Topological spin moiré: stability, dynamics, and controllability $\begin{array}{cccc}\n1 & C & C \\
1 & C & C\n\end{array} \quad \begin{array}{cccc}\n1 & C & C \\
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1 & C & C\n\end{array} \quad \begin{array}{cccc}\n1 &$

Yukitoshi Motome THE UNIVERSITY OF TOKYO

Plan of this talk

-
-
- ๏ superstructure, topology, and emergent electromagnetic field ๏ spin moiré engineering: type, number, amplitude of waves, twist angle, phase shift ๏ complete topological phase diagram for 2D skyrmions and 3D torons

itinerant frustration

- ๏ localized spin systems vs itinerant electron systems ๏ effective long-range/multiple spin interactions by the Fermi surface effects
- ๏ applications to 2D skyrmion crystals and 3D toron crystals

spin moiré

Moiré

figures are taken from Wikipedia

Superstructures are flexibly controlled by many parameters:

number of superposed waves, pitches, amplitudes, relative phases, twist angles, etc.

๏ interference pattern generated by a superposition of multiple waves

๏ moiré superstructures in twisted 2D van der Waals materials Kosterlitz–Thouless transition in two-dimensional superconductors23. erstructures in twisted 2D van de ϵ 10th acture of the environce 2D van ach degeneracy (for the case of \mathcal{L}_2 electrons) from the valleys in the original from the original from the origin-Waals materials zone17,18,25,26. In the vicinity of ∠2 electrons of ∠2 electrons of ∠2 electrons of √2 electrons o Prauno niut celuio

the integration \mathcal{L}_max is similar mechanism for the gaps is similar mechanism

figure is taken from Wikipedia **Exercise 1998 Y. Cao et al., Nature 556, 43 (2018)**

(blue). The curves exhibit the curves exhibit the typical V-shaped conductance near charge conductance near charge neutrality of the neutral plant in the dotted *a*nd the line of the dotted and insulating and insulation and in superlattice bandgaps *n*=±*n*s, which correspond to filling ±4 electrons superconducting domes are observed next to the half-filling state, which is the half-filling state, which is t
The half-filling state, which is the half-filling state, which is the half-filling state, which is the half-fi n in moiré superstructures twistronics by modulated flat band and electron correlation in moiré superstructures

Moiré in condensed matter physics In addition, we did not observe any appreciable superconductivity \mathcal{A} condonced mot (*E*F>0). In Fig. 1e we show the current–voltage (*I*–*Vxx*, where *Vxx* is the four-probe voltage in Fig. 1a) curves of device \sim between, there are conductance minima at ±2 and ±3 electrons per \overline{a} measure minima are associated with many \overline{a} $t \in I$ in Figure . The couplomb energy and the reduced kinetic structure \mathbb{R}^n $e \leftarrow e \leftarrow e$ near the magic angle; the

dimensional superconductor. The inset shows a tentative fit of the

Spin moiré

๏ interference pattern generated by a superposition of spin density waves

๏ richer variety of textures due to the vector fields ๏ richer physics due to the topological properties

Emergent electromagnetic field

๏ Spin moiré harbors a superstructure of emergent electric and magnetic fields. \rightarrow interesting magnetic, electric, optical, and transport phenomena

Experimental realization ly interactions governs the dimensions governs the dimensionality, topology, and \mathbf{I} density of the spin experiment. and heterostructures can be one promising approach toward realizable approach toward realiza $r_{\rm{max}} = 1$. magnetic singularities, magnetic singularities, magnetic generation would be expected a singular contract of the singular stress of the singular contract of the singular stress of the singular contract of the singular cont

๏ Skyrmion and toron crystals have been found in many materials, e.g., in B20 compounds. ϵ Skyrmion and toron crystals have been found in many materials ϵ of in R20 compounds

tion of novel topological spin structures by introducing additional sur-

Y. Fujishiro, N. Kanazawa, and Y. Tokura, Appl. Phys. Lett. 116, 090501 (2020)

The strong coupling among the strong coupling among the spin-charge-lattices would allow \sim

Motivation-1

Based on the spin moiré picture for multiple spin density waves, to investigate the effect of modulations of moiré parameters to explore spin textures that have been overlooked in the previous studies

-
-
-
- to systematically elucidate possible phase transitions in both magnetic and topological aspects

- ๏ advantages of spin moiré, compare to the structural moiré in twisted 2D materials
- more variety of moiré patterns owing to vector (spinor) fields
	- nontrivial topology
	- emergent electromagnetic fields, leading to exotic quantum phenomena - possible to make more than two-dimensional moiré
	-
	- possible to control by external fields, like magnetic field, pressure, and temperature
- ๏ various ways of moiré modulations
	- types of waves (ex. helix or sinusoidal)

- number of waves
- amplitude of waves

- angle between propagation directions

- etc.

トポロジカル磁気構造におけるスピンモアレエンジェアの研究がある。 そのメンジェアレエンジニアリング 33 キャランモアレエンジニアリング 33 キャランモアレエンジニアリング 33 キャランモアレエンジニアリング 33 キャランモア 和 *h*EMF .図中の整数は磁気ヘッジホッグの総数 *N*mを表す. rich variety of magnetic and topological properties, quantum transport, and optical properties

Spin moiré engineering 螺旋の重ね合わせからなる HL(3*q*-HL)(上段中)が得られ るが,*J* ′ を弱くしていくと(*J* ′/*J*<1),*z* 方向に伝搬する波 の振幅が小さくなり,*J* ′/*J*≃0.9 付近でそれが消失すること

}

K. Shimizu, S. Okumura, Y. Kato, and Y. Motome, Phys. Rev. B **103**, 054427 (2021)

K. Shimizu, S. Okumura, Y. Kato, and Y. Motome, Phys. Rev. B **103**, 184421 (2021)

- relative phases of waves

K. Shimizu, S. Okumura, Y. Kato, and Y. Motome, Phys. Rev. B **105**, 224405 (2022)

我々は,Θ に加えて全磁化 *m* の変化も考慮することで, Kotaro Shimizu

Hyperspace representation

K. Shimizu, S. Okumura, Y. Kato, and Y. Motome, Phys. Rev. B **105**, 224405 (2022)

Topological phase diagram for 2D skyrmions

K Shimizu S Okumura V Kato and V Motomo Dhys Dov R . Offittiza, O. Oramaa, T. Rato, and T. Motorne, Fiftys. Hev.

K. Shimizu, S. Okumura, Y. Kato, and Y. Motome, Phys. Rev. B 105, 224405 (2022) determined by *^N*m, with the contour plot of [−]*b*¯*z*, on the plane of *^m* K. Shimizu, S. Okumura, Y. Kato, and Y. Motome, Phys. Rev. B 105, 224405 (2022) ³ (*N*^m = 16), (b) *m* = 0 and ϕ˜ = π (*N*^m = 16), (c) *m* = 0.4 and ϕ˜ = 0 (*N*^m = 8), (d) *m* = 0.7 K. Shimizu, S. Okumura, Y. Kato, and Y. Motome, Phys. Rev. B **105**, 224405 (2022) btome, Phys. Rev. B **105**, 224405 (2022)

Short summary

- ๏ The spin moiré picture is useful to explore more variety of topological spin crystals. - We can exploit the analogy with conventional moiré.
	- There are many advantages of spin moiré, compare to the structural moiré in twisted 2D materials.
- ๏ We demonstrated the usefulness of the spin moiré picture for the phase shift in topological spin crystals.
	- complete topological phase diagrams for 2D skyrmions and 3D torons - unprecedented topological phases with higher-order skyrmions and high-density torons
	-

itinerant frustration

 $d = 1$ and $d = 1$ for the quantum between α for the quantum between α

ARTICLE NATURE COMMUNICATIONS | https://doi.org/10.1038/s41467-019-13675-4

configuration, namely J $\mathcal{L}_{\mathcal{A}}$. H $\mathcal{L}_{\mathcal{A}}$ is the H k $\mathcal{L}_{\mathcal{A}}$ is the H k $\mathcal{L}_{\mathcal{A}}$

configuration, namely J α . H β . H β is the H k β

 $\mathcal{H} \subset \mathcal{H}$ and $\mathcal{H} \subset \mathcal{H}$ and $\mathcal{H} \subset \mathcal{H}$ in the new material EuPtSi.

New generation: nanometer-scale skymions 30 **a d H//c** $nnomationc1$ compared to H<H¹ and vanishes for H>H2. Application of the P4 \bigcap \bigcap \bigcap \bigcap \bigcap PHYSICAL REVIEW LETTERS 120, 097201 (2018) 5 Faculty of Science, University of the Ryukyus, Nishihara, Okinawa 903-0213, Japan \mathbb{R} | \mathbb{R} - \mathbb{R} = \mathbb{R} \mathbb{R} \mathbb{R} \mathbb{R} \blacksquare \sim MNSI WITH \sim \blacksquare at 2 K \blacksquare in the magnetic transitions at the magnetic transitions at the magnetic transitions \blacksquare and \mathcal{L} closed in the ðH; TÞ phase and is observed in a wide temperature range from 3.6 to 0.5 K. In this magnetic phase known resulting in the existence of a characteristic additional Hall \overline{r} $T \cup T \cup T \cup T \cup T$ provision peak structure, (d) (f) resulting in the existence of a characteristic additional Hall \overline{r} $T \cup T$ indicates $C \cup T$ provision peak structure, and the structure, as shown in Fig. 3(b). The structure of t mong n

multi-q modulation describes magnetic structure in the

New generation: nanometer-scale torons

dill, dit di $I = \frac{1}{\sqrt{2}}$ \mathbf{m} in not only nonce dill, ill il \mathcal{I} contrast, \mathcal{I} and \mathcal{I} ad \sim few nm, in not only no Corresponding Author inserted at the bottom right indicate the directions of magnetic $\mathsf{M}\sim\mathsf{f}\mathsf{e}\mathsf{W}$ in m in od \sim few nm, in not only n Corresponding Author \blacksquare inserted at the bottom right indicate the directions of magnetic the directions of magnetic magnet $\Omega_{\rm C} \sim {\rm few\ nm\,\,}10^{-5}$

S. Ishiwata *et al.*, Phys. Rev B 101, 134406 (2020)

\mathbf{m} in not only noncentrosymmetric hut ntrosymmatric hut a ricios y minicipione du c robust or pinned excitations of spin hedgehogs even in the ν troum motric hut a ricions de l'alternie du contra robust or pinned excitations of spin hedgehogs even in the extremely short period \sim few nm, in not only noncentrosymmetric but also centrosymmetric systems

Motivation-2

- ๏ What is the stabilization mechanism for the new-generation topological spin crystals?
	- Conventional mechanism based on the Dzyaloshinskii-Moriya interaction does not explain the extremely short-period textures in centrosymmetric systems.
	- In general, magnetic frustration is active and able to stabilize short-period spin textures even in centrosymmetric systems in the absence of spin-orbit coupling, but it is discussed mostly for insulating systems.
	-

➡ What is a generic mechanism in metallic magnets? What is the role of itinerant electrons?

our proposal: *itinerant frustration*

as an underlying mechanism to generate frustrated/multiple-spin interactions

review: S. Hayami and Y. Motome, J. Phys.: Condens. Matter **33**, 443001 (2021)

Frustration in localized spin systems

 \cdot S_{-q}

Frustration in itinerant electron systems

 \mathbf{F} and (c) and (c) \mathbf{F} and (c) The contour plots of the bare susceptibility \mathbf{F} and (c) \mathbf{F}

Localized vs itinerant frustration second order (Sec. II C) and fourth order (Sec. II D) of *J* , and \mathcal{L} spin in the form in the form in the form in the form in Eq. (8) is generic and in Eq. (8) is general to the following structure \mathbf{a}

-
-
-

๏ itinerant magnets

$$
\mathcal{H} = \sum_{\mathbf{q}} (-J^2 \chi_{\mathbf{q}}^{(0)}) \mathbf{S}_{\mathbf{q}} \cdot \mathbf{S}_{-\mathbf{q}}
$$

degeneracy lifting by …?

➡ multiple-spin interactions arising from

higher-order perturbation in *J*/*t*

$$
J_{{\bf q}_1,{\bf q}_2,{\bf q}_3,{\bf q}_4,\dots}({\bf S}_{{\bf q}_1}\cdot {\bf S}_{{\bf q}_2})({\bf S}_{{\bf q}_3}\cdot {\bf S}_{{\bf q}_4})\cdots
$$

4-spin interaction $\frac{1}{\sqrt{2}}$ from left to right. The vertices with wave lines denote the vertices with wave lines denote the vertices $\frac{1}{\sqrt{2}}$

multiple maxima in $\chi_{\bf q}^{(0)} \rightarrow$ frustration at any order of *J* . In the following sections, we discuss $\frac{1}{100}$ for specific form of such interactions by form of $\frac{1}{100}$ second order (Sec. II C) and fourth order (Sec. II D) of *J* , and

well described in momentum space: intrinsically long-range perturbative expansion of the free energy in Eq. (5): *n* = 1, 2, and 3

relevant wave vectors are dictated by the Fermi surface nesting propagators of itinerant electrons, *G***k**. • size of spin textures can be extremely small • inversion symmetry breaking is not necessary (unlike DMI)

e. J. Phys.: Condens. Matter 33. 443001 (2021) $\frac{3}{2}$. $\frac{3}{2}$. review: S. Hayami and Y. Motome, J. Phys.: Condens. Matter 33, 443001 (2021)

2-spin interaction (RKKY)

spontaneous formation of 3**q** skyrmion crystal with a high skyrmion number of *N*sk=2

^q are located at *Q*¹ and *Q*² in α is a spontaneous formation $\mathcal{C}_{\mathbf{f}}$ represent the fraction $\mathcal{C}_{\mathbf{f}}$ corresponding to $\mathcal{C}_{\mathbf{f}}$

R. Ozawa, S. Hayami, and Y. Motome, Phys. Rev. Lett. **118**, 147205 (2017) *^q* in (a) and (c). Reprinted fgure with permission from [23], Copyright (2017)

Control of skyrmion size by Fermi surface イナコ

d

AARTWAY Winnipeg

)–1

Harry

)

N. D. Khanh et al., Nat. Nanotech. 15, 444 (2020)

Square skyrn of crystal in GdRu2Si2 **Square skyrm of the crystal in Gaku₂Si₂** 2 LI JULUI II \sim $\overline{}$

T (K)

 \mathbf{r} <u>(e)</u> **j** ant flustra

L-TEM images, respectively, and the magnetic reflections \mathcal{L}_max

T (K)

 $s_{\rm eff}$ show a square-lattice-like pattern but with an opposite black/ which contrast. By performing the transport-of-intensity equation $\mathcal{L}_{\mathcal{A}}$ analysis 15 based on the spatial distribution of the spatial distribution of the spatial distribution of the s

Square skyrmion crystal: theory

*Q*¹ direction, similar to the double-*Q* CS state discussed in

 \mathcal{L} direction, similar to the double- \mathcal{L} state discussed in the double- \mathcal{L}

$$
\int_{\nu}S_{-{\bf Q}_{\nu}}^{\beta}+\frac{\Lambda}{N}\big(\sum_{\alpha\beta}\Gamma_{\bf Q_{\nu}}^{\alpha\beta}S_{\bf Q_{\nu}}^{\alpha}S_{-{\bf Q}_{\nu}}^{\beta}\big)^{2}\bigg]
$$

*Q*¹ direction, similar to the double-*Q* CS state discussed in

 $\frac{1}{2}$ **AYYYA** $1 + 1 + 1 + 1 =$ REFER $K/K/K$ $+ + + + + +$ TYVYA,

ITYVYA, $1 + 1 + 1$

REFER

2-spin interaction (RKKY) 4-spin biquadratic interaction

Comparison between experiments and theory **Q1 (b) 2Q2 Q**

A

FP $\frac{1}{2}$ uy
. **2Q2** inset) has been removed; see Supplemental Fig. S2 [28]. (b) Same R. Wiesendanger, S. Seki, and K. von Bergmann, Phys. Rev. Materials **8**, 064404 (2024)J. Spethmann, N. D. Khanh, H. Yoshimochi, R. Takagi, S. Hayami, Y. Motome, at 1.9 T. (c) Same at 2.3 T. (c) Same at 2.3 T. (c) Same insets show the insets show the insets show the inset

and S. Seki, Nat. Commun. **11**, 592 .
Y. Yasui, C. J. Butler, N. D. Khanh, S. Hayami, T. Nomoto, T. Hanaguri, Y. Motome, T. Hasui, C. J. Sp n. Arita, T. Arima, Y. Tokura, and S. Seki, Nat. Commun. **11**, 5925 (2020)
R. Arita, T. Arima, Y. Tokura, and S. Seki, Nat. Commun. **11**, 5925 (2020) Phase I.

. (20) d (green circle), **Q**¹ + **Q**² (black square), 2**Q**² (orange diamond), and **04**, 144404 (2021). (**Q** S. Hayami and Y. Motome, Phys. Rev. B **104**, 144404 (2021)

$-$ evid 1.4 1.2 4 | *V*
|
| H FT of charge density **i l l i l i l i l i l i** *l* **i i** $\frac{1}{2}$ $\frac{1}{2}$ second $\frac{1}{2}$ terminated surface in the Si-terminated $\mathfrak{u}\mathfrak{g}\mathfrak{e}$ coupling strong set $\mathfrak{g}\mathfrak{e}$ stripe pattern for $\frac{1}{1}$, see Fig. 1(a). However, while this term is the phase I; see Fig. 1(a). However, while this term is the phase I; see Fig. 1(a). However, which is the phase I; see Fig. 1(a). However, which is evidence for spin-charge coupling good agreement with theory the magnetic state it does not provide evidence for a multi-**Q** charge modulations concomitant with spin textures:

NATURE COMMUNICATIONS | https://doi.org/10.1038/s41467-020-19751-020-19751-4 ARTICLE ARTICLE ARTICLE ARTICLE A
ARTICLE ARTICLE ARTICL

β ⁰⁰ agrecticite with the or

N. Kanazawa *et al.*, Phys. Rev. B 86, 134425 (2012); Y. Fujishiro *et al.*, Nat. Commun. 10, 1059 (2019) 10 0*H*_c. Phys. Rev. B 86, 134425 (2012): Y $\overline{\text{II}}$ 0.8 50 (3q-HL), which is described by three orthogonal q-vectors, which are pinned along 〈100〉 crystal axes at zero H. In a–c the red (blue) arrows drawn in each *^T* (K)¹⁰⁰ \overline{O} 12); Y. Fujishiro *et al.*, Nat. Commun. **10**, 1059 (2019) SkL, which is approximately described by the in-plane superposition F_1 Commun. 10. 1059 (2019). $\sum_{i=1}^n$

\mathcal{L} in the apical directions of a regular tetrahedron. The axes at zero H. Hedgehogs and anti-hedgehogs are at facecentre of functant nustrations $w = +1$ $w = -1$ period \sim 2-3 nm: importance of itinerant frustration? $w = +1$ $w = -1$ $\text{SINR}:$ $w = +1$ $w = -1$

 $\mathcal{L} = \frac{1}{2} \sum_{i=1}^{n} \frac{1}{2} \sum$

 d domain helical state, while the first-principles calculation \mathcal{L}

 $w = \frac{44}{10}$

Application to 3D toron crystals A n i Lhured \bullet and anti-hedgehogs and anti-hedgeholgs connected by skyrmion strings. This state communical models of the description of the description of the top three helical models of the term \blacksquare two-dimensional (2D) nanometric vortex-like structure consist- \sim directions of local spins. Instead, it is necessary to internal spins. In the introducer is necessary to internal spins. In the introducer is necessary to internal spins. In the internal spins of the introducer is ne hedgehog point defects, which are three-dimensional (3D) UII LU UID LUI UIL UI JU **q**4 **q**3

 $Z - J$ $\overline{}$ **portar** $1 - 2 - 3$ nm: impo

parameters including interatomic distances. Nevertheless, such a

 $\arctan^2\theta$ have been used widely, e.g., for *d* electron systems, at the level as a contract model for the Hubbard-type model for the Hubbard-type models, which is not the Hubbard-type m ladratic interaction and DM-type interaction stabilizes toron cl (similar results for cubic-3q toron crystal) S. Okumura, S. Hayami, Y. Kato, and Y. Motome, Phys. Rev. B **101**, 144416 (2020) spatial inversion symmetry breaking in \mathbb{R} Cooperation between biquadratic interaction and DM-type interaction stabilizes toron crystals. propriesting control
p $\frac{1}{2}$ is the center position of the unit cube and the un normal unit vector of the plants of the p
Plants of the plants of th into two triangles and taking the sum of the solid and taking the sum of the solid angles of the soli i sjädilizes toron crystals. for the (a) 4*Q* and (b) 3*Q* cases. 4*Q*(3*Q*)-HL, 4*Q*(3*Q*)-NC, 2*Q*-VC, 2*Q*-CS, and 1*Q*-H represent the chiral 4*Q* (3*Q*) hedgehog lattice, the nonchiral 4*Q* (3*Q*), the 2*Q* vortex crystal, the 2*Q* chiral stripe, and the 1*Q* helical states, respectively. (similar results for cubic-3**q** toron crystal)

Toron crystals: theory

K $\frac{d\mathbf{A}}{N}(\mathbf{S}_{\mathbf{Q}_{\nu}}\cdot\mathbf{S}_{-\mathbf{Q}_{\nu}})^{2}-i\mathbf{D}_{\nu}\cdot(\mathbf{S}_{\mathbf{Q}_{\nu}}\times\mathbf{S}_{-\mathbf{Q}_{\nu}})$ i

Shun Okumura

Toron crystals: magnetic field MAGNETIC HEDGEHOG LATTICES IN … PHYSICAL REVIEW B **101**, 144416 (2020)

purple, gray, and white regions represent the 4*Q*-HLs (*N*^m ̸= 0), the noncoplanar 4*Q* states (*N*^m = 0), the 1*Q* conical states, and the forced

topological transitions by pair annihilation of hedgehogs and antihedgehogs depending on the ${\sf h}$ direction F_{20} icugenogs and antineugenogs uepending on d

S. Okumura, S. Hayami, Y. Kato, and Y. Motome, Phys. Rev. B **101**, 144416 (2020); Y. Kato and Y. Motome, Phys. Rev. B **107**, 094437 (2023) *m* in Eq. (6); the magnetic susceptibility χ in Eq. (7); the magnetic moments with wave vector **Q**η, *m***^Q**^η in Eq. (8); the number of monopoles and antimonopoles, *N*^m in Eq. (13); and the net scalar spin chirality χsc in Eq. (16) (note that −χsc is plotted in the figure). The green, v. B 101, 144416 (2020); Y. Kato and Y. Motome, Phys. Rev. B 107, 094437 (2023) by the gray arrow) in the 4*Q* case: (a) *h* = 0*.*00, (b) *h* = 0*.*57, (c) *h* =

Toron cr

๏ experiment in SrFeO3

to states with 12 and 3 domains, respectively. The schematic crystal structure and quadruple-*q* vectors viewed along [111] are shown on the S. Ishiwata *et al.*, Phys. Rev B 84, 054427 (2011) s lehiwata *et al* Phys Rey R **101** 134406 (2020). S. Ishiwata *et al.*, Phys. Rev B 101, 134406 (2020)

S*j*

S. Okumura, S. Hayami, Y. Kato, and Y. Motome, in preparation a variety of toron crystals even in the centrosymmetric case, including high-density toron crystals found in the spin moiré analysis field. 4*Q*-H HL, 4*Q*(3*Q*)-S HLs, 2*Q*-CS, and 1*Q*-H repre- σ ⁻¹ σ ⁻¹ σ ⁻¹ sinusoid₂, the double- σ ₂, the doubleralitative understanding of the expert quantum entrerolanding of the experimental pilled diagram of ont co, hedgehog monopole antihedgehog antimonopole qualitative understanding of the experimental phase diagram of SrFeO3

Zero-to-perfect toron Hall effect

zero toron Hall effect (only longitudinal) perfect toron Hall effect (only transverse)

K. Shimizu, S. Okumura, Y. Kato, and Y. Motome, preprint (arXiv:2407.02983)

Lattice discretization significantly affects the motions of short-period torons.

Short summary

- ๏ We have proposed a new mechanism to stabilize topological spin textures, *itinerant frustration*. - generic mechanism for generating frustrated/multi-spin interactions in metallic magnets - many applications for new-generation topological spin crystals with extremely short period in
-
- centrosymmetric systems
- ๏ We presented applications to 2D skyrmion crystals and 3D toron crystals.
	- 2D: higher-order skyrmions, flexible change of the magnetic period, application to GdRu₂Si₂
	- 3D: application to MnSi1-*x*Gd*x*, phase diagrams, effect of magnetic fields, unconventional currentinduced motions of torons

Conclusion

spin moiré picture

๏ superstructure, topology, and emergent electromagnetic field

-
- ๏ spin moiré engineering: type, number, amplitude of waves, twist angle, phase shift ๏ complete topological phase diagram for 2D skyrmions and 3D torons

itinerant frustration

- ๏ localized spin systems vs itinerant electron systems
- ๏ effective long-range/multiple spin interactions by the Fermi surface effects
- ๏ applications to 2D skyrmion crystals and 3D toron crystals

Both concepts will be useful for further exploration of topological spin superstructures

and their physical properties.

➡ new multi-**q** states, new topological transitions, topological responses, etc.

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	- *Others:*
	- Kipton Barros (LANL), Gia-Wei Chern (Virginia), Cristian D. Batista (Tennessee), …
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