Quantum Geometry and Transport of Collective Excitations
in (Non-)Magnetic Insulators, Ingelheim, Germany, May 7, 2025

Chiral-phonon-induced spin polarizations

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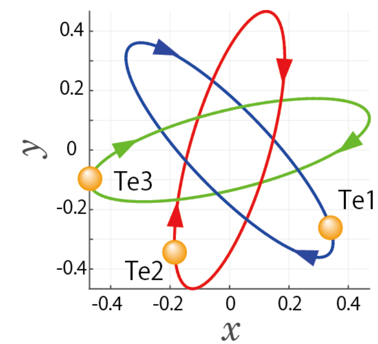
S. Iwasaki (Okayama)
J. Kishine (Open U. of Japan)

Chimera Quasiparticle

Outline

Phonons have angular momenta
→ They can be converted into electron spins



1) Phonon angular momentum in chiral systems

Zhang, Murakami, PRR 4, L012024 (2022)
Ishito, Mao, Kousaka, Togawa, Iwasaki, Zhang,
Murakami, Kishine, Satoh, Nat. Phys.19, 35 (2022)

2) Conversion of phonon angular momentum into spins

Hamada, Murakami, Phys. Rev. Research 2, 023275 (2020)
Yao, Murakami, Phys. Rev. B 105, 184412 (2022)
Yao, Murakami, J. Phys. Soc. Jpn. 93, 034708 (2024)
Yao, Murakami, Phys. Rev. B111,134414 (2025)

3) Coupling between the surface acounstic waves and magnetostatic waves

Kono, Yamamoto, Murakami, in preparation

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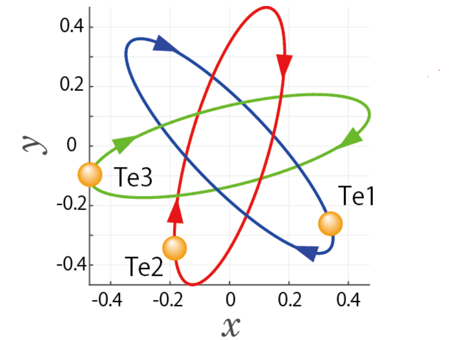

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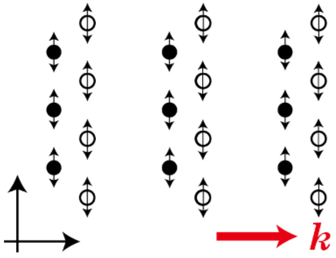
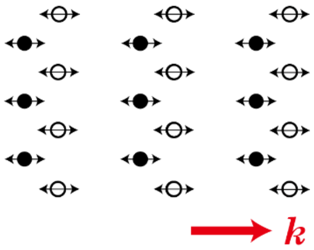
Chiral phonons

Angular momentum of phonons

L. Zhang, Q. Niu, PRL 112, 085503(2014)

- Wavevector // symmetry axis
  → longitudinal & transverse phonons

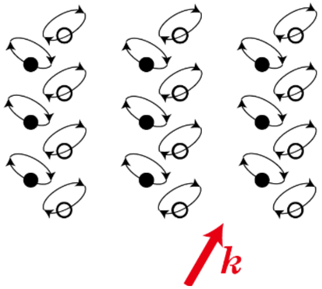




- Wavevector in general directions / crystal with low symmetry

  → mixture between longitudinal & transverse phonons

  → rotational motions = angular momentum

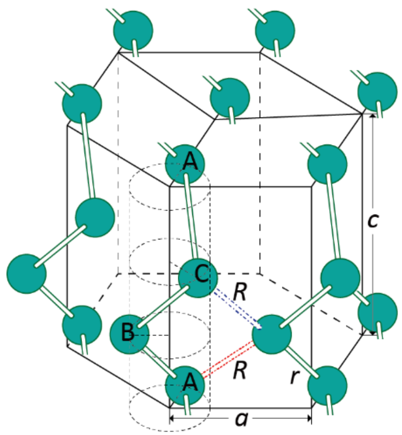


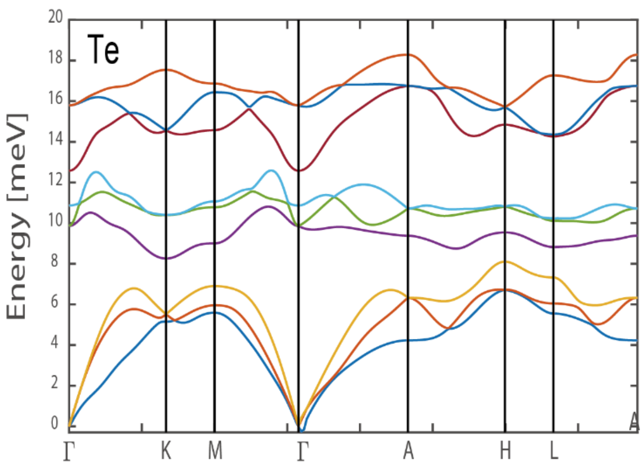
Chiral phonons: phonons with angular momentum

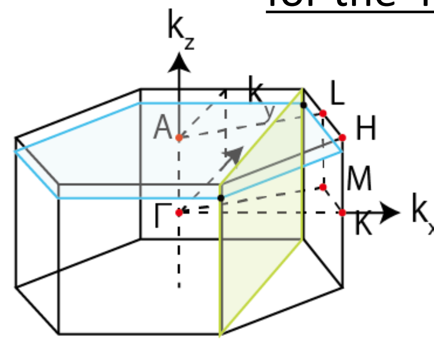
Tellurium (Te)

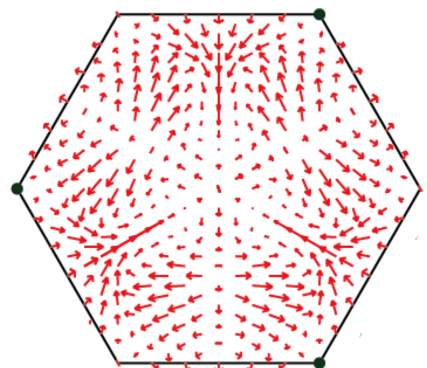
Distribution of angular momemtum for the 4th lowest band

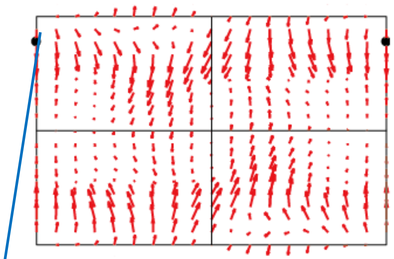
(e.g.) Right-handed crystal

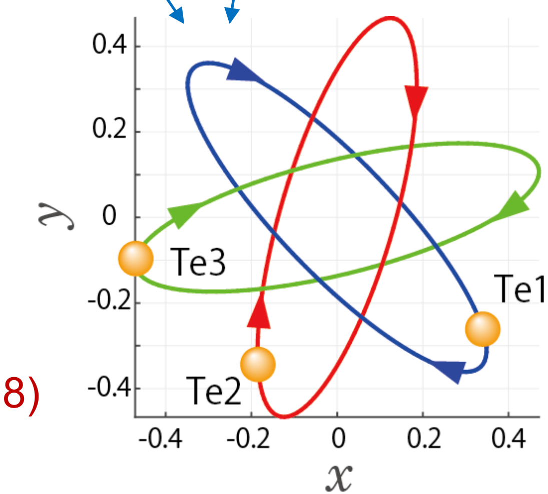












Hamada, Minamitani, Hirayama, Murakami, PRL (2018)

Chiral phonons in chiral crysltal: alpha-HgS

Ishito, Mao, Kousaka, Togawa, Iwasaki, Zhang,
Murakami, Kishine, Satoh, Nat. Phys.19, 35 (2022)

nature physics

Letter https://doi.org/10.1038/s41567-022-01790-x

Truly chiral phonons in α-HgS

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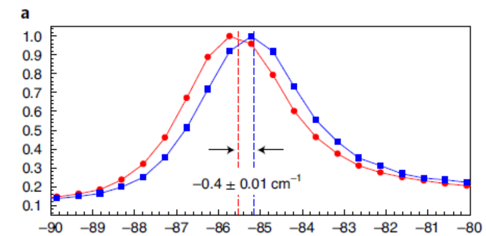
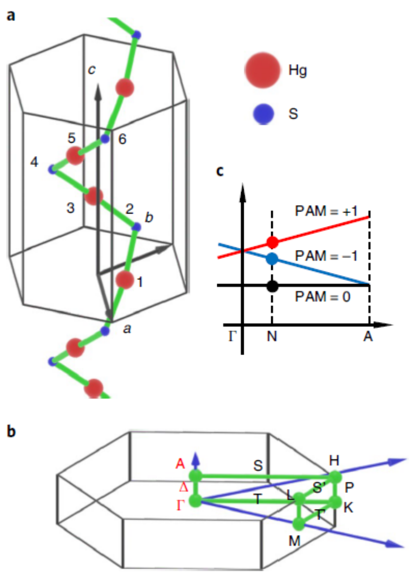
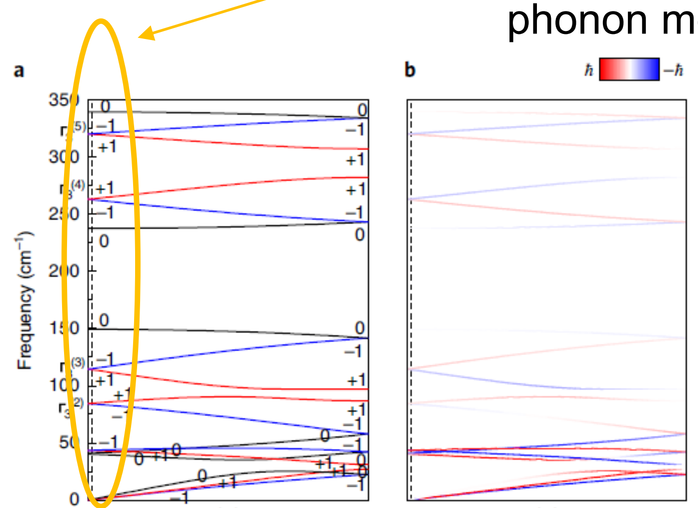
Kyosuke Ishito[1,7], Huiling Mao[1,7], Yusuke Kousaka[2,3], Yoshihiko Togawa[2], Satoshi Iwasaki[3], Tiantian Zhang[1,4], Shuichi Murakami[1,4], Jun-ichiro Kishine[5,6] and Takuya Satoh[1]



Phonon modes are split away from k=0
-- Raman + circularly polarized light

→ Pseudoangular momentum for each phonon mode is observed





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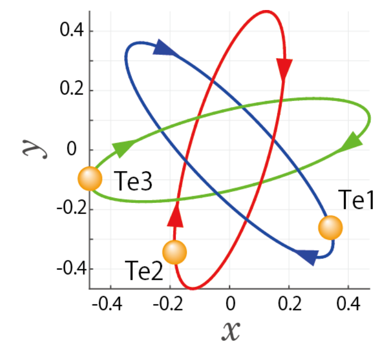

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Angular momentum of phonons

L. Zhang, Q. Niu, PRL 112, 085503(2014)

$$J_{phonon} = \sum_{l,\kappa} (\boldsymbol{u}_{l\kappa} \times \dot{\boldsymbol{u}}_{l\kappa})$$

$\boldsymbol{u}_{l\kappa}$: displacement vector of κth atom from its equilibrium position

Microscopic local rotation
= Phonon angular momentum

Second quantization

$$u_l = \sum_{\boldsymbol{k}\sigma} \varepsilon_{\boldsymbol{k}\sigma}\, e^{i(\boldsymbol{R}_l\cdot\boldsymbol{k}-\omega_{k\sigma}t)} \sqrt{\frac{\hbar}{2\omega_{k\sigma}N}}\, a_{k\sigma} + h.c.$$

$\boldsymbol{k}$: wave vector

σ: mode

a: annihilation operator

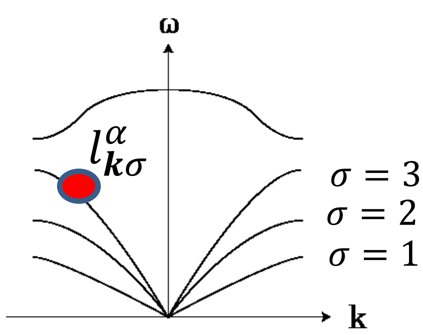
$\varepsilon_{\boldsymbol{k}\sigma}$: polarization vector

Total phonon angular momenta

$$J_{\mathrm{ph}}^{\alpha} = \sum_{\boldsymbol{k}\sigma} l_{\boldsymbol{k}\sigma}^{\alpha} \left[f(\omega_{\boldsymbol{k}\sigma}) + \frac{1}{2} \right]$$

$$l_{\boldsymbol{k}\sigma}^{\alpha} = \left(\varepsilon_{\boldsymbol{k}\sigma}^{\dagger} M_{\alpha} \varepsilon_{\boldsymbol{k}\sigma}\right)\hbar \qquad M^z = I_{M\times M} \otimes \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$f(\omega)$: Bose distribution function



Angular momentum of phonons induced by heat current

crystals with time-reversion symmetry $l_\sigma^\alpha(\boldsymbol{k}) = -l_\sigma^\alpha(-\boldsymbol{k})$

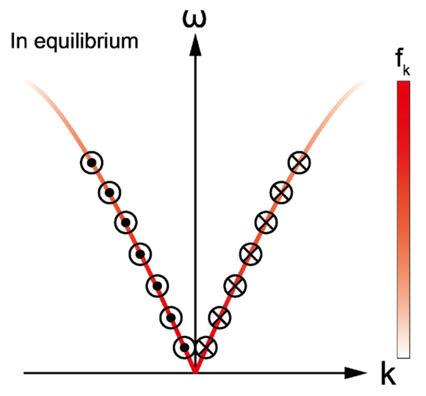
⟶ Total angular momentum is zero in equilibrium.
It becomes nonzero under nonzero temperature gradient

Equilibrium

$$J_{\mathrm{ph}}^\alpha = \sum_{\boldsymbol{k}\sigma} l_{\boldsymbol{k}\sigma}^\alpha \left[f(\omega_{\boldsymbol{k}\sigma}) + \frac{1}{2} \right]$$

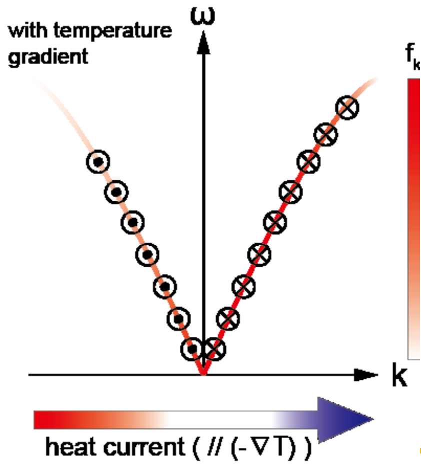
With temperature gradient

$$J_{ph}^\alpha = -\tau \sum_{k,\sigma} \left\{ l_\sigma^\alpha(\boldsymbol{k}) \left(\frac{\partial \omega_\sigma(\boldsymbol{k})}{\partial k_\beta} \right) \frac{\partial f(\omega_\sigma(\boldsymbol{k}))}{\partial T} \right\} \left(\frac{\partial T}{\partial x_\beta} \right)$$



Boltzmann approximation

$$f(\omega) \to f(\omega) - \tau v_\sigma^\beta(\boldsymbol{k}) \frac{\partial f}{\partial T} \frac{\partial T}{\partial x_\beta}$$



Phonon angular momentum is proportional to a temperature gradient

Phonon thermal Edelstein effect

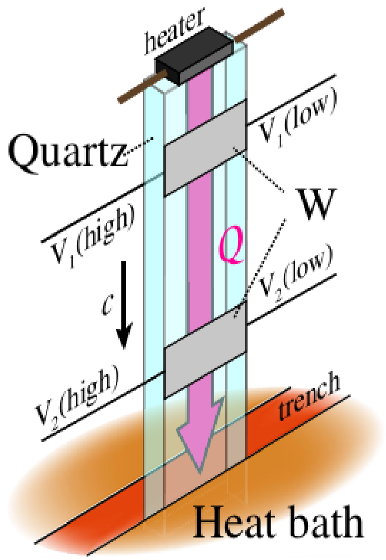
Hamada, Minamitani, Hirayama, Murakami, PRL 121, 175301 (2018)

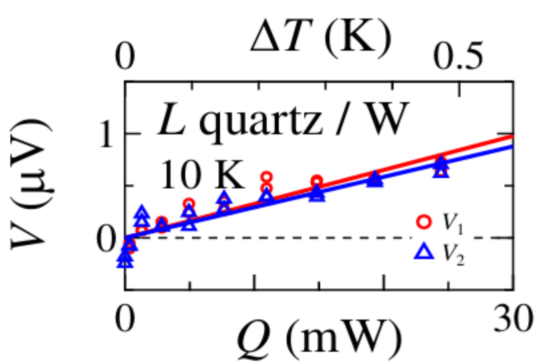
Chiral phonons in chiral crysltal: alpha-HgS

Ohe et al., Phys. Rev. Lett. 132, 056302 (2024)

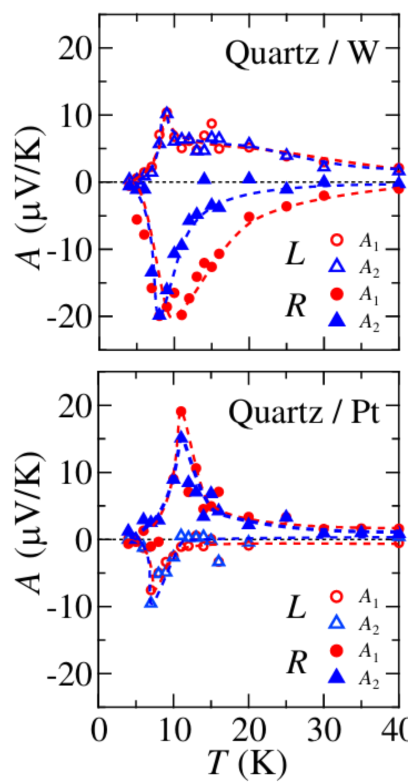
heat current in quartz → chiral phonons

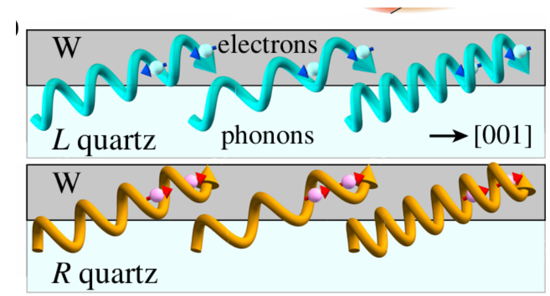
→ voltage in W electrode via SHE/OHE

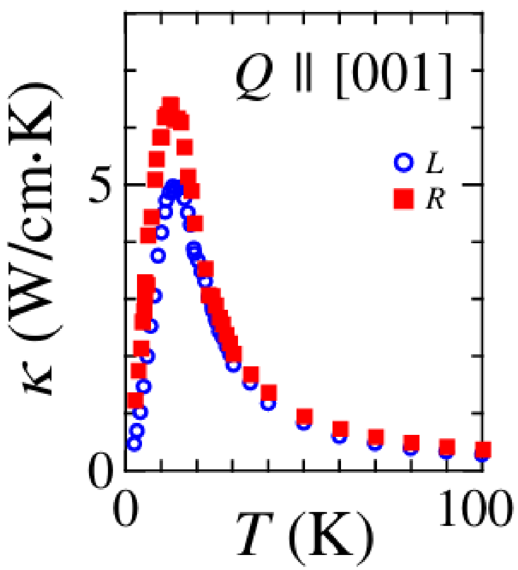




Opposite signals between W and Pt







Phonon angular momentum → Electron spin ?

(Note: they are both axial vectors)

Conversion from rotation to electron spins

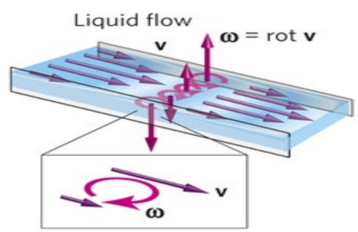
Mechanical generation of spins

Mechanical rotation→
converted into electron spins

c.f. previous works

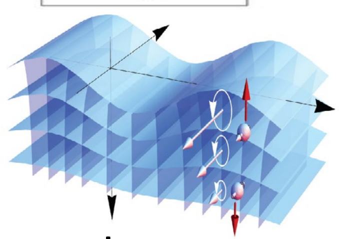
- Einstein-de Haas effect/Barnett effect
- Vortices in liquid metal

  Takahashi et al., Nature Physics **12**, 52 (2016).



- Surface Acoustic Wave

  Matsuo, Ieda, Harii, Saitoh, and Maekawa, Phys. Rev. B., 87, 180402 (2013)



- Twisting mode in carbon nanotubes

  Hamada, Yokoyama, Murakami, Phys. Rev. B 92. 060409(R) (2015)

Key : Spin-rotation coupling

(Matsuo et al. PRL 106, 076601 (2011))

$$H_{\mathrm{SRC}} = -\mathbf{S}\cdot\boldsymbol{\Omega}$$

Spin angular momentum

Angular velocity of mechanical rotation

← Dirac equation from rotating frame

Note: it is independent of the SOC of the material

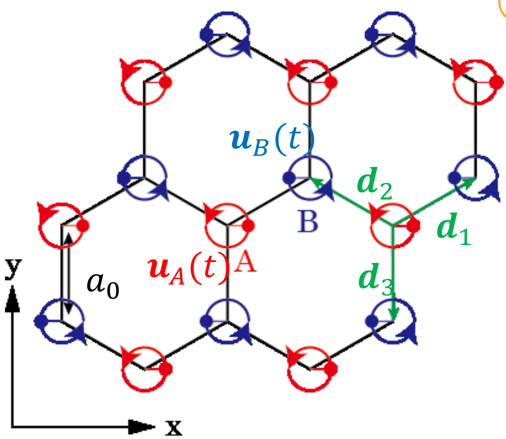
Questions

- What is the microscopic mechanism ?
- Is the SOC needed?

Phonon angular momentum → Electron spin ?

Honeycomb-lattice model with microscopic local rotation
→ dynamically modulate the electronic system

Hamiltonian $H(t) = H_0 + H_t(t) + H_R(t)$



$$H_0 = t\sum_{\langle ij\rangle} c_i^\dagger c_j + \lambda_v \sum_i \xi_i c_i^\dagger c_i + \frac{i\lambda_R}{a_0}\sum_{\langle ij\rangle} c_i^\dagger \left(\boldsymbol{s}\times\boldsymbol{d}_{ij}\right)_z c_j$$

Nearest neighbor hopping | Staggered potential | Rashba SOC

$$H_t(t) = \sum_{\langle ij\rangle} \delta t_{ij}(t) c_i^\dagger c_j \quad \left[\delta t_a(t) = -\frac{\delta t_0}{a_0}\boldsymbol{u}(t)\cdot\boldsymbol{d}_a\right]$$

Dynamical modulation of the hopping

$$H_R(t) = -\frac{i\lambda_R}{a_0^3}\sum_{\langle ij\rangle}\left(\boldsymbol{d}_{ij}\cdot\boldsymbol{u}(t)\right)c_i^\dagger\left(\boldsymbol{s}\times\boldsymbol{d}_{ij}\right)_z c_j + \frac{i\lambda_R}{a_0}\sum_{\langle ij\rangle} c_i^\dagger\left(\boldsymbol{s}\times\boldsymbol{u}(t)\right)_z$$

Dynamical modulation of the Rashba SOC

Phonon frequency: Ω

displacement vector

$$\boldsymbol{u}(t) = \boldsymbol{u}_B(t) - \boldsymbol{u}_A(t)$$

creation and annihilation operator

$$c_i^\dagger = \left(c_{\uparrow,i}^\dagger, c_{\downarrow,i}^\dagger\right), \quad c_i = \left(c_{\uparrow,i}, c_{\downarrow,i}\right)^T$$

Note: the snapshot of the Hamiltonian is non-magnetic

Hamada, Murakami, Phys. Rev. Research 2, 023275 (2020)

Phonon angular momentum → Electron spin ?

We assume that the motion of atoms for phonons is much slower than the motion of electrons.

→ **Adiabatic approximation**

Berry, Proc. R. Soc. Lond. A414, 31 (1987)

(cf.)
In the opposite limit of high frequency driving, we can use the Floquet theory & Magnus expansion.
In the present case of low-frequency driving, they cannot be used.

Adiabatic approximation

Berry, Proc. R. Soc. Lond. A 414, 31 (1987)

$t \to \tau = \Omega t$ (Phonon frequency: Ω)

We approximate the time-evolution operator $U(\tau)$ by iteration using smallness of Ω

$$i\hbar\Omega\partial_\tau\psi(\tau) = H(\tau)\psi(\tau), \quad \psi(\tau) = U(\tau)\psi(0)$$

$$\left[U^\dagger(\tau)H(\tau)U(\tau) - i\hbar\Omega U^\dagger(\tau)\partial_\tau U(\tau)\right]\psi(0) = 0$$

$U(\tau) = \mathrm{R}_0(\tau)R_1(\tau)R_2(\tau)$

$H(\tau)|\epsilon_n(\tau)\rangle = \epsilon_n(\tau)|\epsilon_n(\tau)\rangle$ $\quad R_0(\tau) = \sum_n |\epsilon_n(\tau)\rangle\langle\epsilon_n(0)|$

Instantaneous eigenstate

$$\cdots R_2^\dagger\left[R_1^\dagger\left(\underbrace{R_0^\dagger H(\tau)R_0}_{\widetilde{H}_0(\tau)} - i\hbar\underbrace{\Omega R_0^\dagger\partial_\tau R_0}_{\Omega\widetilde{V}_0(\tau)}\right)R_1 - i\hbar\,\Omega R_1\partial_\tau R_1 - i\hbar\Omega\partial_\tau\right]R_2\cdots$$

$H_1(\tau) = \widetilde{H}_0(\tau) + \Omega\widetilde{V}_0(\tau)$

$H_1(\tau)|\phi_n(\tau)\rangle = E_n(\tau)|\phi_n(\tau)\rangle$ $\quad R_1(\tau) = \sum_n |\phi_n(\tau)\rangle\langle\phi_n(0)|$

$$\cdots R_3^\dagger\left[R_2^\dagger\left(R_1^\dagger H_1(\tau)R_1 - i\hbar\,\Omega R_1\partial_\tau R_1\right)R_2 - i\hbar\Omega \mathrm{R}_2^\dagger\partial_\tau R_2 - i\hbar\Omega\partial_\tau\right]R_3\cdots$$

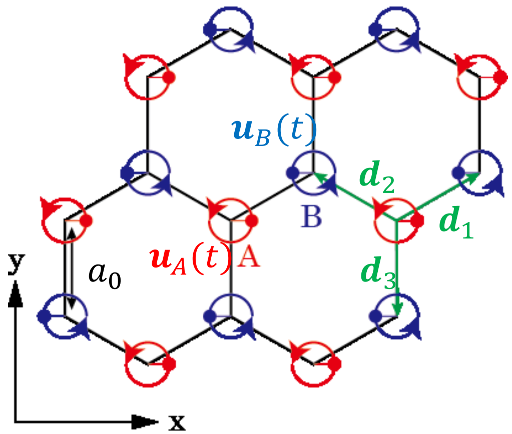
We drop the $O(\Omega^2)$ term

$$R_2(\tau) = R_{2,d}(\tau) = \exp\left(-\frac{i}{\hbar\Omega}\int_0^\tau ds\,\sum_n E_n(s)\,|\phi_n(0)\rangle\,\langle\phi_n(0)|\right)$$

$$\psi(\tau) = R_0(\tau)R_1(\tau)R_{2,d}(\tau)\psi(0)$$

Spin polarization induced by phonon angular momentum

Spin expectation value



$$\langle S_\alpha(\boldsymbol{k},\tau)\rangle_n \simeq \hat{S}_\alpha^{nn}(\boldsymbol{k},\tau) + \hbar\Omega \sum_{m\neq n} \frac{-i\langle \epsilon_m(\boldsymbol{k},\tau)|\partial_\tau H(\boldsymbol{k},\tau)|\epsilon_n(\boldsymbol{k},\tau)\rangle \hat{S}_\alpha^{nm}(\boldsymbol{k},\tau) + i(n \leftrightarrow m)}{\left(\epsilon_n(\boldsymbol{k},\tau) - \epsilon_m(\boldsymbol{k},\tau)\right)^2}$$

Instantaneous term

Geometrical term
(Berry phase)

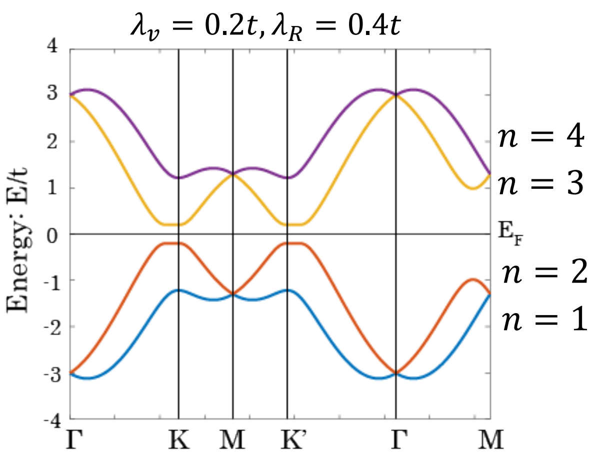
$\hat{S}_\alpha^{nn} = \langle \epsilon_n(\boldsymbol{k},\tau)|\hat{S}_\alpha|\epsilon_n(\boldsymbol{k},\tau)\rangle = 0$

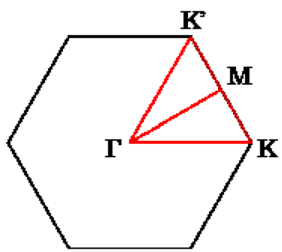
Hamada, Murakami, Phys. Rev. Research 2, 023275 (2020)

Spin polarization induced by phonon angular momentum

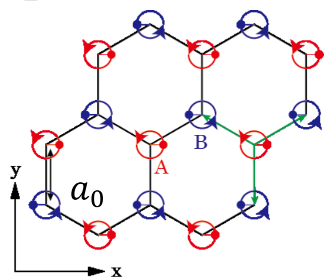
Hamada, Murakami, Phys. Rev. Research 2, 023275 (2020)

Electronic bands
(Without microscopic rotation)

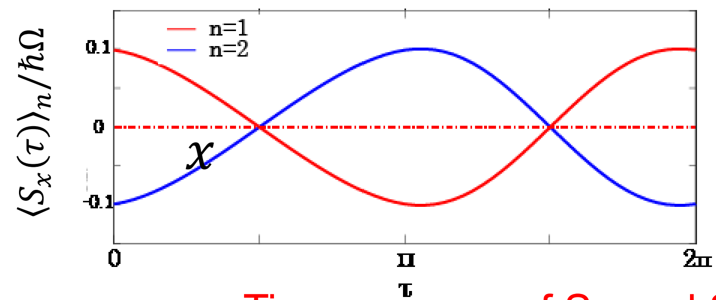




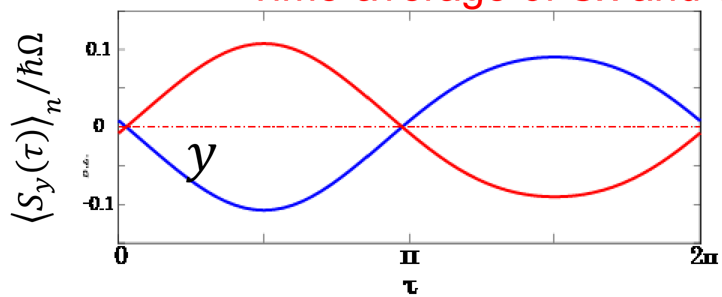
Microscopic rotation

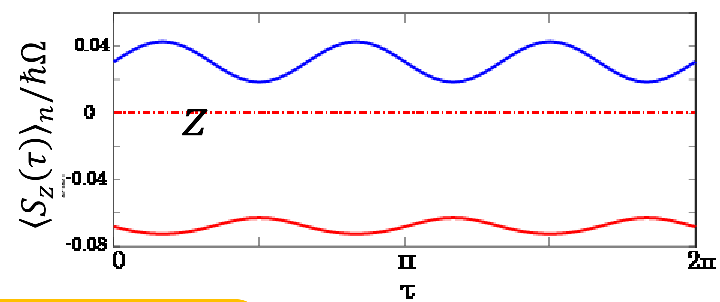


Spin expectation values



Time-average of Sx and Sy is zero





Time-average of Sz is nonzero

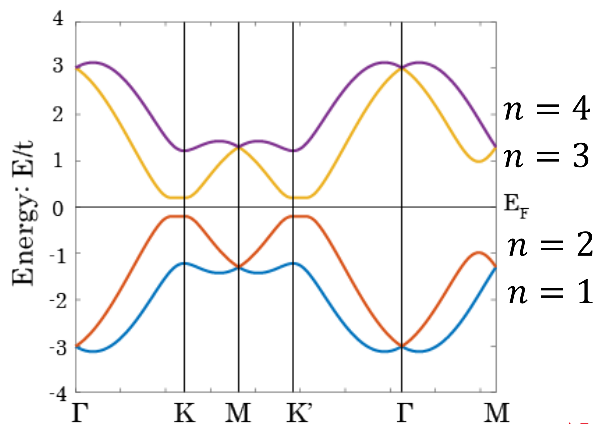
Spin polarization can be induced even in insulators
← interband excitation by dynamical motions of nuclei

Spin polarization induced by phonon angular momentum

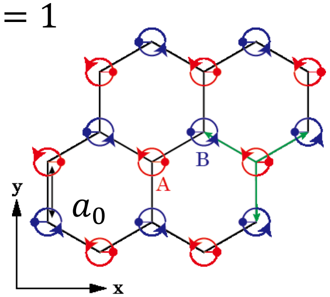
Hamada, Murakami, Phys. Rev. Research 2, 023275 (2020)

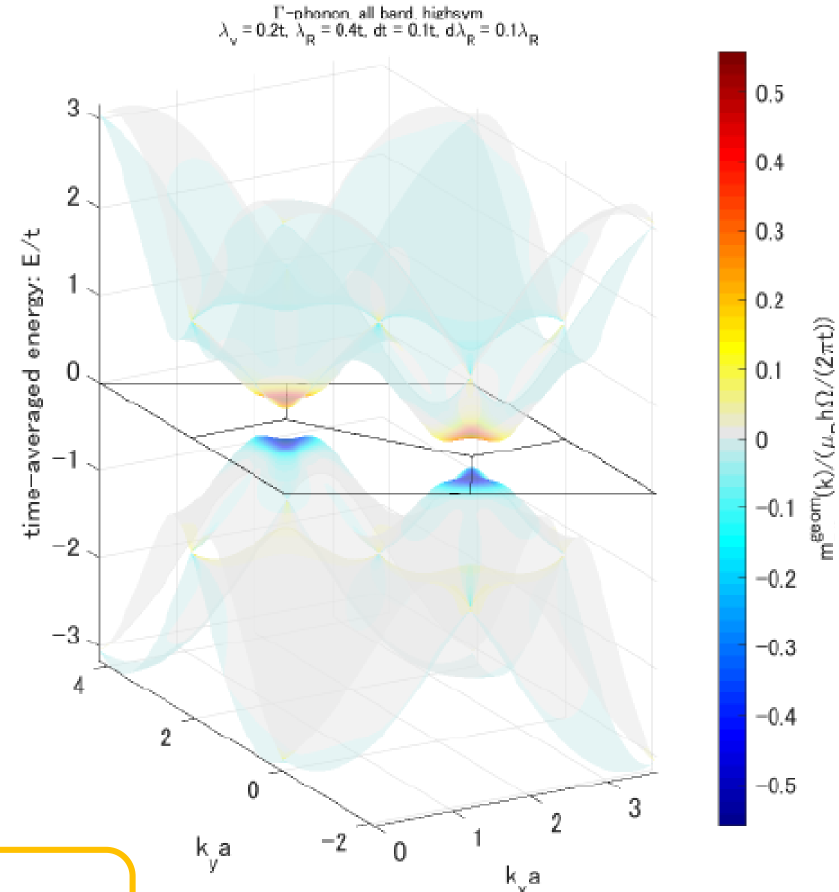
Electronic bands (without phonons)

$\lambda_v = 0.2t, \lambda_R = 0.4t$



Microscopic rotation





Spin polarization can be induced even in insulators
← interband excitation by dynamical motions of nuclei

Cf: chiral phonons → orbital angular momentum
Trifunovic, Ono, Watanabe, Phys. Rev. B **100**, 054408 (2019)

spin magnetization (time-averaged)

Yao, Murakami, Phys. Rev. B111,134414 (2025)

$$\langle \mu^{\alpha}_{s,n} \rangle = \frac{1}{2} J^{\mathrm{ph}}_z \partial_{B_\alpha} \int \frac{d\boldsymbol{k}}{(2\pi)^d} \Omega^{(n)}_{u_x u_y} \Big|_{\boldsymbol{u}=0}. \qquad \Omega^{(n)}_{u_x u_y} = \partial_{u_x} A^{(n)}_{u_y} - \partial_{u_y} A^{(n)}_{u_x}$$

Berry curvature in the phonon coordinates

Cf. orbital magnetization by phonons

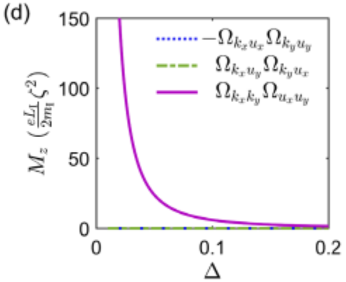
Ren et al., Phys. Rev. Lett.127,186403 (2021)

$$M_z = \frac{e}{2m_{\mathrm{I}}} L_{\mathrm{I}} \int \frac{d\boldsymbol{k}}{(2\pi)^2} \mathrm{Tr}\Omega_{k_\alpha k_\beta u_x u_y}$$

$$\Omega_{k_x k_y u_x u_y} = \Omega_{k_x u_y}\Omega_{k_y u_x} - \Omega_{k_x u_x}\Omega_{k_y u_y} + \Omega_{k_x k_y}\Omega_{u_x u_y}$$

Second Chern form

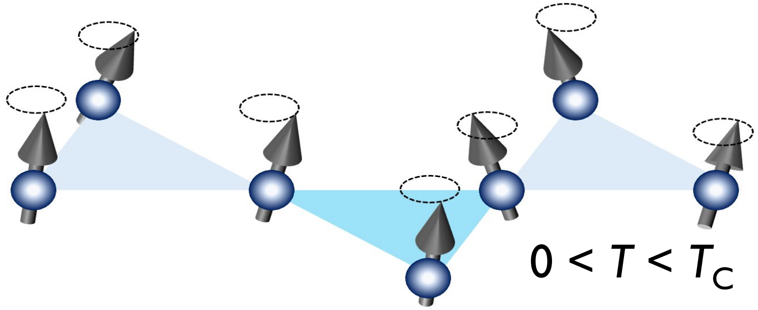
(d)



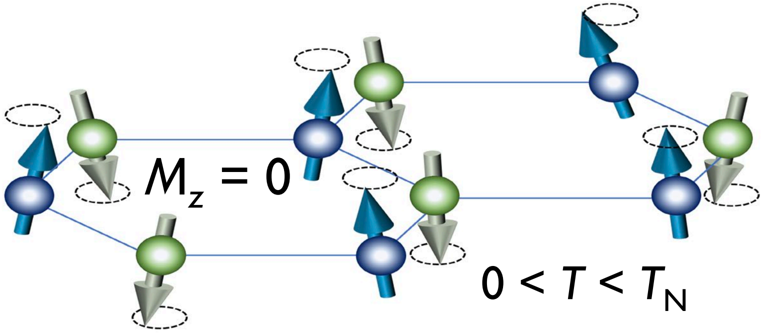
Gapped graphene model

Conversion of chiral phonons into magnons

Ferromagnets



Antiferromagnets



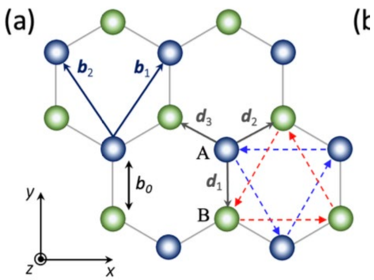
How magnons are affected by chiral phonons?

Yao, Murakami, J. Phys. Soc. Jpn. 93, 034708 (2024)

Model: antiferromagnet on a honeycomb lattice

$$H_0 = H_{\rm ex} + H_{\rm DM} + H_{\rm mag}$$

- ✓ XYZ model $H_{\rm ex} = \sum_{\langle i,j\rangle} \left(J_x S_i^x S_j^x + J_y S_i^y S_j^y + J_z S_i^z S_j^z\right)$
- ✓ DM interaction $H_{\rm DM}$
- ✓ Magnetic field $H_{\rm mag}$



Holstein-Primakoff trans.

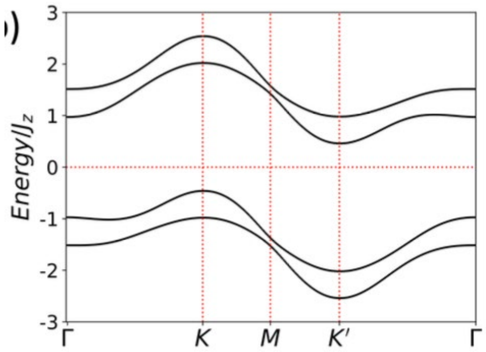
$$S^z_{A,i} = S - b^\dagger_{A,i} b_{A,i}, \qquad S^z_{B,i} = b^\dagger_{B,i} b_{B,i} - S,$$

$$S^+_{A,i} = \left(2S - b^\dagger_{A,i} b_{A,i}\right)^{1/2} b_{A,i} \approx \sqrt{2S} b_{A,i}, \qquad S^+_{B,i} = b^\dagger_{B,i}\left(2S - b^\dagger_{B,i} b_{B,i}\right)^{1/2} \approx \sqrt{2S} b^\dagger_{B,i},$$

$$S^-_{A,i} = b^\dagger_{A,i}\left(2S - b^\dagger_{A,i} b_{A,i}\right)^{1/2} \approx \sqrt{2S} b^\dagger_{A,i}, \qquad S^-_{B,i} = \left(2S - b^\dagger_{B,i} b_{B,i}\right)^{1/2} b_{B,i} \approx \sqrt{2S} b_{B,i},$$

- Bogoliubov-de Gennes Hamiltonian

$$\begin{aligned} H_0 = &\left(3J_z S - g\mu_B \tilde{H}\right) \sum_i \left(b^\dagger_{A,i} b_{A,i} + b^\dagger_{B,i} b_{B,i}\right) \\ &+ \frac{S}{2}(J_x - J_y) \sum_{\langle i,j\rangle} \left(b_{A,i} b^\dagger_{B,i} + b^\dagger_{A,i} b_{B,i}\right) \\ &+ \frac{S}{2}(J_x + J_y) \sum_{\langle i,j\rangle} \left(b_{A,i} b_{B,i} + b^\dagger_{A,i} b^\dagger_{B,i}\right) \\ &+ D_A S \sum_{\langle\langle i,j\rangle\rangle} i\nu_A^{ij} \left(b_{A,i} b^\dagger_{A,j} - b^\dagger_{A,i} b_{A,j}\right) \\ &- D_B S \sum_{\langle\langle i,j\rangle\rangle} i\nu_B^{ij} \left(b_{B,i} b^\dagger_{B,j} - b^\dagger_{B,i} b_{B,j}\right), \end{aligned}$$



Chiral phonons → dynamically modulates spin-spin interaction

Magnon excitation by chiral phonos

- Change in the magnon number : geometric term

$$N_n^{\mathrm{geom}}(\tau) = \frac{\hbar\omega}{S} \sum_{m(\neq n)} \sum_{\boldsymbol{k}} \left\{ \frac{\hat{N}_{nm}(\boldsymbol{k},\tau) A_{mn}(\boldsymbol{k},\tau)}{E_{n,\boldsymbol{k}}(\tau) - E_{m,\boldsymbol{k}}(\tau)} \right\} f_{n,\boldsymbol{k}}$$

Bose dist. fun.

In isotropic model, chiral phonons will not change the magnon number

⇓

We set J_x, J_y, and J_z to be different

spin rotation symmetry

$$H = \sum_{\langle ij \rangle} -J\vec{S}_i \cdot \vec{S}_j + \vec{D}_{ij} \cdot (\vec{S}_i \times \vec{S}_j) - g\mu_{\mathrm{B}}\vec{H} \cdot \sum_i \vec{S}_i$$

$$[S_i^z, H] = 0 \Rightarrow [N_i, H] = 0 \Rightarrow N_n^{\mathrm{geom}} = 0$$

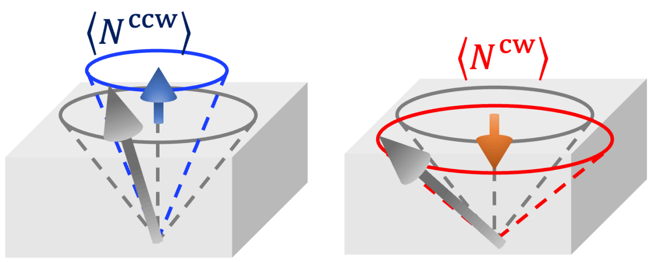
- CW phonons vs CCW phonos

・Bloch Hamiltonian $\mathcal{H}^{\mathrm{CW}}(\boldsymbol{k},\tau) = \mathcal{H}^{\mathrm{CCW}}(\boldsymbol{k},-\tau) \Rightarrow$

$$\hat{N}_{nm}^{\mathrm{CW}}(\boldsymbol{k},\tau) = \hat{N}_{nm}^{\mathrm{CCW}}(\boldsymbol{k},-\tau)$$

$$A_{mn}^{\mathrm{CW}}(\boldsymbol{k},\tau) = -A_{mn}^{\mathrm{CCW}}(\boldsymbol{k},-\tau)$$

・Number of magnon $N_n^{\mathrm{CW}}(\tau) = -N_n^{\mathrm{CCW}}(-\tau)$



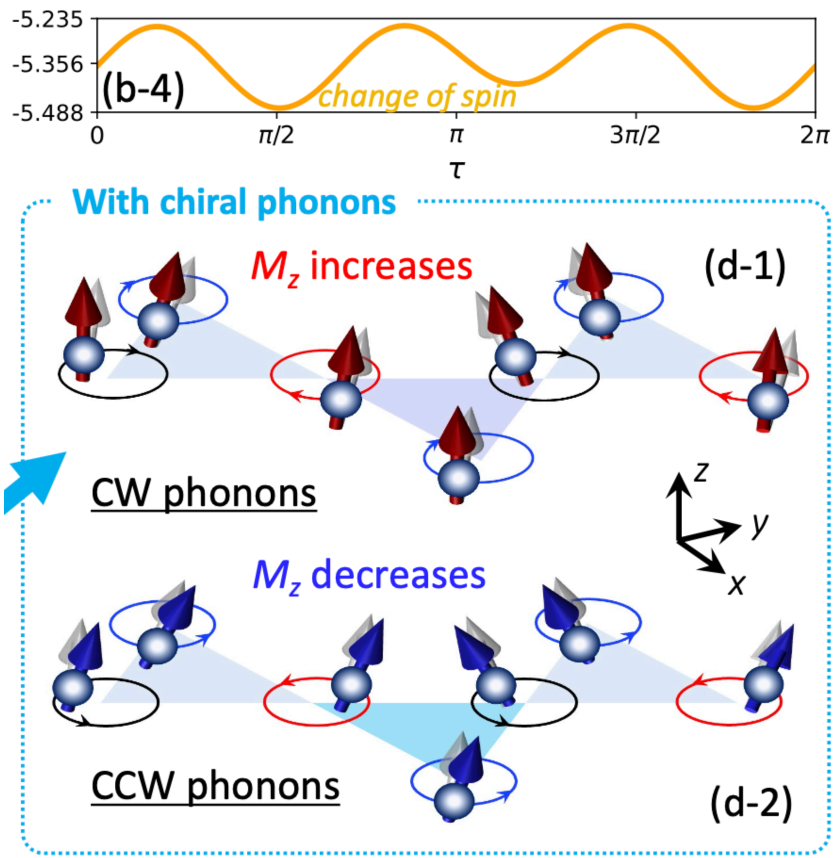
$\langle N^{\mathrm{cw}} \rangle = -\langle N^{\mathrm{ccw}} \rangle$

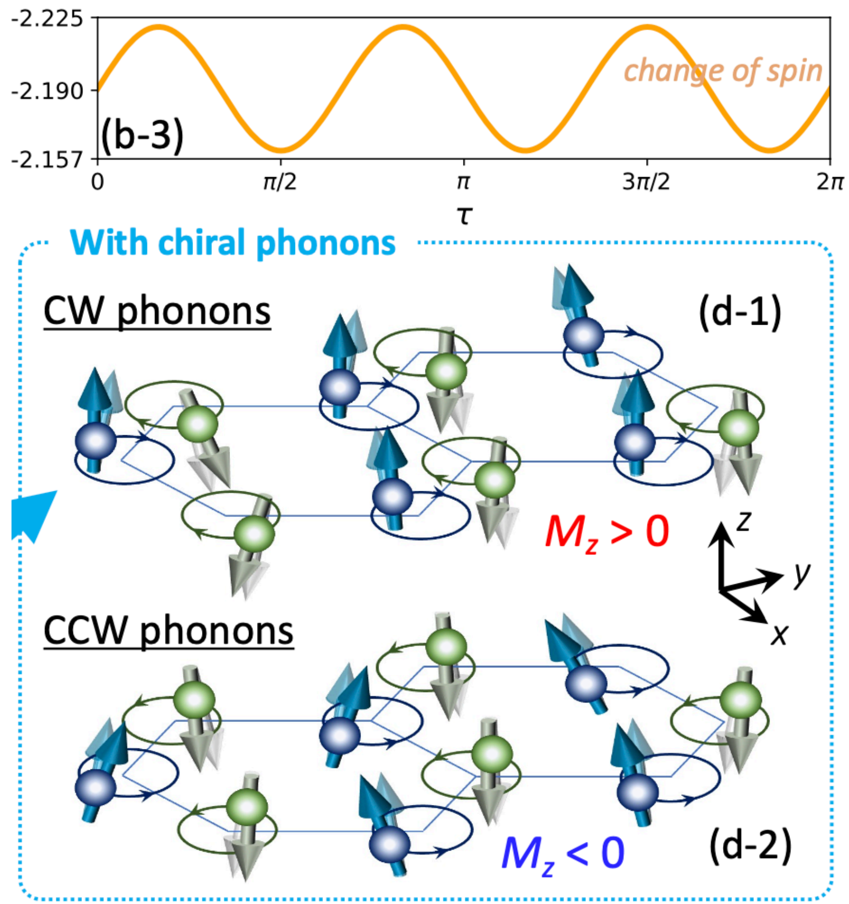
Results

Yao, Murakami, J. Phys. Soc. Jpn. 93, 034708 (2024)
see also Kei Yamamoto JPSJ News Comments 21, 07 (2024).

Ferromagnets

Antiferromagnets

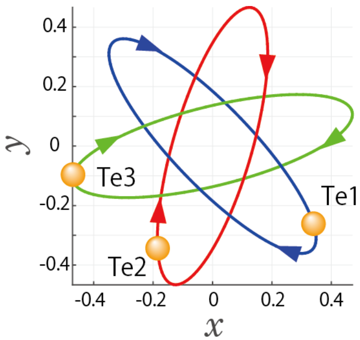




Chiral phonons induce nonzero magnetization.
The sign of the magnetization
depends on the direction of the chiral phonons.

Outline

Phonons have angular momenta
→ They can be converted into electron spins



1) Phonon angular momentum in chiral systems

Zhang, Murakami, PRR 4, L012024 (2022)
Ishito, Mao, Kousaka, Togawa, Iwasaki, Zhang, Murakami, Kishine, Satoh, Nat. Phys.19, 35 (2022)

2) Conversion of phonon angular momentum into spins

Hamada, Murakami, Phys. Rev. Research 2, 023275 (2020)
Yao, Murakami, Phys. Rev. B 105, 184412 (2022)
Yao, Murakami, J. Phys. Soc. Jpn. 93, 034708 (2024)
Yao, Murakami, Phys. Rev. B111,134414 (2025)

3) Coupling between the surface acounstic waves and magnetostatic waves

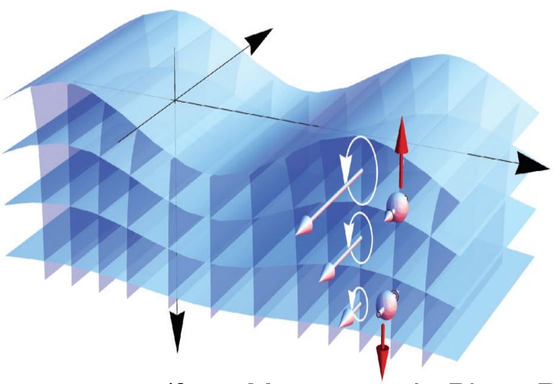
Kono, Yamamoto, Murakami, in preparation

Chiral phonons in larger length scale ?

↓

Coupling of surface acoustic waves and magnetostatic waves through magnetic dipole interaction

(work in progress…)



(from Matsuo et al., Phys. Rev. B 87, 180402 (2013)

- R. Kono, K. Yamamoto, S. Murakami, in preparation

Coupling between SAW and magnons via dipolar coupling

(R. Kono, K. Yamamoto, S. Murakami, in preparation)

Quantum mechanics (nm scale)

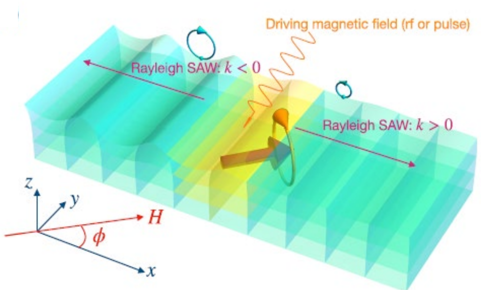
Chiral Phonons ⟷ Magnons

↑ Spin-orbit interaction

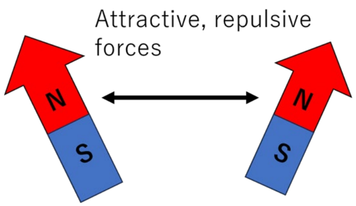
Classical (μm scale)

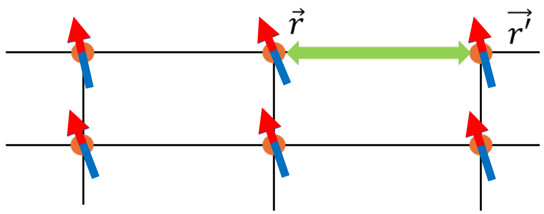
Surface acoustic waves ⟷ Magnetostatic waves

↑ Magnetic dipole interaction



magnetostatic waves (classical) ← dipolar interaction

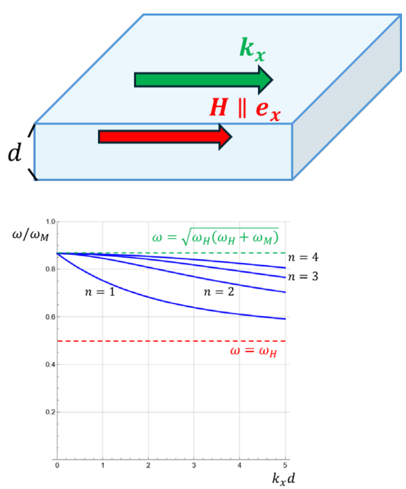




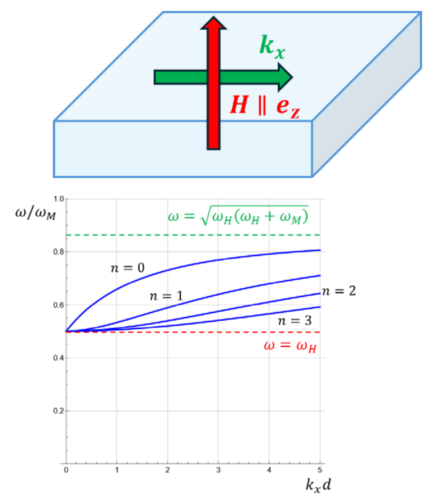
Dipolar interaction
- It plays the role of SOC
- Dominant in the μm scale

$$F_{\text{dipole}} = \frac{\mu_0}{8\pi}\int dV_{\boldsymbol{r}}\int dV_{\boldsymbol{r}'}\left[\frac{\boldsymbol{M}(\boldsymbol{r})\cdot\boldsymbol{M}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^3} - 3\frac{\{\boldsymbol{M}(\boldsymbol{r})\cdot(\boldsymbol{r}-\boldsymbol{r}')\}\{\boldsymbol{M}(\boldsymbol{r}')\cdot(\boldsymbol{r}-\boldsymbol{r}')\}}{|\boldsymbol{r}-\boldsymbol{r}'|^5}\right]$$

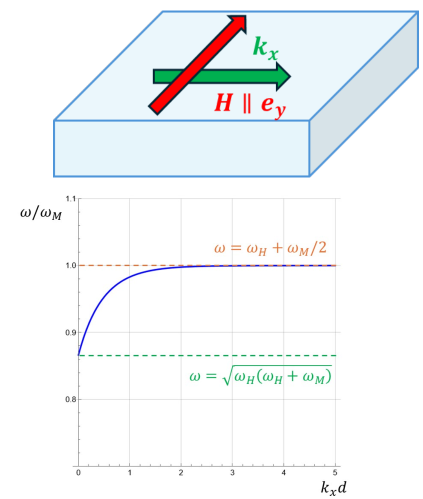
Various types of magnetostatic waves in a slab ferromagnet



Magnetostatic Backward Volume Wave



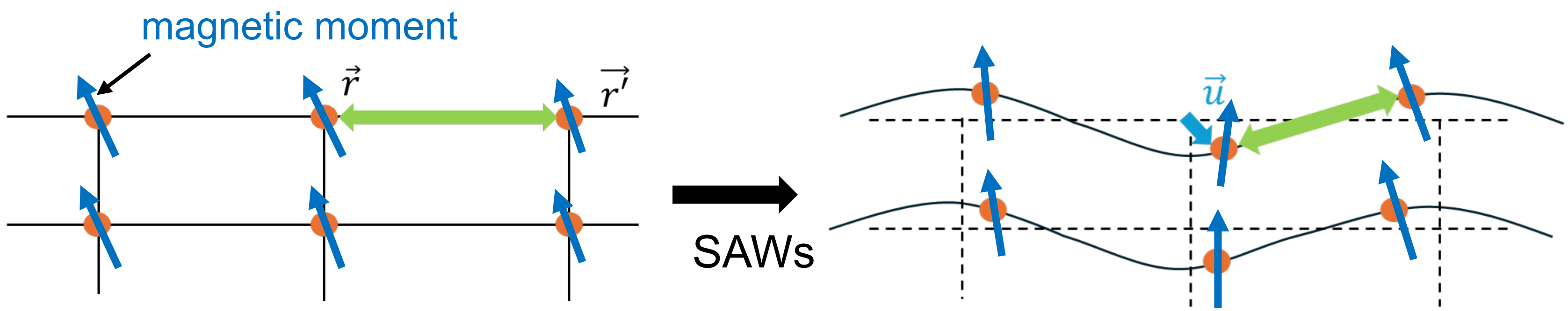
Magnetostatic Forward Volume Wave



Magnetostatic Surface Wave

This study :

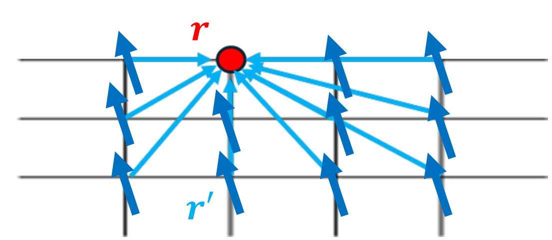
We derive the coupled equations of motion for SAWs and MSWs in a slab



Magnetic dipole interaction = Long range interaction

$$F_{\mathrm{dipole}} = \frac{\mu_0}{8\pi} \int d\boldsymbol{r} \int d\boldsymbol{r}' \left[\frac{\boldsymbol{M}(\boldsymbol{r}) \cdot \boldsymbol{M}(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|^3} - 3 \frac{\{\boldsymbol{M}(\boldsymbol{r}) \cdot (\boldsymbol{r} - \boldsymbol{r}')\}\{\boldsymbol{M}(\boldsymbol{r}') \cdot (\boldsymbol{r} - \boldsymbol{r}')\}}{|\boldsymbol{r} - \boldsymbol{r}'|^5} \right]$$

Technical difficulties : Integration in $\boldsymbol{r}'$
Singularity at $\boldsymbol{r} = \boldsymbol{r}'$



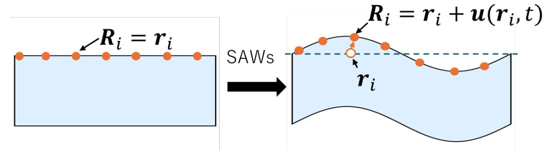
Method

Derivation of the equations of motion

Free energy :

$$F_{\text{dipole}} = \frac{\mu_0}{8\pi}\iint d\boldsymbol{R}d\boldsymbol{R}'\left[\frac{\boldsymbol{M}(\boldsymbol{R})\cdot\boldsymbol{M}(\boldsymbol{R}')}{|\boldsymbol{R}-\boldsymbol{R}'|^3} - 3\frac{\{\boldsymbol{M}(\boldsymbol{R})\cdot(\boldsymbol{R}-\boldsymbol{R}')\}\{\boldsymbol{M}(\boldsymbol{R}')\cdot(\boldsymbol{R}-\boldsymbol{R}')\}}{|\boldsymbol{R}-\boldsymbol{R}'|^5}\right]$$

variable transformation



Kei Yamamoto, and Sadamichi Maekawa, Ann. Phys. 536, 5, 2300395 (2023).

$$F_{\text{dipole}} = \frac{\mu_0}{8\pi}\iint d\boldsymbol{r}d\boldsymbol{r}'\left[\frac{\boldsymbol{M}_0(\boldsymbol{r})\cdot\boldsymbol{M}_0(\boldsymbol{r}')}{|\boldsymbol{R}(\boldsymbol{r})-\boldsymbol{R}(\boldsymbol{r}')|^3} - 3\frac{\{\boldsymbol{M}_0(\boldsymbol{r})\cdot(\boldsymbol{R}(\boldsymbol{r})-\boldsymbol{R}(\boldsymbol{r}'))\}\{\boldsymbol{M}_0(\boldsymbol{r}')\cdot(\boldsymbol{R}(\boldsymbol{r})-\boldsymbol{R}(\boldsymbol{r}'))\}}{|\boldsymbol{R}(\boldsymbol{r})-\boldsymbol{R}(\boldsymbol{r}')|^5}\right]$$

Equations of motion

$$\rho\ddot{\boldsymbol{u}}(\boldsymbol{r}) = \underbrace{\mu\nabla^2\boldsymbol{u}(\boldsymbol{r}) + (\lambda+\mu)\nabla\cdot(\nabla\cdot\boldsymbol{u}(\boldsymbol{r}))}_{\text{general elastic body}} \underbrace{- \frac{\mu_0}{4\pi}\boldsymbol{M}_0(\boldsymbol{r})\cdot\int d\boldsymbol{r}'\frac{\delta \boldsymbol{T}(\boldsymbol{r},\boldsymbol{r}')}{\delta\boldsymbol{u}(\boldsymbol{r})}}_{\boldsymbol{F}_{\text{dipole}}}$$

$$\dot{\boldsymbol{M}}_0(\boldsymbol{r}) = \gamma\mu_0\boldsymbol{M}_0(\boldsymbol{r})\times\left(\boldsymbol{H}\underbrace{-\frac{1}{4\pi}\int d\boldsymbol{r}'\boldsymbol{T}(\boldsymbol{r},\boldsymbol{r}')}_{\boldsymbol{h}_{\text{dipole}}}\right)$$

ρ: mass density
$\mu,\ \lambda$: Lamé's constants

(the energy conservation law is satisfied)

$$\left(\boldsymbol{T}(\boldsymbol{r},\boldsymbol{r}') = \frac{\boldsymbol{M}_0(\boldsymbol{r}')}{|\boldsymbol{R}(\boldsymbol{r})-\boldsymbol{R}(\boldsymbol{r}')|^3} - 3\frac{(\boldsymbol{R}(\boldsymbol{r})-\boldsymbol{R}(\boldsymbol{r}'))\{\boldsymbol{M}_0(\boldsymbol{r}')\cdot(\boldsymbol{R}(\boldsymbol{r})-\boldsymbol{R}(\boldsymbol{r}'))\}}{|\boldsymbol{R}(\boldsymbol{r})-\boldsymbol{R}(\boldsymbol{r}')|^5}\right)$$

equations of motion

$$\rho \ddot{\boldsymbol{u}}(\boldsymbol{r}) = \mu \nabla^2 \boldsymbol{u}(\boldsymbol{r}) + (\lambda + \mu)\nabla \cdot (\nabla \cdot \boldsymbol{u}(\boldsymbol{r})) + \boldsymbol{F}_{\rm dipole}$$
$$\dot{\boldsymbol{M}}_0(\boldsymbol{r}) = \gamma \mu_0 \boldsymbol{M}_0(\boldsymbol{r}) \times (\boldsymbol{H} + \boldsymbol{h}_{\rm dipole})$$

$$\begin{pmatrix} F_\xi^{\rm dipole} \\ F_\zeta^{\rm dipole} \end{pmatrix} = -\mu_0 M_s^2 \int d\xi' \left\{ W^n \begin{pmatrix} n_\xi(\xi') \\ n_\zeta(\xi') \end{pmatrix} + W^u \begin{pmatrix} u_\xi(\xi') \\ u_\zeta(\xi') \end{pmatrix} \right\}$$

$$\boldsymbol{h}_{\rm dipole} = -M_s \cos\theta \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + M_s \int d\xi' \left\{ G^n \begin{pmatrix} n_\xi(\xi') \\ n_\zeta(\xi') \end{pmatrix} + G^u \begin{pmatrix} u_\xi(\xi') \\ u_\zeta(\xi') \end{pmatrix} \right\}$$

$$W^n_{\xi\xi} = \cos\theta \left(kG_Q(\xi,\xi') + \delta(\xi-\xi')\frac{\partial}{\partial \xi'} \right) + ik\sin\theta\cos\varphi\,(G_P(\xi,\xi') - \delta(\xi-\xi'))$$
$$W^n_{\xi\zeta} = ik\cos\theta\,(G_P(\xi,\xi') - \delta(\xi-\xi')) - k\sin\theta\cos\varphi G_Q(\xi,\xi')$$
$$W^n_{\zeta\xi} = ik\cos\theta\,(G_P(\xi,\xi') - \delta(\xi-\xi')) - k\sin\theta\cos\varphi G_Q(\xi,\xi')$$
$$W^n_{\zeta\zeta} = -k\cos\theta G_Q(\xi,\xi') - ik\sin\theta\cos\varphi G_P(\xi,\xi')$$

magnon→phonon

$$G^n_{\xi\xi} = G_P(\xi,\xi') - \delta(\xi-\xi')$$
$$G^n_{\xi\zeta} = iG_Q(\xi,\xi')$$
$$G^n_{\zeta\xi} = iG_Q(\xi,\xi')$$
$$G^n_{\zeta\zeta} = -G_P(\xi,\xi')$$

magnon→magnon

$$W^u_{\xi\xi} = k^2(\cos^2\theta - \sin^2\theta\cos^2\varphi)G_P + 2ik\sin\theta\cos\theta\cos\varphi G_Q$$

phonon→phonon

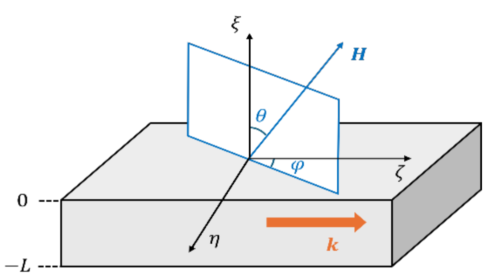
Details will be presented elsewhere

$$W^u_{\zeta\xi} = ik^2(\cos^2\theta - \sin^2\theta\cos^2\varphi)G_Q - 2k^2\sin\theta\cos\theta\cos\varphi G_P$$
$$+ 2k^2\sin\theta\cos\theta\cos\varphi\delta(\xi-\xi') - ik(\sin^2\theta\sin^2\varphi - \cos^2\theta)\delta(\xi-\xi')\frac{\partial}{\partial\xi'}$$

$$G^u_{\xi\xi} = \cos\theta \left(kG_Q(\xi,\xi') + \delta(\xi-\xi')\frac{\partial}{\partial \xi'} \right) + ik\sin\theta\cos\varphi\,(G_P(\xi,\xi') - \delta(\xi-\xi'))$$
$$G^u_{\xi\zeta} = ik\cos\theta\,(G_P(\xi,\xi') - \delta(\xi-\xi')) - k\sin\theta\cos\varphi G_Q(\xi,\xi')$$
$$G^u_{\zeta\xi} = ik\cos\theta\,(G_P(\xi,\xi') - \delta(\xi-\xi')) - k\sin\theta\cos\varphi G_Q(\xi,\xi')$$
$$G^u_{\zeta\zeta} = -k\cos\theta G_Q(\xi,\xi') - ik\sin\theta\cos\varphi G_P(\xi,\xi')$$

$$W^u_{\xi\zeta} = ik^2(\cos^2\theta - \sin^2\theta\cos^2\varphi)G_Q - 2k^2\sin\theta\cos\theta\cos\varphi G_P$$
$$+ 2k^2\sin\theta\cos\theta\cos\varphi\delta(\xi-\xi') - ik(\sin^2\theta\sin^2\varphi - \cos^2\theta)\delta(\xi-\xi')\frac{\partial}{\partial\xi'}$$
$$W^u_{\zeta\zeta} = -k^2(\cos^2\theta - \sin^2\theta\cos^2\varphi)G_P - 2ik^2\sin\theta\cos\theta\cos\varphi G_Q + k^2(\cos^2\theta - \sin^2\theta\sin^2\varphi)\delta(\xi-\xi')$$

phonon→magnon



- $(\theta,\ \varphi) = (\pi/2,\ \pi/2),\ (\pi/2,\ 3\pi/2)$ (Surface wave)

  → phonon→magnon, magnon→phonon $= 0$, phonon→phonon $\neq 0$

  ⟹ Only surface acoustic waves are modulated

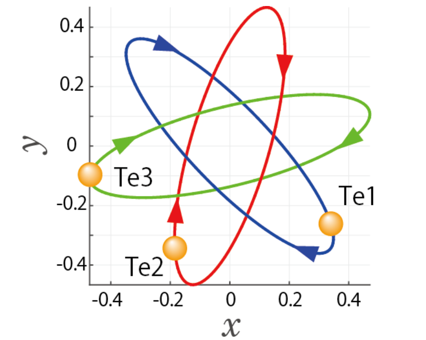
- $(\theta,\ \varphi) = (0,\ \varphi),\ (\pi/2,\ 0),\ (\pi/2,\ \pi)$ (Volume waves)

  ⟹ Phonons and magnons are coupled

Future work: Numerically calculate the dispersions of coupled waves

Conclusions

Phonons have angular momenta
→ They can be converted into electron spins



1) Phonon angular momentum in chiral systems

Zhang, Murakami, PRR 4, L012024 (2022)
Ishito, Mao, Kousaka, Togawa, Iwasaki, Zhang, Murakami, Kishine, Satoh, Nat. Phys.19, 35 (2022)
Y. Suzuki and S. Murakami arXiv:2501.07871

2) Conversion of phonon angular momentum into spins

Hamada, Murakami, Phys. Rev. Research 2, 023275 (2020)
Yao, Murakami, Phys. Rev. B 105, 184412 (2022)
Yao, Murakami, J. Phys. Soc. Jpn. 93, 034708 (2024)
Yao, Murakami, Phys. Rev. B111,134414 (2025)

3) Coupling between the surface acounstic waves and magnetostatic waves

Kono, Yamamoto, Murakami, in preparation

Chimera
Quasiparticle