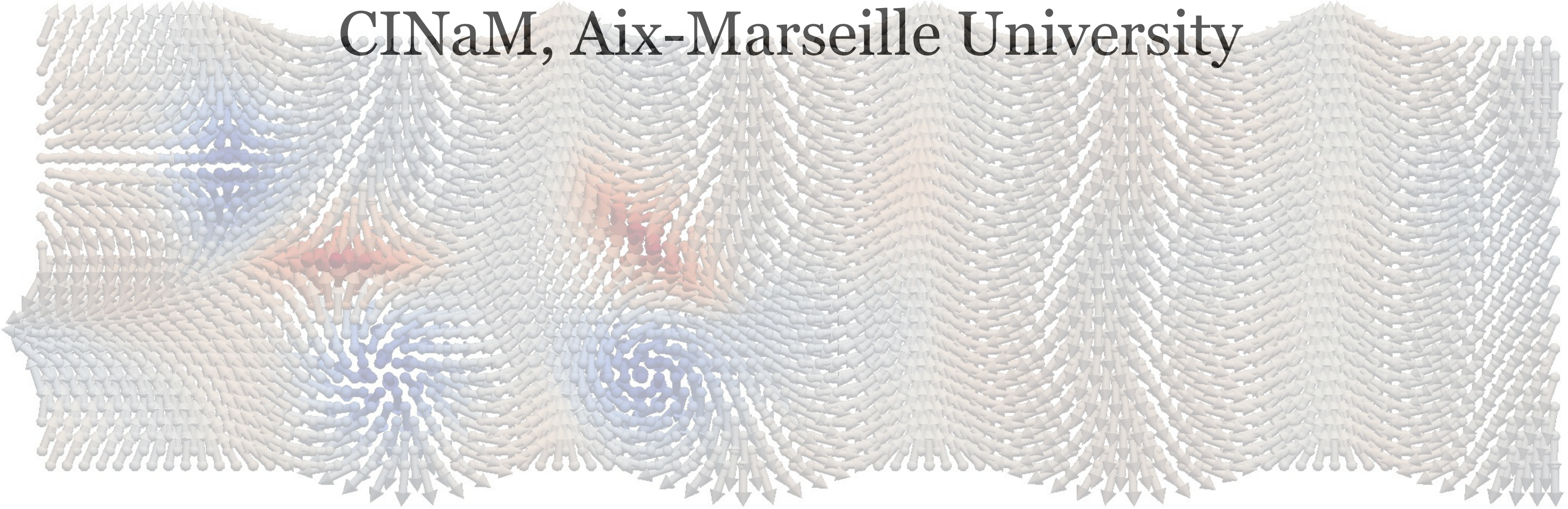
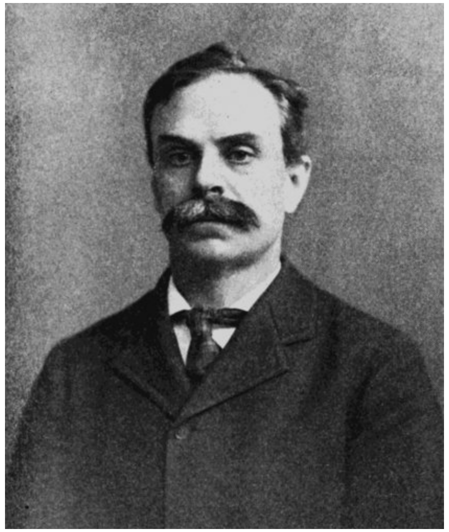
Interacting magnons in magnetic insulators

K. Sourounis, L. Saleem, V. Guémard, A. Manchon
CINaM, Aix-Marseille University

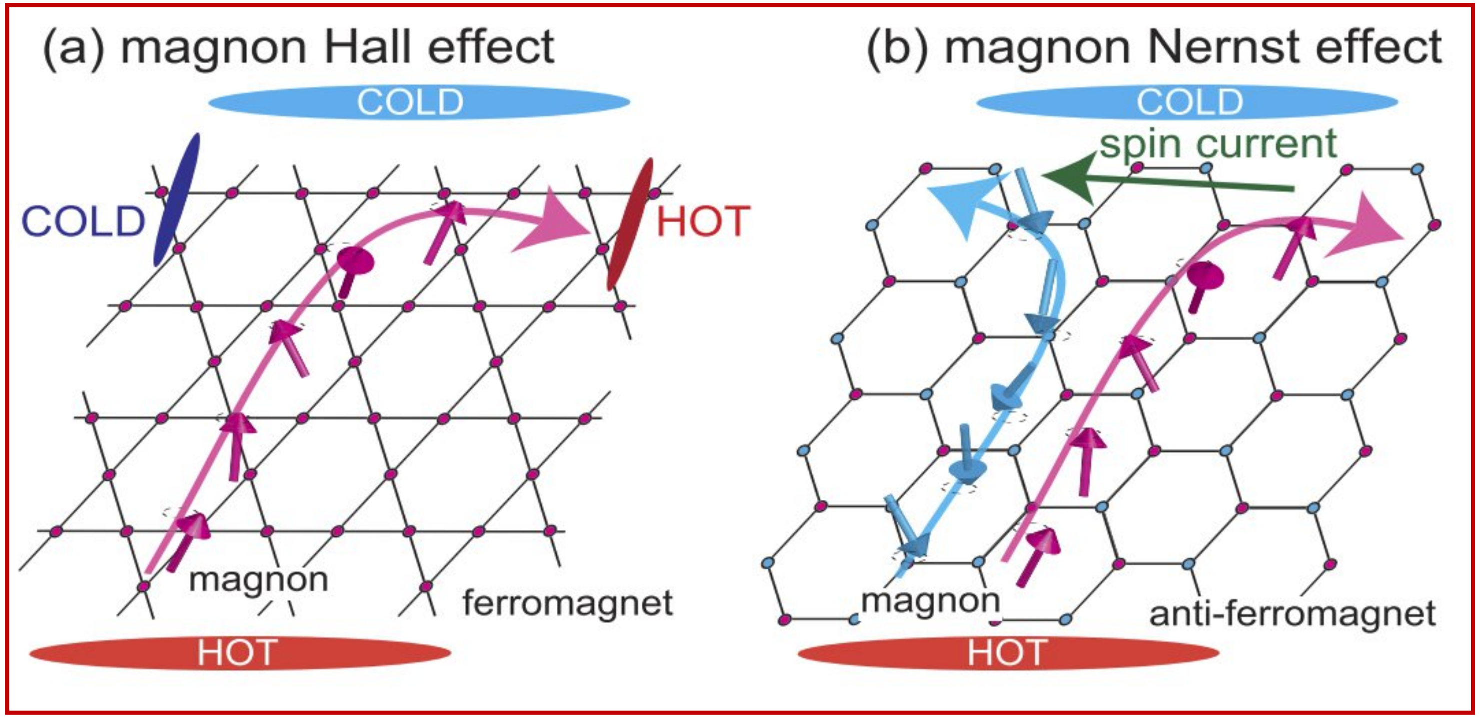


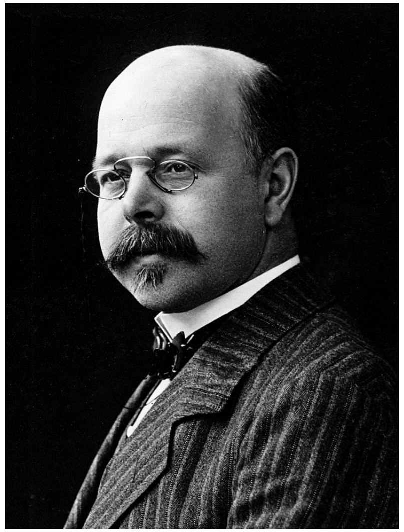
agence nationale de la recherche
A*Midex Initiative d'excellence Aix-Marseille
Co-funded by the European Union
ORBITAL ELECTRONICS
FRANCE 2030 PROGRAMME DE RECHERCHE SPINTRONIQUE

Anomalous transport of magnons



Edwin Hall





Walther Nernst

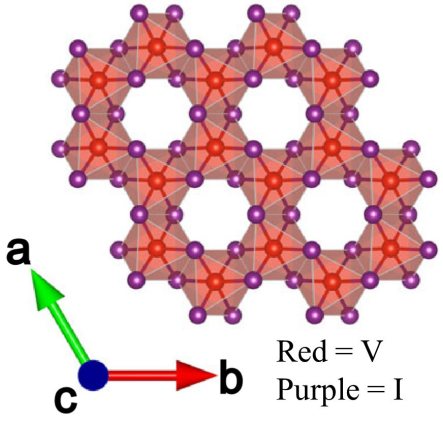
$$\sigma_{xx} = \frac{1}{2\alpha_0 k_B T^2} \sum_n^N \int d\mathbf{k} v_{x,n}^2 \frac{e^{\hbar\epsilon_{n,\mathbf{k}}/k_B T}}{(e^{\hbar\epsilon_{n,\mathbf{k}}/k_B T} - 1)^2},$$

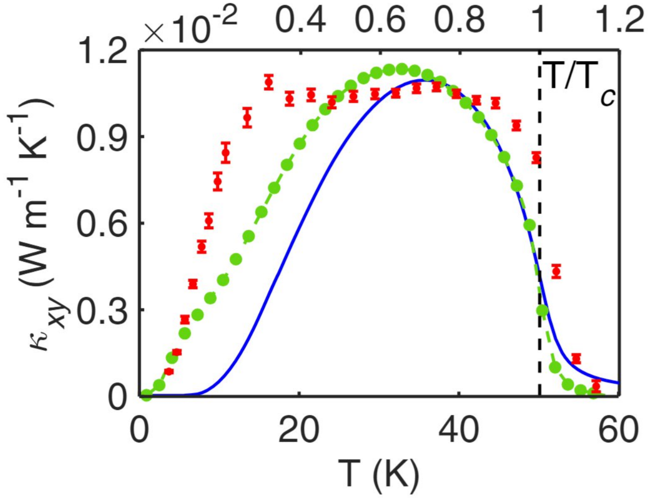
$$\sigma_{xy} = k_B T \sum_n^N \int d\mathbf{k} \Omega_{n,\mathbf{k}}^{xy} (c_1(n_b(\epsilon_{n,\mathbf{k}})) - c_1(n_b(\epsilon_{n,-\mathbf{k}}))),$$

Matsumoto et al. PRB 84, 184406 (2011)

Anomalous transport of magnons

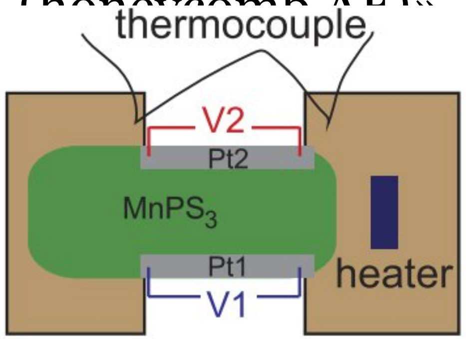
Thermal Hall effect in VI_3
(honeycomb FM)

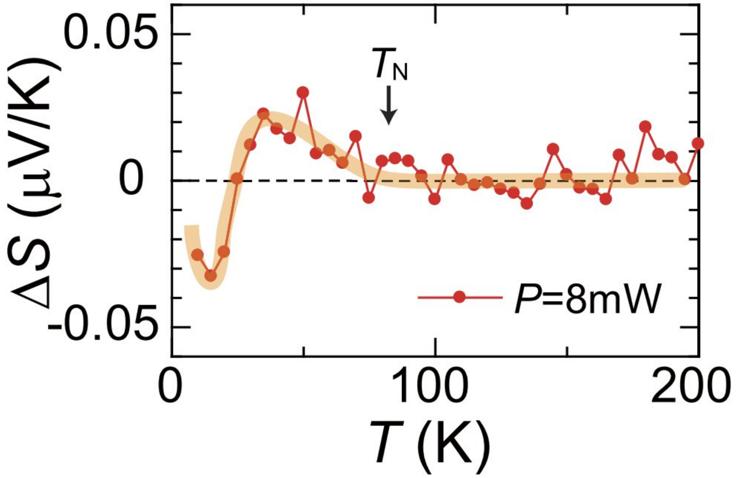




Zhang et al. PRL 127, 247202 (2021)

« Experimental evidence consistent with a magnon Nernst effect in the antiferromagnetic insulator $MnPS_3$
(honeycomb AF)»

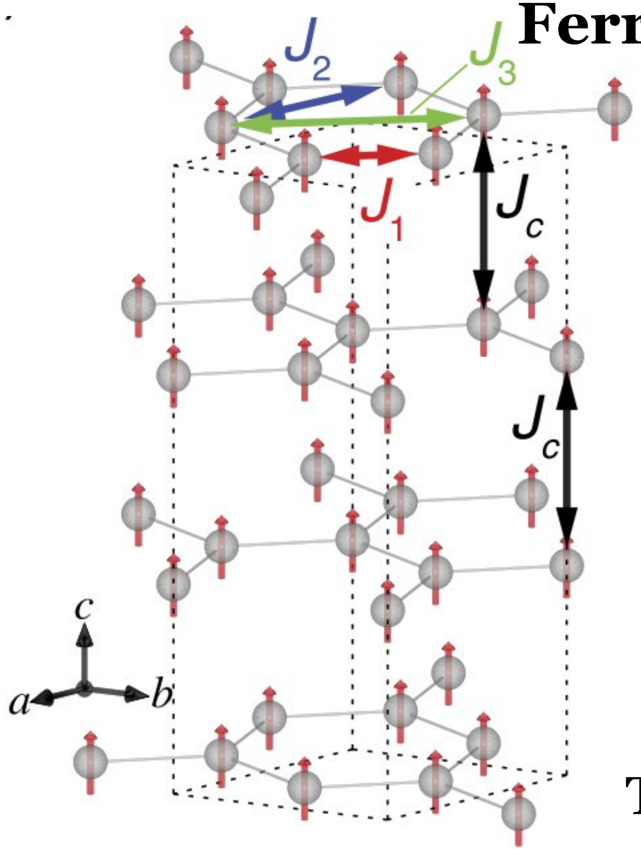


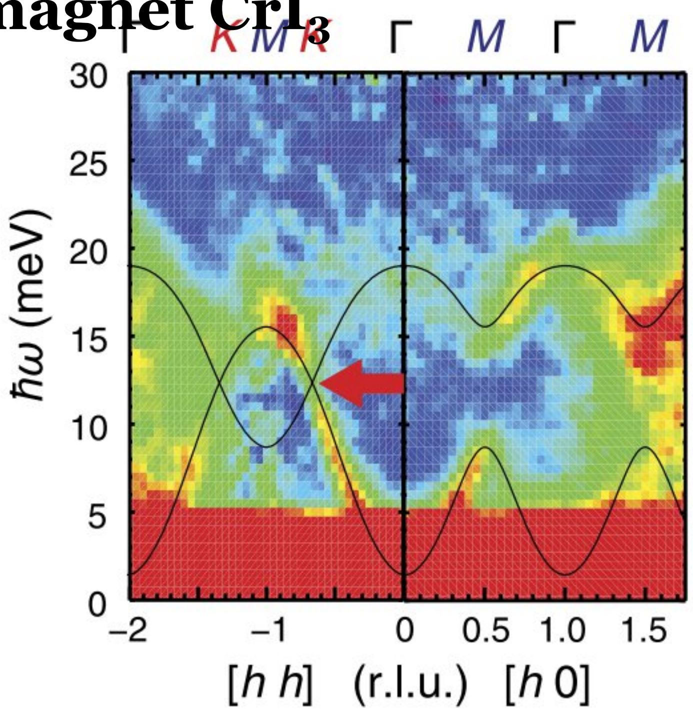


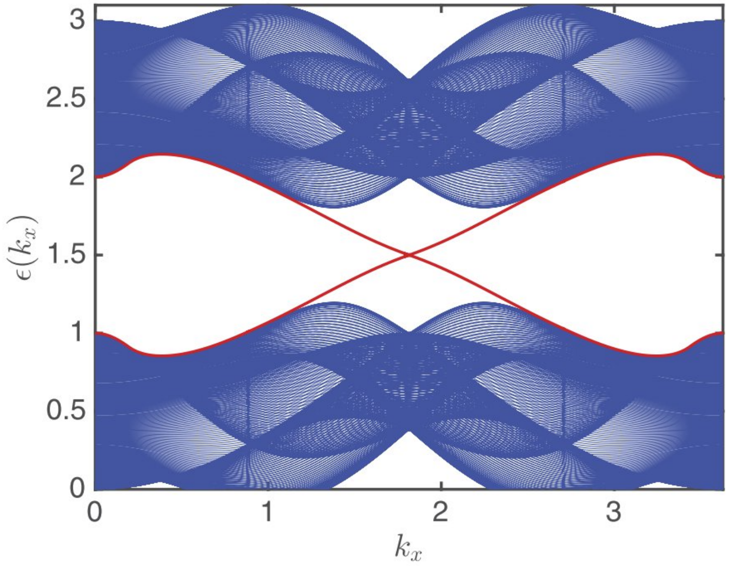
Shiomi et al. PRB 96, 134425 (2017)

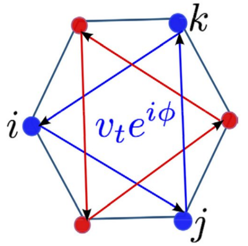
Topological transport of magnons

Topological Spin Excitations in Honeycomb Ferromagnet CrI_3









Owerre JJPCM 28, 386001 (2016)

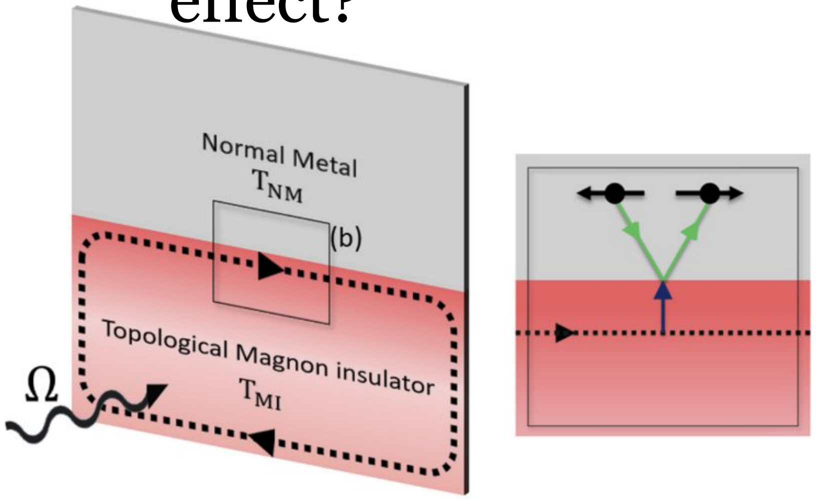
The acoustic and optical spin-wave bands are separated from each other by approximately 4 meV, **most likely arising from** the next-nearest-neighbor DM interaction that breaks inversion symmetry of the lattice. **This may lead to a nontrivial topological magnon insulator** with magnon edge states, analogous to topological insulators in electronic systems but without electric Ohmic heating.

Chen et al. Phys Rev X 8, 041028

Detecting edge magnons?

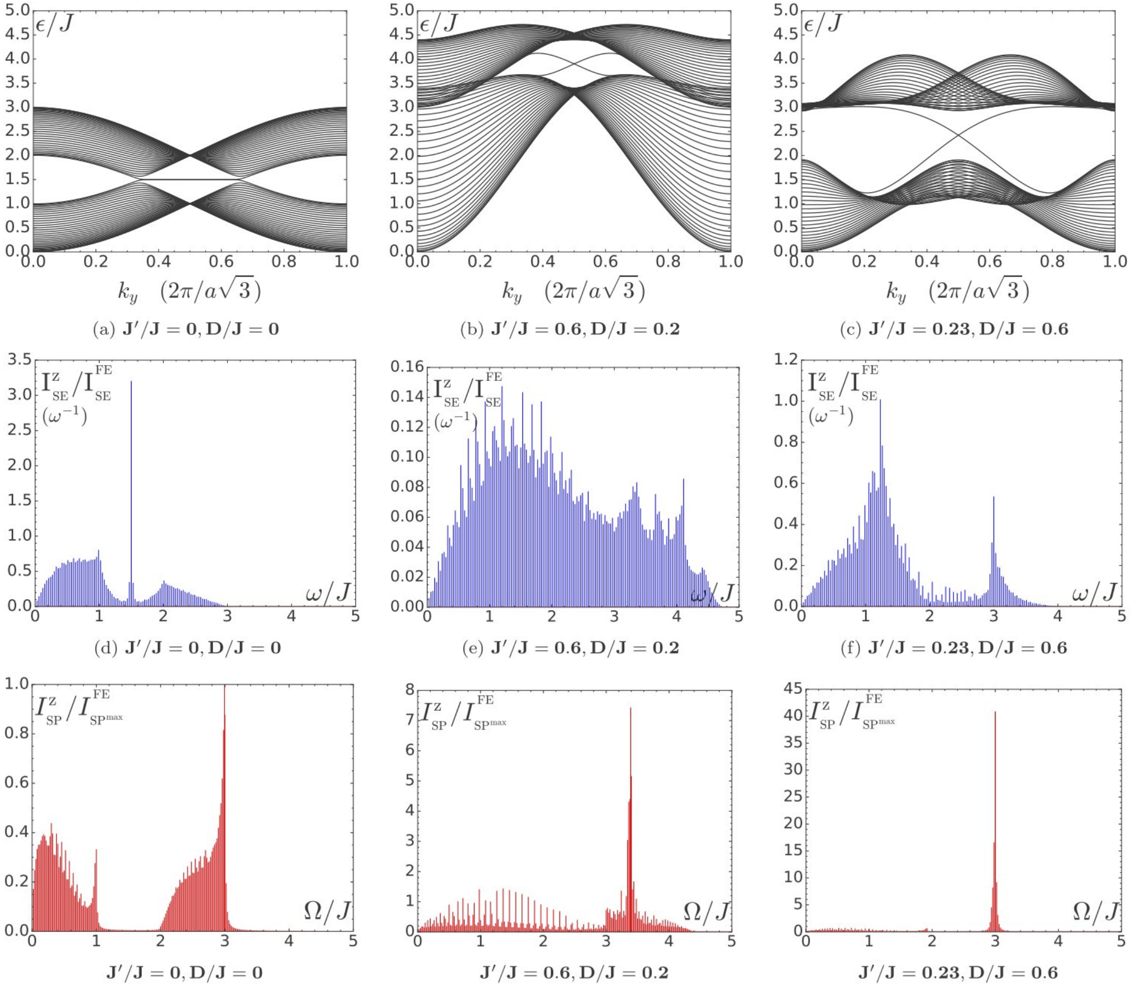
V. Guemard

Can we probe magnonic edge states using spin pumping or spin Seebeck effect?



(a) (b)

$$\langle \hat{I}^{\alpha} \rangle = \frac{\hbar}{2} \sum_{\mathbf{qk},ij} |J_{\mathbf{q},\mathbf{k}}|^2 (P^{\dagger}_{\mathbf{k}} \Upsilon^{\alpha,i} P_{\mathbf{k}})_{jj}$$
$$\times \int \frac{d\omega}{2\pi} \mathrm{Im}\chi^R_i(\mathbf{q},\omega) \mathrm{Im}G^R_{[j]}(\eta_j \mathbf{k},\omega)$$
$$\times \eta_j [n_B(\omega, T_{\mathrm{NM}}) - n_B(\omega, T_{\mathrm{MI}})],$$



Spin Seebeck

Spin pumping
k_y=0, not topologic

V. Guemard, AM, PRB 105, 054433 (2022)

I. Quantum field theory for thermal transport of magnons
II. Role of magnon-magnon interactions on magnon transport
III. Interaction-driven topological phase transitions
IV. Electron-magnon interactions in altermagnets

I. Quantum field theory for thermal transport of magnons

II. Role of magnon-magnon interactions on magnon transport

III. Interaction-driven topological phase transitions

IV. Electron-magnon interactions in altermagnets

How to model thermally-driven transport?

Luttinger

Thermal gradient is a statistical force that affects particles through a distribution function.

It does not h … ltonian description.

$$\nabla\Psi = \frac{\nabla T}{T}\ ; \quad H'(t) = \int d\boldsymbol{r}\ h(\boldsymbol{r})\Psi(\boldsymbol{r},t)$$

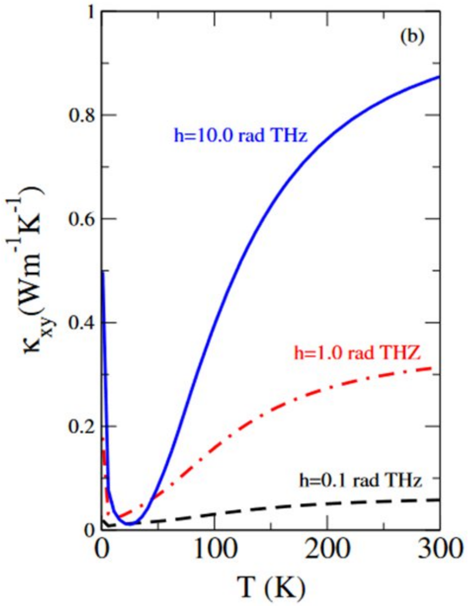
- Luttinger's theory is based on a microscopic formula which inherently includes both the physical transport currents J_Q and **unphysical magnetization currents** J_Q^M.

$$\tilde{J}_Q = J_Q + J_Q^M$$

- Divergency of the thermal Hall conductivity appears at T=0K

- The correct thermal conductivity must include the contributi **heat magnetization** current. The complete tensor must be

$$\kappa_{ij} = \tilde{\kappa}_{ij} + \epsilon_{ijk}\frac{2M_Q^k}{T}$$



Agarwalla Eur. Phys. J. B 81, 197 (2011)

Matsumoto, Murakami, PRB 89, 054420 (2014)
Qin, Niu, and Shi, PRL 107, 236601 (2011).

How to model thermally-driven transpor

L. Saleem

Representing the thermal gradient by a vector potential

$$\frac{\nabla T}{T} = \nabla\phi(t,\mathbf{r}) + \partial_t \mathbf{A}_T(t,\mathbf{r})$$

Moreno, Colemaa arXiv:cond-mat/9603079
Tatara, PRL 114, 196601 (2015)

« Thermal » Electromagnetic field

$$\mathbf{E}_T(t,\mathbf{r}) \equiv -\partial_t \mathbf{A}_T(t,\mathbf{r}) - \nabla\phi(t,\mathbf{r}),$$

$$\mathbf{B}_T(t,\mathbf{r}) \equiv \nabla \times \mathbf{A}_T(t,\mathbf{r}).$$

Gauge-invariant minimal coupling

$$\mathbf{p} \rightarrow \mathbf{p} - i\hbar \mathbf{A}_T \partial_t$$

Heat current

$$\mathbf{J}_Q(t,\mathbf{r}) = \pm i \mathrm{Tr}[\hat{\mathbf{J}}_Q(t,\mathbf{r}) \hat{G}^<_{\mathbf{E}_T}(t,\mathbf{r})].$$

Heat magnetization

$$\mathbf{M}_Q = -\left(\frac{\partial \Omega}{\partial \mathbf{B}_T}\right)_{\mu,T}$$

- Naturally accounts for diamagnetic heat currents.
- Separation of intrinsic and extrinsic contributions
- Readily applicable to multiband systems
- Easily extended to other transport quantities

Saleem, Schwingenschlögl, AM PRB 109, 134415 (2024)

How to model thermally-driven transport?

$$\kappa_{ij} = \tilde{\kappa}_{ij} + \frac{2}{T}\epsilon_{ijk}M_Q^k,$$

$$\tilde{\kappa}_{ij} = \pm\frac{i\hbar}{T}\mathrm{Tr}\underbrace{\int\frac{d\pi^0}{2\pi}\pi^0\hat{v}_i(\hat{G}^<_{\mathbf{E}_T,\mathrm{I}}f_\mp + \hat{G}^<_{\mathbf{E}_T,\mathrm{II}}f'_\mp)_j}_{\text{Heat current contribution}},$$

$$\mathbf{M}_Q = \mp\frac{i\hbar}{\beta^2}\mathrm{Tr}\underbrace{\int_\infty^\beta d\beta\int\frac{d\pi^0}{2\pi}\pi^0\hat{G}^<_{\mathbf{B}_T,\mathrm{I}}\beta f_\mp}_{\text{Heat magnetization contribution}}$$

$$\hat{G}^{R(A)}_{\mathbf{E}_T} = \hat{G}^{R(A)}_0\left\{\hat{\Sigma}^{R(A)}_{\mathbf{E}_T} + \frac{i\pi^0}{2}\left[\partial_{\pi^0}\left(\hat{G}^{R(A)}_0\right)^{-1}\hat{G}^{R(A)}_0\nabla_\pi\left(\hat{G}^{R(A)}_0\right)^{-1} - \nabla_\pi\left(\hat{G}^{R(A)}_0\right)^{-1}\hat{G}^{R(A)}_0\partial_{\pi^0}\left(\hat{G}^{R(A)}_0\right)^{-1}\right]\right\}\hat{G}^{R(A)}_0,$$

$$\hat{G}^{R(A)}_{\mathbf{B}_T} = \hat{G}^{R(A)}_0\left\{\hat{\Sigma}^{R(A)}_{\mathbf{B}_T} + \frac{i\pi^0}{2}\left[\nabla_\pi\left(\hat{G}^{R(A)}_0\right)^{-1}\times\hat{G}^{R(A)}_0\nabla_\pi\left(\hat{G}^{R(A)}_0\right)^{-1}\right]\right\}\hat{G}^{R(A)}_0,$$

$$\begin{aligned}\hat{G}^<_{\mathbf{E}_T} &= \mp(\hat{G}^A_{\mathbf{E}_T} - \hat{G}^R_{\mathbf{E}_T})f_\mp + \hat{G}^R_0[\hat{\Sigma}^<_{\mathbf{E}_T} \pm (\hat{\Sigma}^A_{\mathbf{E}_T} - \hat{\Sigma}^R_{\mathbf{E}_T})f_\mp]\hat{G}^A_0\\ &\pm\frac{i\pi^0}{2}\hat{G}^R_0\left[(\hat{\Sigma}^A_0 - \hat{\Sigma}^R_0)\hat{G}^A_0\nabla_\pi(\hat{G}^A_0)^{-1} - \nabla_\pi(\hat{G}^R_0)^{-1}\hat{G}^R_0(\hat{\Sigma}^A_0 - \hat{\Sigma}^R_0)\right]\hat{G}^A_0 f'_\mp,\\ \hat{G}^<_{\mathbf{B}_T} &= \mp(\hat{G}^A_{\mathbf{B}_T} - \hat{G}^R_{\mathbf{B}_T})f_\mp + \hat{G}^R_0[\hat{\Sigma}^<_{\mathbf{B}_T} \pm (\hat{\Sigma}^A_{\mathbf{B}_T} - \hat{\Sigma}^R_{\mathbf{B}_T})f_\mp]\hat{G}^A_0,\end{aligned}$$

Saleem, Schwingenschlögl, AM PRB 109, 134415 (2024)

I. Quantum field theory for thermal transport of magnons

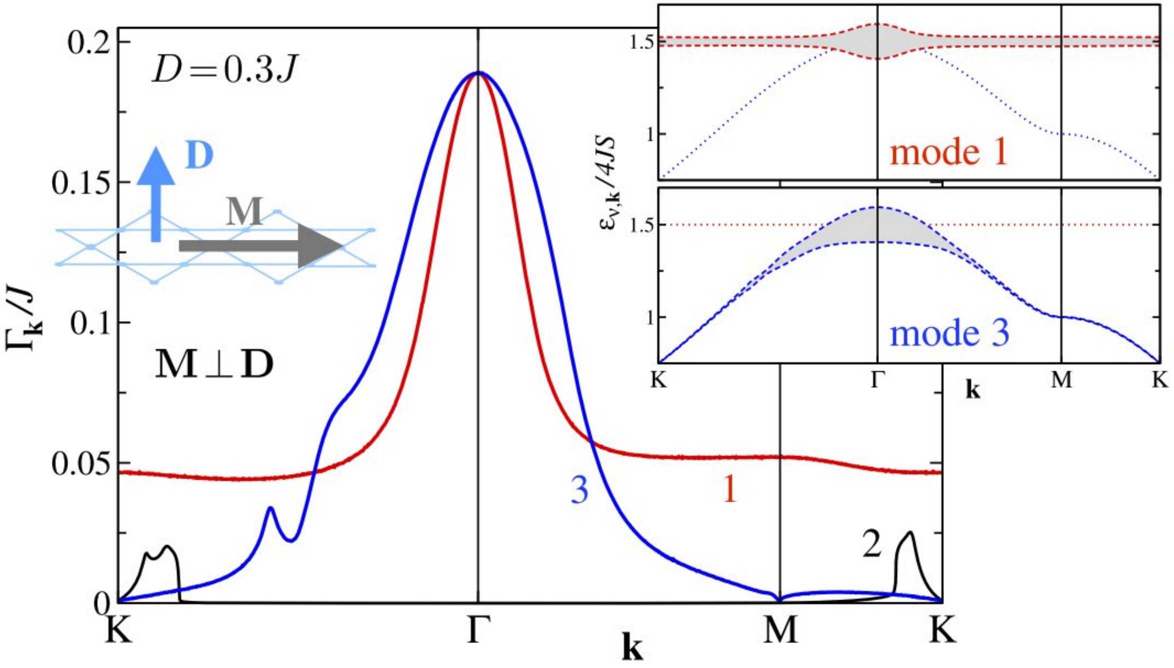
II. Role of magnon-magnon interactions on magnon transport

III. Interaction-driven topological phase transitions

IV. Electron-magnon interactions in altermagnets

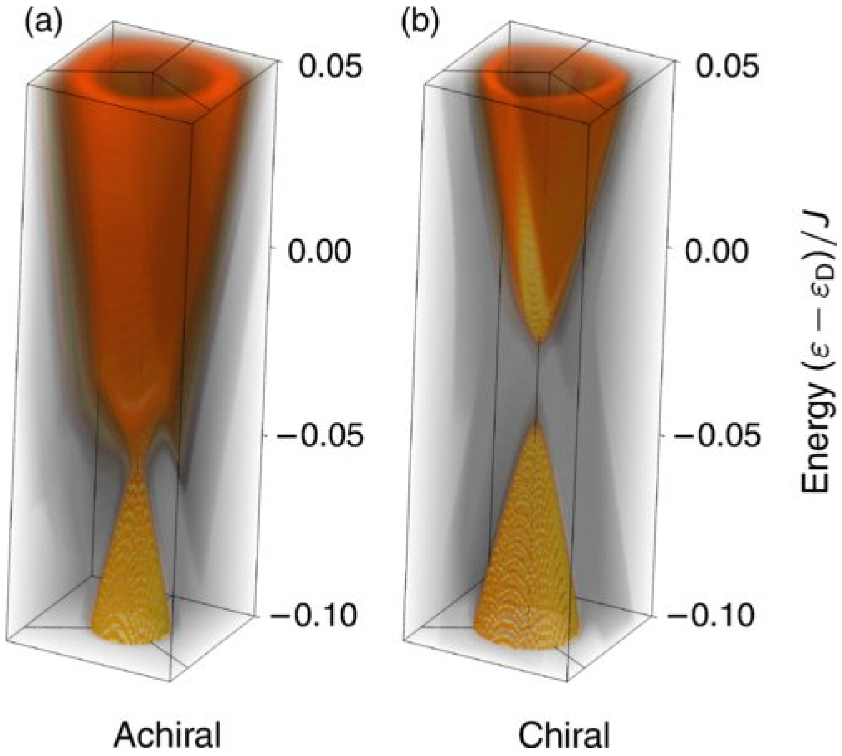
Interacting magnons in antiferromagnets

DMI, necessary to induce topology, induces magnon damping



Chernyshev PRL 117, 187203 (2016)

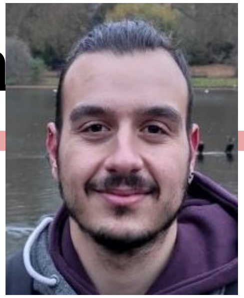
…but interactions can stabilize topological phas



Mook et al. PRX 11, 021061 (2021)

Most work on thermal transport of magnon neglect interactions
Strong magnon damping at Γ point…what about Hall effect?

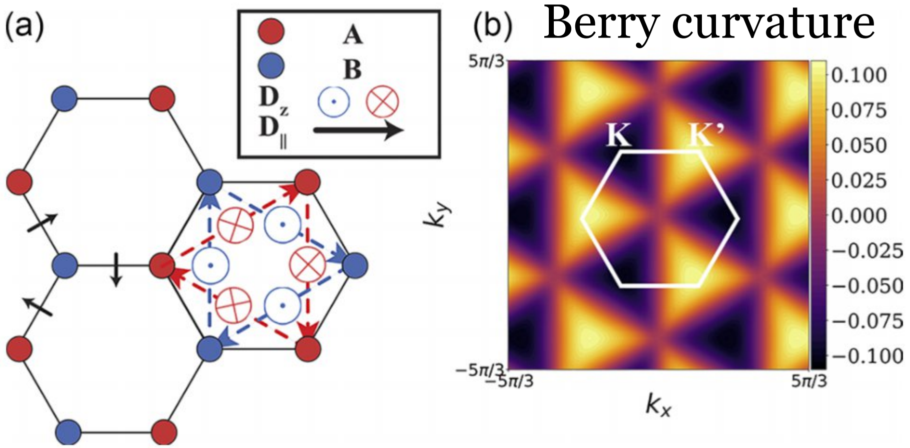
Interacting magnons in antiferromagn



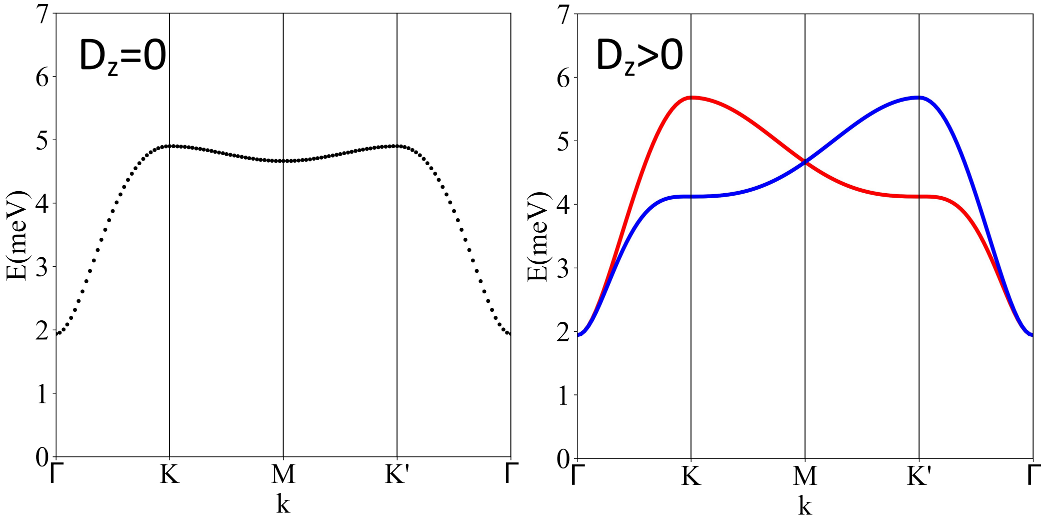
K. Sourounis

$$H = J\sum_{\langle ij\rangle}\mathbf{S}_{A,i}\cdot\mathbf{S}_{B,j} + K\sum_{i}\left(S_i^z\right)^2$$
$$+D_z\sum_{\langle\langle ij\rangle\rangle} z\cdot(\mathbf{S}_i\times\mathbf{S}_j) + D_{||}\sum_{\langle ij\rangle}\boldsymbol{\eta}_{ij}\cdot(\mathbf{S}_{A,i}\times\mathbf{S}_{B,j})$$

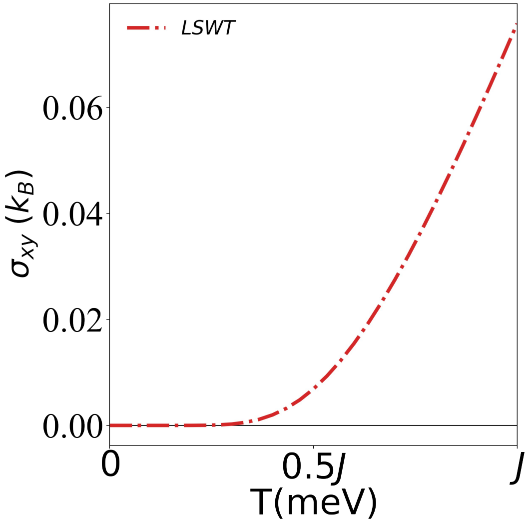
Turned on upon interactions



Non-interacting honeycomb antiferromagnet



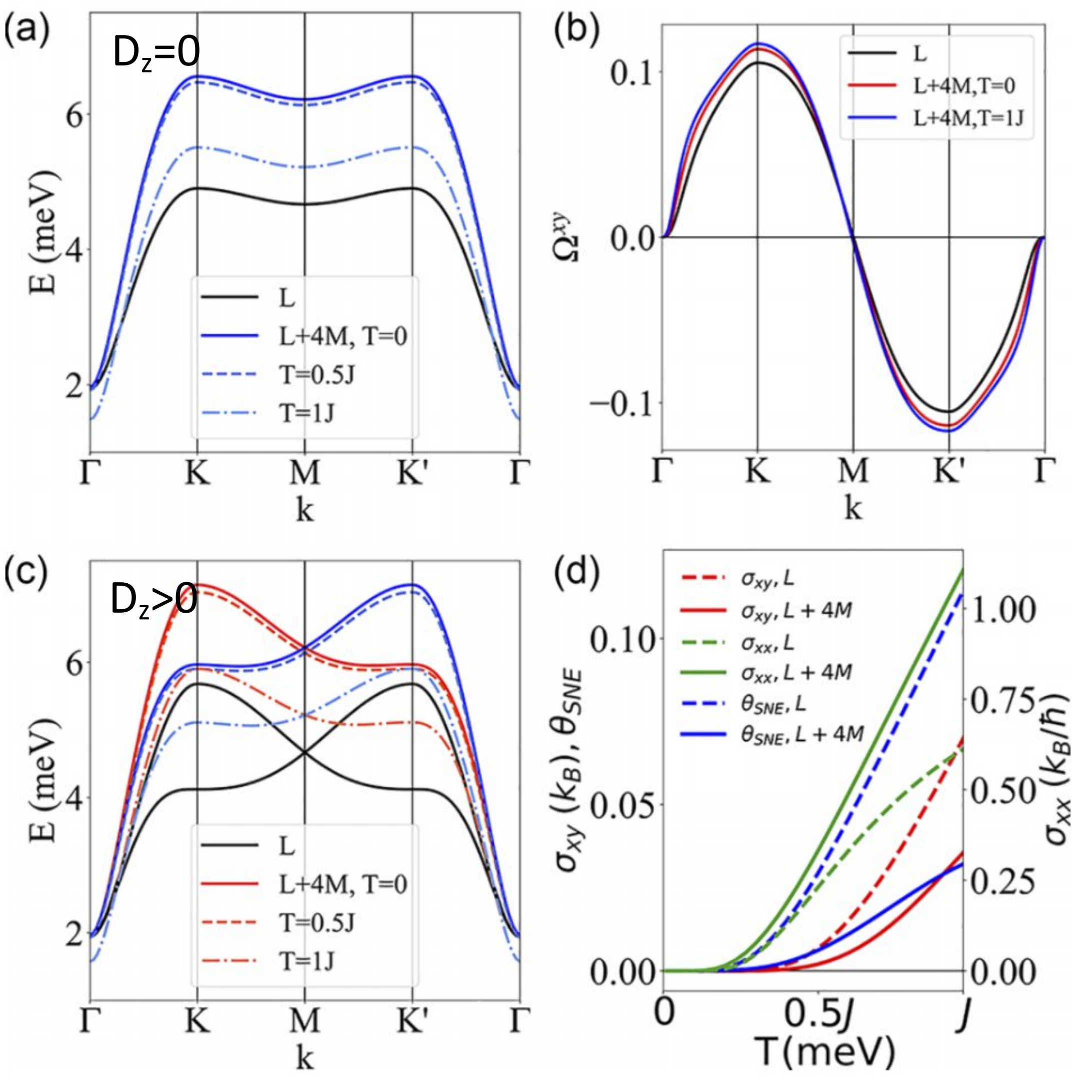
Spin Nernst effect

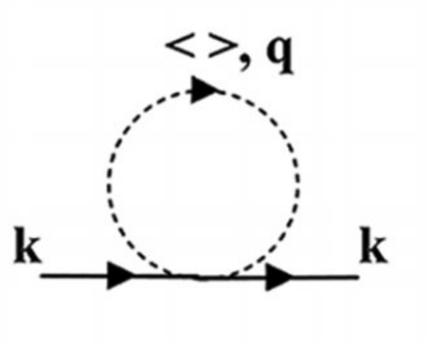


See also Cheng et al. PRL (2016); Zyuzin, Kovalev PRL (2016)

Sourounis, AM PRB 110, 054429 (2024)

Interacting magnons in antiferromagnets





4-magnon interaction

- Mean-field theory (equiv to Hartree-Fock)

- Quantum contribution (T=0)..unique to AF

increases the bandwidth and enhance the magnon velocity

- Thermal contribution (T>0)

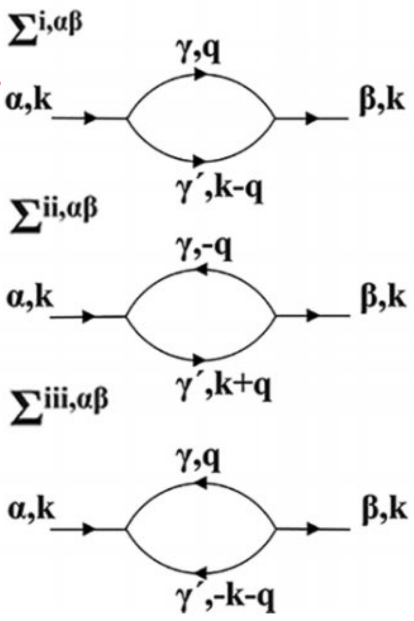
Narrows the magnon bandwidth

Sourounis, AM PRB 110, 054429 (2024)

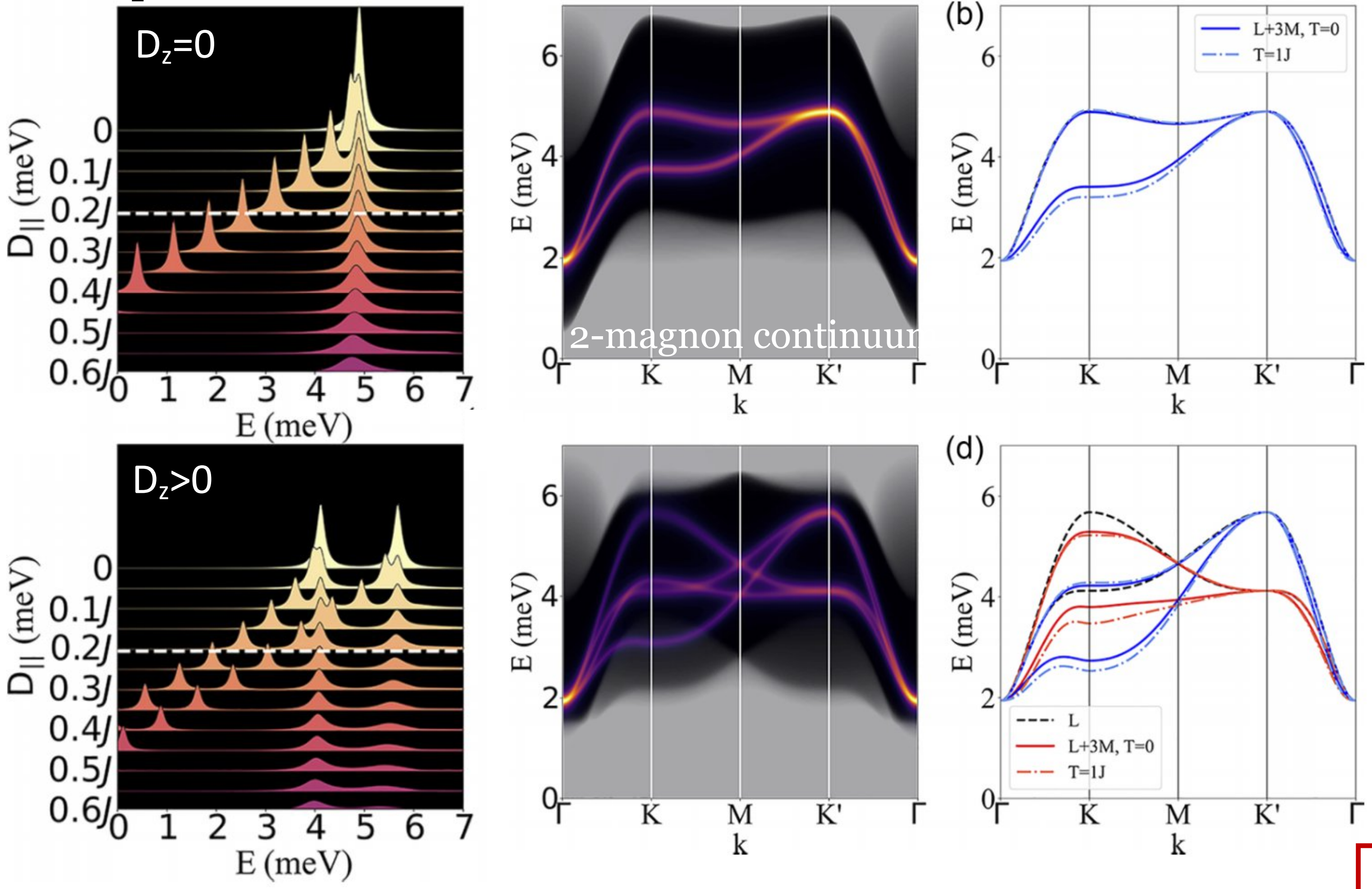
Interacting magnons in antiferromagnets

$$D_{||} \sum_{\langle ij \rangle} \boldsymbol{\eta}_{ij} \cdot (\mathbf{S}_{A,i} \times \mathbf{S}_{B,j})$$

3-magnon interactions



Spectral function at K



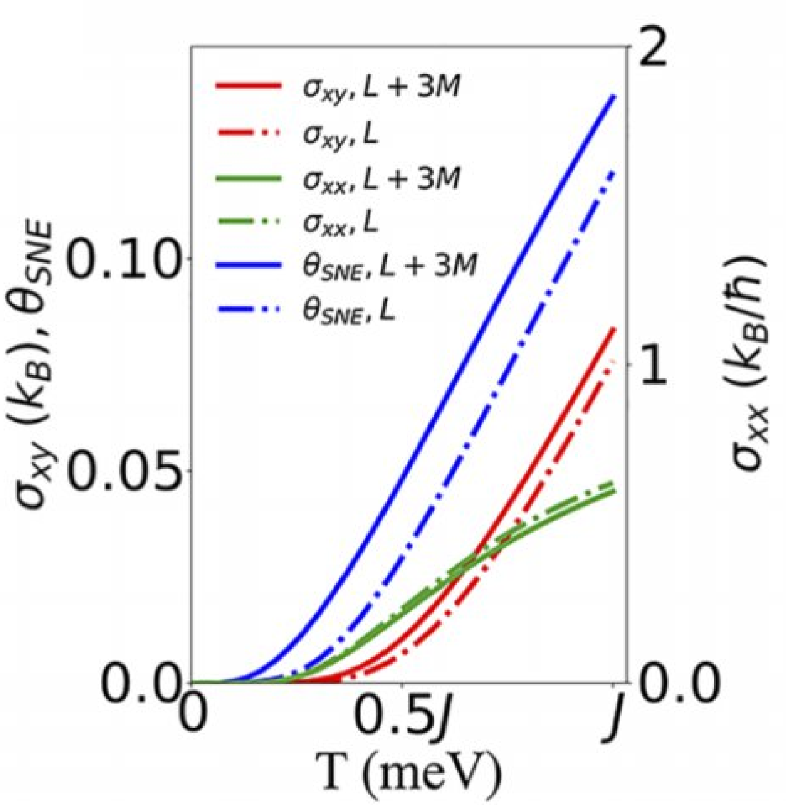
3-magnon interaction

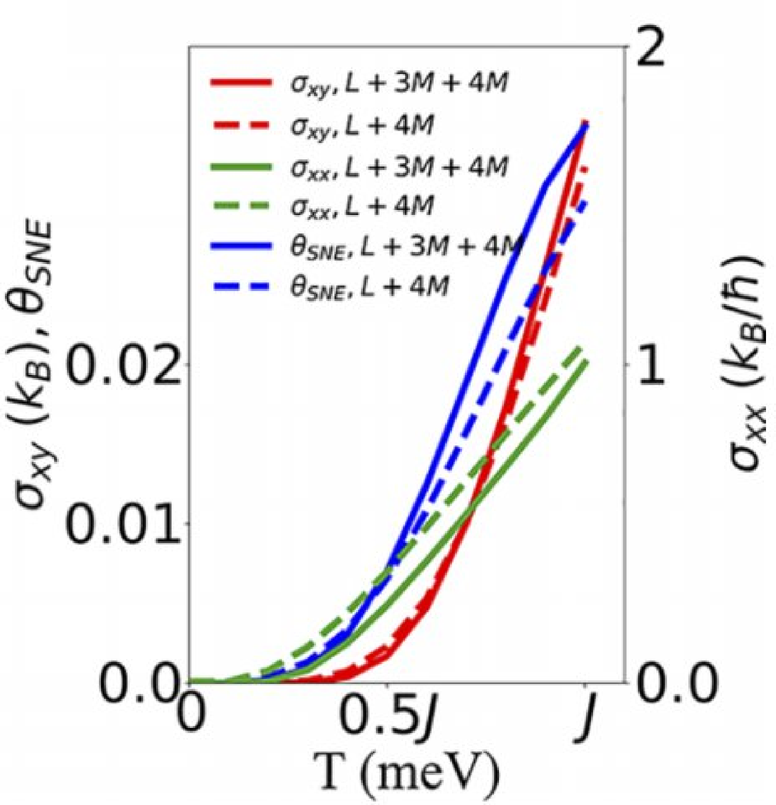
- Self-energy
- Induces magnon decay
- Strong influence of temperature
- No influence on Berry curvature

Sourounis, AM PRB 110, 054429 (2024)

Interacting magnons in antiferromagnets

3-magnon + 4-magnon interactions





Sourounis, AM PRB 110, 054429 (2024)

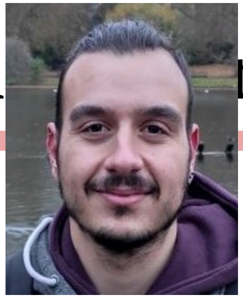
I. Quantum field theory for thermal transport of magnons

II. Role of magnon-magnon interactions on magnon transport

III. Interaction-driven topological phase transitions

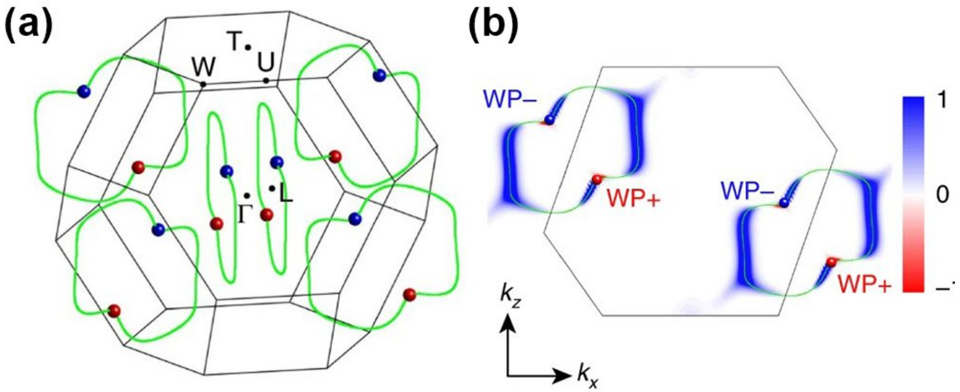
IV. Electron-magnon interactions in altermagnets

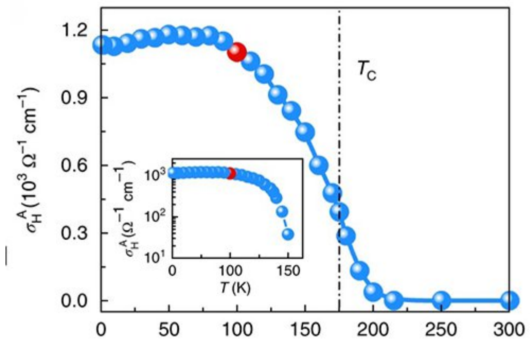
Topological phase transitions in magnetic Weyl m



K. Sourounis

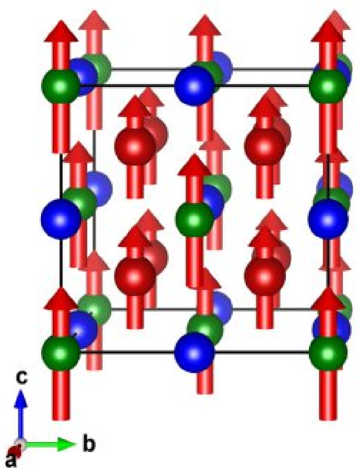
Ferromagnetic Weyl semimetal with intrinsic magnetic order Co_3SnS_2



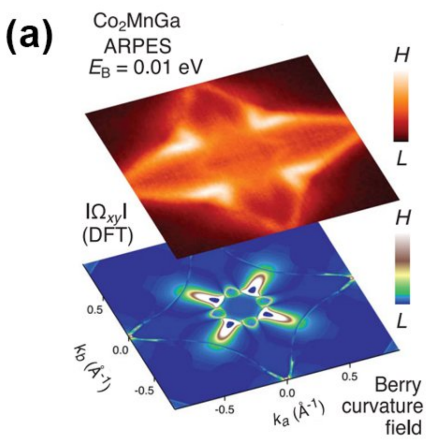


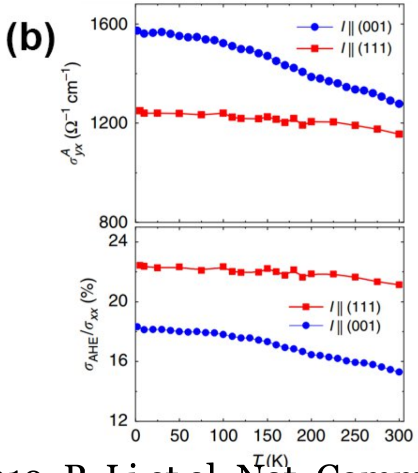
E. Liu et al., Nat. Phys. 2018

Cubic Heusler Weyl Semimetals, Co_2Mn_2X (X=Ga, Al)



Cubic Heusler: K. Manna et al., PRX 2018, J. Noky et al. Comp. Mat. 2020





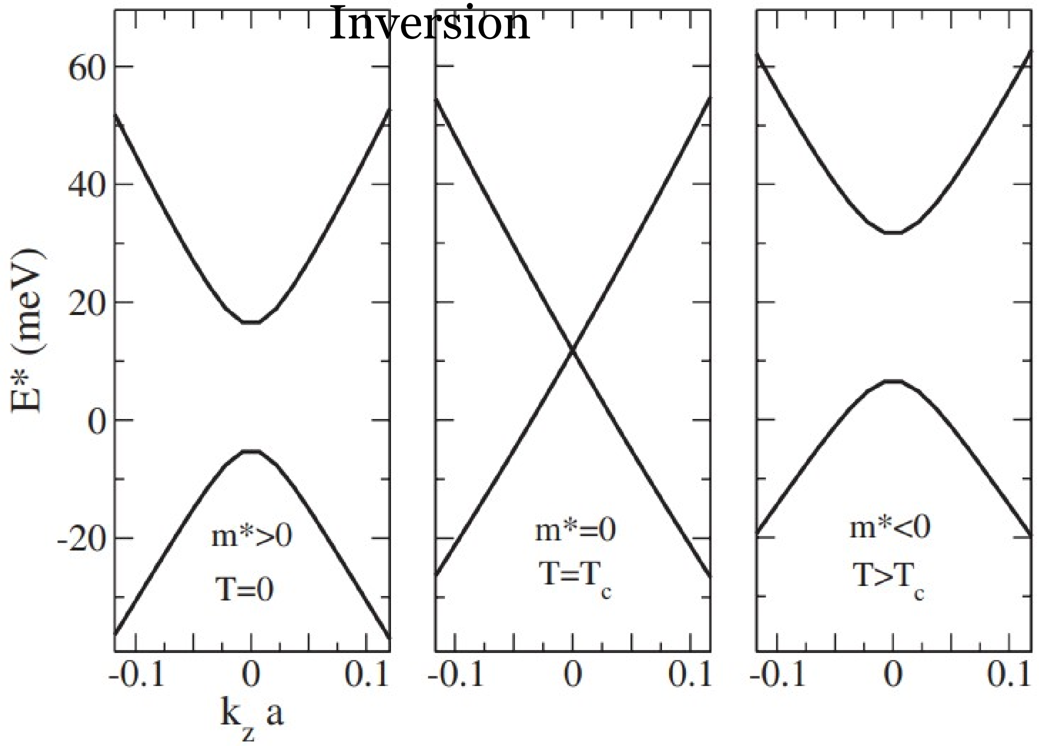
I. Belopolski et al., Science 2019, P. Li et al. Nat. Comm. 2020

Topological phase transitions in magnetic Weyl semim

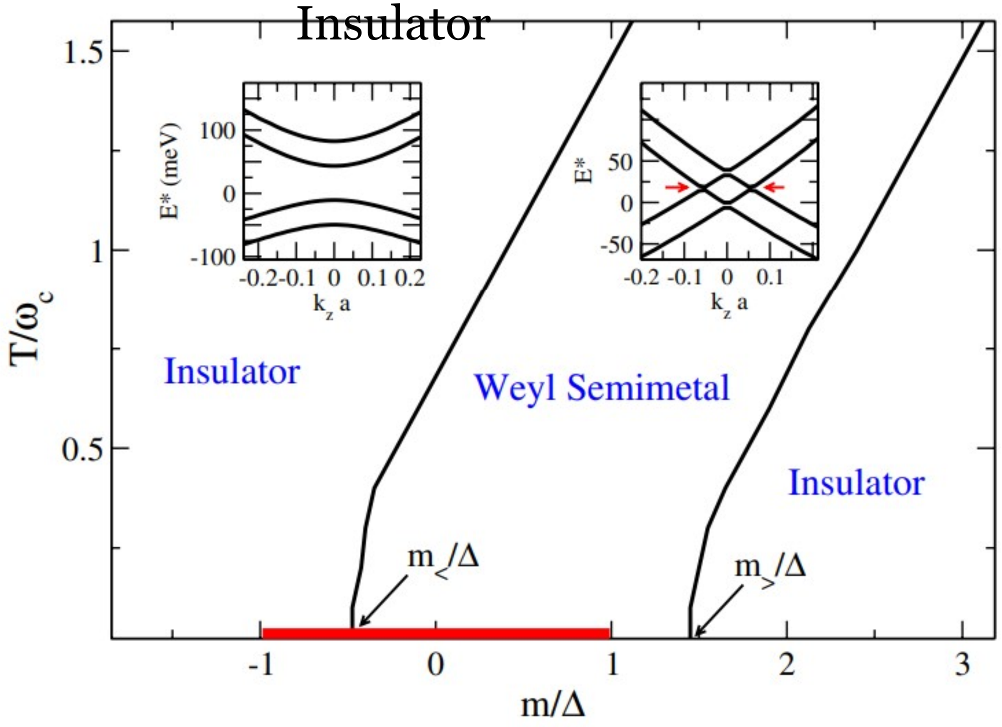
$$h^{(0)}(\mathbf{k}) = \epsilon_{0,\mathbf{k}}\mathbf{1}_4 + \mathbf{d}_\mathbf{k}\cdot\boldsymbol{\sigma}\boldsymbol{\tau}^x + M_\mathbf{k}\mathbf{1}_2\boldsymbol{\tau}^z,$$
$$\hat{\Sigma}(i\omega_n, \mathbf{0}) = \Sigma_0(i\omega_n)\mathbf{1}_4 + \Sigma_z(i\omega_n)\mathbf{1}_2\boldsymbol{\tau}^z,$$

Electron-phonon interaction acts on orbital space

Dirac Mass Inversion



WSM / Trivial Insulator

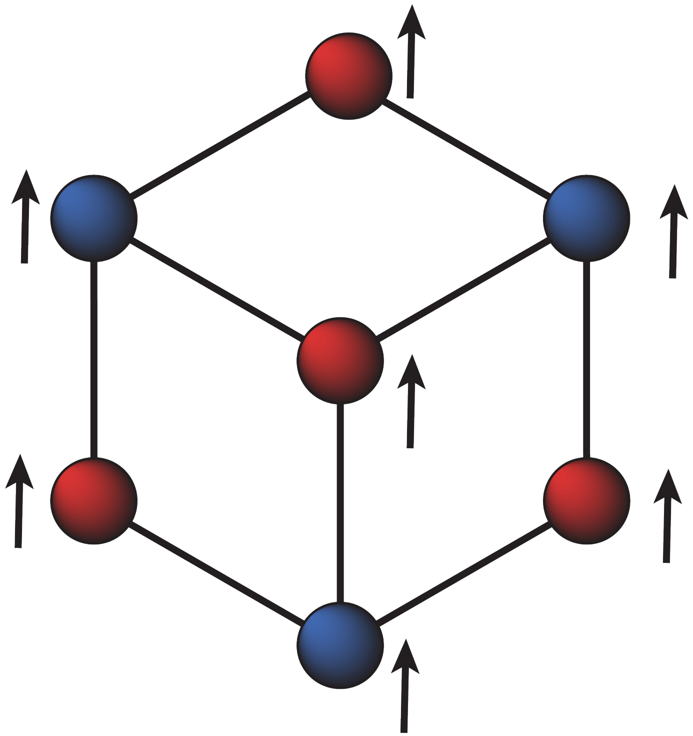


Garate PRL 110, 046402 (2013)

Topological phase transitions in magnetic Weyl semim

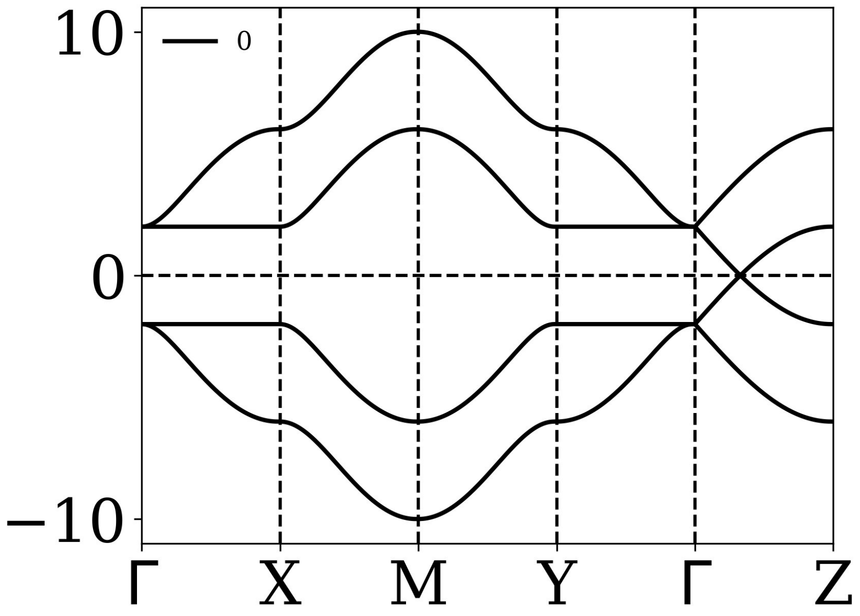
Ferromagnetic Weyl semimetal model

$$H_k = \boldsymbol{d_k} \cdot \boldsymbol{\sigma}\tau_x + M_k\tau_z + \Delta\langle S^z\rangle\sigma_z$$



Weyl Points can be tuned by m: Mass and Δ: Magnetization

m=0 ➔ Weyl point along GZ



$$d_k = -2\lambda \sin(k_i) \ , M_k = m + 2t\sum_i (3 - \cos(k_i))$$

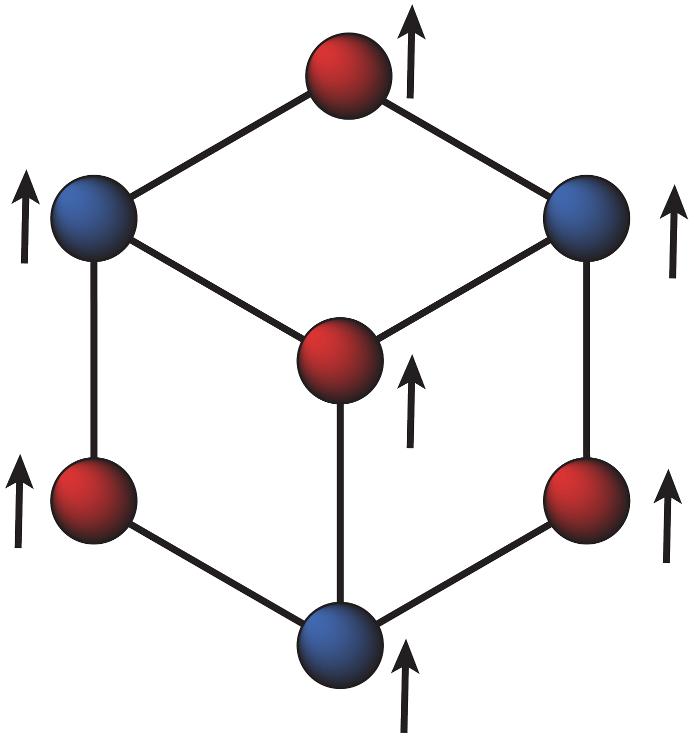
τ_i: Orbital Space σ_i: Spin Space

Sourounis, AM arXiv:2412.17044 (2025)

Topological phase transitions in magnetic Weyl semim

Ferromagnetic Weyl semimetal model

$$H_k = \boldsymbol{d_k} \cdot \boldsymbol{\sigma}\tau_x + M_k\tau_z + \Delta\langle S^z\rangle\sigma_z$$

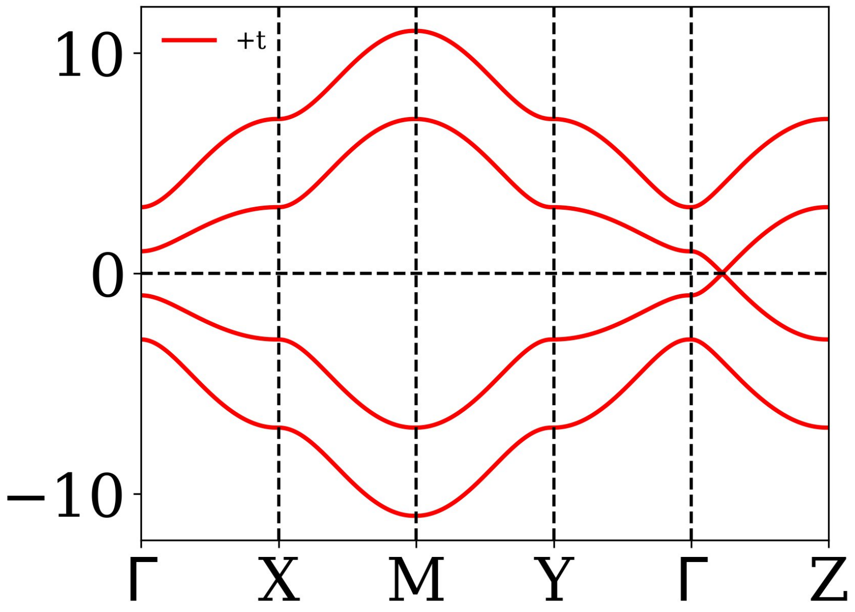


$$d_k = -2\lambda \sin(k_i) \;, M_k = m + 2t\sum_i (3 - \cos(k_i))$$

τ_i: Orbital Space σ_i: Spin Space

Weyl Points can be tuned by m: Mass and Δ: Magnetization

m=+1 ➔ Weyl point along GZ

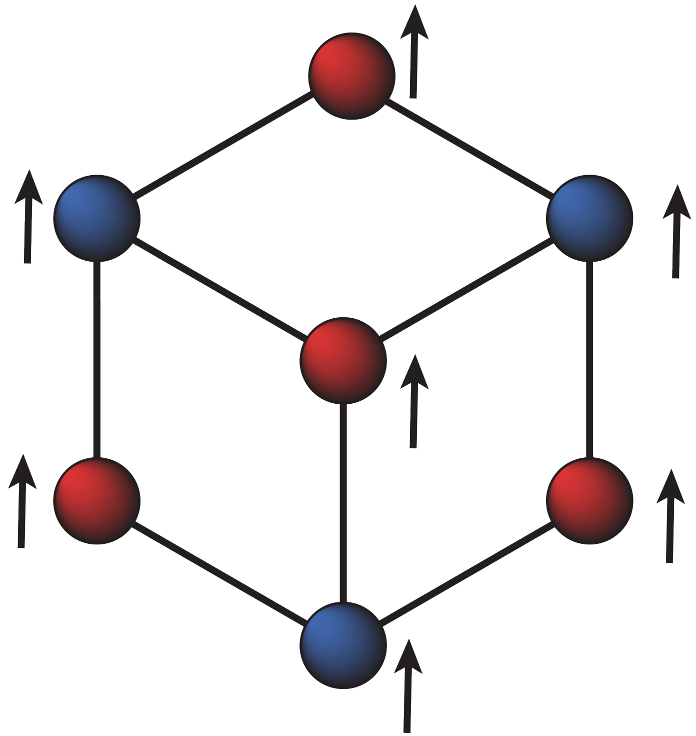


Sourounis, AM arXiv:2412.17044 (2025)

Topological phase transitions in magnetic Weyl semim

Ferromagnetic Weyl semimetal model

$$H_k = \boldsymbol{d_k} \cdot \boldsymbol{\sigma}\tau_x + M_k\tau_z + \Delta\langle S^z\rangle\sigma_z$$

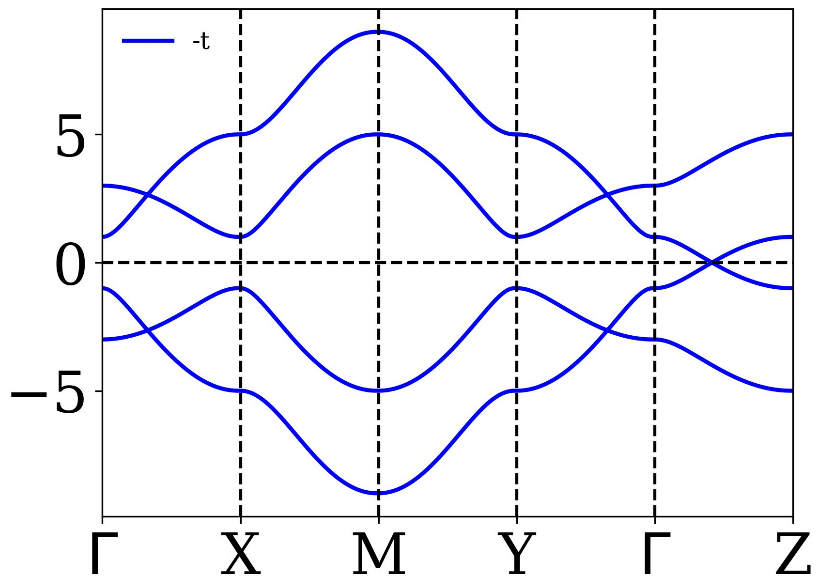


$$d_k = -2\lambda \sin(k_i) \;\; ,M_k = m + 2t\sum_i (3 - \cos(k_i))$$

τ_i: Orbital Space σ_i: Spin Space

Weyl Points can be tuned by m: Mass and Δ: Magnetization

m=-1 ➔ Weyl point in (X,Y) plane



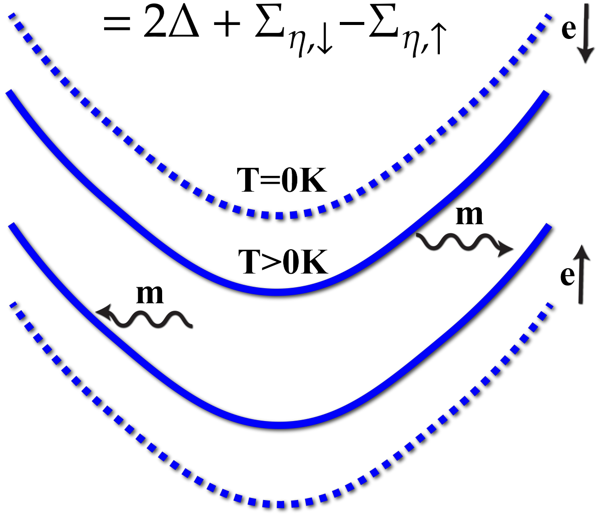
Sourounis, AM arXiv:2412.17044 (2025)

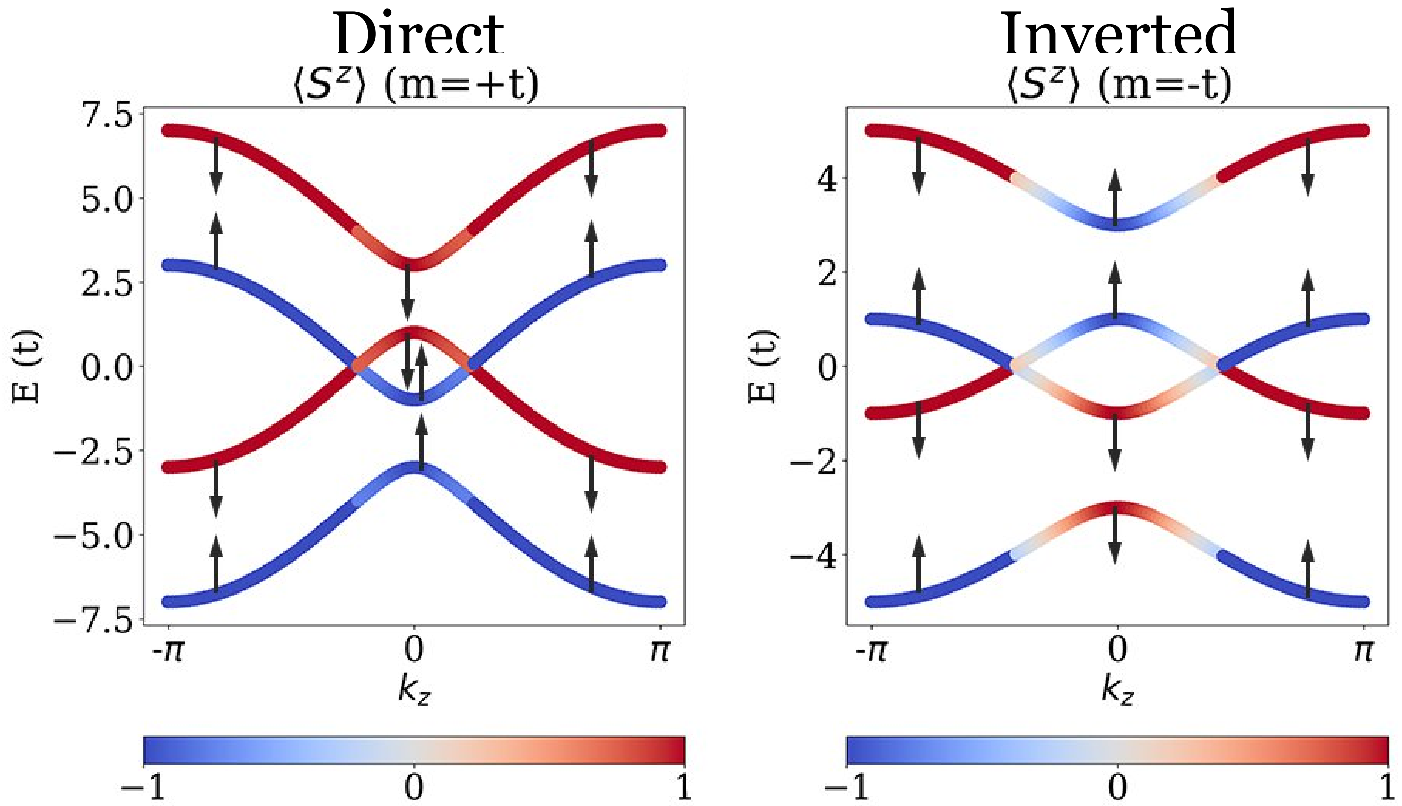
Topological phase transitions in magnetic Weyl semim

Electron-Magnon Interaction acts on spin space

Hertz, Edwards - JoP (1973)

$$\Delta_{eff} = 2\Delta + \Sigma_{\eta,\downarrow} - \Sigma_{\eta,\uparrow}$$





Sourounis, AM arXiv:2412.17044 (2025)

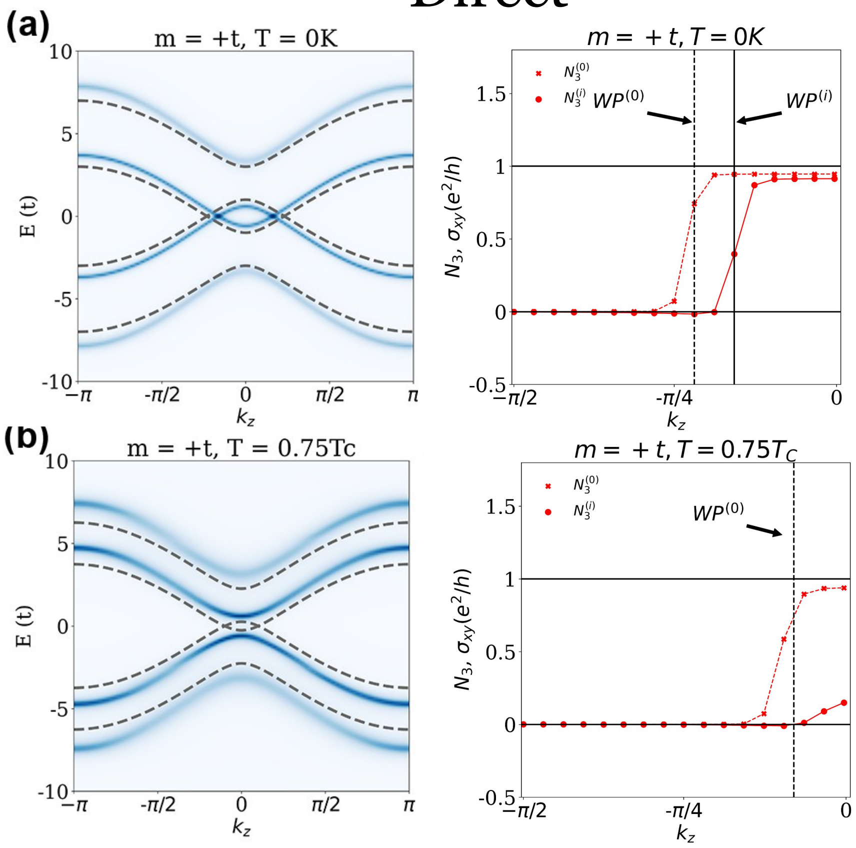
Topological phase transitions in magnetic Weyl semim

Interacting topology

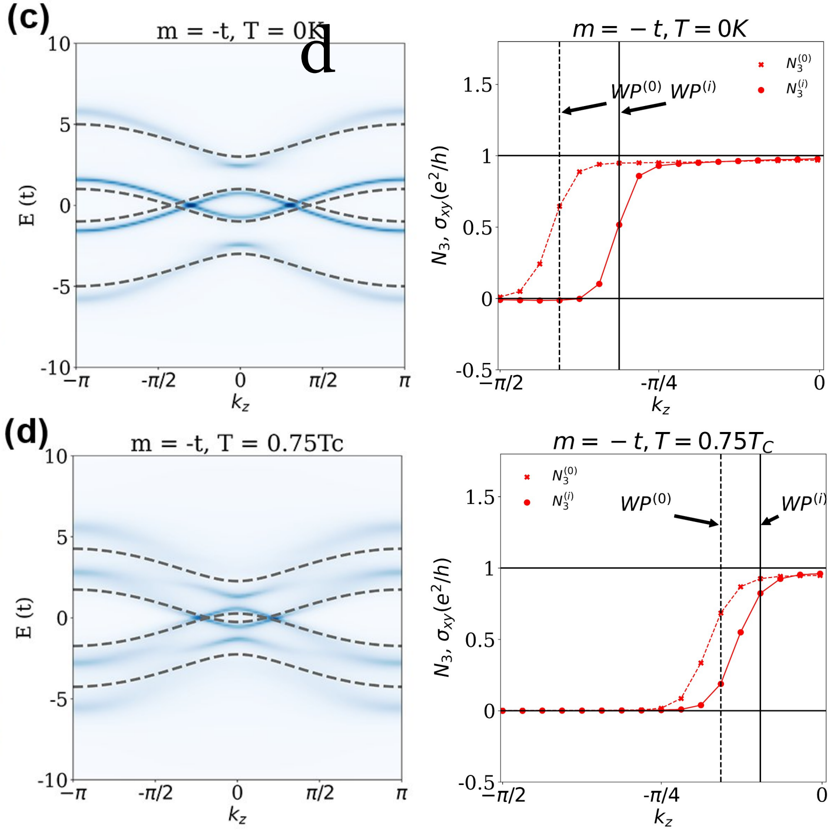
$$N_3(k_z) = \frac{e^2}{h}\int d\eta \int dk_x dk_y \times \mathrm{Tr}[G^T_{\mathbf{k}}(\partial_\eta G^{-1}_{\mathbf{k}})G^T_{\mathbf{k}}(\partial_{k_x} G^{-1}_{\mathbf{k}})G^T_{\mathbf{k}}(\partial_{k_y} G^{-1}_{\mathbf{k}}) - (x \leftrightarrow y)],$$

Zhao et al. PRL 131, 106601 (2023)

Direct



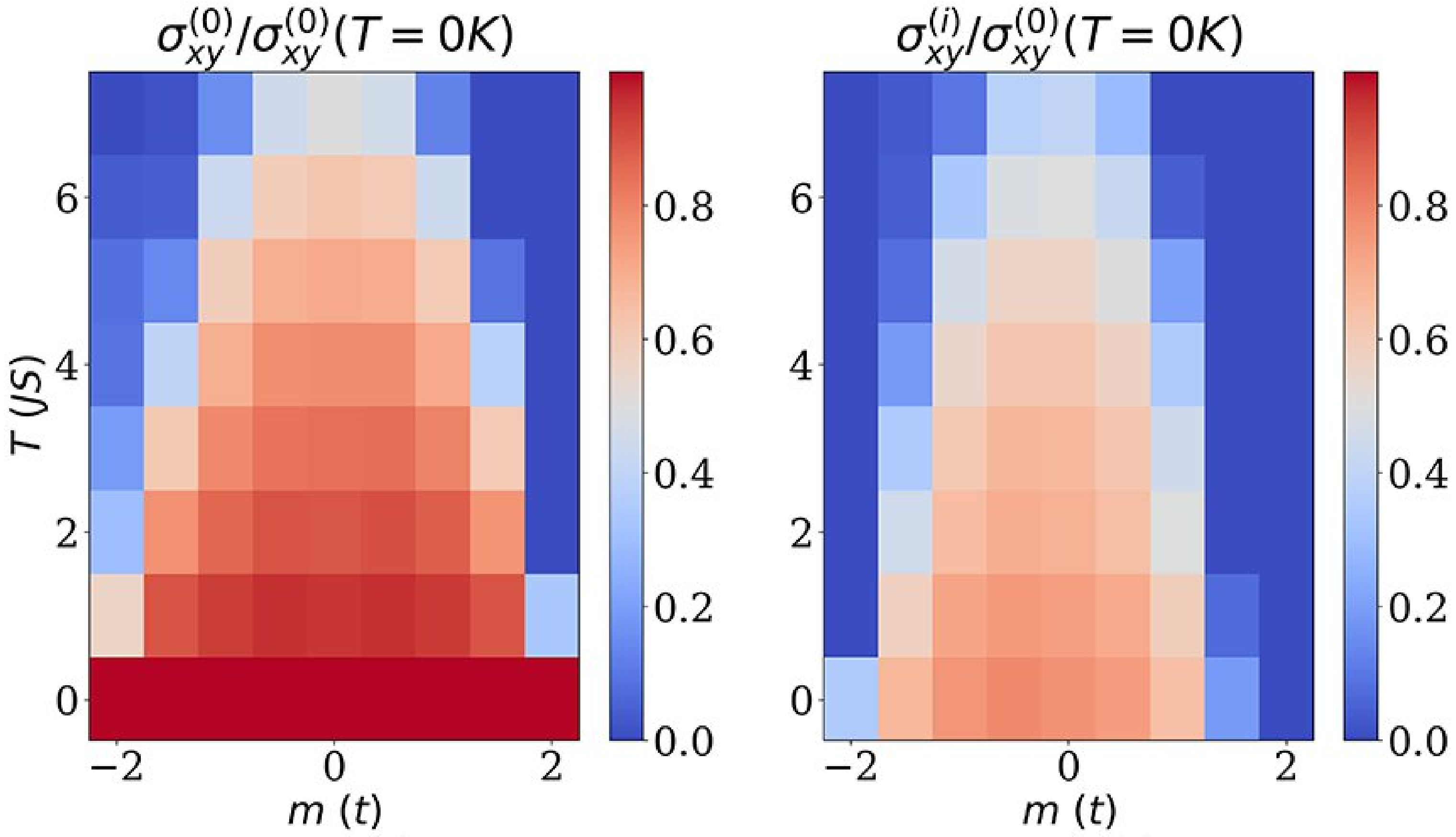
Inverte
d



Sourounis, AM arXiv:2412.17044 (2025)

Topological phase transitions in magnetic Weyl semim

$\Sigma_k = 0$ $\Sigma_k > 0$



Trivial (m<0): Conductivity faster than magnetization $\langle S^z \rangle$

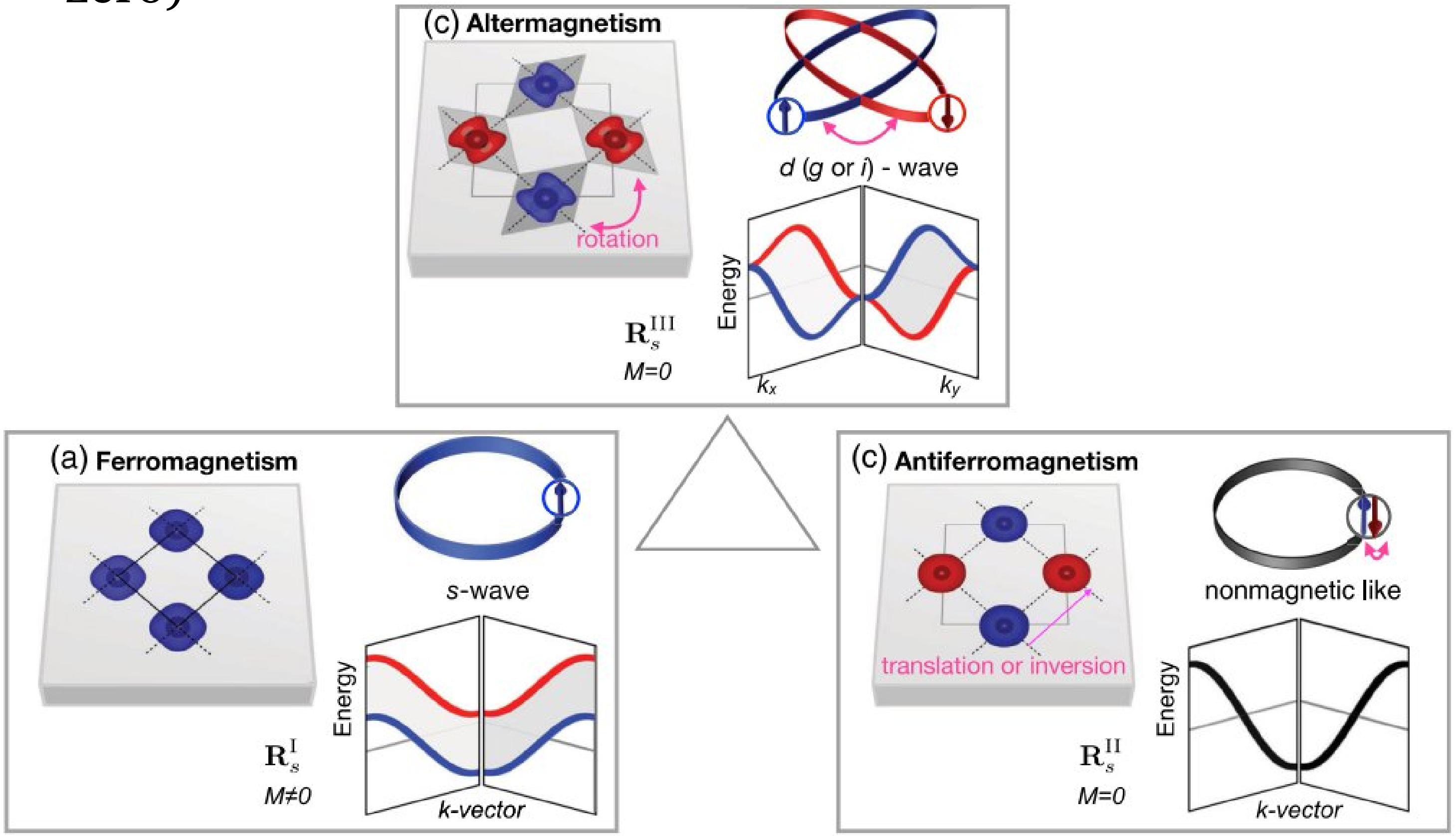
Inverted (m>0): Conductivity decreases as magnetization $\langle S^z \rangle$

Sourounis, AM arXiv:2412.17044 (2025)


I. Quantum field theory for thermal transport of magnons
II. Role of magnon-magnon interactions on magnon transport
III. Interaction-driven topological phase transitions
IV. Electron-magnon interactions in altermagnets

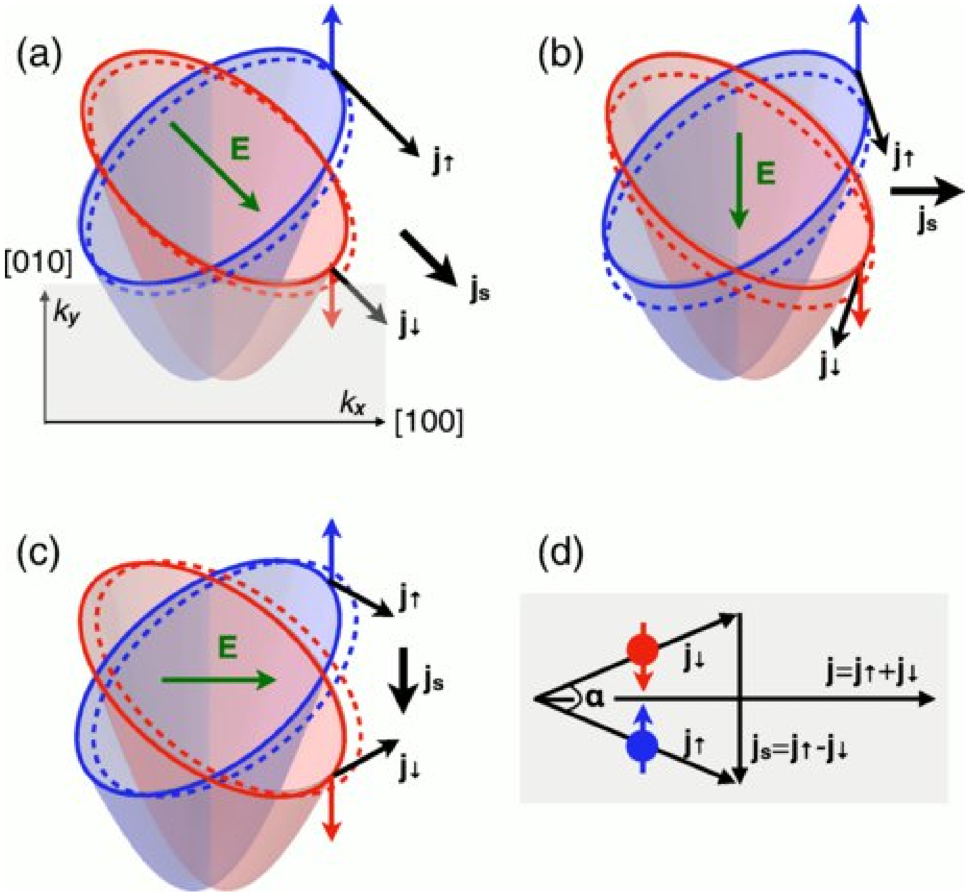
Magnon spin splitter in altermagnets

Spin-Split Antiferromagnets (Net magnetization equals to zero)



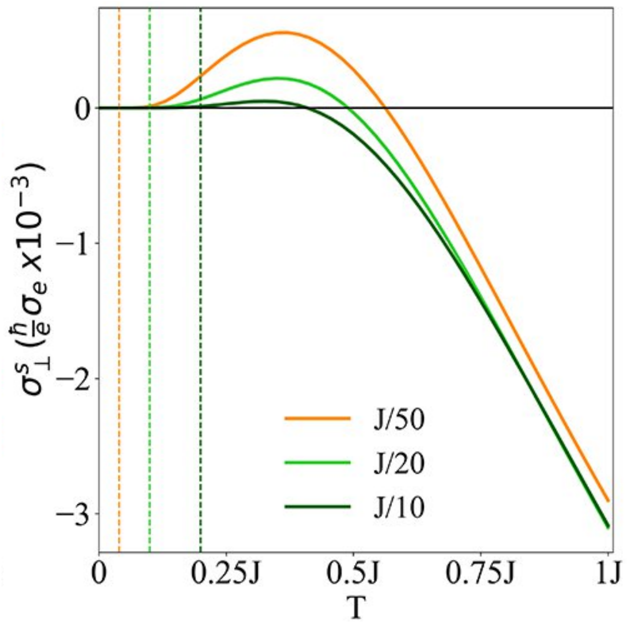
L Šmejkal et al. , PRX 12, 040501 (2022)

Magnon spin splitter in altermagnets



$$
\begin{aligned}
H_{\boldsymbol{k}}^{em} &= -\sqrt{\frac{I^2 S}{2N}} \sum_{\boldsymbol{q}} \left(W_{\boldsymbol{q}}^{+} \gamma_{\boldsymbol{q},+} + W_{\boldsymbol{q}}^{-} \gamma^{\dagger}_{-\boldsymbol{q},-} \right) c^{\dagger}_{\boldsymbol{k}+\boldsymbol{q},\downarrow} c_{\boldsymbol{k},\uparrow} \\
&+ \left(W_{\boldsymbol{q}}^{+} \gamma^{\dagger}_{-\boldsymbol{q},+} + W_{\boldsymbol{q}}^{-} \gamma_{\boldsymbol{q},-} \right) c^{\dagger}_{\boldsymbol{k}+\boldsymbol{q},\uparrow} c_{\boldsymbol{k},\downarrow} + h.c., \qquad (4)
\end{aligned}
$$

$$
\mathbf{J}_m^{\nu} = \frac{\hbar}{\Omega_m} \sum_{\boldsymbol{q}} \boldsymbol{v}_{\boldsymbol{q}}^{\nu} \partial_{\omega_{\boldsymbol{q},\nu}} n^{0}_{\boldsymbol{q},\nu} g_{\boldsymbol{q}}^{\nu},
$$



As temperature increases, the magnons are thermally excited:

- Current is active above the magnon gap (2K)
- Current increases as temperature increases
- As the temperature approaches the Neel limit magnons lose energy

Sourounis, AM PRB 111, 134448 (2025)

Thank you for your attention!

anr agence nationale de la recherche

A*Midex Initiative d'excellence Aix-Marseille

Co-funded by the European Union

ORBITAL ELECTRONICS

FRANCE 2030 PROGRAMME DE RECHERCHE SPINTRONIQUE