
D. Hardt


R. Doostani


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A. Rosch

Active Magnetic Matter: Propelling Ferrimagnetic Domain Walls by Dynamical Frustration
https://arxiv.org/abs/2405.14320

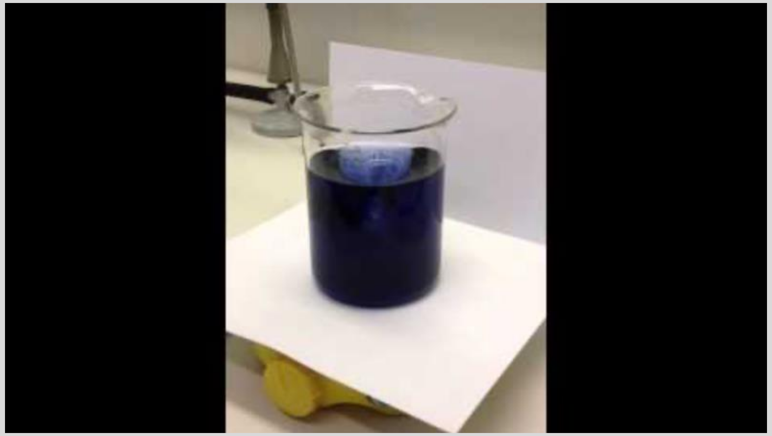
Active Magnetic Matter

Nina del Ser
University of Cologne, Caltech

Active matter systems

- Constituent units use local energy to induce, e.g., motion
- Far from equilibrium
- Flocking, long-range order, new phase transitions..

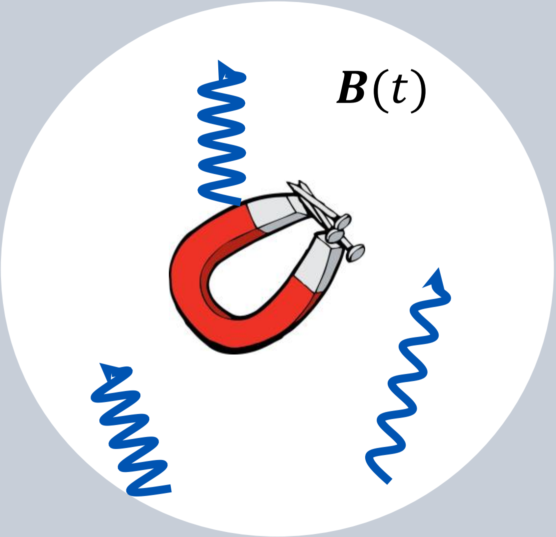
Flock of starlings



Briggs-Rauscher reaction

Plan of the talk

1. Identify a magnetic system with “active” units
2. Investigate dynamical properties of a single unit
3. Many units together?



Energy injected into system by oscillating magnetic field $\boldsymbol{B}(t)$

Spin Dynamics

- Dynamics governed by the **L**andau **L**ifshitz **G**ilbert (**LLG**) equation (1955)

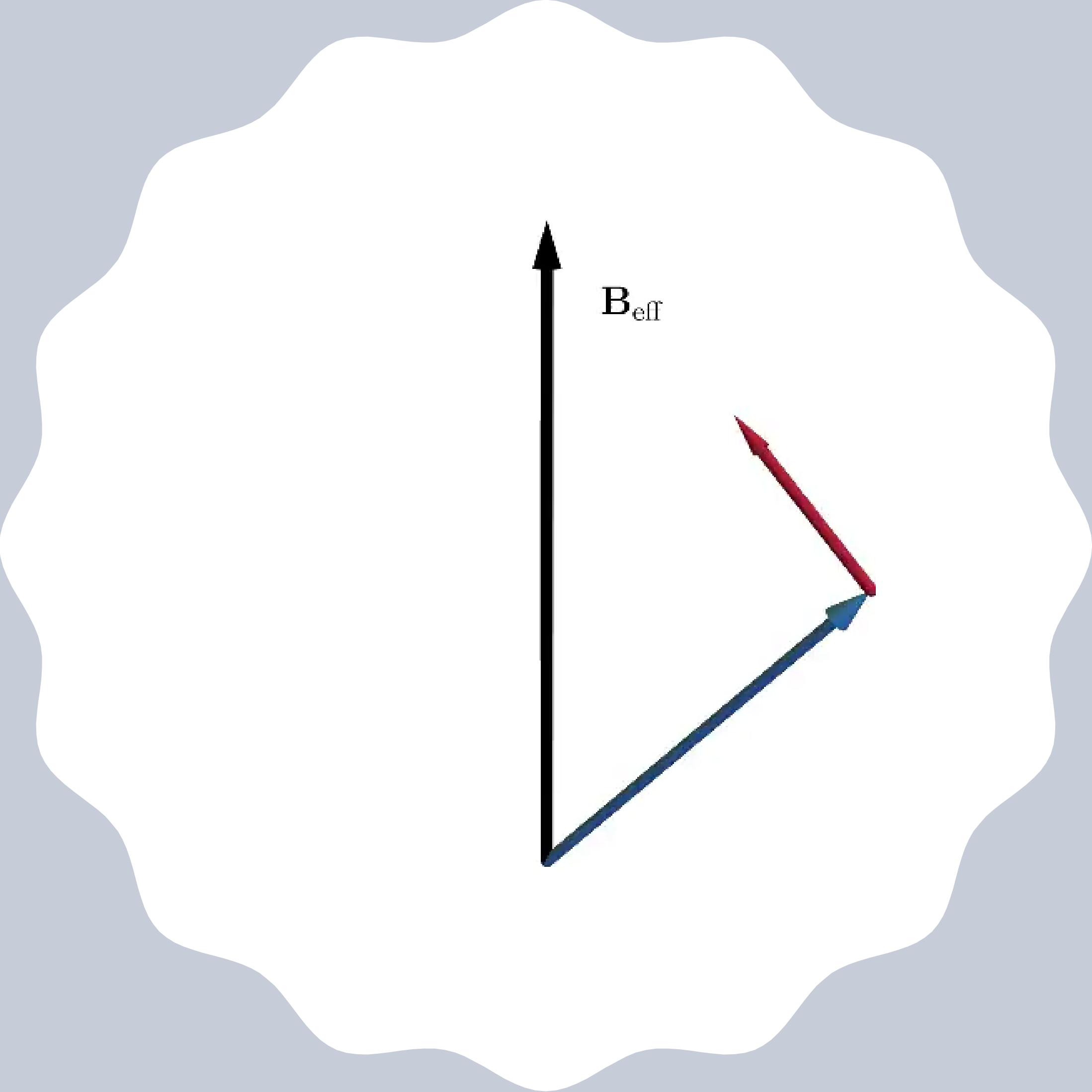
gyromagnetic ratio

damping constant

$$\dot{\boldsymbol{n}} = \gamma \boldsymbol{n} \times \boldsymbol{B}_{\text{eff}} - \frac{\gamma}{|\gamma|} \boldsymbol{\alpha}\, \dot{\boldsymbol{n}} \times \boldsymbol{n}$$

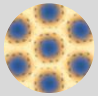
effective magnetic field,

$$= -\frac{1}{M_0} \frac{\delta F[\boldsymbol{n}]}{\delta \boldsymbol{n}}$$



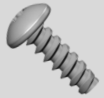
Goldstone Mode Activatio

For more details, see:



Imaging the Ultrafast Coherent Control of a Skyrmion Crystal
P. Tengdin et al. , *PRX 12, 041030 (2022)*

Skyrmion Jellyfish in Driven Chiral Magnets
N. del Ser & V. Lohani, *SciPost Phys. 15,065 (2023)*



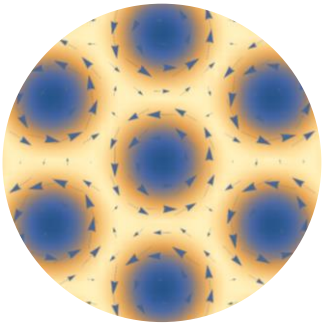
Archimedean Screw in Driven Chiral Magnets
N. del Ser, L. Heinen & A. Rosch, *SciPost Phys. 11, 009 (2021)*

- Magnetic textures have translational ar
- Driving the system out of equilibrium u lasers, activates these

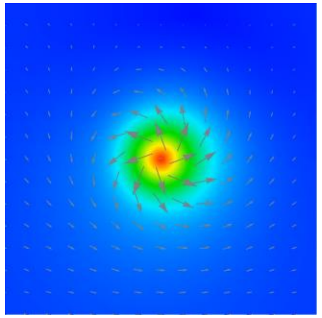
$$\boldsymbol{B}(t) = \boldsymbol{B}\cos\omega t$$

Goldstone mode $\varphi = A\,t, \quad A \propto B^2$

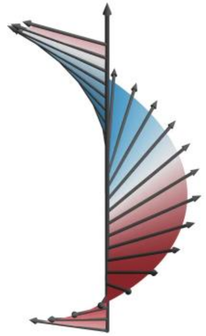
A linear in power injected into the system!



rotational



translational



translational /rotational

In this talk, apply this principle to **driven ferrimagnets**…

1. Identify magnetic system with “active” units

A ferrimagnetic chain

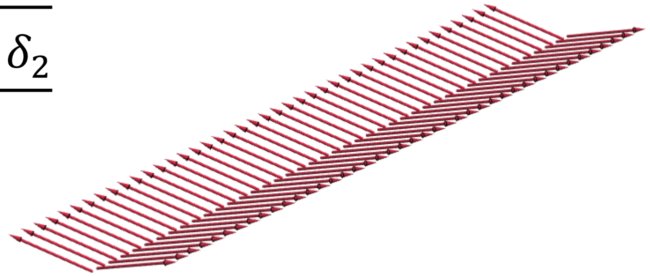
Magnetite – a naturally occurring ferrimagnet

antiferromagnetic　　　　　　anisotropies

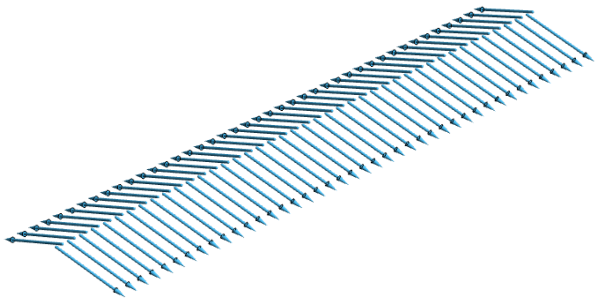
$$H = J \sum_{\langle i,j \rangle} S_i^x S_j^x + S_i^y S_j^y - \Delta S_i^z S_j^z + \sum_i \frac{\delta_2}{2} (S_i^z)^2 + \frac{\delta_4}{2} (S_i^z)^4$$

ferromagnetic

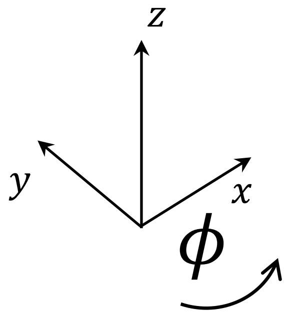
$$S_*^z = \pm \sqrt{\frac{2J(\Delta - 1) - \delta_2}{\delta_4}}$$



$S_*^z > 0$



$S_*^z < 0$



ϕ is a Goldstone mode!

1. Identify magnetic system with “active” units

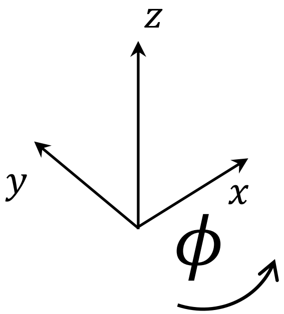
A driven ferrimagnetic chain

$B_z(t) = B_0 \cos(\omega t)$

$$H = J \sum_{\langle i,j \rangle} S_i^x S_j^x + S_i^y S_j^y - \Delta S_i^z S_j^z + \sum_i \frac{\delta_2}{2}(S_i^z)^2 + \frac{\delta_4}{2}(S_i^z)^4 - g_i B_z(t)\, S_i^z$$

$$\phi = \omega_{\mathrm{rot}} t$$

Goldstone mode activated!



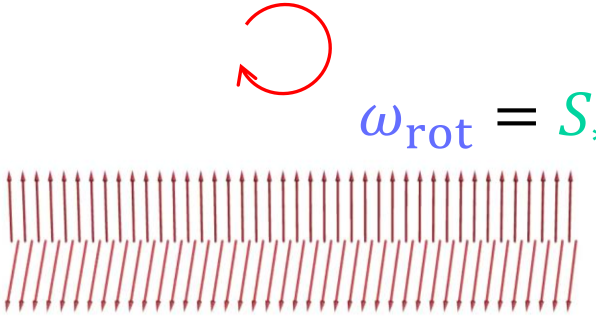
ϕ is a Goldstone mode!

A driven ferrimagnetic chain

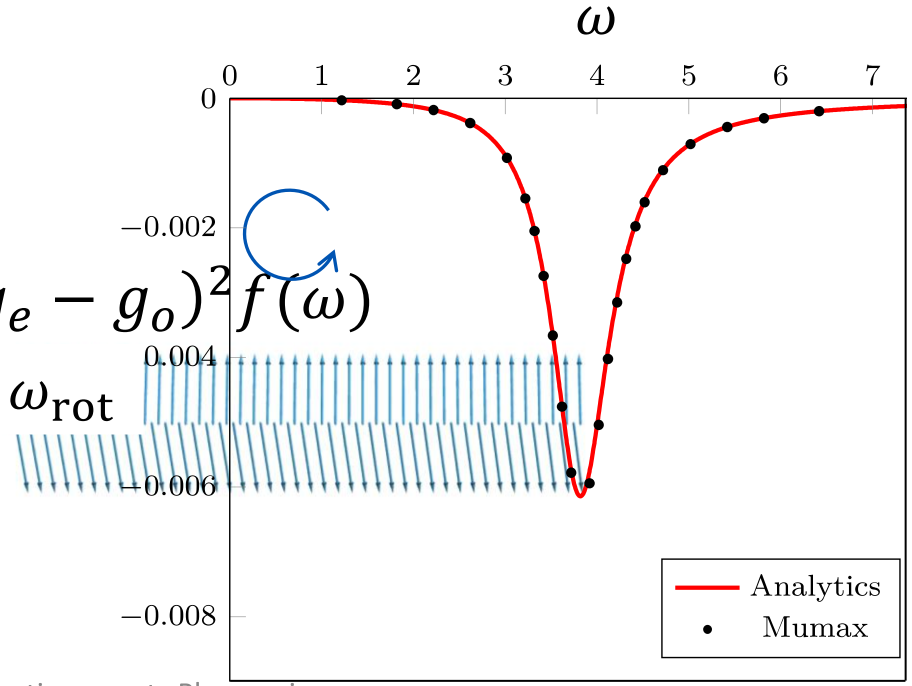
$$B_z(t) = B_0 \cos(\omega t)$$

$$H = J \sum_{\langle i,j \rangle} S_i^x S_j^x + S_i^y S_j^y - \Delta S_i^z S_j^z + \sum_i \frac{\delta_2}{2} (S_i^z)^2 + \frac{\delta_4}{2} (S_i^z)^4 - g_i B_z(t)\, S_i^z$$

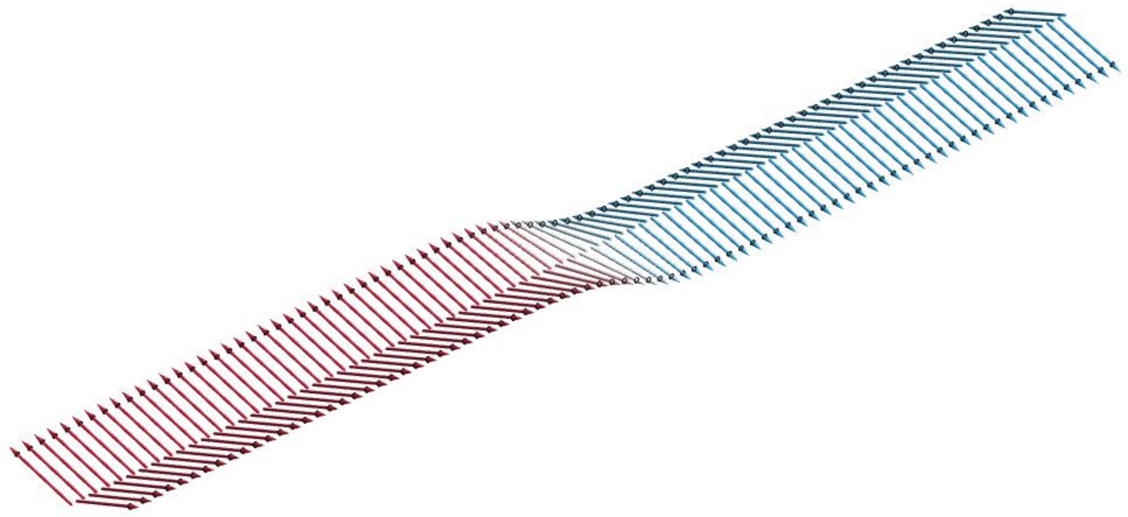
switch to top view



$$\omega_{\mathrm{rot}} = S_*^z\, B_0^2 (g_e - g_o)^2 f(\omega)$$

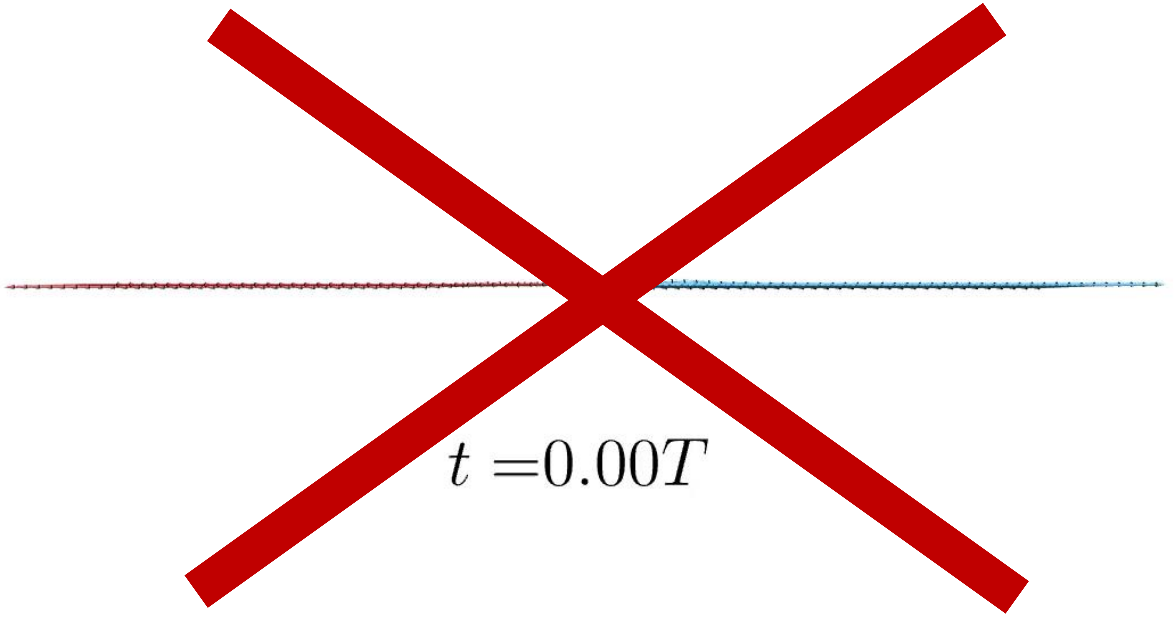


What if we glue them together before driving?



(ferrimagnetic domain wall)

Dynamical Frustration



$$T = \frac{2\pi}{\omega_{\mathrm{rot}}}$$

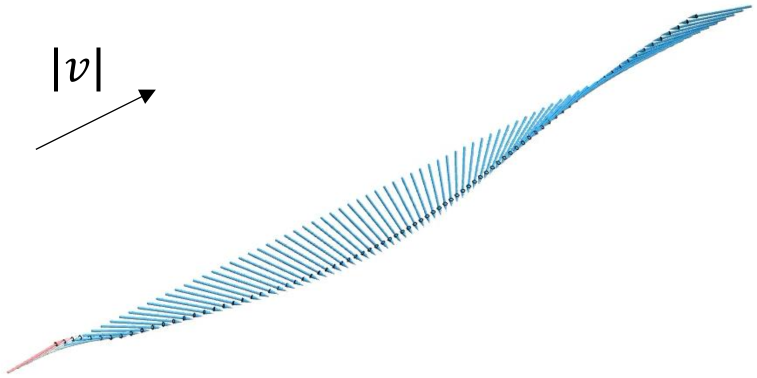
Every $\frac{T}{2}$, antiferromagnetic spins at the centre align!

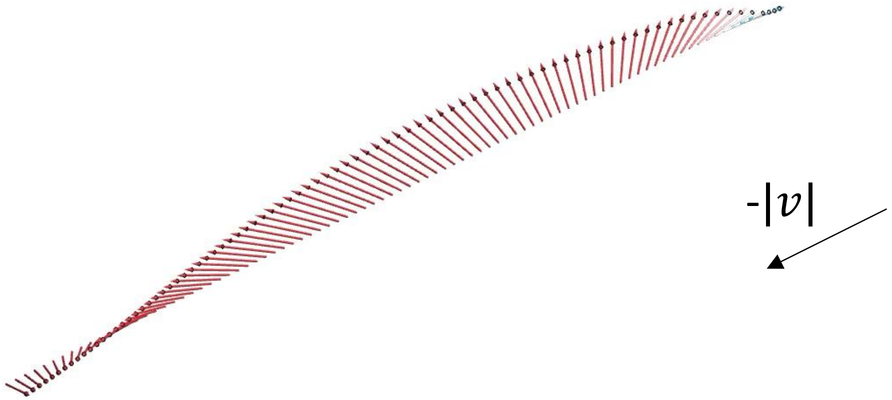
How to resolve this paradox?

1. Identify magnetic system with “active” units

Solution: domain wall starts to move!

(Note: only *even* spins shown)



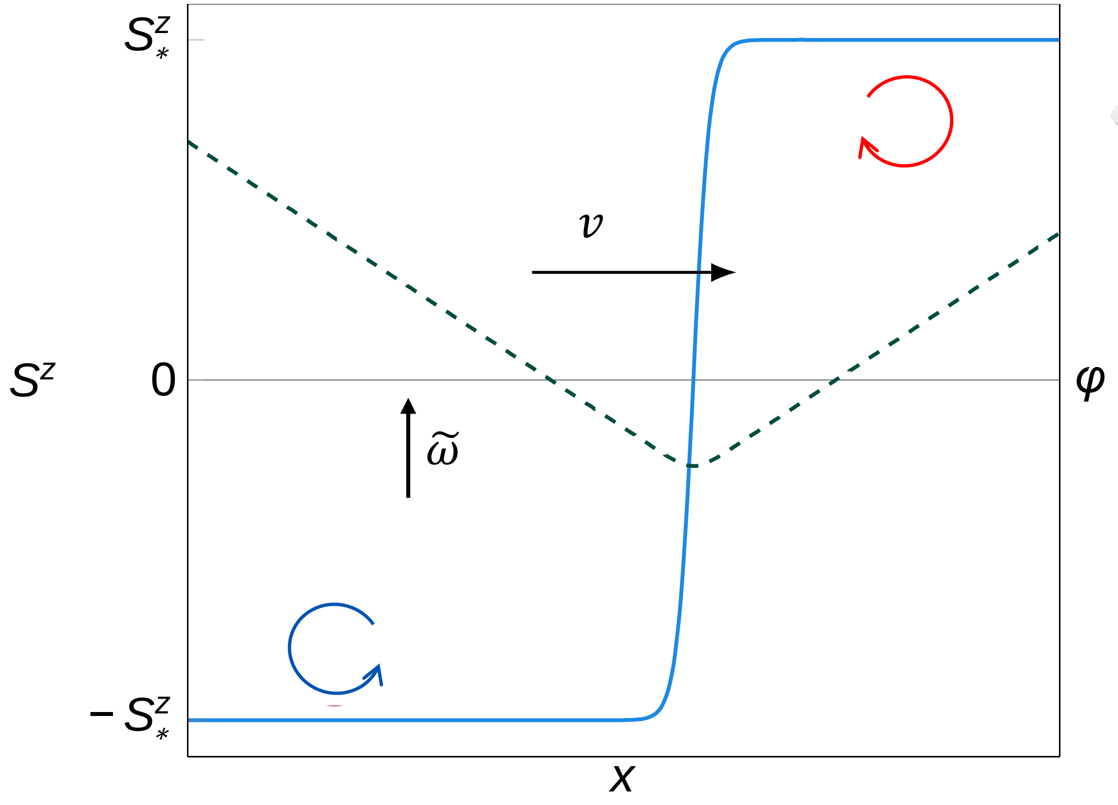


Remarkably, **both** $\pm|v|$ allowed!

→ ‘**active**’ motion

Next task: calculate v

Calculate v



1) Conserved Noether current
(valid if system undamped)

$$\int dx\, \dot{S}^z + \nabla_x j = 0$$

$$\downarrow$$

$$v S^z|_{-\infty}^{+\infty} = j|$$

2) Moving ansatz

$$S^z = S^z(x - vt),$$
$$\Phi = \phi(x - vt) + \tilde{\omega} t$$

ω_{rot} linear in driving power
$\rightarrow$ **ultrafast** motion!

$$\rightarrow \quad v = \pm\sqrt{\frac{\rho_s \omega_{\mathrm{rot}}}{S^z_*}}$$

What about damping?..

Calculate v for finite α

Need to solve some horrible coupled non-linear equations..

$$
\begin{aligned}
-\mathrm{sign}(\gamma)\dot{\phi}_{e/o}\sin(\theta_{e/o}) - \alpha\dot{\theta}_{e/o} = &-\sin(\theta_{e/o})\left(\delta_2\cos(\theta_{e/o}) + \delta_4\cos^3(\theta_{e/o})\right) \\
&+ 2J\left(\Delta\sin(\theta_{e/o})\cos(\theta_{o/e}) - \cos(\theta_{e/o})\sin(\theta_{o/e})\cos(\phi_{e/o}-\phi_{o/e})\right) \\
&- a^2 J\Bigg(\\
&\theta''_{o/e}(\Delta\sin(\theta_{e/o})\sin(\theta_{o/e}) + \cos(\theta_{e/o})\cos(\theta_{o/e})\cos(\phi_{e/o}-\phi_{o/e})) \\
&+ (\theta'_{o/e})^2(\Delta\sin(\theta_{e/o})\cos(\theta_{o/e}) - \cos(\theta_{e/o})\sin(\theta_{o/e})\cos(\phi_{e/o}-\phi_{o/e})) \\
&+ 2\theta'_{o/e}\phi'_{o/e}\cos(\theta_{e/o})\cos(\theta_{o/e})\sin(\phi_{e/o}-\phi_{o/e}) \\
&+ \cos(\theta_{e/o})\sin(\theta_{o/e})\phi''_{o/e}(x)\sin(\phi_{e/o}-\phi_{o/e}) \\
&- \cos(\theta_{e/o})\sin(\theta_{o/e})(\phi'_{o/e})^2\cos(\phi_{e/o}-\phi_{o/e})\Bigg) + \epsilon g_{e/o}B_1(t)\sin(\theta_{e/o}), \\
\mathrm{sign}(\gamma)\sin(\theta_{e/o})\dot{\theta}_{e/o} - \alpha\dot{\phi}_{e/o}\sin^2(\theta_{e/o}) = &\; 2J\sin(\theta_e)\sin(\theta_o)\sin(\phi_{e/o}-\phi_{o/e}) \\
&- Ja^2\sin(\theta_{e/o})\Bigg(\sin(\phi_{e/o}-\phi_{o/e})\left(\sin(\theta_{o/e})\phi'^2_{o/e} - \theta''_{o/e}\cos(\theta_{o/e})\right) \\
&+ 2\theta'_{o/e}\cos(\theta_{o/e})\phi'_{o/e}\cos(\phi_{e/o}-\phi_{o/e}) + \theta'^2_{o/e}\sin(\theta_{o/e})\sin(\phi_{e/o}-\phi_{o/e}) \\
&+ \sin(\theta_{o/e})\phi''_{o/e}\cos(\phi_{e/o}-\phi_{o/e})\Bigg).
\end{aligned}
$$

Calculate v for finite α

Amazingly, an approximate solution exists (details in paper):

$$\longrightarrow \qquad v = \pm\left(\sqrt{\left(\frac{\alpha\rho_s}{\xi_0\eta}\right)^2 + \left(\frac{\rho_s\omega_{\mathrm{rot}}}{S_*^z}\right)} - \frac{\alpha\rho_s}{\xi_0\eta}\right)$$

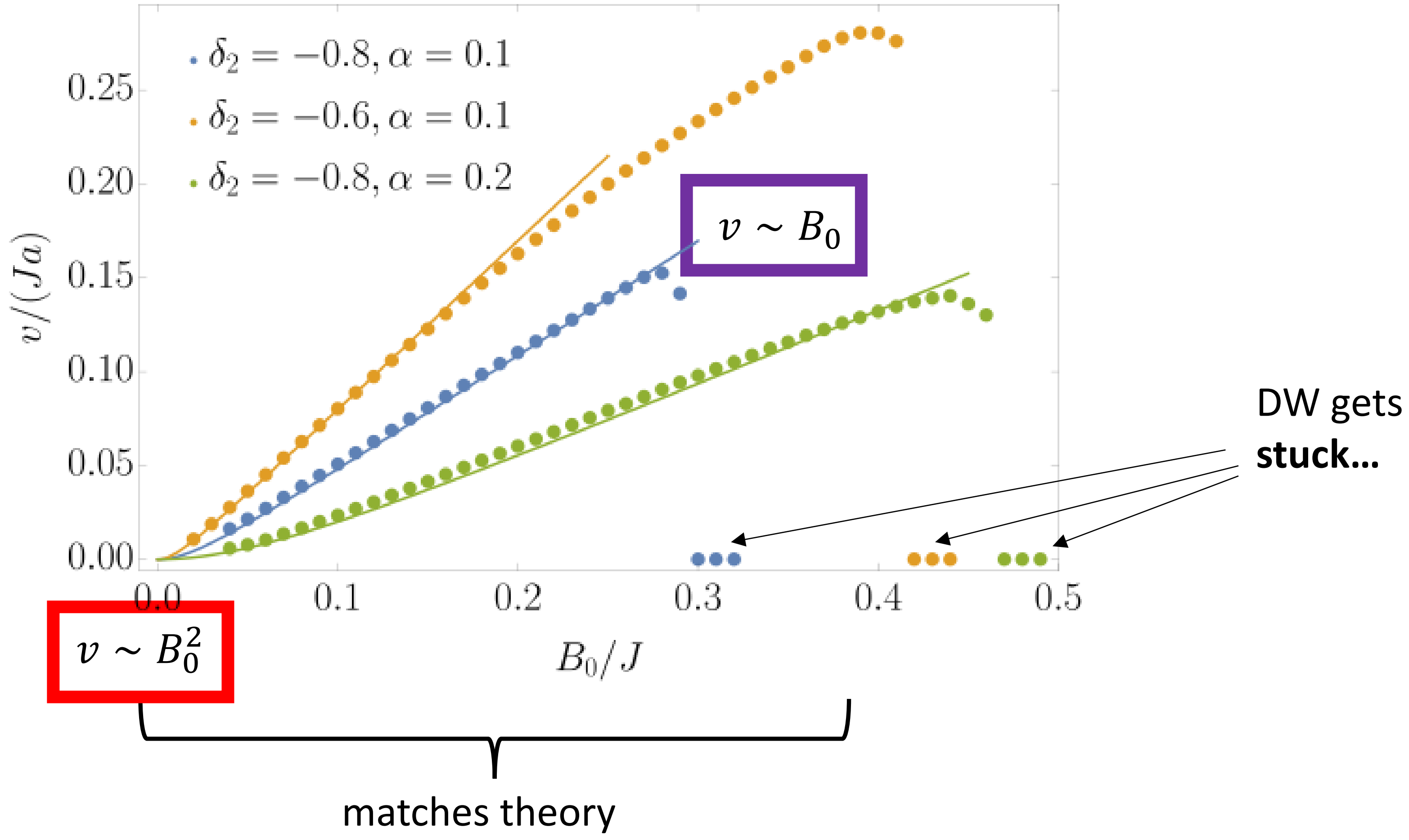
ρ_s is the spin stiffness,
ξ_0 the domain wall width,
η a prefactor known analytically

$$\boxed{v \sim \sqrt{\omega_{\mathrm{rot}}} \sim B_0}$$

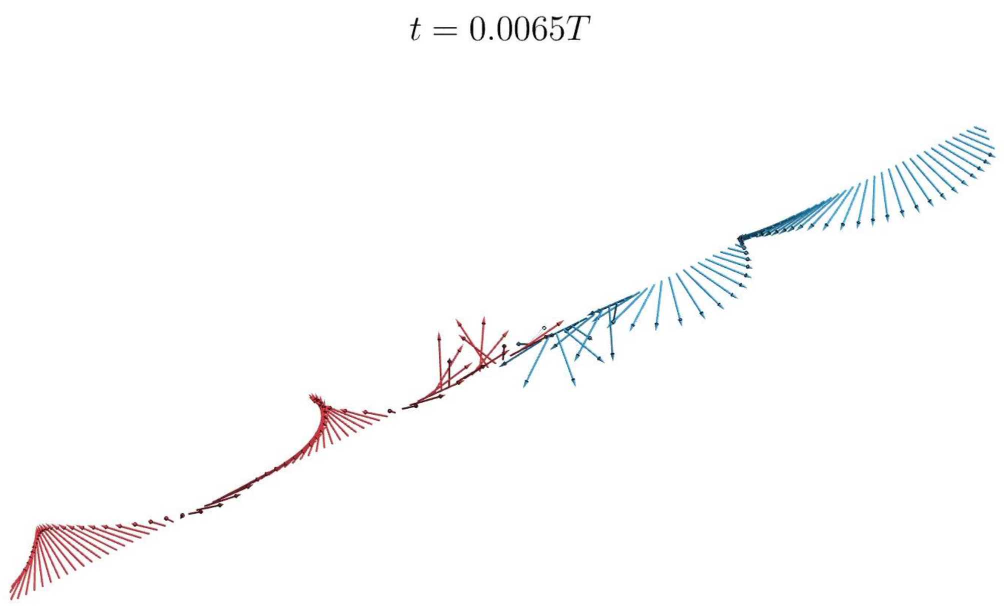
$$\boxed{v \sim \frac{\omega_{\mathrm{rot}}}{\alpha} \sim \frac{B_0^2}{\alpha}}$$

Qualitatively new regime!

v as a function of the driving strength B_0



Increase B_0?..

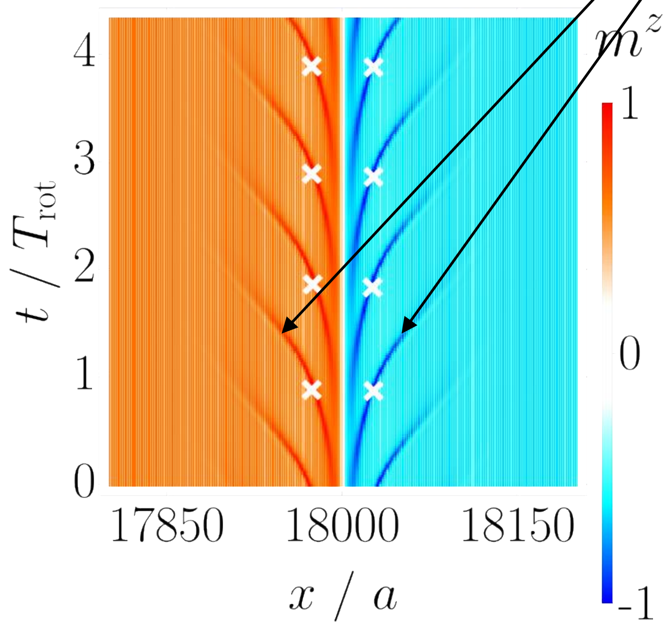


domain wall gets **stuck**!

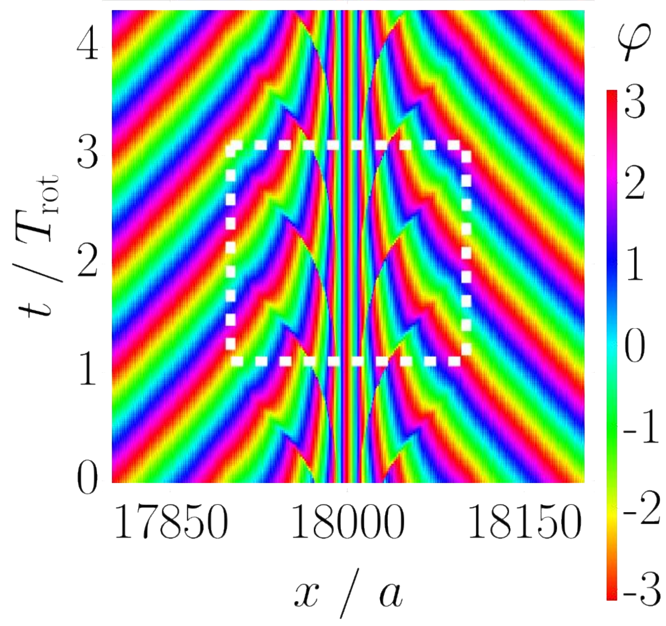
Stuck Solution

At higher B_0, domain wall gets **stuck**

Tension build up from spins far from the DW rotating released every T_{rot} as two sharp pulses!

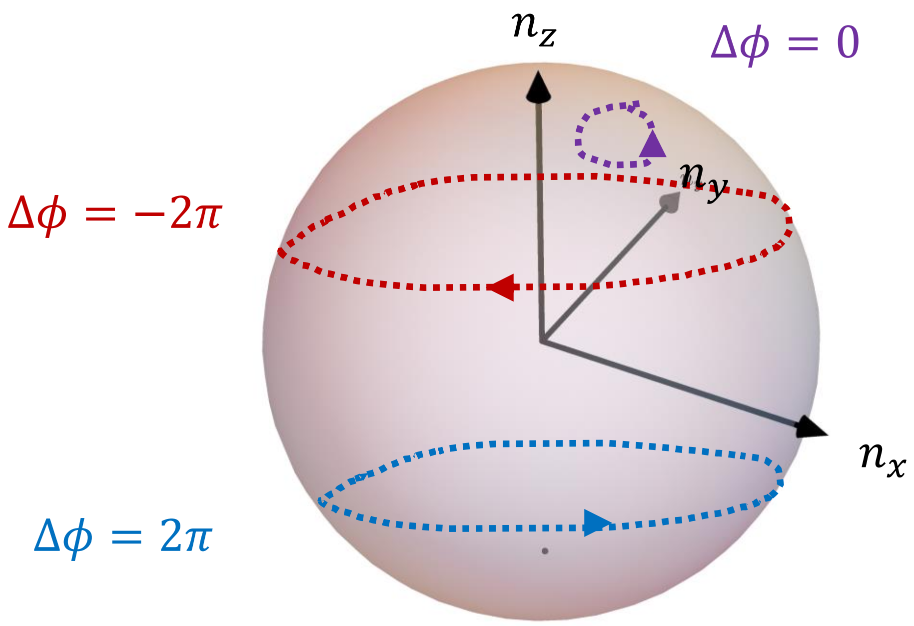


$S^z(x,t)$



$\phi(x,t)$

Topological Considerations



Define winding number

space time indices

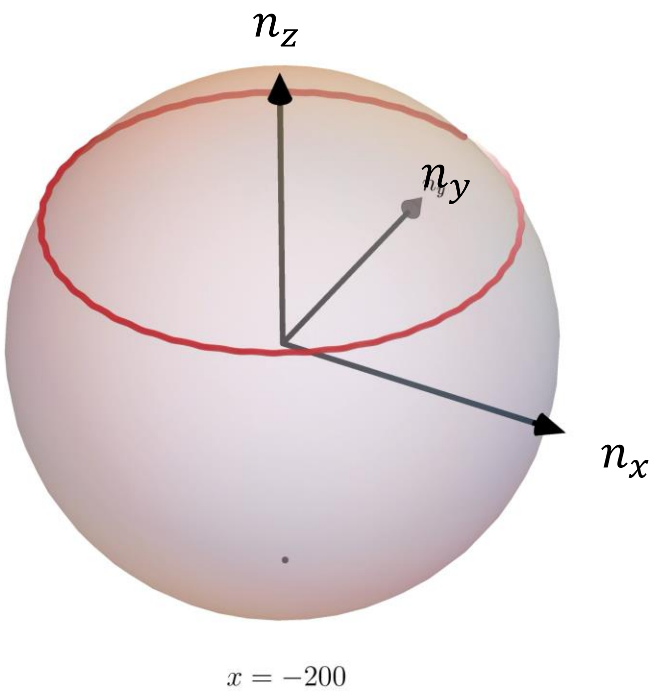
$$W = \frac{1}{2\pi}\int \mathrm{d}\boldsymbol{q} \cdot \nabla_q \phi \qquad \mathrm{d}\boldsymbol{q} = (\mathrm{d}t, \mathrm{d}x), \quad \nabla_q = (\partial_t, \partial_x)$$

Winding number only **changes** when n_z-axis is **crossed**!

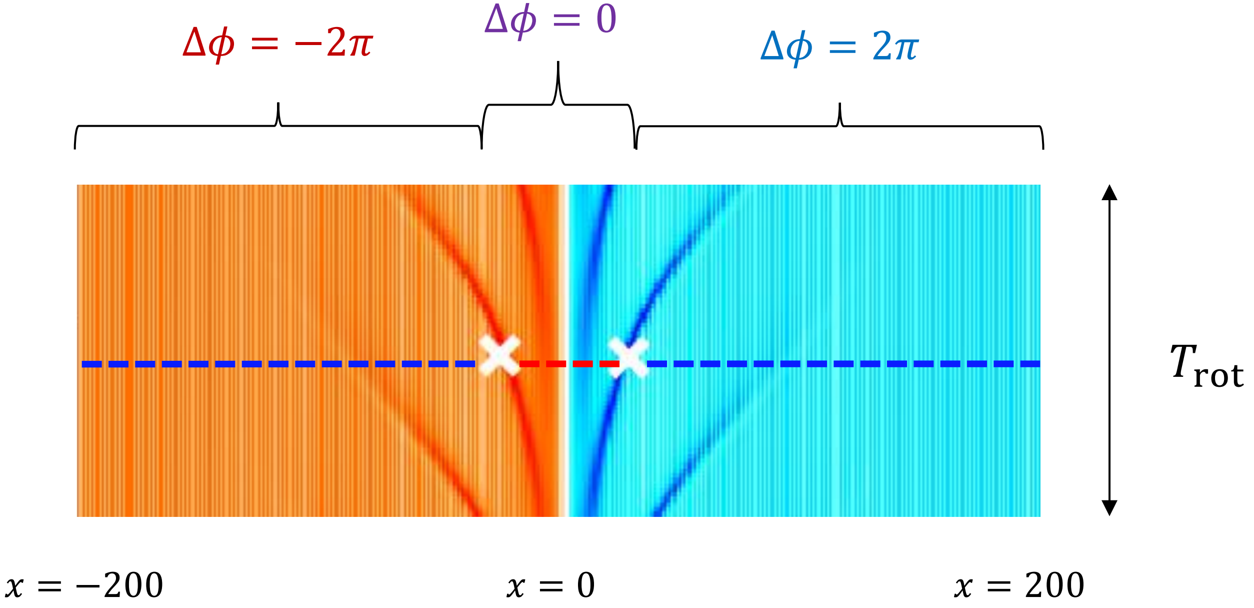
Stuck Solution: Topology

Plot of $\boldsymbol{n}(x,t) = \frac{\boldsymbol{S}(x,t)}{S_0}$

over 1 T_{rot}

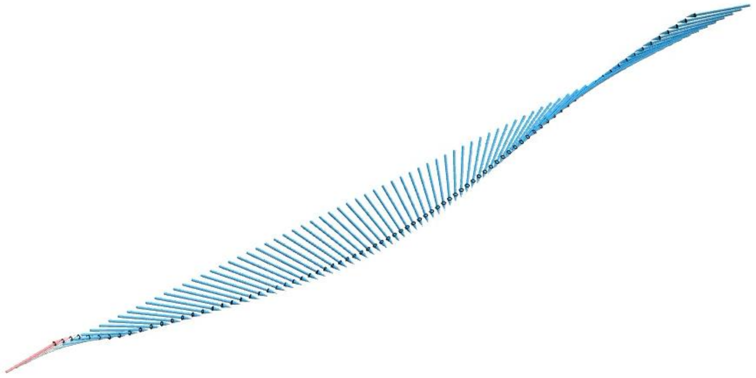


Scan as a function of x



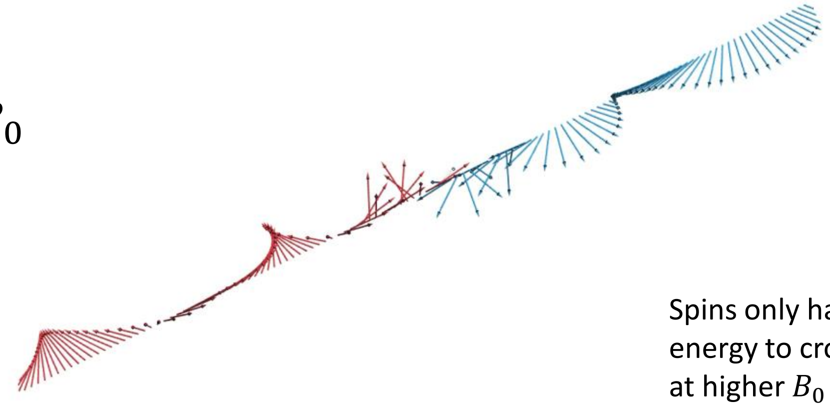
Topology of Moving vs. Stuck

moving



Increase B_0

stuck



Spins only have enough energy to cross the z-axis at higher B_0!

- Spins don’t have enough energy to cross z-axis
- W **is conserved everywhere** – finite v needed

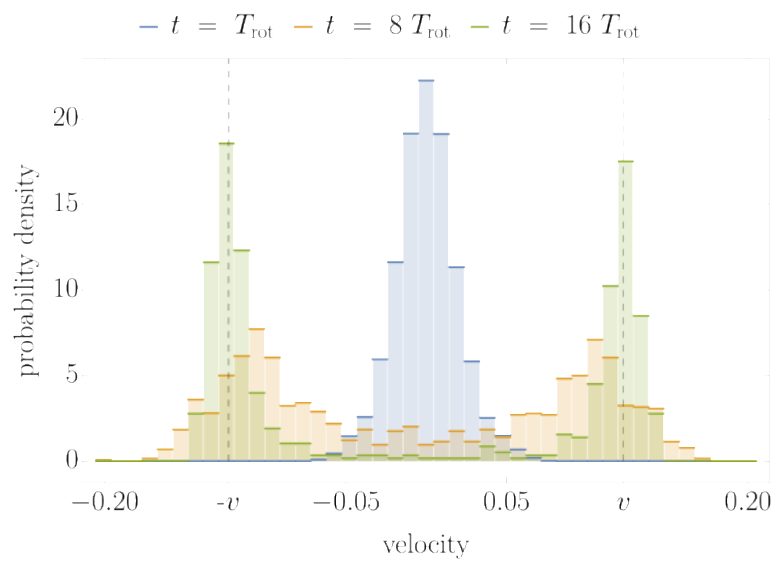
- Spins have enough energy to cross z-axis
- W is **approximately** conserved
- At space-time vortex locations, W jumps by quantised multiples of 2π
- No need for finite v

Many Domain Walls

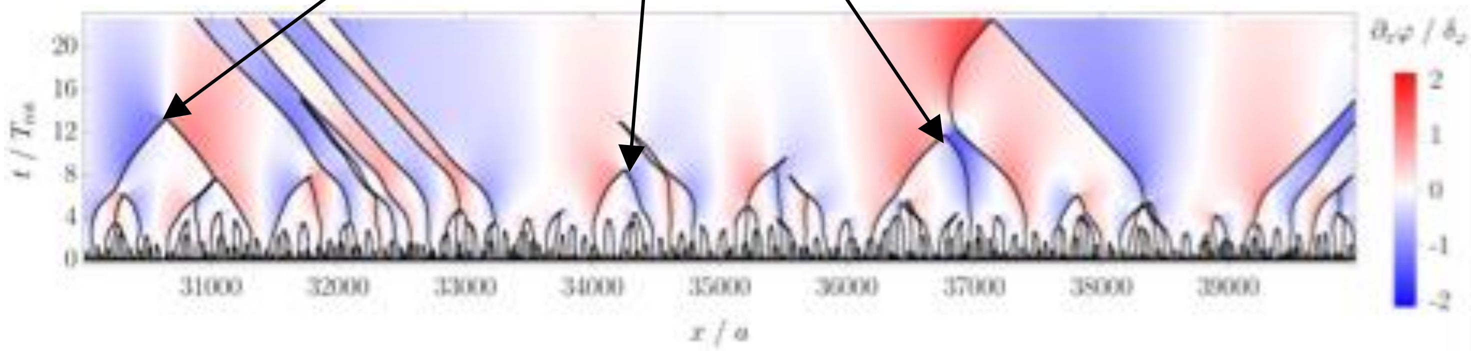
- Investigate numerically what happens in a driven system containing many domain walls
- Compare to equilibrium case (no driving) at finite temperature

Many Driven Domain Walls

- Start with antiferromagnetic initial state with a small random S^z component
- Create domain walls by quenching to ferrimagnet
- Domain walls move and annihilate when they meet

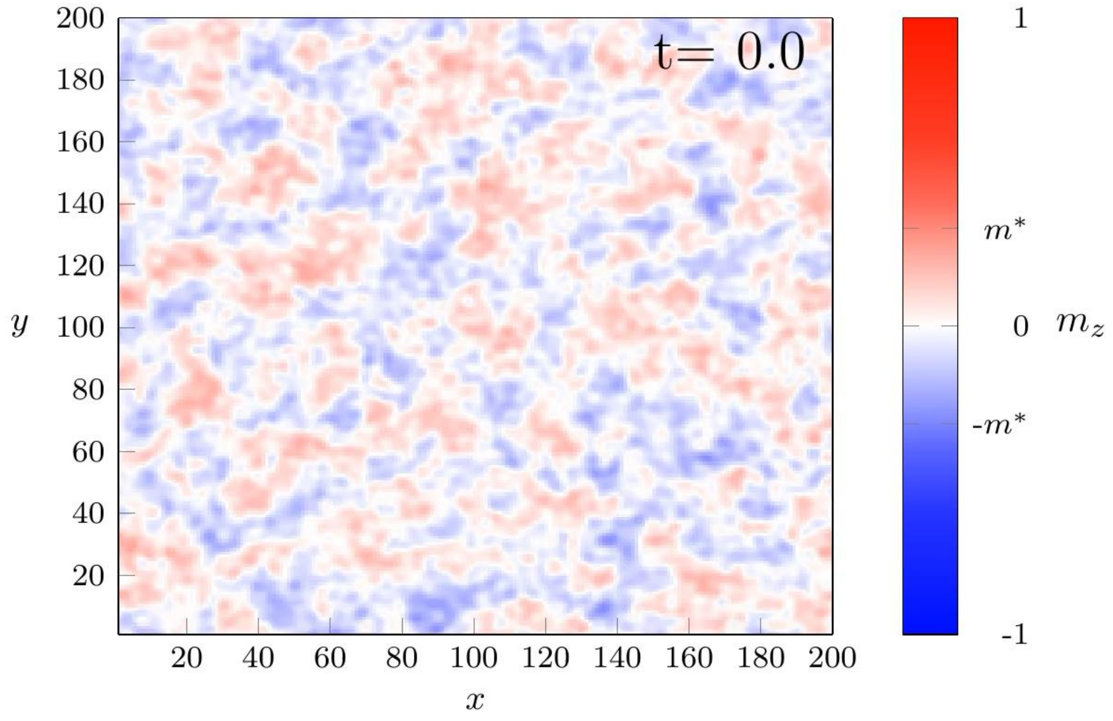


Velocity distribution of DWs

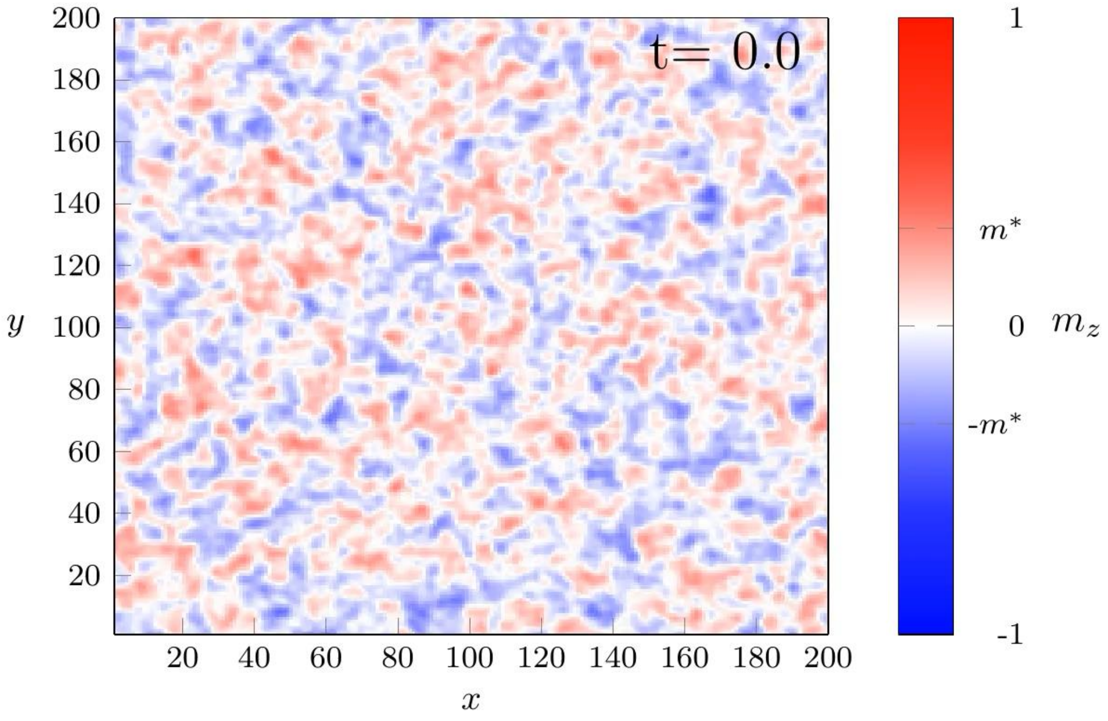


DW world lines and phase gradient $\phi(x)$

2D Domain Wall Party



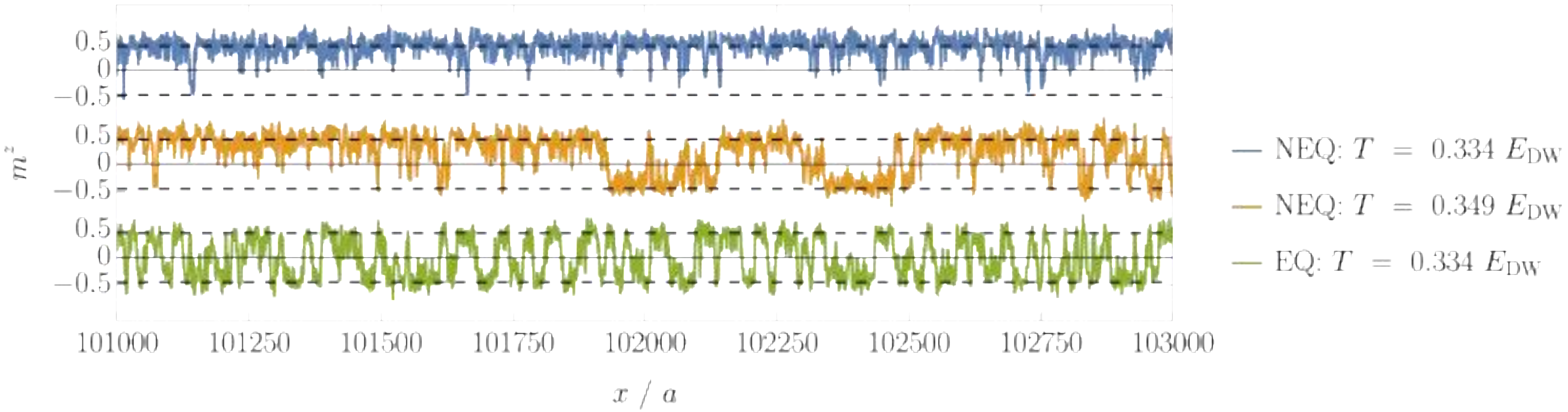
noisy, no drive



noisy, driven

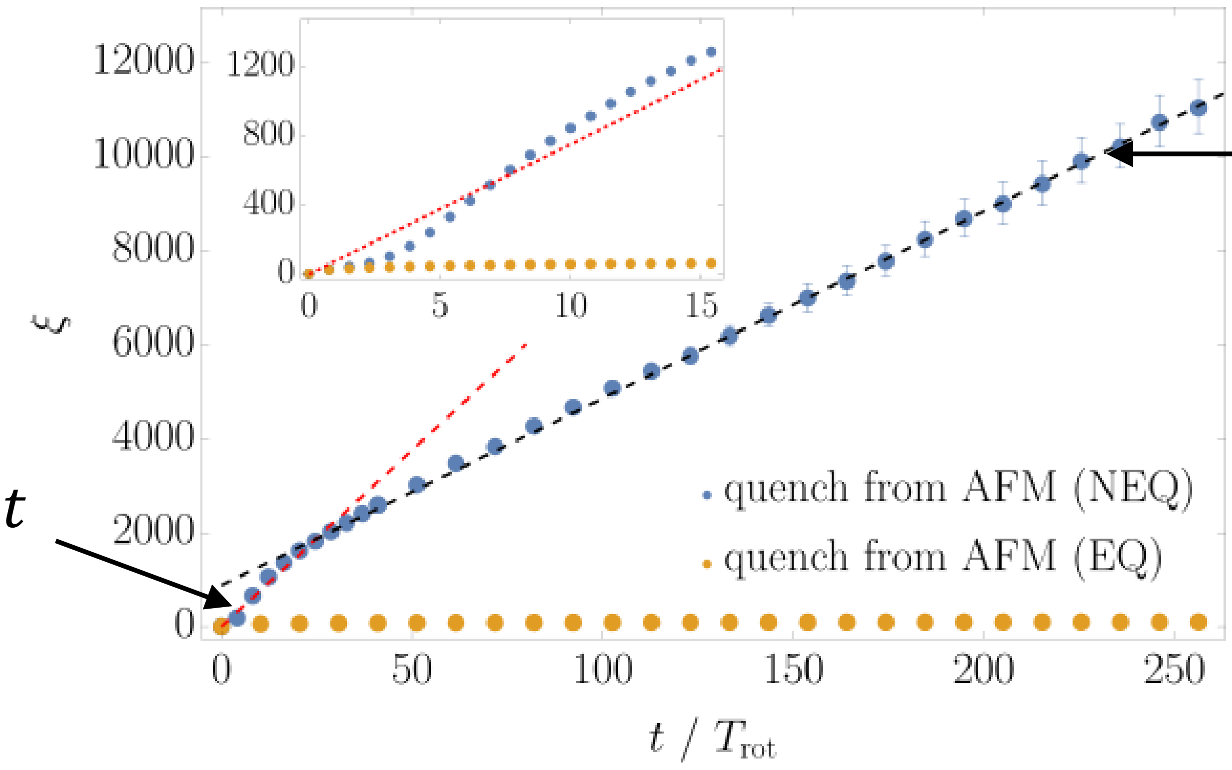
Correlation Length Driven vs Non-Driven

- Number of DWs n_{DW} decreases over time → Correlation length $\xi = 1/n_{\mathrm{DW}}$ increases
- ξ in driven system is about 10^5 vs 20 sites in the non-driven system at similar T!



Time Dependence of Correlation Length I

- Domain walls move and annihilate when they meet
- $\xi = 1/n_{\mathrm{DW}}$ **grows very fast** compared to equilibrium system



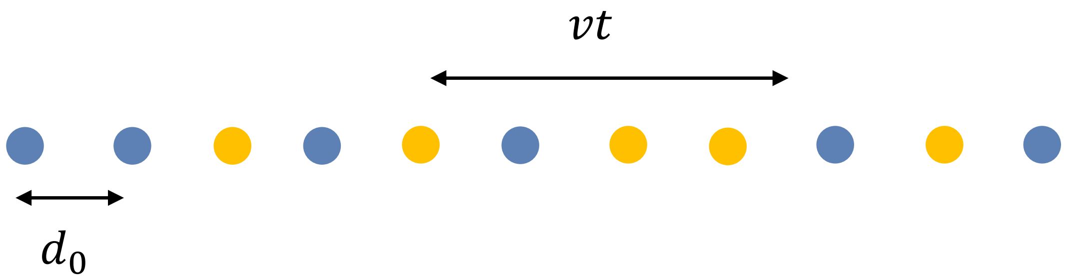
$\xi \sim v\, t$

$\xi \sim \mu\, v\, t \quad \mu < 1$

Not what is expected for independently moving DWs!

Time Dependence of Correlation Length II

Independently right- and left-moving domain walls:



Linear growth of $\xi(t) = \mu\, vt$ in driven system suggests the DWs are **interacting**!

$N_{R/L} \sim \frac{vt}{d_0}$ right- and left-moving DWs can collide after time t in an area vt

→

$N_{R/L}$ is normally distributed with width $\sqrt{N_{R/L}}$

→

$\sqrt{N_{R/L}}$ DWs left after time t in area vt.

→

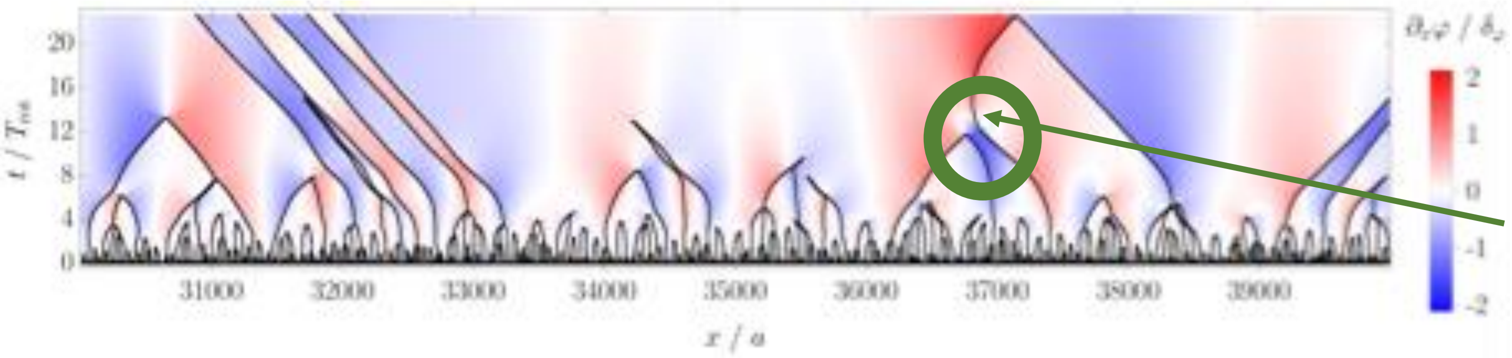
$$\boxed{\xi(t) \sim \frac{\sqrt{N_{R/L}}}{d_0} \sim \sqrt{t}}$$

Healing mechanism

- Remember the (approximately) conserved winding number W?
  → Define top. charge $\rho_{\text{top}} = \partial_x \phi$.
- Associated top. current is $j_{\text{top}} \approx \frac{\omega_{\text{rot}}}{2\pi}$ far from domain wall
  → Build up of ρ_{top} at centre of DW and when 2 DWs meet and annihilate.
- ρ_{top} decays very slowly by diffusion. Enough time to repel incoming DW.

$$\dot{S}^z + \nabla_x j = 0$$

Spin current $j \sim \partial_x \phi$ too!



Incoming 3rd DW repelled by build up of ρ_{top}

Summary

1. Driving a ferrimagnet with $\boldsymbol{B}(t)$ makes it "active"
2. Active units are domain walls moving at $\pm v$ (direction chosen by spontaneous symmetry breaking)
3. At higher B, domain wall gets stuck, emergence of space-time vortices.
4. $W = \frac{1}{2\pi} \int d\boldsymbol{q} \cdot \nabla_q \phi$ (approximately) conserved
5. Dynamics of many domain walls **very different** in driven vs. equilibrium cases

Thanks for listening!

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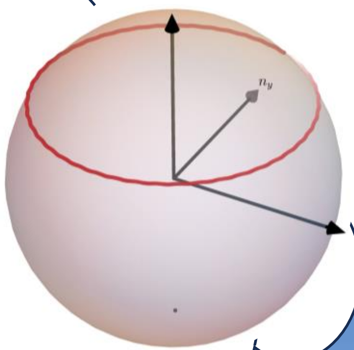

Questions?