Higher rank gauge fields to produce thermal Hall effects

Chisa HOTTA
The University of Tokyo

When and why the magnons can have a finite thermal Hall effect ?

Masataka Kawano

magnons

Ryo Makuta

SOC Hubbard skyrmions

M.Kawano, C. Hotta, Phys. Rev. B **99**, 054422 (2019)

M.Kawano, Y. Onose, C. Hotta, Communications Physics **2**, 27 (2019)

M. Kawano, C. Hotta, Phys. Rev. B **100**, 174402 (2019)

H. Takeda, M. Kawano, ... M. Yamashita, C. Hotta, Nature Comm. **15**, 566 (2024)

R. Makuta, C. Hotta Phys. Rev. Res. **6**, 023133 (2024)

M. Kawano, C. Hotta, Phys. Rev. B **107**, 045123(2024)

R. Makuta, C. Hotta, arXiv: 2504.05166

Gauge fields and fluxes

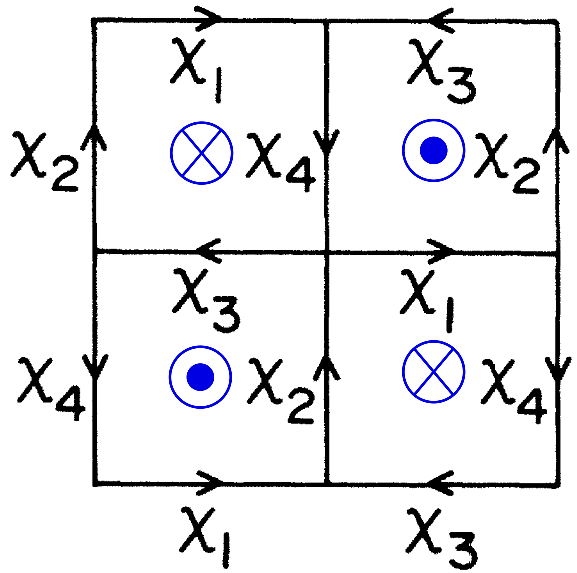
U(1) gauge fields

The transfer integral can have a Peierls phase.

Affleck-Marston, PRB **37**,3774 (1988)

$$H = \sum_{(i,j)} [(n/J)|\chi_{ij}|^2 + (c_{j\alpha}^{\dagger} c_{i\alpha} \chi_{ij} + \mathrm{H.c.})]$$

$$\chi_{ij} \rightarrow e^{i\varphi_{ij}}$$



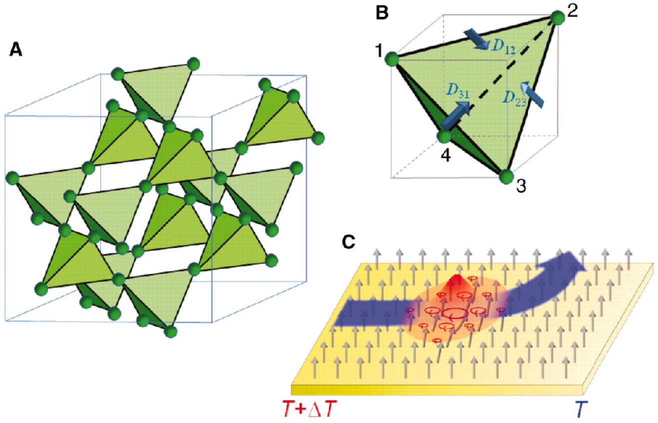
π -flux state

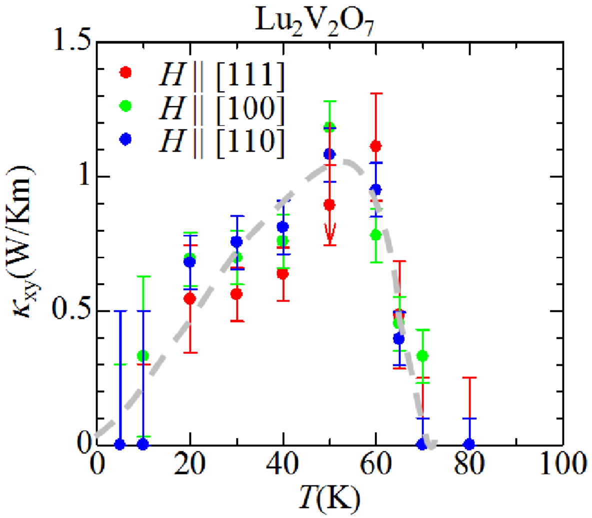
Dirac point, High *Tc* superconductivity.

Observation of the Magnon Hall Effect

Y. Onose,[1,2*] T. Ideue,[1] H. Katsura,[3] Y. Shiomi,[1,4] N. Nagaosa,[1,4] Y. Tokura[1,2,4]

SCIENCE 329(2010)





pyrochlore ferromagnet

T_C =70K

Thermal Hall conductivity κ_{xy} appears below T_c.

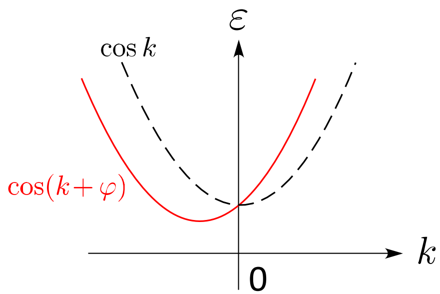
Theories:

S. Fujimoto, PRL **103**, 047203 (2009)

H. Katsura, N. Nagaosa, P. A. Lee, PRL **104**, 066403 (2010).

R. Matsumoto, S. Murakami , PRL 106, 197202, PRB **84**, 184406 (2011)

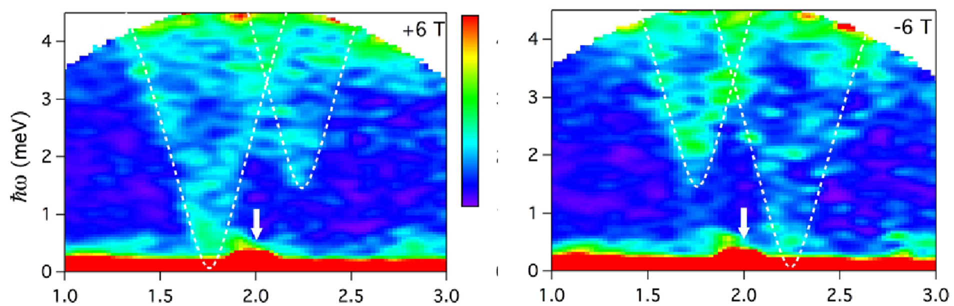
nonreciprocity

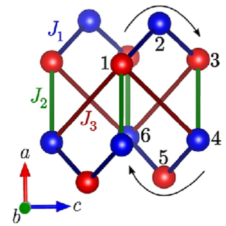


Melcher (1973) Kataoka(1987)

Zakeri et al., (2010), Iguchi et al.(2015)

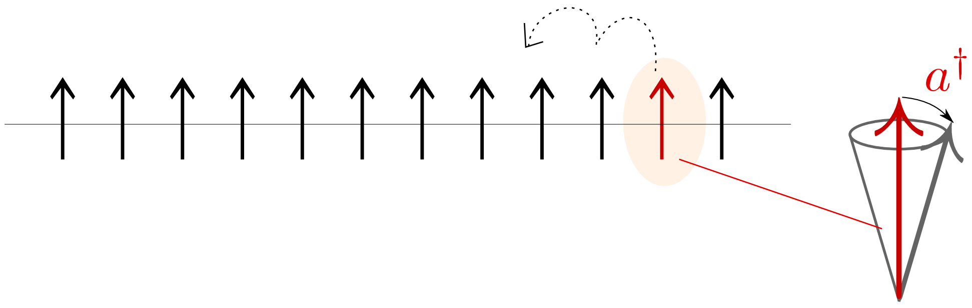
α -$Cu_2V_2O_7$





Gitgeatpong, et al 2017

Ferromagons



Heisenberg $J\, S_i \cdot S_j \;=\; J \;\; (a_i^\dagger a_j + a_j^\dagger a_i)$

DM $D_z\,[\, S_i \times S_j \,] = D_z\, \frac{\mathrm{i}}{2}\,(a_i^\dagger a_j - a_j^\dagger a_i)$

$$J\boldsymbol{S}_i \cdot \boldsymbol{S}_j + \boldsymbol{D} \cdot [\boldsymbol{S}_i \times \boldsymbol{S}_j] \simeq \sqrt{J^2 + (D_{i,j})^2}\left(\mathrm{e}^{i\varphi_{ij}}\, a_i a_j^\dagger + \mathrm{h.c.}\right) + \cdots$$

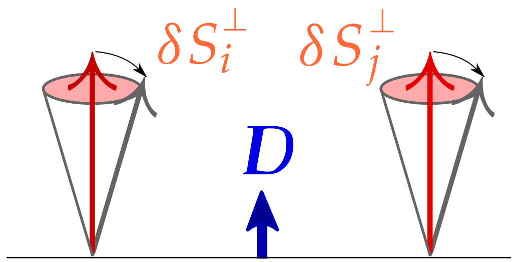
$$\varphi_{i,j} = \arctan(D/J)$$

U(1) gauge field

Fictitious magnetic field induces nonreciprocity & thermal Hall effect

We need DM vector to have a component parallel to the Ferro moment.

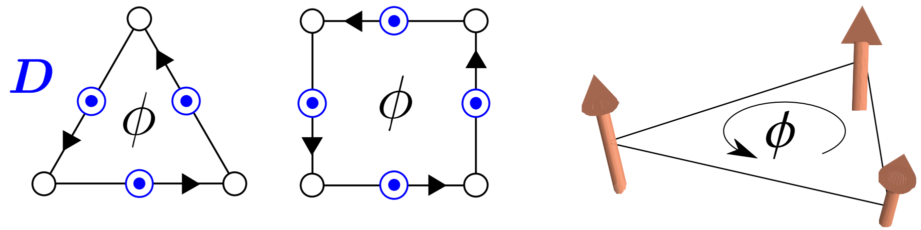
$$\boldsymbol{D} \cdot [\, S_i \times S_j \,] \simeq D_z\,[\, \delta S_i^\perp \times \delta S_j^\perp \,]$$



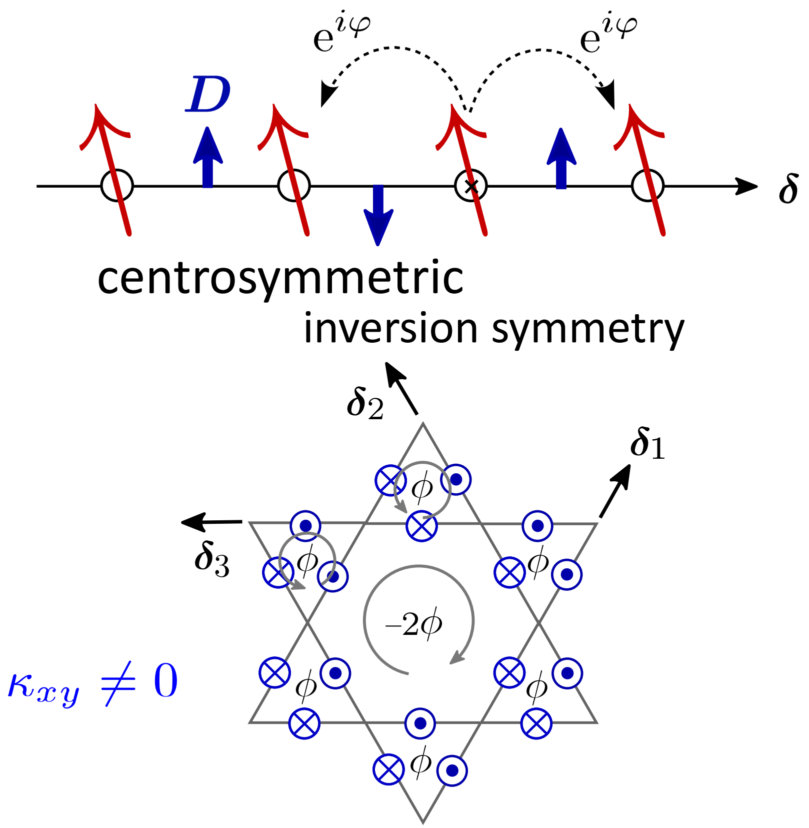
xy fluctuation

DM rules for U(1) gauge fields

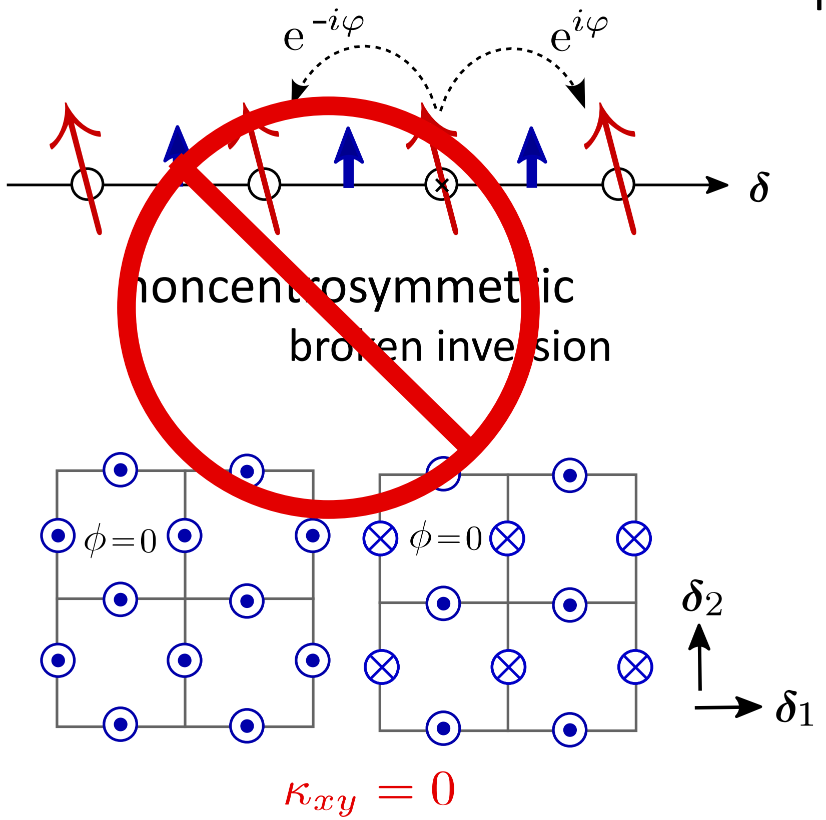
- Fictitious magnetic flux generated by ...
    - staggered DM interaction
    - noncoplaner spins placed on a closed path



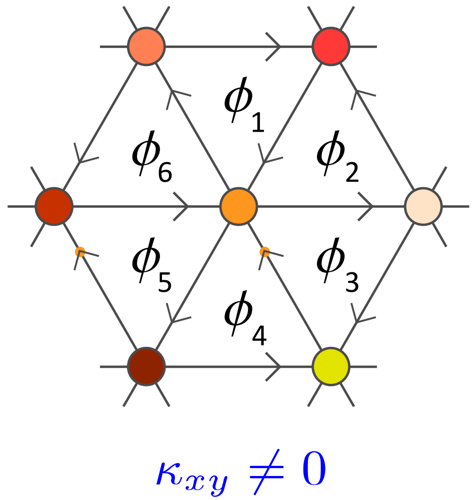
staggered DM



uniform DM



skyrmions
noncoplaner magnets



Avoid flux cancellation

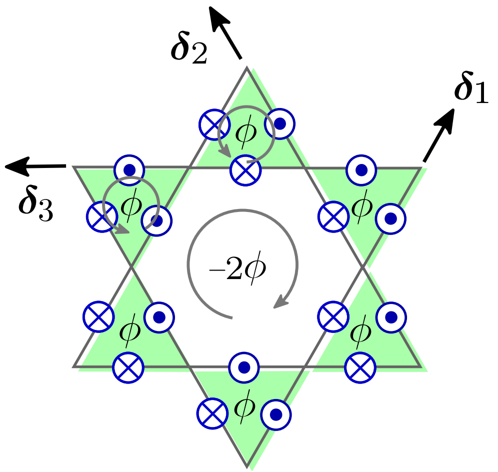
It is NOGO only for ferromagnets.

- All cases have net zero flux.
- Corner-shared lattice have thermal Hall, edge-shaerd lattices, not.

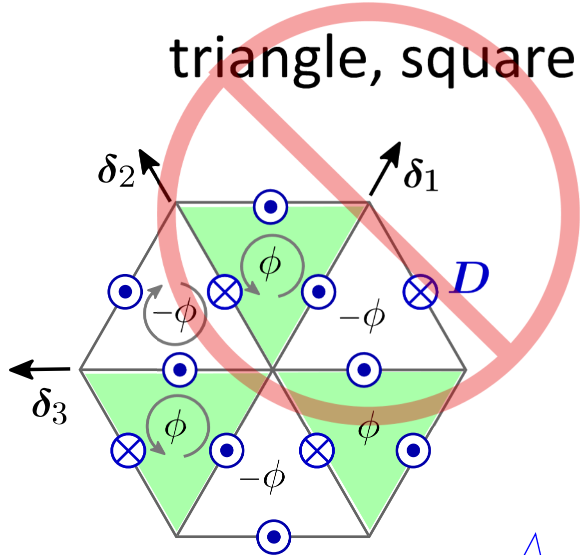
NOGO theorem

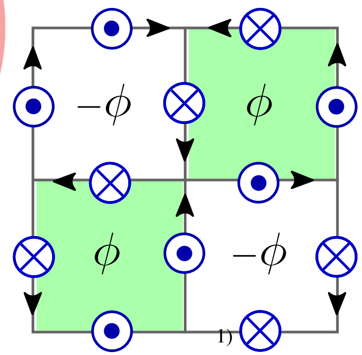
Katsura, et. al (2010)
Ideue, et. al. PRB85, 134411 (2012)

pyrochlore, kagome $\kappa_{xy} \neq 0$



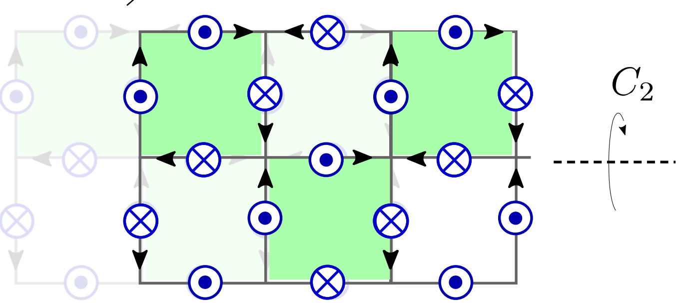
triangle, square $\kappa_{xy} = 0$

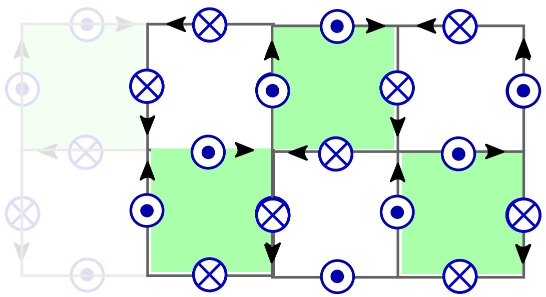




symmetry operation that relates $-\phi$ and ϕ fluxes,

half-period translation $\Longrightarrow$



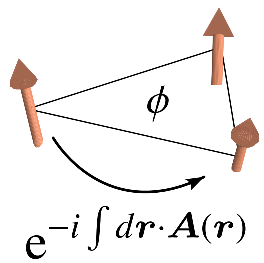


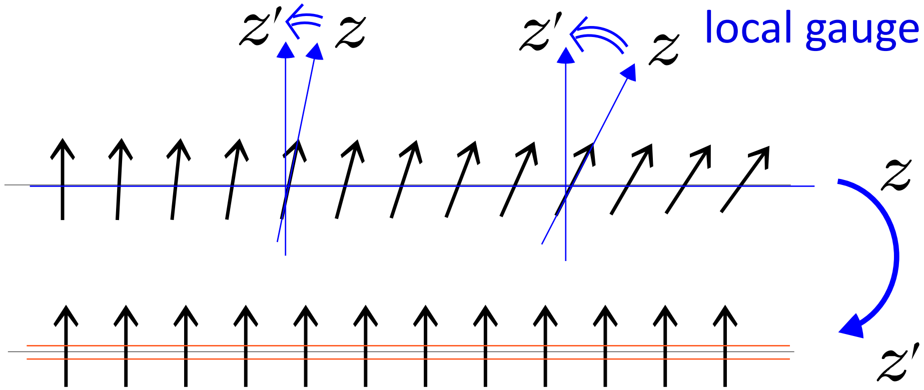
flux goes back but $\kappa_{xy} \rightarrow -\kappa_{xy}$

$\kappa_{xy} = 0$

U(1) gauge for incommensurate case

non-coplaner texture





local gauge transformation

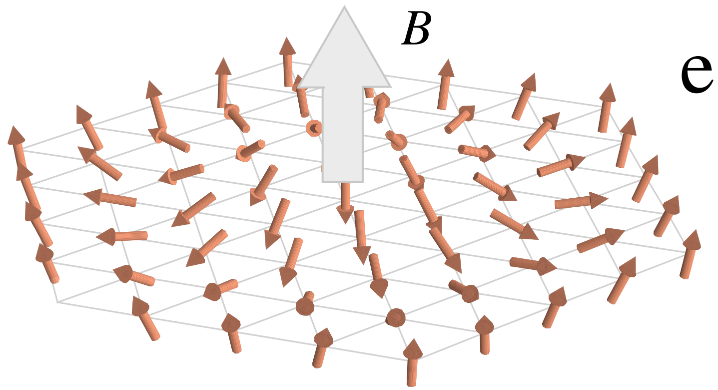
z points in various directions

z' form a global coordinate

interactions are dressed with U(1) gauge

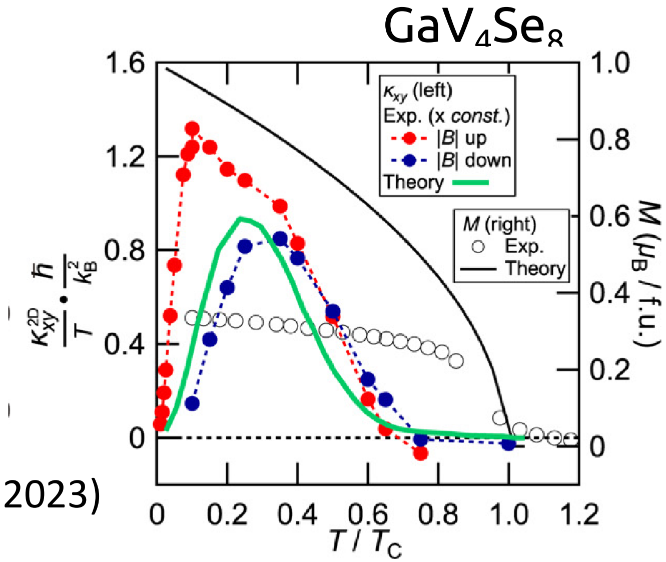
Slowly varying ordered moments generate a vector potential

scalar U(1) gauge field



$$e^{-i\int d\boldsymbol{r}\cdot\boldsymbol{A}(\boldsymbol{r})}$$

skyrmions



Hoogdalem, et.al. PRB 87, 024402 (2013)

Akazawa,et.al.(2023)

What about antiferromagnets?

Masataka Kawano, Chisa Hotta, Phys. Rev. B **99**, 054422 (2019)

Masataka Kawano, Yoshinori Onose, Chisa Hotta, Communications Physics **2**, 27 (2019)

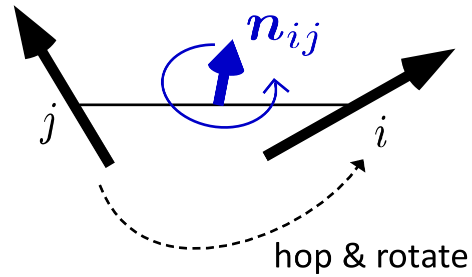
Masataka Kawano, Chisa Hotta, Phys. Rev. B **100**, 174402 (2019)

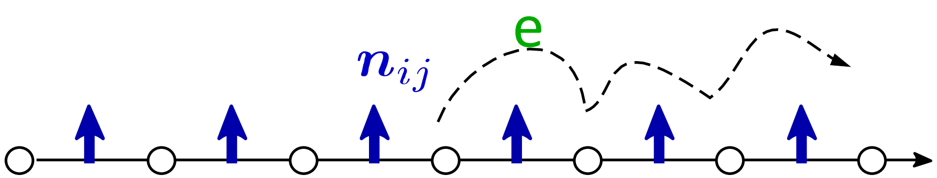
SU(2) gauge: spin orbit coupled electrons

$$\mathcal{H} = \sum_{\langle i,j\rangle}\sum_{\alpha,\beta} -t_{\mathrm{eff}}\,(c^\dagger_{i\uparrow},\, c^\dagger_{i\downarrow})\; e^{-i\frac{\theta}{2}\boldsymbol{n}_{ij}\cdot\boldsymbol{\sigma}} \begin{pmatrix} c_{j\uparrow}\\ c_{j\downarrow}\end{pmatrix}$$

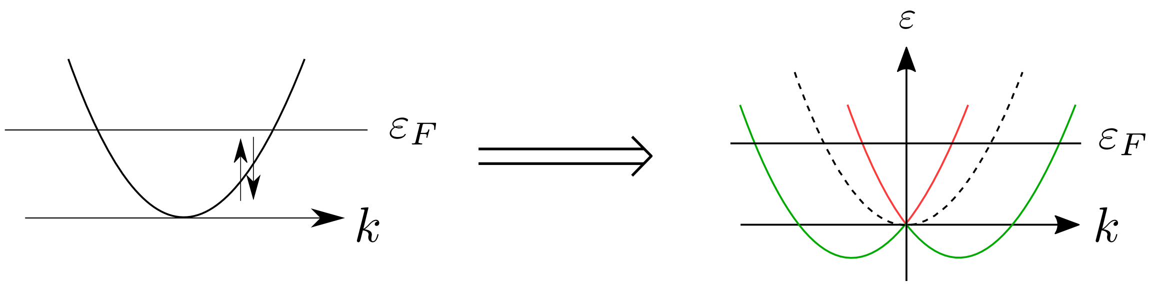
2x2 matrix **SU(2) gauge field**

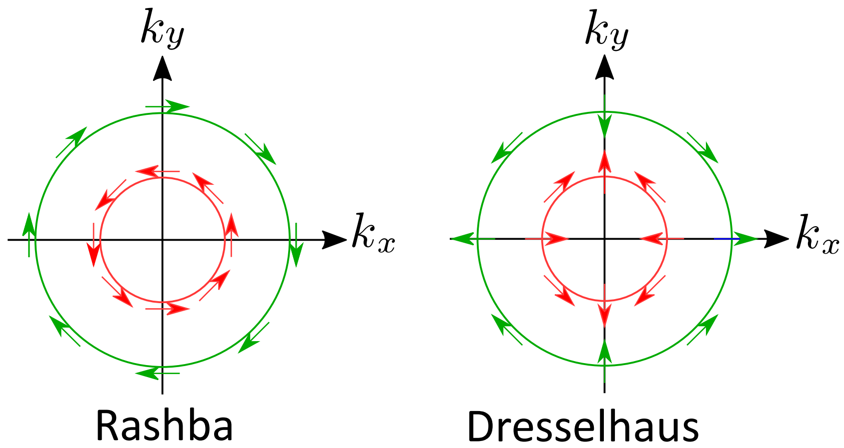
rotation of electron spin about the $\boldsymbol{n}_{ij}$ -axis





SOC + broken inversion symmetry = spin splitting





Can we realize it in Insulators?

"spin momentum locking"

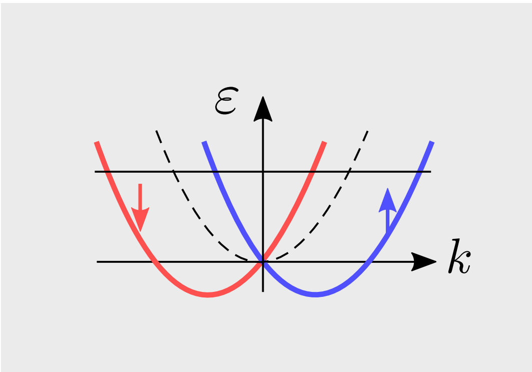
Degrees of freedom and gauges

electrons

up & down spins

SO coupling

↓



Rashba-Dresselhaus effect

Anomalous Hall effect: SU(2) gauge

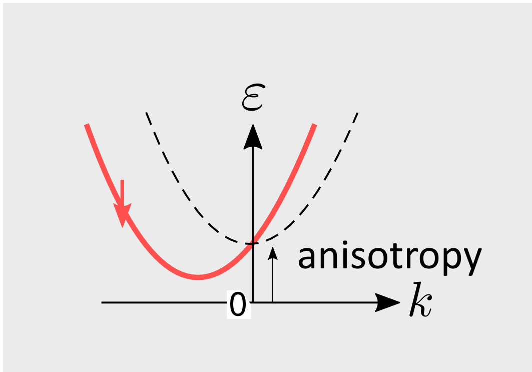
Spin Hall effect :topological insulators

ferro magnons

single component

DM interaction
noncoplaner spins

↓



Nonreciprocity

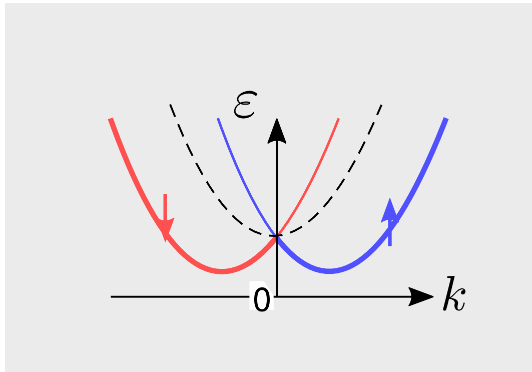
thermal Hall effect:
U(1) gauge

antiferro magnons

A & B sublattices

DM interaction / other effects

↓

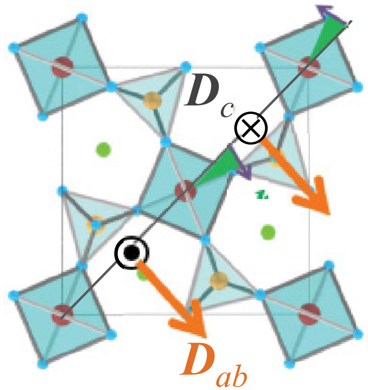


Rashba-Dresselhaus effect

“Anomalous” thermal Hall : SU(2) gauge

topological phase & edge states

$Ba_2MnGe_2O_7$



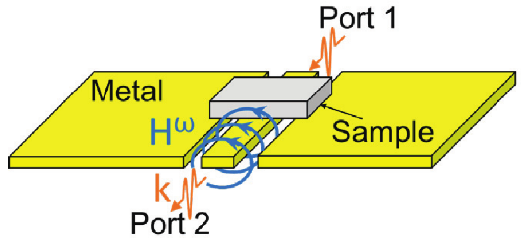
Mn^{2+}

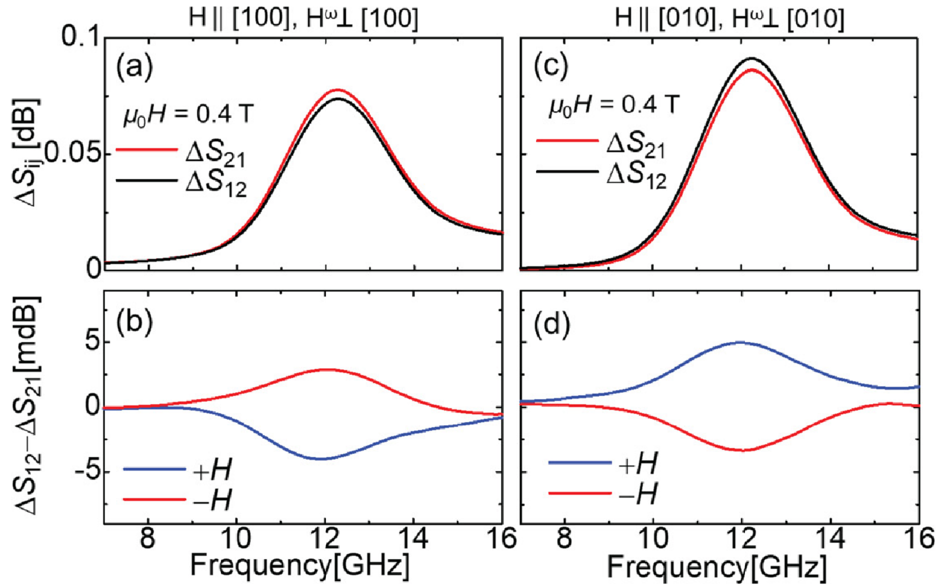
$S = 5/2$ square lattice

- Easy-plane antiferromagnet

J =27 μ eV

Onose group 2018



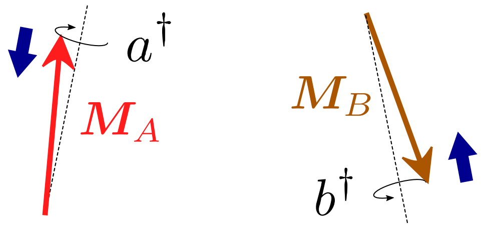


microwave nonreciprocity

Iguchi, et.al. PRB **98**, 064416 (2018)

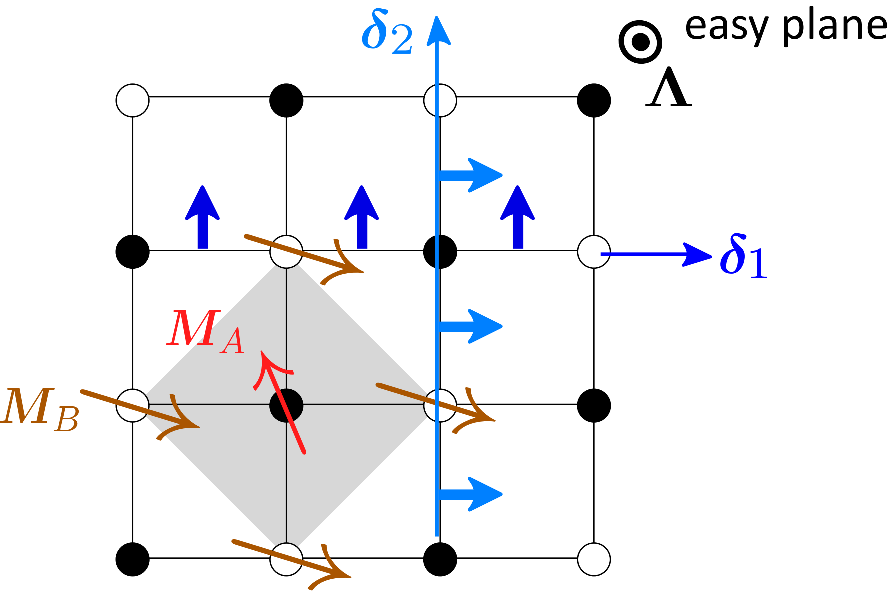
magnon Hall (experimental), not yet.

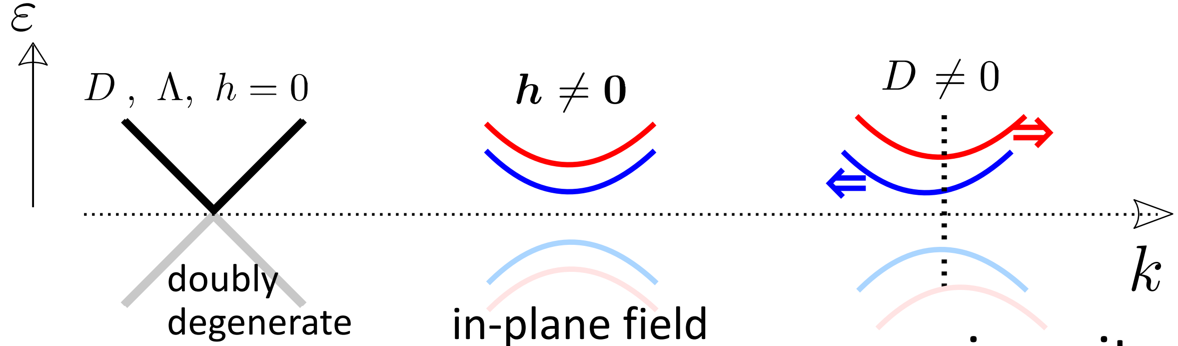
Rashba antiferromagnons



$$\mathcal{H} = J\sum_{\langle i,j\rangle} \boldsymbol{S}_i\cdot\boldsymbol{S}_j + \Lambda\sum_i (S_i^z)^2 + \sum_{\langle i,j\rangle}\boldsymbol{D}_{ij}\cdot[\boldsymbol{S}_i\times\boldsymbol{S}_j]$$

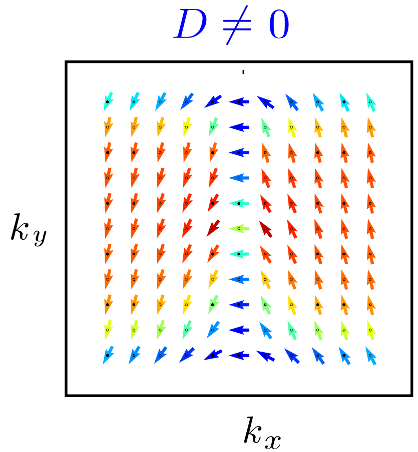
Rashba type DM



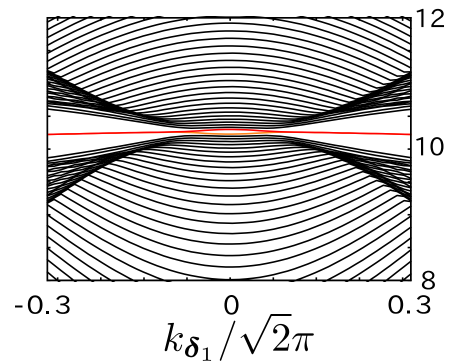


nonreciprocity
is realized

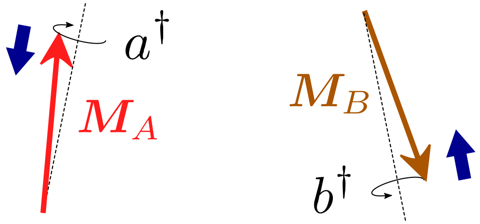
spin textures appear.



We get a topological
edge state

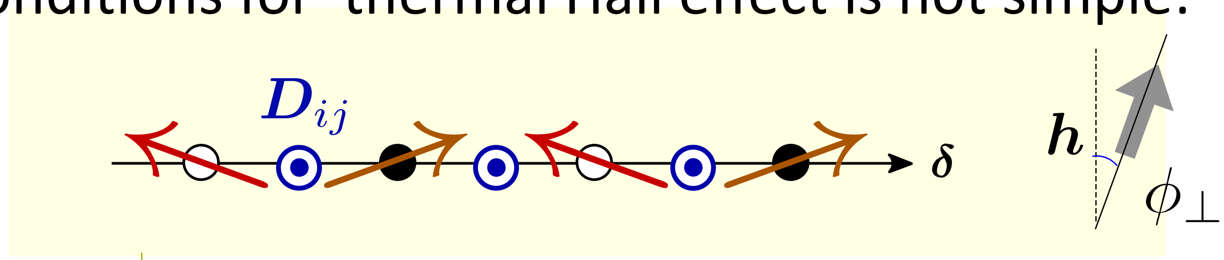


Rashba antiferromagnons

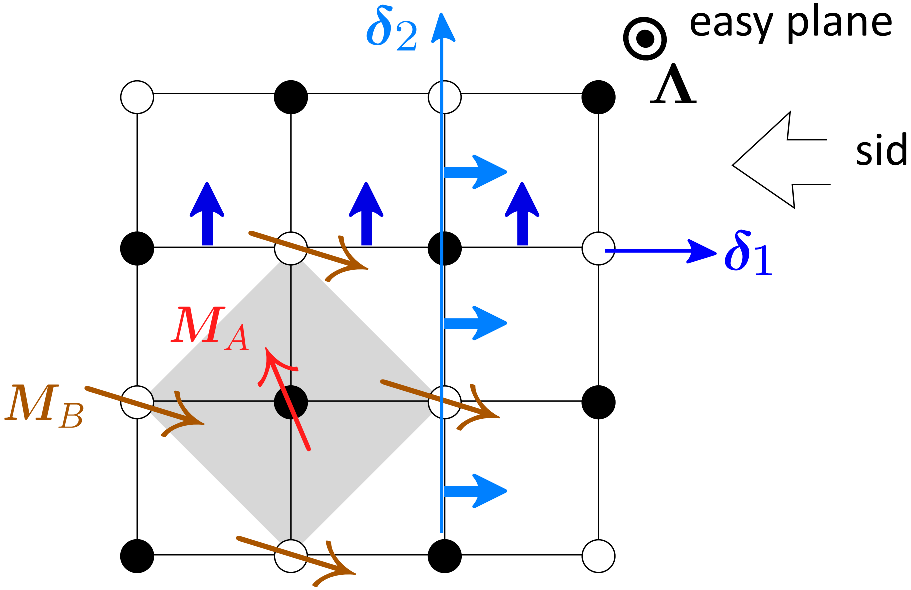


$$\mathcal{H} = J\sum_{\langle i,j\rangle} \boldsymbol{S}_i\cdot\boldsymbol{S}_j + \Lambda\sum_i (S_i^z)^2 + \sum_{\langle i,j\rangle}\boldsymbol{D}_{ij}\cdot[\boldsymbol{S}_i\times\boldsymbol{S}_j]$$

Conditions for thermal Hall effect is not simple:



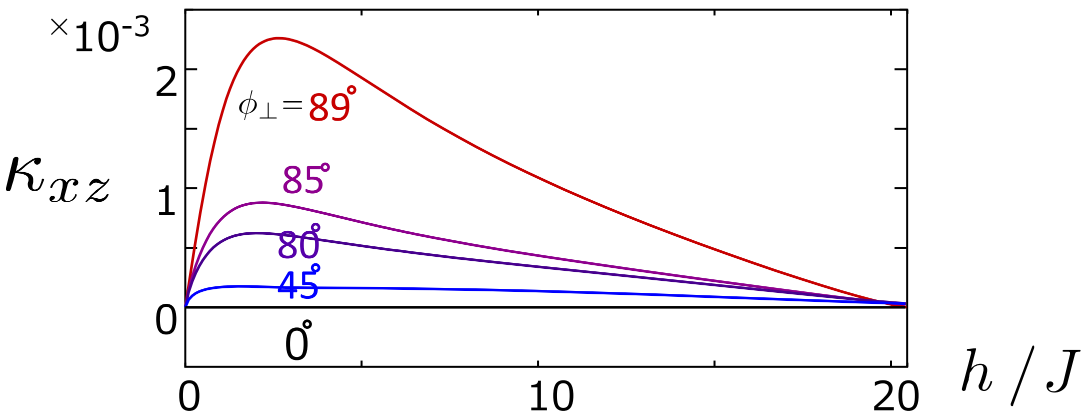
Rashba type DM



DM interactions are $\boldsymbol{D}\perp\boldsymbol{M}$

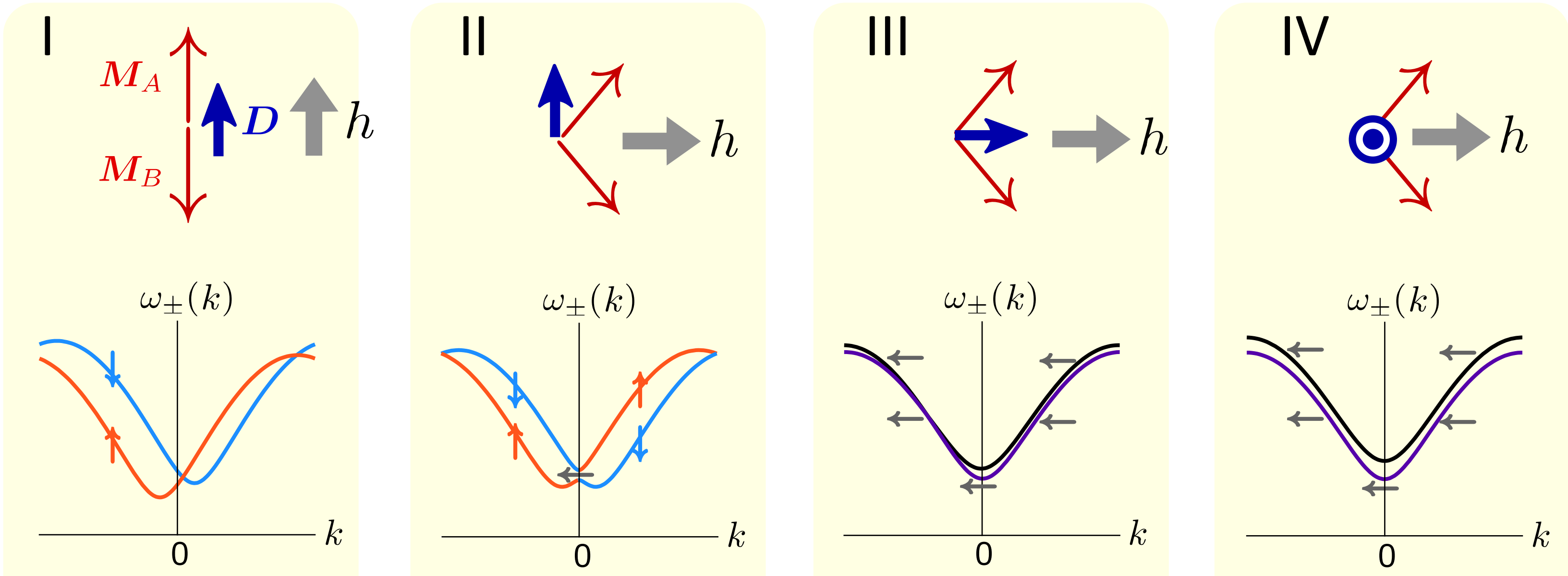
(remind that for ferromagnets, $\boldsymbol{D}\parallel\boldsymbol{M}$)

We need a field to cant the spins off the plane.



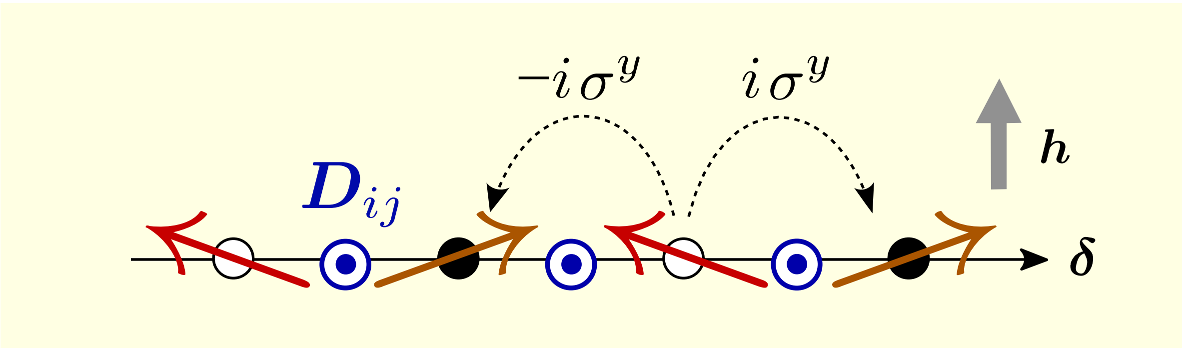
Rashba antiferromagnons

Full classifications about DM and AF and magnetic field are done.

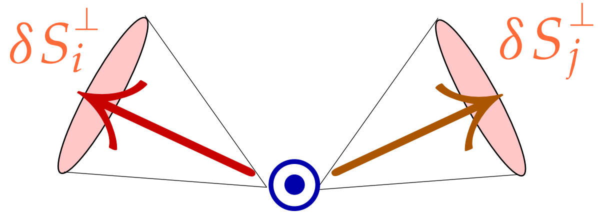


	I	II	III	IV
spin-momentum locking textures	×	○	△	△
nonreciprocity	○	×	○	×
thermal Hall	×	×	×	○

SU(2) gauge



$$\boldsymbol{D}\cdot[\,\delta S_i^{\perp}\times\delta S_j^{\perp}\,]$$



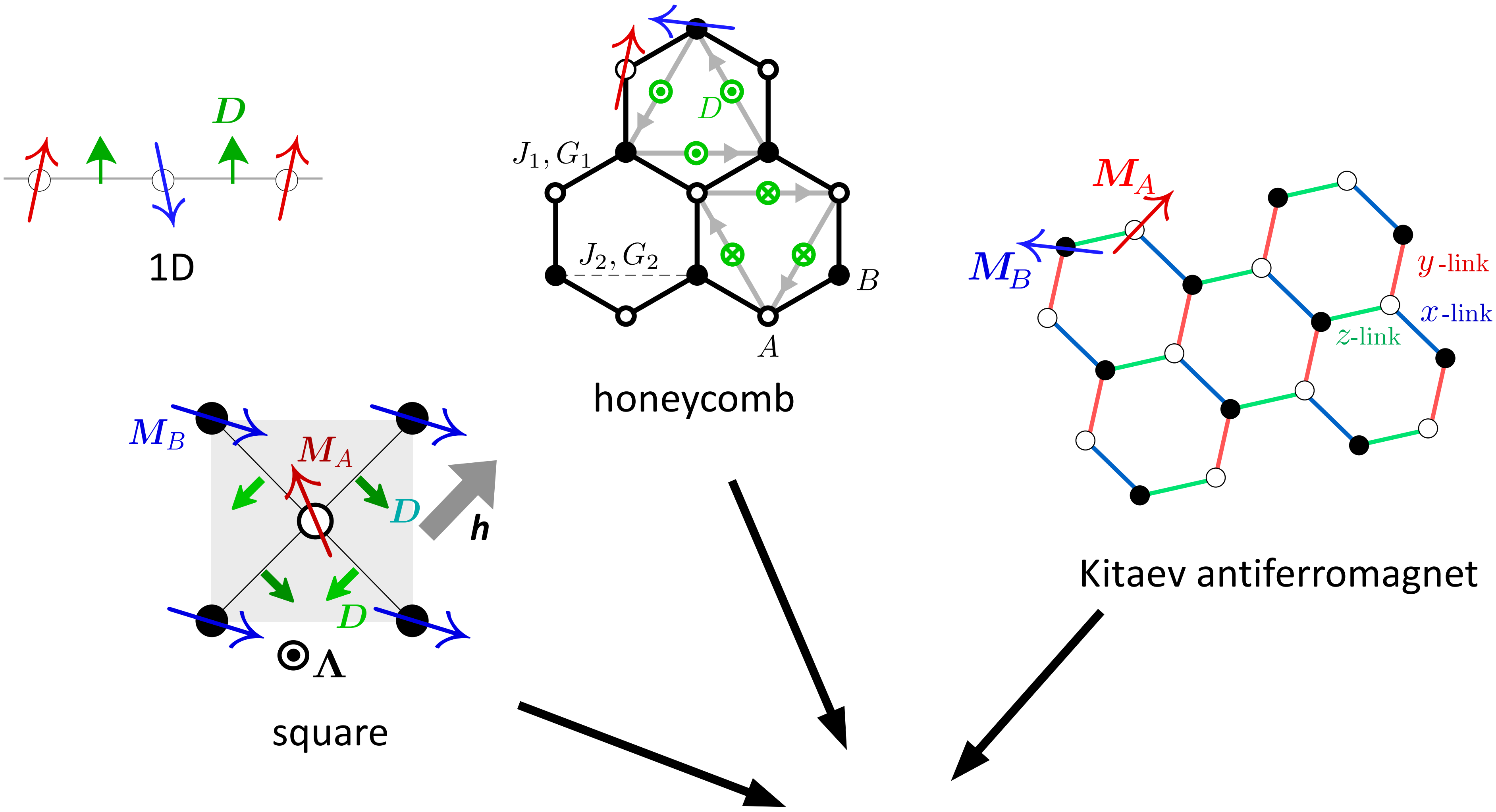
SU(2) gauge

hopping “flips" the pseudo-spins

- ferromagnon $\longrightarrow$ U(1),
- two species of magnons $\longrightarrow$ SU(2) as they mix via “pseudo-SOC”

At this moment in 2018, we were not perfectly sure whether this could be the general underlying picture.

Any two-sublattice AF canbe ...



put into the general SU(2) framework

M. Kawano, C. Hotta, Phys. Rev. B **100**, 174402 (2019)

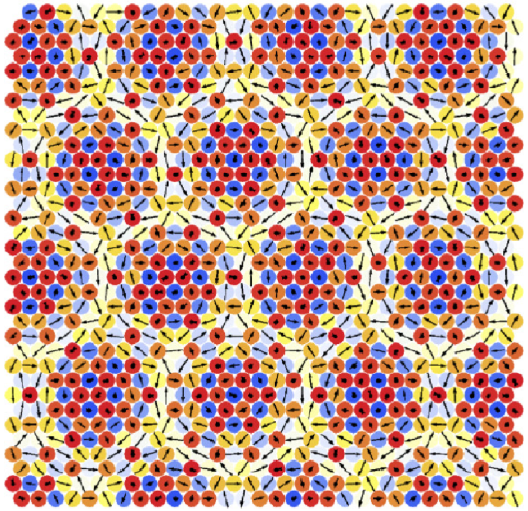
Skyrmions

H. Takeda, M. Kawano,
K. Tamura, M. Akazawa, J. Yan, T.Waki, H. Nakamura , K. Sato,Y. Narumi, M. Hagiwara
M. Yamashita, C. Hotta **Nature Comm. 15, 566 (2024)**

Antiferro skyrmions

insulating antiferromagnet

Rosales,et.al. PRB 92, 214439 (2015)



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Fractional antiferromagnetic skyrmion lattice induced by anisotropic couplings

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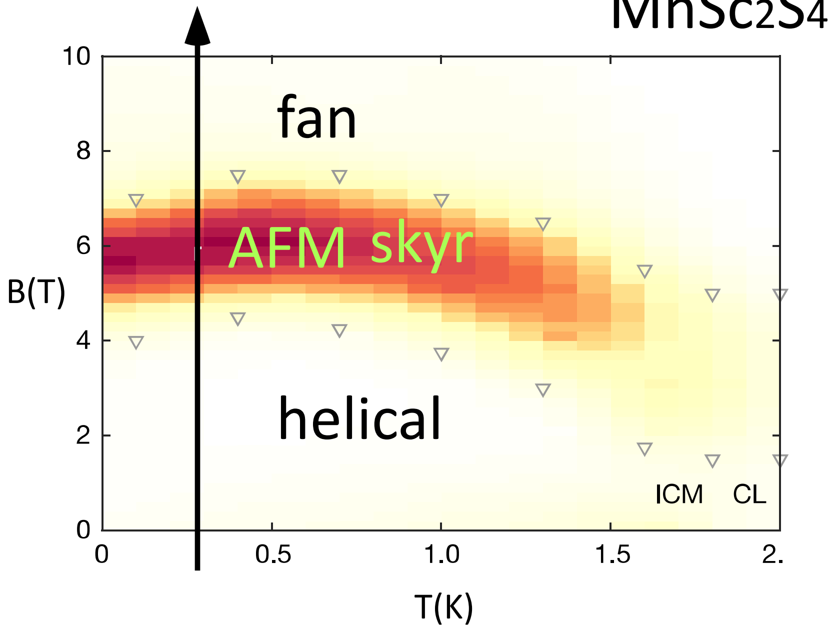
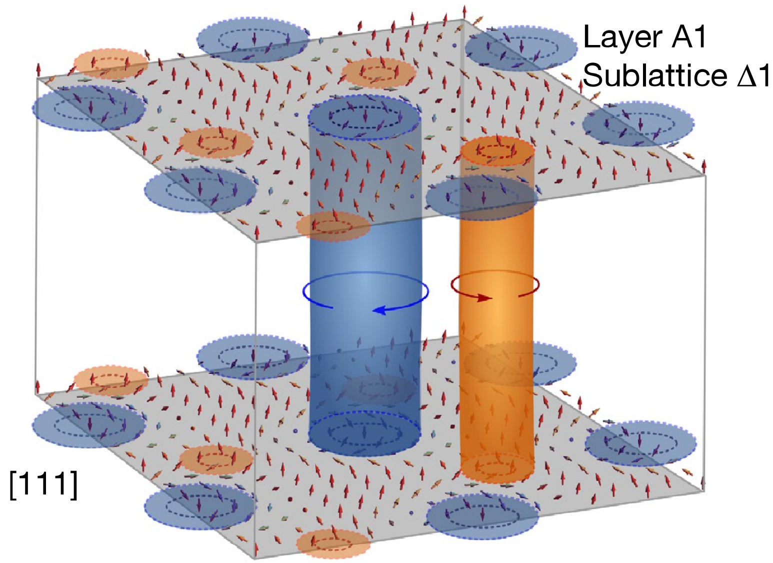
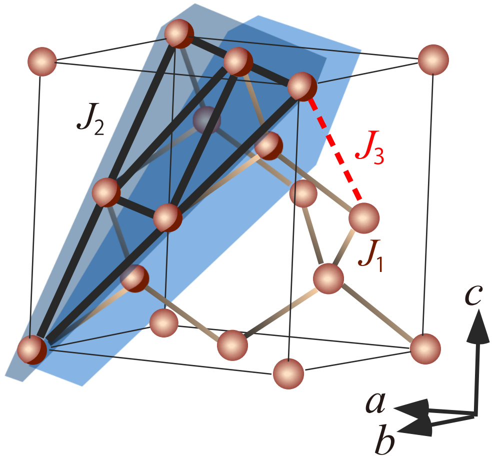
Shang Gao[1,2,3,17,18], H. Diego Rosales[4,5,6], Flavia A. Gómez Albarracín[4,5,6], Vladimir Tsurkan[7,8], Guratinder Kaur[1,2], Tom Fennell[1], Paul Steffens[9], Martin Boehm[9], Petr Čermák[10,11], Astrid Schneidewind[10], Eric Ressouche[12], Daniel C. Cabra[4,5,13], Christian Rüegg[2,14,15,16] & Oksana Zaharko[1]✉

$MnSc_2S_4$



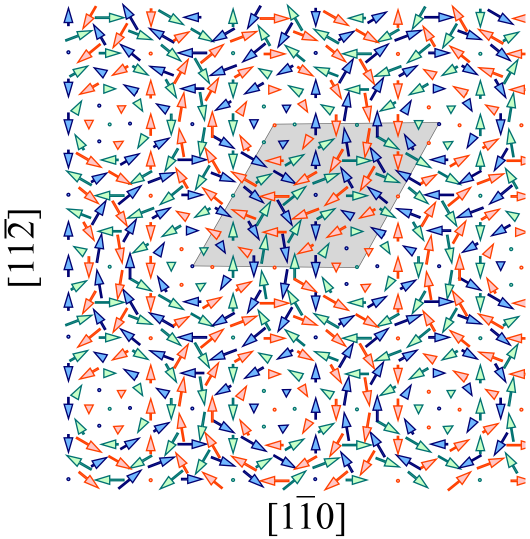


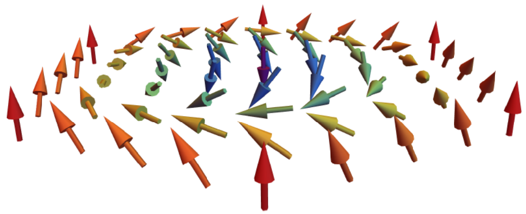
$MnSc_2S_4$



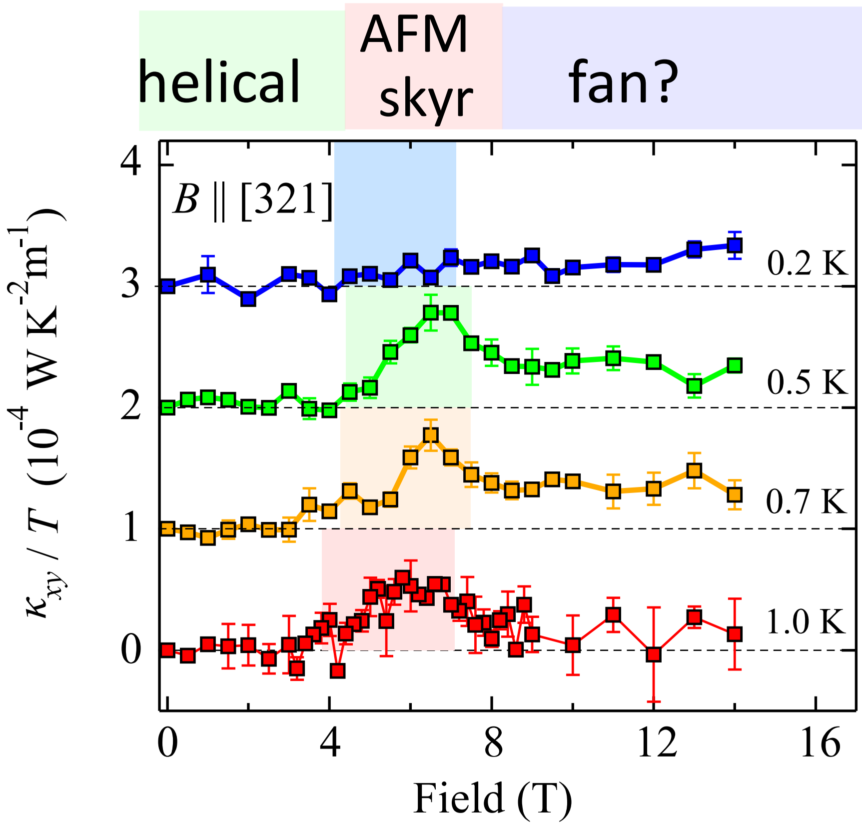
Mn^{2+} (S = 5/2)

3D diamond lattice

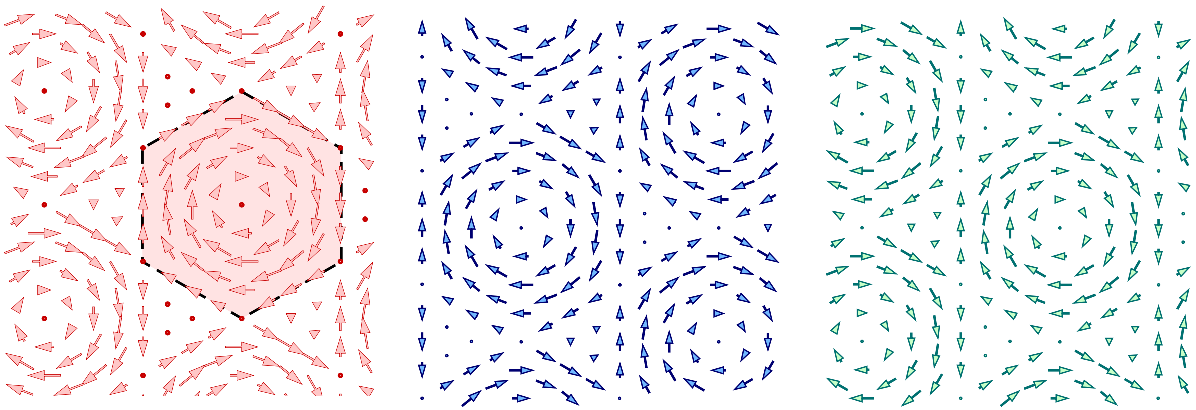




Takeda & Yamashita (ISSP)



three skyrmion sublattices

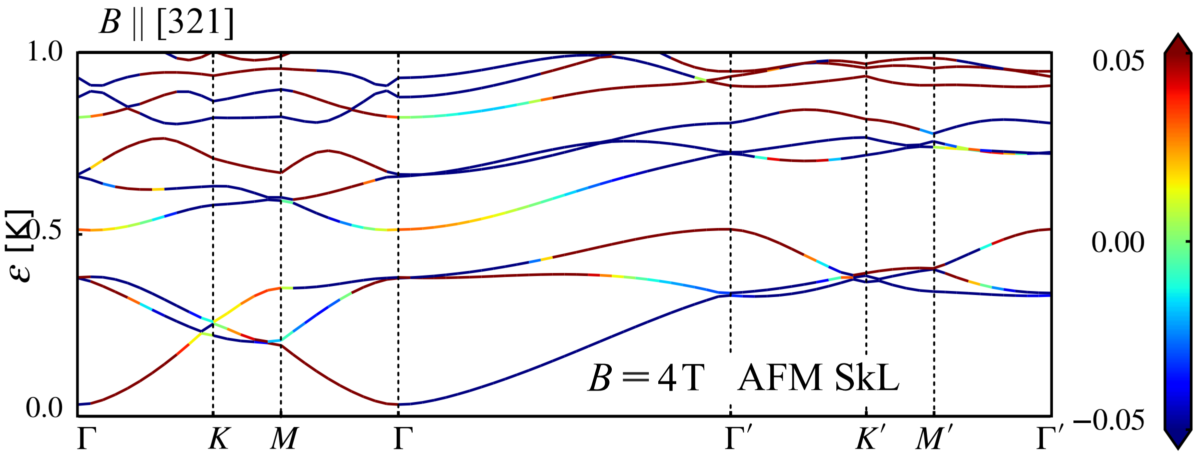


Thermal Hall effect observed only in the AF-skyrmion phase

$MnSc_2S_4$

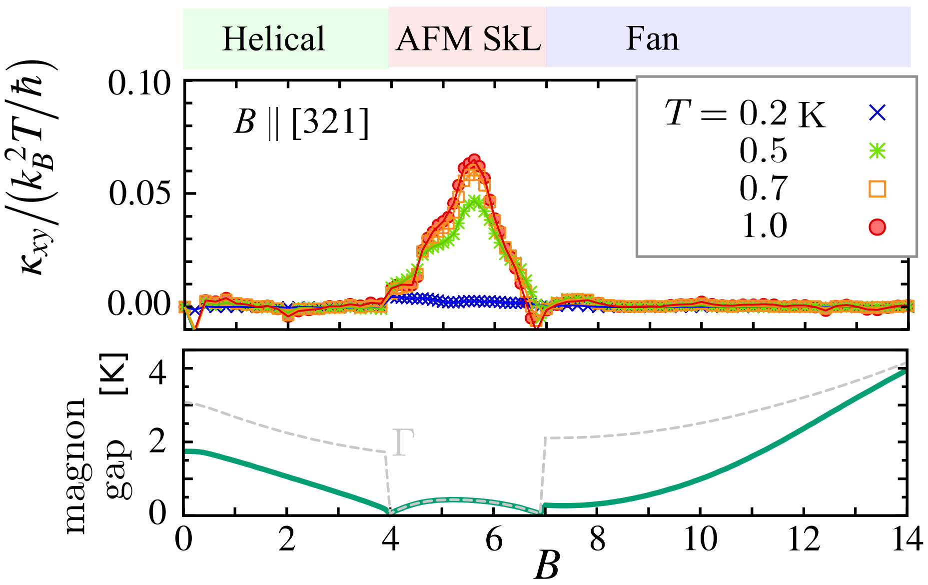
spin-wave calculation

$N_c = 384$ sites/unit cell



$\Omega_{xy}^{(n)}(\boldsymbol{k})$

Berry curvature



$$\mathcal{H} = \sum_{\boldsymbol{r},\boldsymbol{\delta}_l} \frac{J_l}{2} \boldsymbol{S}_{\boldsymbol{r}} \cdot \boldsymbol{S}_{\boldsymbol{r}+\boldsymbol{\delta}_l} + \frac{3}{2} J_{\parallel} \sum_{\boldsymbol{r},\boldsymbol{\delta}_1} (\boldsymbol{S}_{\boldsymbol{r}} \cdot \hat{\boldsymbol{\delta}_1})(\boldsymbol{S}_{\boldsymbol{r}+\boldsymbol{\delta}_l} \cdot \hat{\boldsymbol{\delta}_1})$$

$$+ A_4 \sum_{\boldsymbol{r},\mu=x,y,z} (S_{\boldsymbol{r}}^{\mu})^4 - g\mu_B \sum_{\boldsymbol{r}} \boldsymbol{B} \cdot \boldsymbol{S}_{\boldsymbol{r}}$$

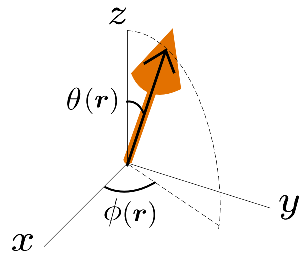
→ Berry curvature

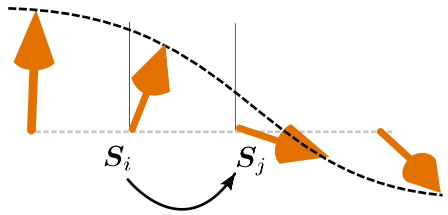
$$\kappa_{xy} = -\frac{k_B^2 T}{\hbar} \int_{\mathrm{BZ}} \frac{d^3\boldsymbol{k}}{(2\pi)^3} \sum_{n=1}^{N_s} c_2[f(\varepsilon_n(\boldsymbol{k}))]\, \Omega_{xy}^{(n)}(\boldsymbol{k})$$

thermal Hall coefficient

U(1) gauge fields of three sublattices

Slowly varying ordered moments generate a vector potential.



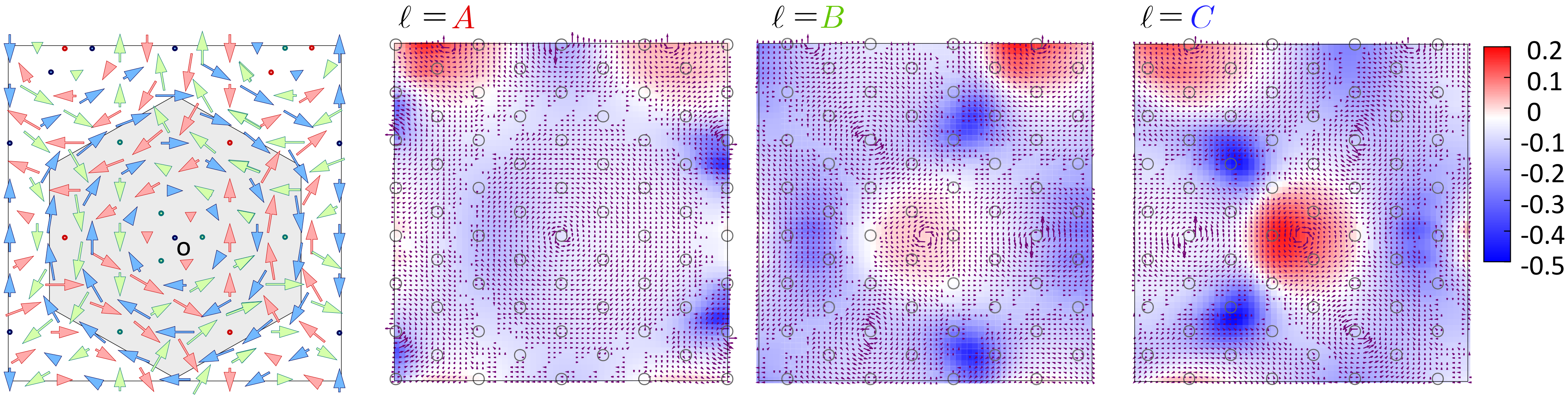


$$e^{-i\int d\boldsymbol{r}\cdot\boldsymbol{A}(\boldsymbol{r})}$$

$$\rightarrow \boldsymbol{A}_\ell(\boldsymbol{r}) = -\cos\theta_\ell(\boldsymbol{r})\,\nabla\phi_\ell(\boldsymbol{r})$$

$$B_{\mathrm{skyr},\ell}(\boldsymbol{r}) = \partial_x A_y(\boldsymbol{r}) - \partial_y A_x(\boldsymbol{r})$$

U(1) gauge field.



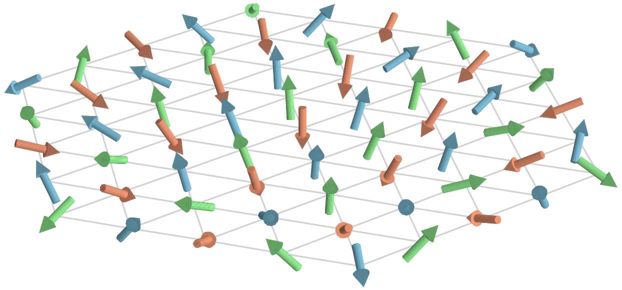
$$\sum_{\ell=1}^{3} B_{\mathrm{skyr},\ell}(\boldsymbol{r}) \simeq 0$$ Net U(1) becomes zero

Triangular lattice: pi-rotation will convert the flux back to it was.

$\kappa_{xy} \rightarrow -\kappa_{xy}$ $\kappa_{xy} = 0$? NO

SU(3) gauge field

3sublattice behave as “pseudo-spins”



$$\mathrm{e}^{-i\int d\boldsymbol{r}\cdot \boldsymbol{T}(\boldsymbol{r})}$$

$$\hat{\mathcal{H}} = J \sum_{\langle i,j \rangle} \hat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{S}}_j + \cdots ,$$

$$\hat{\boldsymbol{s}}(\boldsymbol{r}) \simeq \sqrt{\frac{S}{v}} \left(\hat{b}(\boldsymbol{r}) \boldsymbol{e}^{-}(\boldsymbol{r}) + \mathrm{H.c.} \right) + \left(\frac{S}{v} - \hat{b}^{\dagger}(\boldsymbol{r}) \hat{b}(\boldsymbol{r}) \right) \boldsymbol{m}(\boldsymbol{r})$$

three species of bosons

3x3 Hermitian&Traceless matrix

SU(3) gauge field

$$\hat{\mathcal{H}}_{\mathrm{eff}} \simeq \frac{1}{2} \int d^2\boldsymbol{r}\, \hat{\Psi}^{\dagger}(\boldsymbol{r}) \left[\frac{3}{2} J_2 S a^2 \left(\boldsymbol{\nabla} I_{3\times3} - i\boldsymbol{T}(\boldsymbol{r}) \right)^2 \right] \hat{\Psi}(\boldsymbol{r}) + \cdots$$

$$\hat{\Psi}(\boldsymbol{r}) = \begin{bmatrix} \hat{b}_1(\boldsymbol{r}) \\ \hat{b}_2(\boldsymbol{r}) \\ \hat{b}_3(\boldsymbol{r}) \end{bmatrix}$$

$\Updownarrow$

U(1) gauge field

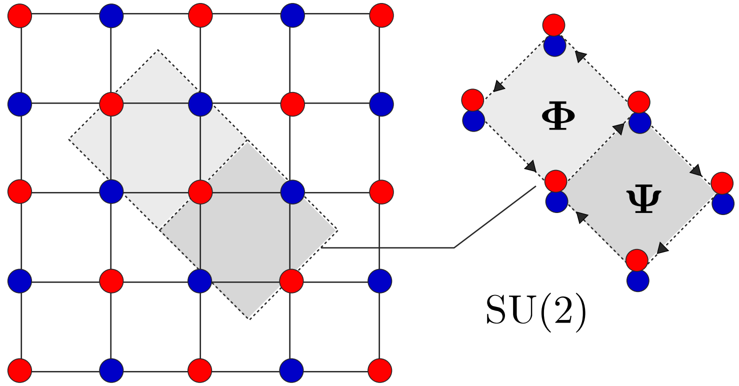
$$\hat{\mathcal{H}}_{\mathrm{eff}}^{\mathrm{FM}} \simeq \frac{3}{2} J S a^2 \int d^2\boldsymbol{r}\, \hat{b}^{\dagger}(\boldsymbol{r}) \left(\boldsymbol{\nabla} - i\boldsymbol{A}(\boldsymbol{r}) \right)^2 \hat{b}(\boldsymbol{r})$$

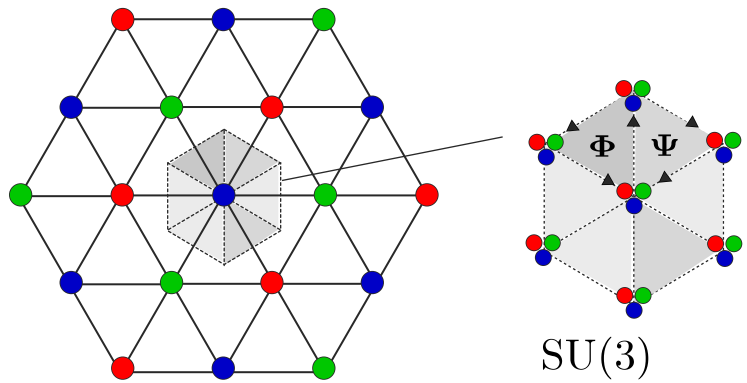
Field theory details:

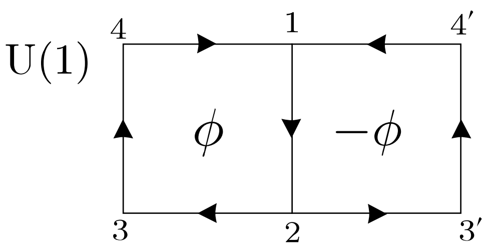
Kawano, arXiv/ 2403.11655, 2502.11924

Gauge conditions

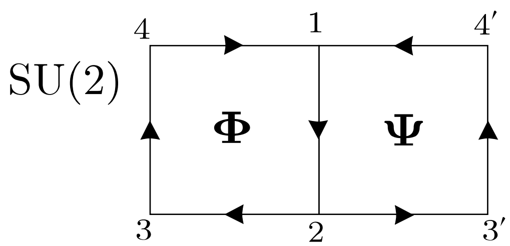
	species of bosons	gauge field	gauges on sublattice
Ferromagnet	1	U(1)	
Antiferromagnet	2	SU(2) ←	U(1)⊗U(1)
Antiferromagnet	3	SU(3) ←	U(1)⊗U(1)⊗U(1)



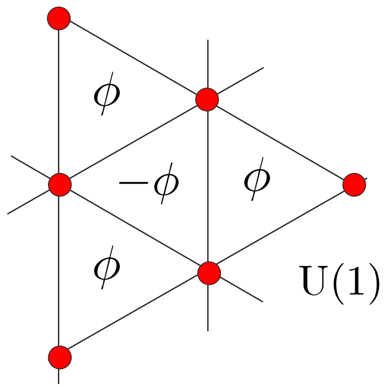




$$U_{12}U_{23}U_{34}U_{41} = e^{i\phi}$$



$$\boldsymbol{T}_{12}\boldsymbol{T}_{23}\boldsymbol{T}_{34}\boldsymbol{T}_{41} = e^{i\boldsymbol{\Phi}}$$

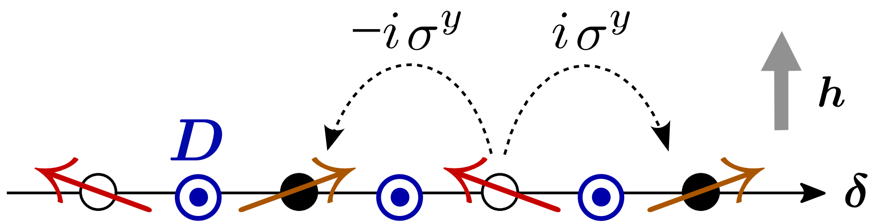


Higher rank gauges are non commutative,
and give net nonzero flux. They do not cancel out.

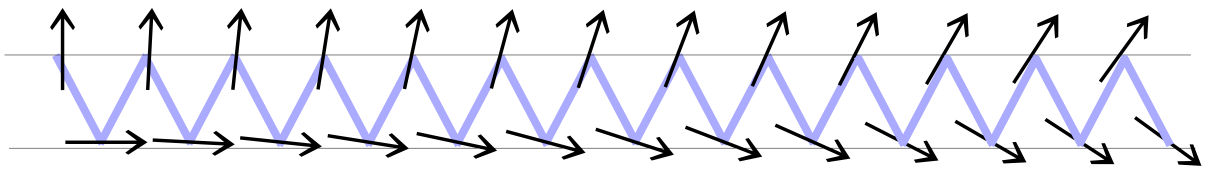
For what condition can we turn on the higher rank gauge coupling?

Gauge conditions

Generally, if you have more than two species of "well-defined magnons", just try to couple them.



We need to cant the spins off the plane to switch on SU(2) gauge.

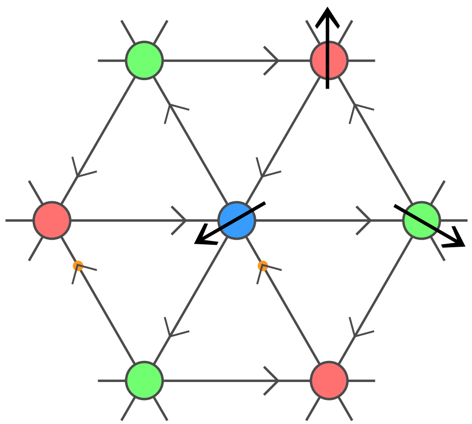


U(1)

U(1)

incommensurate orders

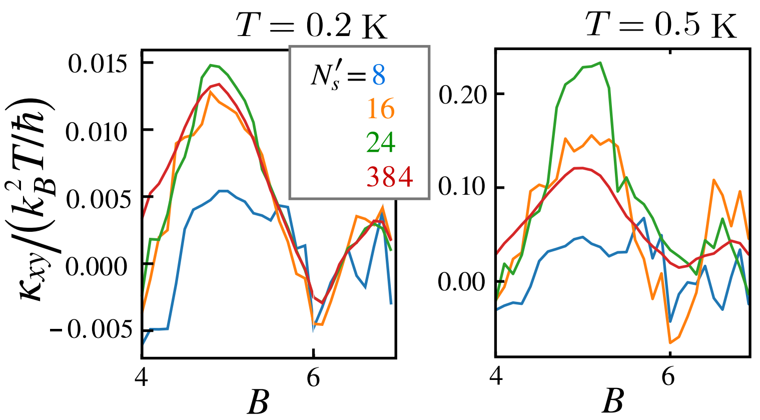
Because the spin z-axis is warped, straightforward to couple them.

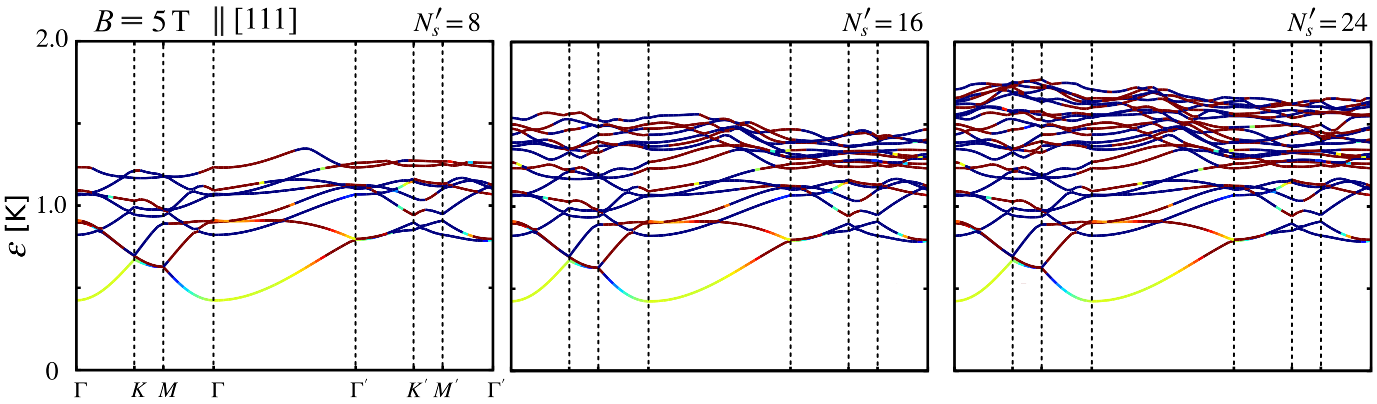


Simple coplaner 3sublattce 120 deg Neel order does not work.

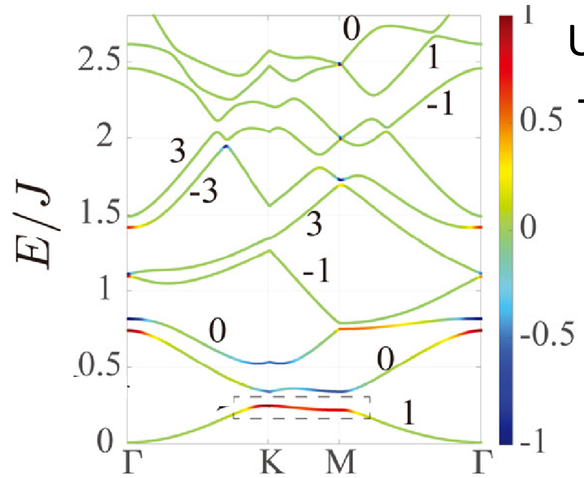
Remarks

Large contribution from very high energy branches.



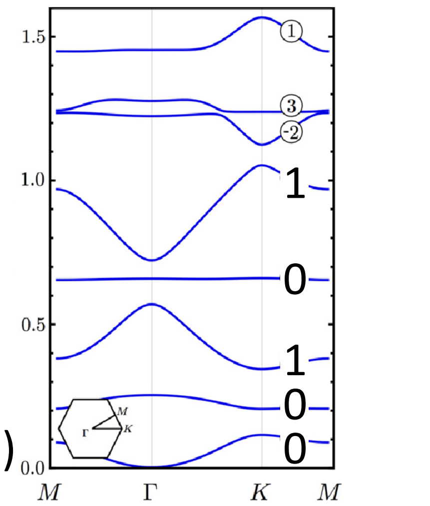


U(1) or SU(3) gauge theories do not give proper details on Berry curvatures.



U(1) gauge theory

This Chern# do not agree with spin wave theory.



spin wave

Roldan-Molina *et al.* New J. Phys. **18** 045015(2016)

Back to electrons

R. Makuta, C. Hotta, PRR 6, 023133 (2024)

M. Kawano, C. Hotta, PRB 107, 045123(2024)

R. Makuta, C. Hotta , arXiv: 2504.05166

SOC generate DM terms

$$\mathcal{H} = \sum_{\langle i,j\rangle}\sum_{\alpha,\beta} -t_{\mathrm{eff}}\,(c^{\dagger}_{i\uparrow},\, c^{\dagger}_{i\downarrow})\; \mathrm{e}^{-i\frac{\theta}{2}\boldsymbol{n}_{ij}\cdot\boldsymbol{\sigma}} \begin{pmatrix} c_{j\uparrow} \\ c_{j\downarrow} \end{pmatrix} + U\sum_{i} n_{i,\uparrow}n_{i,\downarrow}$$

SU(2) gauge field

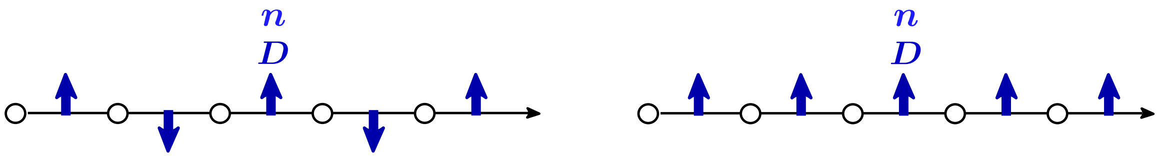
half-filling

$U \to \infty$

Magnetic Mott insulator 2nd order

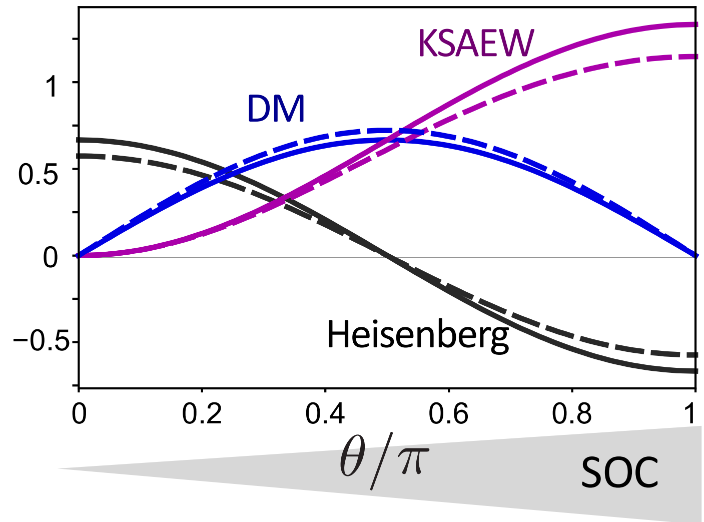
$$\mathcal{H}^{(2)} = \sum_{\langle i,j\rangle} J\boldsymbol{S}_i\cdot\boldsymbol{S}_j + \sum_{\langle i,j\rangle}\boldsymbol{D}\cdot\boldsymbol{S}_i\times\boldsymbol{S}_j + \sum_{\langle i,j\rangle} K(\boldsymbol{n}\cdot\boldsymbol{S}_i)(\boldsymbol{n}\cdot\boldsymbol{S}_j)$$

Heisenberg DM KSAEW (Kaplan-Sherhtman-Aharony-Entin-Wohlman)



inversion symmetry kept broken inversion symmetry

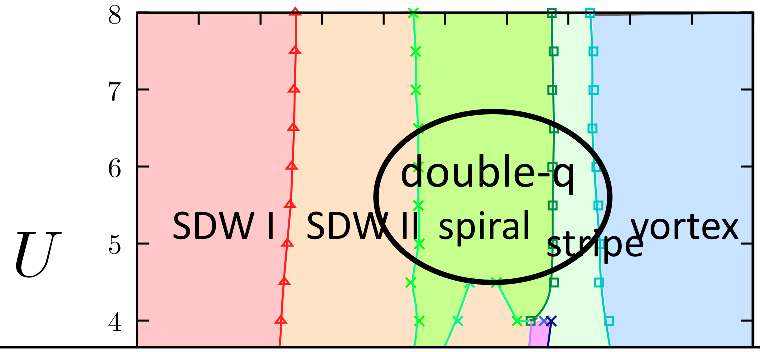
SOC vector // DM vector

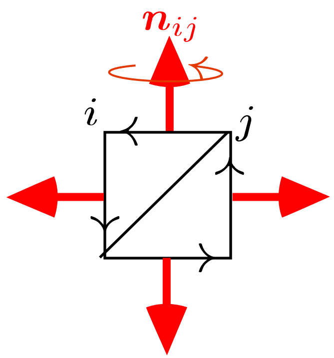


Rashba SOC Hubbard model at 1/2-filling

Square lattice

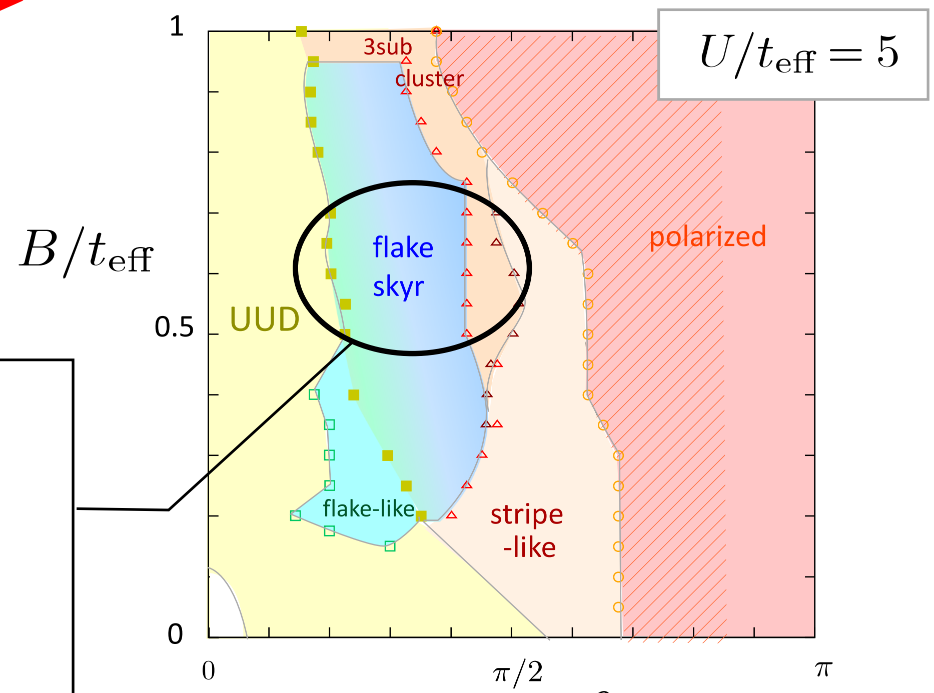
Kawano-Hotta PRB **107**,045123 (2023)

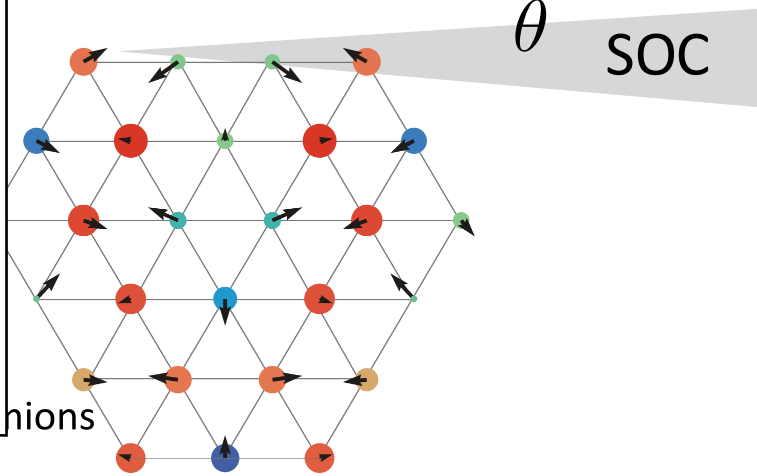


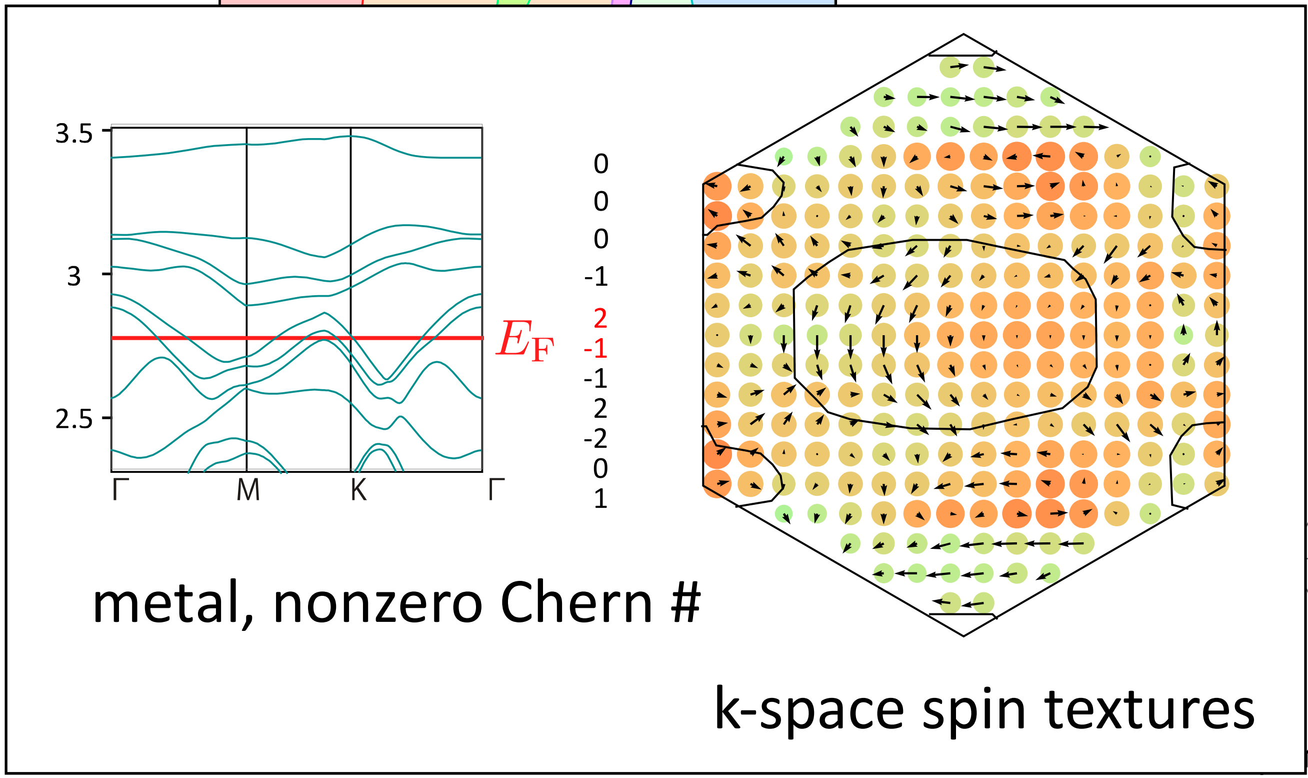


Triangular lattice

Makuta-Hotta PRR **6**,023133 (2024)







SOC Mott ins. → effective spin model

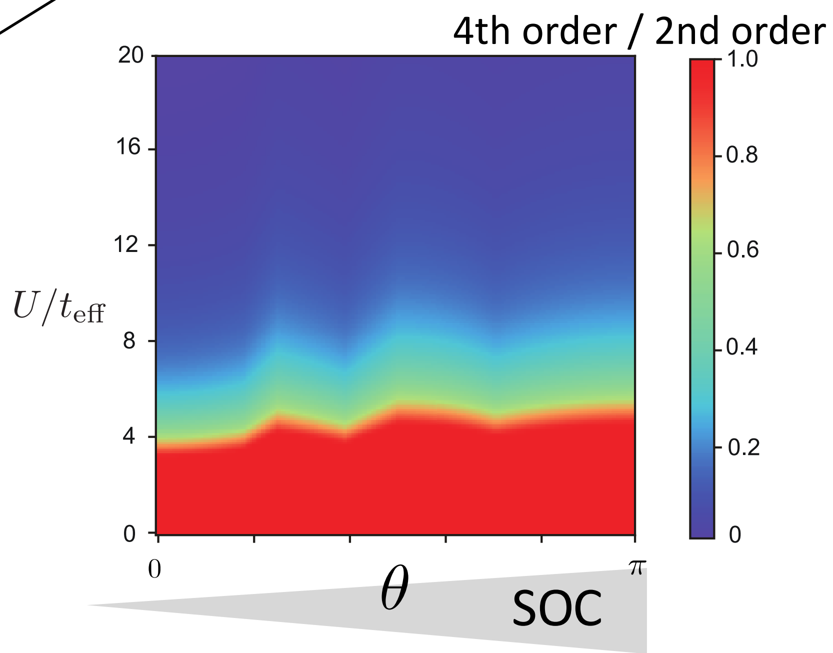
$U \to \infty \quad \mathcal{H}_{\text{eff}} = \mathcal{H}^{(2)} + \mathcal{H}^{(4)}$

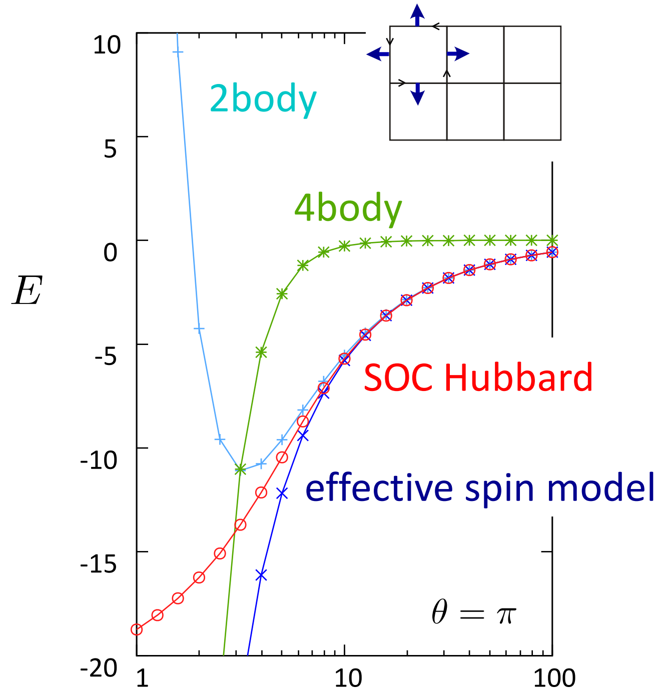
$$\mathcal{H}^{(2)} = \sum_{\langle i,j \rangle} J \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \sum_{\langle i,j \rangle} \boldsymbol{D} \cdot \boldsymbol{S}_i \times \boldsymbol{S}_j + \sum_{\langle i,j \rangle} K (\boldsymbol{n} \cdot \boldsymbol{S}_i)(\boldsymbol{n} \cdot \boldsymbol{S}_j)$$

$$\mathcal{H}^{(4)} = \frac{t_{\text{eff}}^4}{U^3} \sum_{\gamma} \sum_{\langle abcd \rangle} \hat{h}_{\gamma}^{\langle abcd \rangle}$$

- 4th order term is large, and reproduces the Hubbard result well.

ring
ring$^{(\mathrm{d})}$
ring-KS
KS$\times$H$_1$
KS$\times$H$_2$
KS$\times$H$_1^{(\mathrm{d})}$
KS$\times$H$_2^{(\mathrm{d})}$
DM$\times$KS$^{(\mathrm{d})}$
DM$\times$H
DM$\times$H$^{(\mathrm{d})}$
DM$\times$DM
DM$\times$DM$_2$
DM$\times$Ising$^{(\mathrm{d})}$
ring-Γ





Perspectives

Chern number in electronic systems

Anomalous Hall

electrons

spin splitting of bands

spin momentum locking

Spin Hall

Metal

Insulator

Broken inversion symmetry

uniform component of gauge fluxes

Magnons

Berry curvature does not cancel out

spin textures

real space & k space

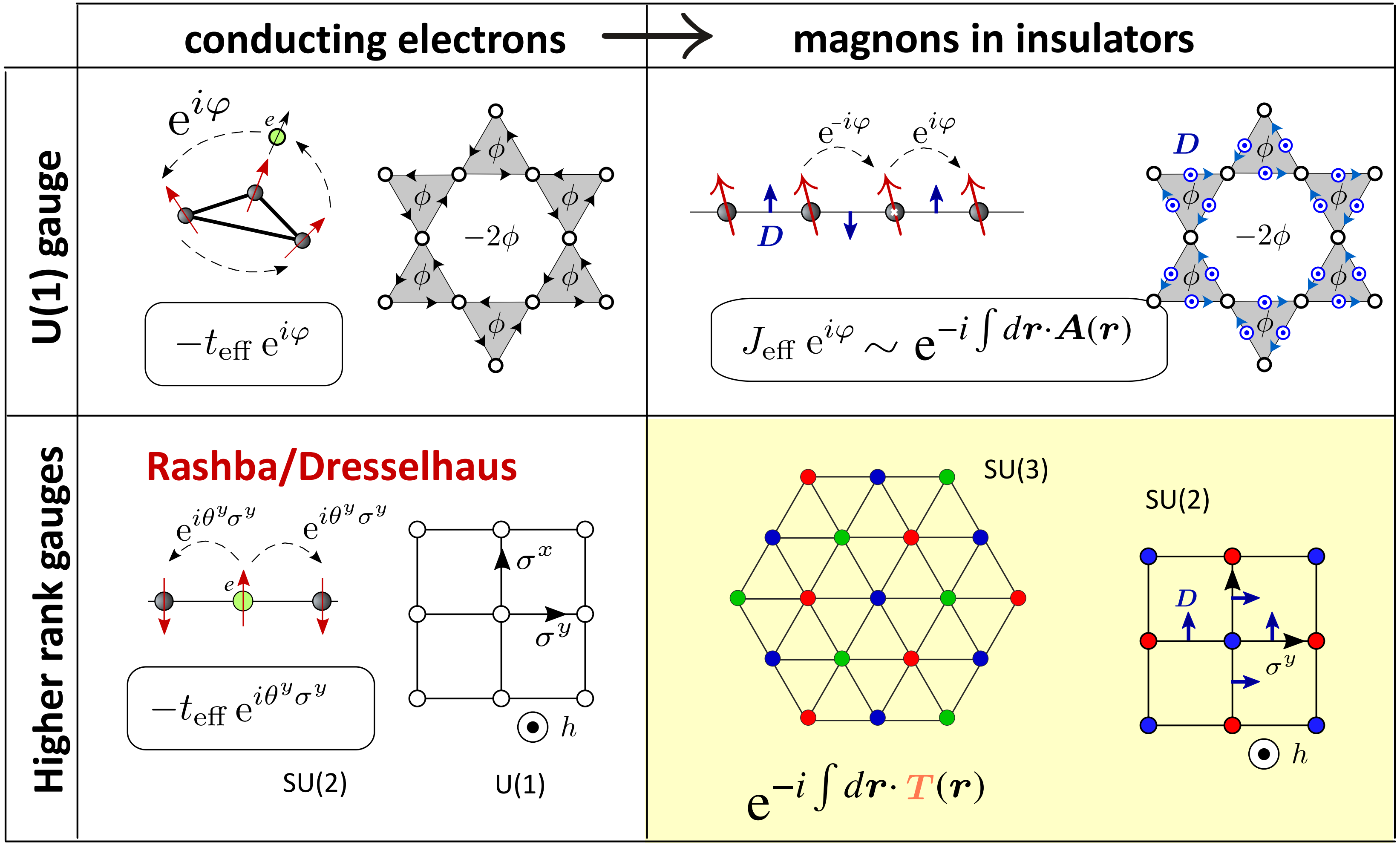
Thermal Hall

Thermal Hall

Summary

Accessible by an effective spin model upto 4th order

$U \to \infty$



To have a thermal Hall effect, prepare more than two species of magnons, and make them couple in terms of high rank gauge.