

aftermagnon



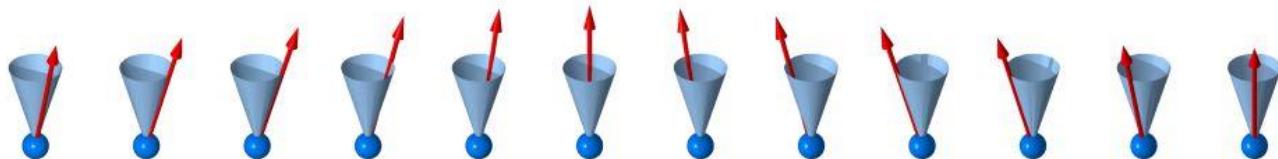
what can happen to a magnon?

- I. [subjective] overview of m-m interactions
- II. novella on disorder
- III. common message

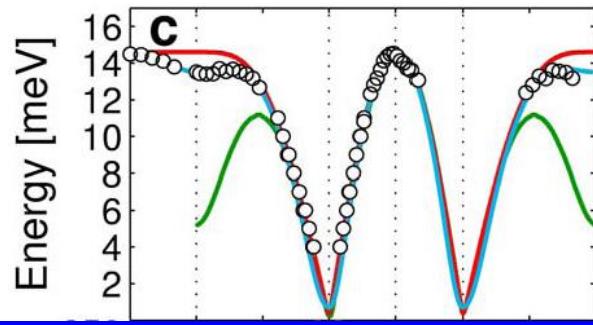
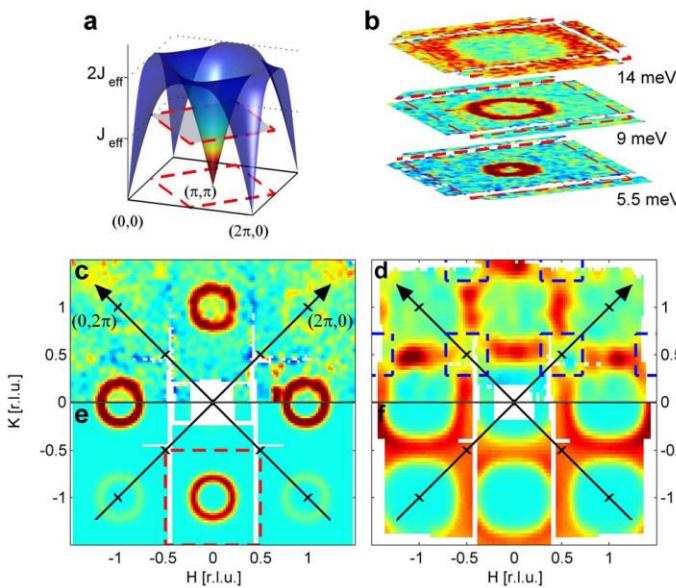


what IS magnon?

standard lore ...



- spin waves, 1930, *Felix Bloch*
 - quantized theory: *Holstein-Primakoff, Anderson-Kubo, Dyson, ...*
 - 1957, experiment, INS: *Brockhouse, ...*
- sharp peaks = quasiparticles = quanta of spin waves
well defined elementary excitations in $D>1$



2D spin-1/2 quantum antiferromagnet copper deuteroformate tetradeuterate (CFTD)



nuances ...

Zur Theorie des Ferromagnetismus.

Von W. Heisenberg in Leipzig.

Zur Theorie des Ferromagnetismus.

Von F. Bloch, zurzeit in Utrecht.

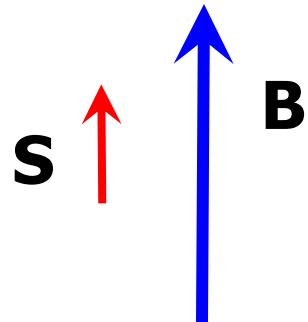
- Heisenberg, 1928, (where is the H-model??)
- Bloch, 1930
- **metals, no classical waves**
- no word "Spinwellen"
- Bethe, 1931, B-ansatz, \Rightarrow first time word "Spinwellen" for Bloch's solution
- HP, Anderson, Kubo, Dyson, never used "magnon"; where is Neel, LL?
- **1D:** Jordan-Wigner, 1928, fermions, Bethe, 1931 (is it a fermionic description)?
- **1941:** first magnon is a fermion (*)

fermion or boson?

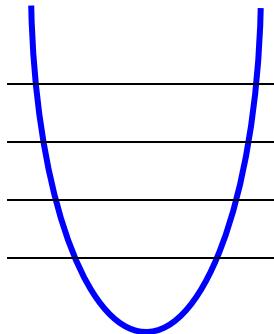
magnons \approx bosons, why natural?

$$[S_{\vec{r}}^{\alpha}, S_{\vec{r}'}^{\beta}] = \emptyset \quad \tilde{r} \neq \tilde{r}'$$

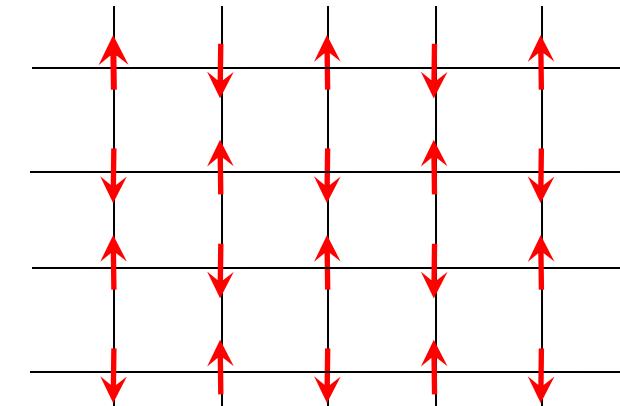
- spin operators commute on different sites!
- **order!** \Rightarrow effective (Weiss) field for each spin
local raising (lowering) operators \Rightarrow bosons
- **ordered state** \Rightarrow classical wave \Rightarrow quantization is **bosonic**

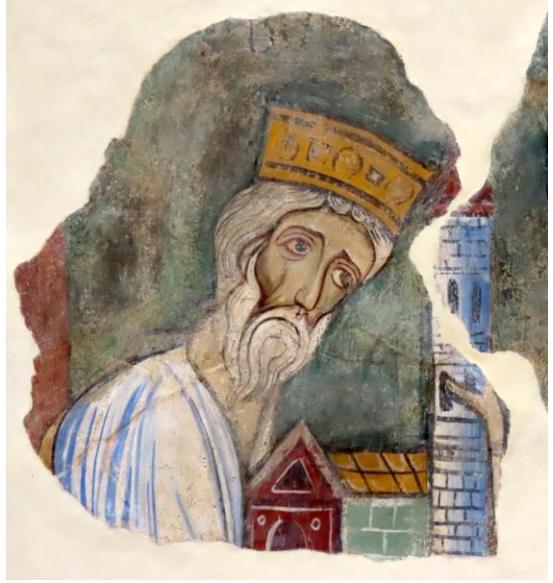


$$\mathcal{H} = -\mu_B \mathbf{B} \cdot \mathbf{S}$$



$$S^z = S - b^\dagger b, \\ S^+ \approx b, \quad S^- \approx b^\dagger$$





I. bosonic path

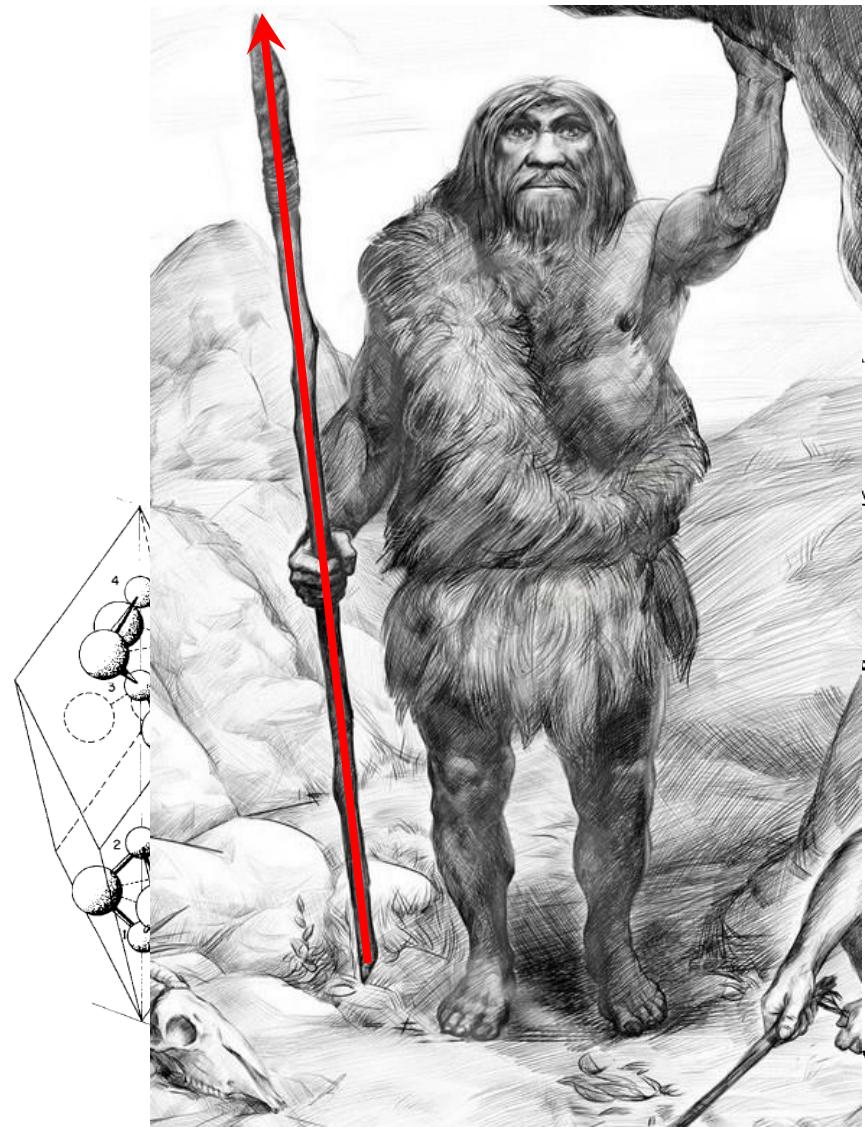
The Virgin, two Apostles and Offering of a church to a saint

Burgundy, late 12th century

Christ in Majesty. Among them is the king Boson (with the letters "BOS" above him) who entrusts the establishment he founded to saint Stephen

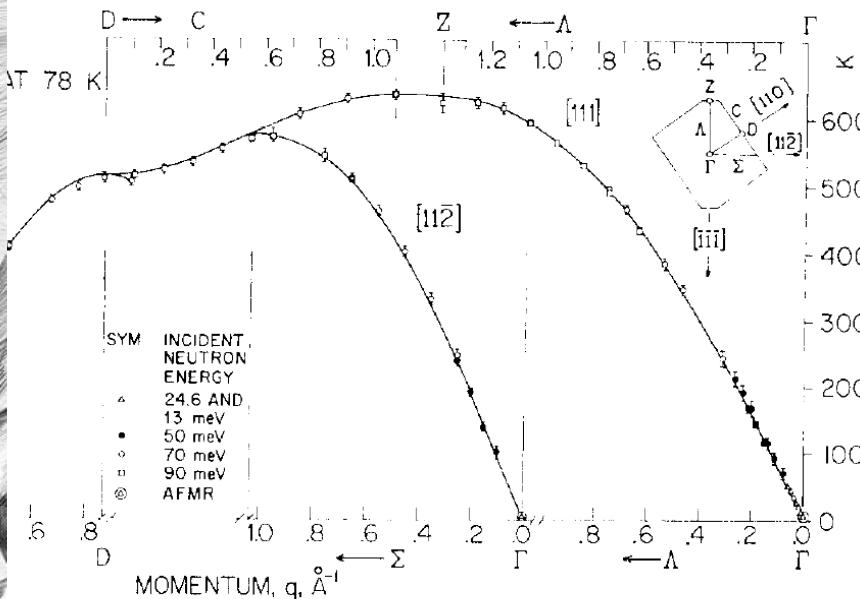
Musée de Cluny, Paris

magnons in Cr_2O_3 = CrO-magnons ...



- where they there all the time?
- indeed, must have existed for long time
- are like humans, \Rightarrow first you care how they look
- then \Rightarrow how they interact (with others)

MUELSEN, M. T. HUTCHINGS AND G. SHIRANE

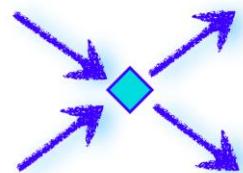


Energy dispersion relation $E(\mathbf{q})$ in several directions as obtained

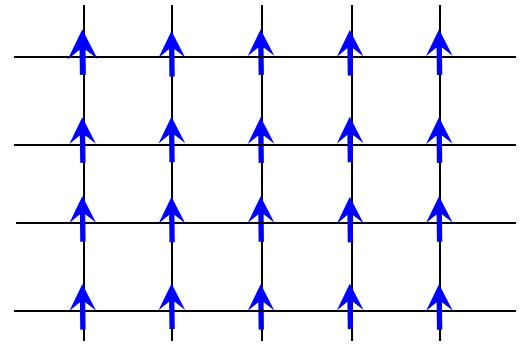
how do “bosons” interact?, I

- normal Bose gas \Rightarrow density-density
 - # of bosons preserved

$$\hat{\mathcal{H}}_0 = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} b_{\mathbf{k}}^\dagger b_{\mathbf{k}} + \frac{1}{4} \sum_{\mathbf{k}_i} V_{\mathbf{k}_1, \mathbf{k}_2; \mathbf{k}_3, \mathbf{k}_4}^{(1)} b_{\mathbf{k}_1}^\dagger b_{\mathbf{k}_2}^\dagger b_{\mathbf{k}_3} b_{\mathbf{k}_4}$$



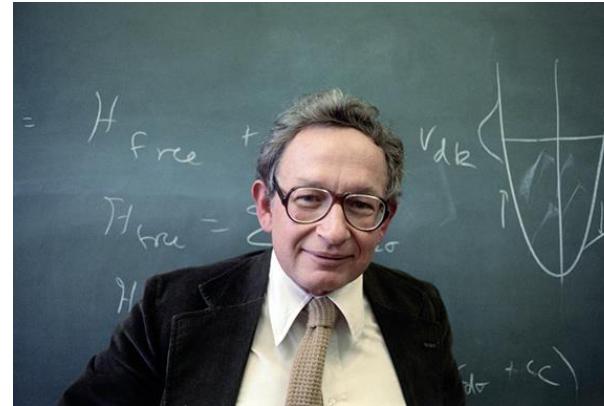
- exactly that for **Heisenberg** ferromagnet
 - GS and excitations are e-states of S_{tot}^z
 - **GS** $S_{\text{tot}}^z |GS\rangle = SN |GS\rangle$
 - magnon $\Rightarrow \Delta S^z = -1$
 - magnon # preserved



- is simplest = general
and complicated = special?
- or, simplest = restrictive
and complicated = general?

more is different

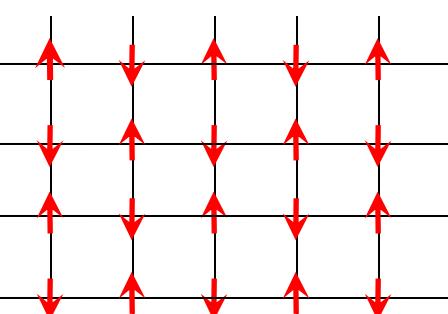
PWA



how do bosons interact? in an AF, II

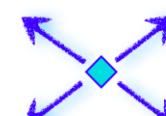
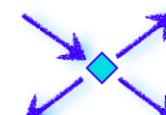
- in an antiferromagnet (**define AF!**)
 - GS is the superposition of states with different S_{tot} [PWA'82]
 - no definite spin for a magnon!
 - magnon # **not** preserved: $a \Rightarrow ub + vb^\dagger$
- true for **collinear Heisenberg** or **XXZ** models
- only $2 \Rightarrow 2, 1 \Rightarrow 3, 0 \Rightarrow 4$, parity or U(1)
- seems similar to BEC problem, **but it is not!**
- here $\Rightarrow \langle a \rangle \sim S^+ = 0$, no canting, but $\langle aa \rangle \neq 0$

$$\hat{\mathcal{H}}_0 = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} b_{\mathbf{k}}^\dagger b_{\mathbf{k}} + \frac{1}{4} \sum_{\mathbf{k}_i} V_{\mathbf{k}_1, \mathbf{k}_2; \mathbf{k}_3, \mathbf{k}_4}^{(1)} b_{\mathbf{k}_1}^\dagger b_{\mathbf{k}_2}^\dagger b_{\mathbf{k}_3} b_{\mathbf{k}_4} + \dots$$



$$+ \frac{1}{3!} \sum_{\mathbf{k}_i} V_{\mathbf{k}_1, \mathbf{k}_2; \mathbf{k}_3, \mathbf{k}_4}^{(2)} b_{\mathbf{k}_1}^\dagger b_{\mathbf{k}_2}^\dagger b_{\mathbf{k}_3}^\dagger b_{\mathbf{k}_4} + \text{H.c.}$$

$$+ \frac{1}{4!} \sum_{\mathbf{k}_i} V_{\mathbf{k}_1, \mathbf{k}_2; \mathbf{k}_3, \mathbf{k}_4}^{(3)} b_{\mathbf{k}_1}^\dagger b_{\mathbf{k}_2}^\dagger b_{\mathbf{k}_3}^\dagger b_{\mathbf{k}_4} + \text{H.c.}$$



PHYSICAL REVIEW B VOLUME 3, NUMBER 3 1 FEBRUARY 1971

Dynamics of an Antiferromagnet at Low Temperatures: Spin-Wave Damping and Hydrodynamics*

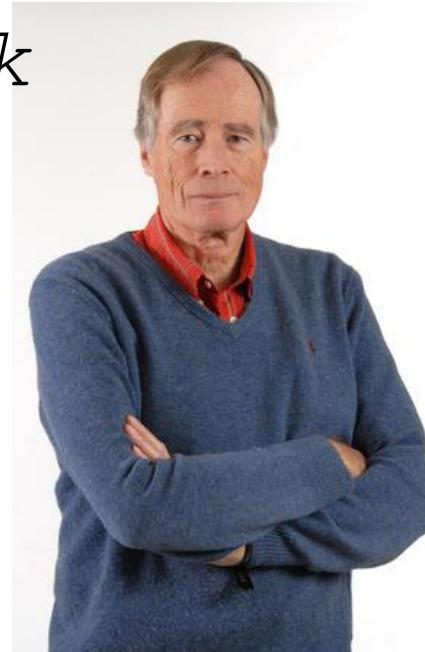
A. B. Harris and D. Kumar
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104
and
B. I. Halperin and P. C. Hohenberg
Bell Telephone Laboratories, Murray Hill, New Jersey 07974
(Received 14 July 1970)

V. G. Bar'yakhtar, V. L. Sobolev, and A. G. Kvirkadze, ZhETP **65**, 790 (1973);
S. M. Rezende and R. M. White, PRB **14**, 2939 (1976); **18**, 2346 (1978);
Yu. A. Kosevich and A. V. Chubukov, Sov. Phys. JETP **64**, 654 (1986)];
S. Tyc and B. I. Halperin, PRB **42**, 2096 (1990);
P. Kopietz, PRB **41**, 9228 (1990).

- interactions I and II (FM and AFM) \Rightarrow universal and **leading**
- dipole-dipole, DM, field-induced canting? \Rightarrow small corrections
- **different from other bosons! BEC, phonons, etc.**

is less also different?

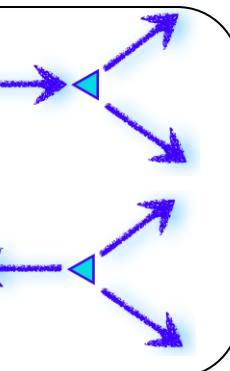
Zach Fisk



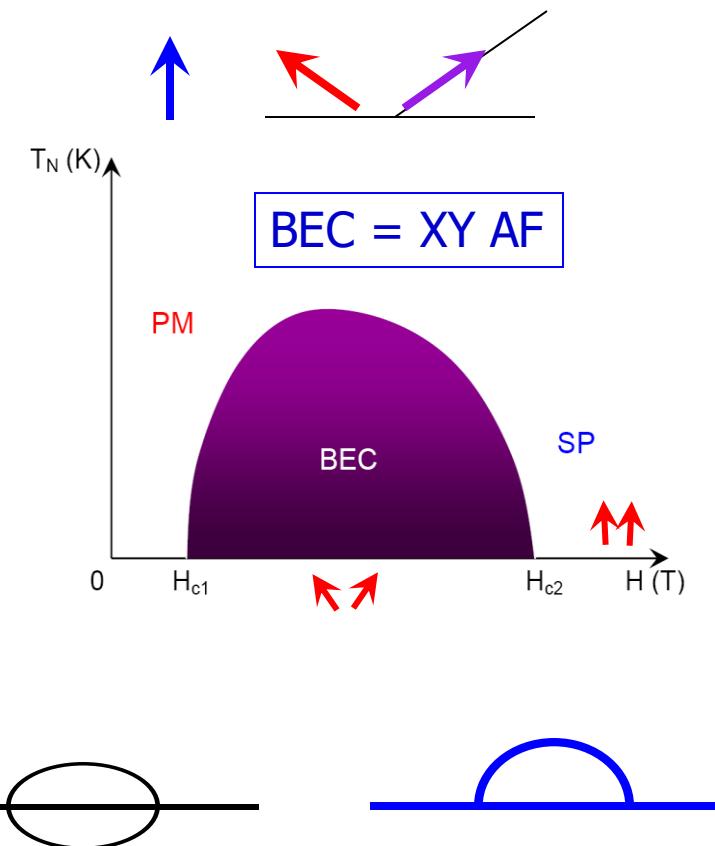
different class of reality: more interactions

- # of bosons **not** preserved $\langle a \rangle \sim S^+ \neq 0$

- Bogolyubov substitution $b_{\mathbf{k}_1}^\dagger b_{\mathbf{k}_2}^\dagger b_{\mathbf{k}_3} b_{\mathbf{k}_4} \Rightarrow b_{\mathbf{k}_1}^\dagger b_{\mathbf{k}_2}^\dagger b_{\mathbf{k}_3} \langle b_0 \rangle$
- cubic anharmonicities** (as for phonons!)

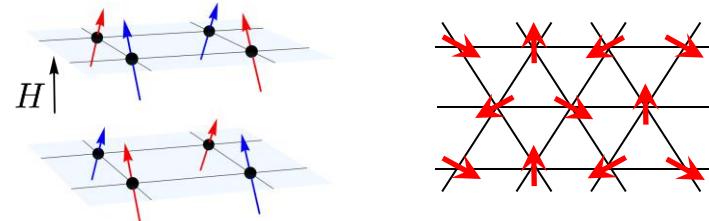
$$\mathcal{H} = \dots + \frac{1}{2!} \sum_{\mathbf{k}_i} V_{\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3}^{(3,1)} b_{\mathbf{k}_1}^\dagger b_{\mathbf{k}_2}^\dagger b_{\mathbf{k}_3} + \text{H.c.} + \frac{1}{3!} \sum_{\mathbf{k}_i} V_{\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3}^{(3,2)} b_{\mathbf{k}_1}^\dagger b_{\mathbf{k}_2}^\dagger b_{\mathbf{k}_3}^\dagger + \text{H.c.}$$


BEC AF: “transverse” magnetization



- new reality?: non-collinear magnets, interactions $\sim J$, not small**

- AFs in a field
- spirals
- DM, dipolar, SOC!

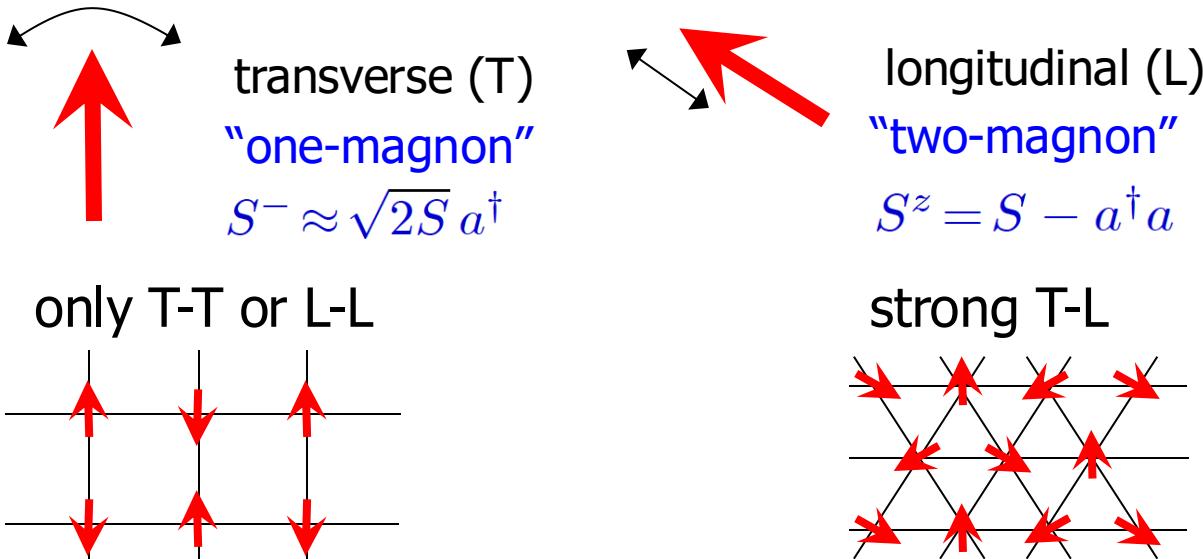


- new phenomena: 1 \Rightarrow 2 decays**

T. Matsubara and H. Matsuda, Prog. Theor. Phys. **16**, 569 (1956);
E. G. Batyev and L. S. Braginskii, Sov. Phys. JETP **60**, 781 (1984).

“collinear” vs “non-collinear”

non-collinearity → transverse-longitudinal coupling → 3-boson terms



$U(1)$ symmetry forbids $S^z S^\pm$ terms

REVIEWS OF MODERN PHYSICS, VOLUME 85, JANUARY-MARCH 2013

Colloquium: Spontaneous magnon decays

M. E. Zhitomirsky

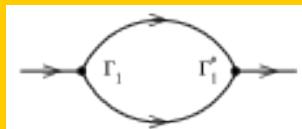
Service de Physique Statistique, Magnétisme et Supraconductivité, UMR-E9001,
CEA-INAC/UJF, 17 rue des Martyrs, 38054 Grenoble cedex 9, France

A. L. Chernyshev

Department of Physics, University of California, Irvine, California 92697, USA

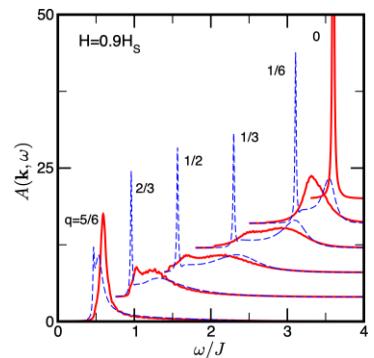
- **SOC** ⇒ **don't need non-collinearity for T-L coupling**
- populated exchange matrix
⇒ **off-diagonal terms**

$$\mathcal{H}_{\text{od}} = \sum_{\langle ij \rangle} \left(\tilde{J}_{ij}^{xz} \tilde{S}_i^x \tilde{S}_j^z + \tilde{J}_{ij}^{yz} \tilde{S}_i^y \tilde{S}_j^z \right)$$

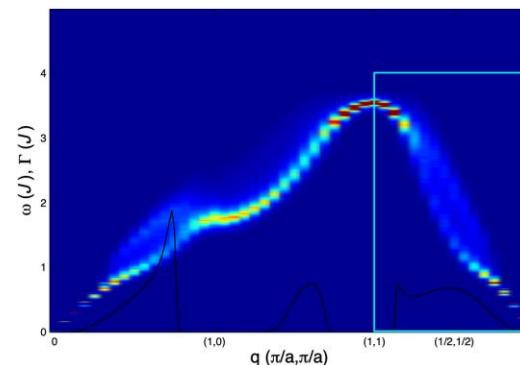


theory vs reality

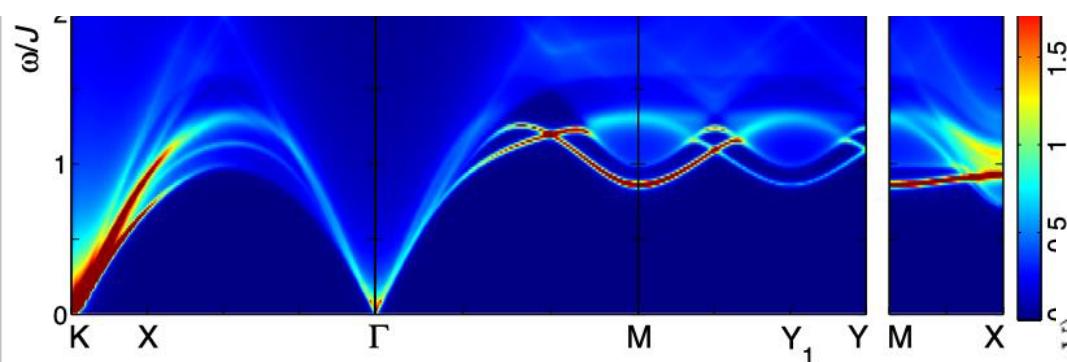
early theory, sq+H



✓ QMC!



triangular ???



○ quantitative comparisons,
needs detailed calculations

LSWT

ED

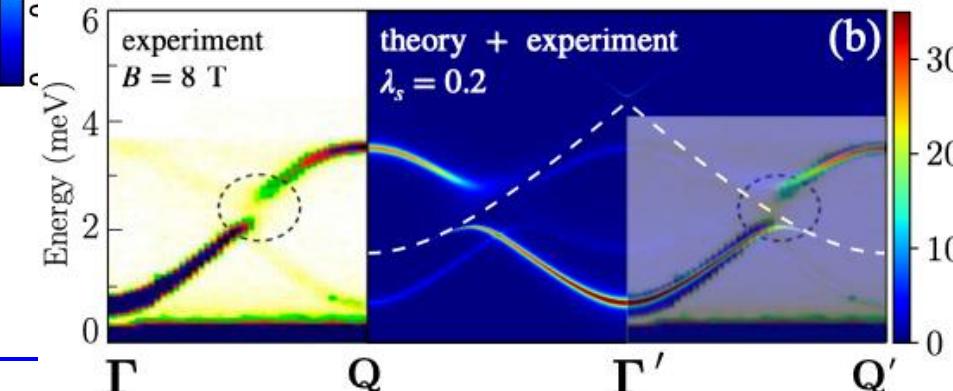
theory

1D Ising chain+ SOC

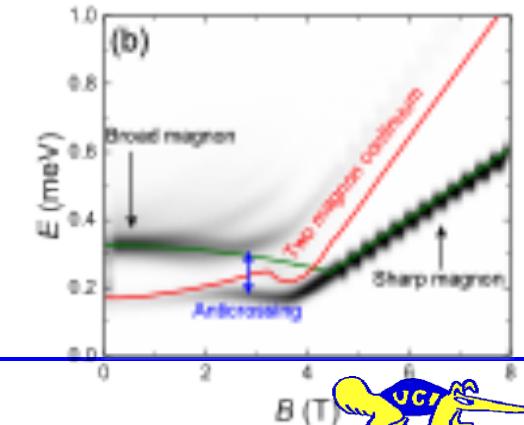
honeycomb, SOC

✓ ED!

✓ some exp!



triangular J1-J2 +H

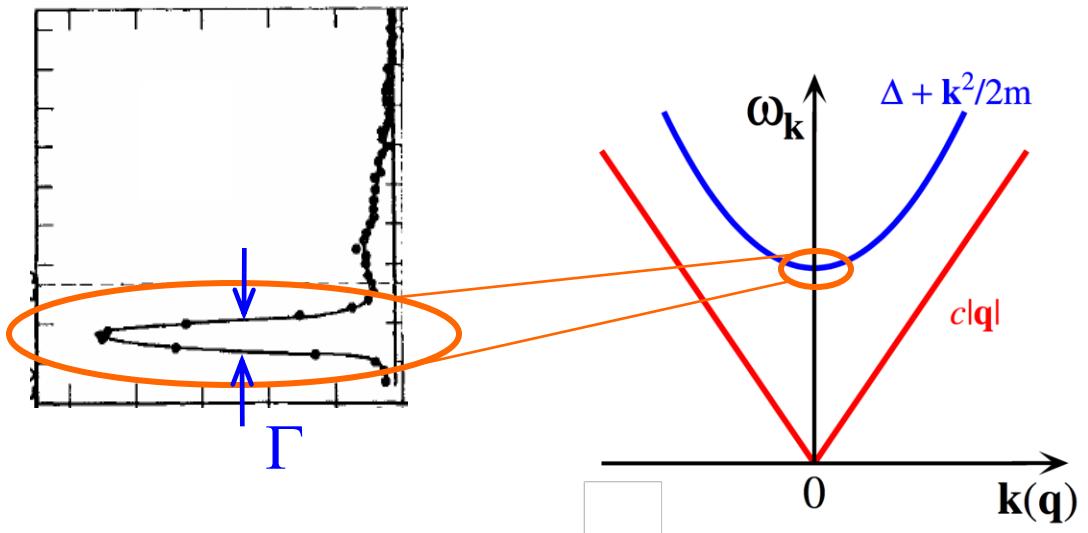


II. disorder?

lifetime of the optical mode, history ...



Mike Zhitomirsky

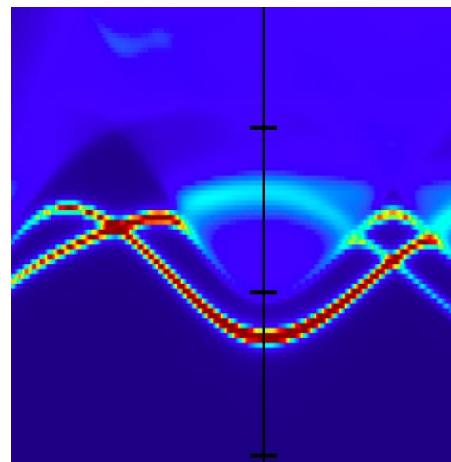
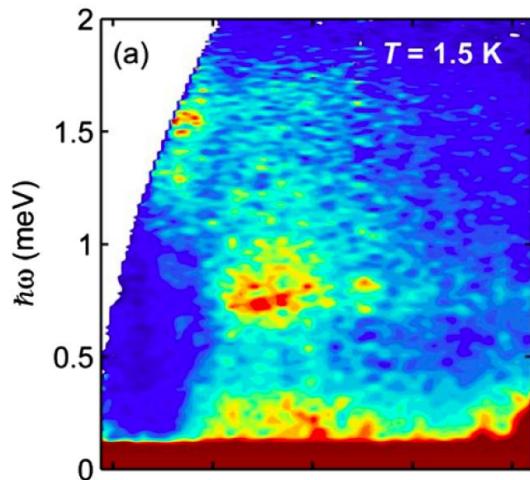


Louis Pierre Regnault

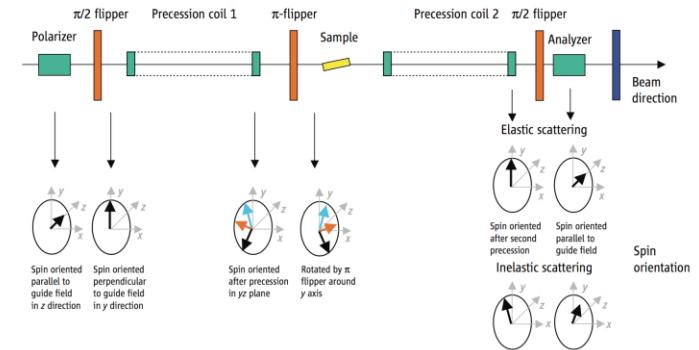
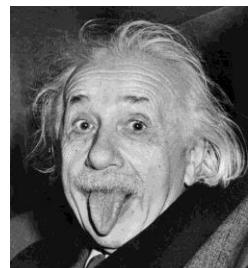


$$\Gamma = AT^\alpha ??$$

neutron-scattering spin-echo



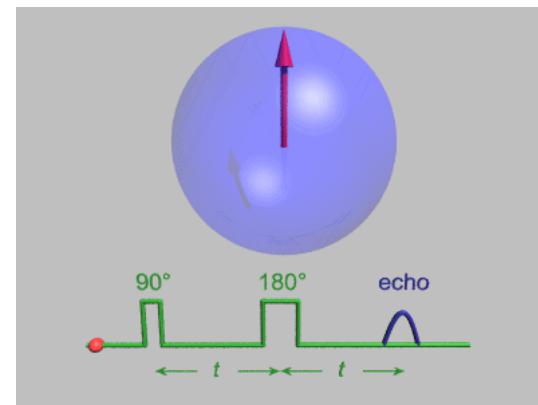
magnon
spinon



APPLIED PHYSICS 1888 VOL 312 SCIENCE 2006

The Neutron Spin-Echo Technique at Full Strength

Joël Mesot



- resolution ~ 1 μeV ($\approx 0.01 \text{ K}$) !

BaNi₂(PO₄)₂

L. P. REGNAULT AND J. ROSSAT-MIGNOD

PHASE TRANSITIONS IN QUASI TWO-DIMENSIONAL PLANAR MAGNETS

L. J. De Jongh (Ed.), *Magnetic Properties of Layered Transition Metal Compounds* 271–321.
© 1990 Kluwer Academic Publishers. Printed in the Netherlands.

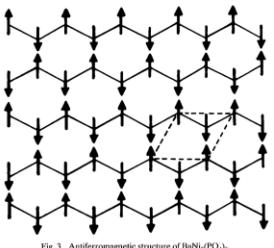
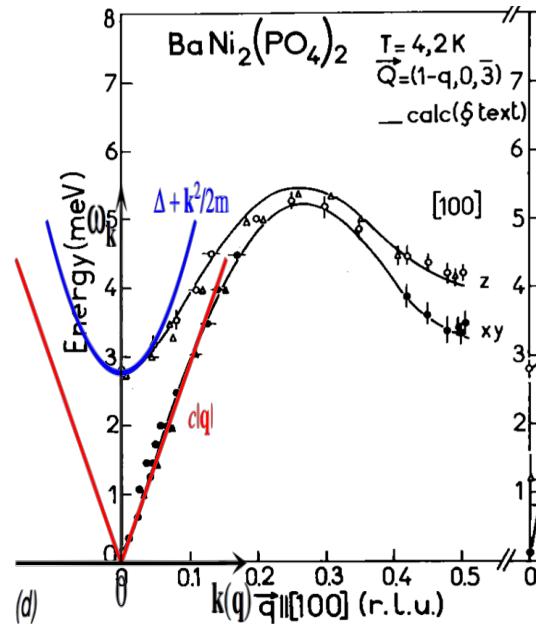
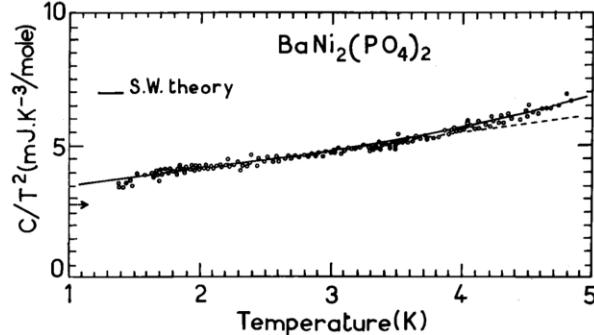
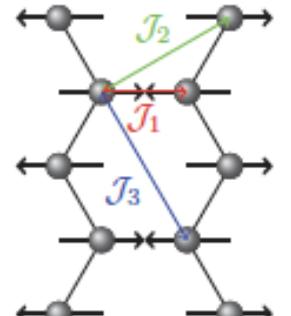
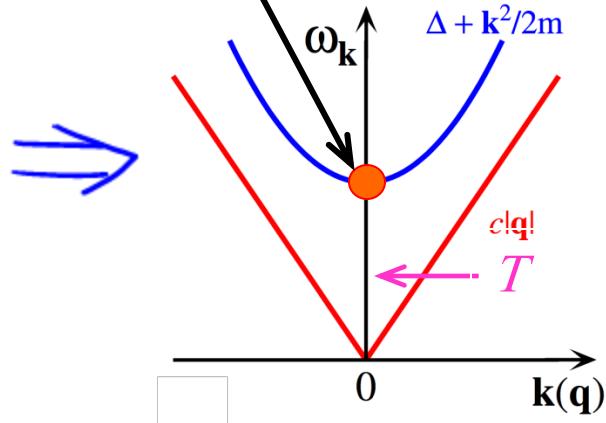
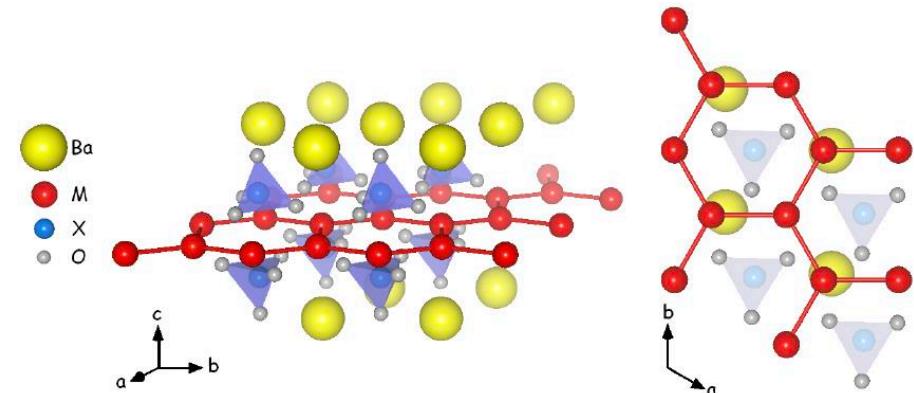


Fig. 3. Antiferromagnetic structure of BaNi₂(PO₄)₂.



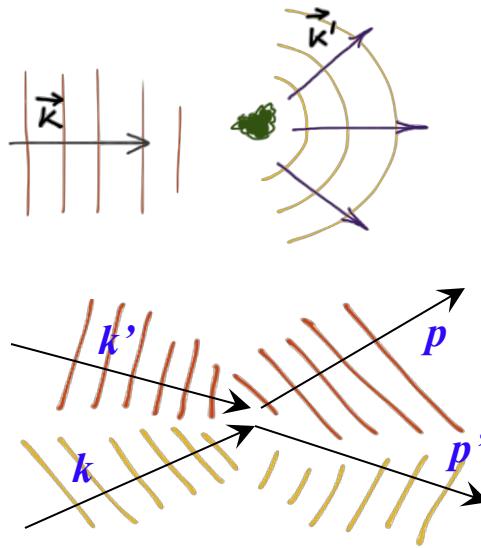
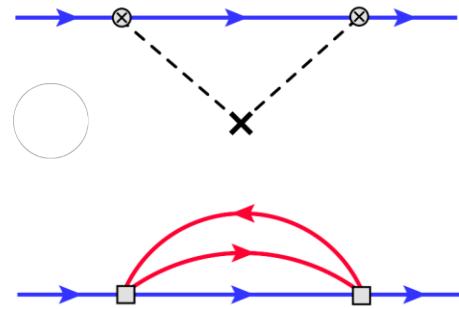
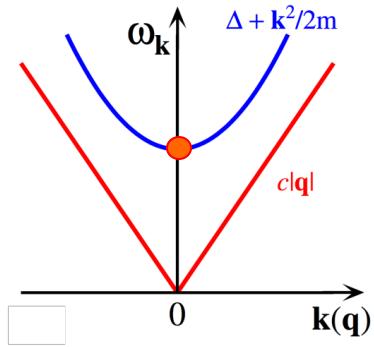
$$\Gamma = ???$$



- $S=1$, 2D, planar (XY)
- honeycomb lattice \Rightarrow gapped and acoustic modes

$$\mathcal{H} = \sum_{\langle i,j \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j + D \sum_i (S^z)^2$$

damping, theory expectations



- linewidth in a “simple” AF
- spin waves: scattering on?
 - impurities
 - themselves

- local distortions $\rightarrow \delta D, \delta J \rightarrow$ conventional impurity scattering (2D):

$$\Gamma_{\mathbf{k}}^{\text{imp}} \approx \Gamma_0 \propto n_{\text{imp}} \overline{\delta D}^2 \frac{m \omega_{\text{max}}^2}{\Delta^2}$$

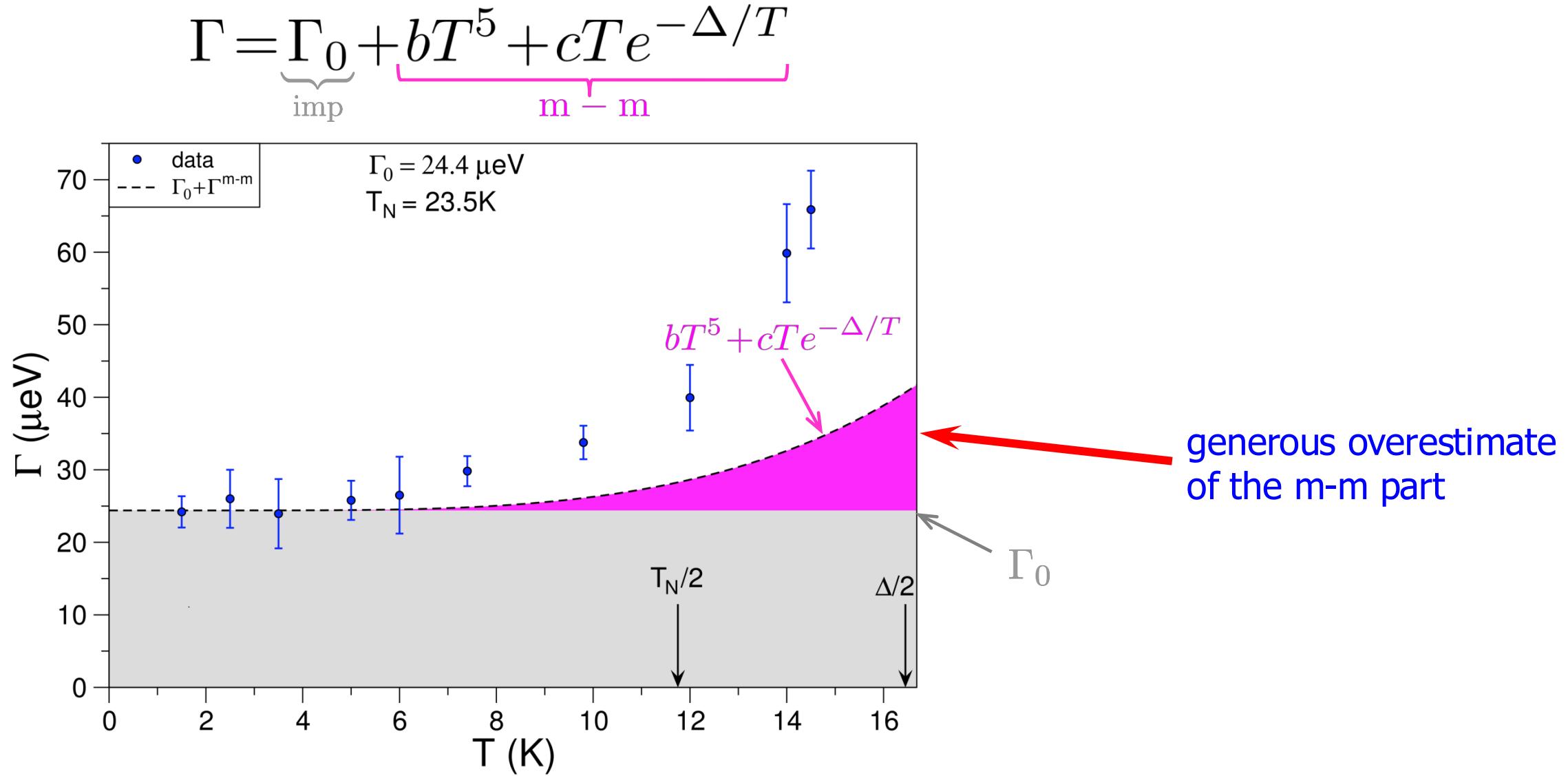
$$\Gamma_{\mathbf{k} \rightarrow 0}^{\text{m-m}} \approx \frac{\pi^3}{15} \frac{\tilde{g}^2}{c} \left(\frac{T}{c} \right)^5$$

- gapped or thermally excited gapless:

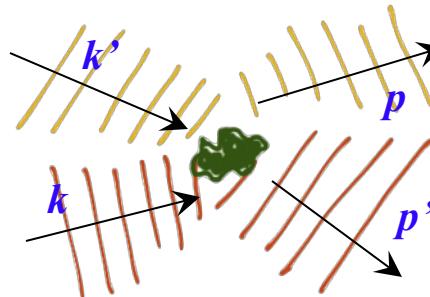
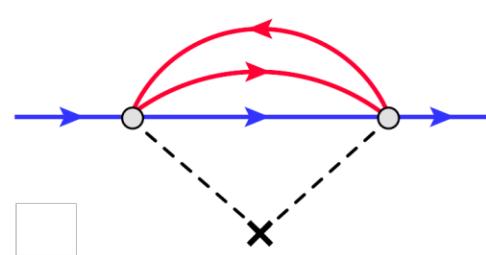
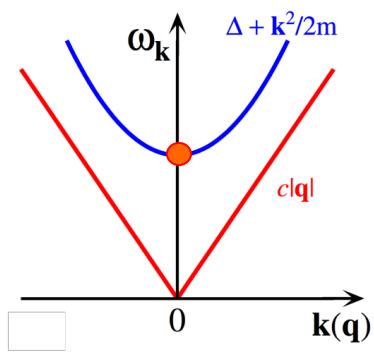
$$\Gamma_{\mathbf{k} \rightarrow 0}^{\beta\beta \rightarrow \beta\beta} \approx \frac{g_\beta^2 m^2 T}{4\pi} e^{-\Delta/T}$$

- **g's for m-m scattering** are known/derivable as all J's and D are known from the spectrum!

standard lore: $\varrho = \varrho_0^{\text{imp}} + \varrho^{ee}(T)$



better theory: impurity-assisted scattering



- impurity facilitates **stronger** m-m scattering

$$V_{\mathbf{k}, \mathbf{q}; \mathbf{k}', \mathbf{q}'}^{\text{imp}} \approx \tilde{g}_{\text{imp}} / \sqrt{\tilde{q}\tilde{q}'} \quad \text{vs} \quad V_{\mathbf{kq}; \mathbf{k}'\mathbf{q}'}^{\text{m-m}} \propto \sqrt{qq'}$$

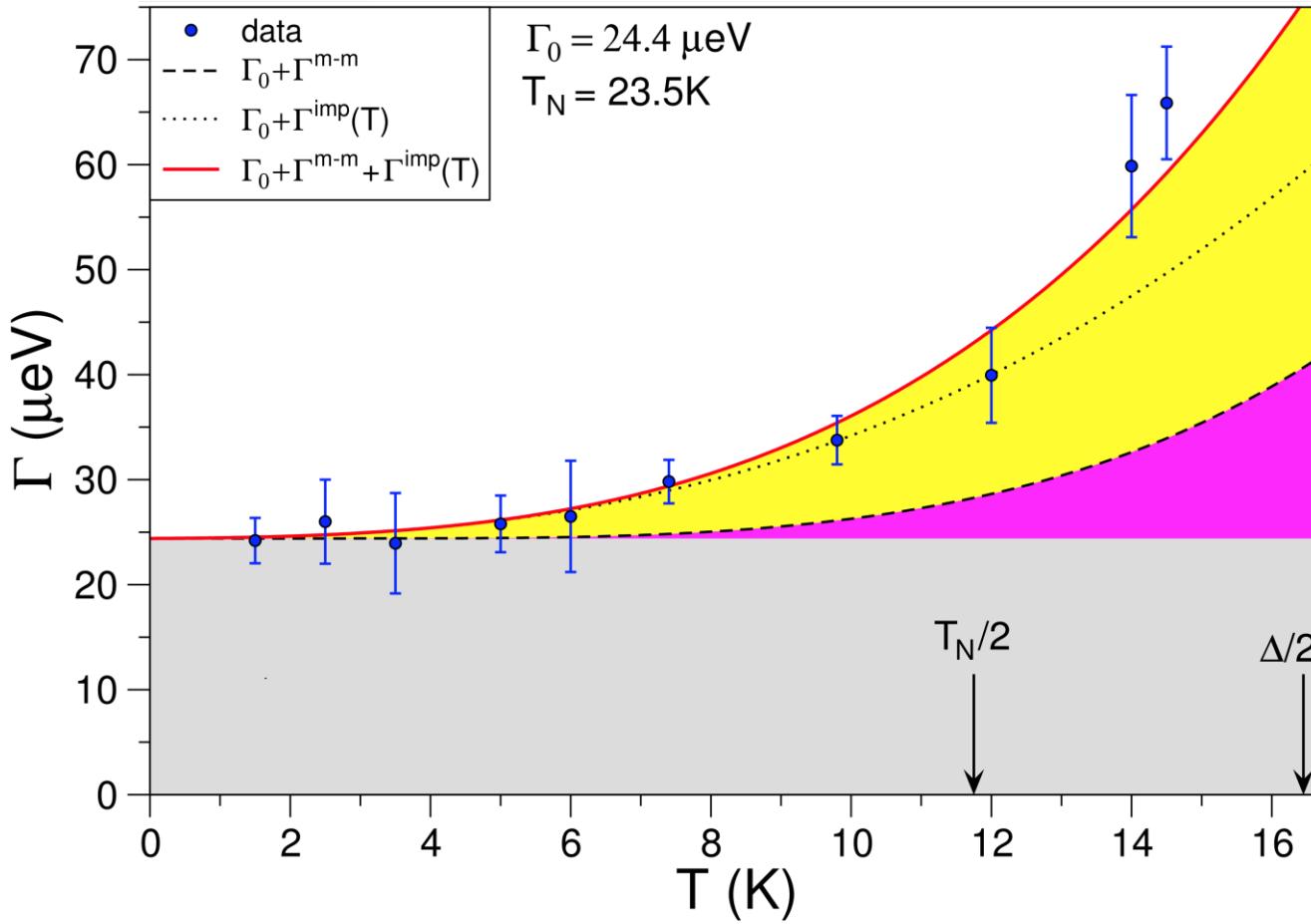
- → **lower power of T** in Γ

$$\boxed{\Gamma_{\mathbf{k} \rightarrow 0}^{\text{imp}, T} \approx \tilde{A} \left(\frac{T}{c} \right)^2 \left[\left(\ln \frac{T}{\omega_0} \right)^2 + \frac{\pi^2}{3} \right]} \quad \boxed{\tilde{A} \sim n_{\text{imp}} \overline{\delta D}^2 m}$$

- reasons for “stronger” potential:
 - “regular” m-m interactions are singular but cancel out
 - impurities violate that cancellation
- “weak Kondo”, optical spin-flip “sits” at impurity, scatters acoustic mode stronger ...

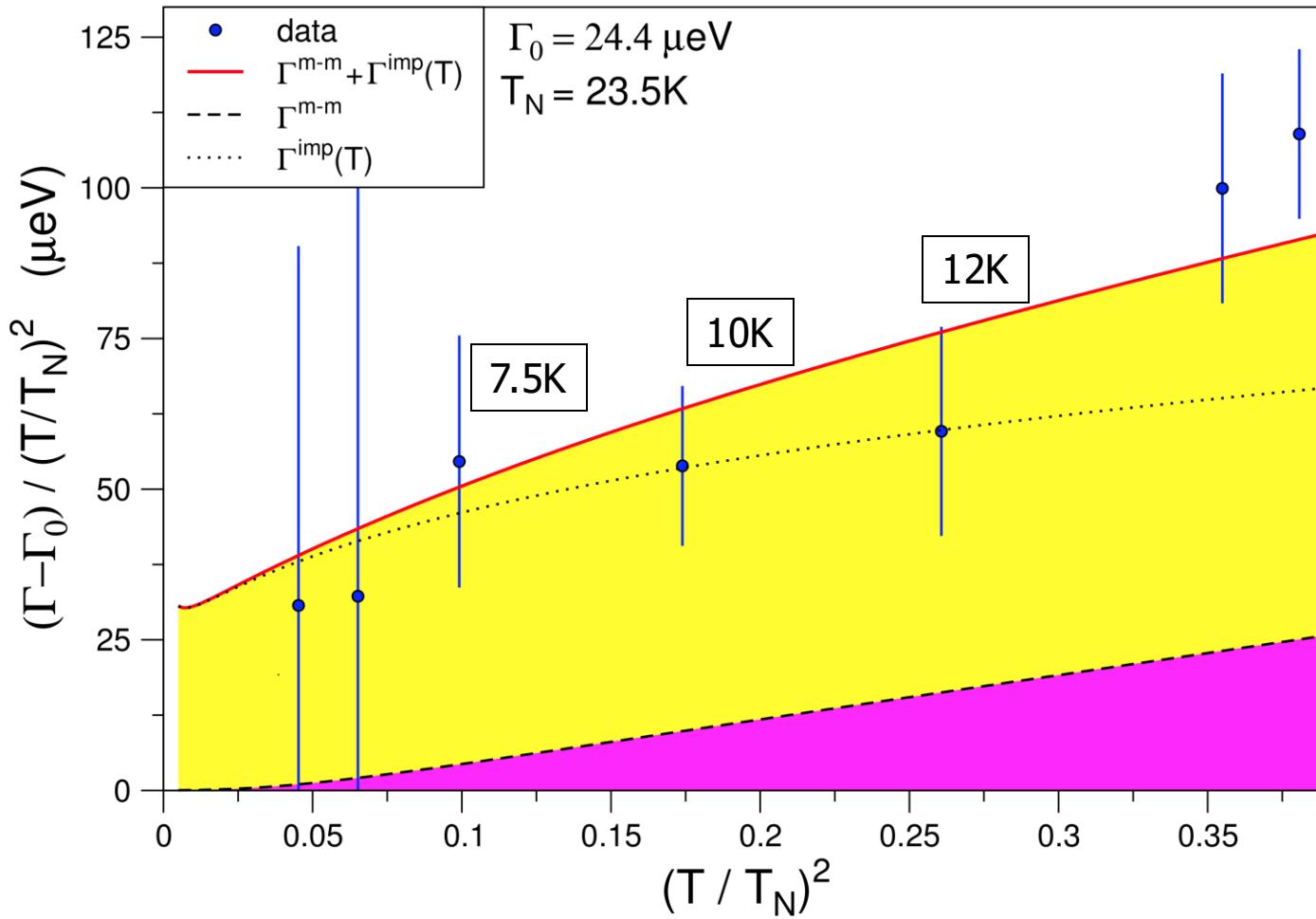
beyond the standard model ...

$$\Gamma = \underbrace{\Gamma_0}_{\text{imp}} + \underbrace{aT^2 \left[\left(\ln \frac{T}{\omega_0} \right)^2 + \frac{\pi^2}{3} \right]}_{\text{imp. finite-}T} + \underbrace{bT^5 + cTe^{-\Delta/T}}_{m-m}$$



“usual” m-m mechanism
is totally **subleading!**

better picture ...



“usual” m-m mechanism
is totally **subleading!**

cross-checks, predictions

T=0 and T>0 impurity terms must be related ($\Gamma_0 \sim \tilde{A} \sim n_{\text{imp}} \overline{\delta D}^2 m$)

true, in our fit: $\Gamma_0 \approx \tilde{A} \approx 25 \mu\text{eV}$

does disorder strength make sense?

estimate: $n_{\text{imp}} (\overline{\delta D}/D)^2 \approx \Gamma_0/\omega_{max} \approx 10^{-2}$

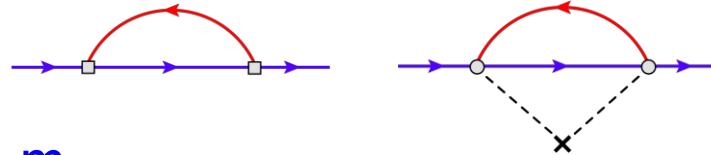
translates into a (very reasonable) statement that in BaNi₂(PO₄)₂, strong modulation of magnetic couplings of order 1 is spread over 1 in 100 unit cells

predictions:

* 3D: $\Gamma_{3D}^{\text{imp}} \propto n_{\text{imp}} T^{9/2}$

* AFs with non-collinear order \rightarrow 3-magnon coupling

new diagrams:



no change in m-m

lower power of T in impurity-induced:

$$\boxed{\Gamma_{\mathbf{k} \rightarrow 0}^{\text{imp},T} \approx \tilde{A}_3 \left(\frac{T}{c} \right) \ln \frac{T}{\omega_0}}$$
$$\tilde{A}_3 \propto n_{\text{imp}} |\delta g_3|^2$$

quo vadis?

- is simplest = general and complicated = special?
- or, simplest = restrictive and complicated = general?
- lower-order anharmonicities = more general, new effects, **1** \Rightarrow **2 vs 1** \Rightarrow **3**
- after many years, the technology is able to test theoretical ideas of HKHH in reality:
 \Rightarrow general case: low- T lifetime is completely dominated by simple disorder
- *"The great tragedy of Science — the slaying of a beautiful hypothesis by an ugly fact."*
Thomas Huxley



conclusions

- ✓ **the world (CM version) is not ideal:**
 - ⇒ symmetries are lowered ⇒ different phenomena
 - ⇒ disorder breaks some locally
- ✓ **there is still universality**
- ✓ **theorists:** embrace reality!



CM="physics of dirt"
["Schmutzphysik"]
Wolfgang Pauli

better picture ...

$$V_{\mathbf{k},\mathbf{q};\mathbf{k}',\mathbf{q}'}^{\text{m-m}} = -2Ju_{\mathbf{k}'}u_{\mathbf{q}'}u_{\mathbf{q}}u_{\mathbf{k}} \left([\text{I}] + d[\text{II}] \right)$$

where terms in the bracket are

$$\begin{aligned} [\text{I}] &= (\gamma_{\mathbf{k}} + \gamma_{\mathbf{q}})(x_{\mathbf{k}'} + x_{\mathbf{q}'}) (1 + x_{\mathbf{k}}x_{\mathbf{q}}) \\ &\quad + (\gamma_{\mathbf{k}'} + \gamma_{\mathbf{q}'})(x_{\mathbf{k}} + x_{\mathbf{q}}) (1 + x_{\mathbf{k}'}x_{\mathbf{q}'}) \\ &\quad + \gamma_{\mathbf{k}}(x_{\mathbf{k}} + x_{\mathbf{q}}x_{\mathbf{k}'}x_{\mathbf{q}'}) + \gamma_{\mathbf{q}}(x_{\mathbf{q}} + x_{\mathbf{k}}x_{\mathbf{k}'}x_{\mathbf{q}'}) \\ &\quad + \gamma_{\mathbf{k}'}(x_{\mathbf{k}'} + x_{\mathbf{q}}x_{\mathbf{k}}x_{\mathbf{q}'}) + \gamma_{\mathbf{q}'}(x_{\mathbf{q}'} + x_{\mathbf{k}}x_{\mathbf{k}'}x_{\mathbf{q}}) \quad (31) \\ &\quad + 2 \left((\gamma_{\mathbf{q}-\mathbf{q}'} + \gamma_{\mathbf{k}-\mathbf{q}'})(1 + x_{\mathbf{k}}x_{\mathbf{q}}x_{\mathbf{k}'}x_{\mathbf{q}'}) \right. \\ &\quad \left. + \gamma_{\mathbf{q}-\mathbf{q}'}(x_{\mathbf{k}}x_{\mathbf{k}'} + x_{\mathbf{q}}x_{\mathbf{q}'}) + \gamma_{\mathbf{k}-\mathbf{q}'}(x_{\mathbf{k}}x_{\mathbf{q}'} + x_{\mathbf{q}}x_{\mathbf{k}'}) \right. \\ &\quad \left. + \gamma_{\mathbf{k}+\mathbf{q}}(x_{\mathbf{k}} + x_{\mathbf{q}})(x_{\mathbf{k}'} + x_{\mathbf{q}'}) \right), \end{aligned}$$

and

$$\begin{aligned} [\text{II}] &= 4 \left(1 + x_{\mathbf{k}}x_{\mathbf{q}}x_{\mathbf{k}'}x_{\mathbf{q}'} + (x_{\mathbf{k}} + x_{\mathbf{q}})(x_{\mathbf{k}'} + x_{\mathbf{q}'}) \right) \\ &\quad - 3 \left(x_{\mathbf{k}} + x_{\mathbf{q}} + x_{\mathbf{k}'} + x_{\mathbf{q}'} \right. \\ &\quad \left. + x_{\mathbf{k}'}x_{\mathbf{q}'}(x_{\mathbf{k}} + x_{\mathbf{q}}) + x_{\mathbf{k}}x_{\mathbf{q}}(x_{\mathbf{k}'} + x_{\mathbf{q}'}) \right), \quad (32) \end{aligned}$$

$$V_{\mathbf{k},\mathbf{q};\mathbf{k}',\mathbf{q}'}^{\text{imp}} = \delta D u_{\mathbf{k}'} u_{\mathbf{q}'} u_{\mathbf{q}} u_{\mathbf{k}} [\text{II}] / 2$$

$$V_{\mathbf{k},\mathbf{q};\mathbf{k}',\mathbf{q}'}^{\text{m-m}} \approx \tilde{g} \sqrt{\tilde{q}\tilde{q}'}$$

$$\tilde{g} = -4J\sqrt{2d} \left(\frac{2+d}{2\sqrt{1+d}} \right)$$

$$V_{\mathbf{k},\mathbf{q};\mathbf{k}',\mathbf{q}'}^{\text{imp}} = \tilde{g}_{\text{imp}} / \sqrt{\tilde{q}\tilde{q}'}$$

Who Named the -ON's?Charles T. Walker¹ and Glen A. Slack²

the ON's

TABLE I. The history of -ON's.

Particle	Author of concept	Author of name
Boson	Particle whose quantum state is symmetric does not obey Pauli principle, Dirac, Heisenberg, 1926.	Dirac, 1947
Electron	Unit of electricity, Stoney, 1874. Electron as subatomic particle, Thomson, 1897. Electron as carrier of electricity in metals, Drude, 1900. Quantum treatment of electron in solids, Pauli, 1927 and Bloch, 1928.	Stoney, 1891
Exciton	Excitation waves which travel through a crystal, Frenkel, 1931.	Frenkel, 1936
Fermion	Particle whose quantum state is antisymmetric obeys Pauli principle. Dirac, 1926.	Dirac, 1947
Magnon	Spin waves, Bloch, 1930. Spin-deviation operators, Holstein and Primakoff, 1940.	Landau, 19?? (Mentioned by Pomeranchuk in 1941.)
Neutron	Neutral particle with mass of proton, Rutherford, 1920.	Rutherford, 1920 or 1921 (Mentioned by Glasson in 1921.)
Phonon	Quantum of elastic waves, Tamm, 1930.	Frenkel, 1932
Photon	Quantum of electromagnetic radiation, Einstein, 1905.	Lewis, 1926
Plasmon	Quantization of plasma oscillations, Bohm and Pines, 1951.	Pines, 1956
Polariton	Quantum mechanical treatment of coupled electromagnetic and lattice fields, Fano, 1956.	Hopfield, 1958
Polaron	Localized electron from electron-lattice interaction, Landau, 1933.	Pekar, 1946
Proton	Hydrogen atoms are the elemental constituent of matter, Prout, 1815.	Rutherford, 1920
Roton	Elementary excitation of the vortex spectrum in liquid helium, Landau, 1941.	Tamm, 19?? (Mentioned by Landau in 1941.)

Now just as for light and electrons it is possible to associate the acoustical waves with certain particles which we shall call "phonons" and to replace the study of the heat oscillations forming these waves by the study of the motion of the corresponding "phonons".*

The footnote is rather amusing:

*It is not in the least intended to convey hereby the impression that such phonons have real existence. On the contrary, the possibility of their introduction rather serves to discredit the belief in the real existence of photons.



word magnon ... visionary, or misguided?

Vol. IV, No. 4

JOURNAL of PHYSICS

1941

THE THERMAL CONDUCTIVITY OF THE PARAMAGNETIC DIELECTRICS AT LOW TEMPERATURES

By I. POMERANCHUK

the magnons. The experimental facts available suggest that the magnons are submitted to the Fermi statistics; namely, when $T \ll \theta_k$ the susceptibility tends to a constant limit, which is of the order of const/θ_k (*). [for $T > \theta_k$, $\chi = \text{const}/(T + \theta_k)$]. Evidently we have here to deal with the Pauli paramagnetism which can be directly obtained from the Fermi distribution. Therefore, we shall assume the Fermi statistics for the magnons*. Due to the Fermi

the lattice. Such magnetic excitations will be called in the following «magnons» (this name was suggested by L. Landau).

* A. Perrier a. H. Kamerlingh Onnes, Leiden Comm., No. 139 (1914).

solid oxygen! ⇒

