Skyrmionic Hopfions:
3D particle-like twisted topologies in light

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LEVERHULME TRUST

UKRI Engineering and Physical Sciences Research Council

UNIVERSITY OF BIRMINGHAM

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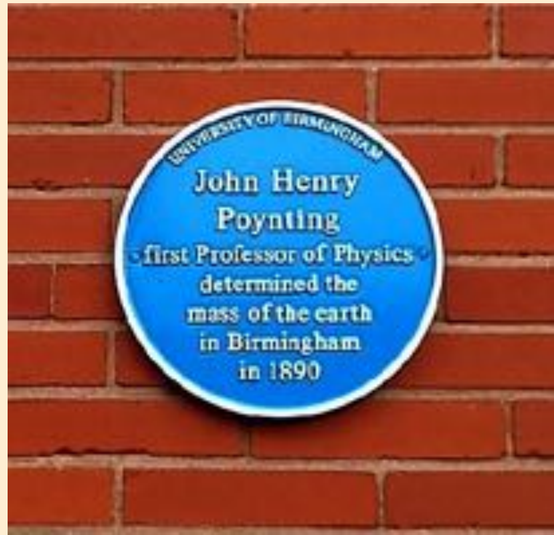

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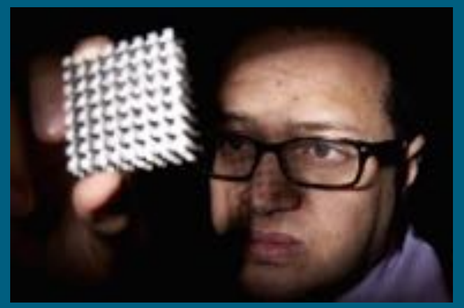
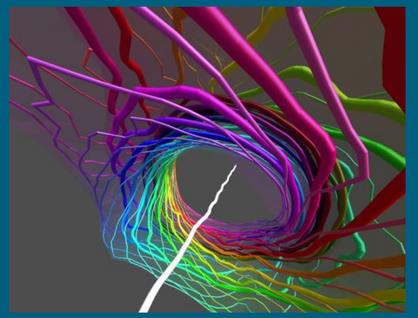
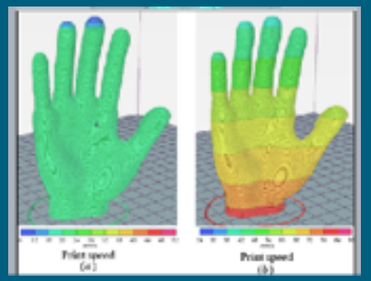






- EM, optics & photonics
- quantum tech, quantum matter
- soft matter, chemistry & life sci
- mathematics, computation & data
- mechanical & vibration engineering
- fabrication & manufacturing

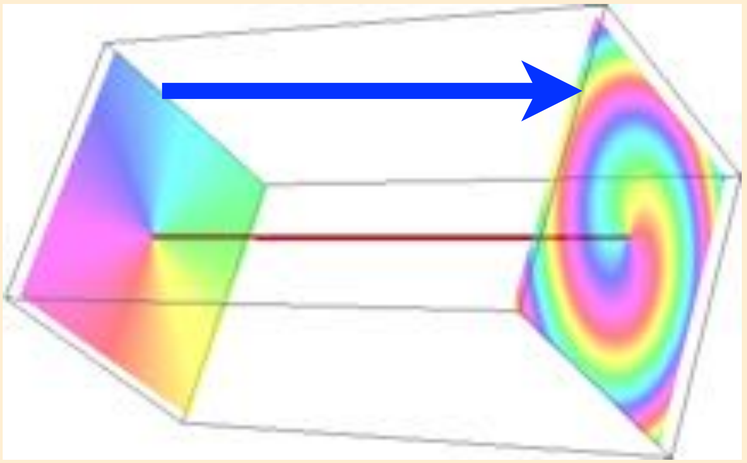
final places for Sept 2024 entry still available!

https://www.birmingham.ac.uk/university/colleges/eps/study/phd/cdt/topological-design-cdt/index.aspx

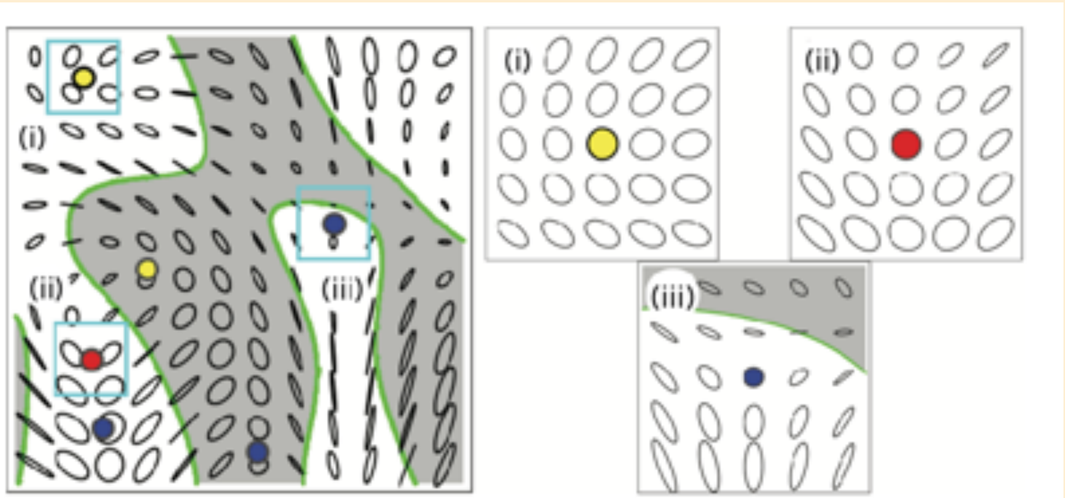
main idea — free space optical topology

create, by ***linear superposition*** (usually of monochromatic modes), "***images***" of nontrivial ***topological configurations*** in light

- "***defects***": special points, lines and surfaces (e.g. scalar vortices, polarisation singularities including C points/lines)



- "***textures***": space/volume filling patterns (e.g. polarisation <~> nematic liquid crsytal texture, 3D Hopf texture <~this talk)



outline

- **context of topologies in free space optical beam**
- structure of the 3D skyrmionic hopfion beam
- some (more) skyrmions background
- 3D optical hypersphere: topology of polarization-phase space
- perspective and discussion

optical “order parameters”

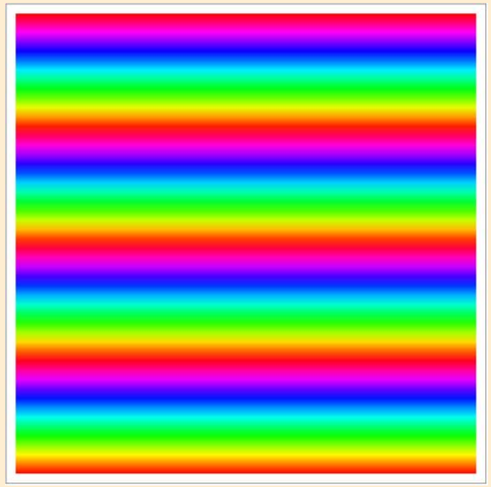
- directions of instantaneous EM field

- monochromatic light, propagating in z-direction,
  take time-independent complex spatial dependence

$$\boldsymbol{E} = \mathrm{Re}\{\mathrm{e}^{\mathrm{i}\omega t}\,\psi(\boldsymbol{r})\,\boldsymbol{e}\}$$

complex scalar amplitude

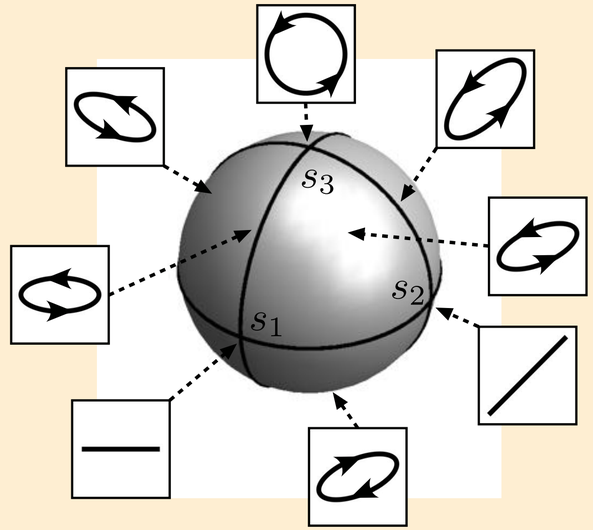
transverse polarization vector (2D complex)



e.g. plane wave

Poincaré sphere (pseudospin)

Stokes parameters ~ cart coords



scalar vortices as wave dislocations

John Nye

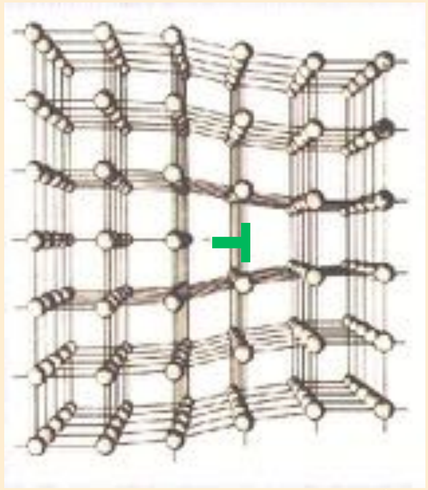
Proc. R. Soc. Lond. A. **336**, 165–190 (1974)
Printed in Great Britain

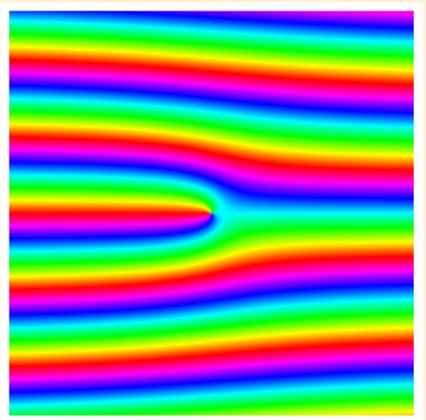
Dislocations in wave trains

BY J. F. NYE AND M. V. BERRY
H. H. Wills Physics Laboratory, University of Bristol

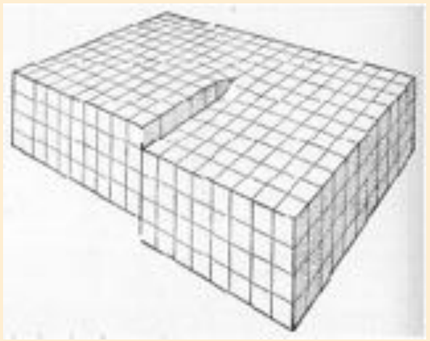
(*Communicated by F. C. Frank, F.R.S. – Received 17 January* 1973)

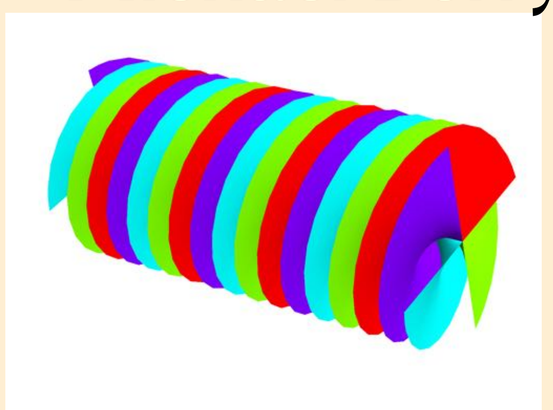
Michael Berry





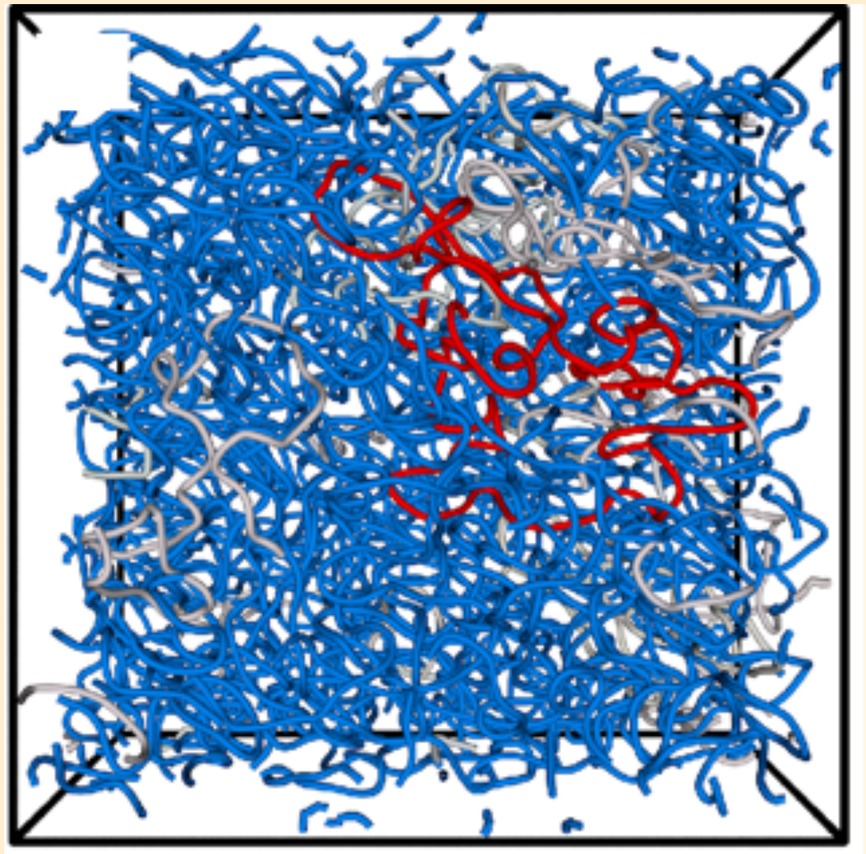
wave edge dislocation:
propagation ⊥ to defect line





wave screw dislocation:
helicoidal wavefronts

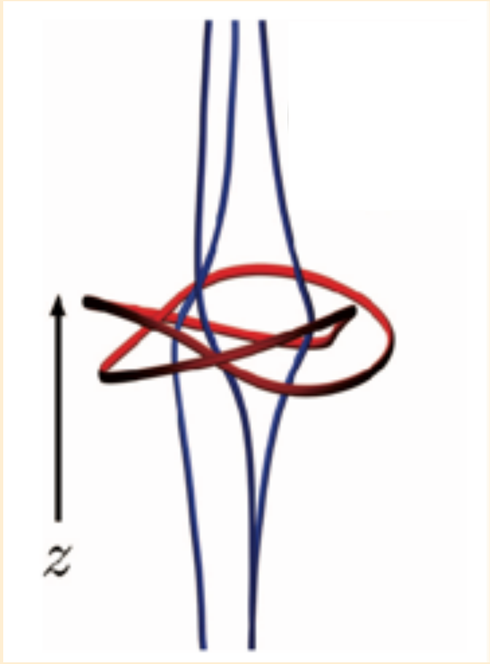
examples of 3D defect configurations



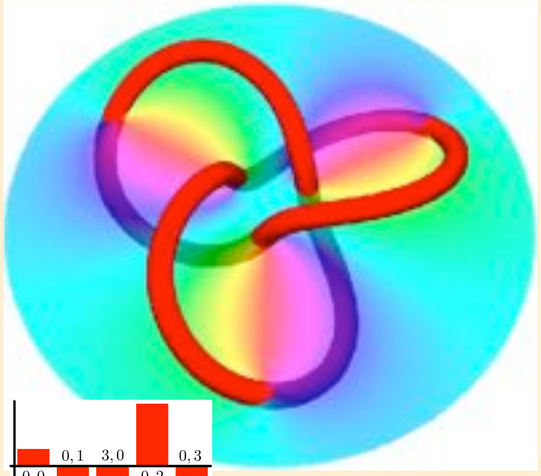
$\leftrightarrow \lambda$

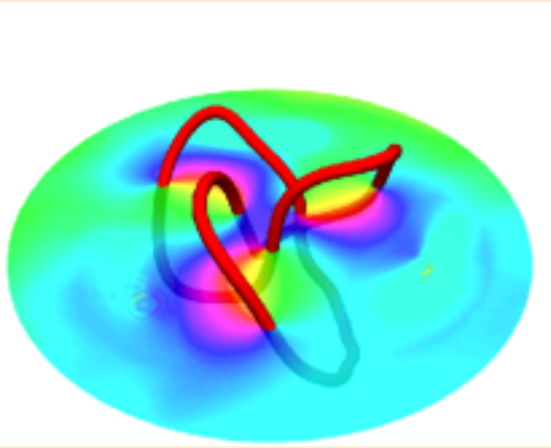
vortex tangles in random wave superposition

(Taylor & MRD 2016)

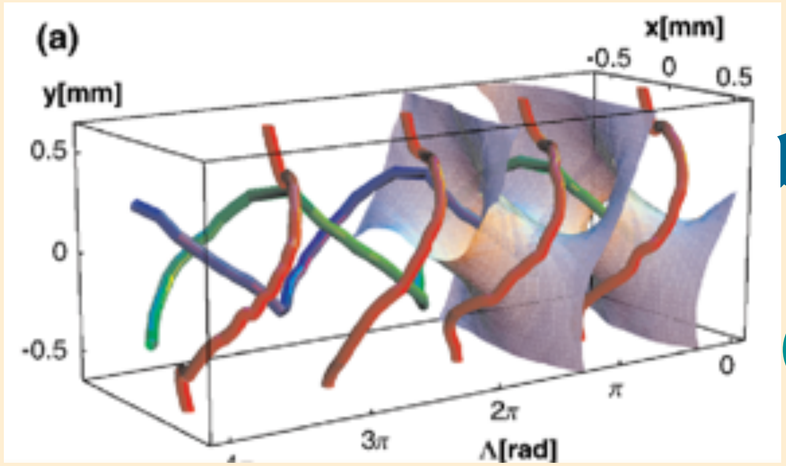


threaded torus knots in Bessel beams (Berry & MRD 2001)



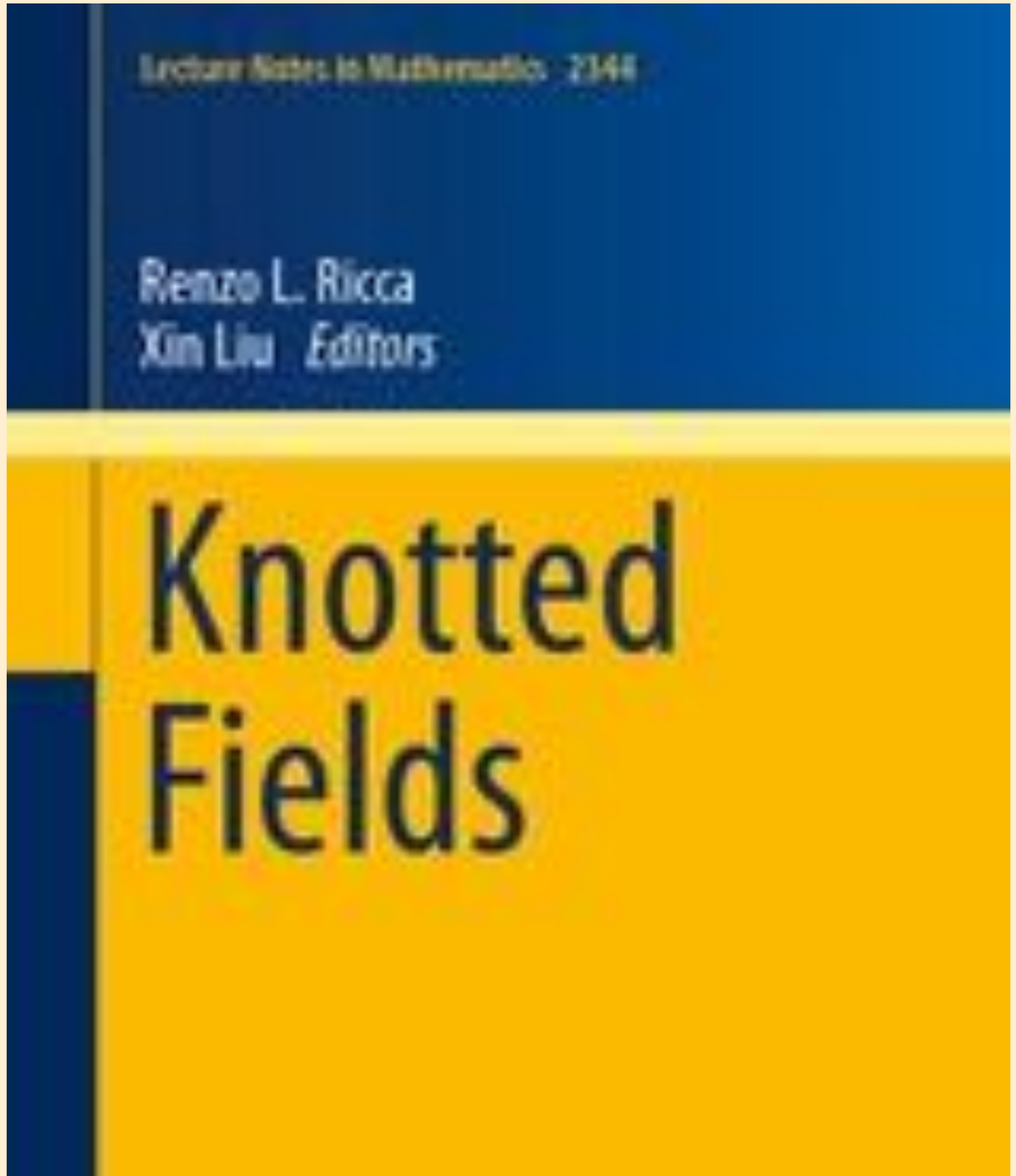


isolated vortex knots (MRD & *al*, 2010)



pol sing helices in birefringent crystal (Flossmann & *al*, 2005)

new edited book on knotted fields



- published summer 2024 (***online now***)
- chapters on knot theory, fibrations, topological quantum field theory, excitable media, magnetic hopfions, classical & quantum fluids, optics

Chapter 6
Designing Knotted Fields in Light and Electromagnetism

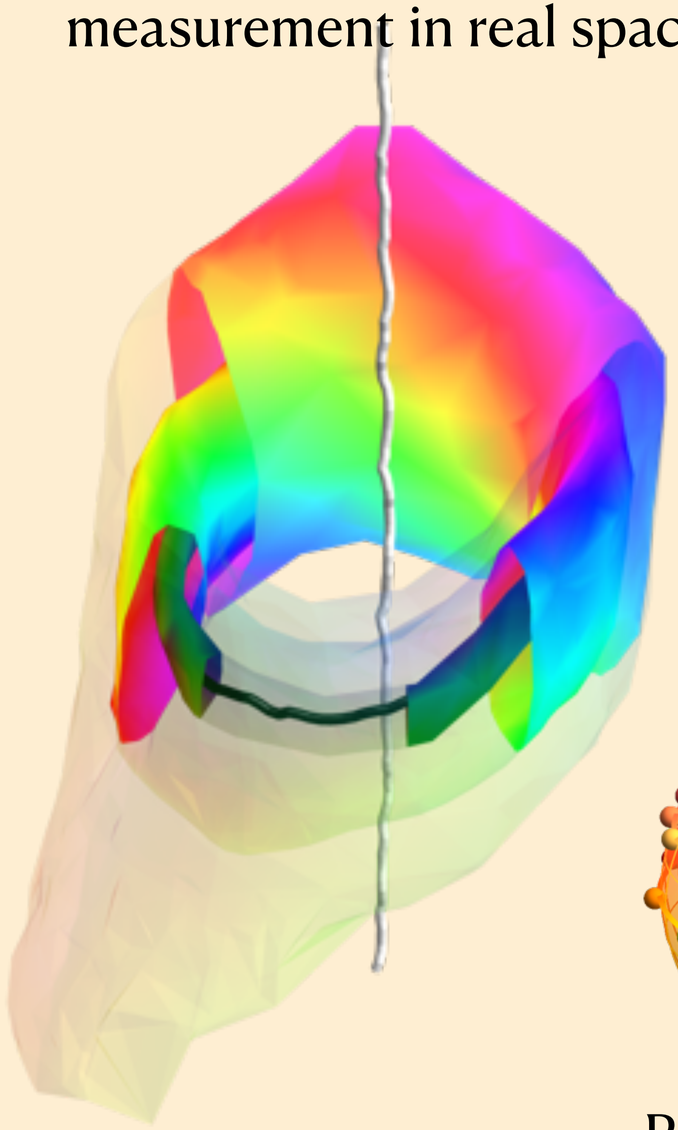
Check for updates

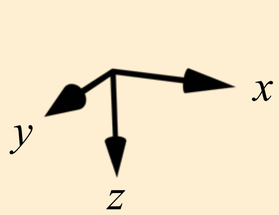
Mark R. Dennis

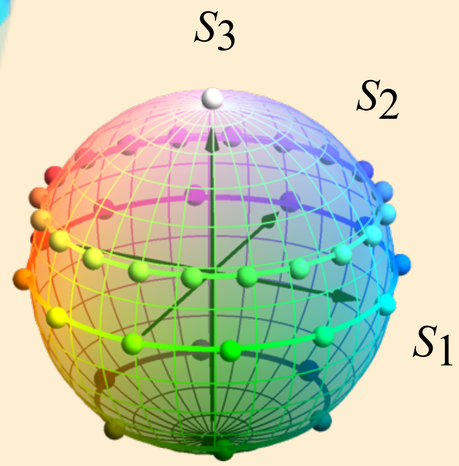
Abstract These lectures survey the subject of knotted fields applied to linear waves and optical fields such as laser light. Starting with a description of the topology

main idea — optical “skyrmionic hopfion”

measurement in real space







Poincaré sphere

we structured the transverse polarisation and phase of a paraxial beam in free space into a “3D topological particle” – we call a ***skyrmionic hopfion***

what does this mean?

relation to 2D skyrmions?

why particle-like?

topological significance?

main refs: Nat Comm **12** 6785 (2021)
Comm Physics **5** 54 (2022)

Danica Sugic
(Birmingham & RIKEN)

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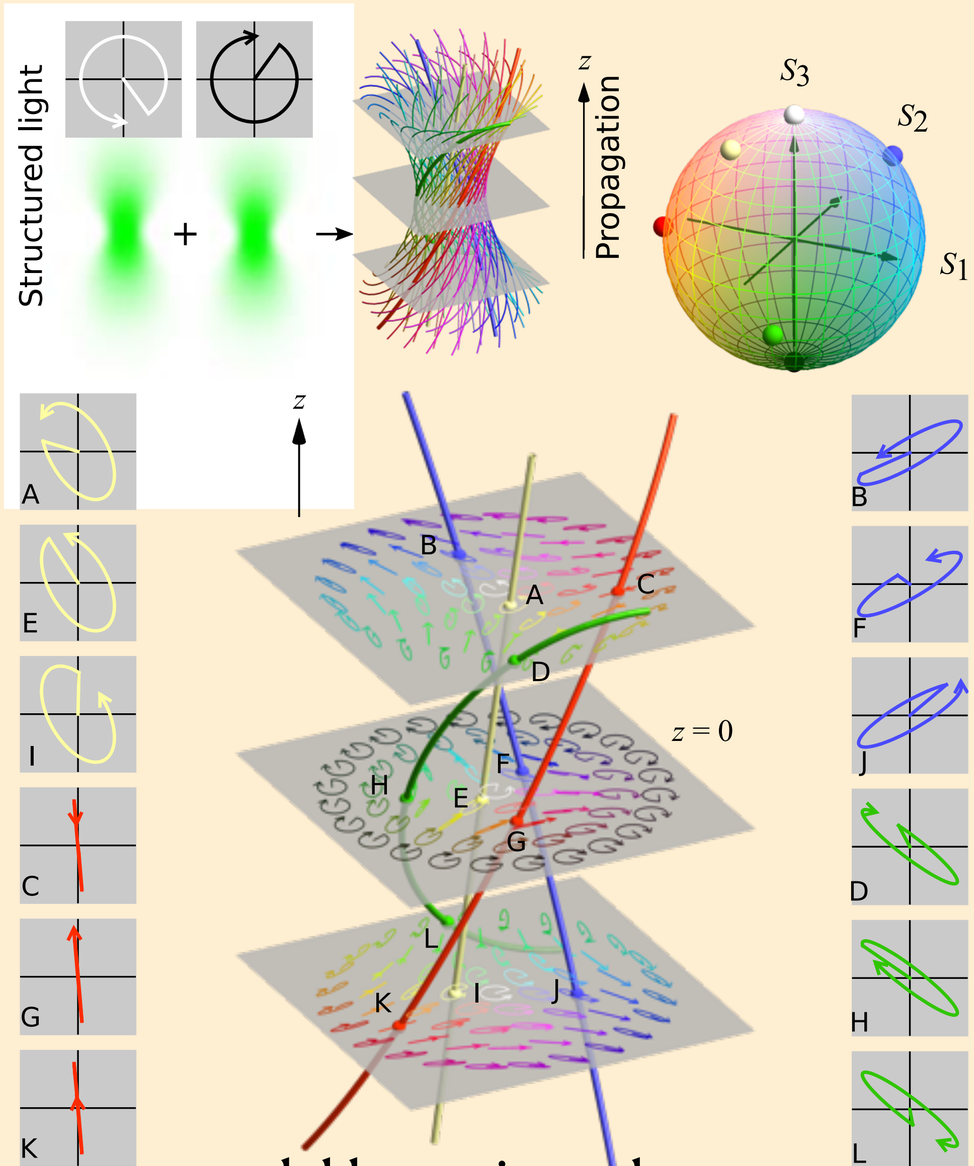
outline

- context of topologies in free space optical beam
- **structure of the 3D skyrmionic hopfion beam**
- some (more) skyrmions background
- 3D optical hypersphere: topology of polarization-phase space
- perspective and discussion

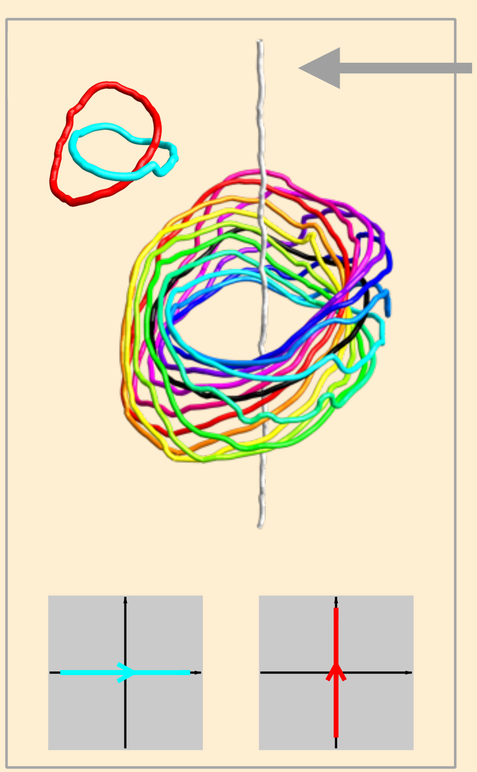
structured polarisation texture



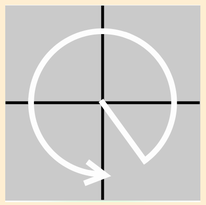
model beam in real space

- structured paraxial beam, free 3D propagation
- different state of elliptic pol at each point in each transverse plane
- same pol states along ***filaments*** in 3D: ensemble of filaments defines a 3D topological ***texture***
- optical phase varies along filaments

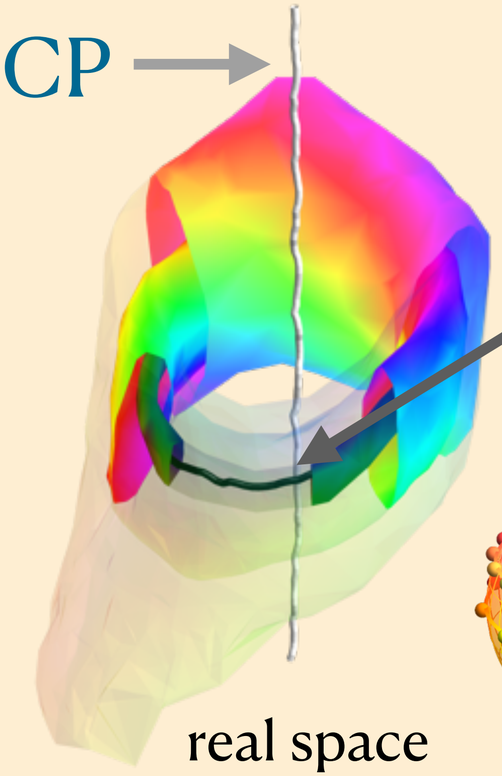
polarisation structure of skyrmionic hopfion



RHCP

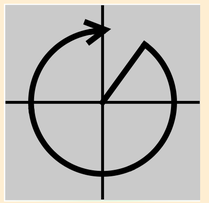


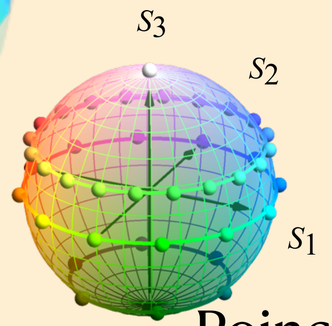
lin pols
on torus



real space
measurement

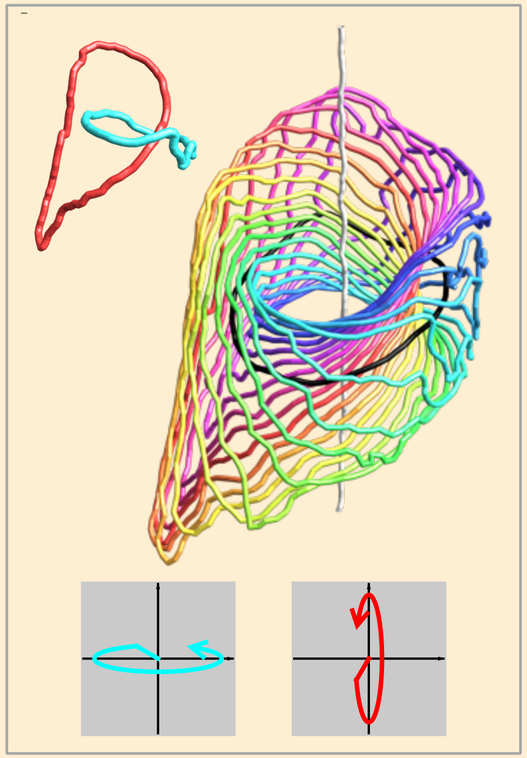
LHCP





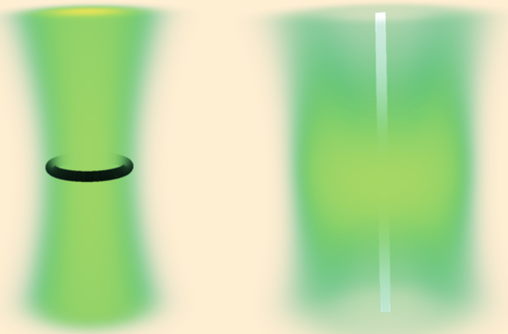
Poincaré
sphere

pols,
fixed S_3
on torus

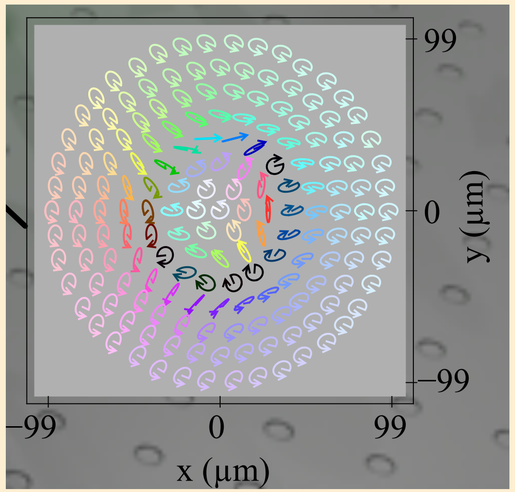


polarisation filaments are ***loops***, lying on ***tori*** (fixed ellipticity, varying azimuth), between the filaments of RH and LH circ pol

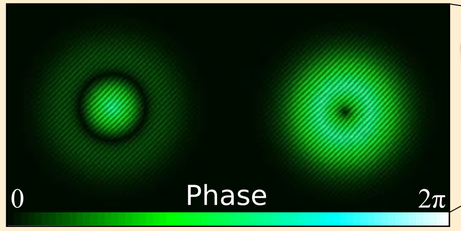
experimental generation and measurement



RH and LH fields

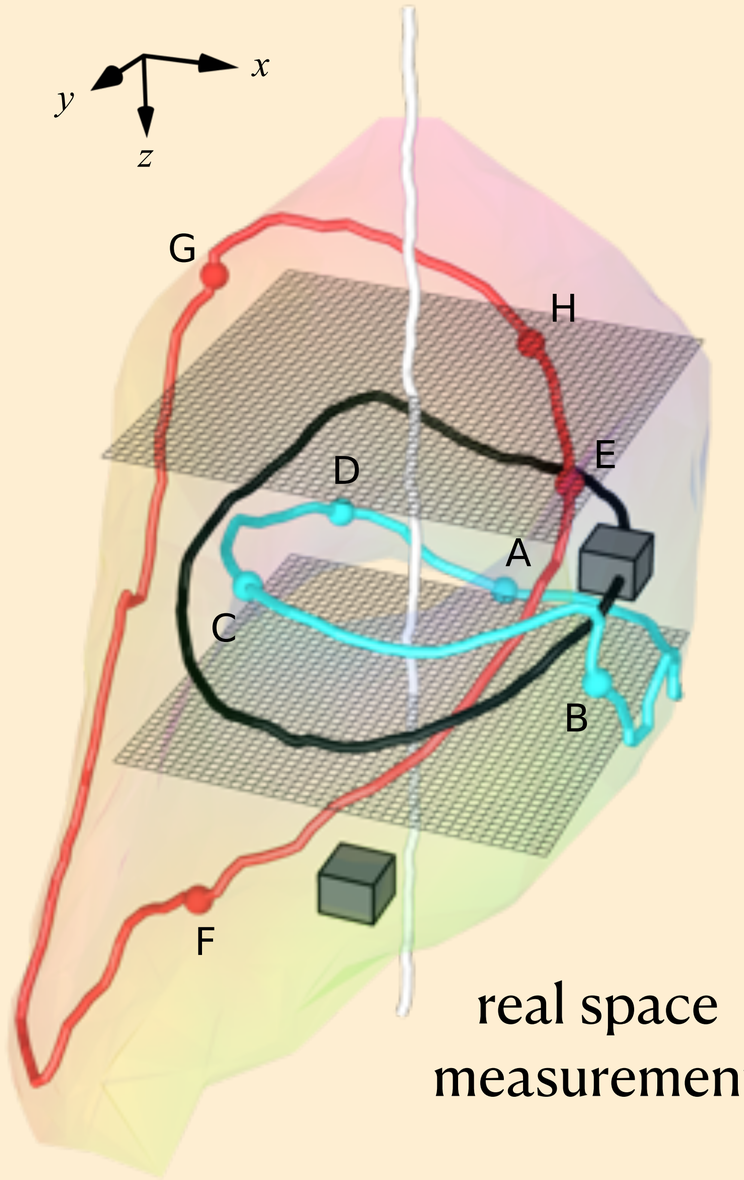


waist plane
(includes filament
of LHCP)



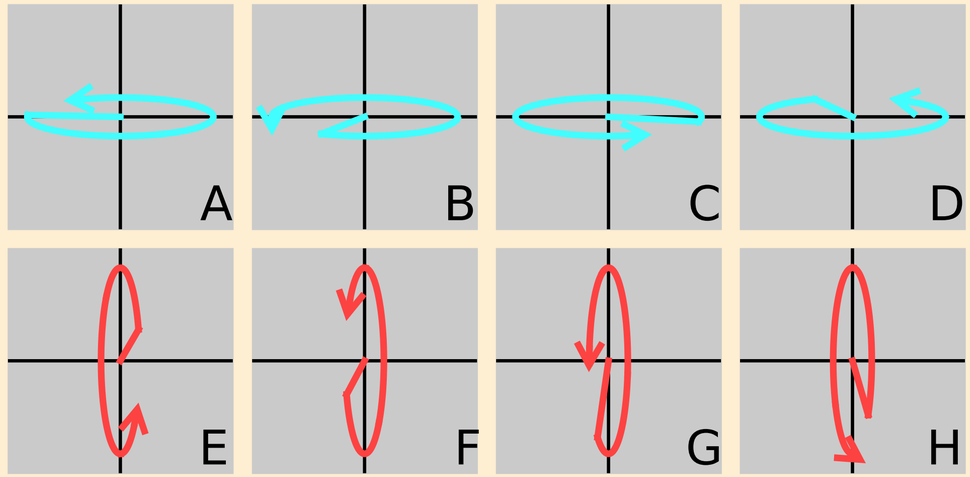
measure
both pol and
phase

extra features of 3D skyrmionic texture



real space measurement

- (almost) every pol filament is a ***loop***
- every pair of loops (any pols) is ***linked***
- around each loop, phase varies thru 2π



outline

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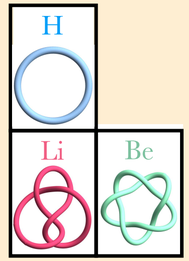
Skyrme and 1D topological solitons

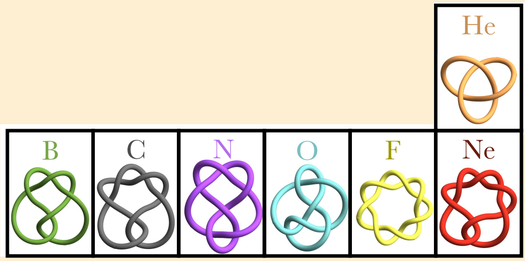
what defines a fundamental particle?

localised — nonlinear soliton eq

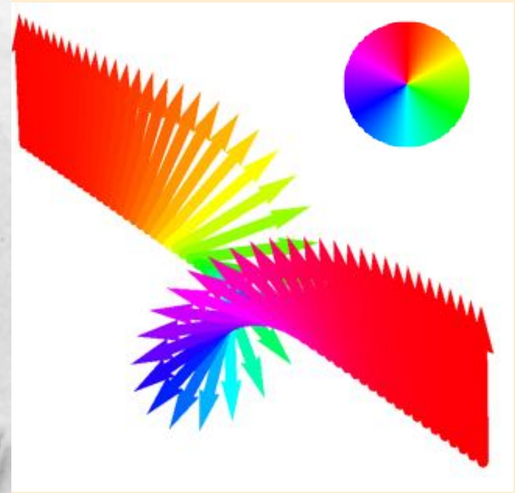
quantised — topological degree

heterogeneous — diff winding numbers



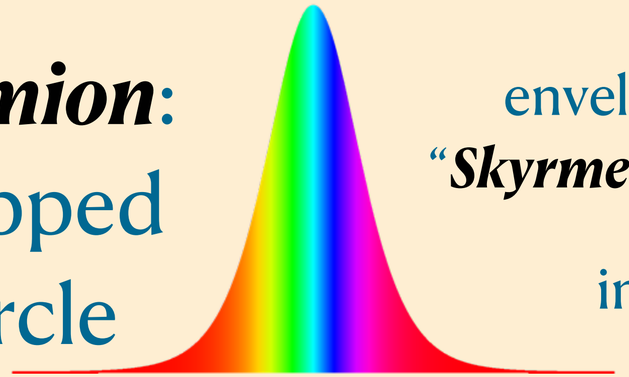


e.g. Kelvin's knotted vortices in the ether



1D ***Skyrmion***: line mapped to a circle

$$\chi : \mathbb{R} \longrightarrow S^1$$



envelope is "***Skyrme density***": $\frac{1}{2\pi}\partial_x \chi$

integrates here to +1

Skyrme density ~ ***mapping's jacobian***

generalises to any dimension

$$\mathbb{R}^n \longrightarrow S^n$$

Skyrme number / ***degree*** found by integrating over all real space

THR Skyrme (1922-1987, Prof @ Birmingham):

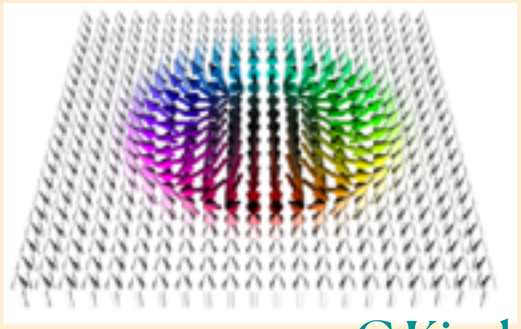
nuclear particles are ***topological solitons***

mapping 2D plane to sphere: “baby” skyrmions

unit 3-vector

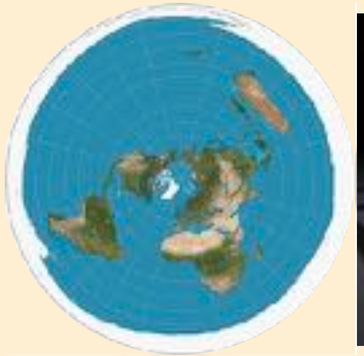
$\boldsymbol{s}$

2-sphere
target space



C Kind

2D degree:
plane wraps
around sphere



H Segerman

2D Skyrme density: $\boldsymbol{s} \cdot \partial_x \boldsymbol{s} \times \partial_y \boldsymbol{s}$

e.g. **spintronics**
magnetic spin direction

localised by nonlinearity

2D skyrmions act as bits
in proposed magnetic
‘racetrack memory’

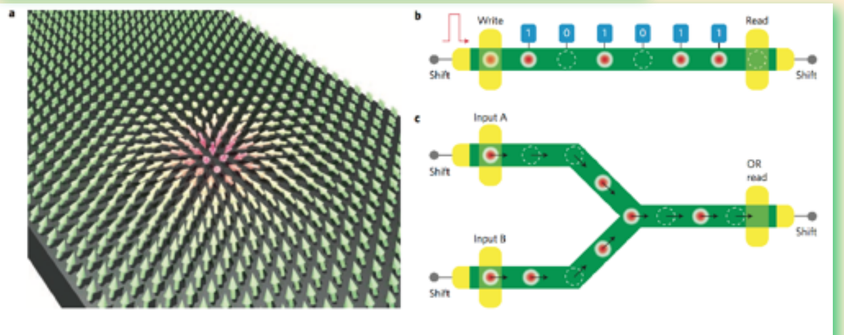
SPINTRONICS

Skyrmionics gets hot

The observation of magnetic skyrmions at room temperature that can be driven by short current pulses at speeds exceeding 100 m s^{-1} raises great expectations for skyrmion-based racetrack memories.

Stefan Krause and Roland Wiesendanger

NATURE MATERIALS | VOL 15 | MAY 2016 | www.nature.com/naturematerials

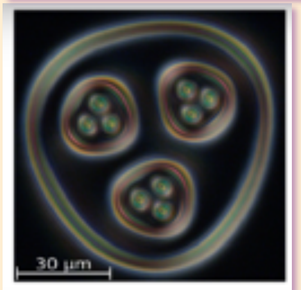


higher 2D Skyrme numbers possible!

nature physics
LETTERS

Two-dimensional skyrmion bags in liquid crystals and ferromagnets

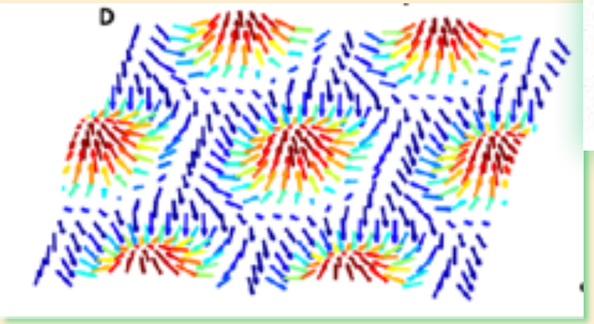
David Foster[1], Charles Kind[2], Paul J. Ackerman[3], Jung-Shen B. Tai[3], Mark R. Dennis[1,4*] and Ivan I. Smalyukh[3,5,6*]



heterogeneous $\Longrightarrow$
skyrmions in
antiskyrmion “bags”

optical baby skyrmions / skyrmionic beams

sphere target space:
direction of ***E*** field



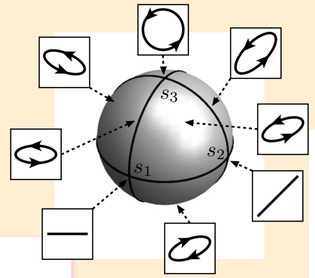
RESEARCH ARTICLE

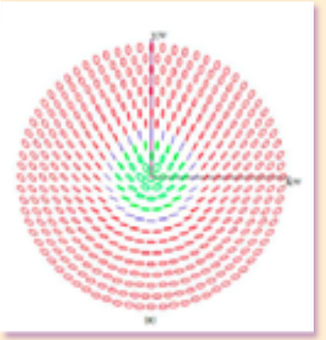
Optical skyrmion lattice in evanescent electromagnetic fields

S. Tsesses[1], E. Ostrovsky[1], K. Cohen[1], B. Gjonaj[2], N. H. Lindner[3], G. Bartal[1,*]
+ See all authors and affiliations

Science 07 Sep 2018:
Vol. 361, Issue 6406, pp. 993-996
DOI: 10.1126/science.aau0227

sphere target space:
Poincaré sphere





Full Poincaré beams

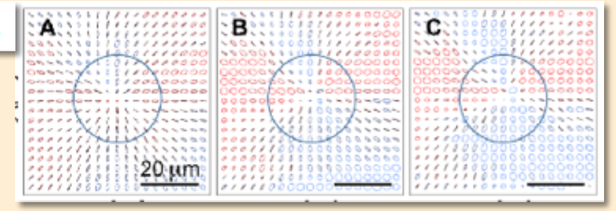
Amber M. Beckley, Thomas G. Brown*, and Miguel A. Alonso

24 May 2010 / Vol. 18, No. 10 / OPTICS EXPRESS 10777

Twist of generalized skyrmions and spin vortices in a polariton superfluid

Stefano Donati[a,b], Lorenzo Dominici[a,1], Galbadrakh Dagvadorj[c], Dario Ballarini[a], Milena De Giorgi[a], Alberto Bramati[d], Giuseppe Gigli[a], Yuri G. Rubo[e], Marzena Hanna Szymańska[c,1], and Daniele Sanvitto[a]

14926–14931 | PNAS | December 27, 2016 | vol. 113 | no. 52

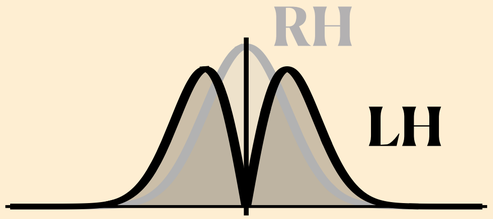


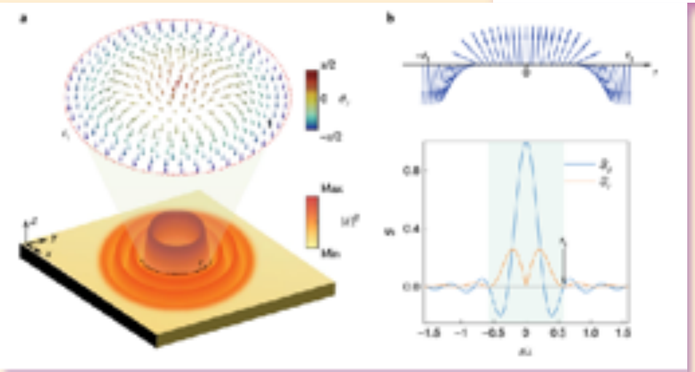
sphere target space:
direction of optical spin

nature physics
LETTERS

Deep-subwavelength features of photonic skyrmions in a confined electromagnetic field with orbital angular momentum

Luping Du[1*], Aiping Yang[1], Anatoly V. Zayats[2*] and Xiaocong Yuan[1*]



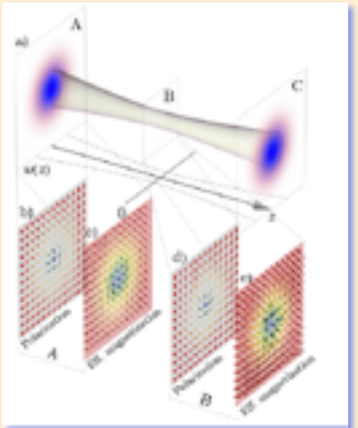


PHYSICAL REVIEW A **102**, 053513 (2020)

Paraxial skyrmionic beams

Sijia Gao,* Fiona C. Speirits, Francesco Castellucci, Sonja Franke-Arnold, and Stephen M. Barnett
School of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, United Kingdom

Jörg B. Götte



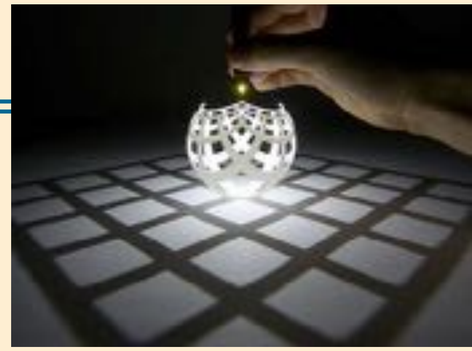
outline

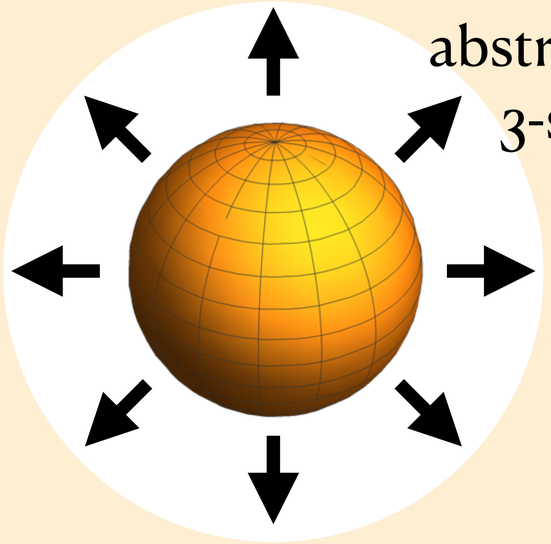
- context of topologies in free space optical beam
- structure of the 3D skyrmionic hopfion beam
- some (more) skyrmions background
- **3D optical hypersphere: topology of polarization-phase space**
- perspective and discussion

anatomy of 3-sphere (hypersphere)

how to visualise a 3-sphere?
wrapping around 3D space?

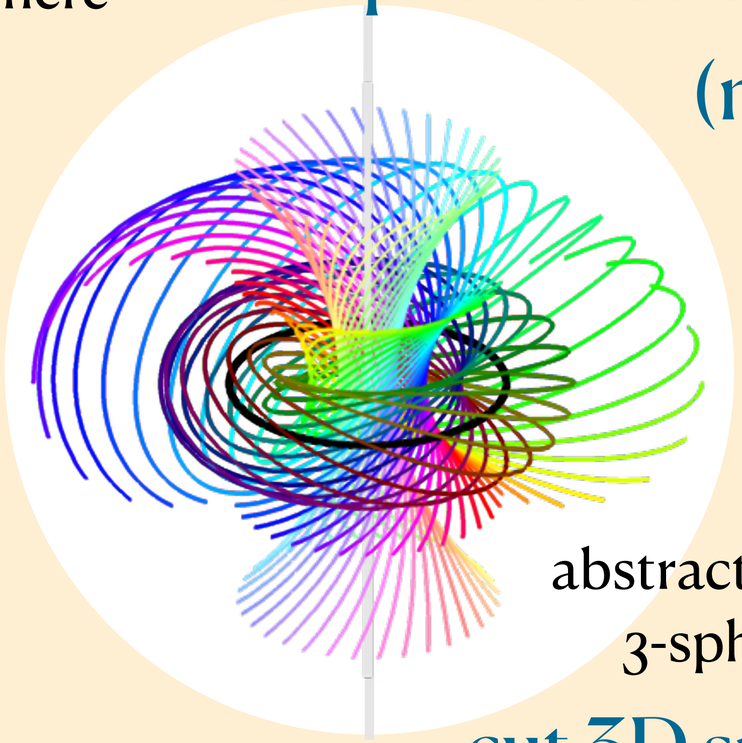
first: map 3-sphere ⟸ 3D with ∞



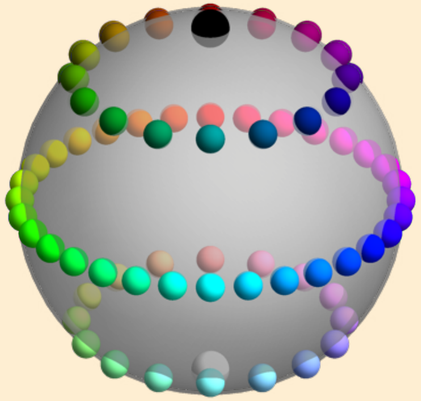


abstract space
3-sphere

3-sphere also supports ***Hopf fibration***
(no lower-dim analogue)



abstract space
3-sphere



each loop corresponds to a point on the 2-sphere

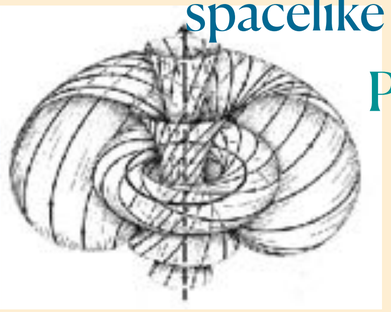
hyperspherical polars:

usual θ, ϕ and radius as $f(\psi)$.

cut 3D space into interlinking loops

hopfions and skyrmionic hopfions

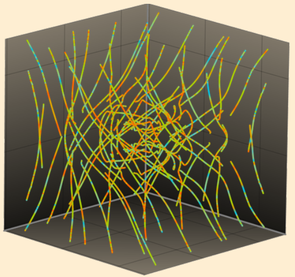
map from 3D to 2-sphere – *hopfions*



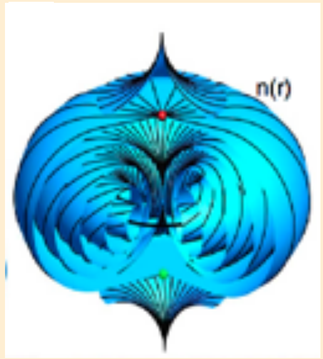
spacelike twistor
Penrose
(1984)

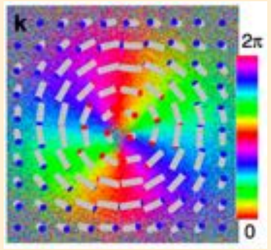
~> various sols of wave eqs,
Hopfion-like current flows

Dirac current
Bialynicki-Birula
et al, PRA (2019)

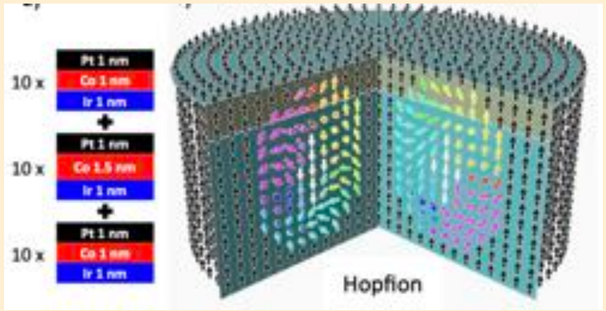


e.g. Hopf-
Gaussian beam
MRD (unpub)



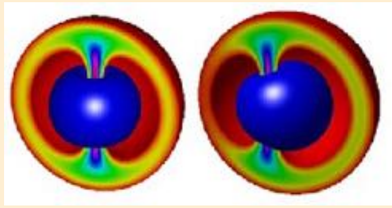


~> hopfions in liquid
crystal textures
Ackerman, van de Lagemaat
& Smalyukh, *Nat Commun* (2015)



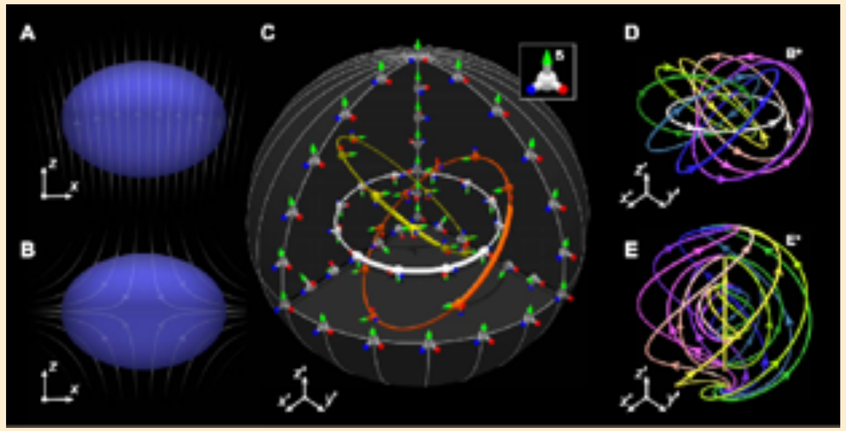
~> hopfions in magnetic multilayers
Kent *et al, Nat Commun* (2021)

to 3-sphere – *3D skyrmions / skyrmionic hopfions*



in BEC (theory)
Ruostekoski &
Savage *PRL* (2003)

in models of high energy
physics, Foster, *PRD* (2011)



in BEC (expt) Lee *et al, Sci Adv* (2018)

the “optical hypersphere”

parameter space of **normalised *E*** field

$$(\mathrm{Re}E_\mathrm{R})^2 + (\mathrm{Im}E_\mathrm{R})^2 + (\mathrm{Re}E_\mathrm{L})^2 + (\mathrm{Im}E_\mathrm{L})^2 = 1$$

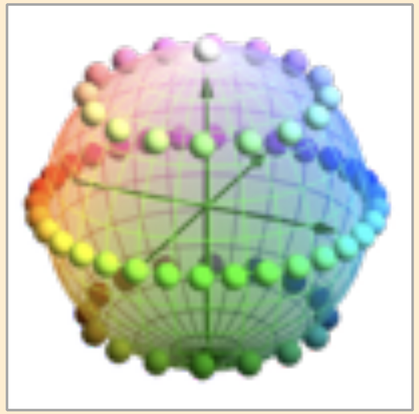
circular field components:

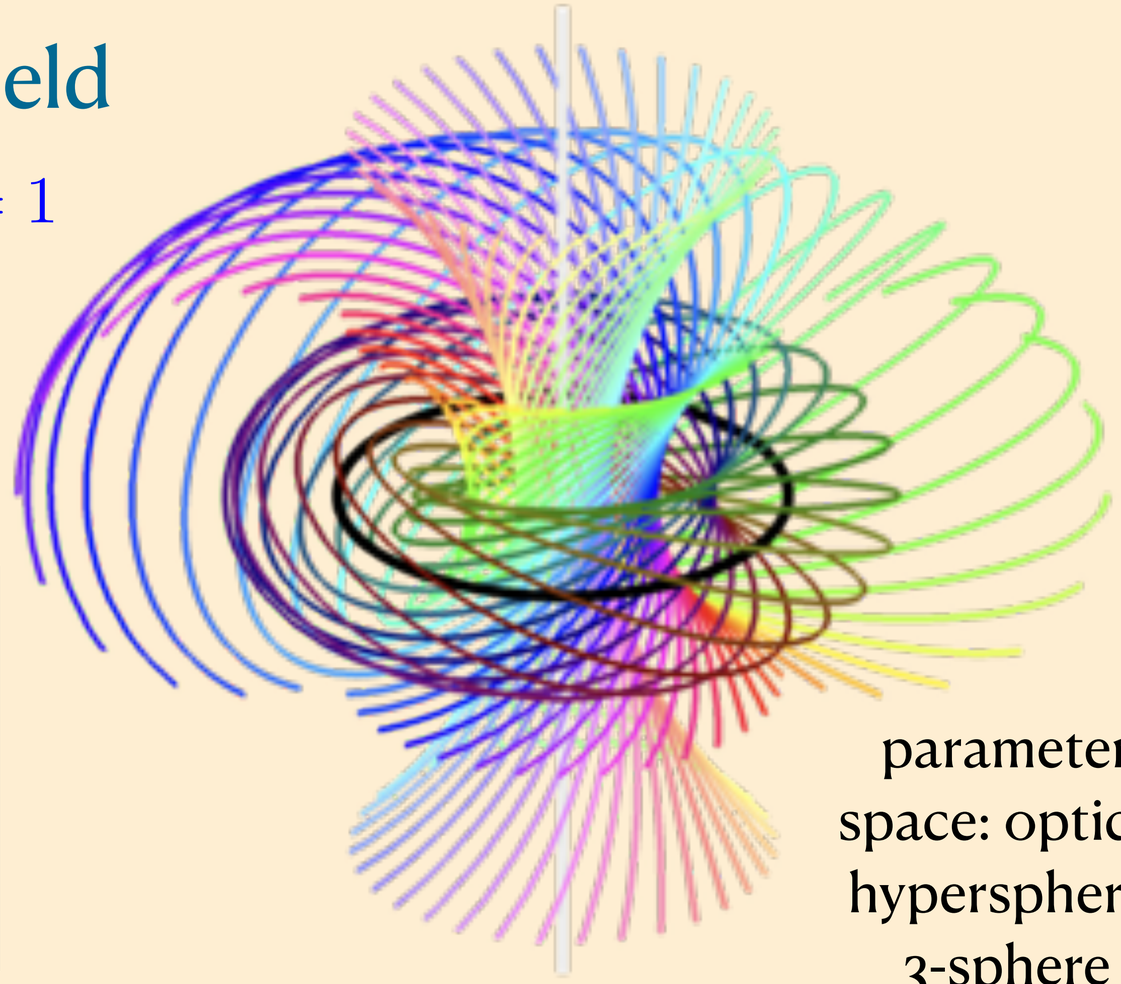
$$E_\mathrm{R} = \cos\frac{\beta}{2}\mathrm{e}^{\frac{\mathrm{i}}{2}(\gamma-\alpha)} \qquad E_\mathrm{L} = \sin\frac{\beta}{2}\mathrm{e}^{\frac{\mathrm{i}}{2}(\gamma+\alpha)}$$

ellipticity $S_3 = \cos\beta$

azimuth $\alpha = \arctan(S_2/S_1)$

phase $\gamma = \arg(E_\mathrm{R}E_\mathrm{L})$



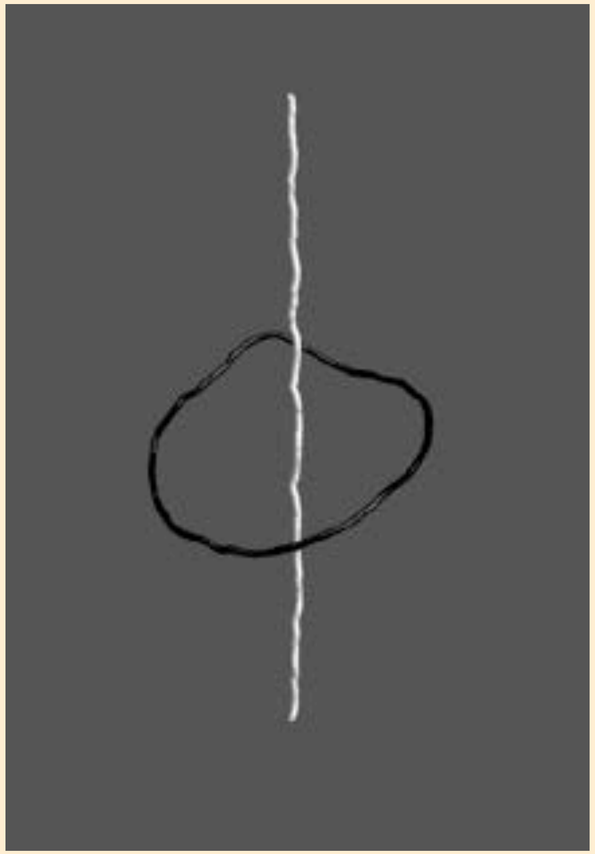


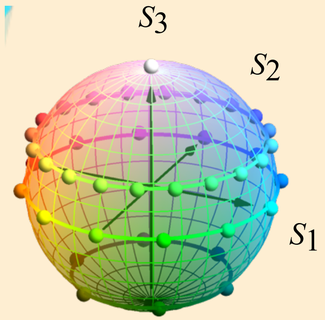
parameter space: optical hypersphere, 3-sphere

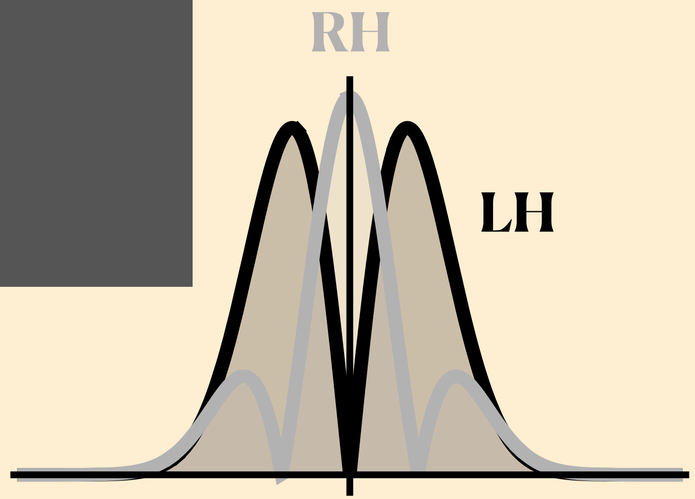
physical state: 3 angles

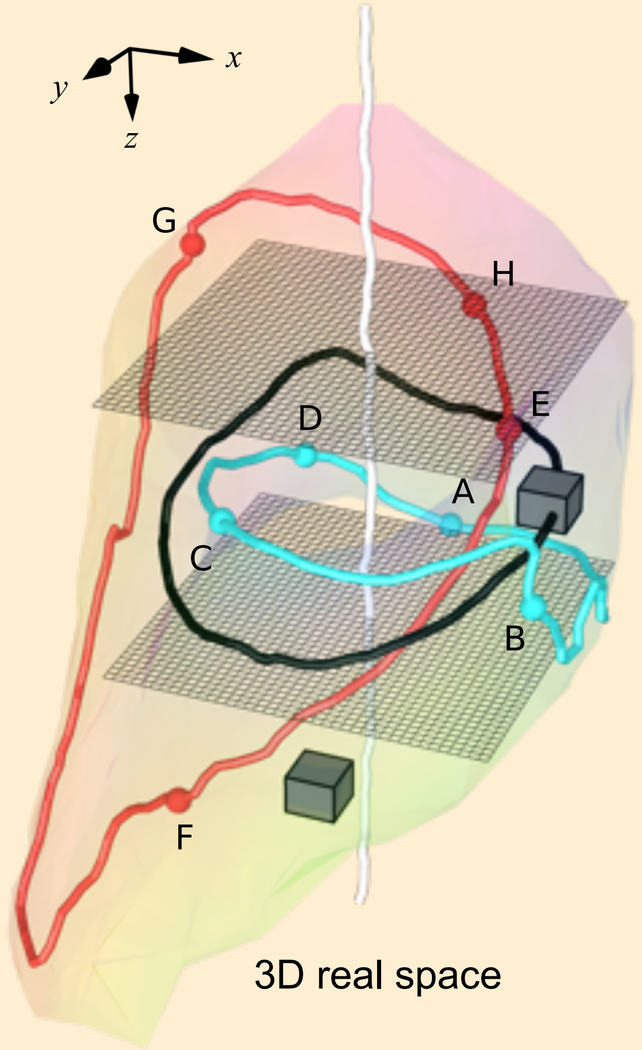
naturally gives Hopf fibration!

skyrmionic hopfion – realises all polarisation/phase states in 3D

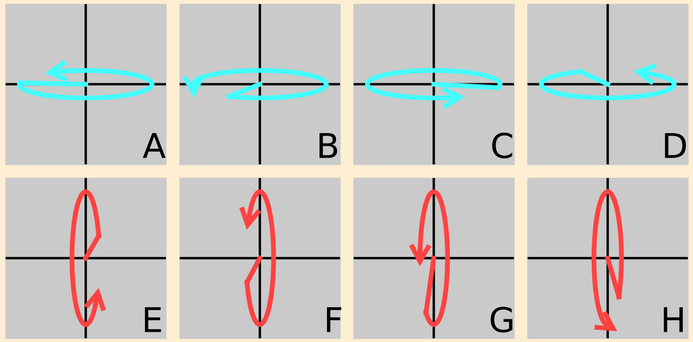


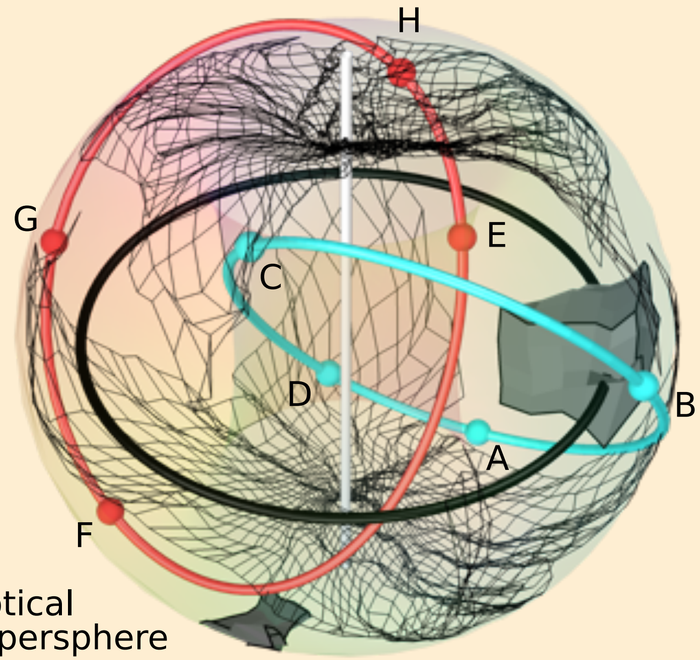






3D real space





Optical hypersphere

3D Skyrme density

degree is integral over all space of cts Skyrme density:

$$\Sigma = \frac{1}{2\pi^2}\det(\vec{n}, \partial_x\vec{n}, \partial_y\vec{n}, \partial_z\vec{n})$$

$$\vec{n} \equiv (\mathrm{Re}E_\mathrm{R}, \mathrm{Im}E_\mathrm{R}, \mathrm{Re}E_\mathrm{L}, \mathrm{Im}E_\mathrm{L})$$

$$= \frac{1}{16\pi^2}\nabla\alpha\cdot\nabla\cos\beta\times\nabla\gamma$$

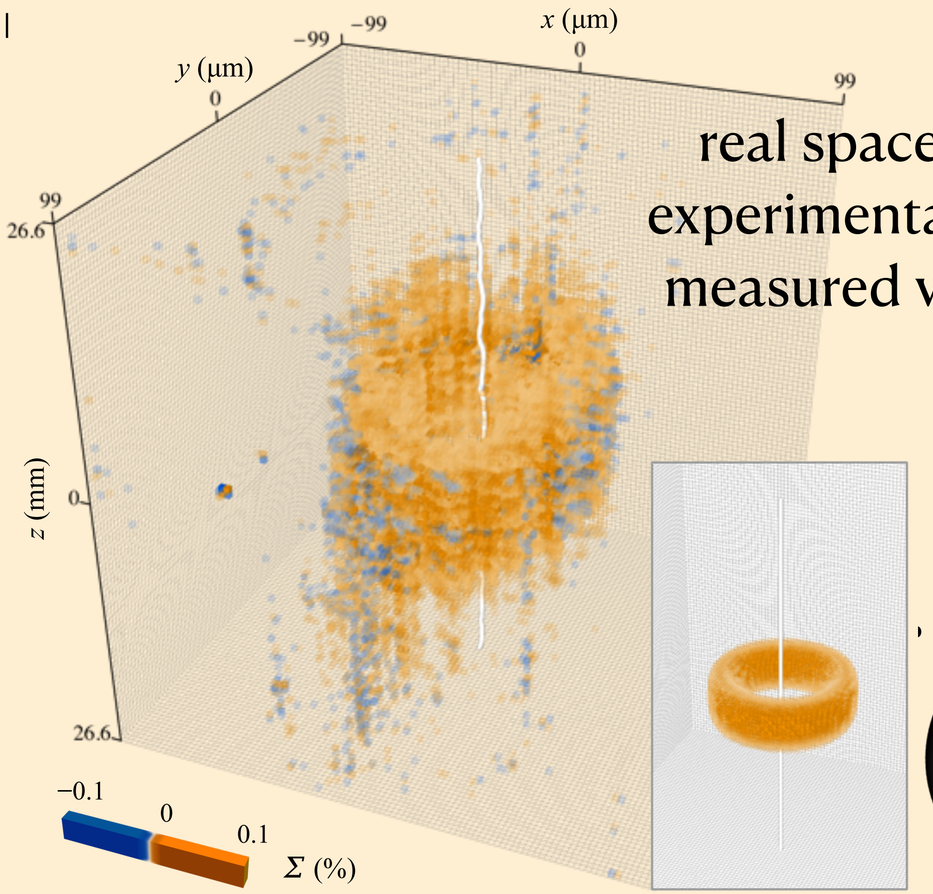
jacobian

$$= \frac{1}{4\pi^2}\boldsymbol{J}\cdot\nabla\times\boldsymbol{J}$$

(fluid) ***helicity*** of orbital current

$$\boldsymbol{J} \equiv \frac{1}{2\pi}\mathrm{Im}\begin{pmatrix}E_\mathrm{R}^* \\ E_\mathrm{L}^*\end{pmatrix}\cdot\boldsymbol{\nabla}\begin{pmatrix}E_\mathrm{R} \\ E_\mathrm{L}\end{pmatrix}$$

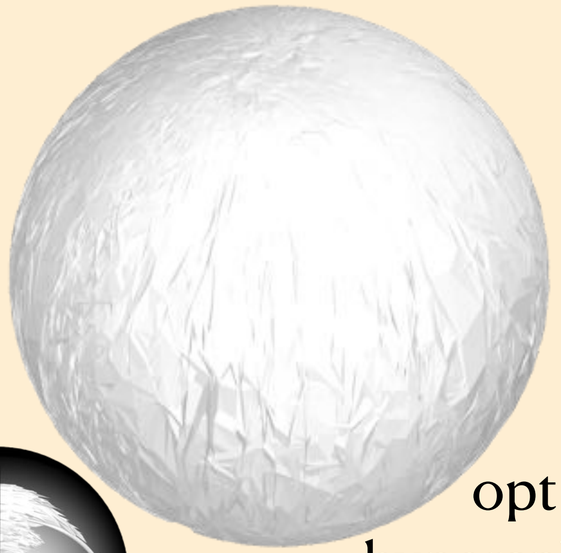
Kedia, Foster, MRD, Irvine *PRL* 2016



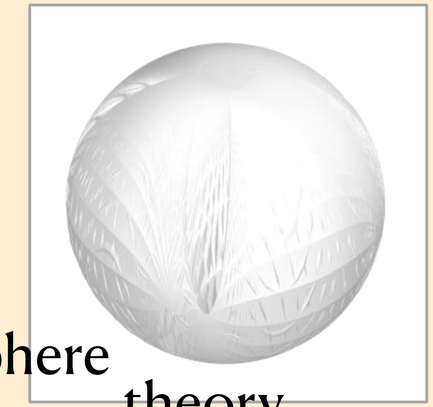
real space: experimentally measured vol

from expt

0.945



opt hypersphere

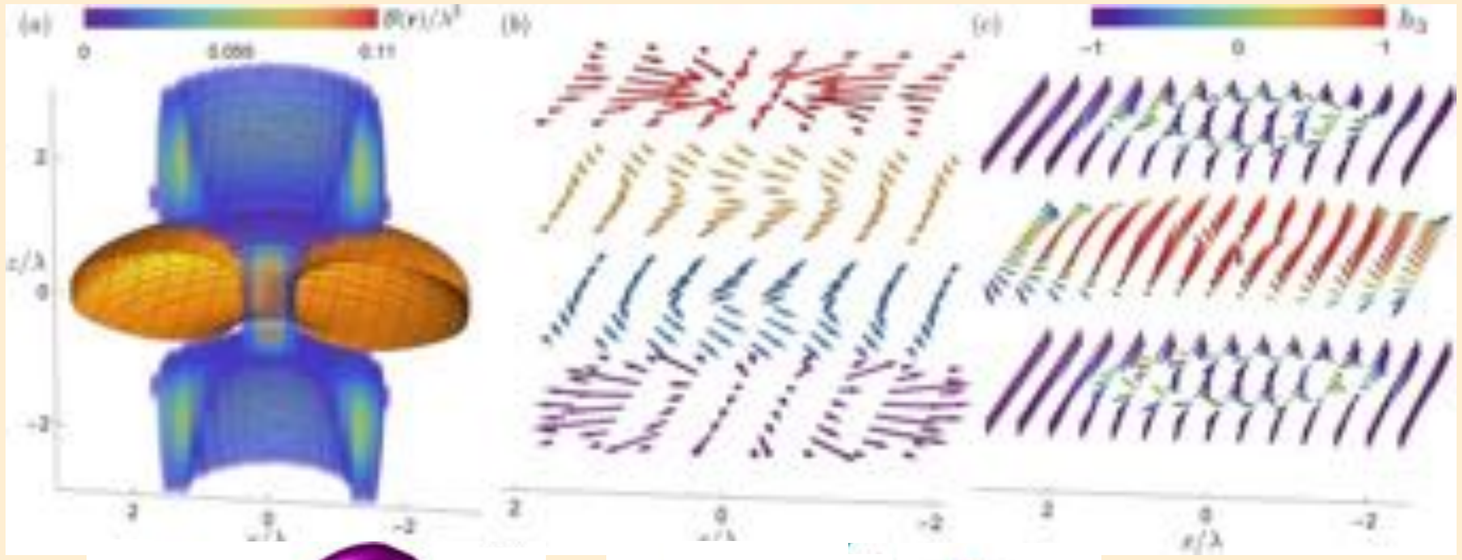


theory

0.997

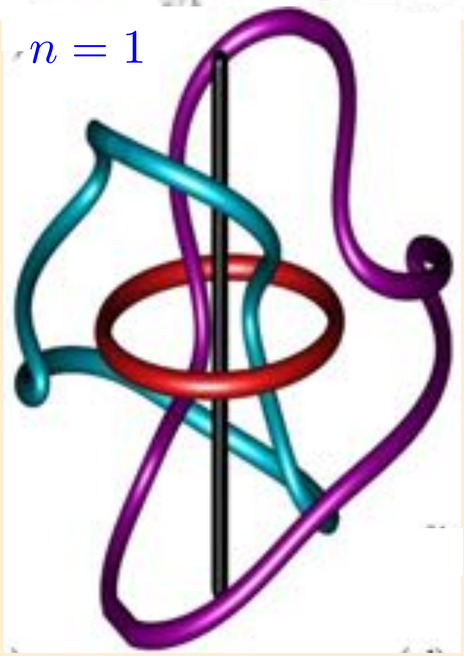
almost fills 3-sphere, localised due to engineered structured field

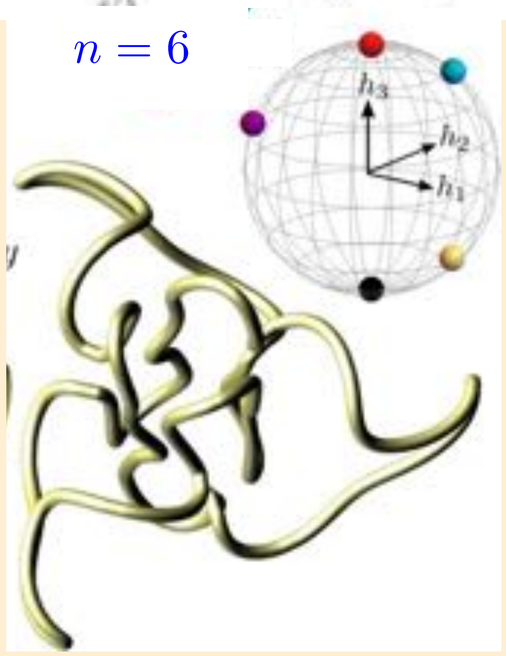
knotted skyrmionic hopfions for embedding in cold atoms



Parmee, MRD, Ruostekoski (2021)

- optical skyrmion field couples to atom field via polarisation density current (artificial gauge field)





- optimised, designed optical skyrmionic hopfion field for imprinting into atom fields:
  - ~ filaments (preimages) always linked
  - ~ designed knots filaments

concluding remarks

normalised optical state corresponds to a point in a hypersphere; Poincaré sphere is base space for Hopf fibration

found and measured a structured beam which realises (almost) all these states within a finite, 3D propagation volume

resulting texture corresponds to a ***topological particle-like*** structured beam: (skyrmionic hopfion of polarization & phase)

localised ✓ beam design

quantised ✓ measured 0.94

heterogeneous ✓ new physics?

> So, as we watch the weaving of the garment of Nature, we resolve it in imagination into threads of ether spangled over with beads of matter. We look still closer, and the beads of matter vanish; they are mere knots and loops in the threads of ether.
>
> J H Poynting, 1899

thank you!