Collective dynamics of topological magnetic defects induced by electromagnons

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Outline

- **Multiferroics, electromagnons and electric control of magnetism**
- **Skyrmion helicity dynamics induced by electromagnon excitation**
- **Translational dynamics of skyrmions and antiskyrmions**

Multiferroics

MM, Multiferroics: different routes to magnetoelectric coupling
npj Spintronics 2, 18 (2024)

Multiferroic $GdFeO_3$

PL_1L_2

Fe^{3+} S=5/2

T_N = 661K

Γ_4	$G_x A_y F_z$

$M_z \propto G_x$

Gd^{3+} S=7/2

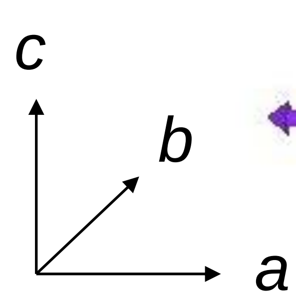
T'_N = 2.5K

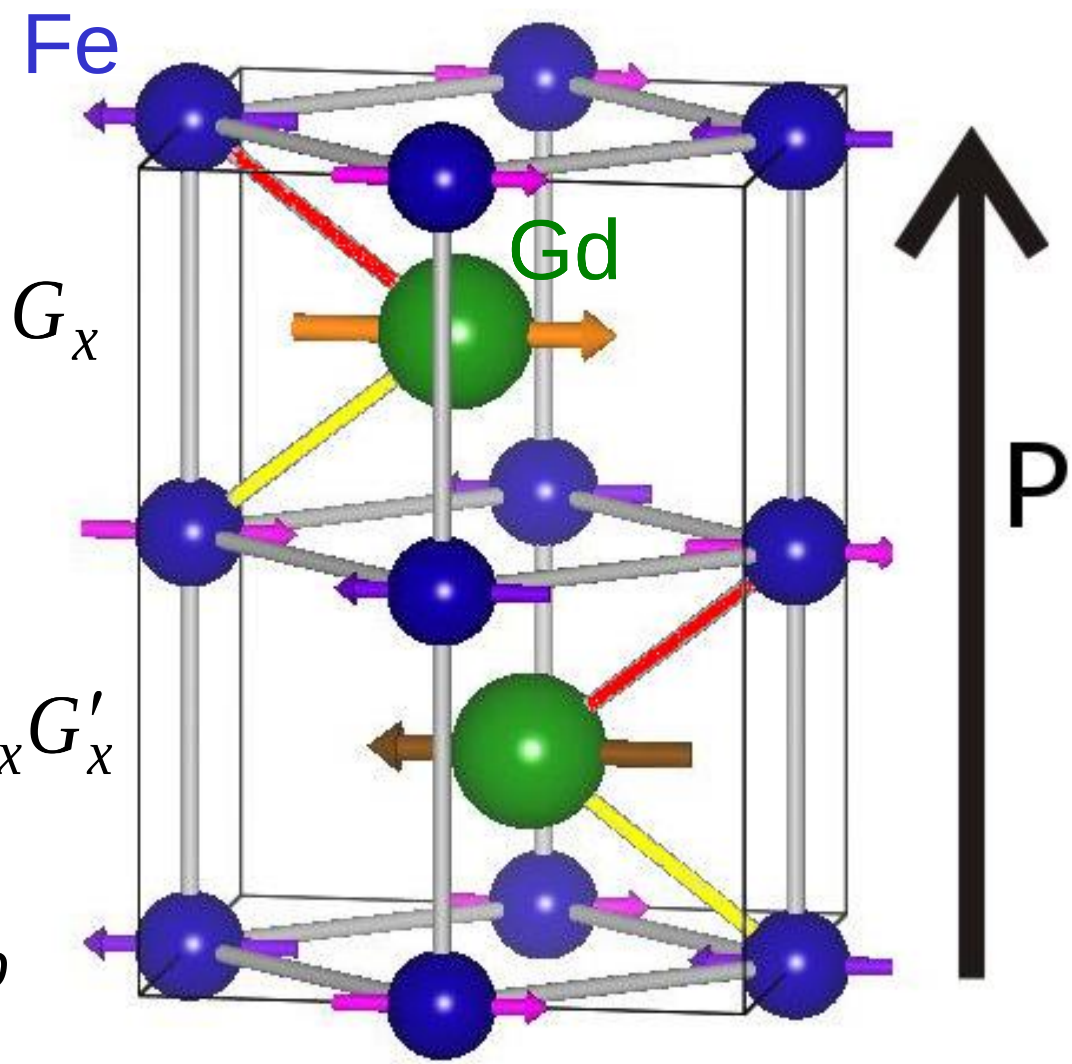
Γ_5	$G'_x A'_y$

$P_z \propto G_x G'_x$

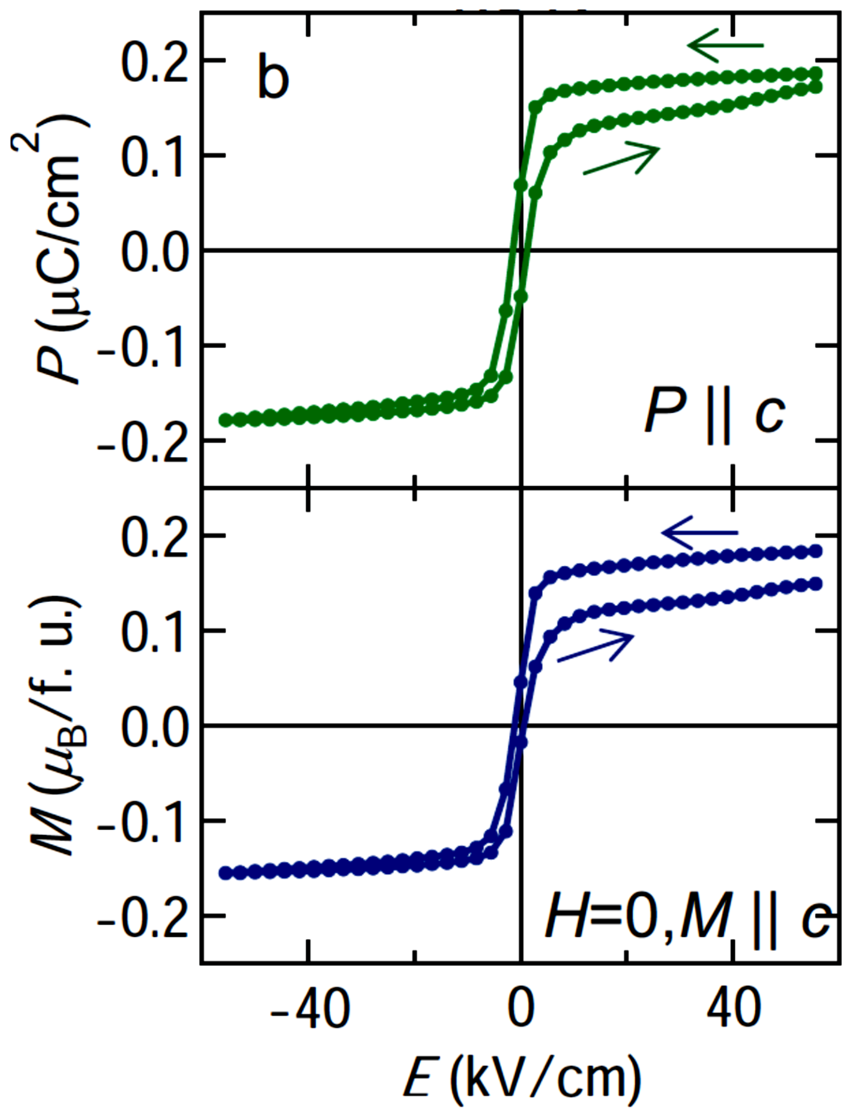
T < T'_N

$P_z \sim 0.1 \mu C\ cm^{-2}$



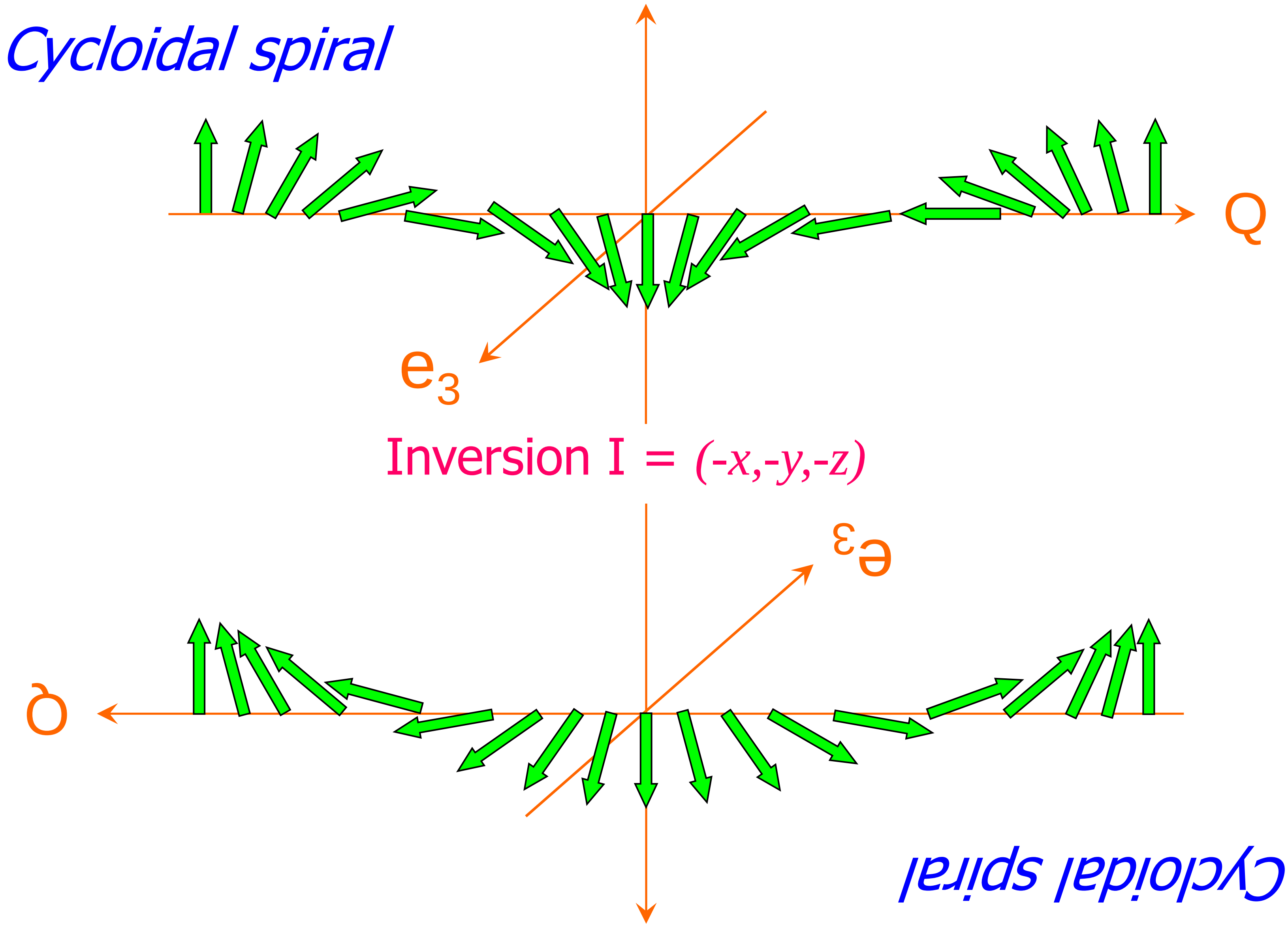


Electric control of magnetization in $Dy_{0.7}Tb_{0.3}FeO_3$

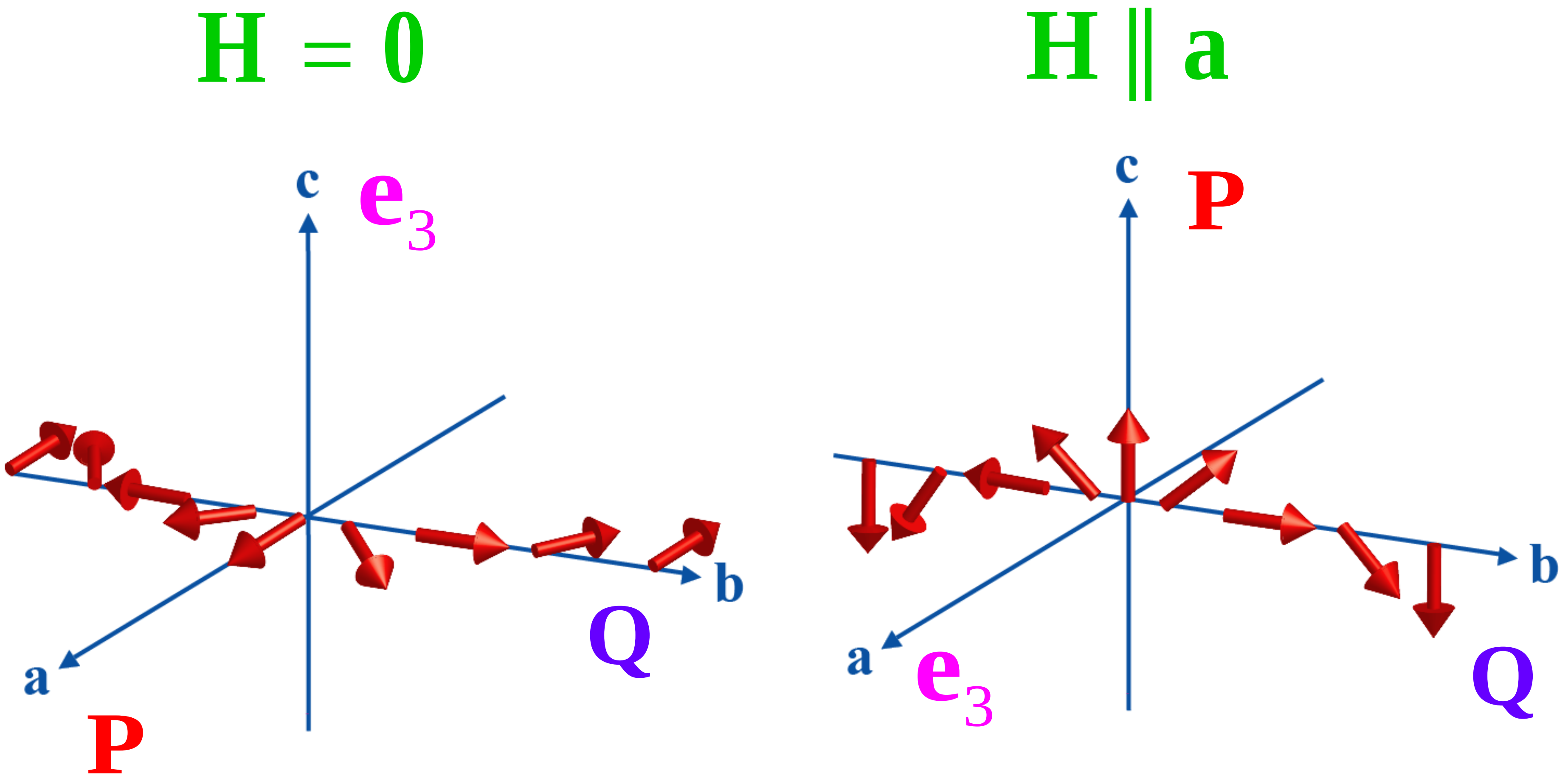


Y. Tokunaga et al Nature Materials ***8*** *838 (2012)*

Breaking of inversion symmetry by spiral spin ordering



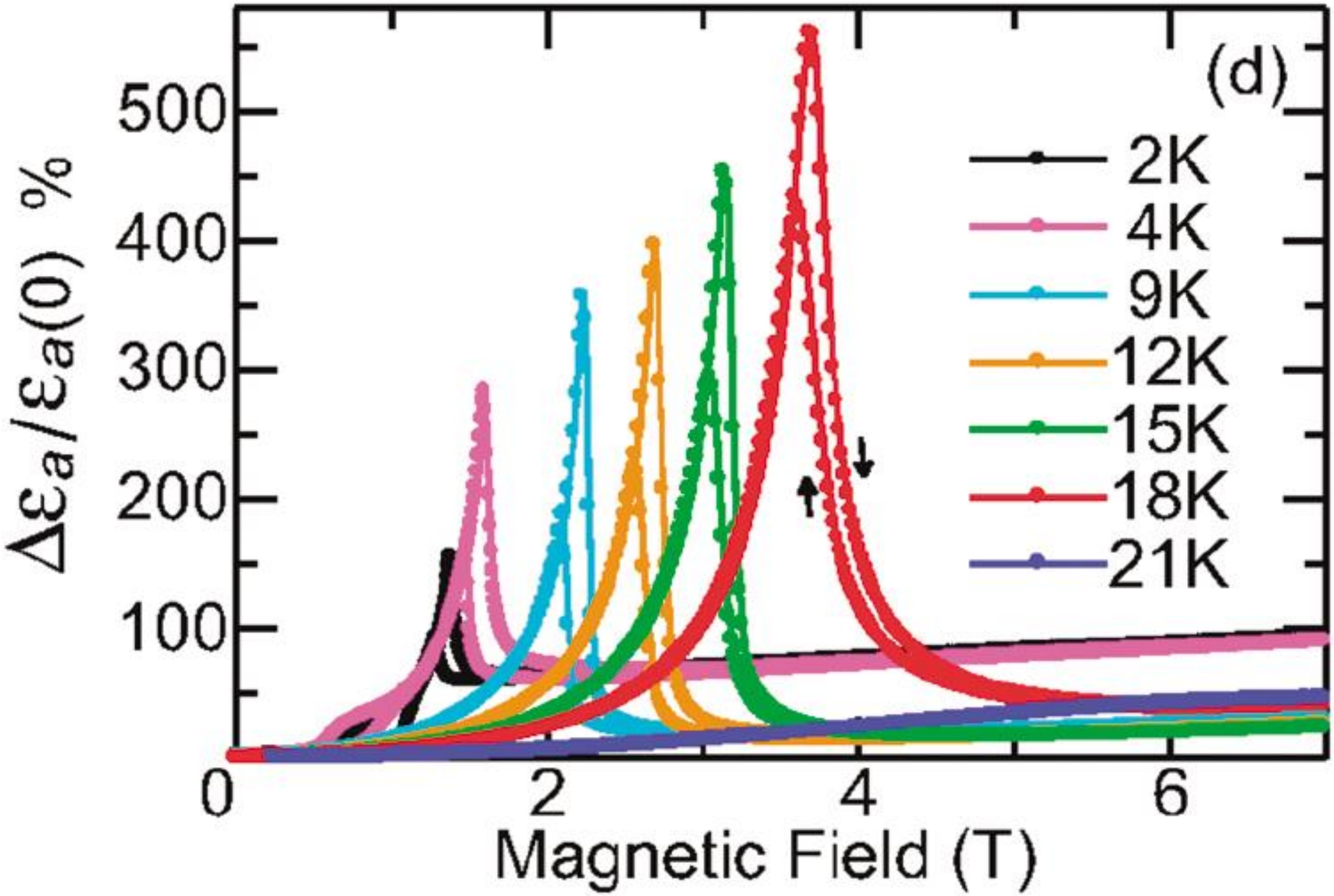
Polarization Flop in $Eu_{1-x}Y_xMnO_3$



$$P \propto e_3 \times Q$$

Spin Flop leads to Polarization Flop

Giant magnetocapacitance effect in $DyMnO_3$



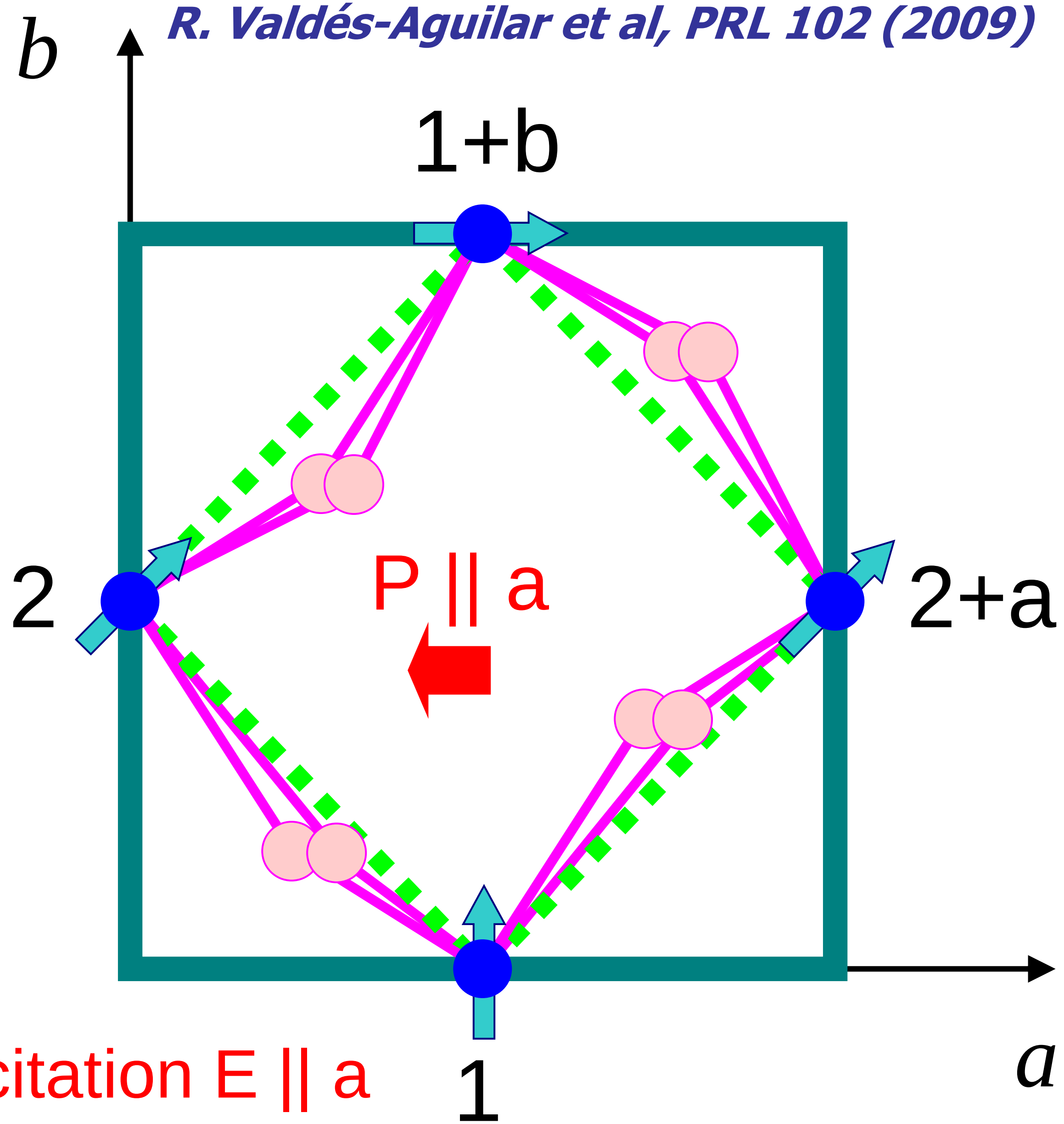
T. Goto et al PRL ***92****, 257201 (2004)*

Electromagnons: spin waves coupled to electric field

$RMnO_3$

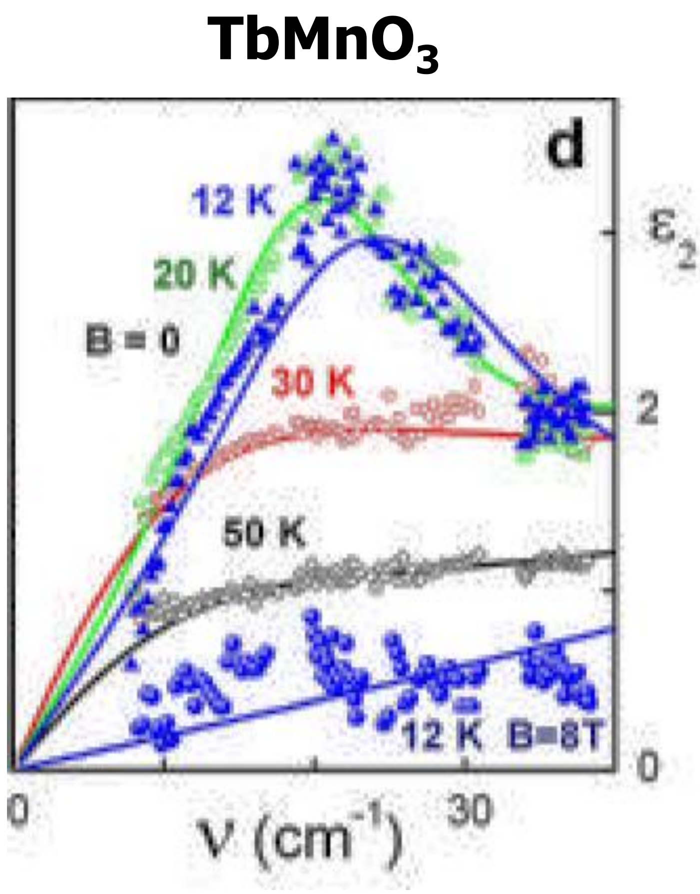
R. Valdés-Aguilar et al, PRL 102 (2009)

Q || b

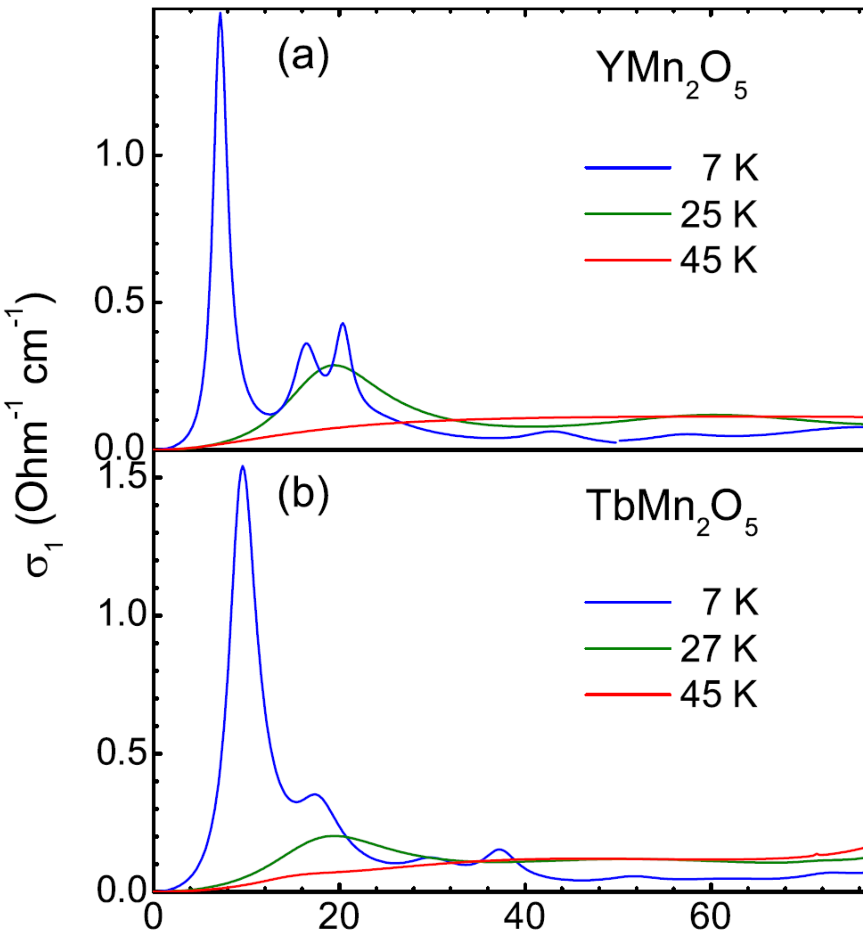


magon excitation E || a

Electromagnons in multiferroics

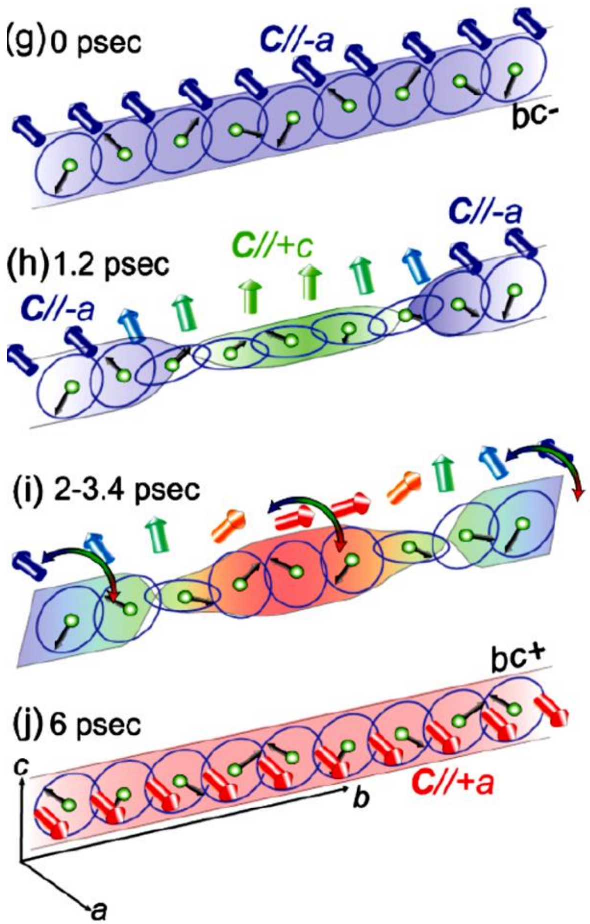


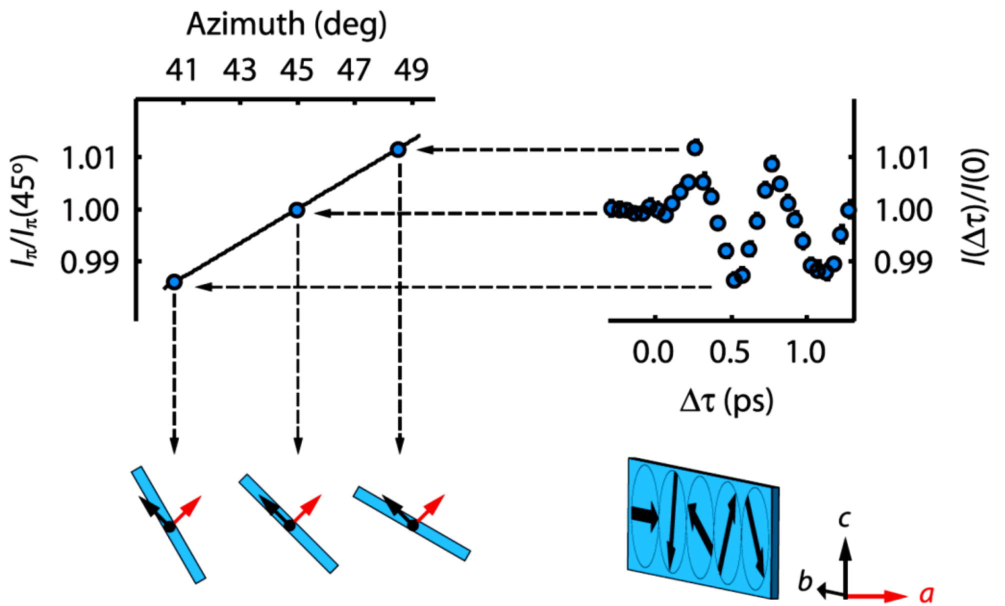
A. Pimenov et al
Nature Mat. (2006)



A.B. Sushkov et al (2006)

E-induced rotation of spiral plane





T. Kubacka, Science 343, 1333 (2014)

M.Mochizuki & N. Nagaosa, PRL 105, 147202 (2010)

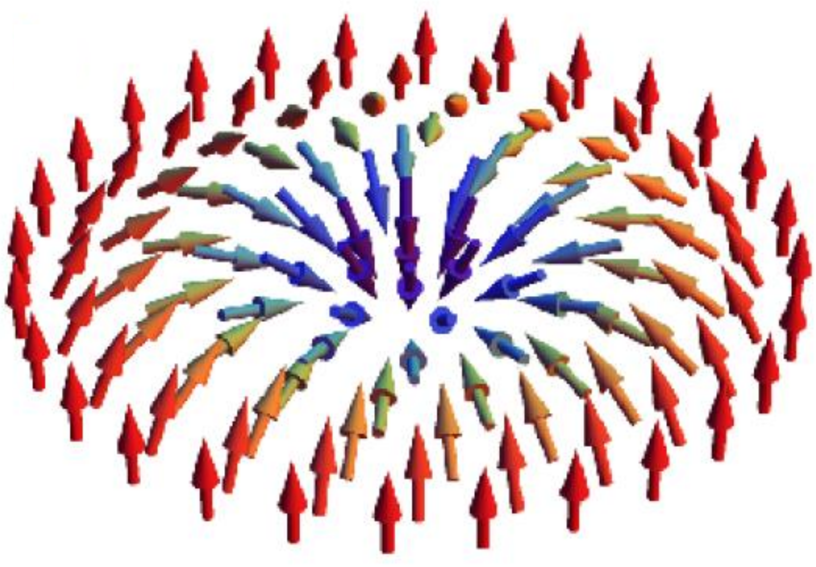
Electrically-excited large-amplitude collective dynamics of topological defects

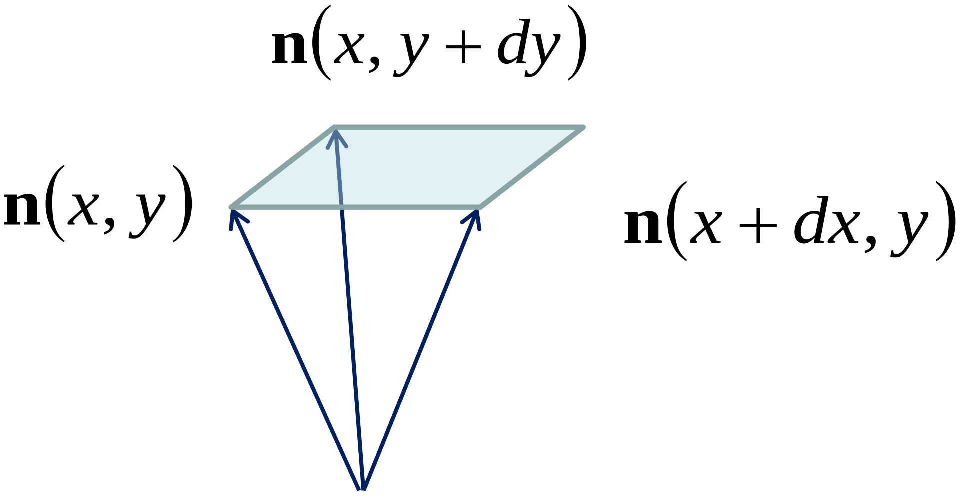
MM, J. Phys. Soc. Jpn. 92, 081005 (2023)
R. Knapman et al. Commun. Phys. 7, 151 (2024)

Topological charge

Pontryagin number:

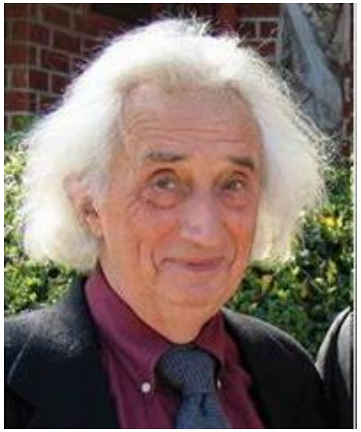
$$Q = \frac{1}{4\pi}\int d^2x\left(\mathbf{n}\cdot\left[\partial_x\mathbf{n}\times\partial_y\mathbf{n}\right]\right) = \frac{1}{4\pi}\int d\Omega = \text{integer \#}$$

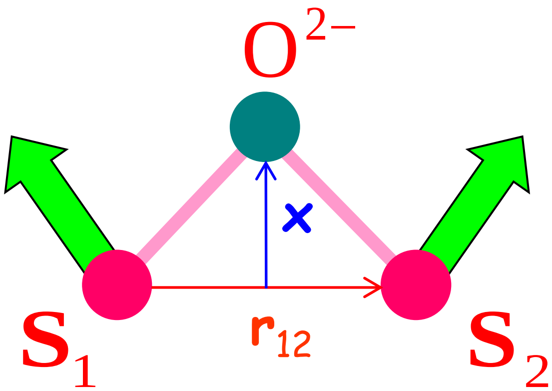




Dzyaloshinskii-Moriya interaction

I. Dzyaloshinskii, Sov. Phys. JETP 19, 960 (1964) T. Moriya, Phys. Rev. 120, 91 (1960)





$$E_{DM} = \mathbf{D}_{12} \cdot [\mathbf{S}_1 \times \mathbf{S}_2]$$

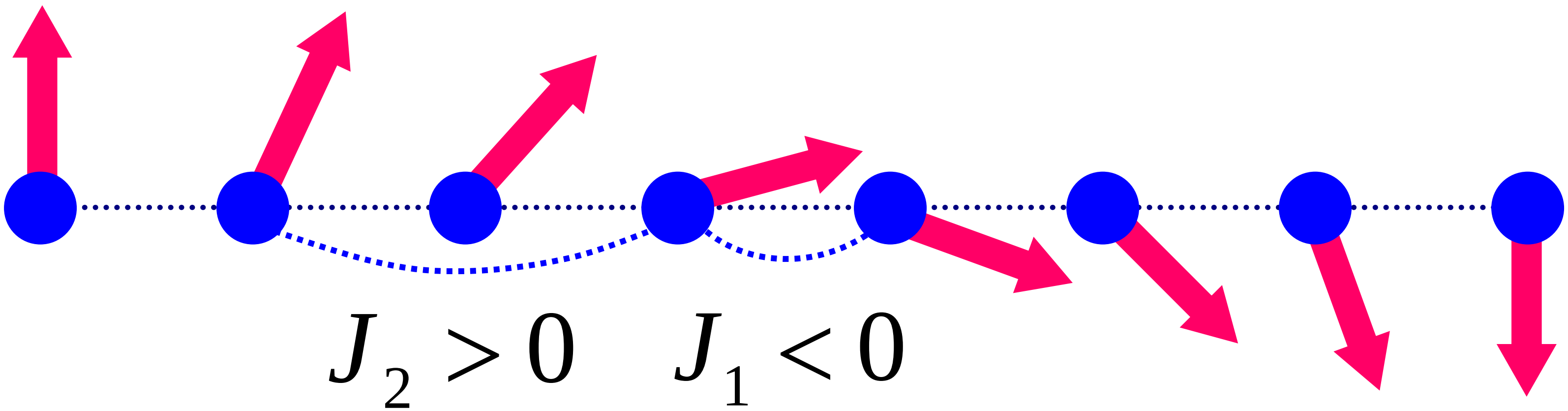
Lifshitz invariants in non-centrosymmetric magnets

$$M_a \partial_b M_c - M_c \partial_b M_a$$

Spin-spiral states

Spirals in centrosymmetric frustrated magnets

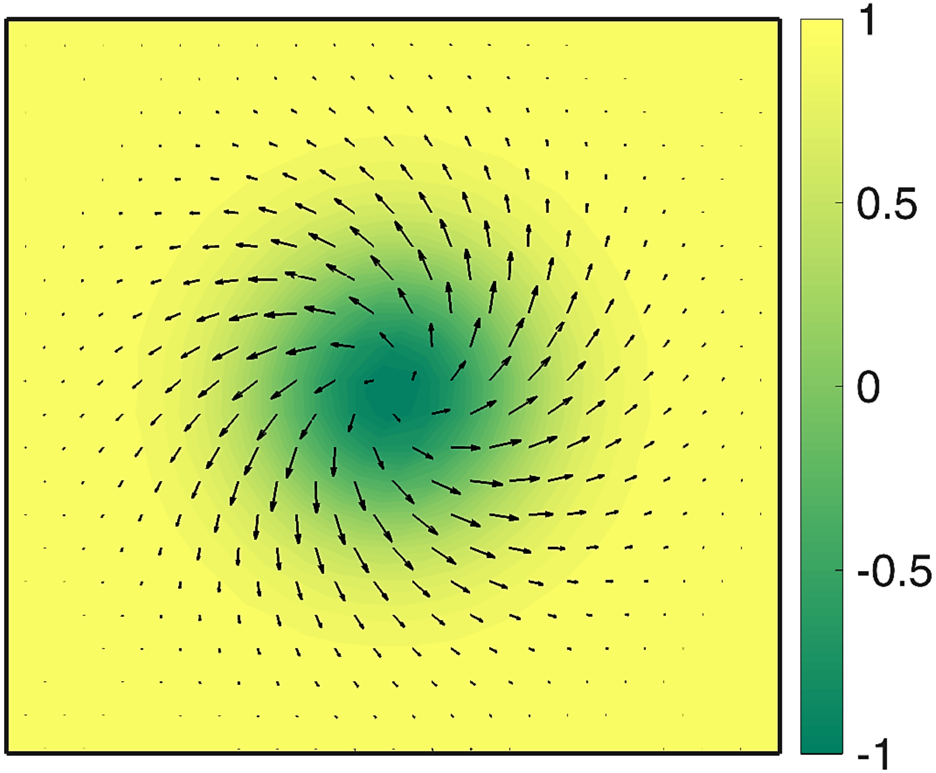
J. J. Villain, Phys. Chem. Solids 11, 303 (1959)
A. J. Yoshimori, Phys. Soc. Jpn. 14, 807 (1959)

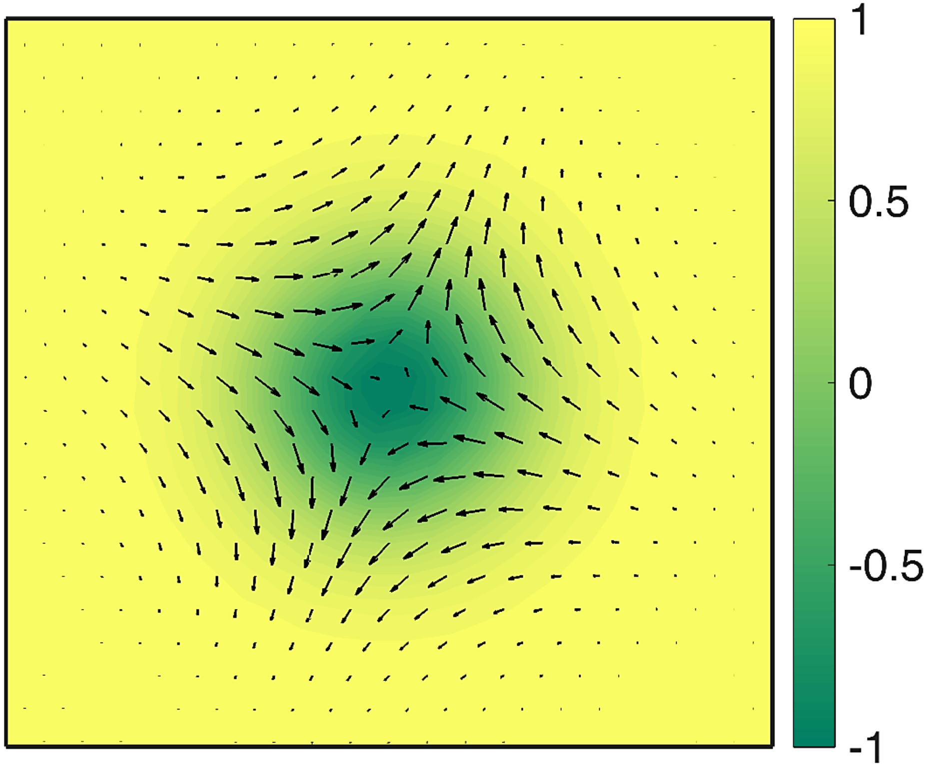


Arbitrary helicity and vorticity of skyrmions in frustrated magnets

$$\Theta = \Theta(r) \qquad \Phi = \mathrm{v}\,\phi + \chi$$

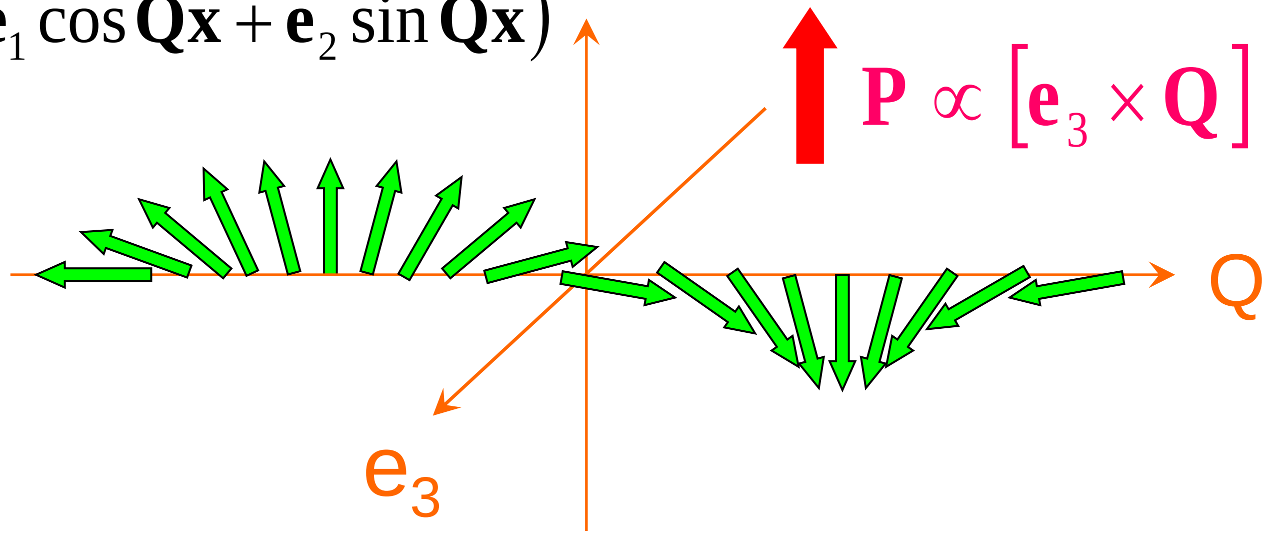
$$Q_{top} = \mathrm{v}\frac{\left(m_z(0) - m_z(\infty)\right)}{2}$$





Polarization induced by spiral

$$\mathbf{M} = M\left(\mathbf{e}_1 \cos \mathbf{Qx} + \mathbf{e}_2 \sin \mathbf{Qx}\right)$$



$$F_{me} = -\lambda E_z (M_z \partial_x M_x - M_x \partial_x M_z)$$

Inverse DM mechanism

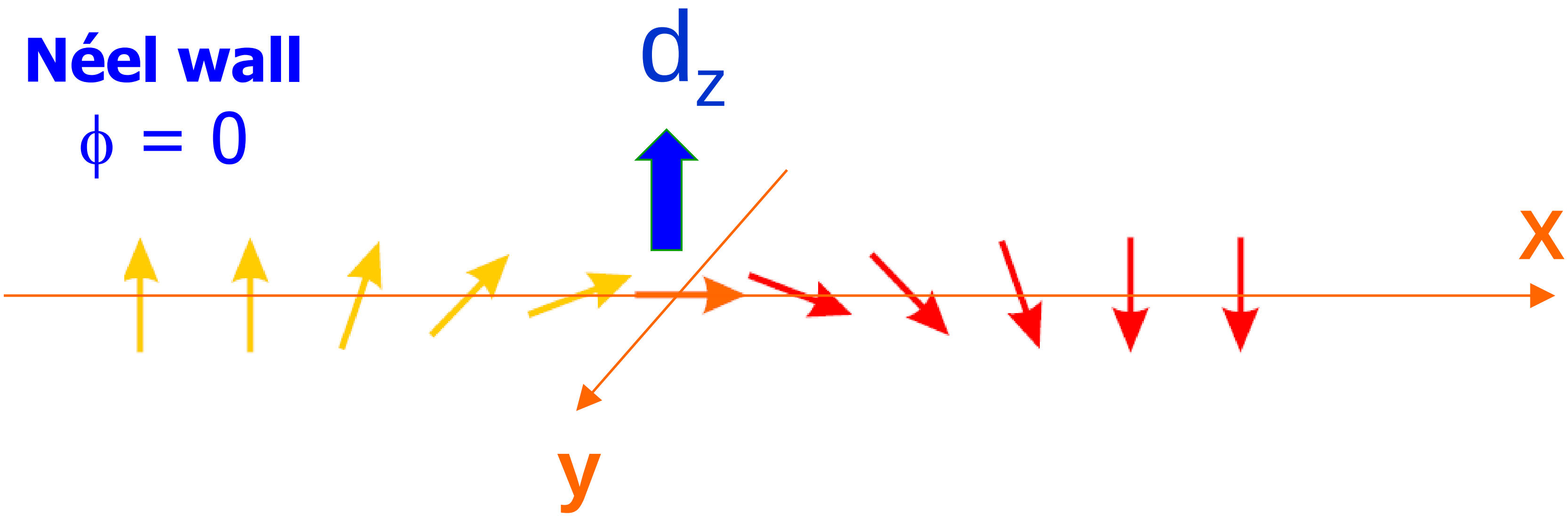
$$P_z = -\frac{\partial F_{me}}{\partial E_z} = \lambda (M_z \partial_x M_x - M_x \partial_x M_z)$$

Bary'akhtar et al, JETP Lett 37, 673 (1983) , Stefanovskii et al, Sov. J. Low Temp. Phys. 12, 478(1986), H. Katsura et al PRL 95, 057205 (2005), Sergienko et al PRB 73, 094434 (2006), M.M. PRL 96, 067601 (2006)

Electric dipole moment of magnetic domain wall

Néel wall

$\phi = 0$



Electric polarization

$$P_z = -\lambda \frac{d\theta}{dx} \cos\phi$$

Electric dipole moment is independent of dw shape

$$d_z = \pm \pi \lambda \cos\phi$$

Skyrmion electric dipole moment

Magneto-electric coupling

$$f_{me} = -g\,\mathbf{E}\cdot\left[\mathbf{m}(\nabla\cdot\mathbf{m}) - (\mathbf{m}\cdot\nabla)\mathbf{m}\right]$$

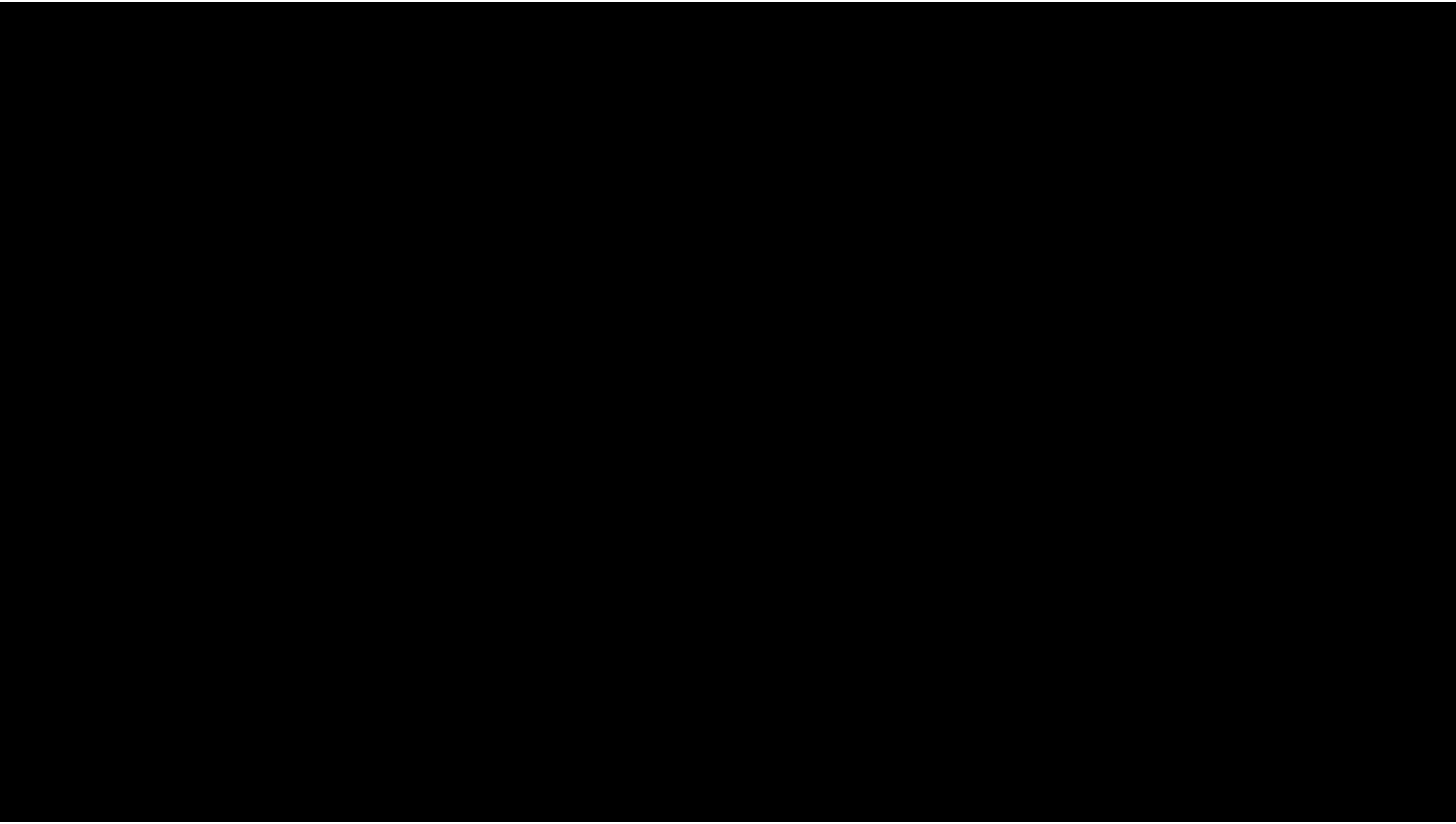
Polarization induced by skyrmion

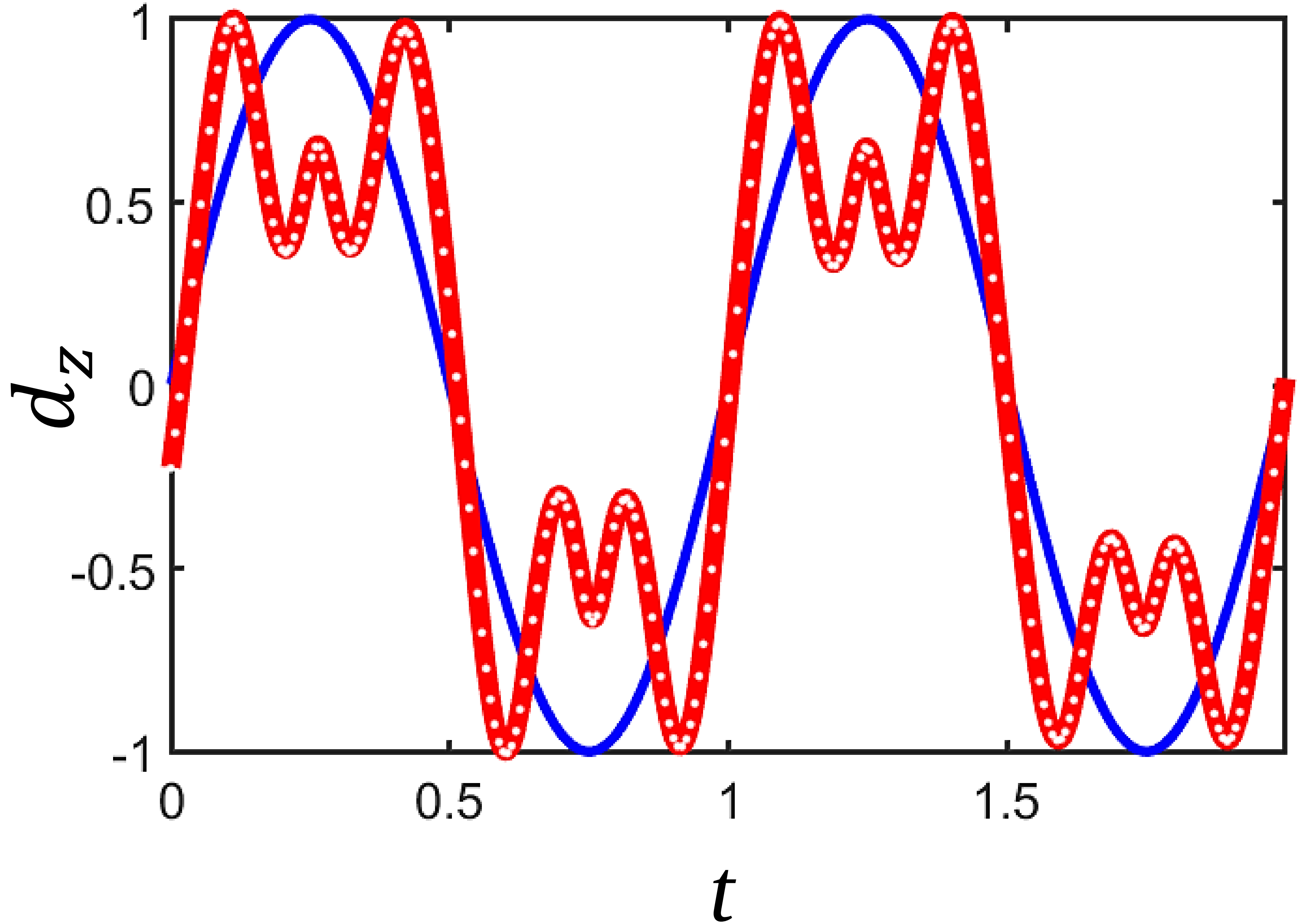
$$P_z = -\frac{\partial f}{\partial E_z} = g\left[\frac{d\Theta}{dr} + \mathrm{v}\,\frac{\sin 2\Theta}{2r}\right]\cos\left((\mathrm{v}-1)\varphi + \chi\right)$$

Dipole moment for vorticity v = +1

$$D_z(\chi) = D(0)\cos\chi$$

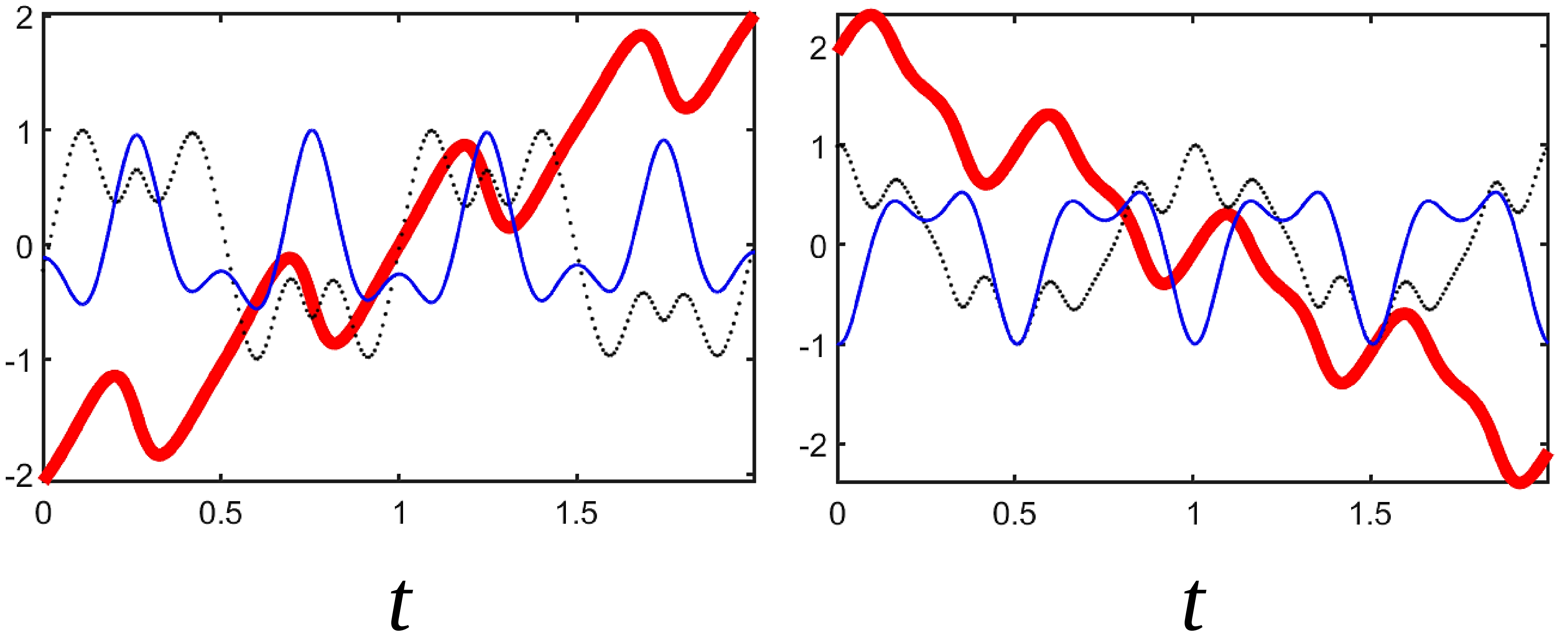
Spin precession in electric field



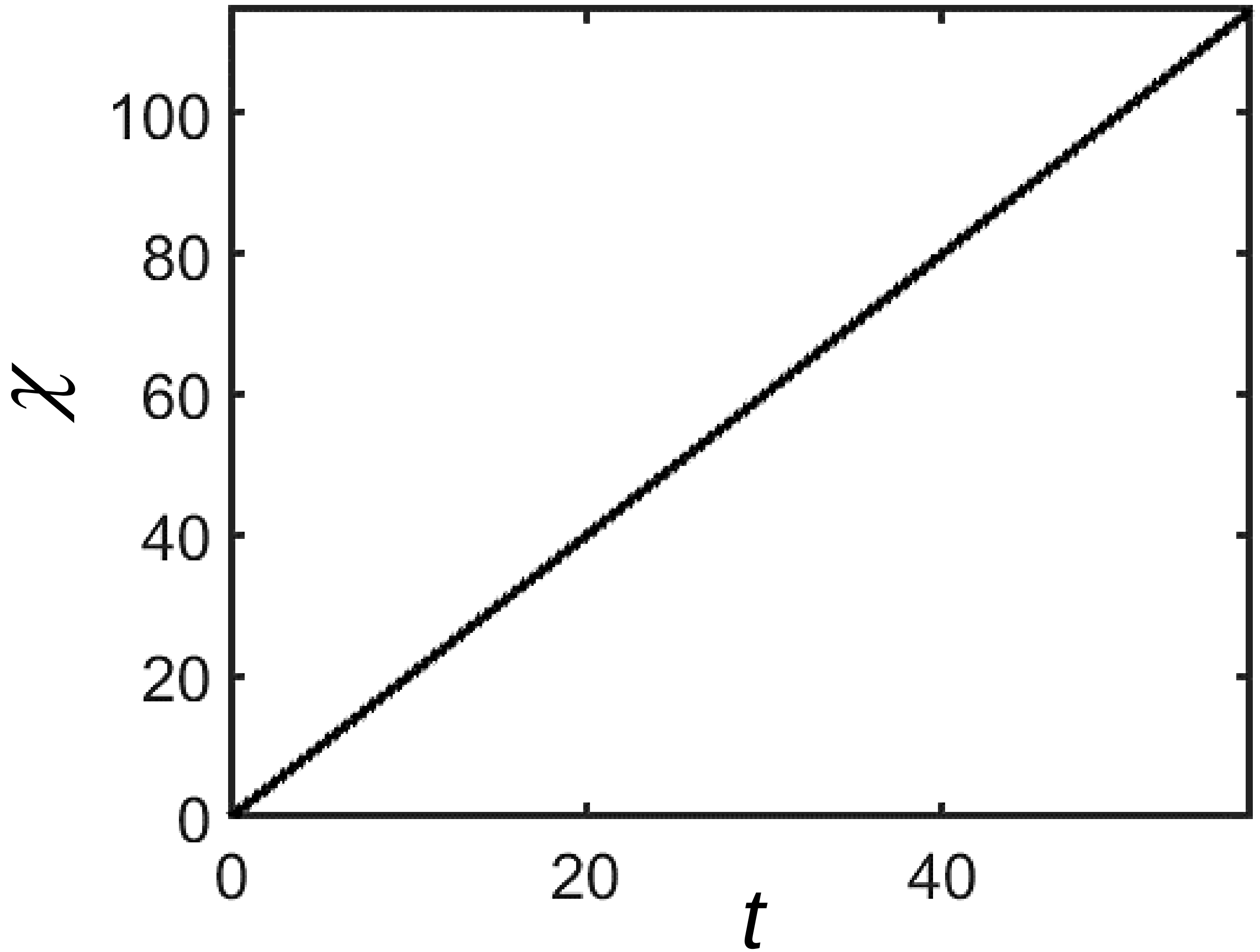


1
0.5
0
-0.5
-1
d_z
0
0.5
1
1.5
t

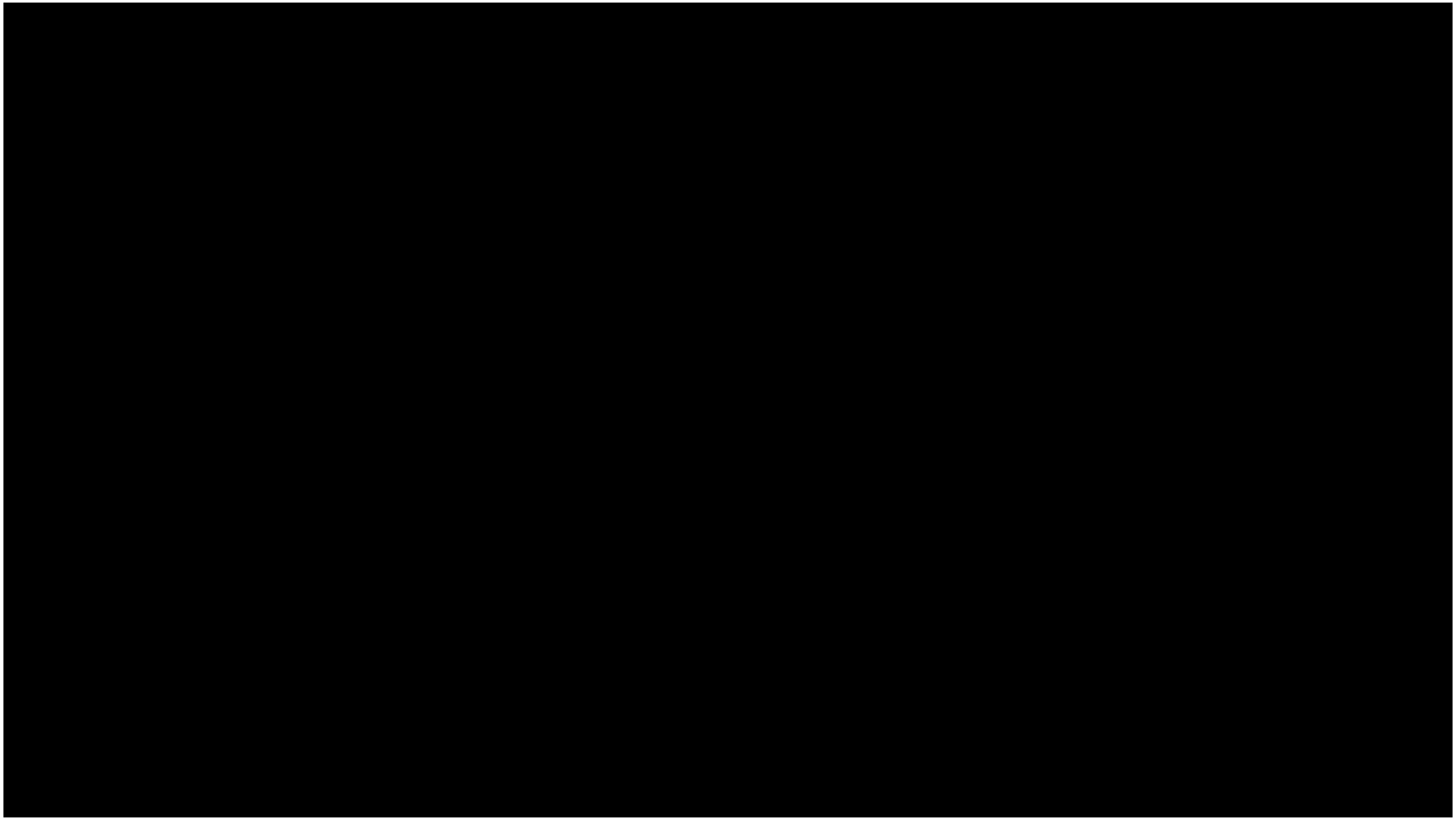
Clockwise and anticlockwise spin rotation



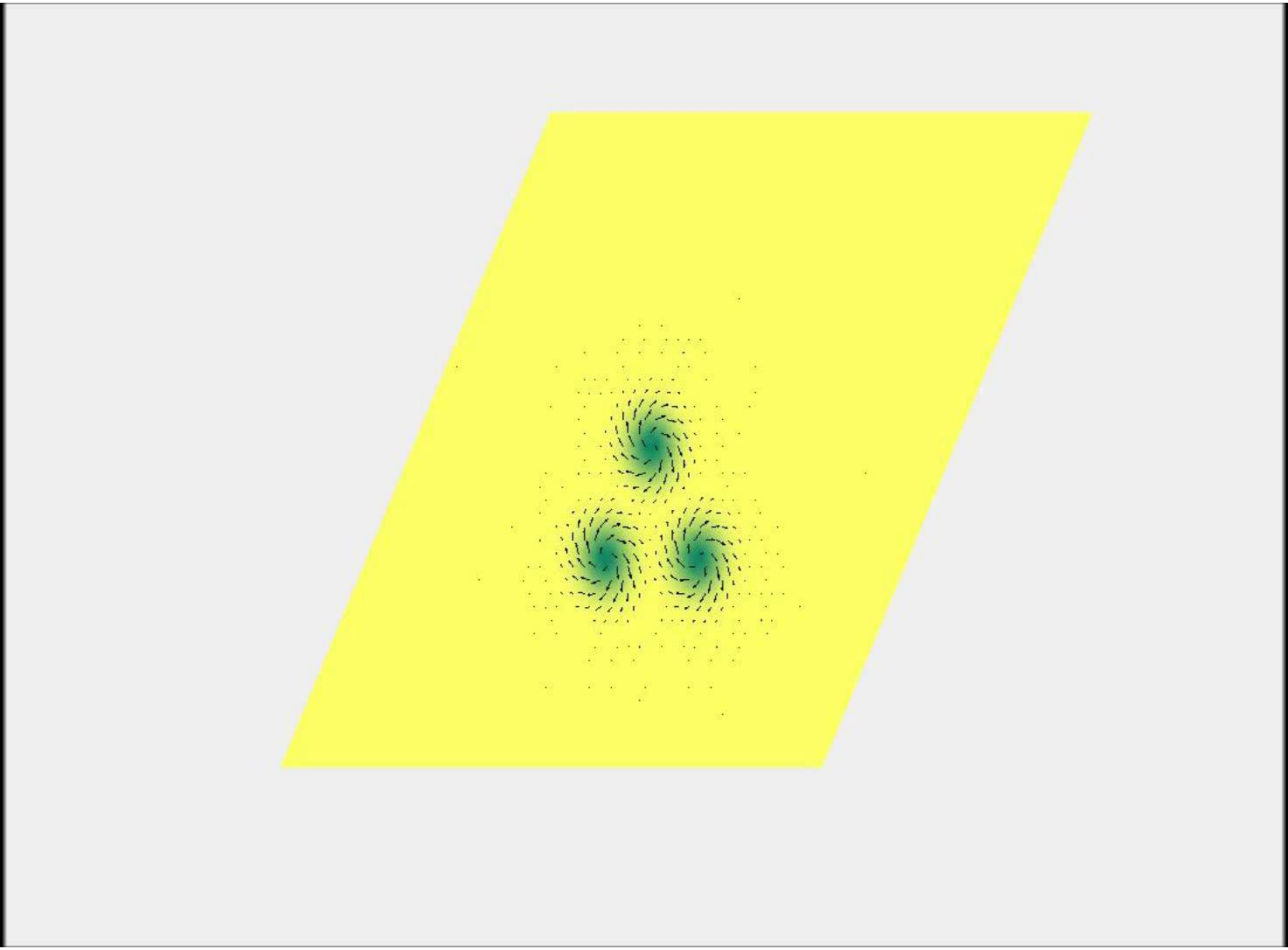
Persistent spin precession



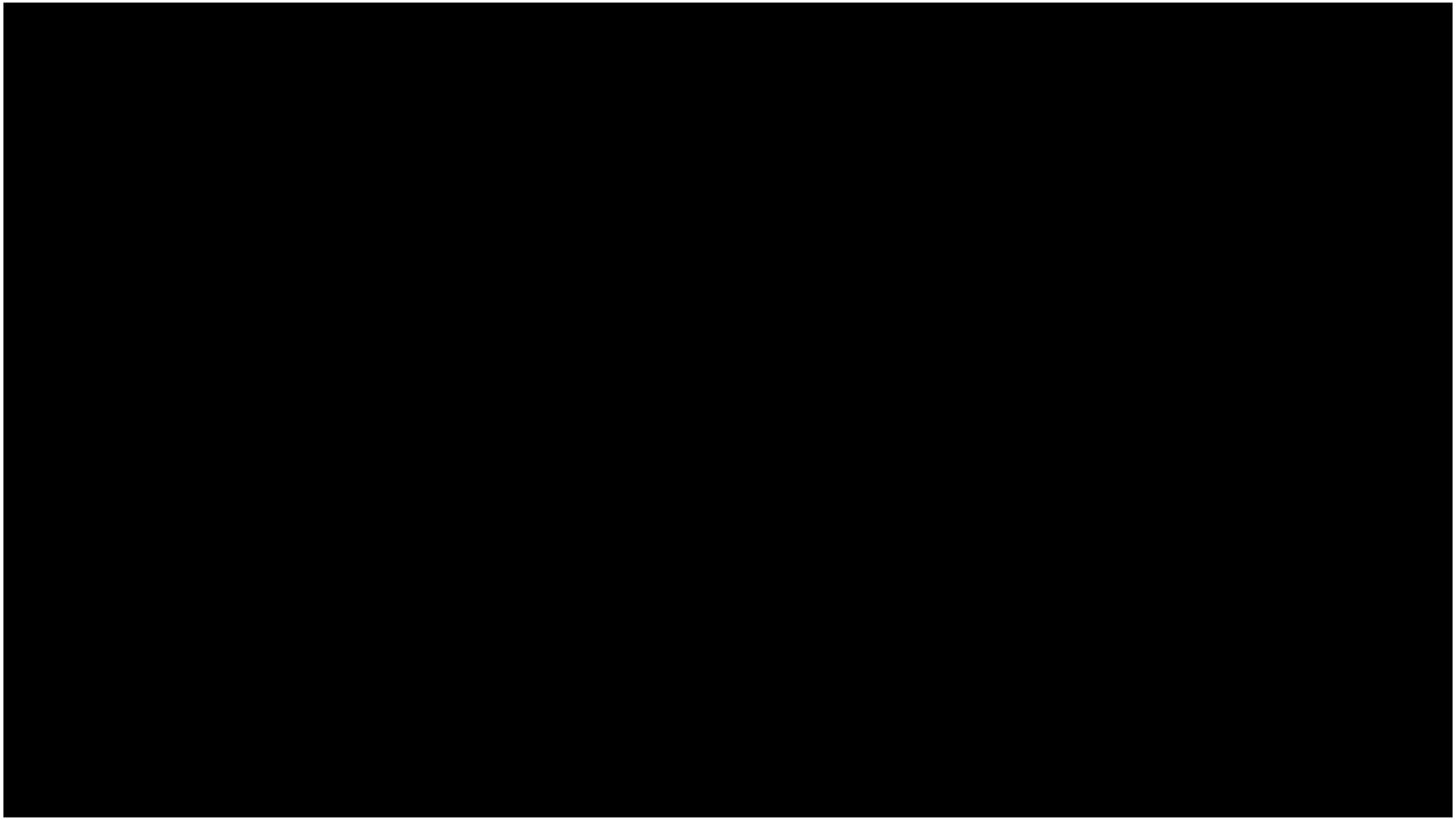
Electrically-induced rotation of skyrmion-skyrmion pair



E_z-excitation of 3 skyrmions



Electrically-induced motion of skyrmion-antiskyrmion pair



Skyrmion`s Electromagnon

Charge and topological densites

Electric polarization due to inverse DM mechanism

$$P_i = \lambda(m_j \partial_j m_i - m_i \partial_j m_j)$$

Electric charge density

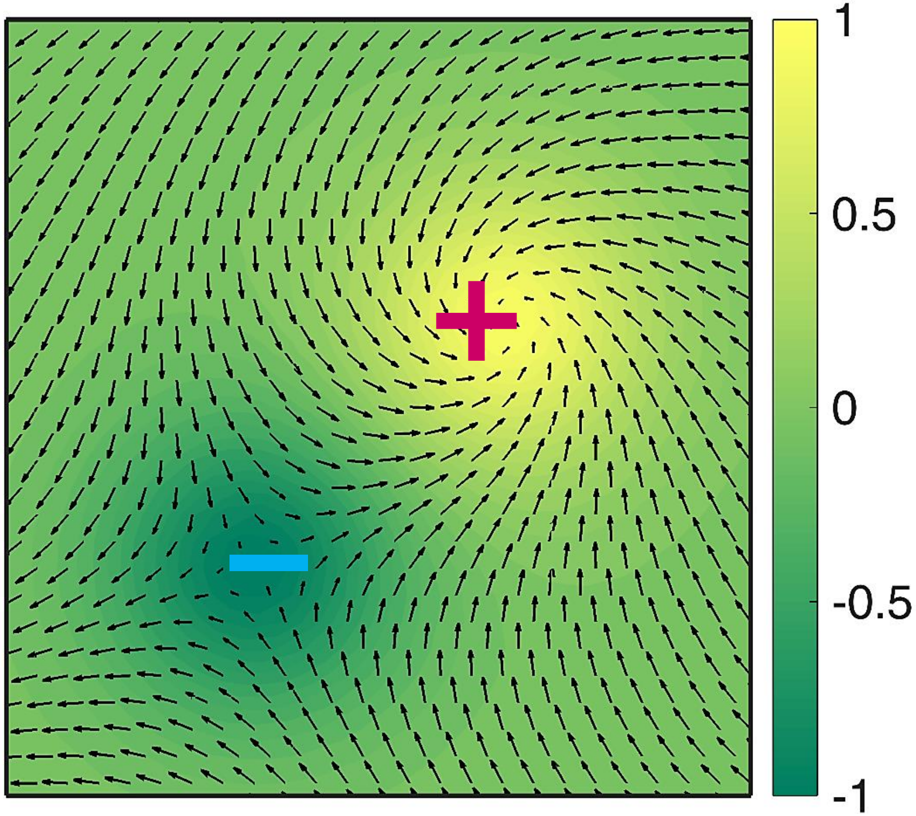
$$\boxed{\rho_e = -\mathrm{div}\,\mathbf{P} = 8\pi\lambda\rho_{top} m_z}$$

Topological charge density

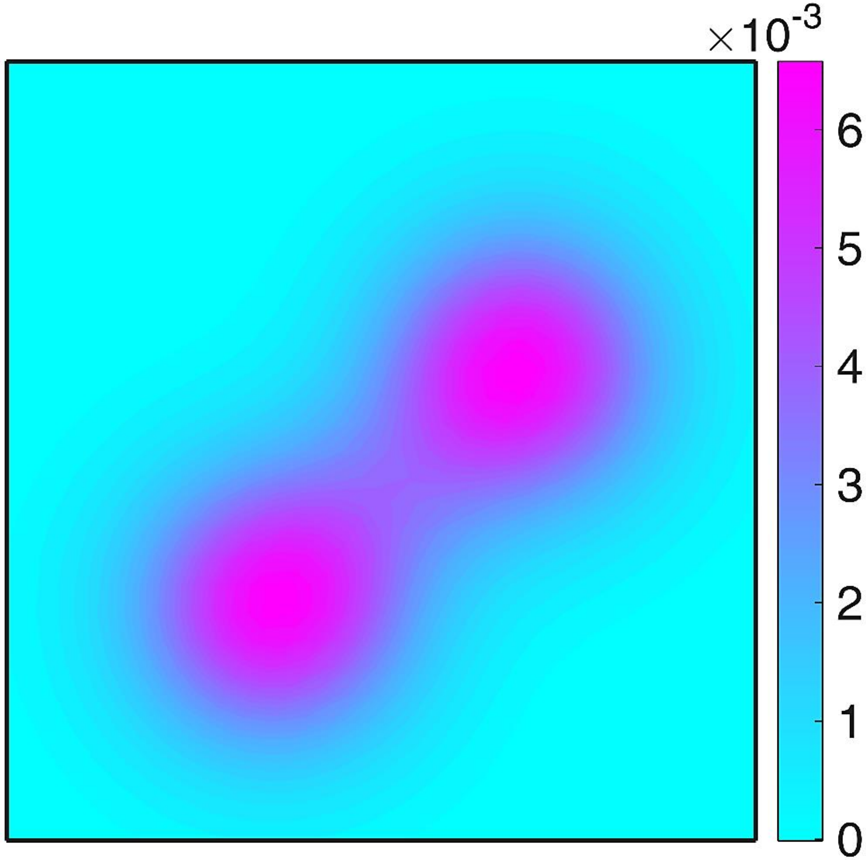
$$\rho_{top} = \frac{1}{4\pi}\left(\mathbf{m} \cdot [\partial_x \mathbf{m} \times \partial_y \mathbf{m}]\right)$$

Bi-merons

Spin configuration

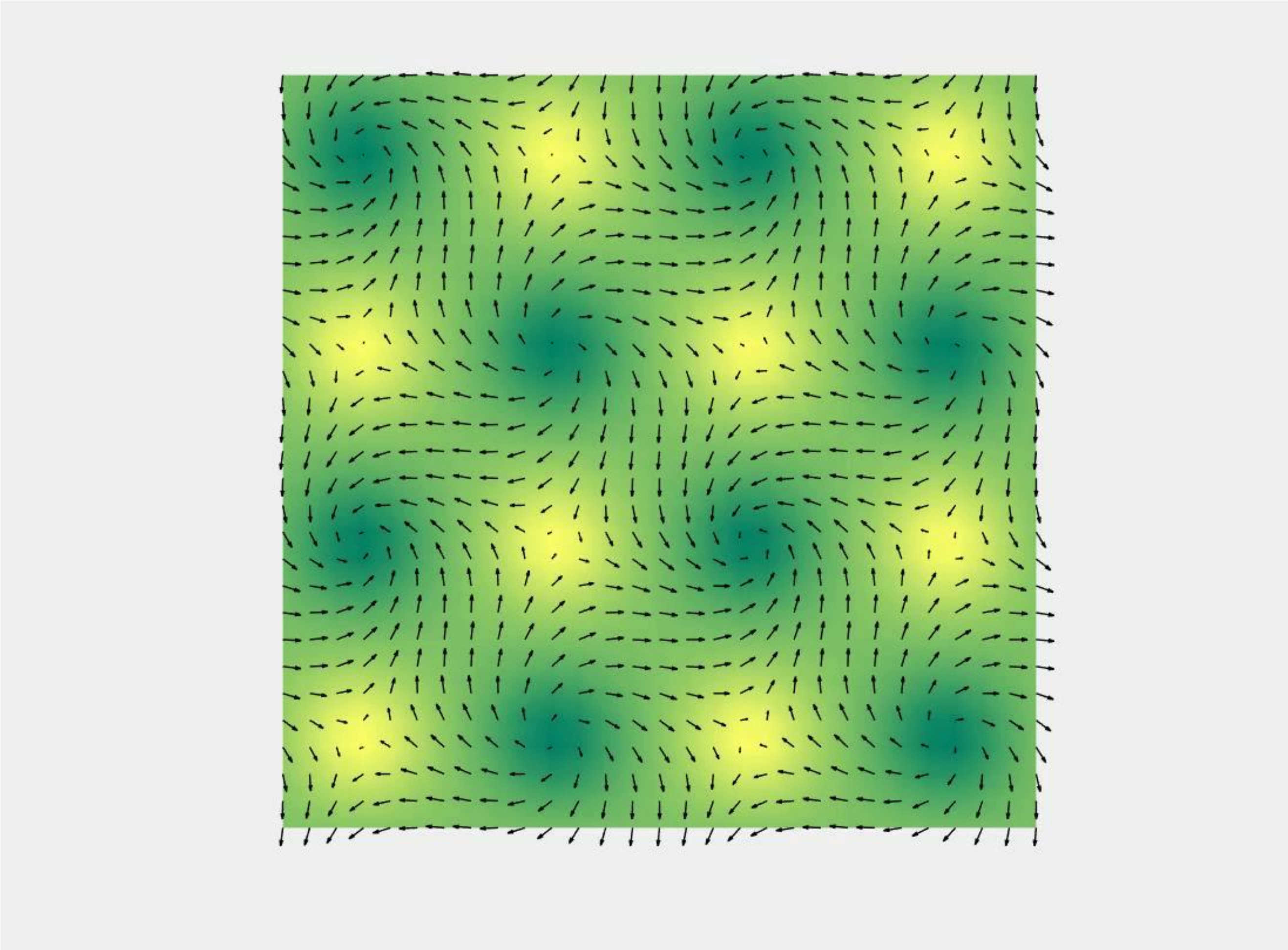


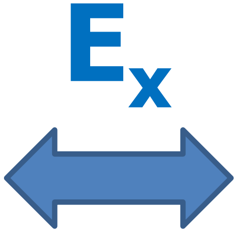
Topological density



MM in "The 2020 skyrmionics roadmap", J. Phys. D: Appl. Phys. 53, 363001 (2020)

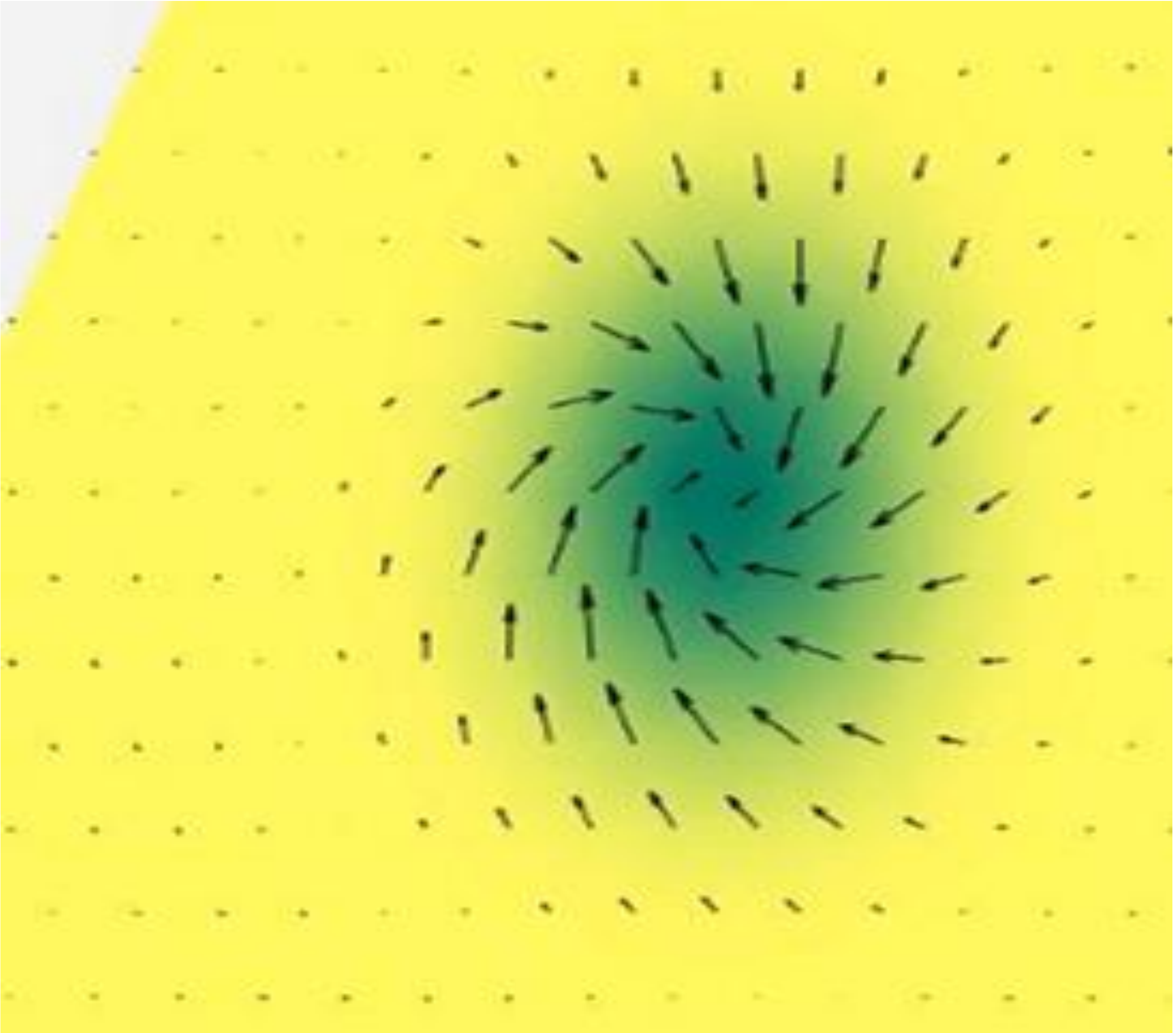
E_x excites plasma y-oscillations



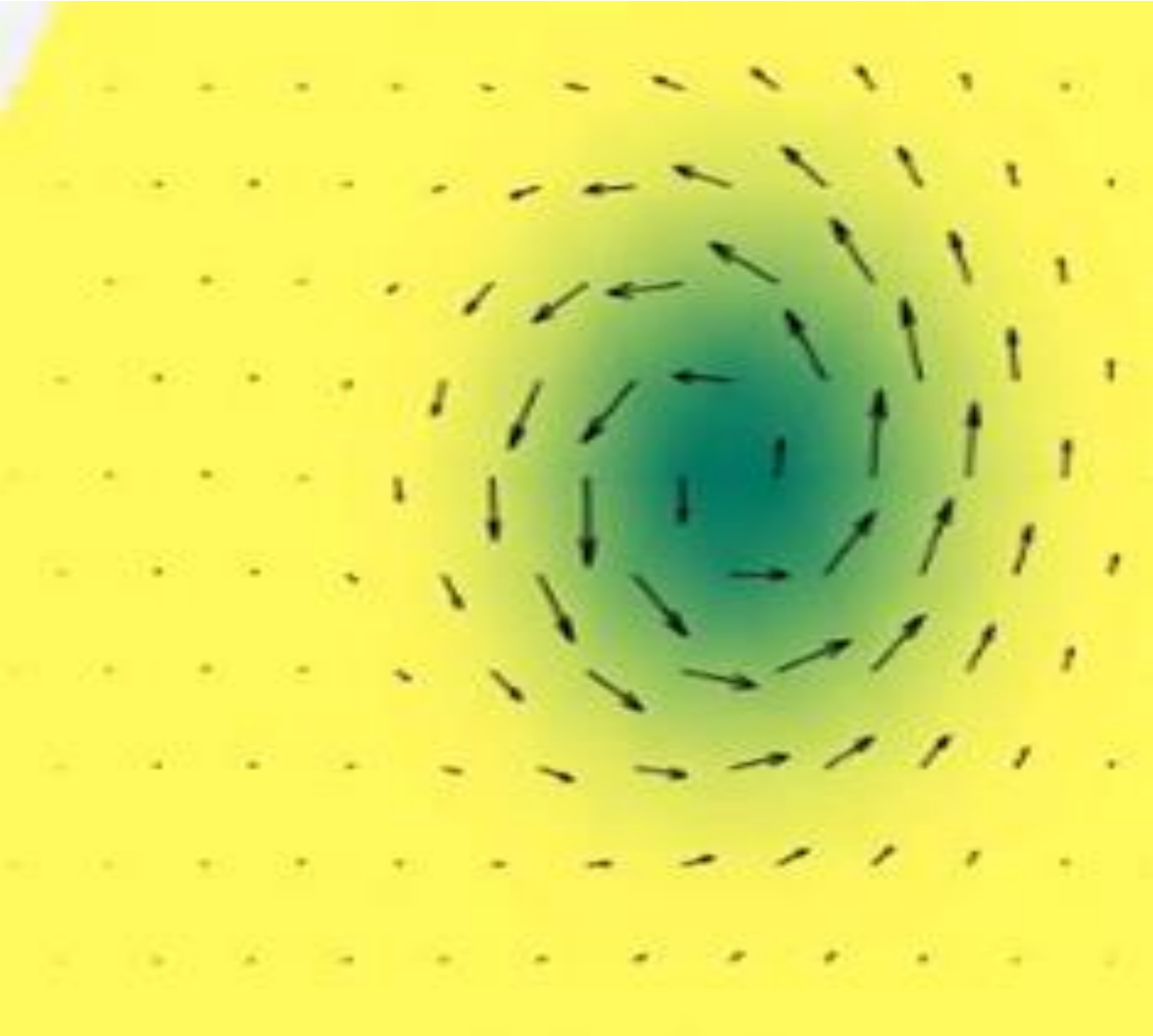


Spin precession in skyrmion excited by in-plane electromagnetic fields

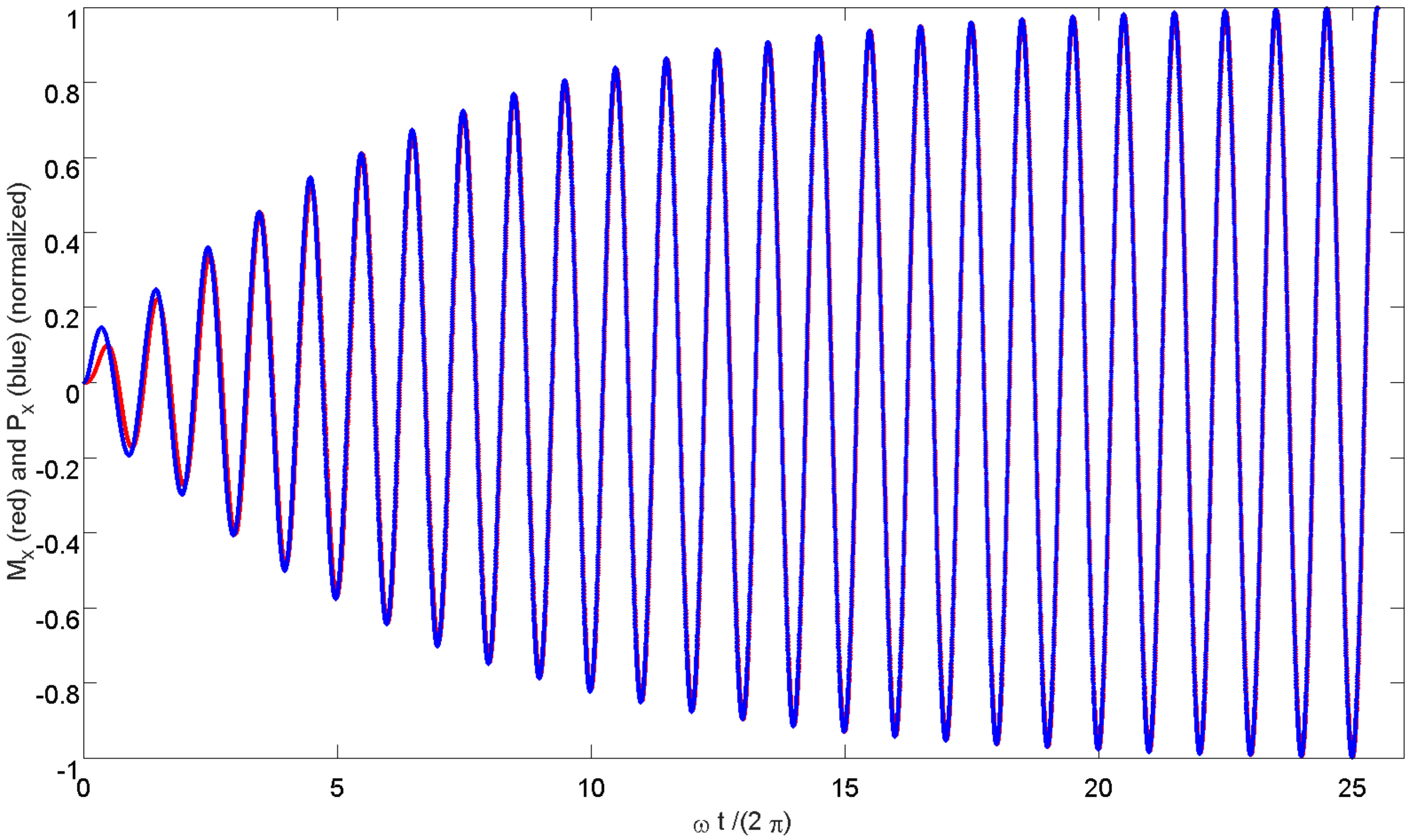
Skyrmion rotation in $h_x = h_0 \cos(\omega t)$



Skyrmion rotation $e_x = e_0\cos(\omega t)$

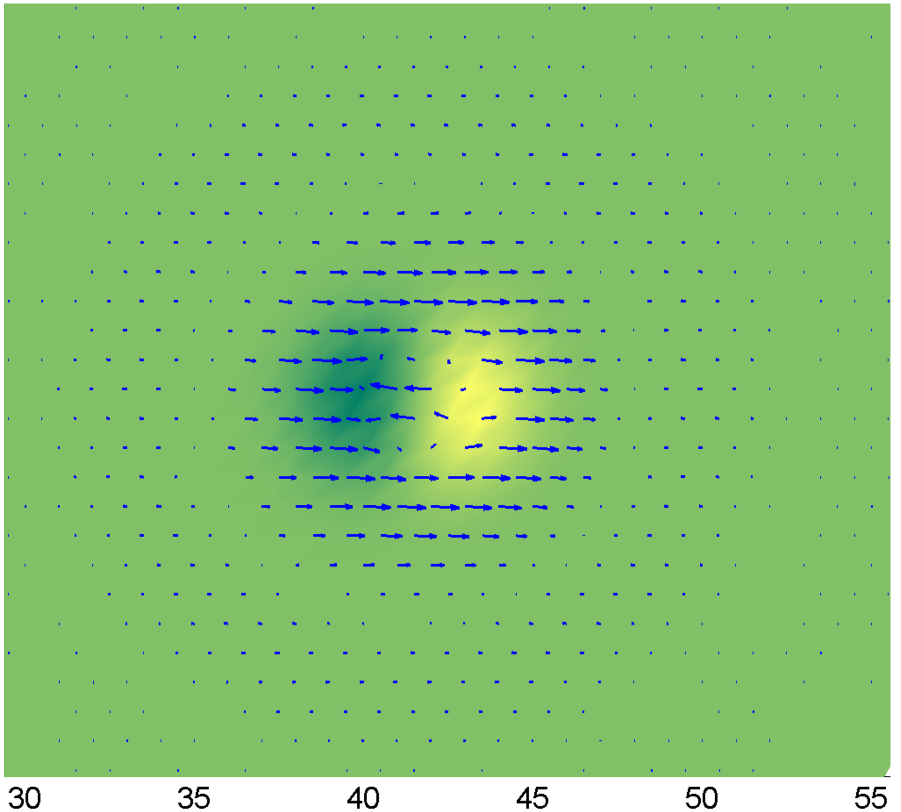


Electromagnon mode



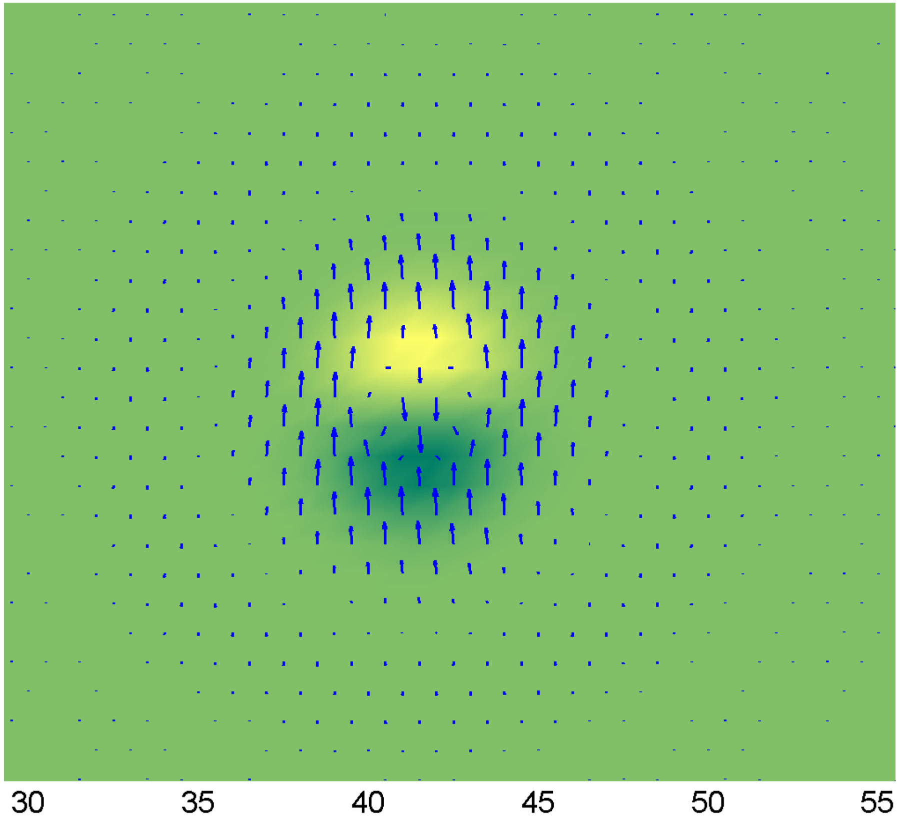
Localized mode with $\omega = 0.1068$

Q_1



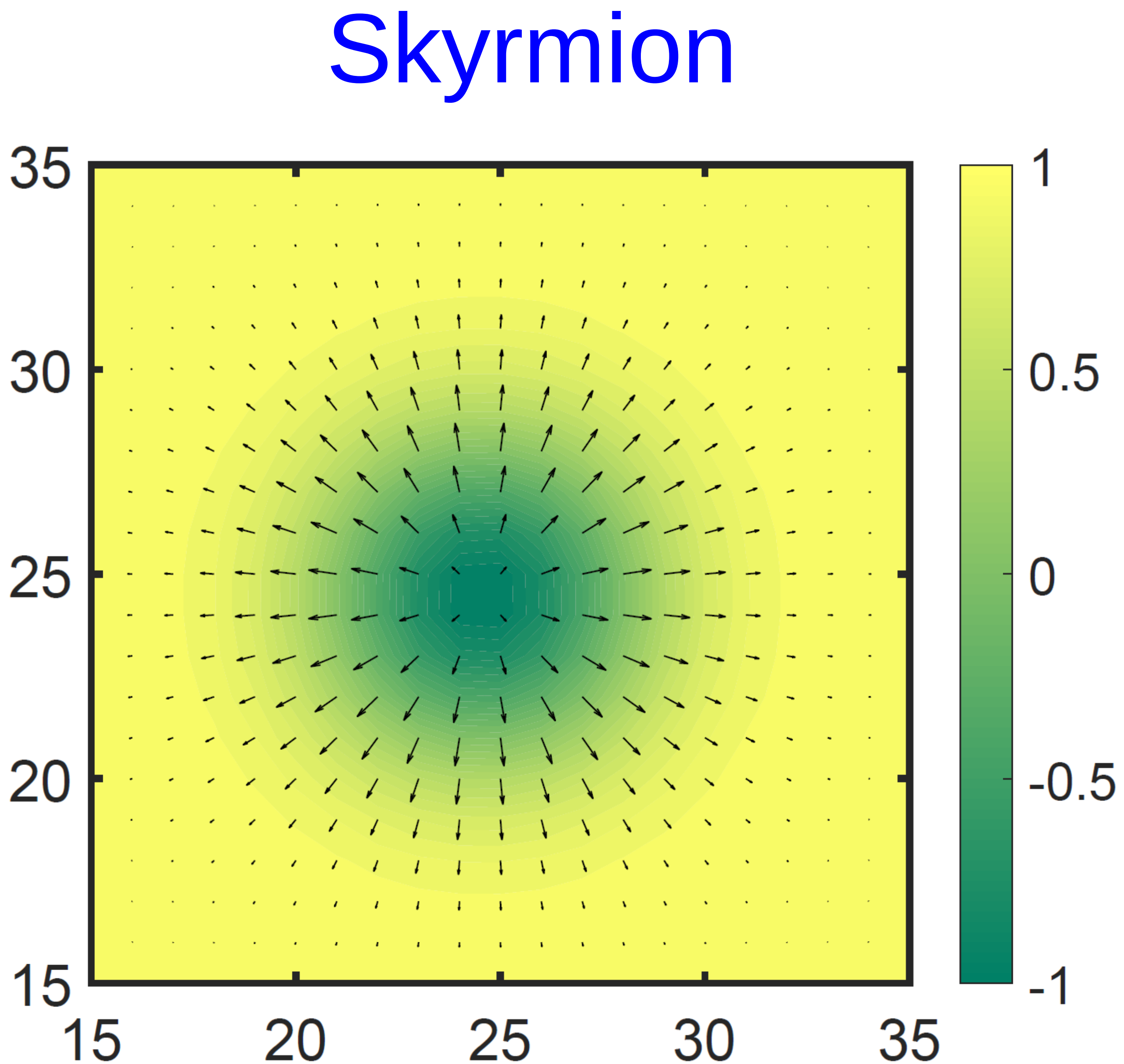
P_x, M_x

Q_2

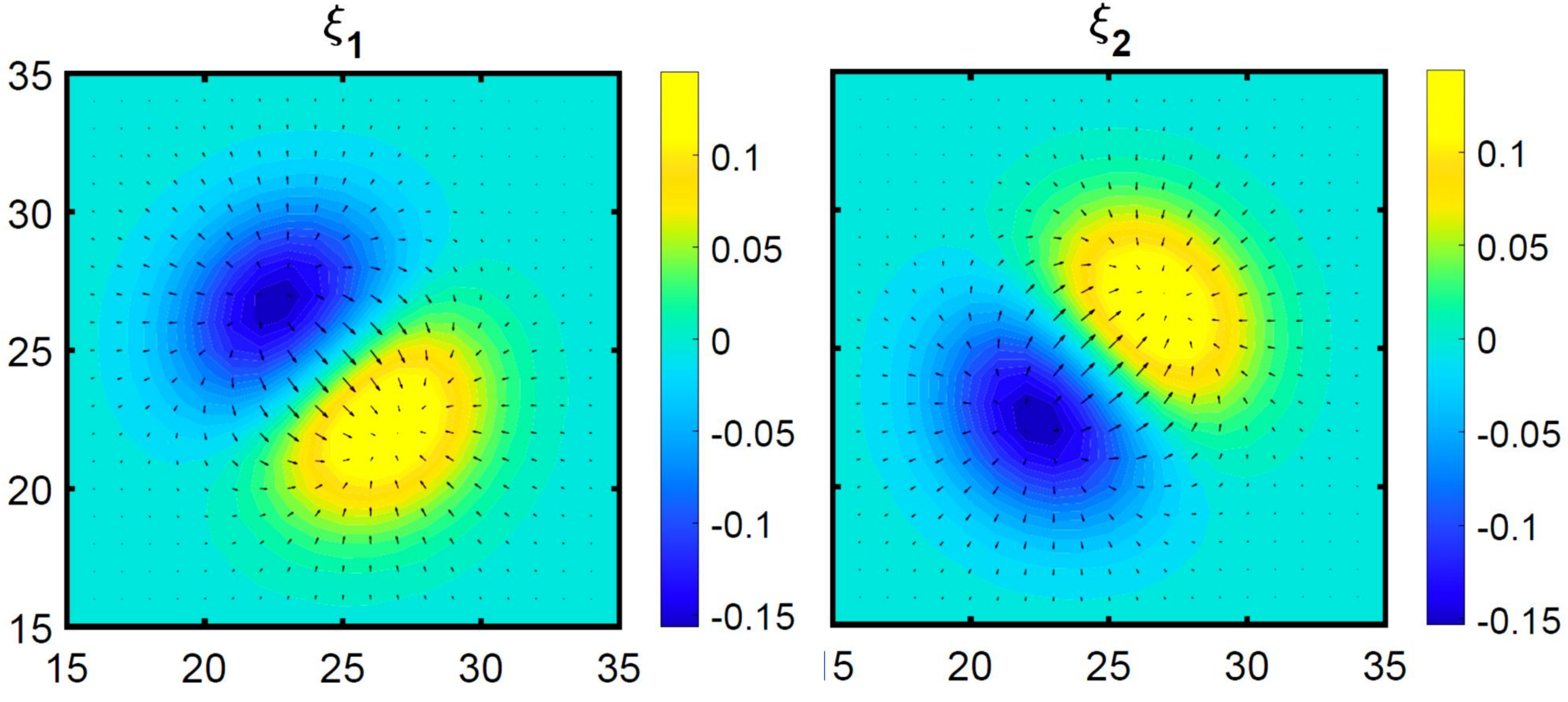


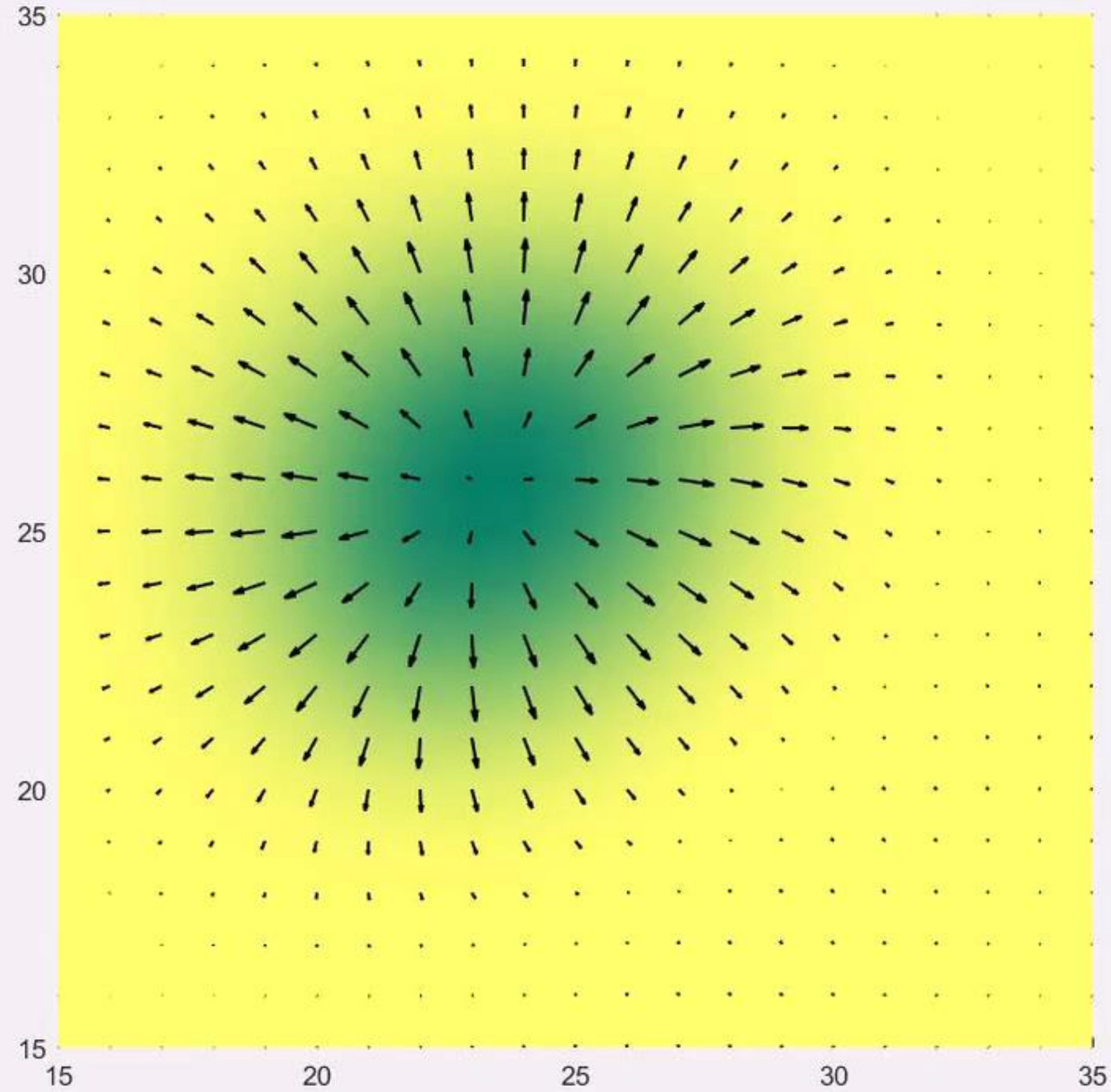
P_y, M_y

Skyrmion excitation modes


Skyrmion
35
30
25
20
15
15
20
25
30
35
1
0.5
0
-0.5
-1

Translation

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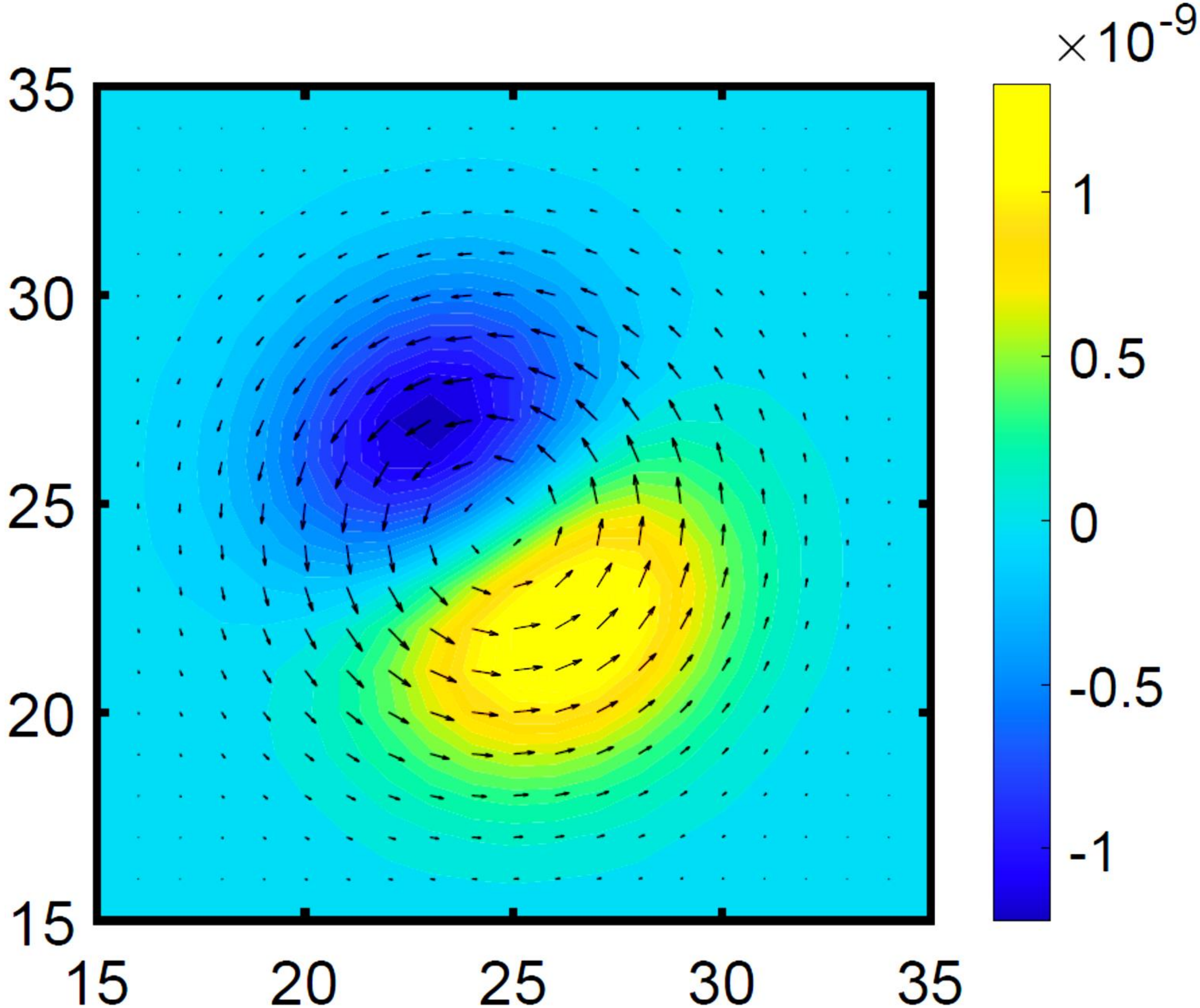
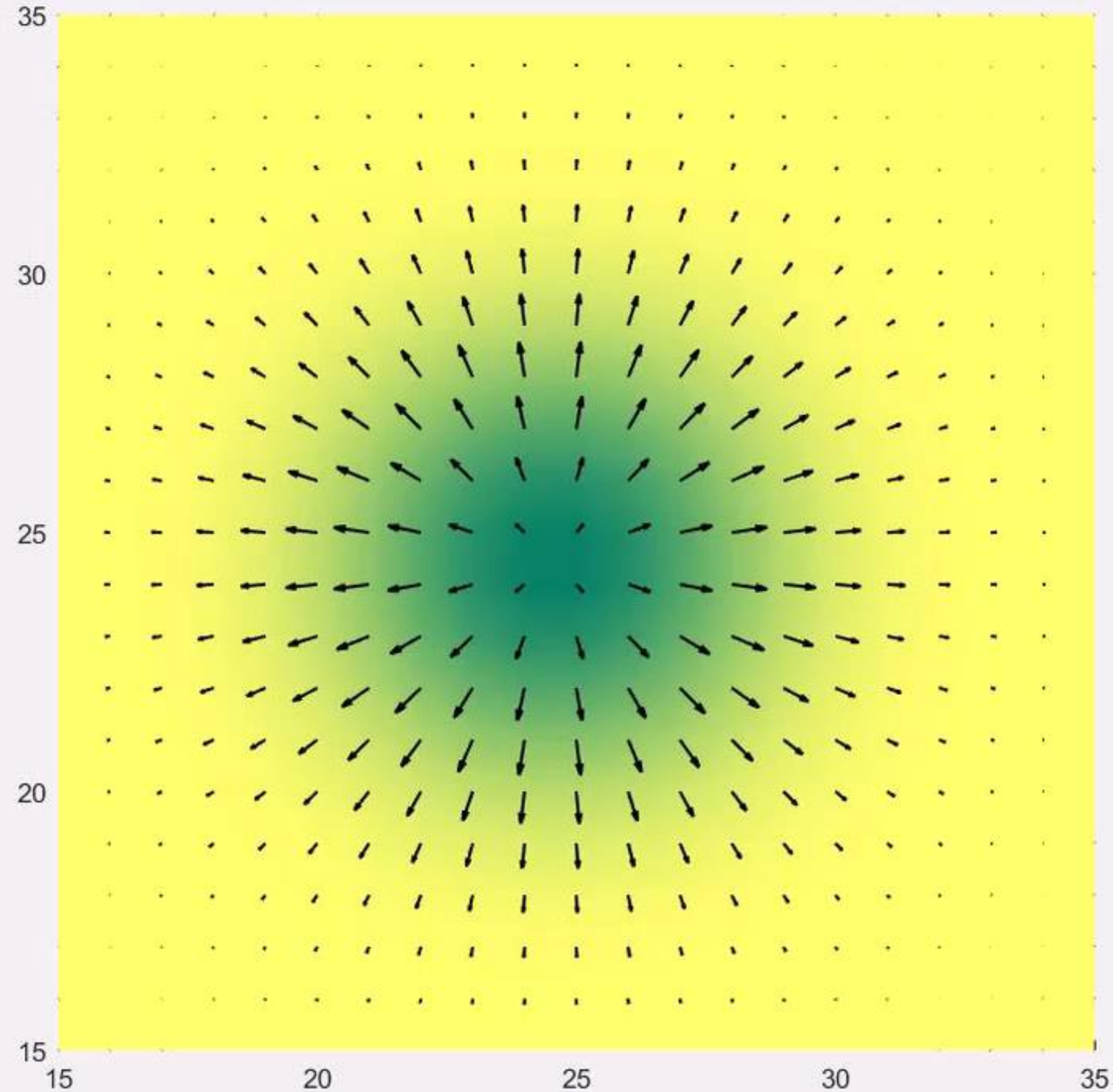
35

Rotation





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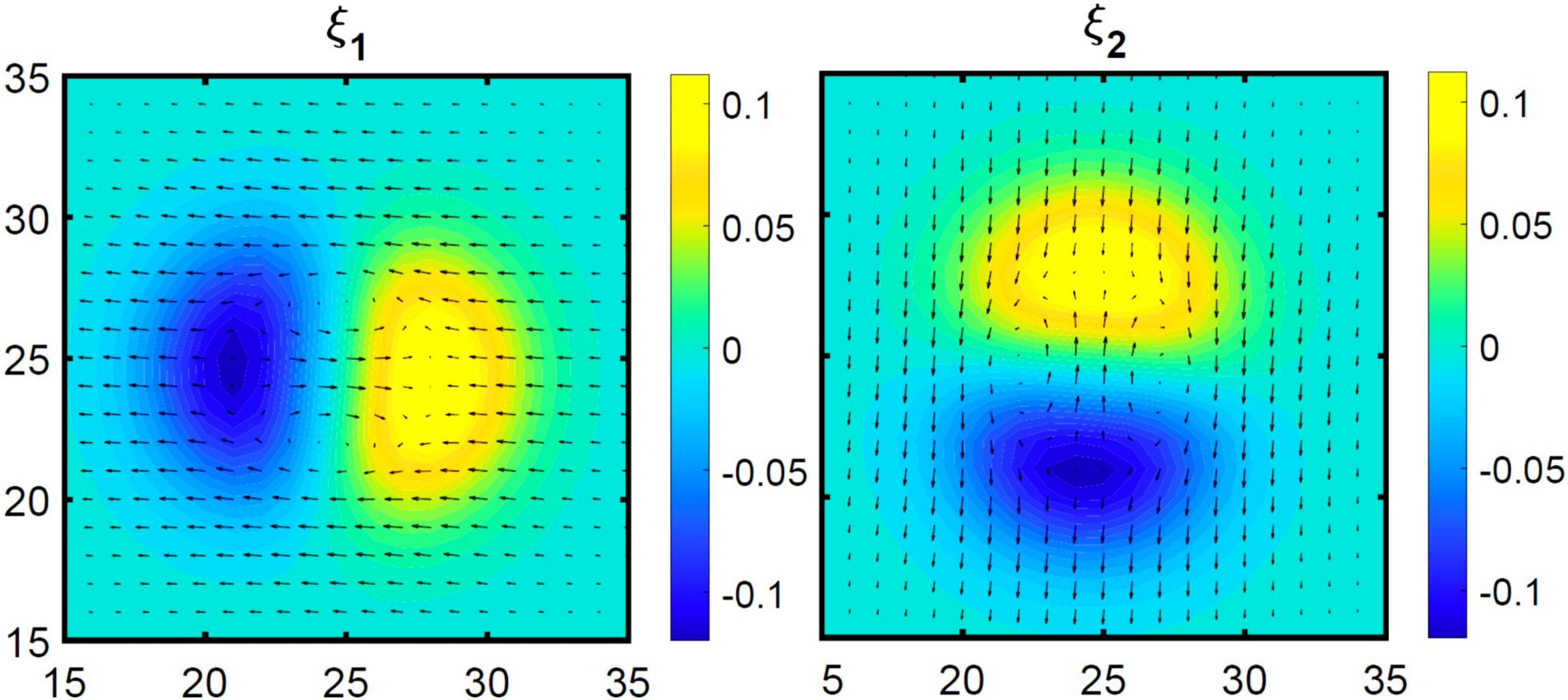
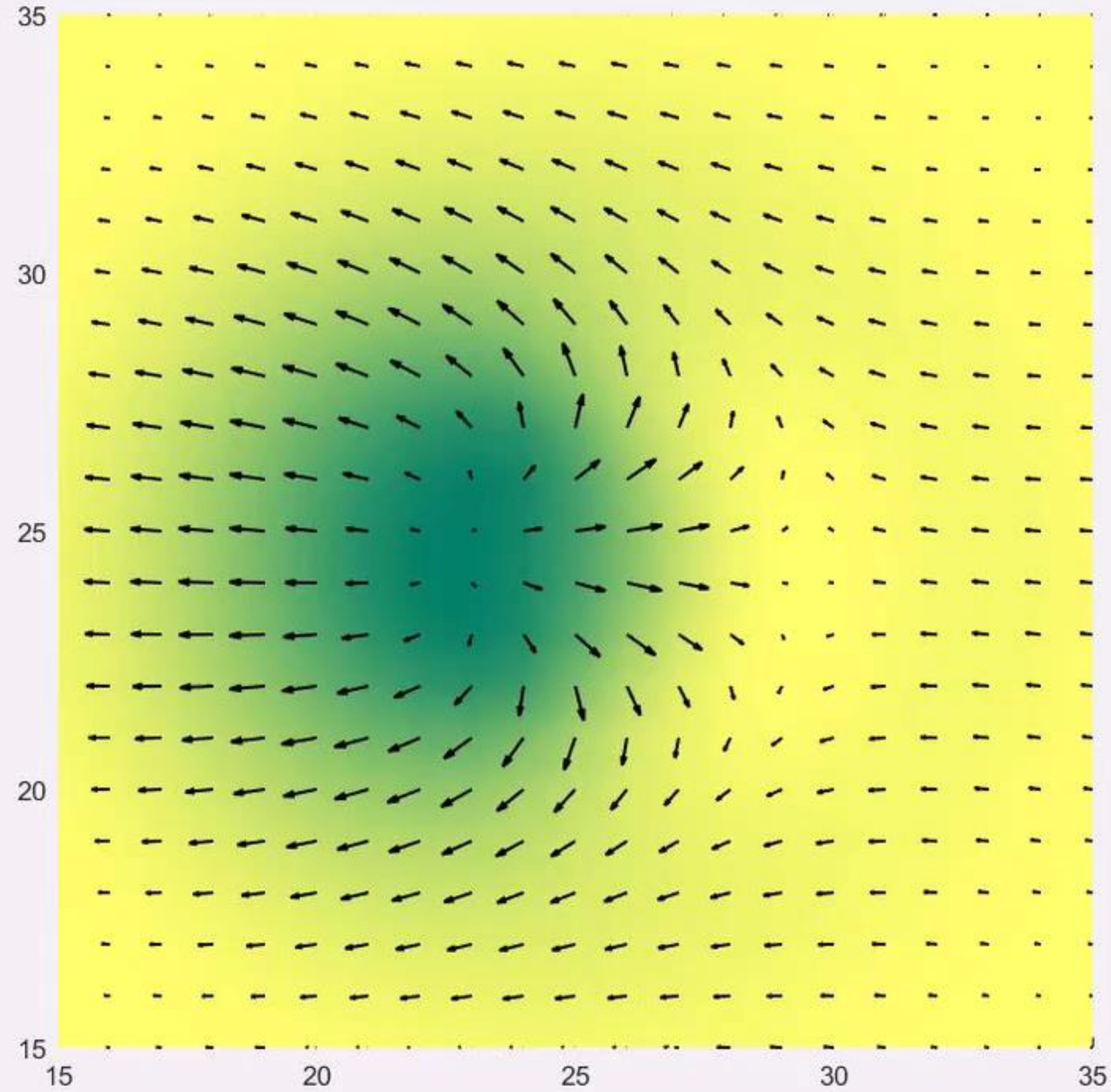
35

Electromagnon





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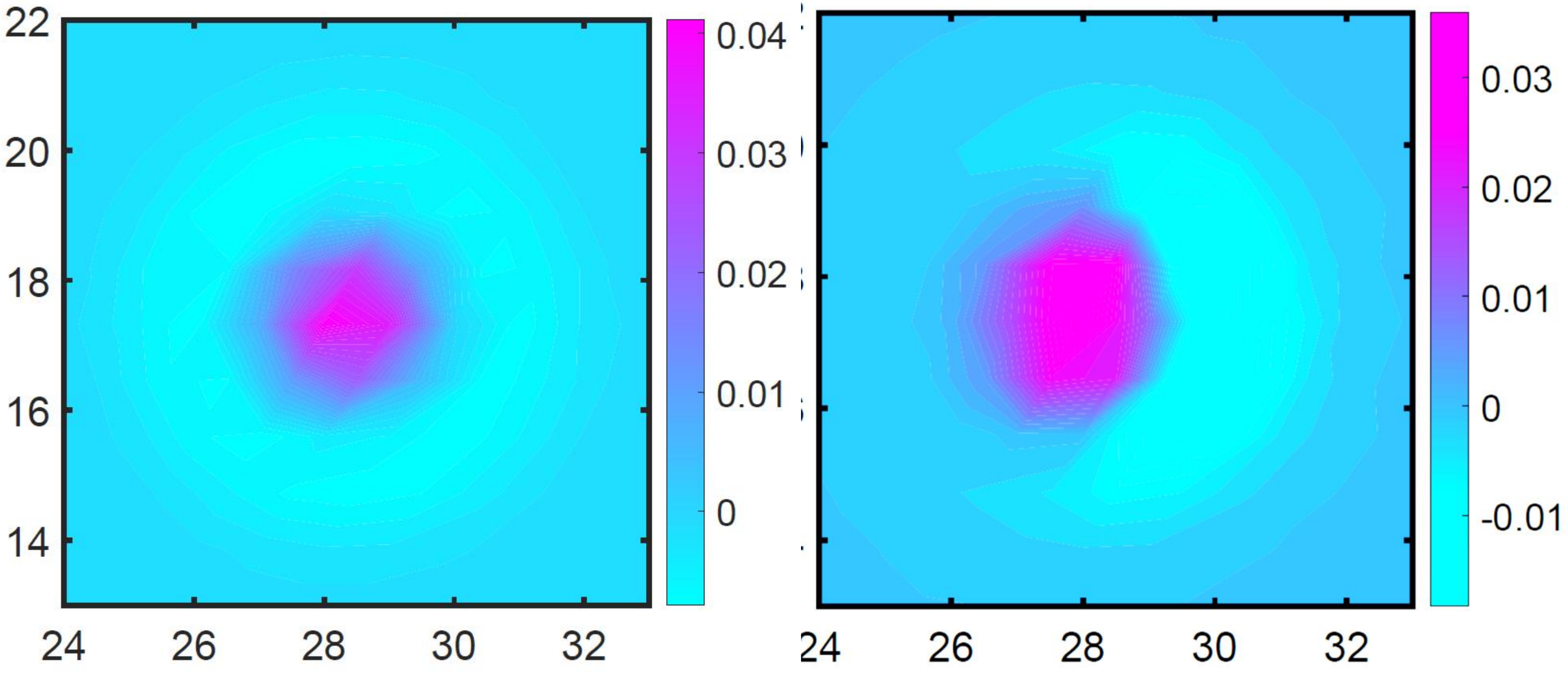
30
35

Charge density



Electromagnon

Skyrmion $\quad \vec{m}(\theta_0, \phi_0) \quad \theta_0 = \theta_0(r) \quad \phi_0 = \varphi + \chi$

Electromagnon mode $\quad \delta\vec{m} = Q_1 \vec{\xi}_1 + Q_2 \vec{\xi}_2 = Q\left(\cos\psi\, \vec{\xi}_1 + \sin\psi\, \vec{\xi}_2\right)$

$$\delta m_x = Q\, h(r) \cos(\psi + \chi)$$

$$\delta m_y = Q\, h(r) \sin(\psi + \chi)$$

$$\delta m_z = -Q\, tg\,\theta_0(r)\, h(r) \sin(\psi - \varphi)$$

Magnetoelectric coupling

$$H_e = g_e \int d^2x\, E_i\, m_i \overleftrightarrow{\partial_j} m_j = -\lambda_e (Q_1 E_x + Q_2 E_y) = -\lambda_e Q (\cos\psi\, E_x + \sin\psi\, E_y)$$

Zeeman interaction

$$H_m = -g_m \int d^2x\, (\vec{H} \cdot \vec{m}) = -\lambda_m Q \left(\cos(\psi + \chi) E_x + \sin(\psi + \chi) E_y\right)$$

Thiele equations for skyrmion in electromagnetic fields

Lagrangian: $L = -\delta M_z \dot{\chi} + \frac{1}{2} G_Q Q^2 \left(\dot{\psi} + \frac{\dot{\chi}}{2} \right) - H$ $\qquad X = Y = 0$

Hamiltonian: $H = H_0(\delta M_z, Q) - H_e - H_m$

$$H_0 \approx \frac{1}{2}\left(K_Q + K_{QM}\,\delta M_z\right) Q^2 + \frac{1}{2} K_M\, \delta M_z^2 + \frac{1}{3} K_M'\, \delta M_z^3$$

Dissipation function: $F = \frac{\alpha}{2}\left[(\Gamma_\chi + \Gamma_\chi' Q^2)\dot{\chi}^2 + \Gamma_M\, \delta \dot{M}_z^2 + \Gamma_Q \left(\dot{Q}^2 + Q^2 \dot{\psi}^2\right) + 2\Gamma_{\psi\chi} Q^2 \dot{\psi}\dot{\chi} \right]$

Equations of motion:

$$-G_Q Q\left(\dot{\psi} + \frac{1}{2}\dot{\chi}\right) + \alpha\left(\Gamma_Q + \Gamma' Q^2\right)\dot{Q} = -\frac{\partial H}{\partial Q}$$

$$G_Q Q \dot{Q} + \alpha Q^2 \left(\Gamma_Q \dot{\psi} + \Gamma_{\psi\chi}\dot{\chi}\right) = -\frac{\partial H}{\partial \psi}$$

$$-\delta \dot{M}_z + \frac{1}{2} G_Q Q \dot{Q} + \alpha\left((\Gamma_\chi + \Gamma_\chi' Q^2)\dot{\chi} + \Gamma_{\psi\chi} Q^2 \dot{\psi} \right) = -\frac{\partial H}{\partial \chi}$$

$$\dot{\chi} + \alpha \Gamma_M\, \delta \dot{M}_z = -\frac{\partial H}{\partial\, \delta M_z}$$

Clockwise and anticlockwise spin precession

Rotating electric field: $(E_x, E_y) = E(\cos\omega t, \sin\omega t)$ $\quad H_e = -\lambda_e Q E \cos(\psi - \omega t)$

$$\psi(t) \approx \psi_0 + \omega t \,, \quad \dot{\psi} = \omega$$

$$-\delta \dot{M}_z + \frac{1}{2} G_Q Q \dot{Q} + \alpha\left((\Gamma_\chi + \Gamma_\chi' Q^2)\dot{\chi} + \Gamma_{\psi\chi} Q^2 \dot{\psi}\right) = -\frac{\partial H}{\partial \chi} = 0 \quad \overset{\text{time average}}{\Rightarrow} \quad \langle \dot{\chi} \rangle = -\frac{\Gamma_{\psi\chi} Q^2}{\Gamma_\chi + \Gamma_\chi' Q^2} \langle \dot{\psi} \rangle$$

total derivatives ($-\delta \dot{M}_z$ and $\frac{1}{2} G_Q Q \dot{Q}$); $\langle \dot{\psi} \rangle = \omega$

Rotating magnetic field: $(H_x, H_y) = H(\cos\omega t, \sin\omega t)$ $\quad H_m = -\lambda_m Q E \cos(\psi + \chi - \omega t)$

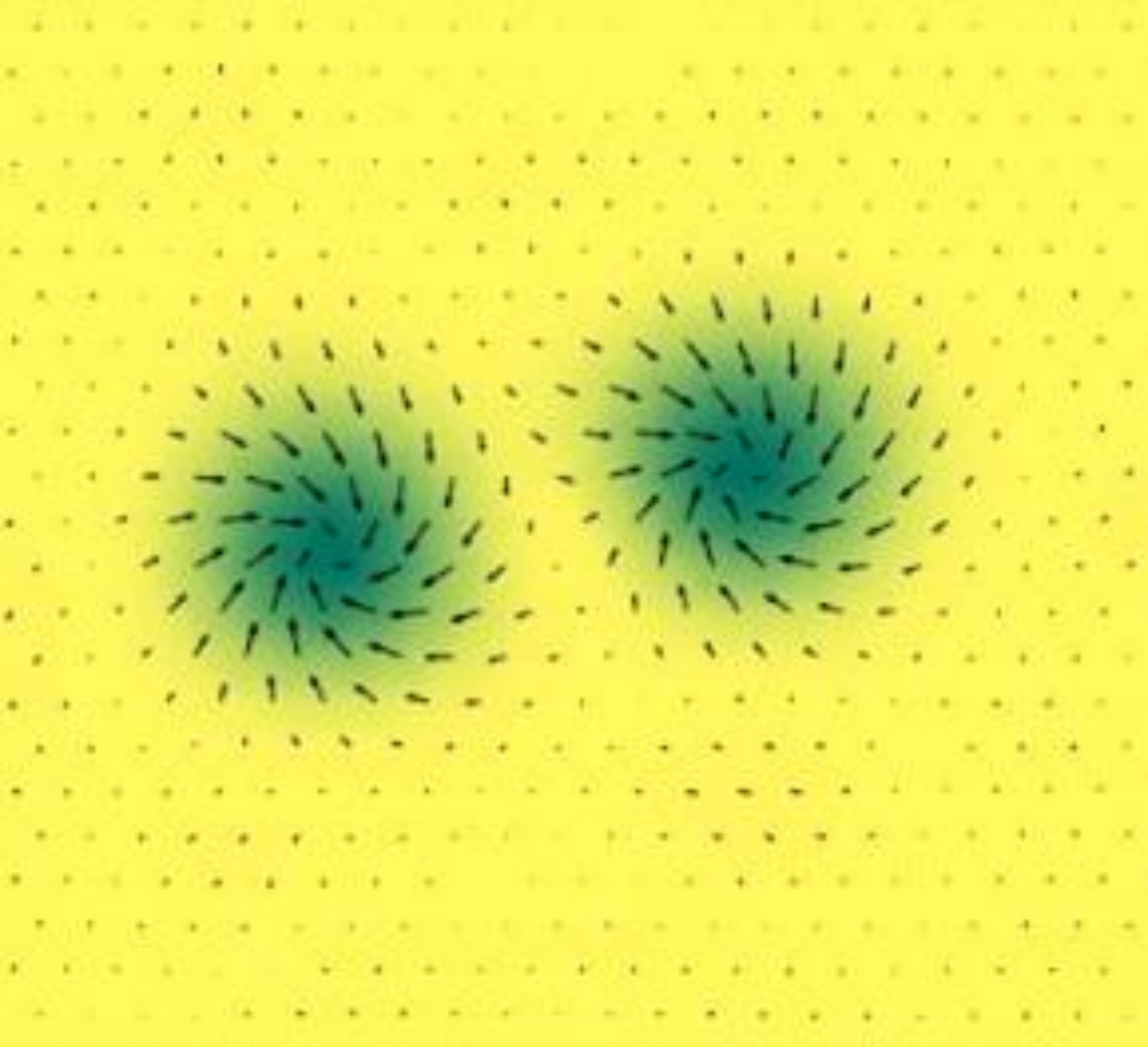
$$\dot{\psi} + \dot{\chi} = \omega$$

$$\alpha\left[\Gamma_Q Q^2 \langle \dot{\psi} \rangle + \Gamma_{\psi\chi} Q^2 \langle \dot{\chi} \rangle\right] = -\frac{\partial H}{\partial \psi}$$

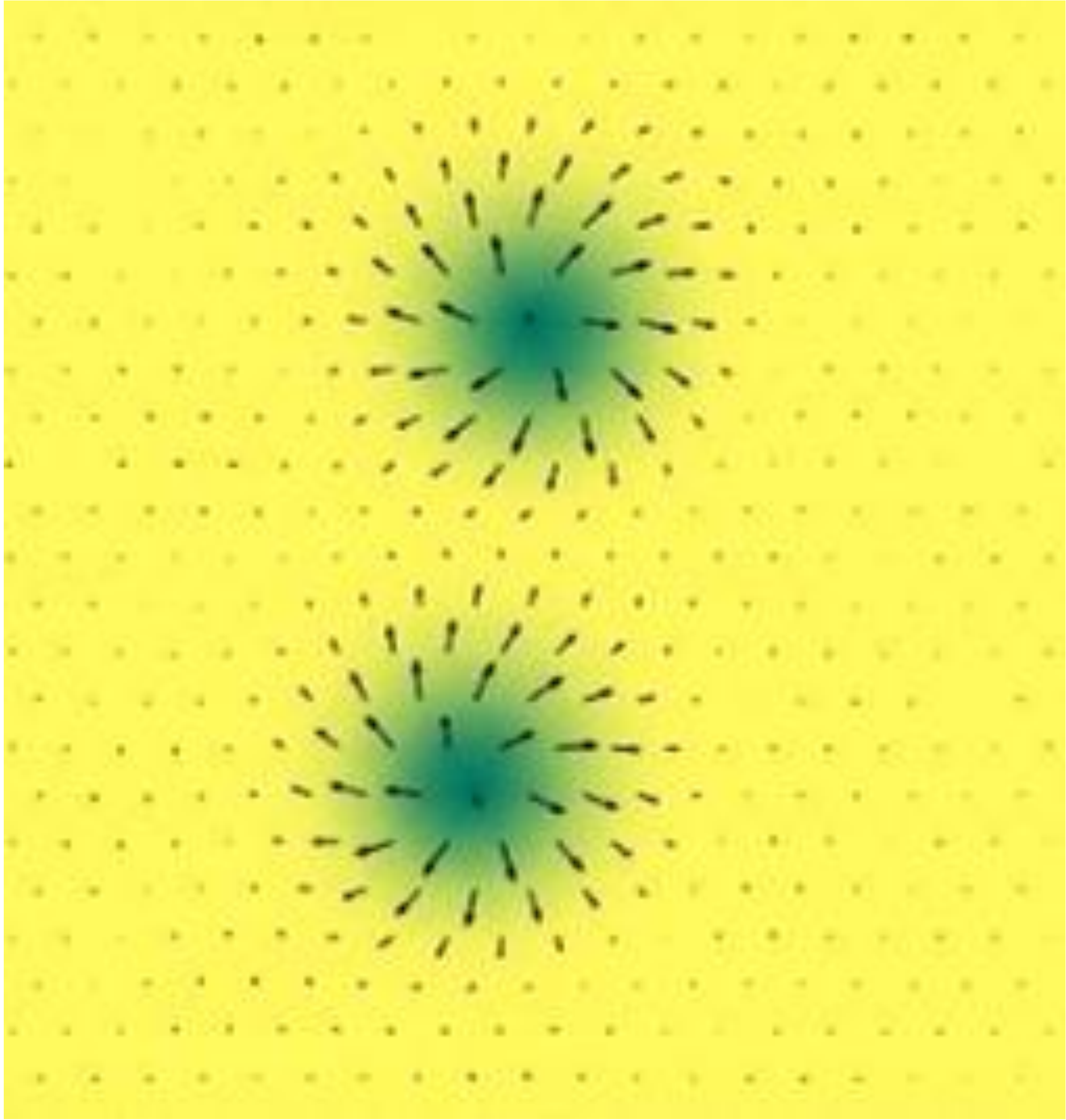
$$\alpha\left[(\Gamma_\chi + \Gamma_\chi' Q^2)\langle \dot{\chi} \rangle + \Gamma_{\psi\chi} Q^2 \langle \dot{\psi} \rangle\right] = -\frac{\partial H}{\partial \chi}$$

$$\frac{\partial H}{\partial \psi} = \frac{\partial H}{\partial \chi} \qquad \langle \dot{\chi} \rangle = +\frac{(\Gamma_Q - \Gamma_{\psi\chi}) Q^2}{\Gamma_\chi + (\Gamma_\chi' - \Gamma_{\psi\chi}) Q^2} \langle \dot{\psi} \rangle$$

Skyrmion pair in $h_x = h_0\cos(\omega t)$

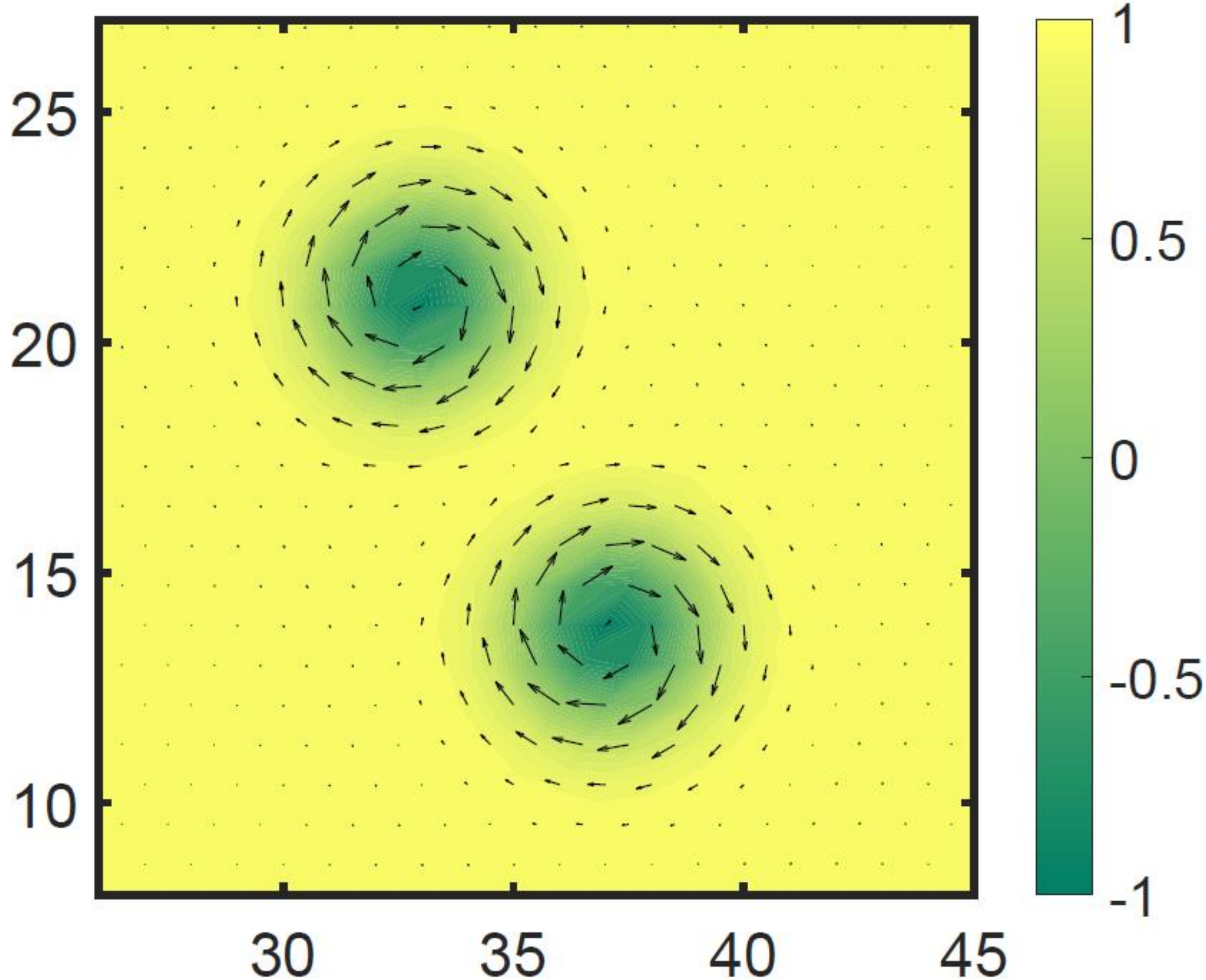


Skyrmion pair in $e_x = e_0\cos(\omega t)$

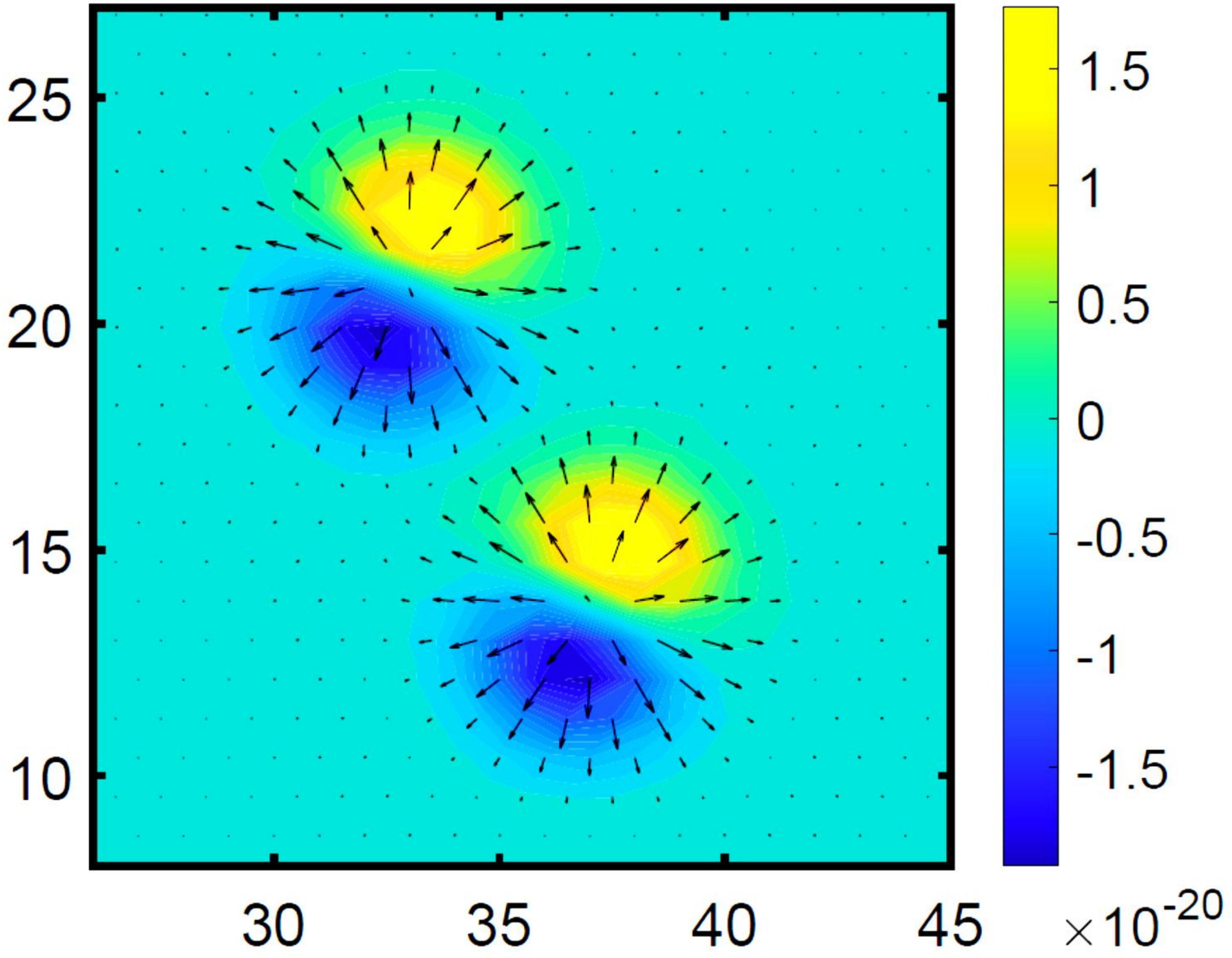


Excitations of skyrmion pair

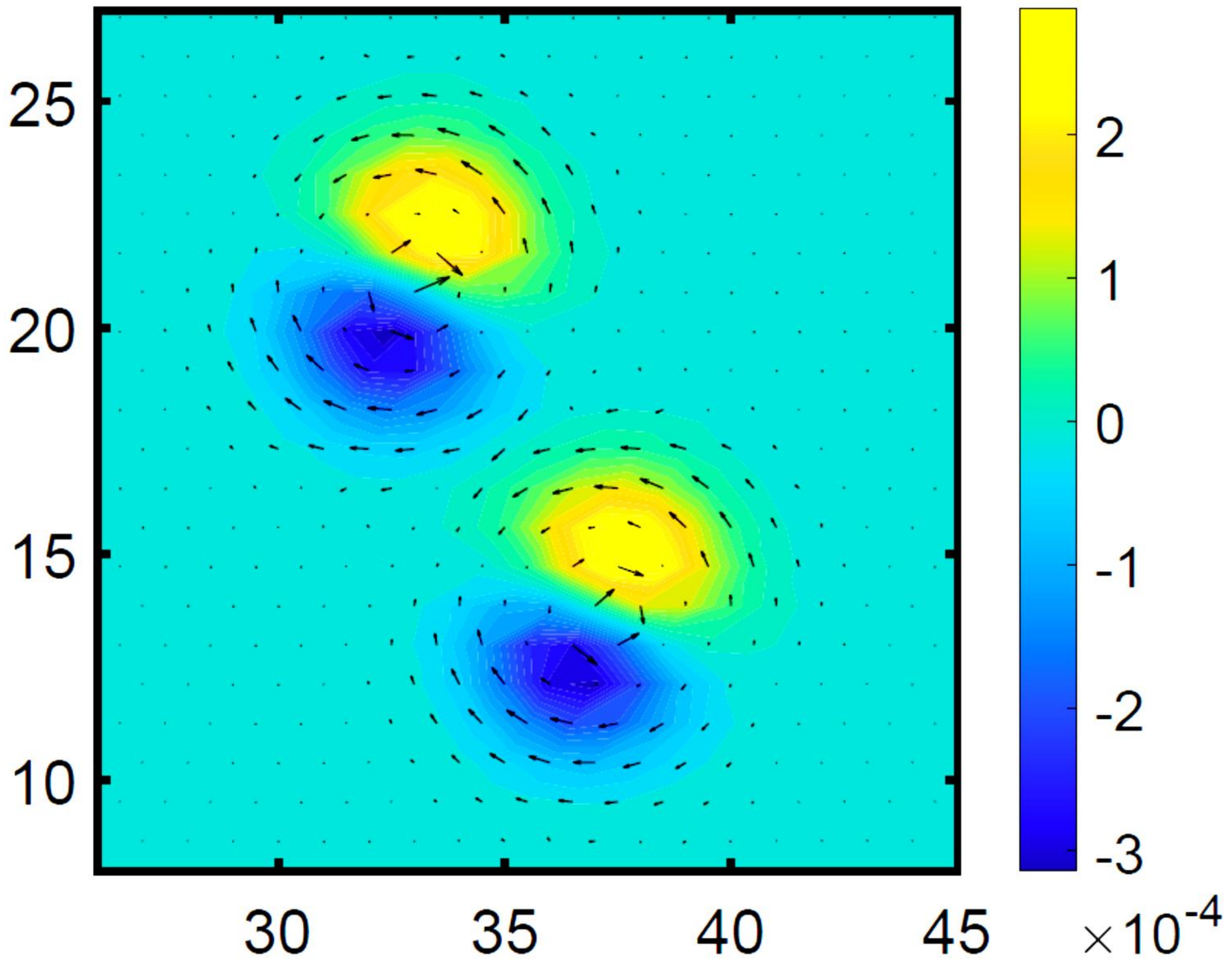
Skyrmion pair



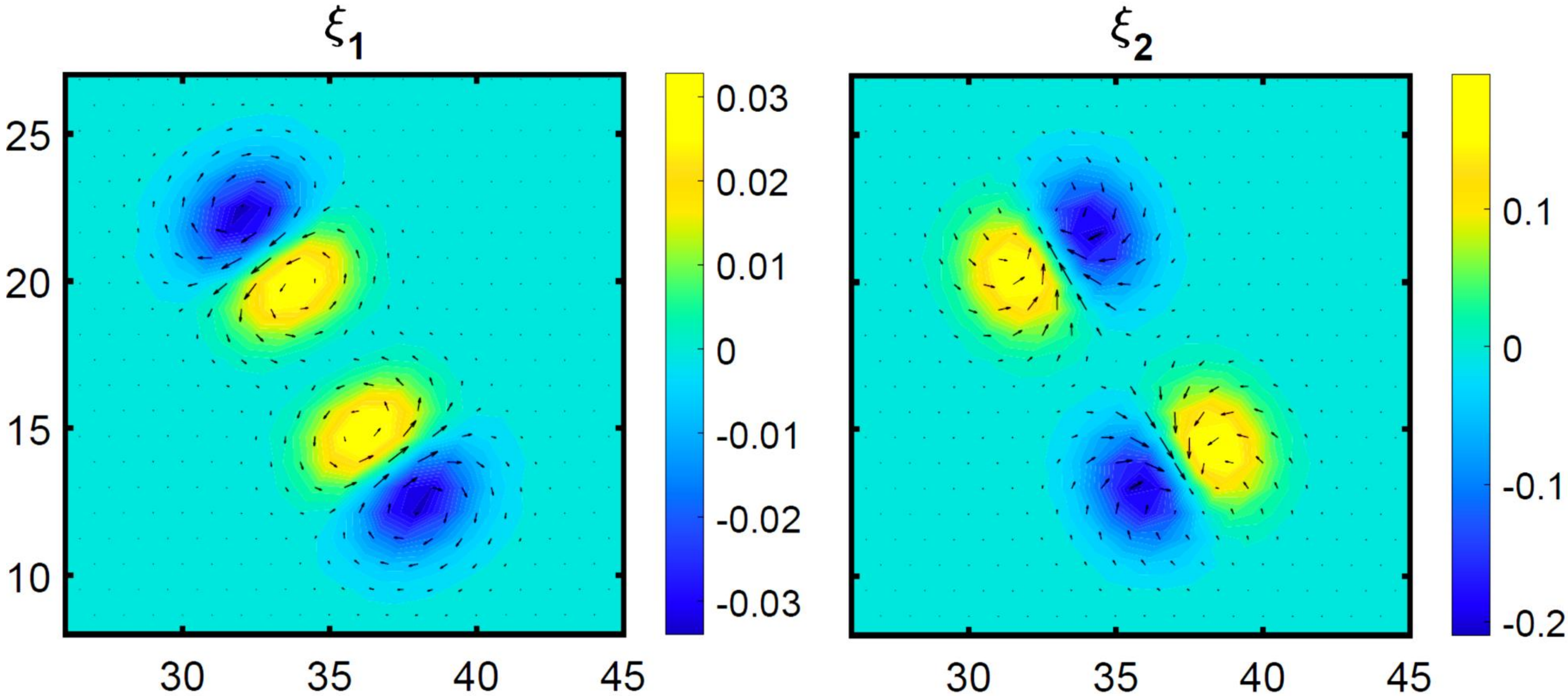
Rotation



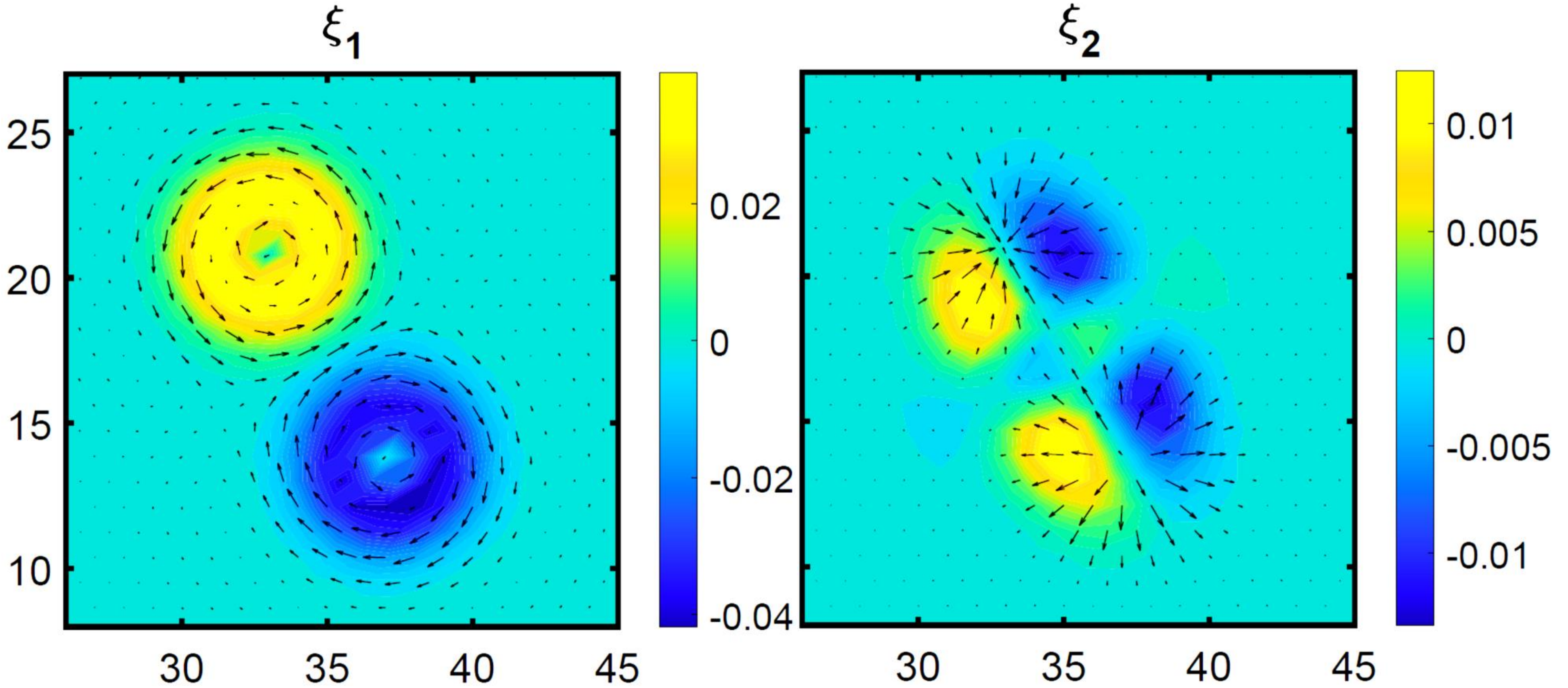
Translation



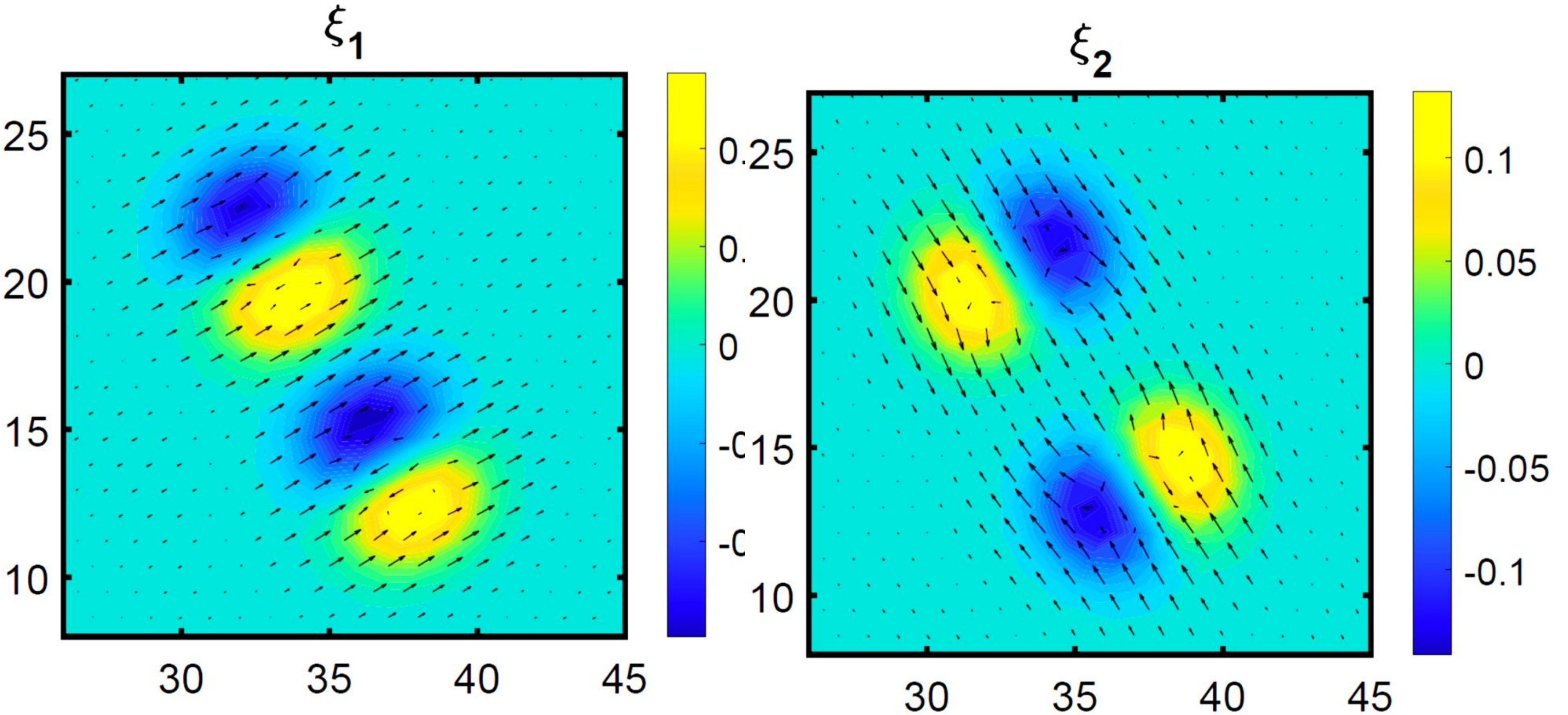
Mode 3



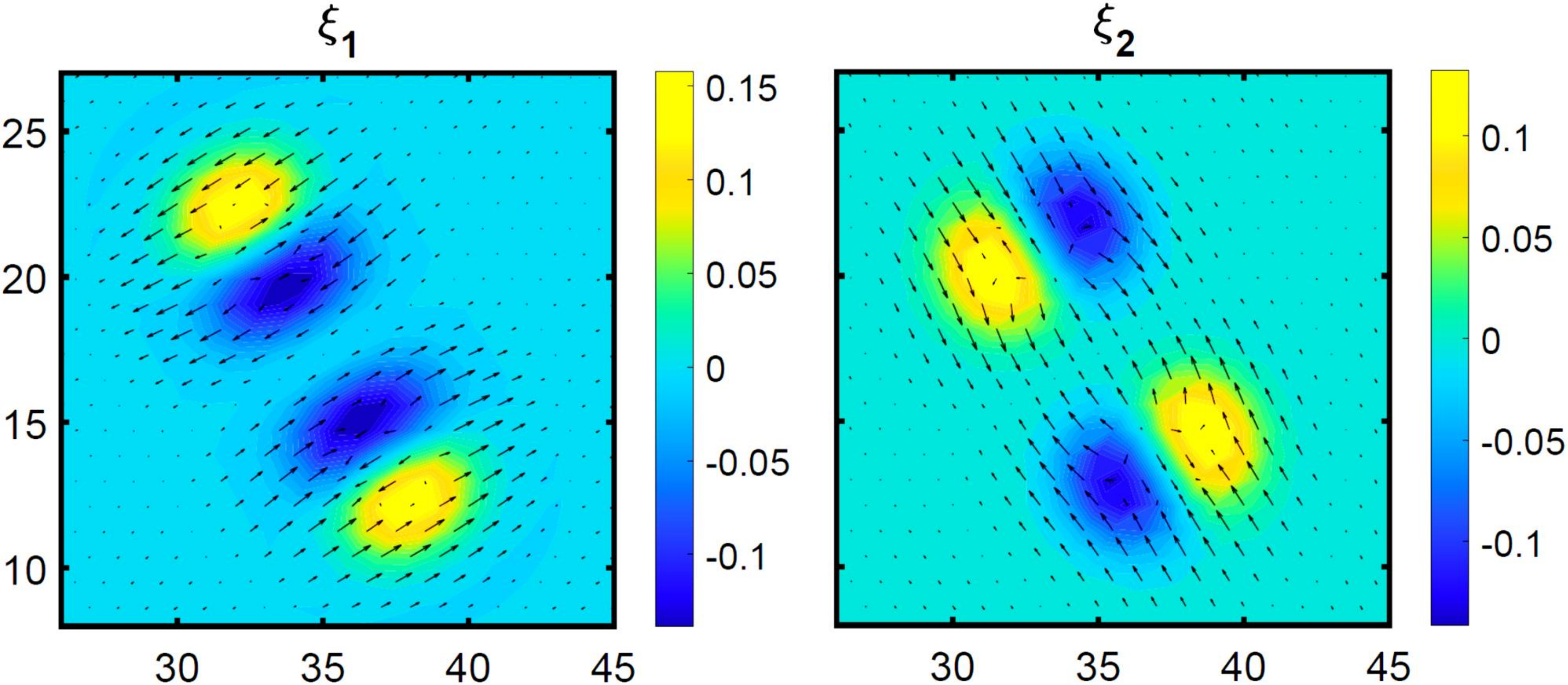
Mode 4



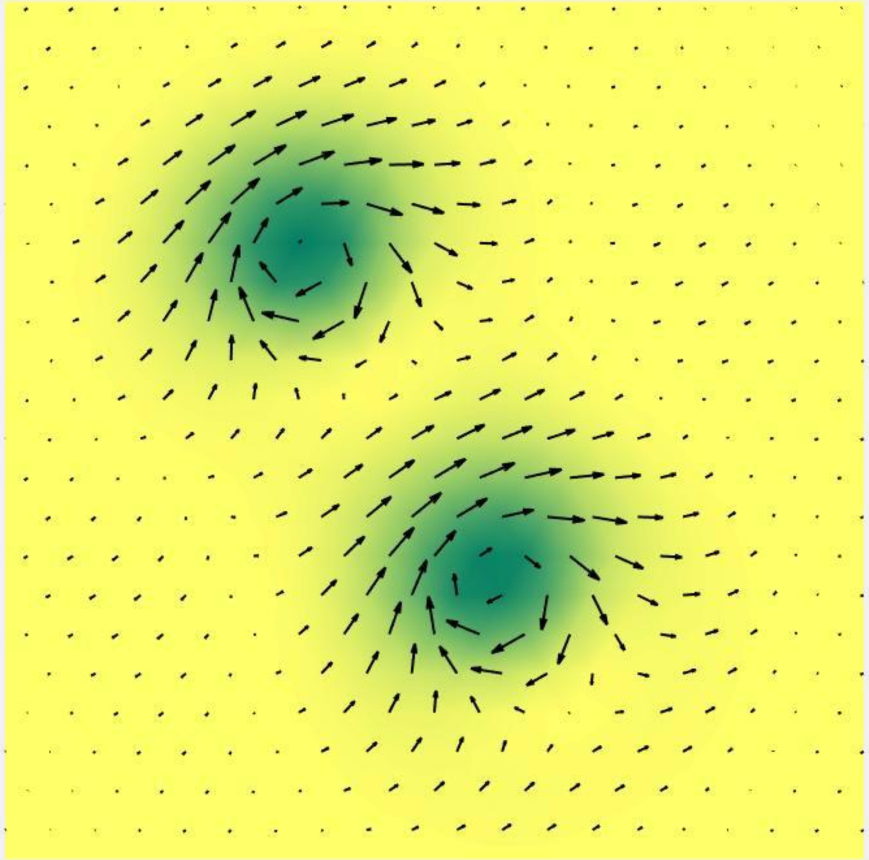
Mode 5 (electromagnon)



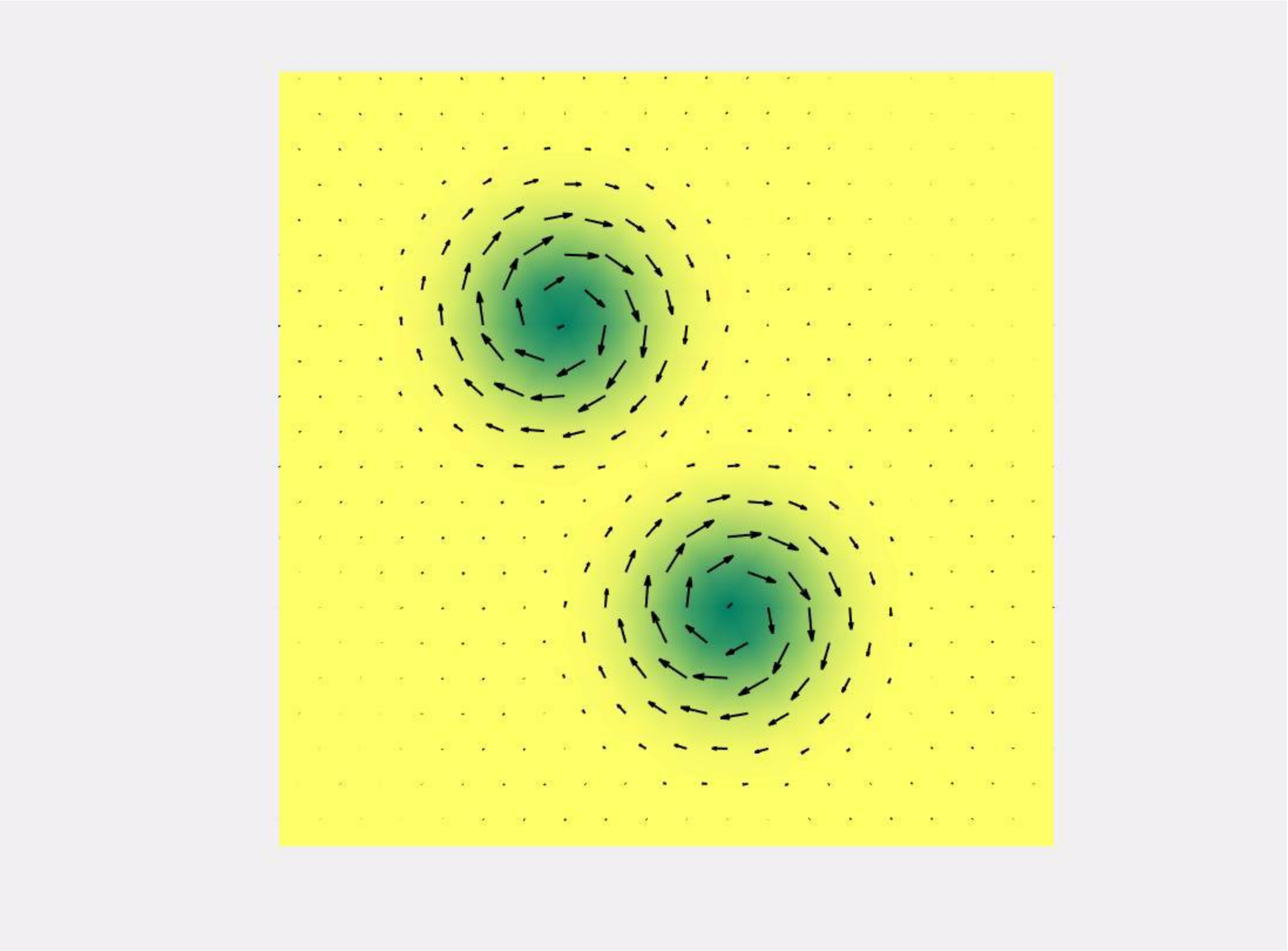
Mode 6



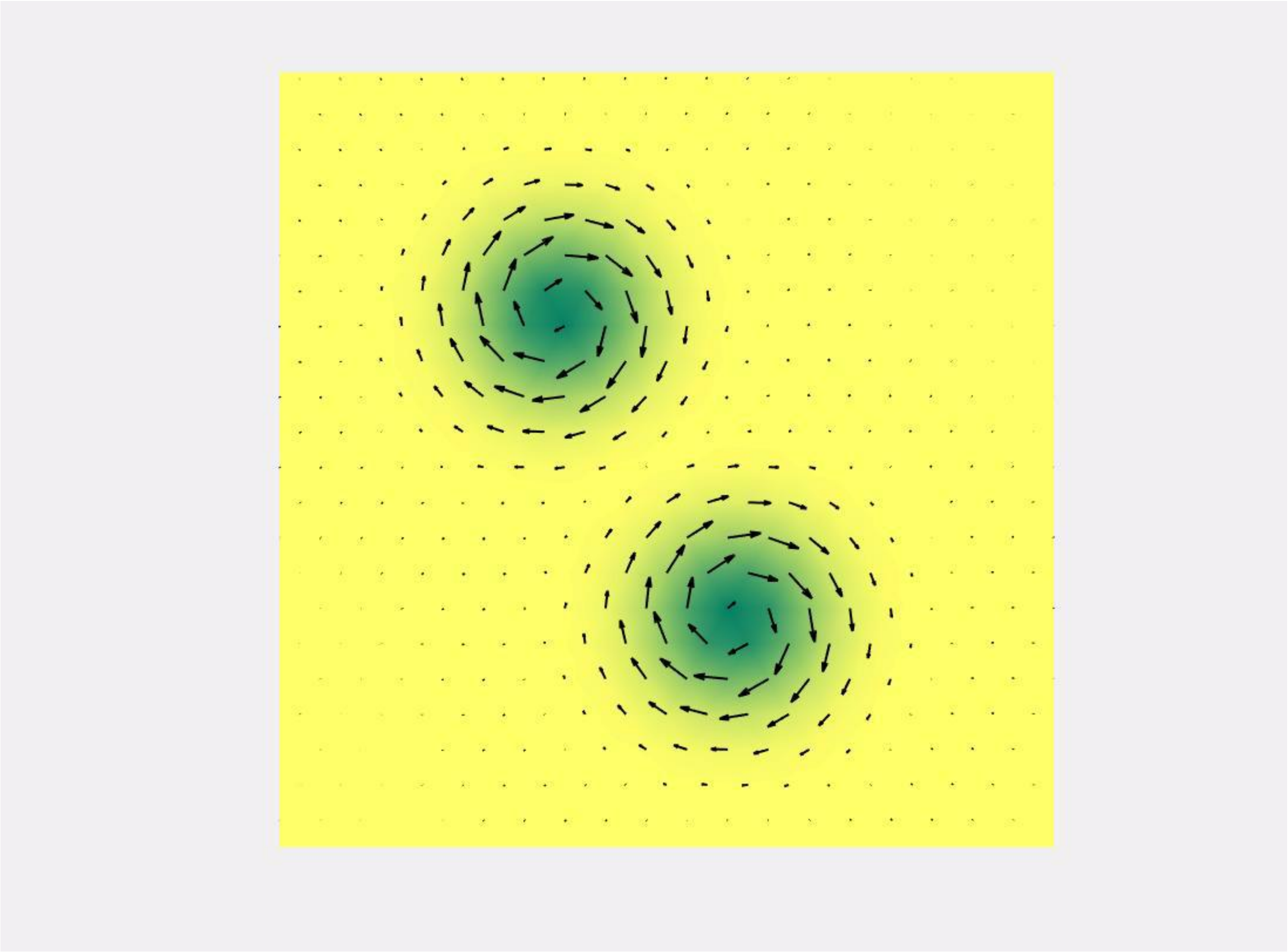
Mode 5: Electromagnon



Rotation

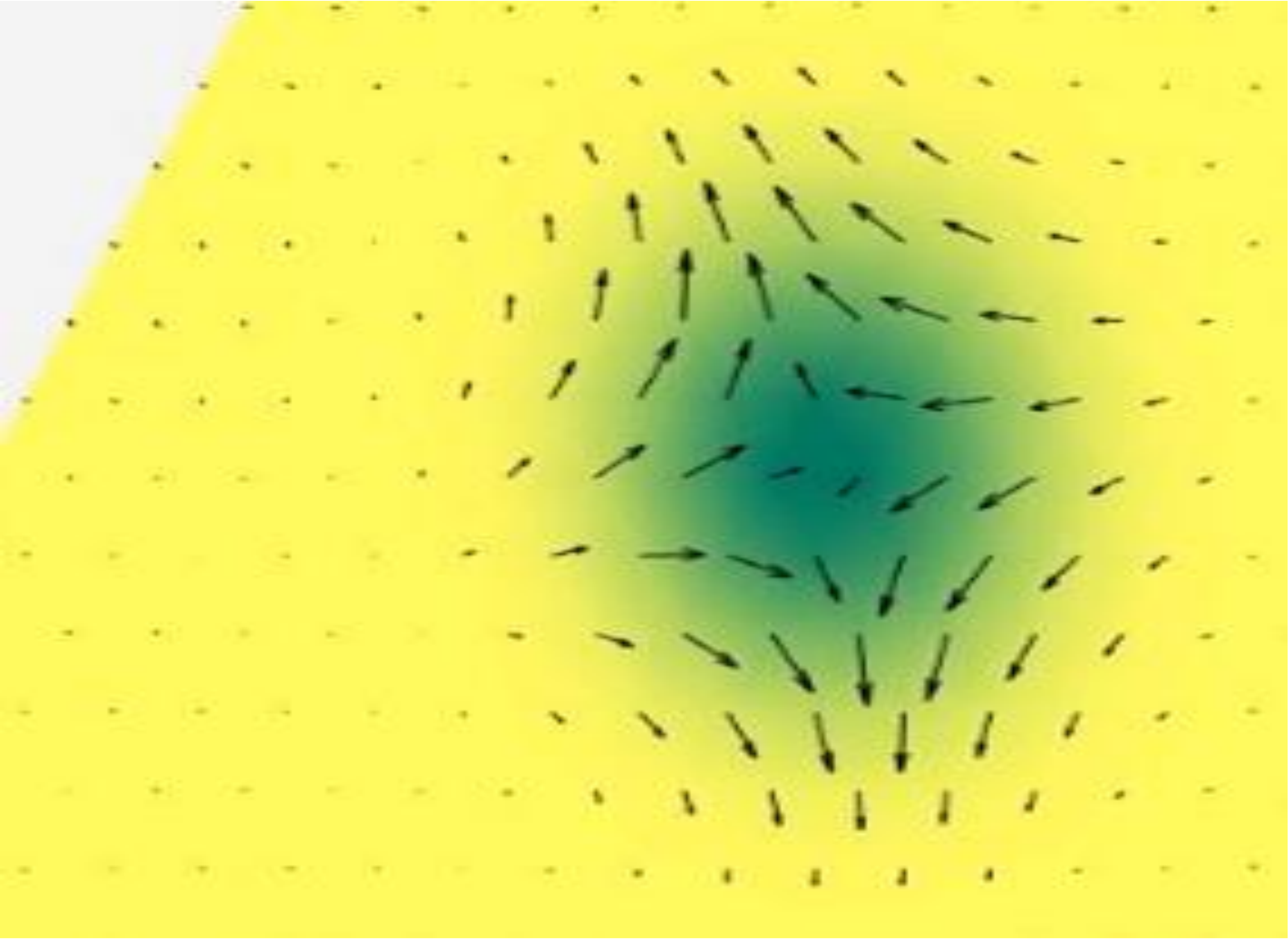


Mode 3

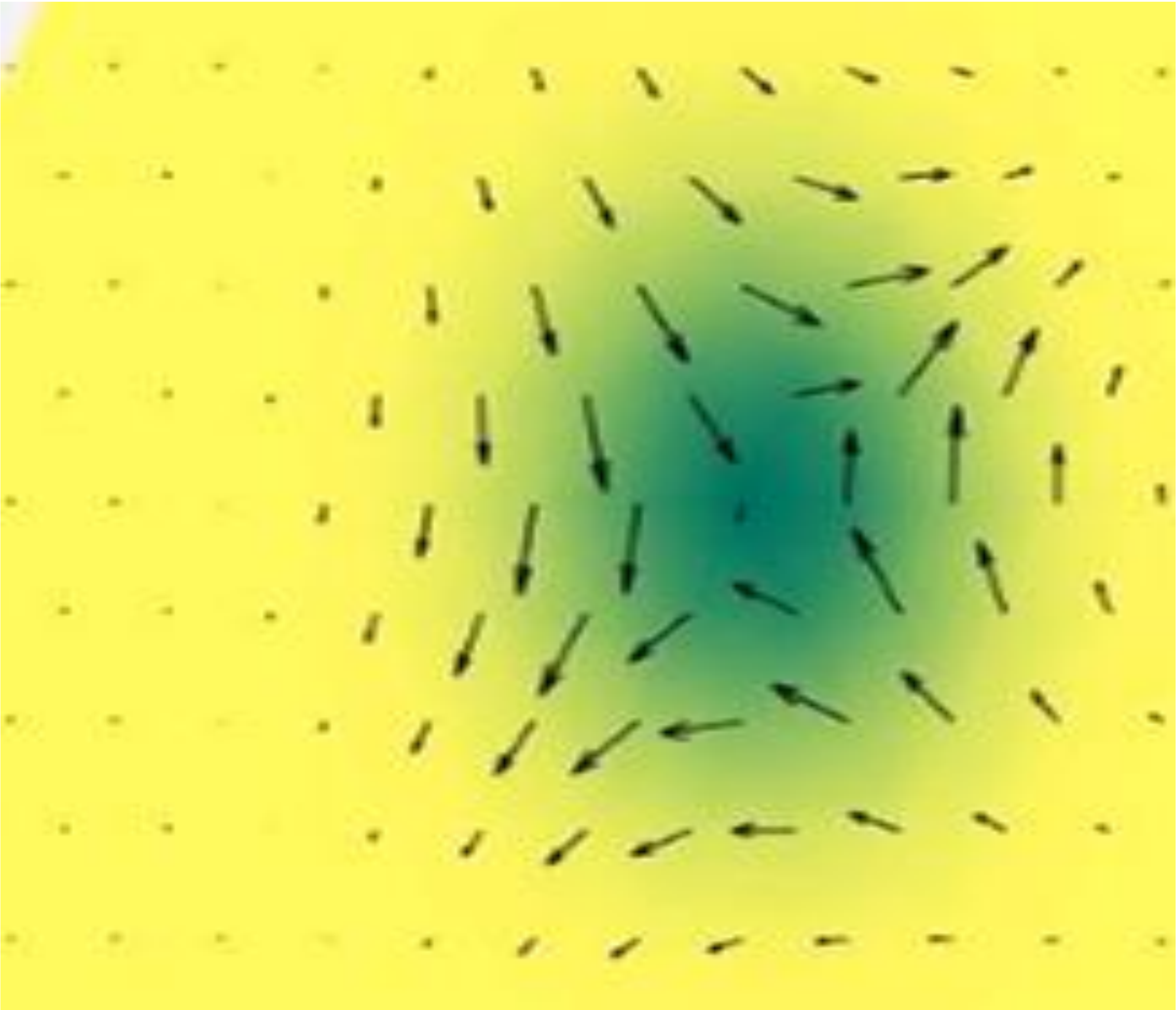


Spin precession in antiskyrmion

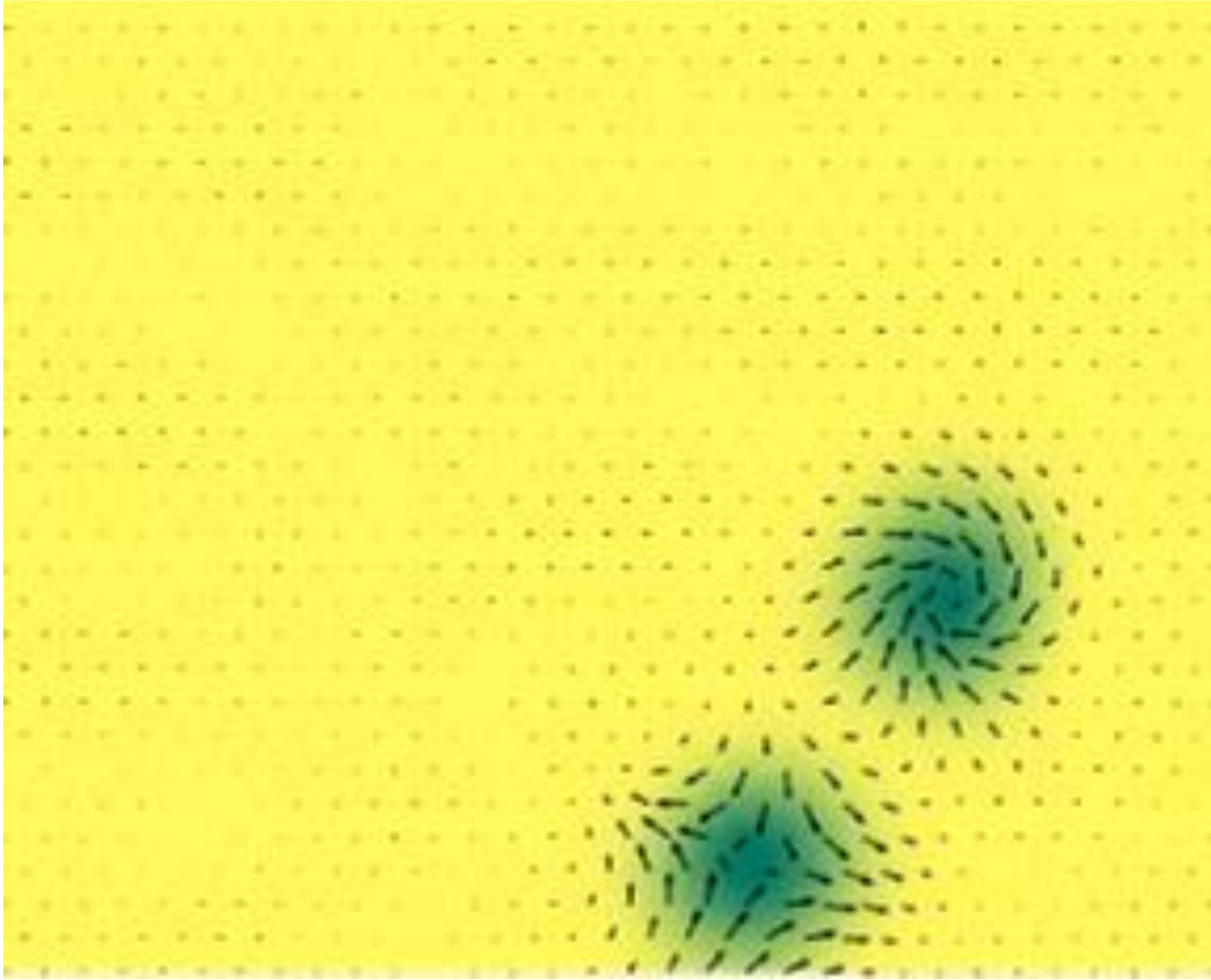
Antiskyrmion rotation $h_x = h_0\cos(\omega t)$



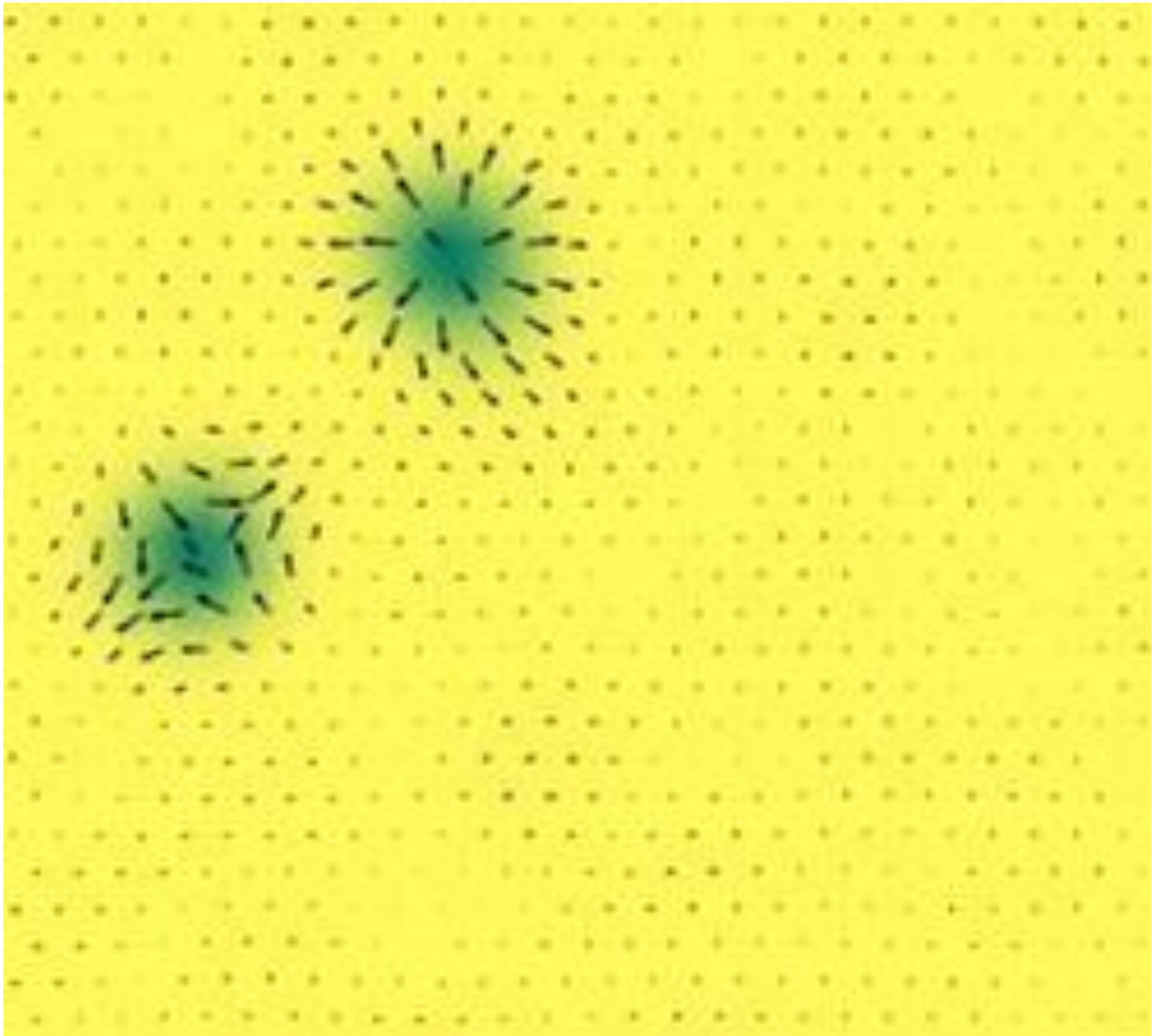
Antiskyrmion rotation $e_x = e_0\cos(\omega t)$



Skyrmion-antiskyrmion pair in $h_x = h_0 \cos(\omega t)$

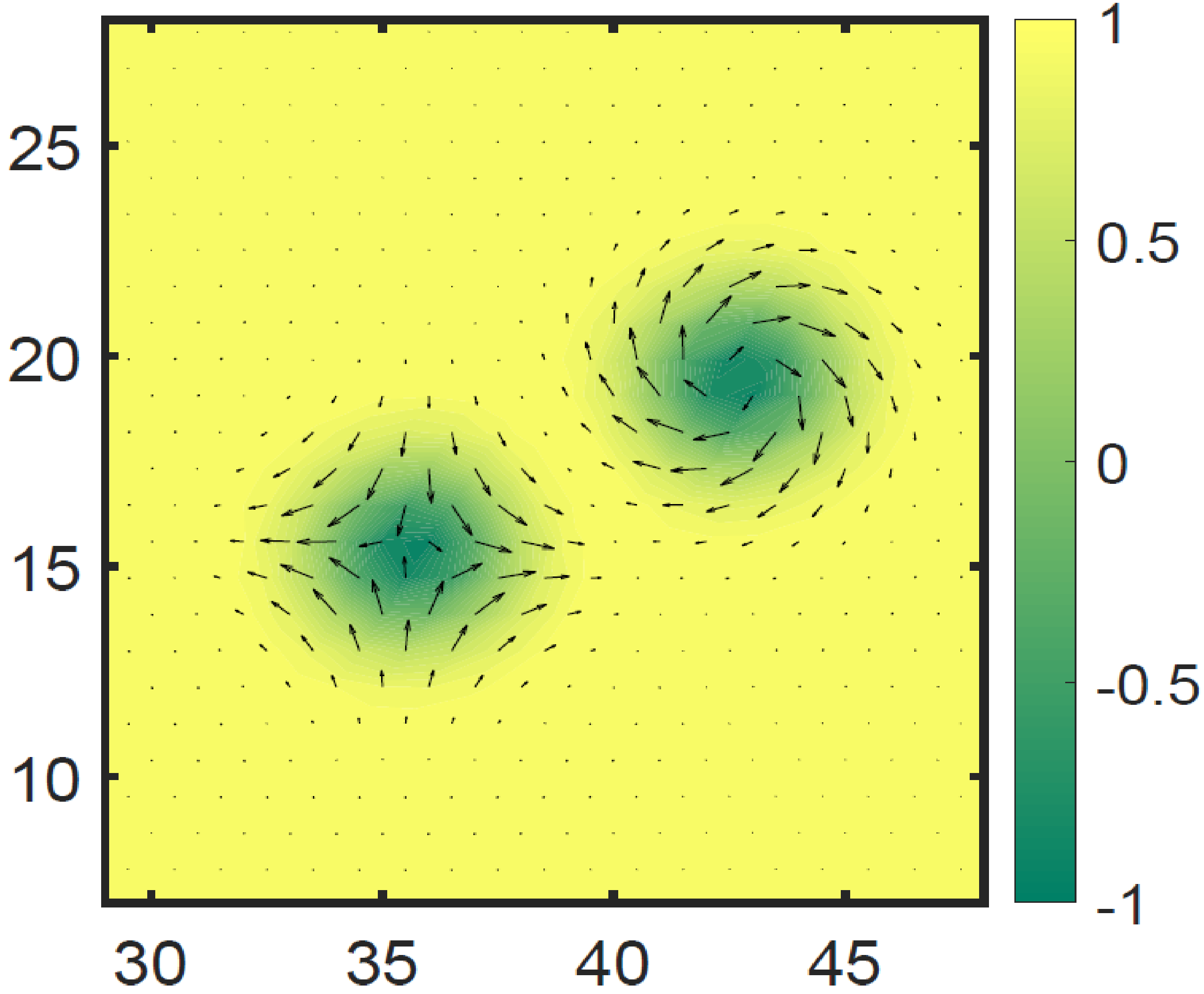


Skyrmion-antiskyrmion pair in $e_x = e_0 \cos(\omega t)$

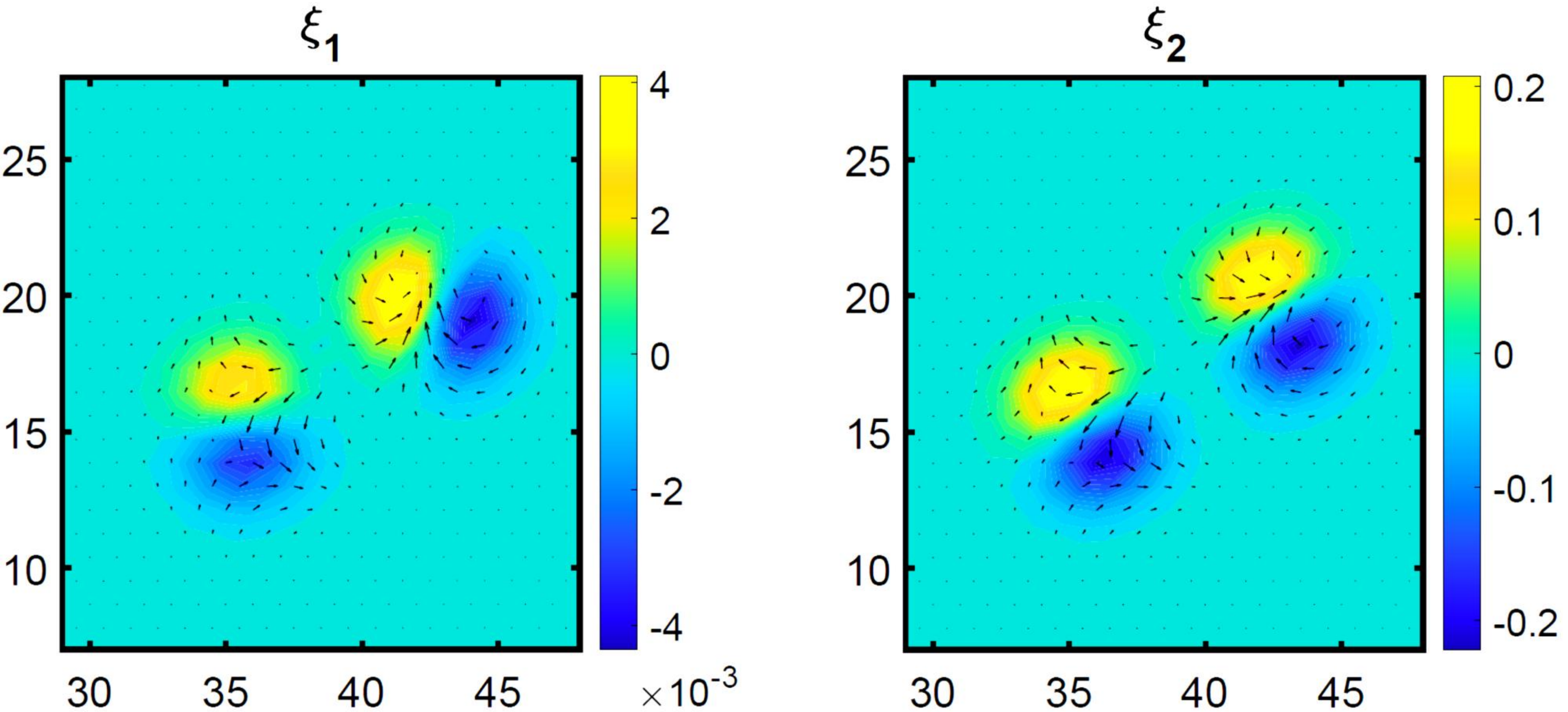


Excitations of skyrmion-antiskyrmion pair

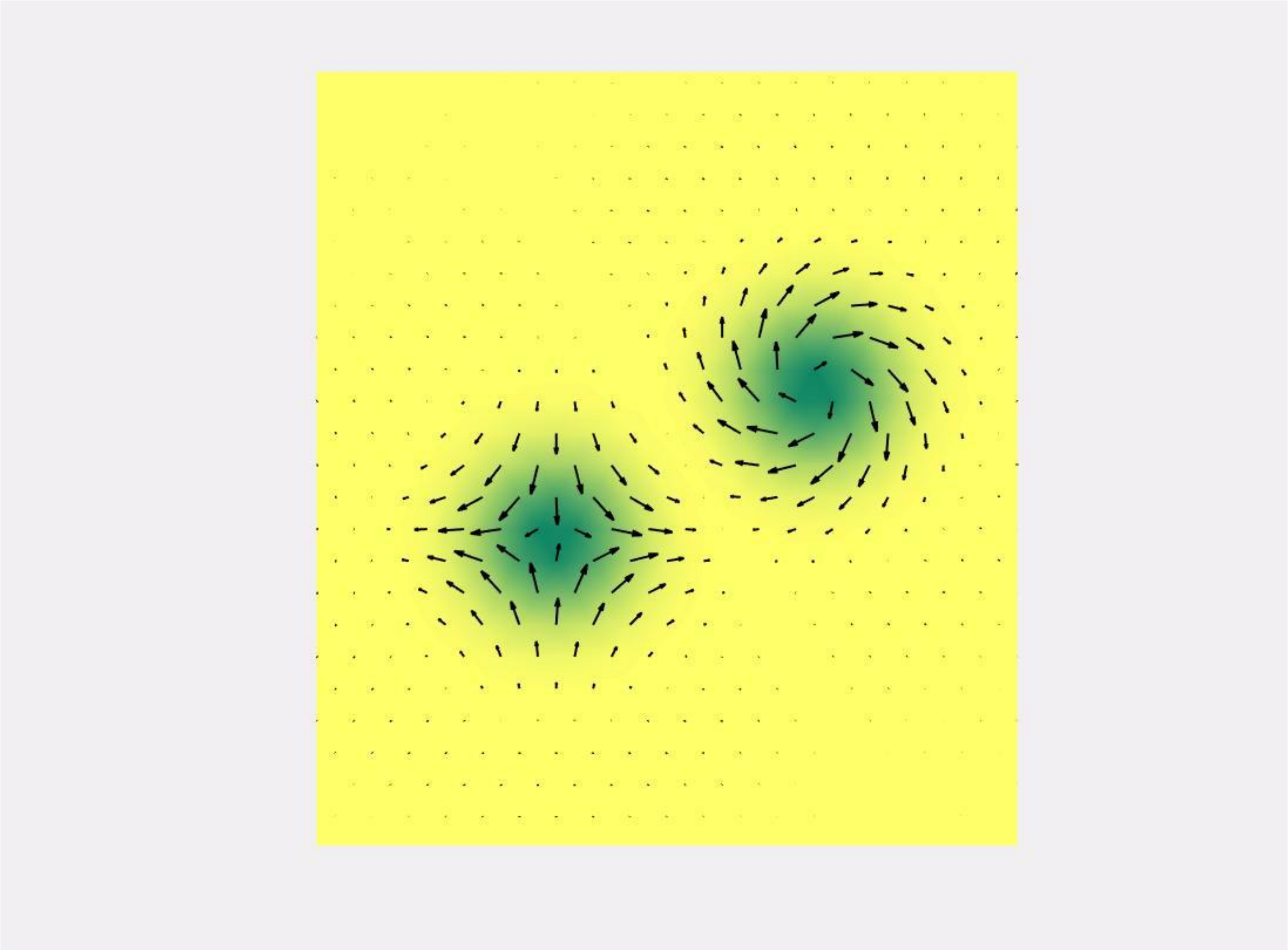
Skyrmion Antiskyrmion pair



Mode 3



Mode 3



Conclusions

- **Magnetic frustration leads to a new collective degree of freedom, skyrmion helicity, which can be excited both electrically and magnetically**
- **Electric polarization and charges induced by non-collinear spin textures allow for electric manipulation of topological magnetic defects**