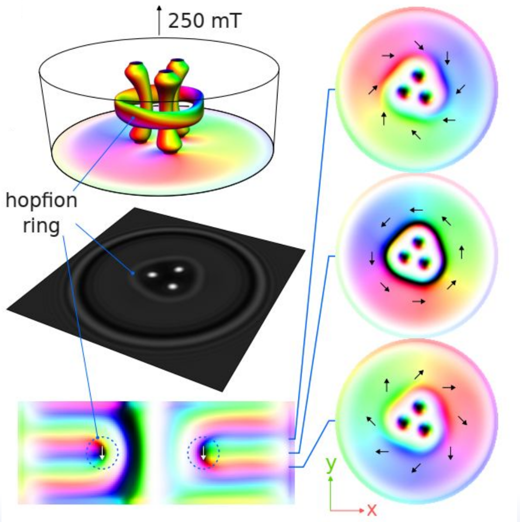
Article

Hopfion rings in a cubic chiral magnet

https://doi.org/10.1038/s41586-023-06658-5
Received: 11 March 2023
Accepted: 20 September 2023
Published online: 22 November 2023
Open access

Fengshan Zheng[1,2,3], Nikolai S. Kiselev[3,4], Filipp N. Rybakov[5], Luyan Yang[6], Wen Shi[2,3], Stefan Blügel[3,4] & Rafal E. Dunin-Borkowski[2,3]

Magnetic skyrmions and hopfions are topological solitons[1]—well-localized field configurations that have gained considerable attention over the past decade owing to their unique particle-like properties, which make them promising objects for

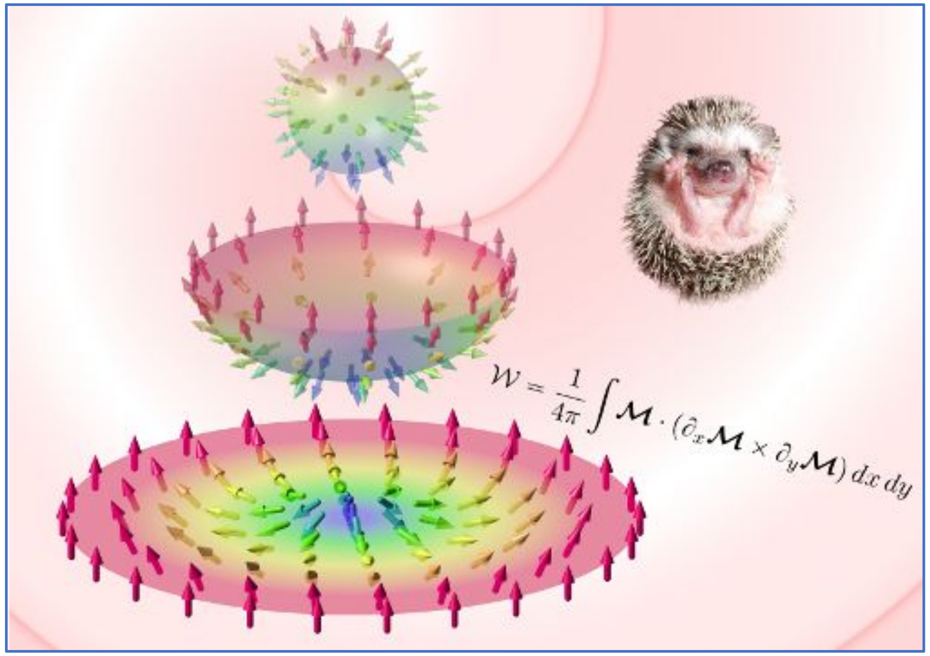


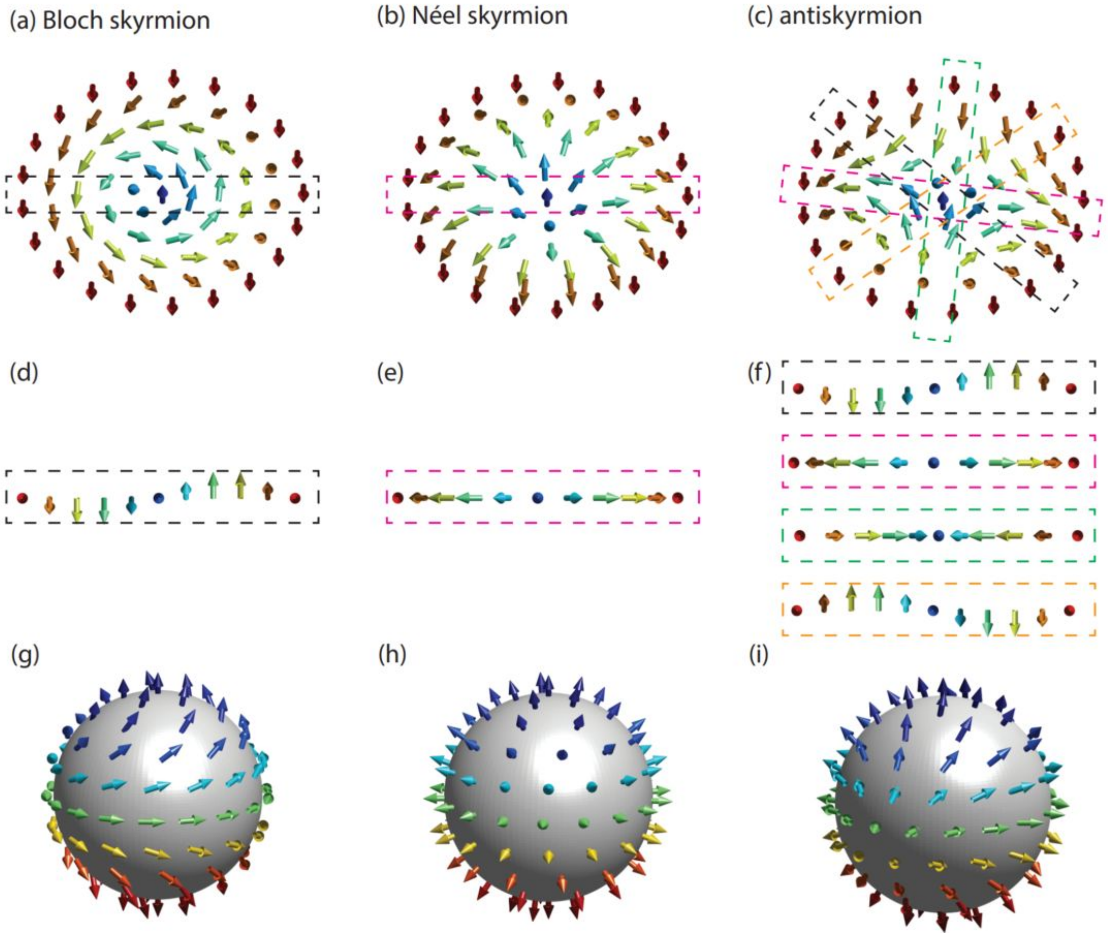
Skyrmions

- $\pi_2(S^2) = \mathbb{Z}$

$$N_{sk} = \frac{1}{4\pi}\int d^2x\, m\cdot(\partial_x m \times \partial_y m)$$

A ferromagnet is described by a magnetization vector $\boldsymbol{m}(\boldsymbol{x},t)$ with a fixed length, $|\boldsymbol{m}(\boldsymbol{x},t)|$ =1, i.e., $\boldsymbol{m} \in \mathbb{S}^2$





Screw dislocations

A new twist in the topology of chiral magnets

Phys. Rev. Lett. 128, 157204 (2022)

Screw dislocations in chiral magnets

TWIST
Topological Whirls In SpinTronics

Theoretical prediction

PHYSICAL REVIEW LETTERS **128,** 157204 (2022)

Screw Dislocations in Chiral Magnets

Maria Azhar,[1] Volodymyr P. Kravchuk,[1,2] and Markus Garst[1,3]
[1] *Institut für Theoretische Festkörperphysik, Karlsruhe Institute of Technology, 76131 Karlsruhe, Germany*
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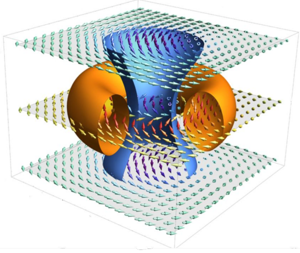

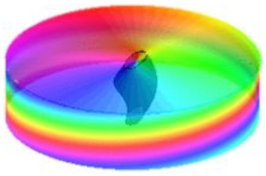

Experimental observation

Article

Hopfion rings in a cubic chiral magnet

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Received: 11 March 2023
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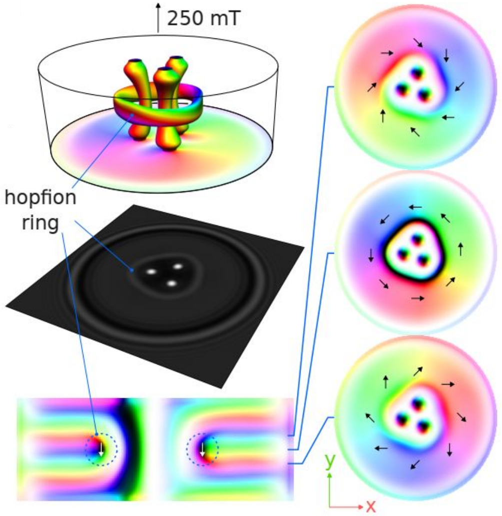
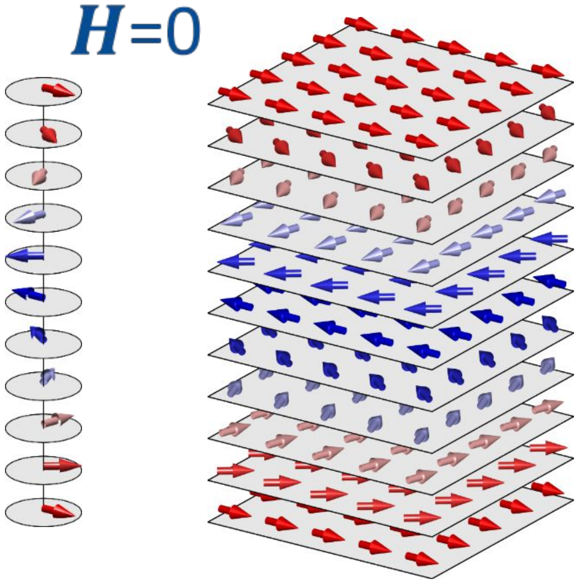
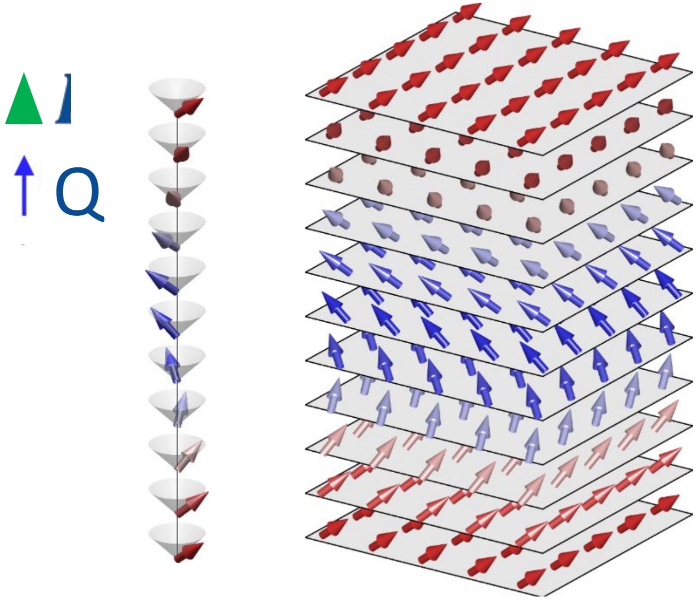


Screw dislocations in chiral magnets



helical spiral



Conical spiral

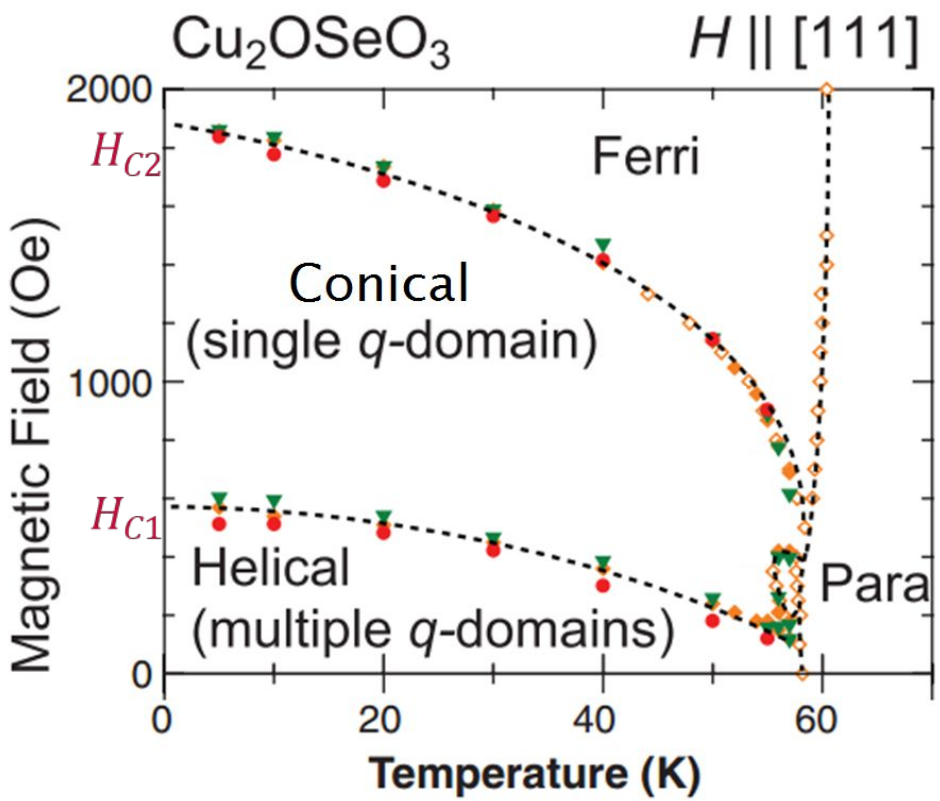
$$\mathcal{E} = A(\boldsymbol{\nabla n})^2 + D\boldsymbol{n}\cdot\boldsymbol{\nabla}\times\boldsymbol{n} - M_s\mu_0\boldsymbol{H}\cdot\boldsymbol{n}$$

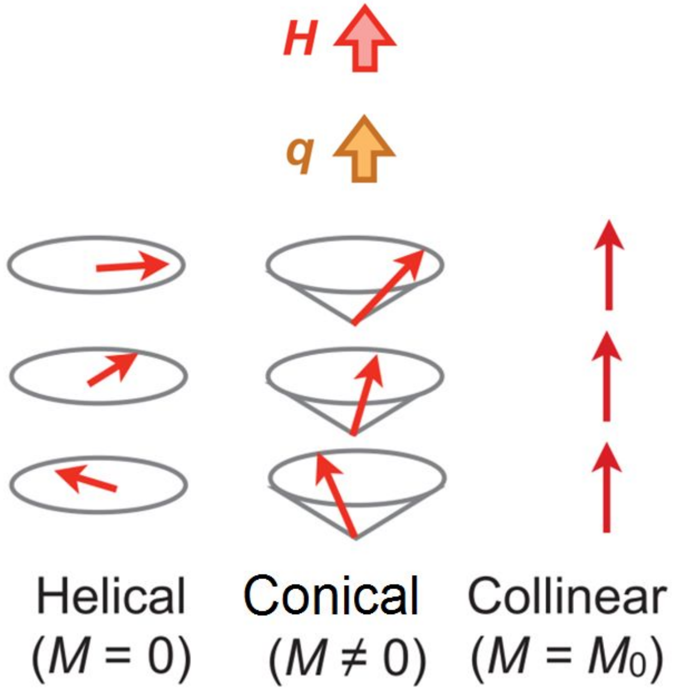
Zeeman energy

helical spiral wavelength λ_h

$$\lambda_h = \frac{2\pi}{q} = 4\pi\frac{A}{D} \qquad q = D/2A$$

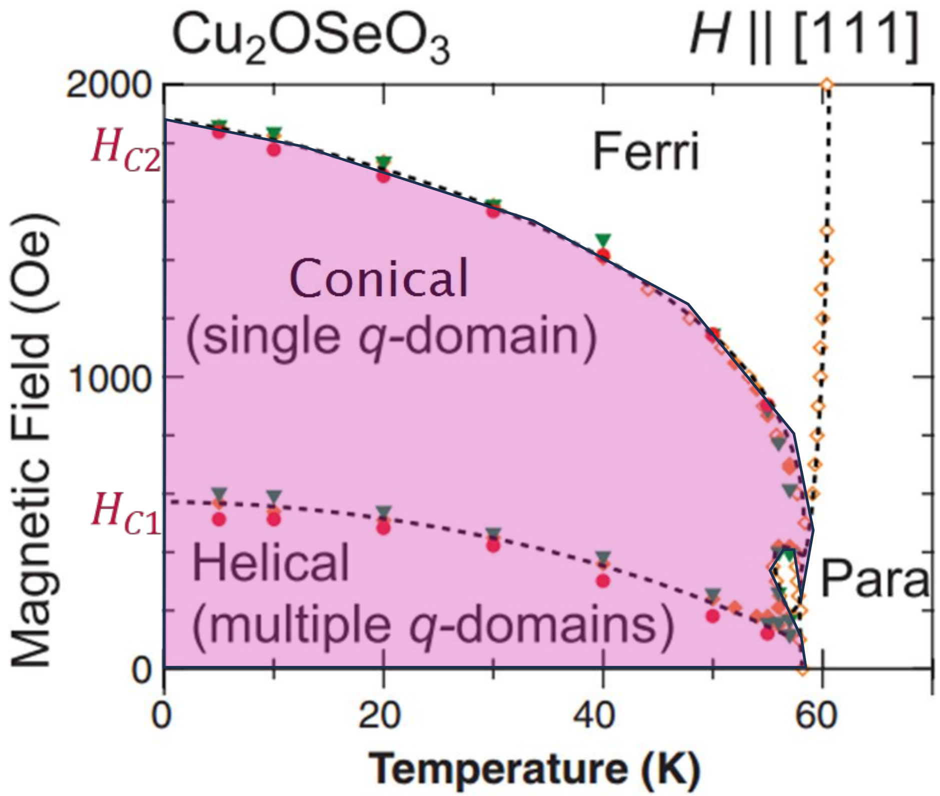
Screw dislocations in chiral magnets





Seki et al Science 336, 198–201 (2012)

Screw dislocations in chiral magnets



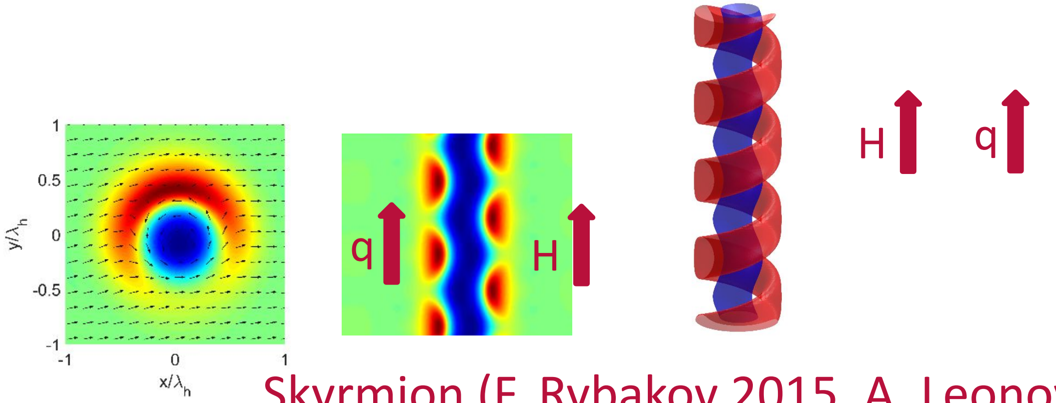
Topological defects in a spiral background:

- Skyrmions [1,2]
- Chiral bobbers [3]
- Toron or dipole string [4,5]
- Hopfions or heliknotons [6,7]
- Edge dislocations [8,9]
- **Screw dislocations [10]**

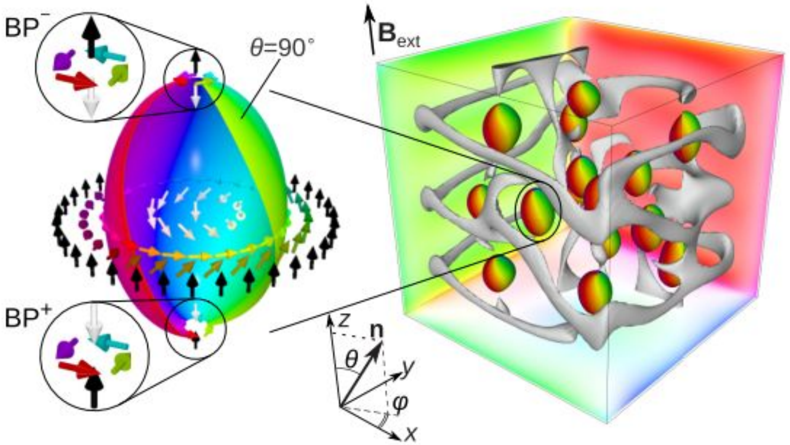
[1] F. Rybakov 2015, [2] A. Leonov 2016
[3] F. Rybakov 2015
[4] A. Leonov 2018, [5] G. Müller 2020
[6] R. Voinescu 2020 [7] V. Kuchkin 2023
[8] Dussaux (2016) [9] Schoenherr (2018)
[10] Azhar 2022

Screw dislocations in chiral magnets

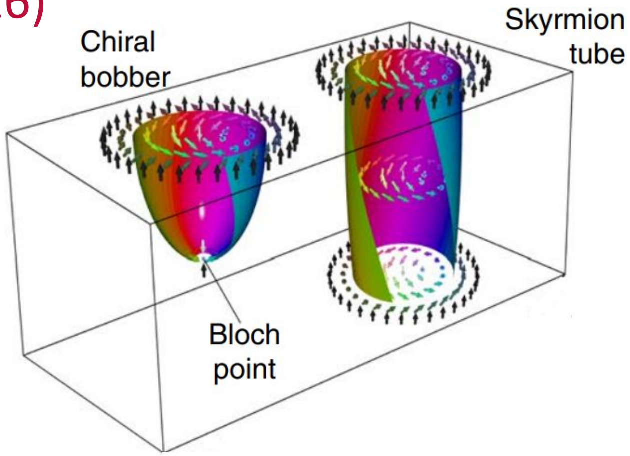
Topological defects in chiral magnets



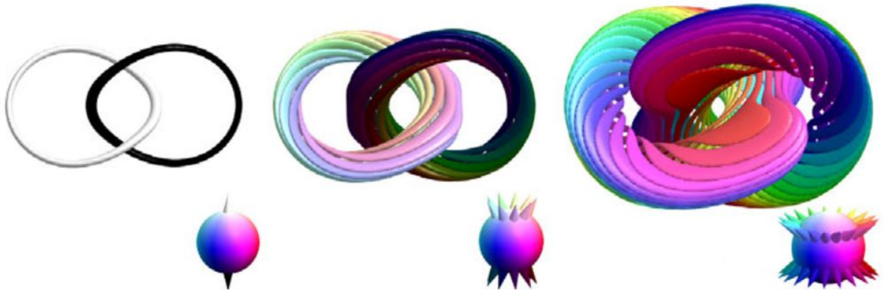
Skyrmion (F. Rybakov 2015, A. Leonov 2016)



Toron or dipole string
(A. Leonov 2018, G. Müller 2020)



Chiral bobbers (F. Rybakov 2015)

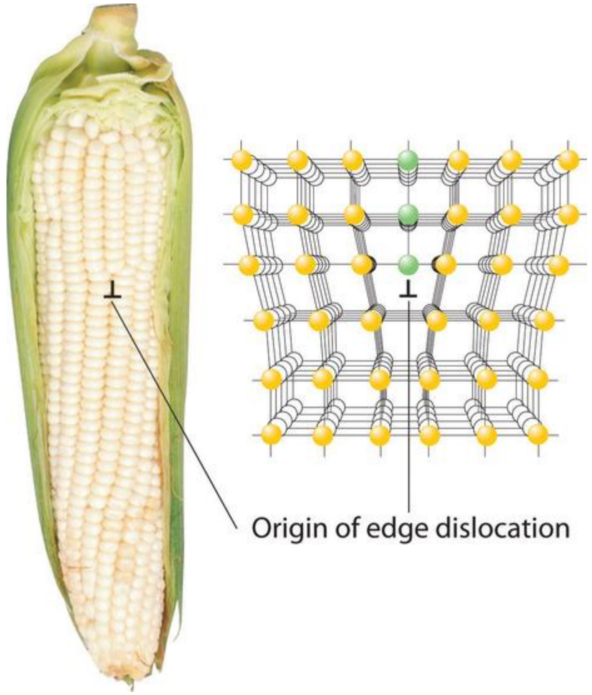


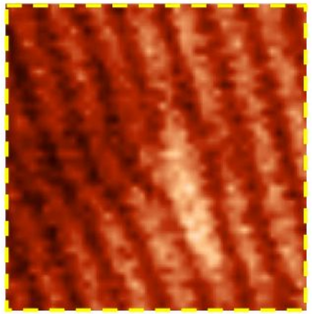
Hopfion/Heliknoton/Hopf soliton
(R. Voinescu 2020)

Screw dislocations in chiral magnets

TWIST — Topological Whirls In SpinTronics

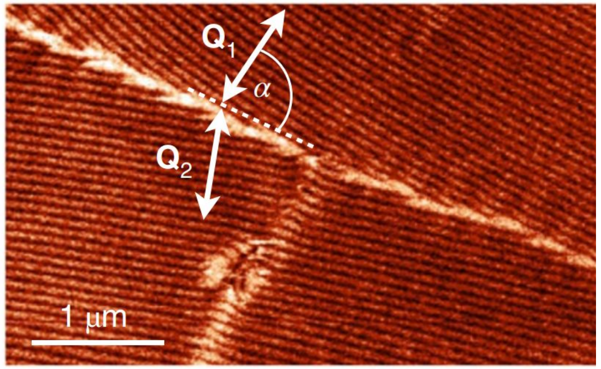
Dislocations are **topological defects**, a consequence of periodic (crystalline or magnetic) order.





MFM image

$$Q = \frac{1}{4\pi}\int d^2x\left(\mathbf{n}\cdot\left[\partial_x\mathbf{n}\times\partial_y\mathbf{n}\right]\right) = 0.5$$



MFM image

Dussaux, Nat Comm (2016), Schoenherr, Nat Phy (2018)

Play a role in helix wavevector reorientation, domain wall relaxation, domain wall mobility, melting or condensation of helical order, ...

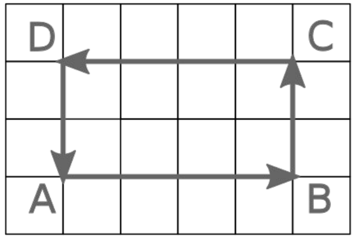
Screw dislocations in chiral magnets

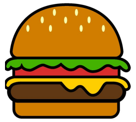
Dislocations are **topological defects**, a consequence of periodic (crystalline or magnetic) order.

quantized Burgers vector = topological protection

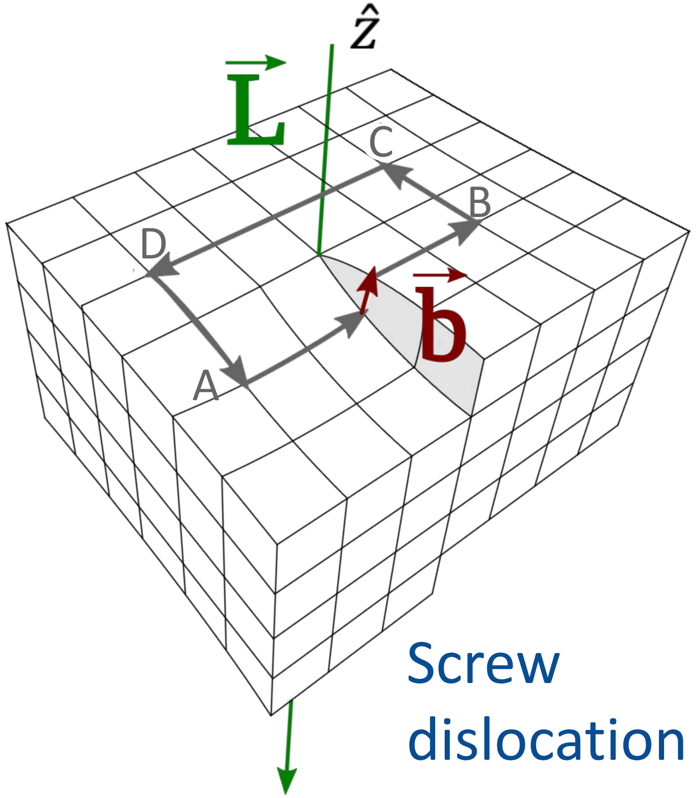
Screw dislocations in chiral magnets

$$\vec{b} = \nu\lambda_h \, \hat{z} \quad \text{for integer } \nu$$
$$\oint d\vec{u} = \vec{b}$$

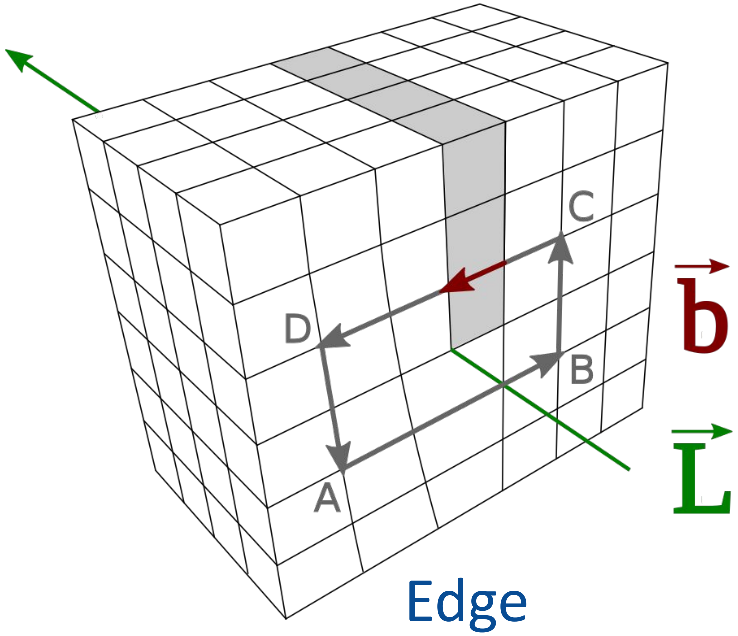




Burgers Vector $\vec{b}$
Dislocation Line Vector $\vec{L}$



Screw dislocation



Edge dislocation

Screw dislocations in chiral magnets

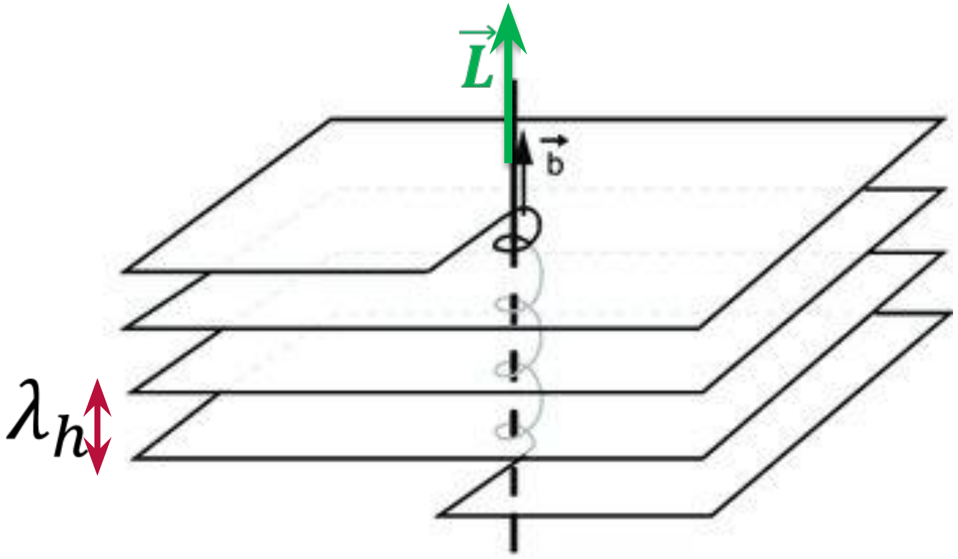
TWIST
Topological Whirls In SpinTronics

$$\vec{b} = \nu\lambda_h\,\hat{z} \quad \text{for integer } \nu$$

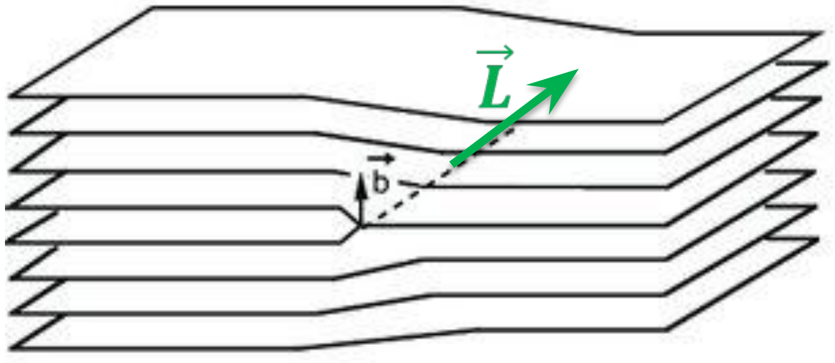
$$\oint d\vec{u} = \vec{b}$$

Burgers Vector $\vec{b}$
Dislocation Line Vector $\vec{L}$

A system with layers can host screw dislocations or edge dislocations.

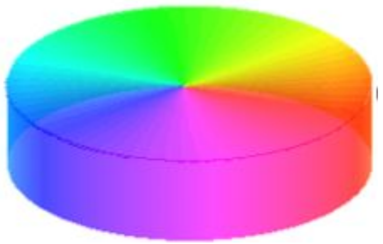


Screw dislocation



Edge dislocation

Screw dislocations in chiral magnets



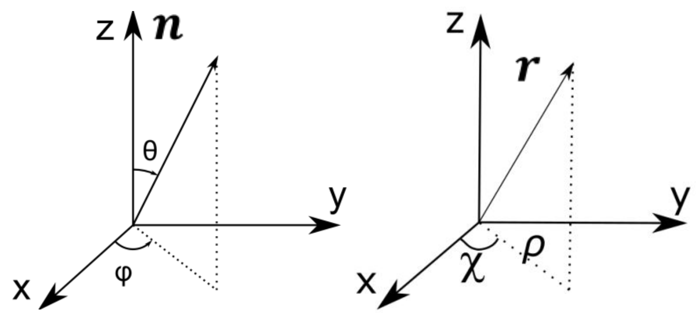
Conventional vortex

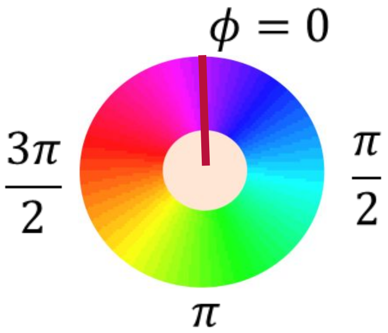
$$\phi = \nu\chi$$

$$\nu = +1$$

$$\boldsymbol{n} = (\cos\phi\sin\theta\,, \sin\phi\ \sin\theta\,, \cos\theta)$$

$$\boldsymbol{r} = (\rho\cos\chi\,, \rho\sin\chi\,, z)$$





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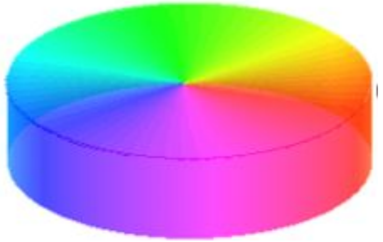
Screw dislocations in chiral magnets



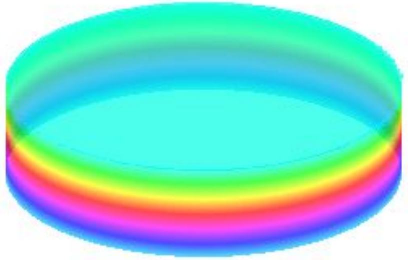
Conventional vortex

$$\phi = \nu\chi$$

$$\nu = +1$$

$$q = \frac{2\pi}{\lambda_h}$$

λ_h

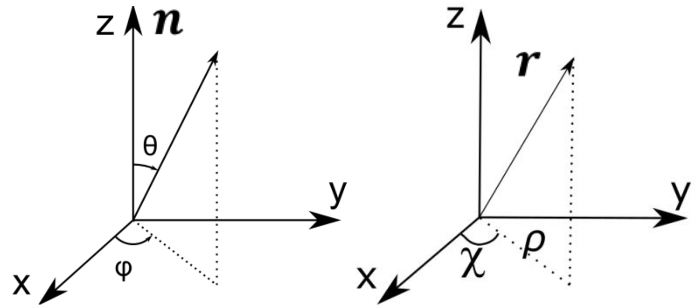


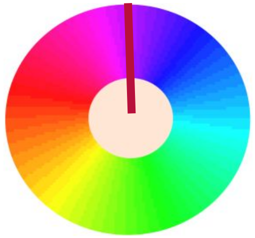
Helical spiral

$$\phi = qz$$

$$\boldsymbol{n} = (\cos\phi\sin\theta, \sin\phi\ \sin\theta, \cos\theta)$$

$$\boldsymbol{r} = (\rho\cos\chi, \rho\sin\chi, z)$$





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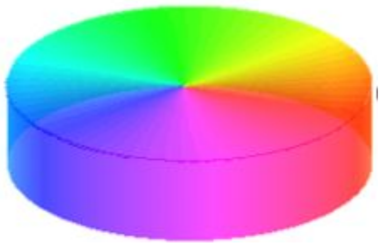
Screw dislocations in chiral magnets

TWIST – Topological Whirls In SpinTronics

$$\boldsymbol{n} = (\cos\phi \sin\theta, \sin\phi \sin\theta, \cos\theta)$$

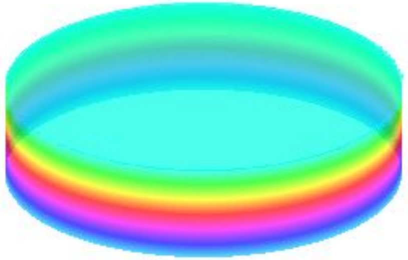
$$\boldsymbol{r} = (\rho \cos\chi, \rho \sin\chi, z)$$



Conventional vortex

$$\phi = \nu\chi$$

$$\nu = +1$$

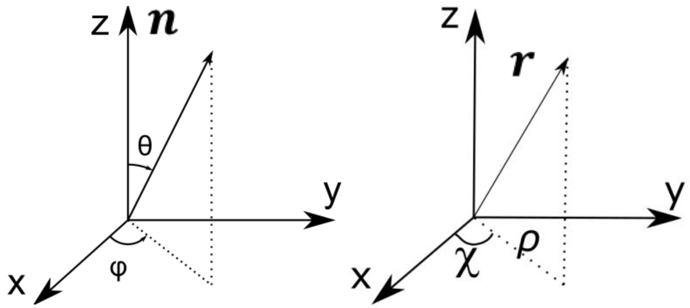


$$q = \frac{2\pi}{\lambda_h}$$

λ_h

Helical spiral

$$\phi = qz$$

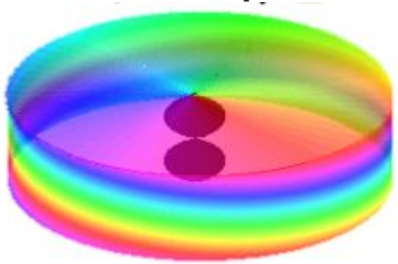


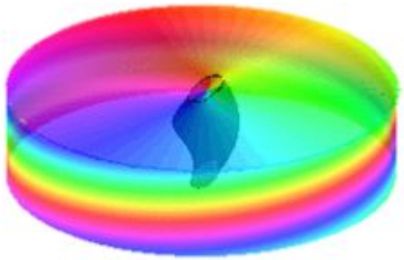
$$\vec{b} = \nu\lambda_h\,\hat{z} \quad \text{for integer } \nu$$

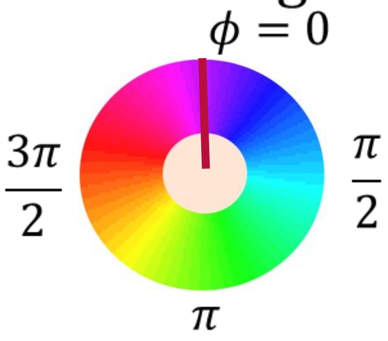
Screw dislocations

$$\phi \cong \nu\chi + qz$$

$\nu = +1$ $\qquad$ $\nu = -1$







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Screw dislocations in chiral magnets

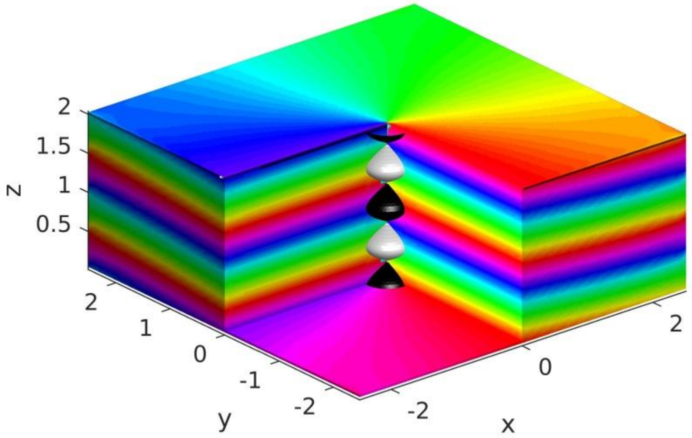
TWIST
Topological Whirls In SpinTronics

$$\mathcal{E} = A(\partial_i \boldsymbol{n})^2 + D\, \boldsymbol{n} \cdot (\nabla \times \boldsymbol{n})$$

$$\theta = \frac{\pi}{2} + \frac{\nu}{q} \frac{\sin(qz + (\nu - 1)\chi)}{\rho} + \mathcal{O}(\rho^{-2})$$

$$\phi = \nu\chi + qz + \mathcal{O}(\rho^{-2})$$

Energy does not scale with the (lateral) system size at zero anisotropy and zero magnetic field, for all values of ν.



Reason: The effective elasticity for the far field has a Landau-Peierls form, typical for lamellar structures in an isotropic environment.

P. M. Chaikin and T. C. Lubensky, *Principles of Condensed Matter Physics*

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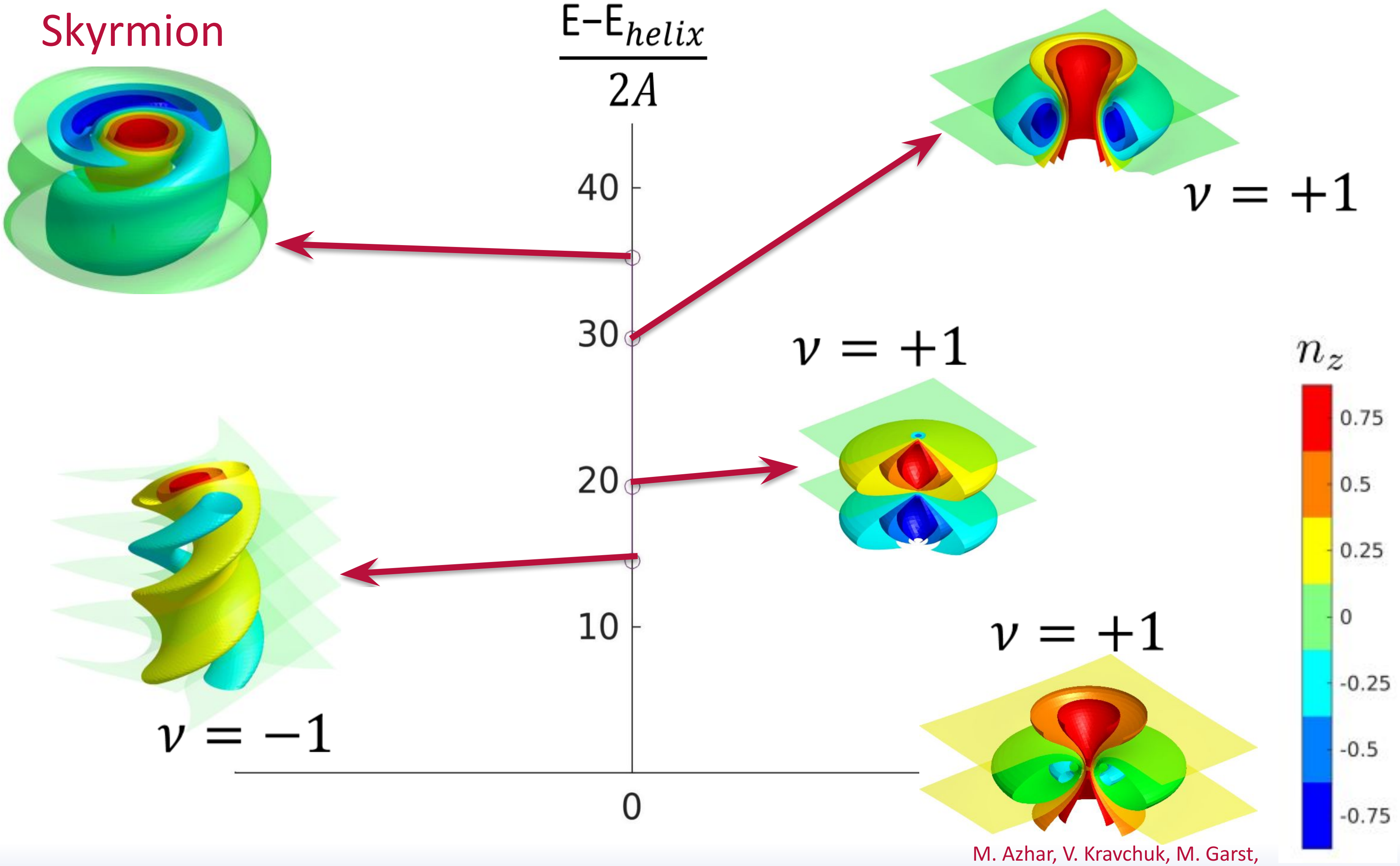
Screw dislocations in chiral magnets



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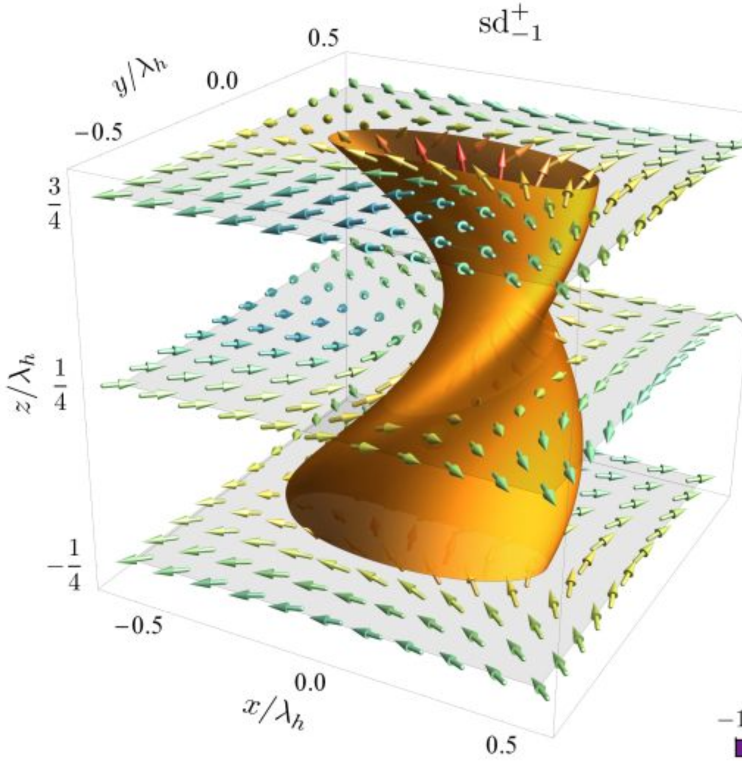
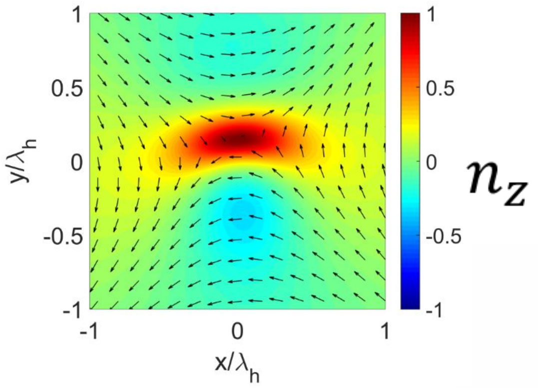
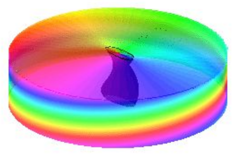
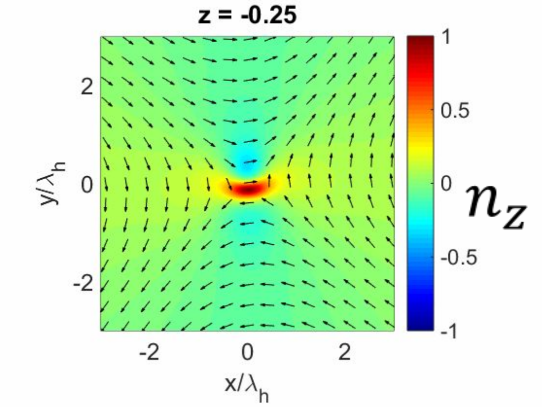
Screw dislocations in chiral magnets

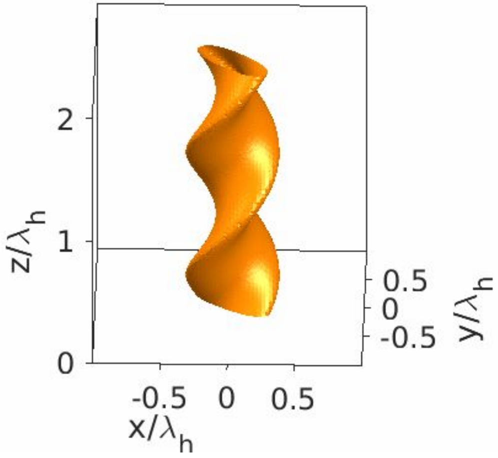
$\nu = -1$ screw dislocation











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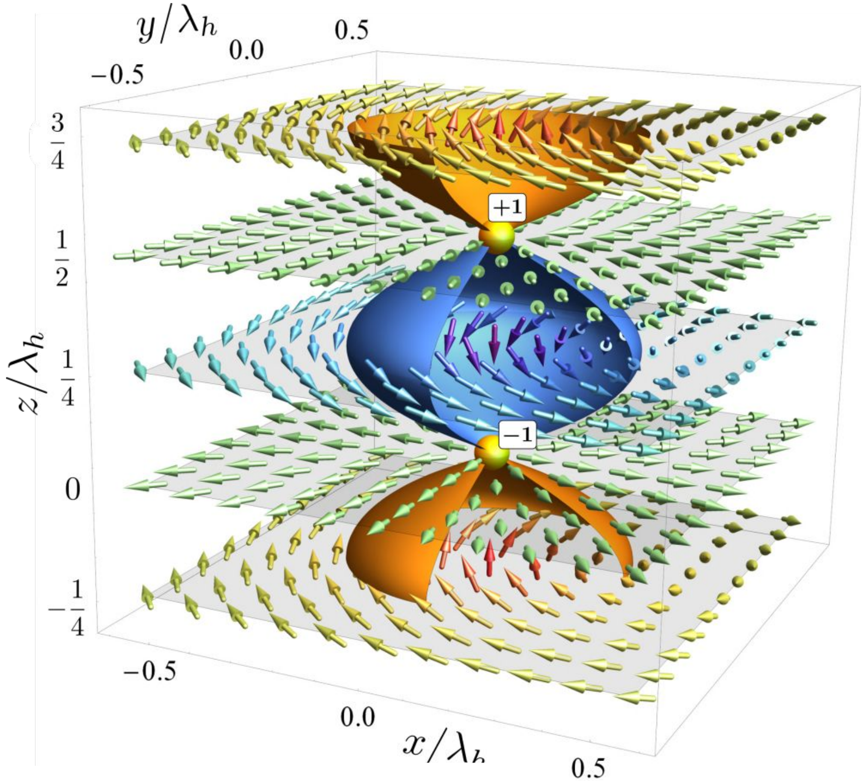
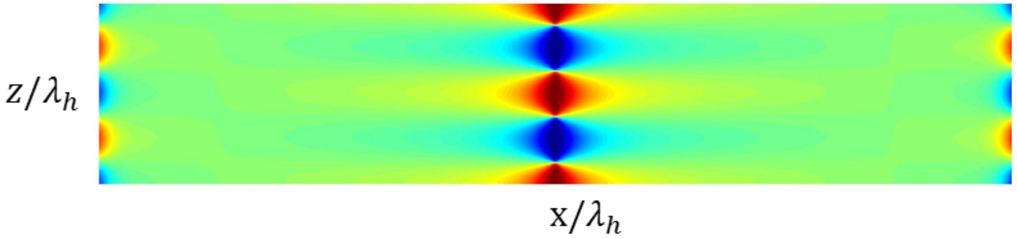
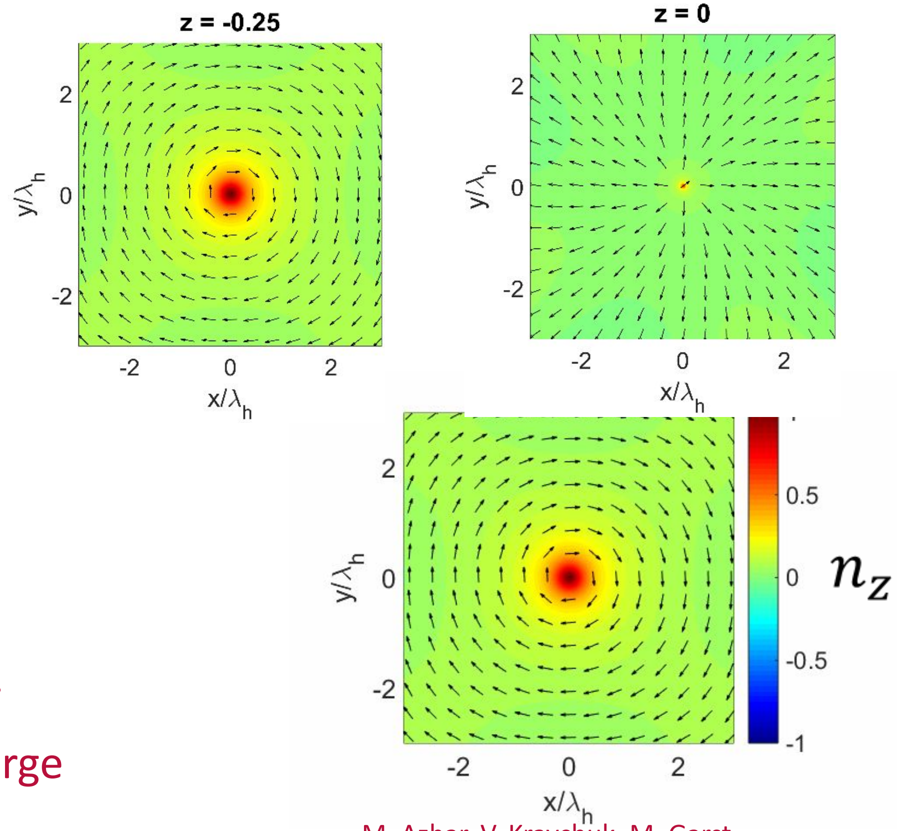
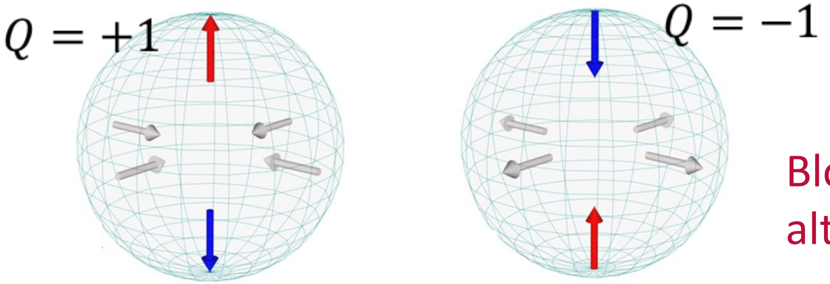
Screw dislocations in chiral magnets

$\nu = +1$









Bloch points of alternating charge

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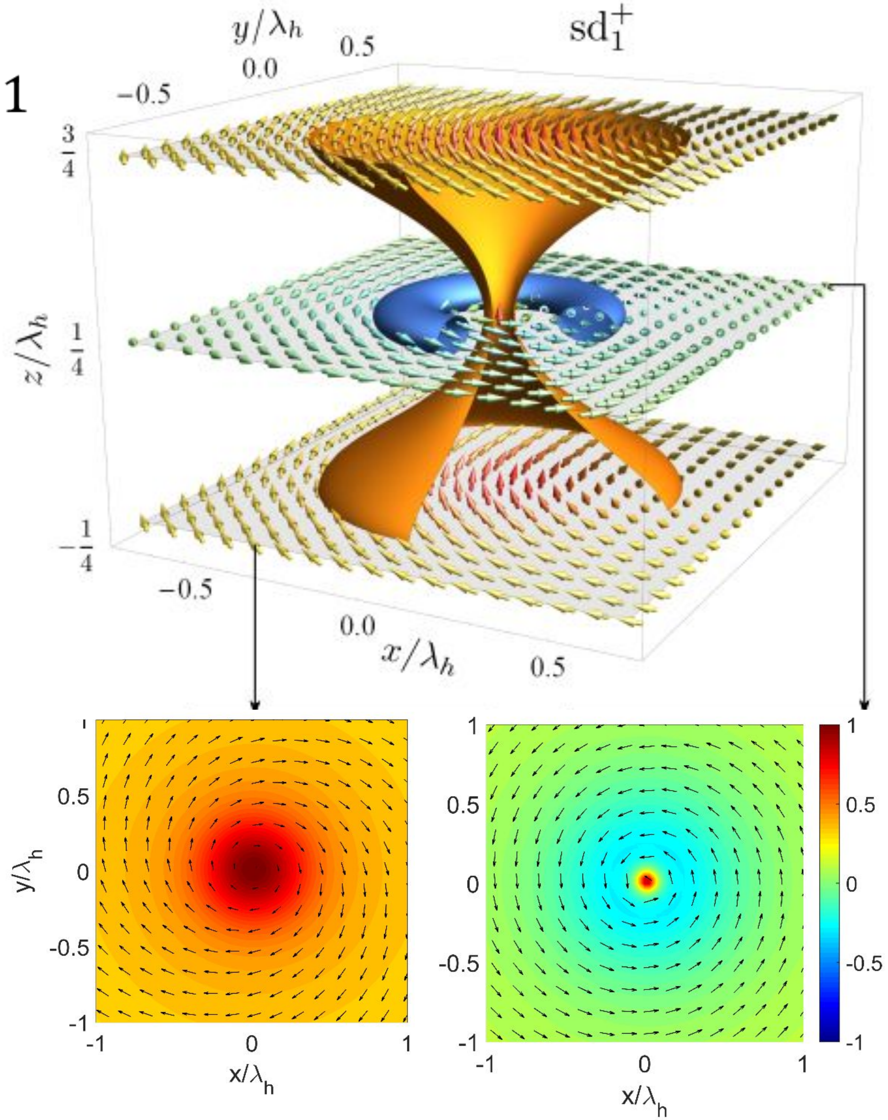
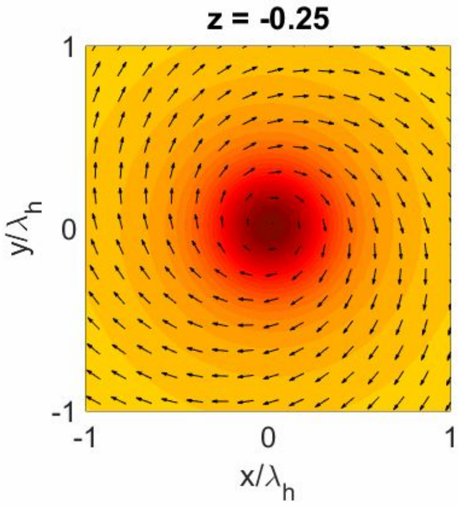
Screw dislocations in chiral magnets

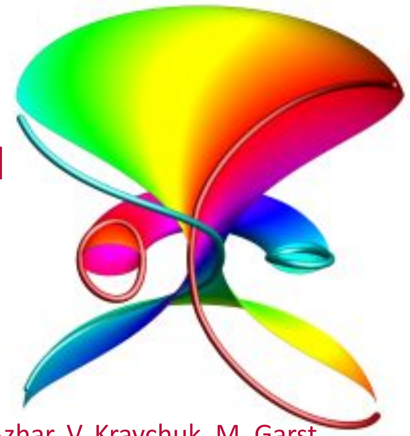
$v = +1$





Different linking numbers for the core and the torus

Fractional hopfion



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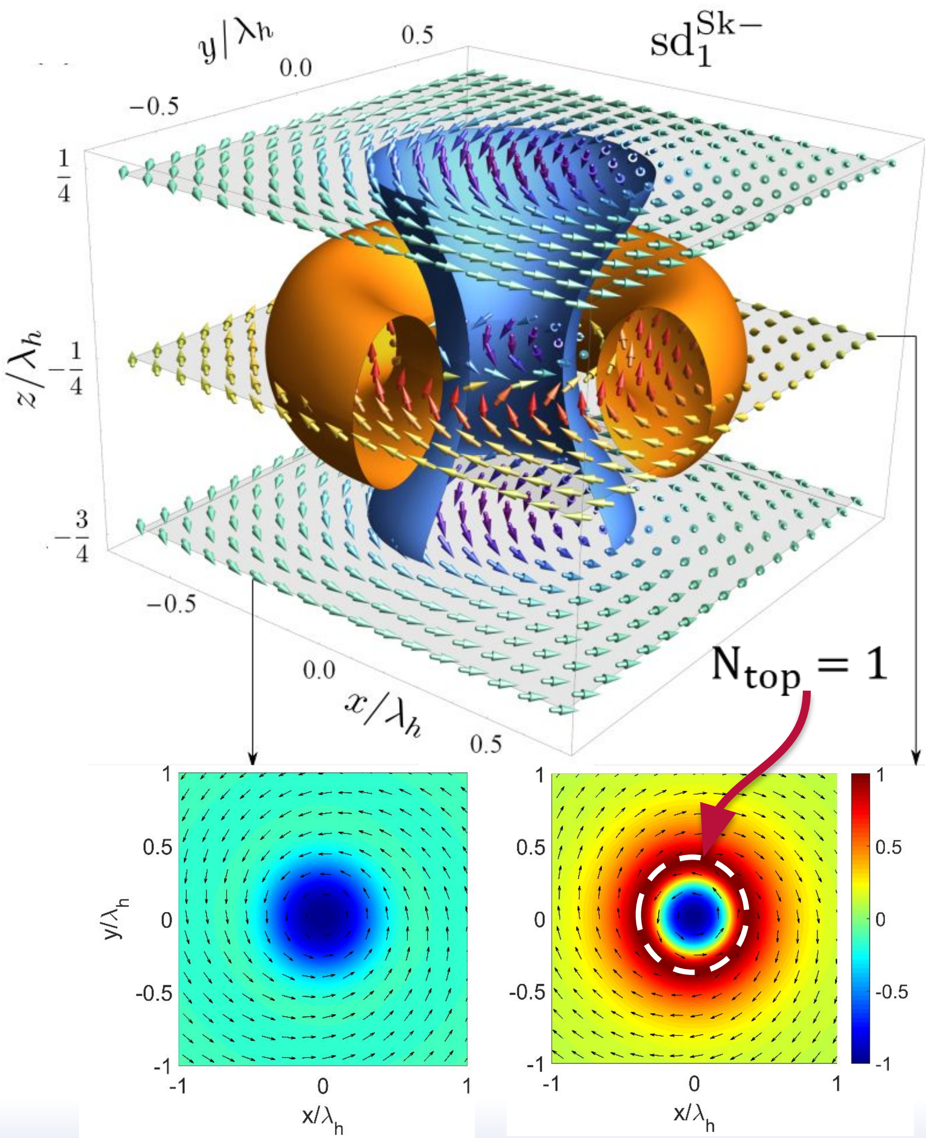
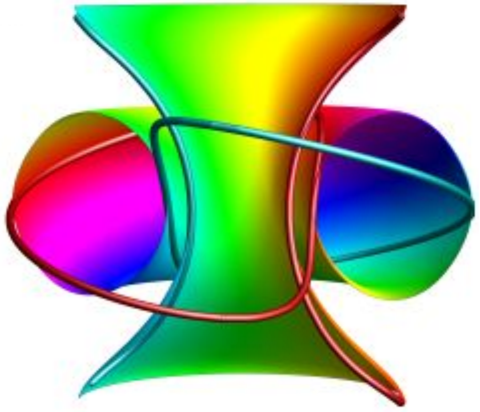
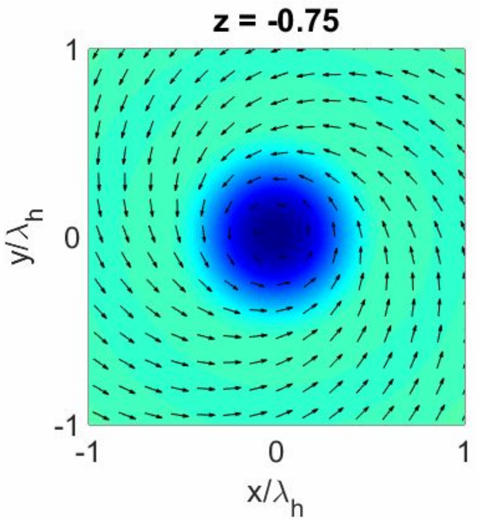
Screw dislocations in chiral magnets



Different linking numbers for the core and the torus

Fractional hopfion





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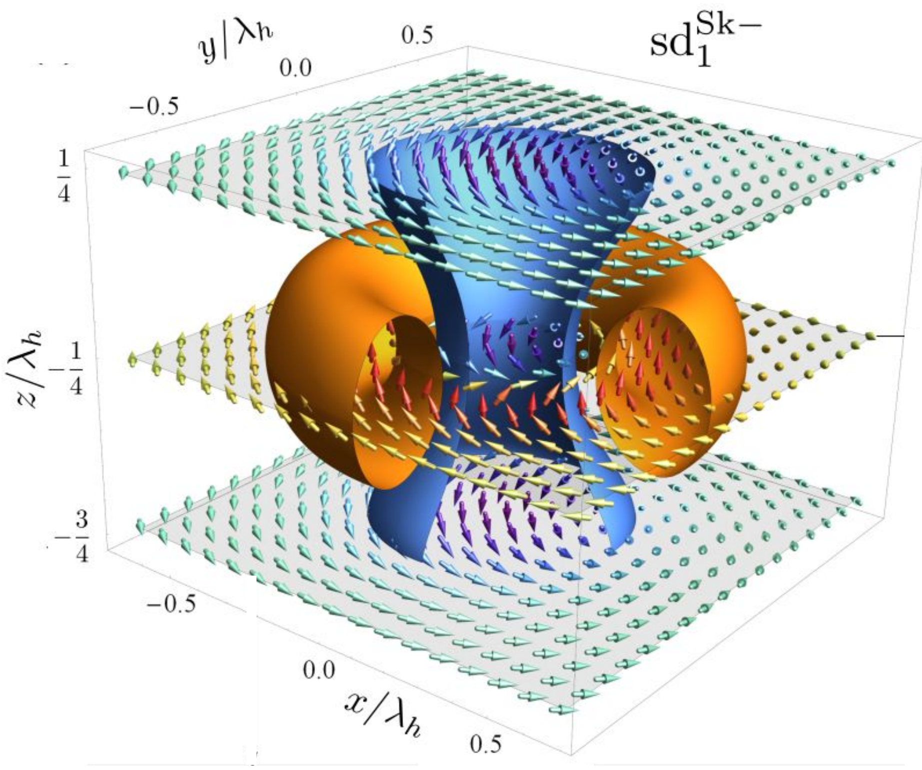
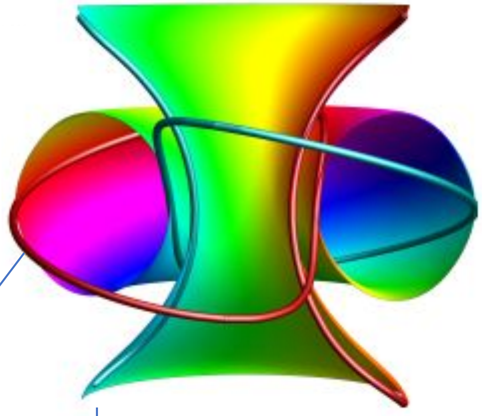
Screw dislocations in chiral magnets

TWIST
Topological Whirls In SpinTronics



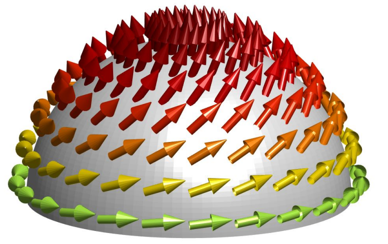
Different linking numbers for the core and the torus

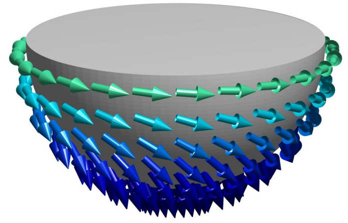


Fractional hopfion

Core maps to lower half-sphere

Torus maps to upper half-sphere





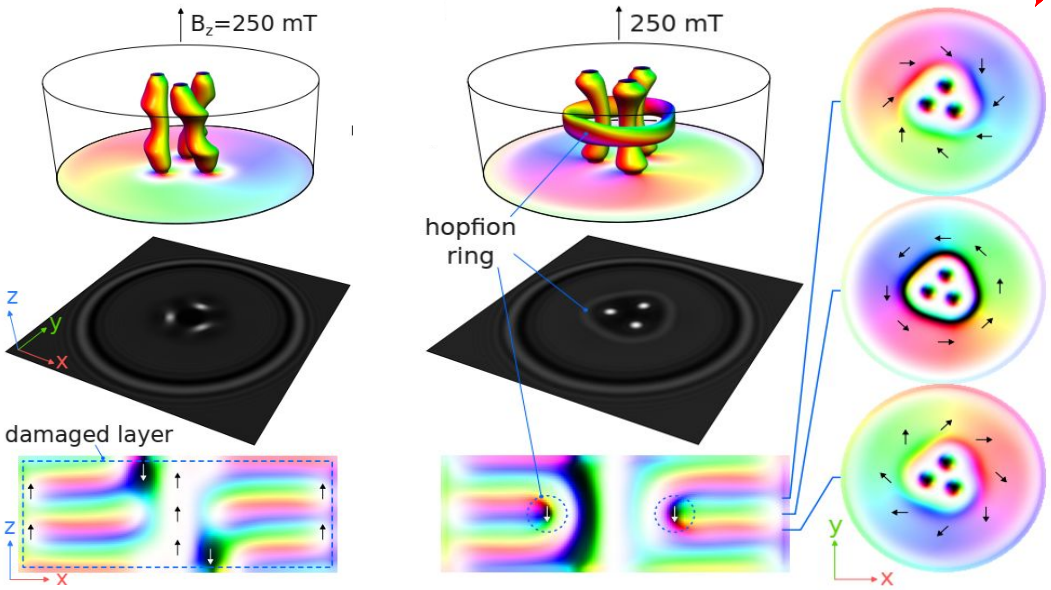
Screw dislocations in chiral magnets

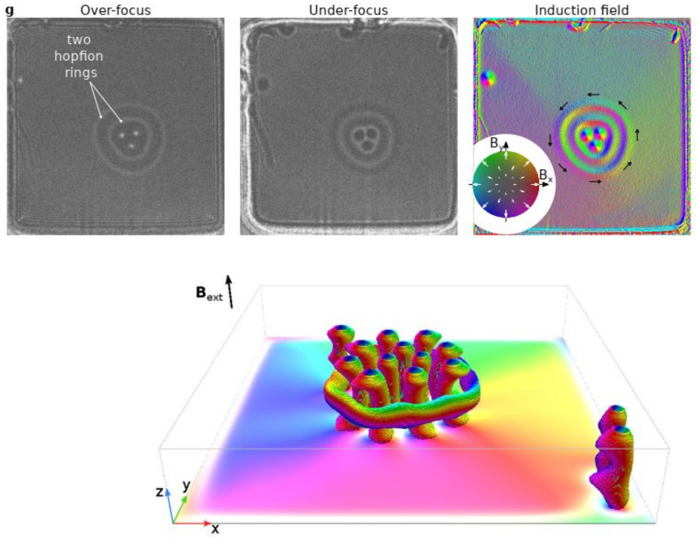
"screw dislocations" in a 180 nm FeGe film.

Zheng, F., Kiselev, N.S., Rybakov, F.N. et al.
"Hopfion rings in a cubic chiral magnet."
Nature 623, 718–723 (2023)

$$\phi = \nu\chi + qz$$

far field

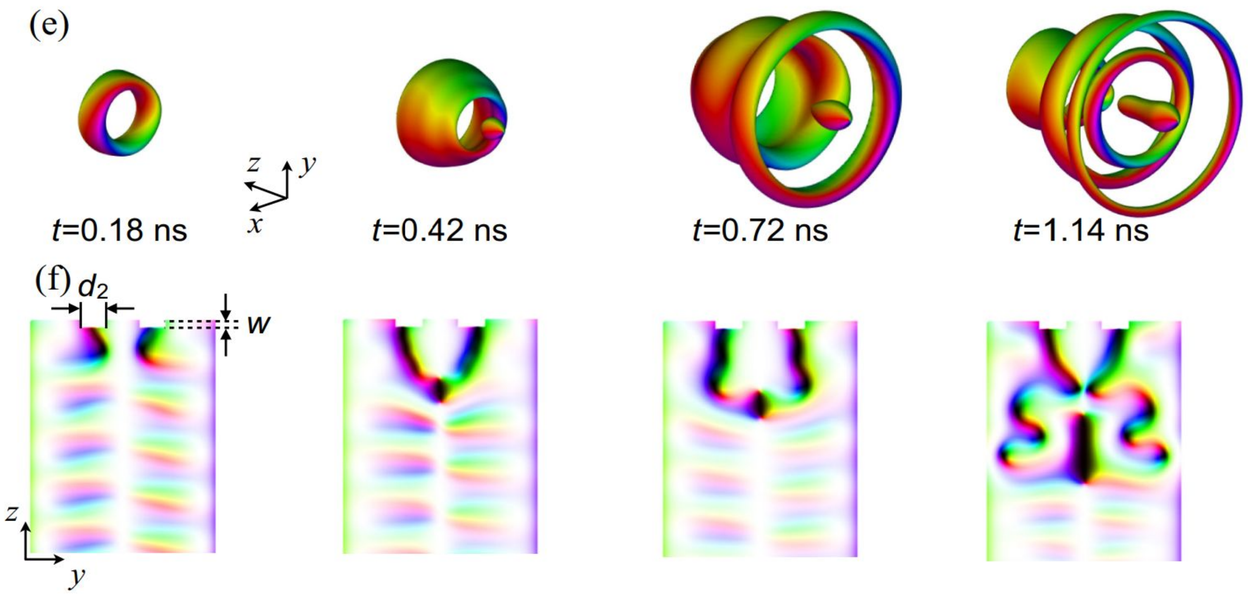




Screw dislocations in chiral magnets

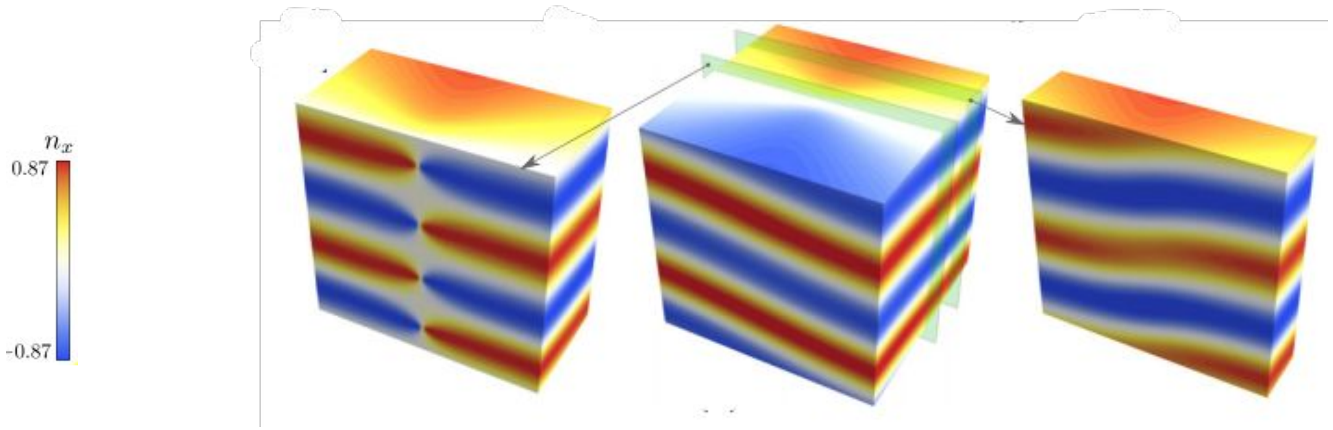
"Current-Induced Creation of Topological Vortex Rings in a Magnetic Nanocylinder"

Liu, Nagaosa, Phys. Rev. Lett. 132, 126701 (2024)



Summary

- A new topological magnetic texture, PRL 128, 157204 (2022)
- Non-integer skyrmion charge and Hopf index
- quantized Burgers vector provides topological stability



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Special thanks: Karin Everschor-Sitte, Alessandro Pignedoli, Ross Knapman, TWIST group