Symmetry in the spin-orbit free limit:
spin groups & representation theory

Hana Schiff

Thursday, May 8th, 2025

SPIN PHENOMENA
INTERDISCIPLINARY CENTER

Quantum Geometry and Transport of Collective Excitations in (Non-)Magnetic Insulators

Overview

- Intro to spin groups
- Co-representation theory of spin point groups
- Do we need spin group theory?
- Landau theories for collinear altermagnets

Collaborators

My advisor!

Judit Romhányi
UC Irvine

Paul McClarty
Université Paris-Saclay

Jeff Rau
University of Windsor

University of Windsor

Intro to spin groups

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An operation $\hat{O}$ that transforms the state $|\psi\rangle$ such that

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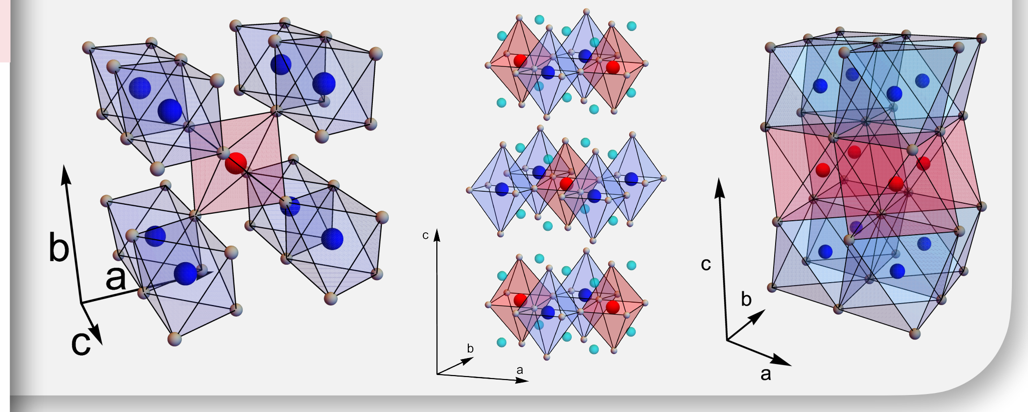
Crystal structure



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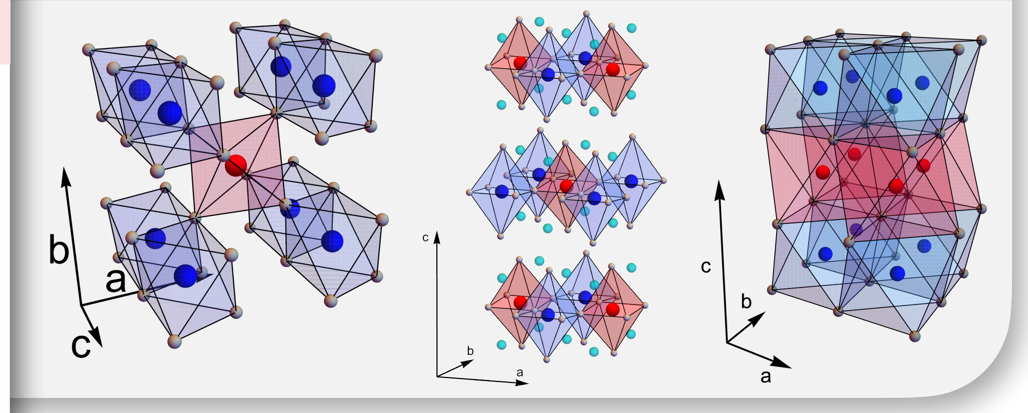
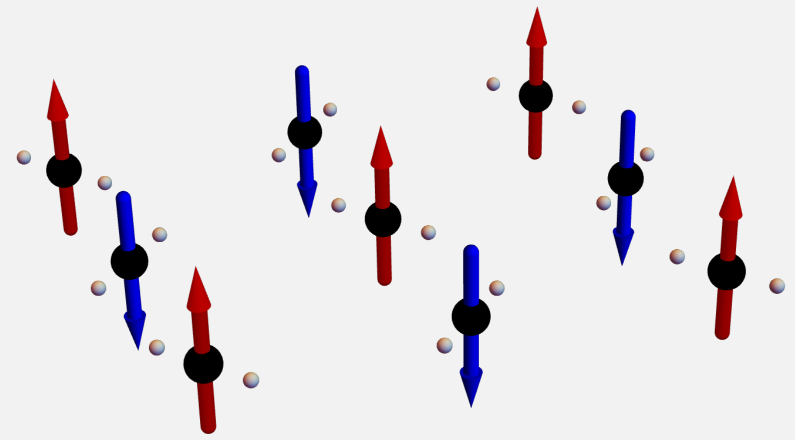


Magnetic order



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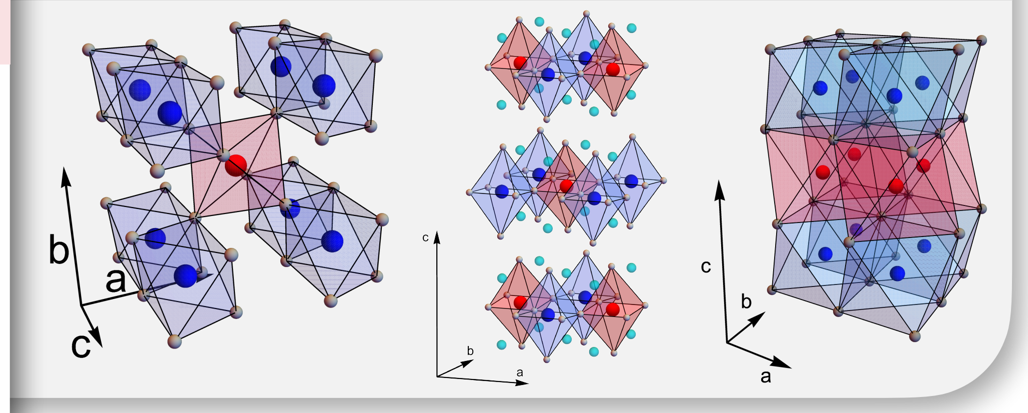

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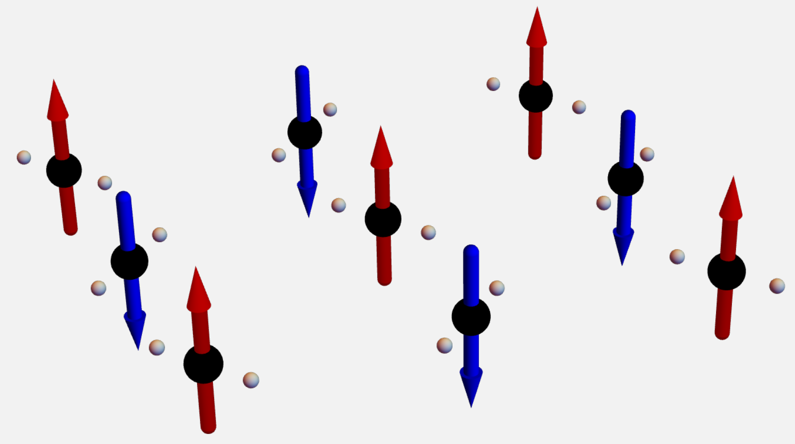
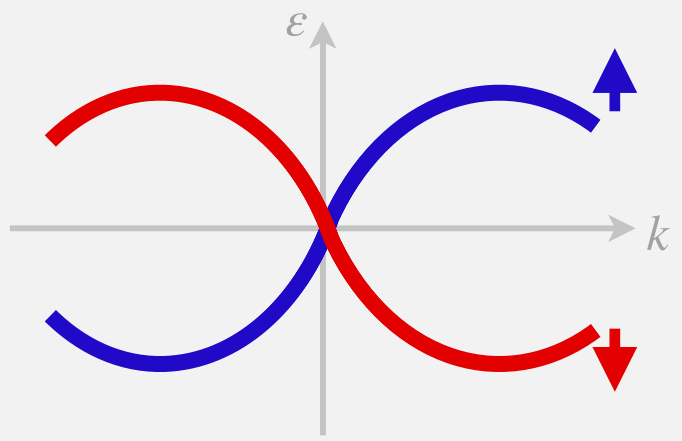


Band structure



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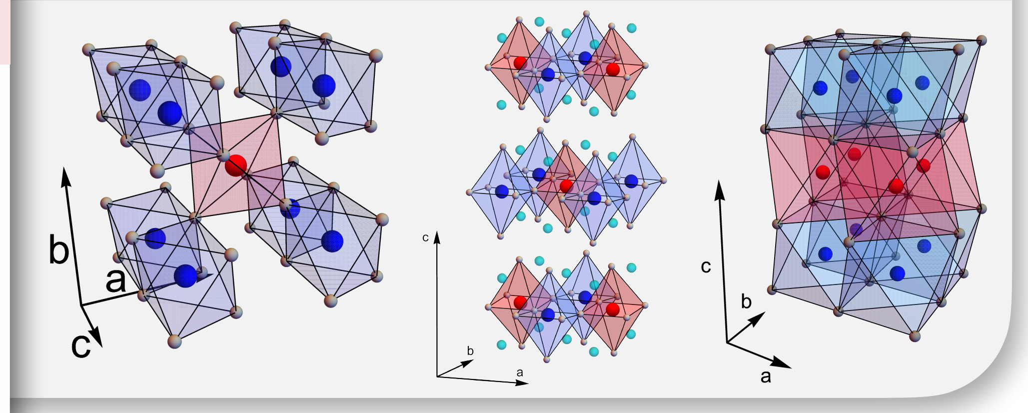

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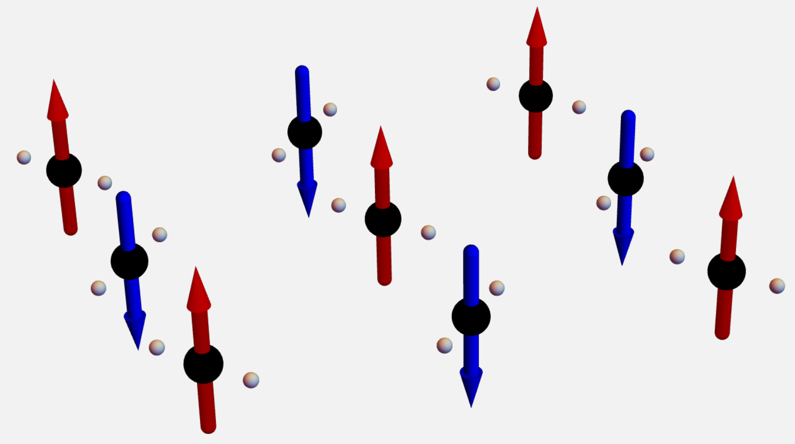

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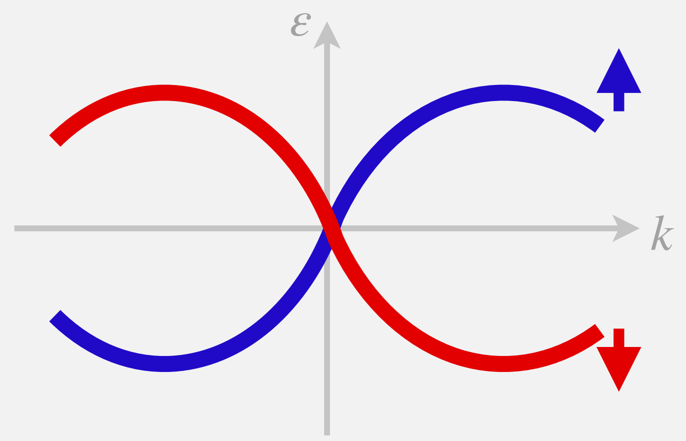

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An operation $\hat{O}$ that transforms the Hamiltonian $\mathscr{H}$ such that

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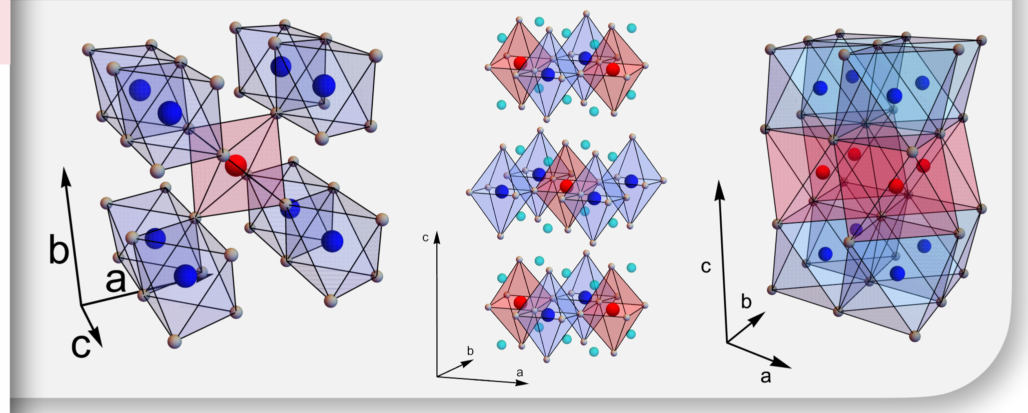

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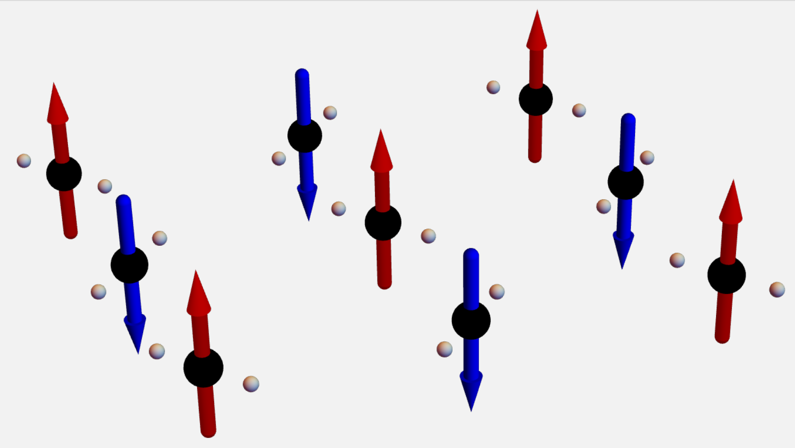

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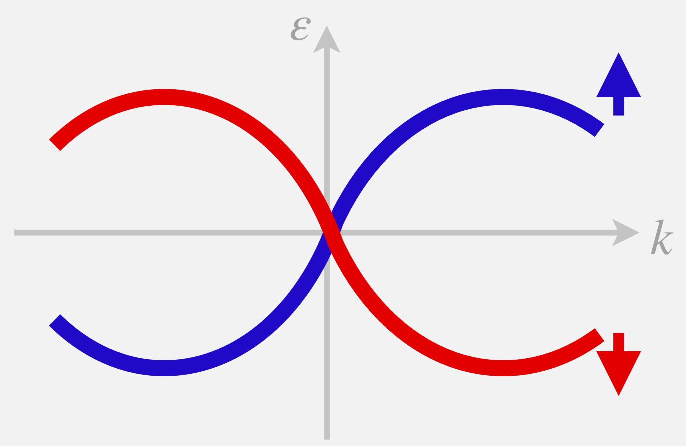

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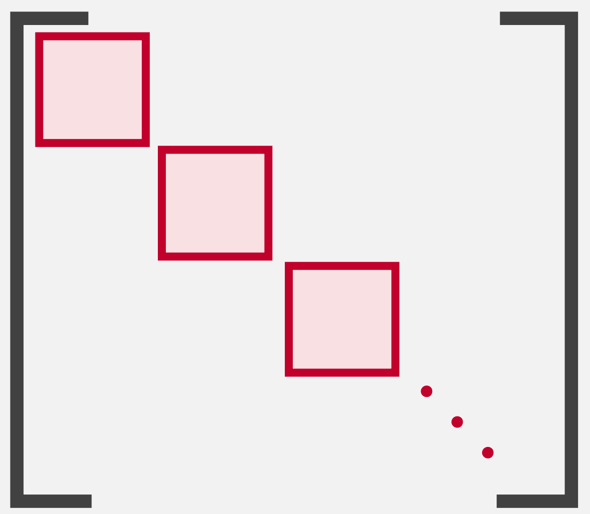
Shared eigenspaces



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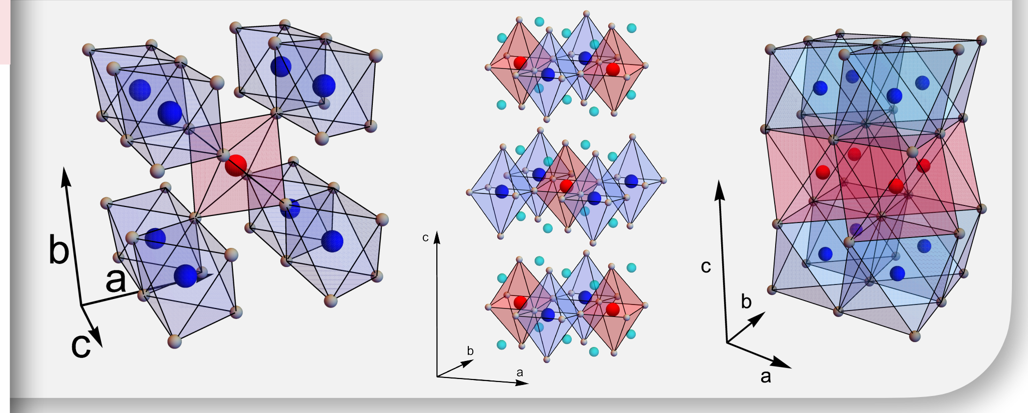

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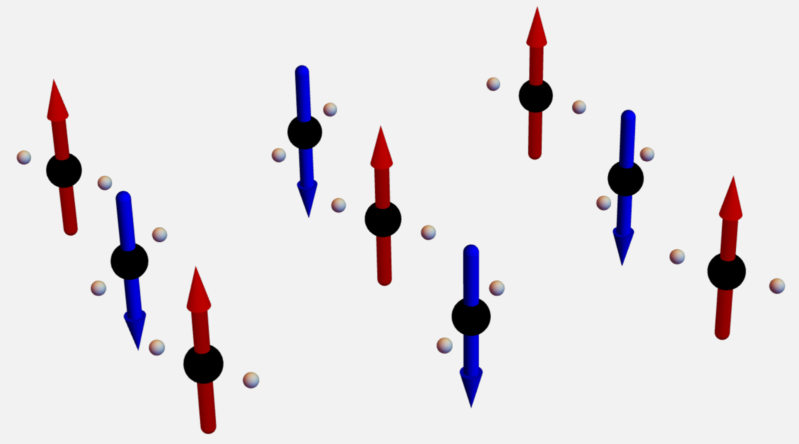

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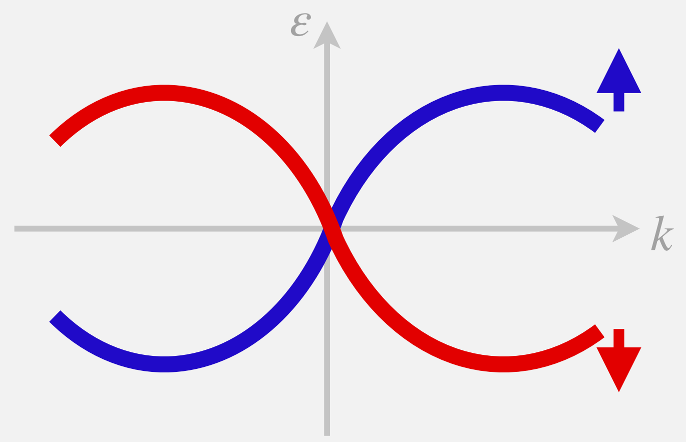

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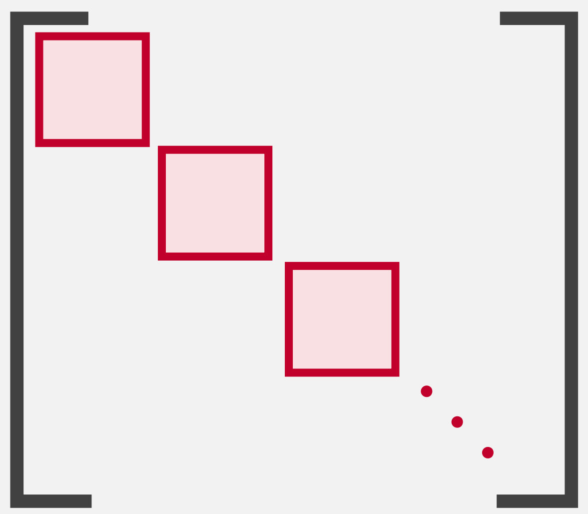
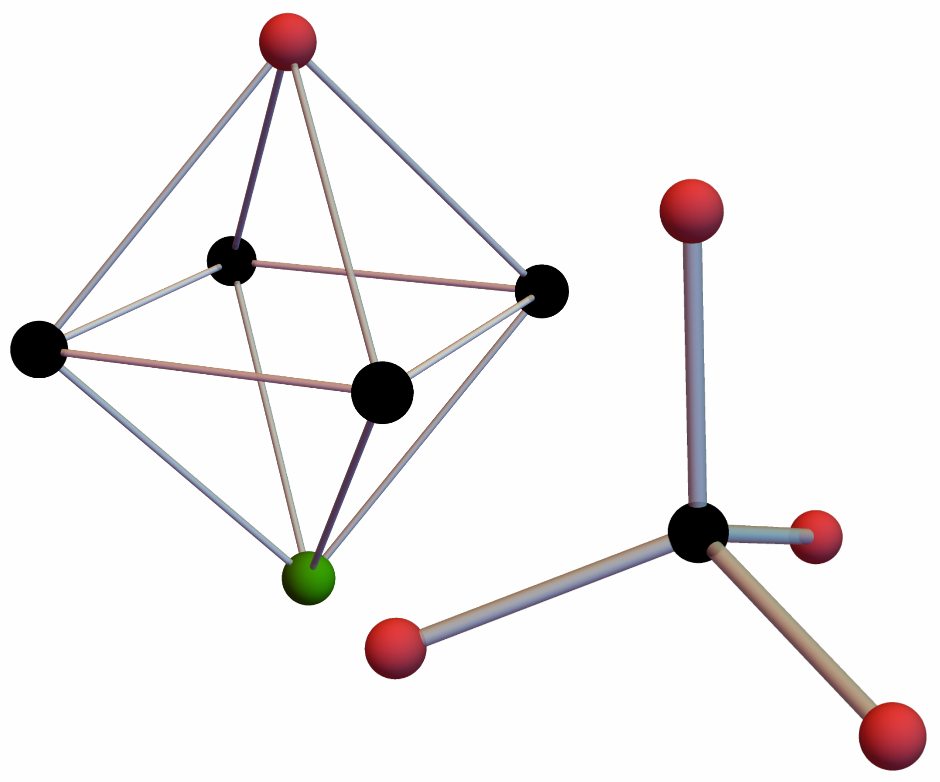


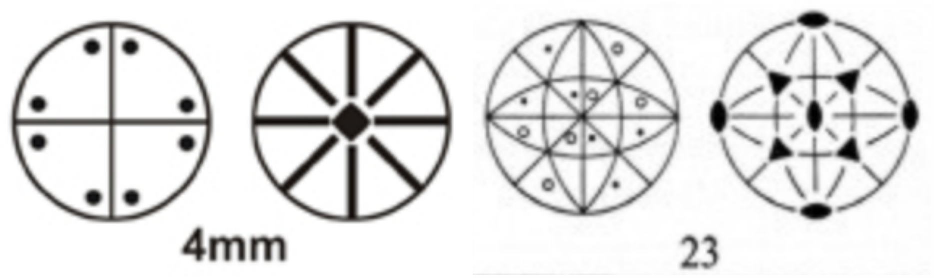
Time dependent Schrödinger eq.

$$\frac{d\hat{O}}{dt} = -i\hbar[\mathscr{H}, \hat{O}]$$

Symmetry

Crystallographic point groups



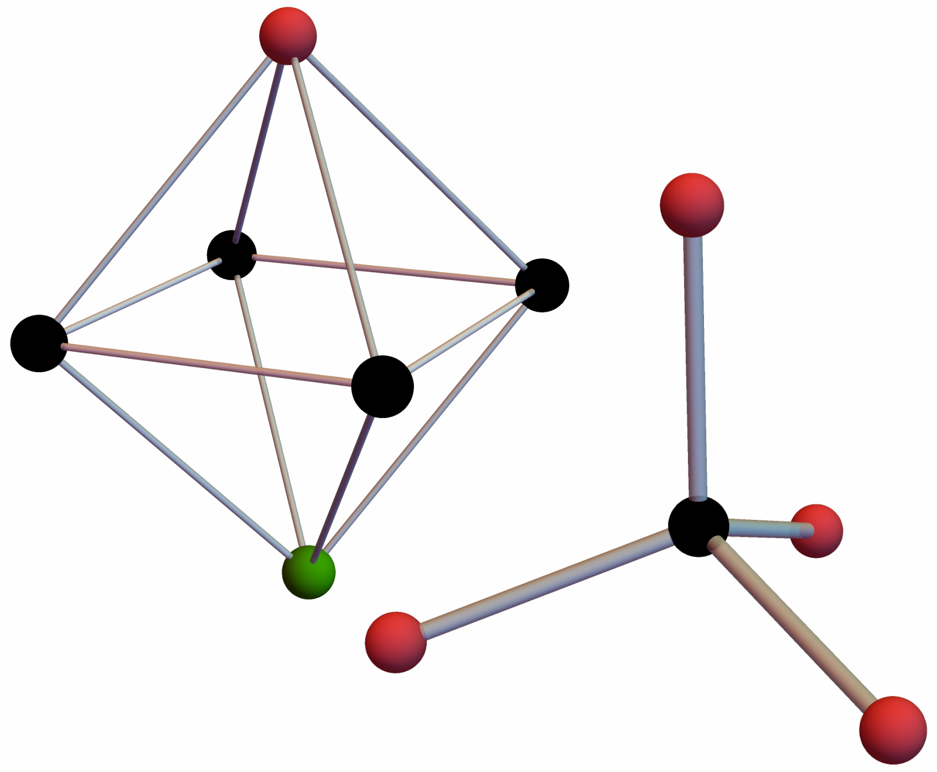


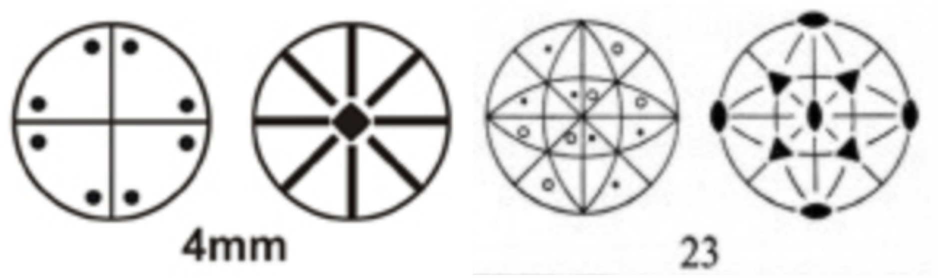
Symmetries of finite figures compatible with a 3D lattice

32 groups

Symmetry

Crystallographic point groups





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1826

Johann F. C. Hessel

1830

Symmetry

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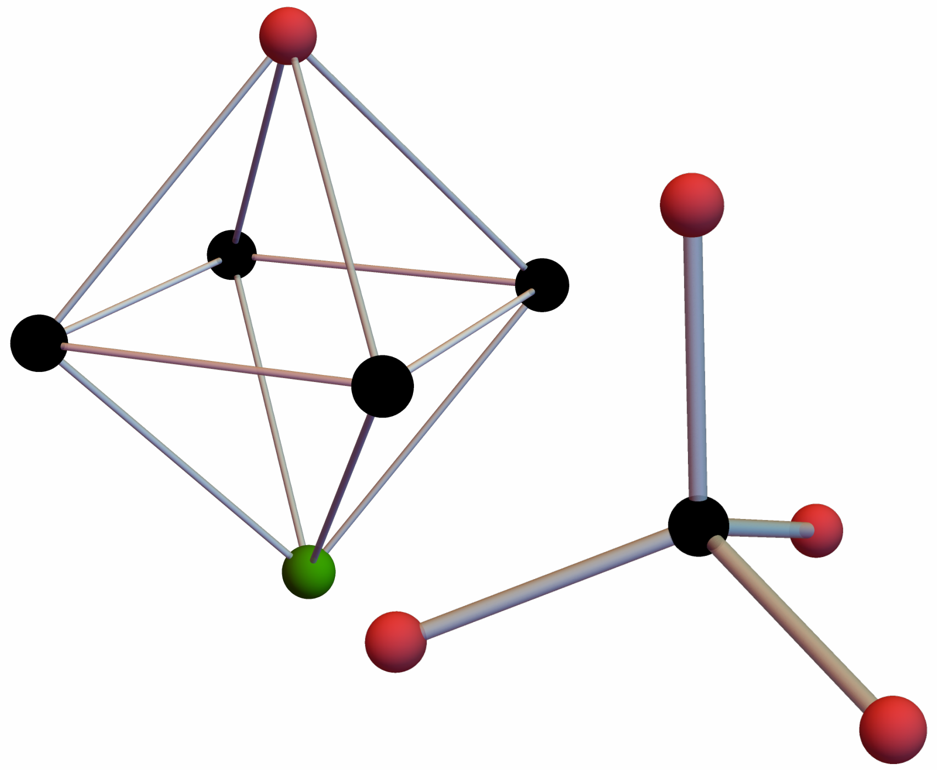

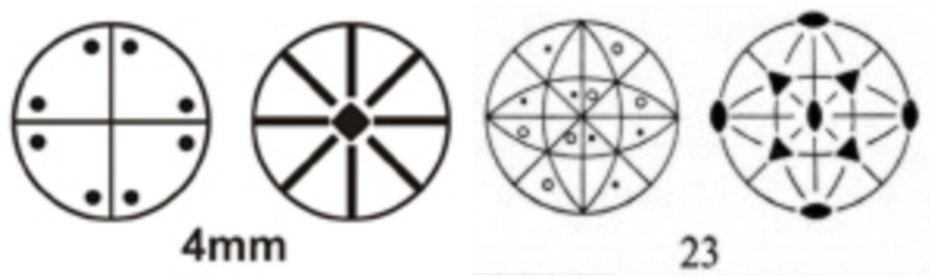

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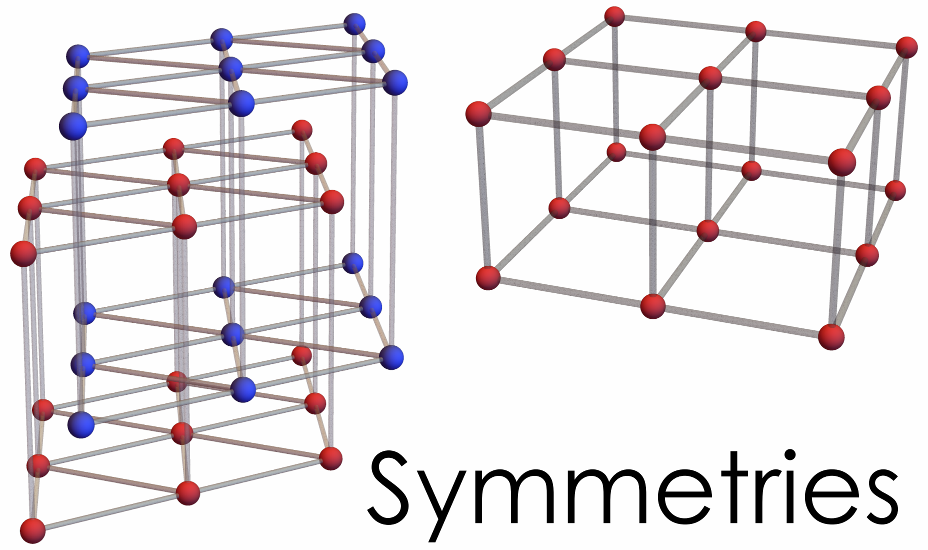
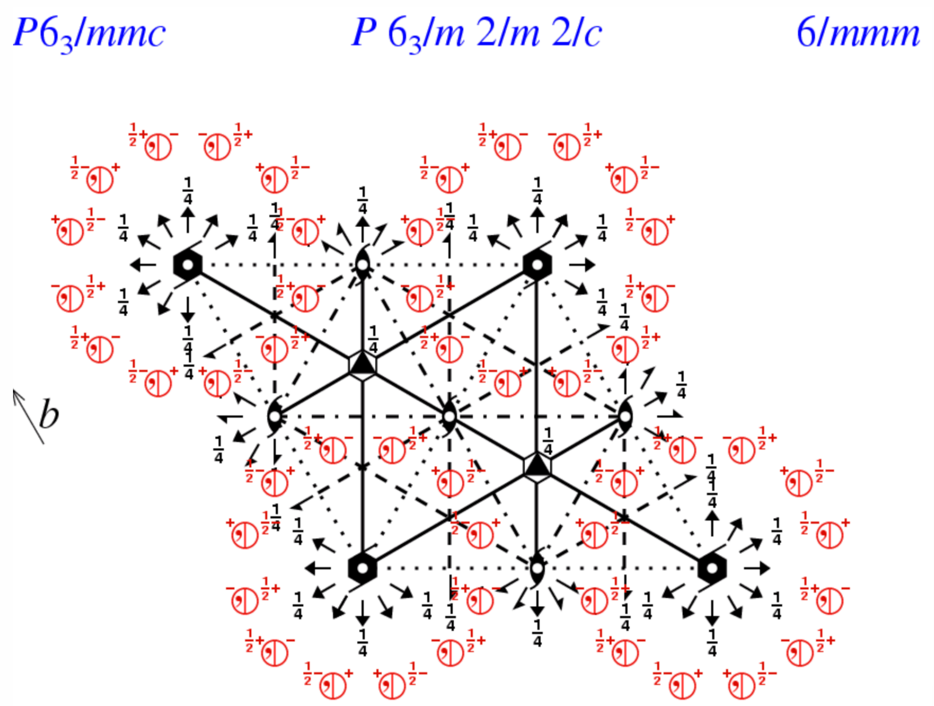


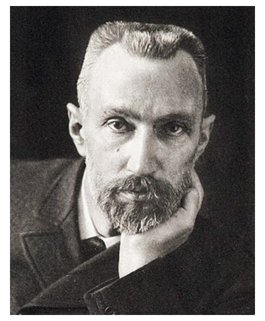
Symmetries of 3D lattices

230 groups

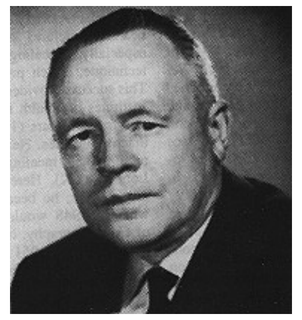


Symmetry

William Barlow



Pierre Curie



Ralph W.G. Wyckoff

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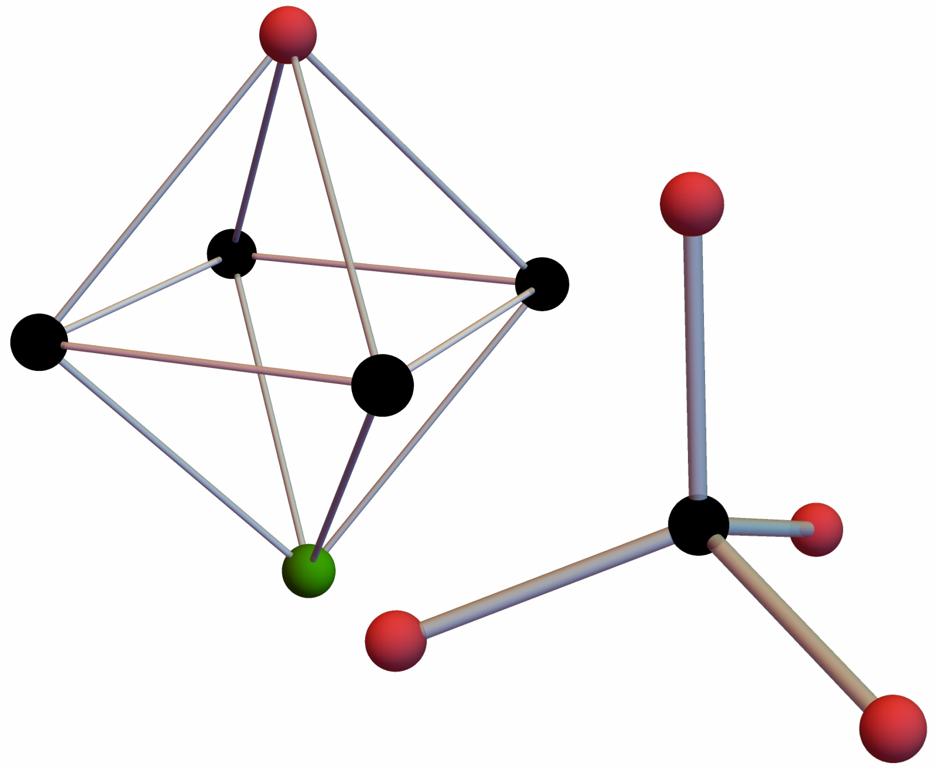
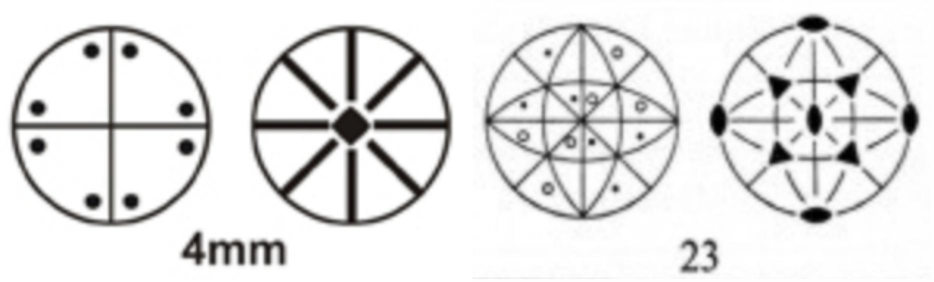
Crystallographic point groups





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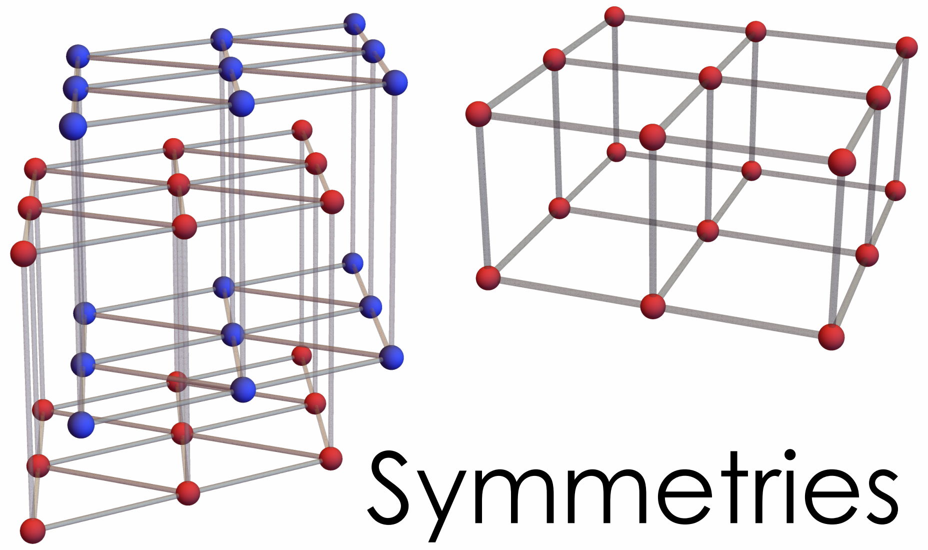

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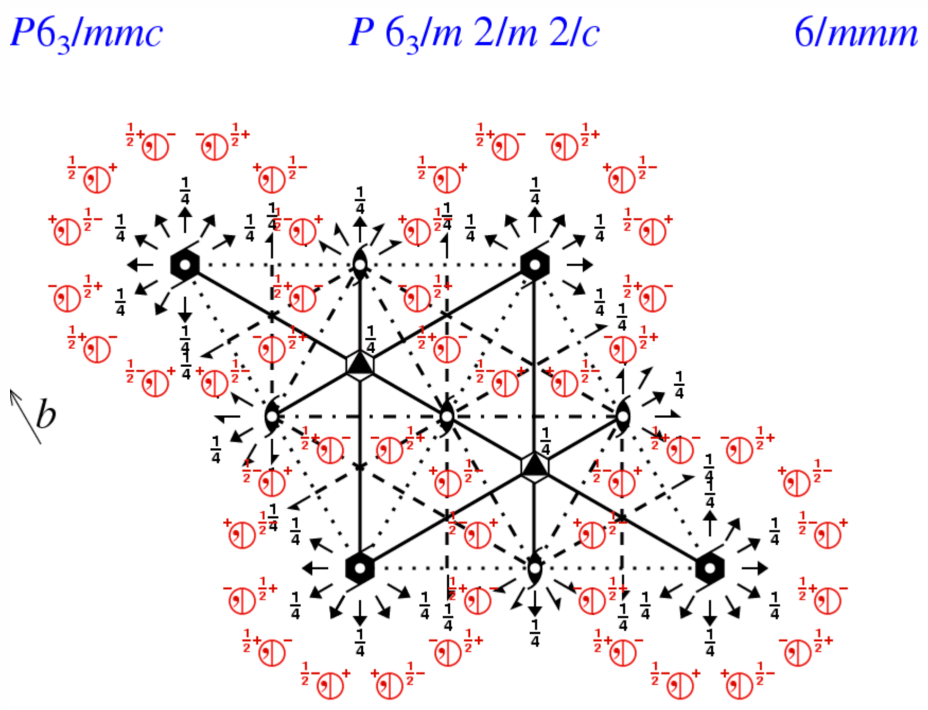

Evgraf Fedorov

1891

Arthur M. Schoenflies

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Symmetry

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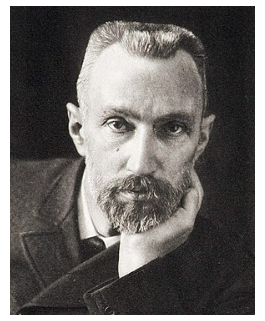

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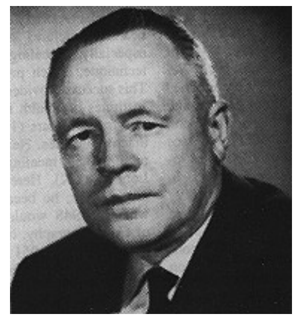

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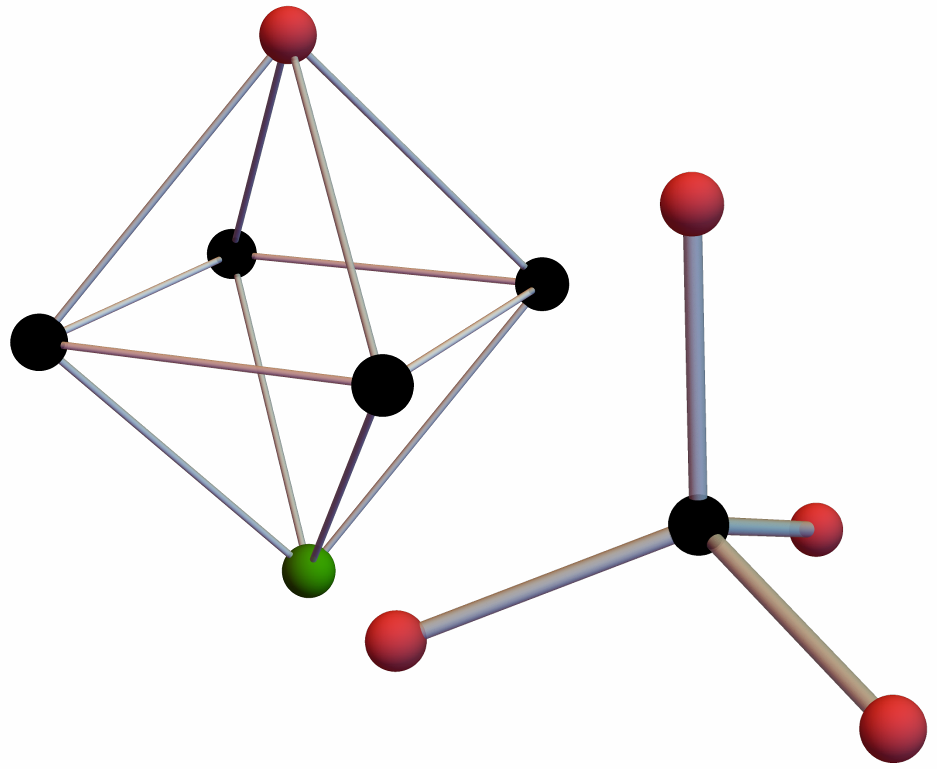

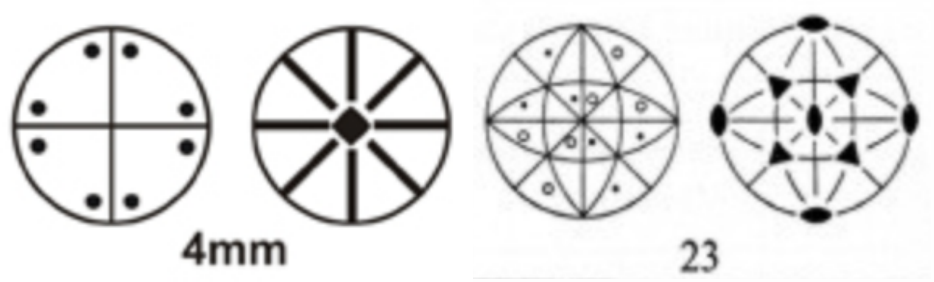

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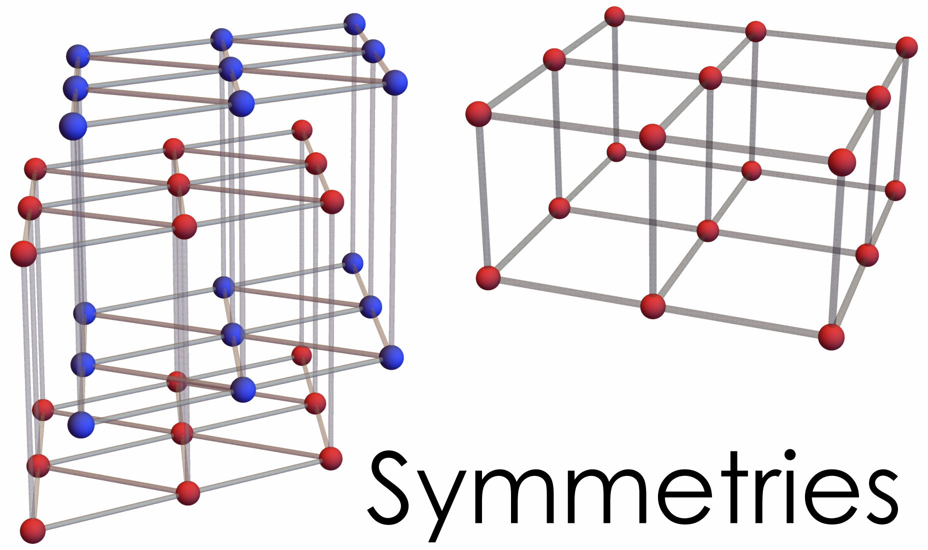

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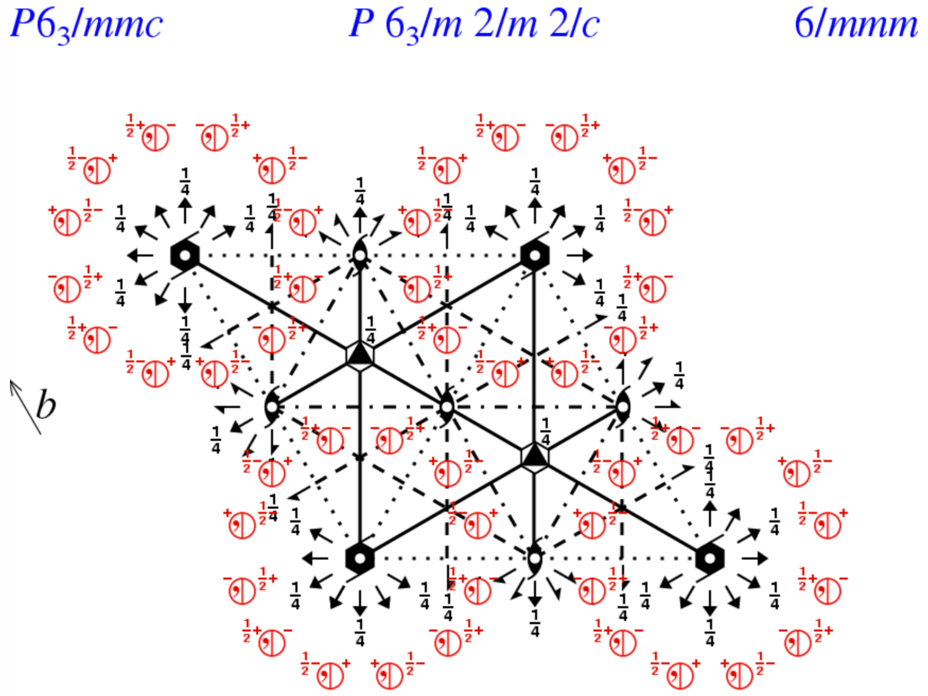

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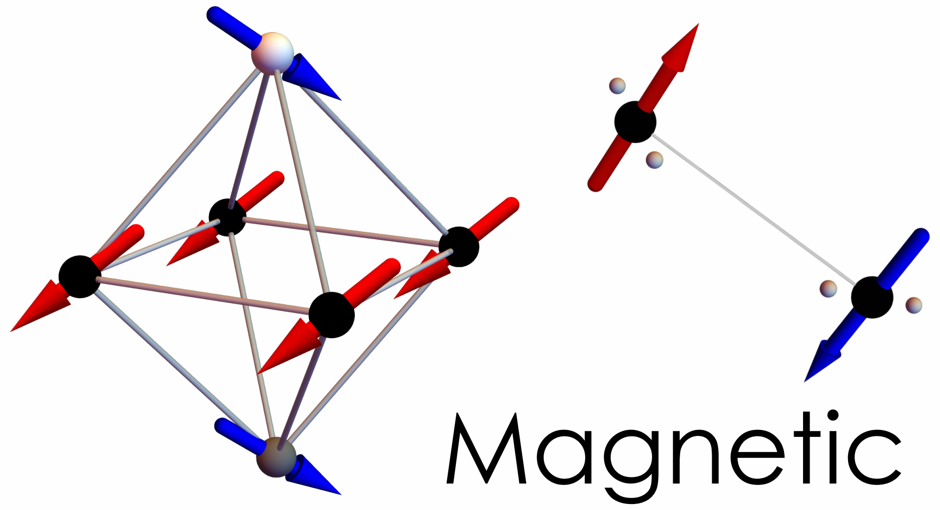
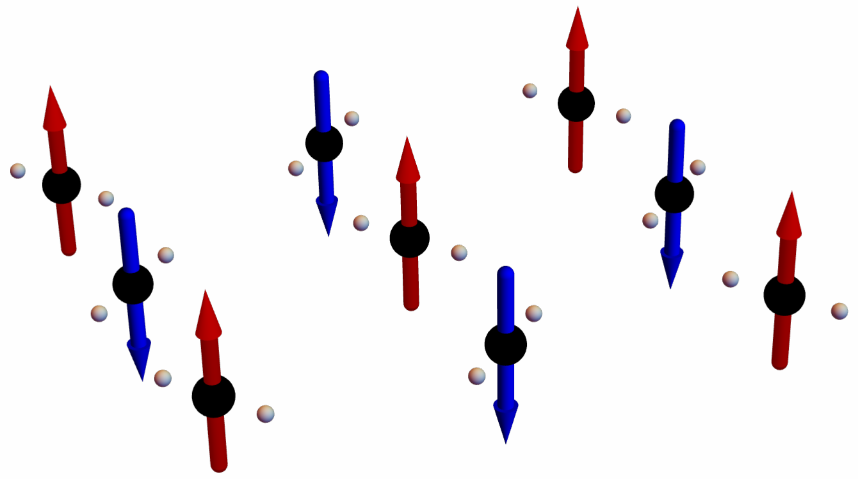
Magnetic groups



Magnetic point groups

122 groups

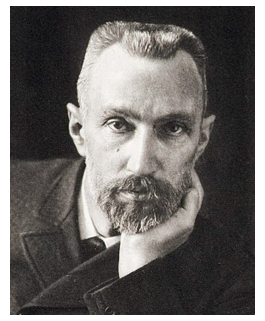
Magnetic space groups



1651 groups

Symmetry


William Barlow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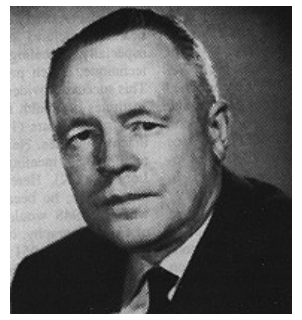
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Alexander M. Zamorzaev


Ben Zion Tavger

V. Zaitsev
E. I. Sokolov
J. O. Dimmock
R. G. Wheeler

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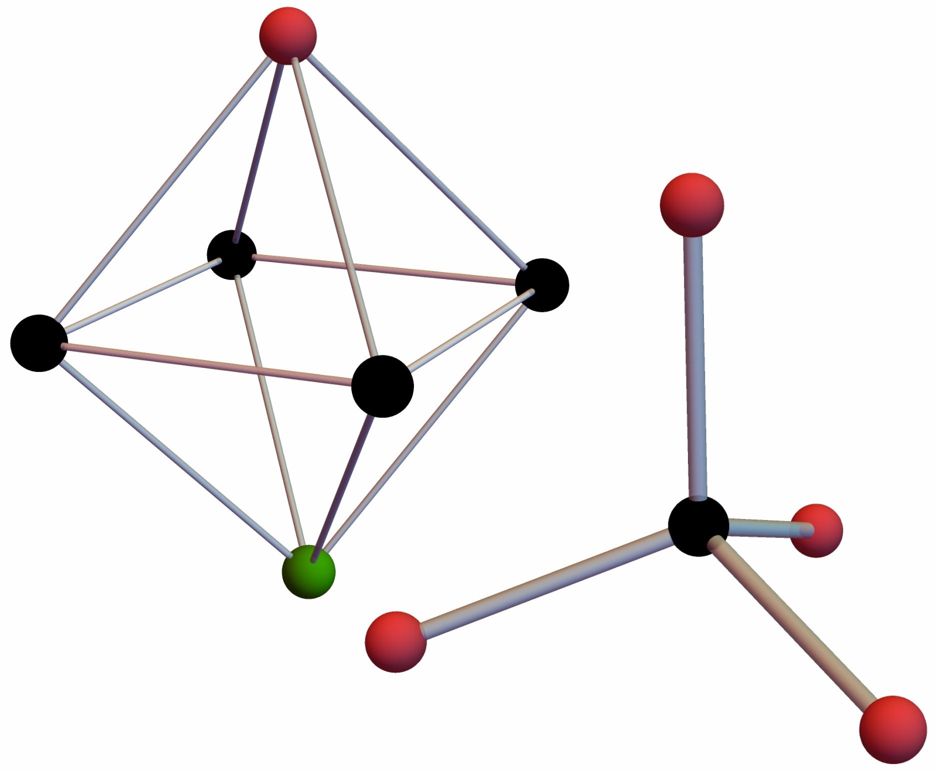
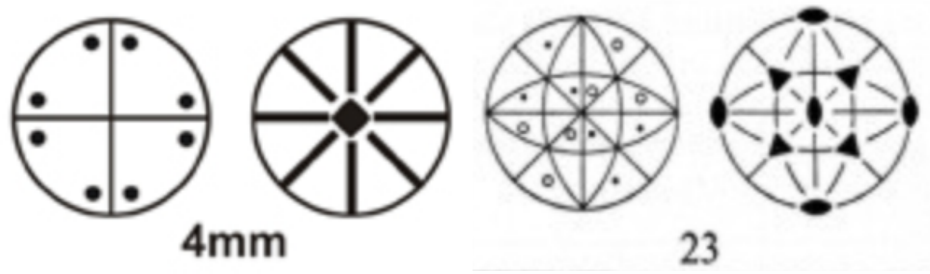
Crystallographic point groups





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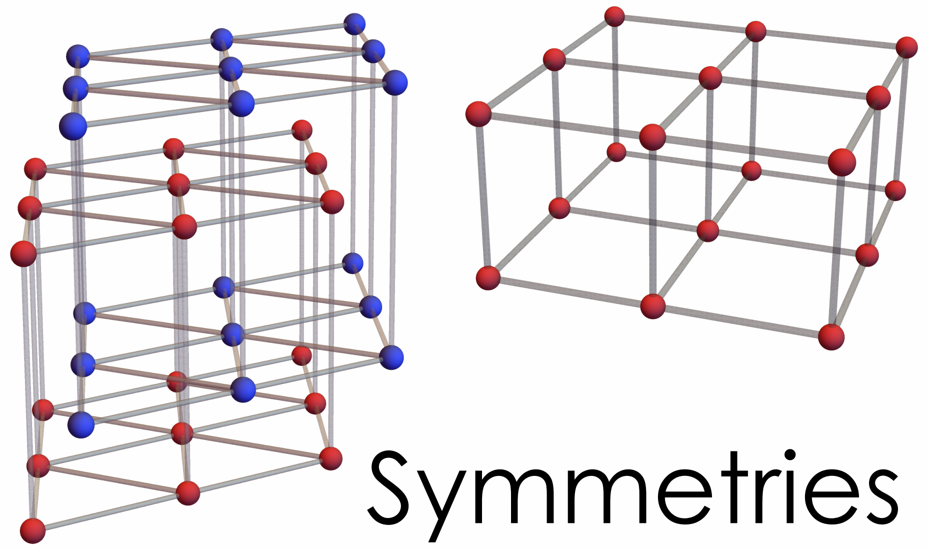

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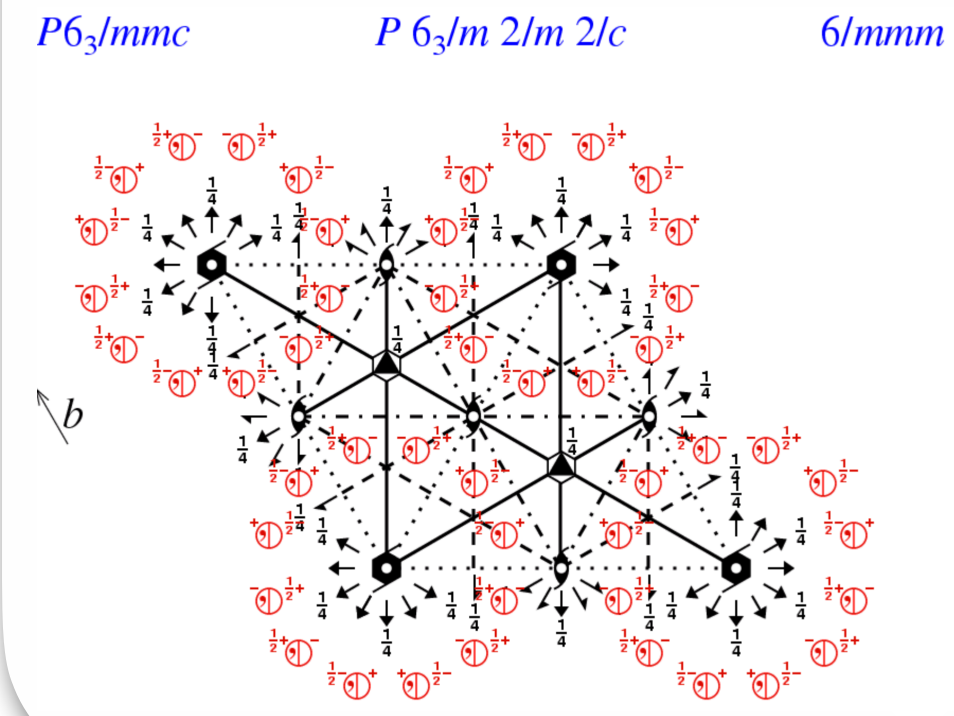

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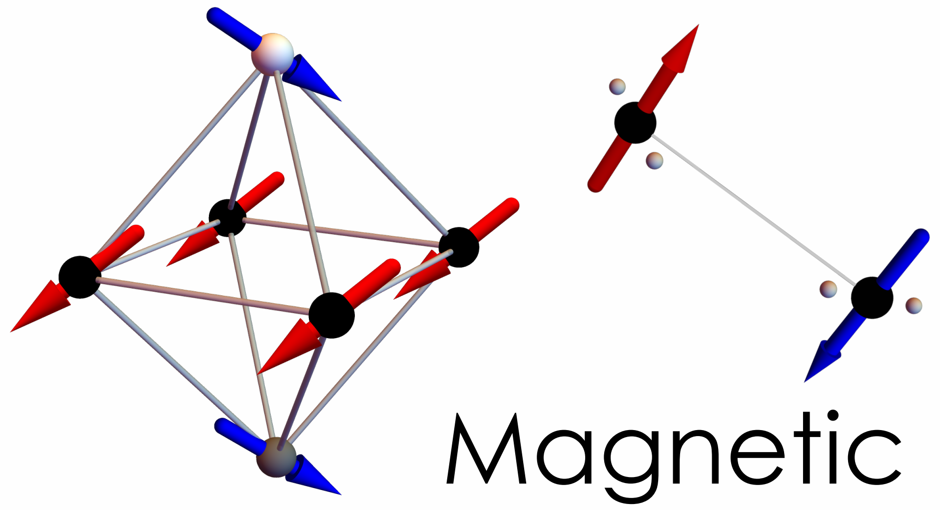
Magnetic groups



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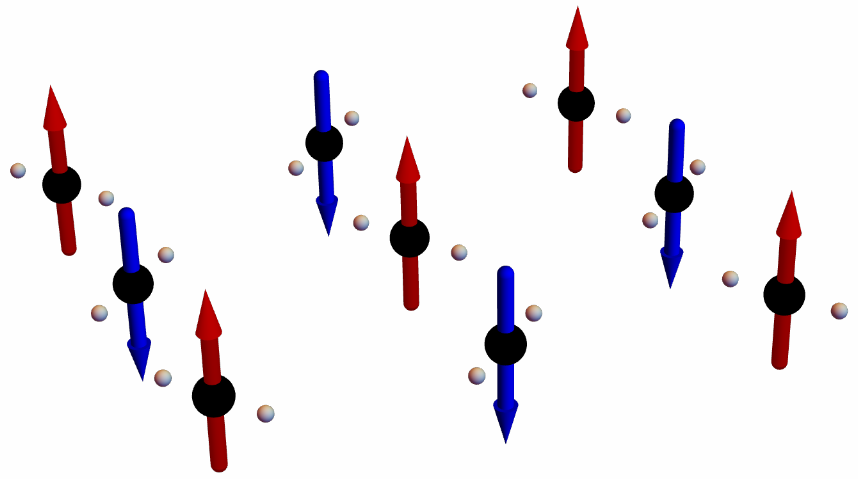


1651 groups

Alexei V. Shubnikov


1951

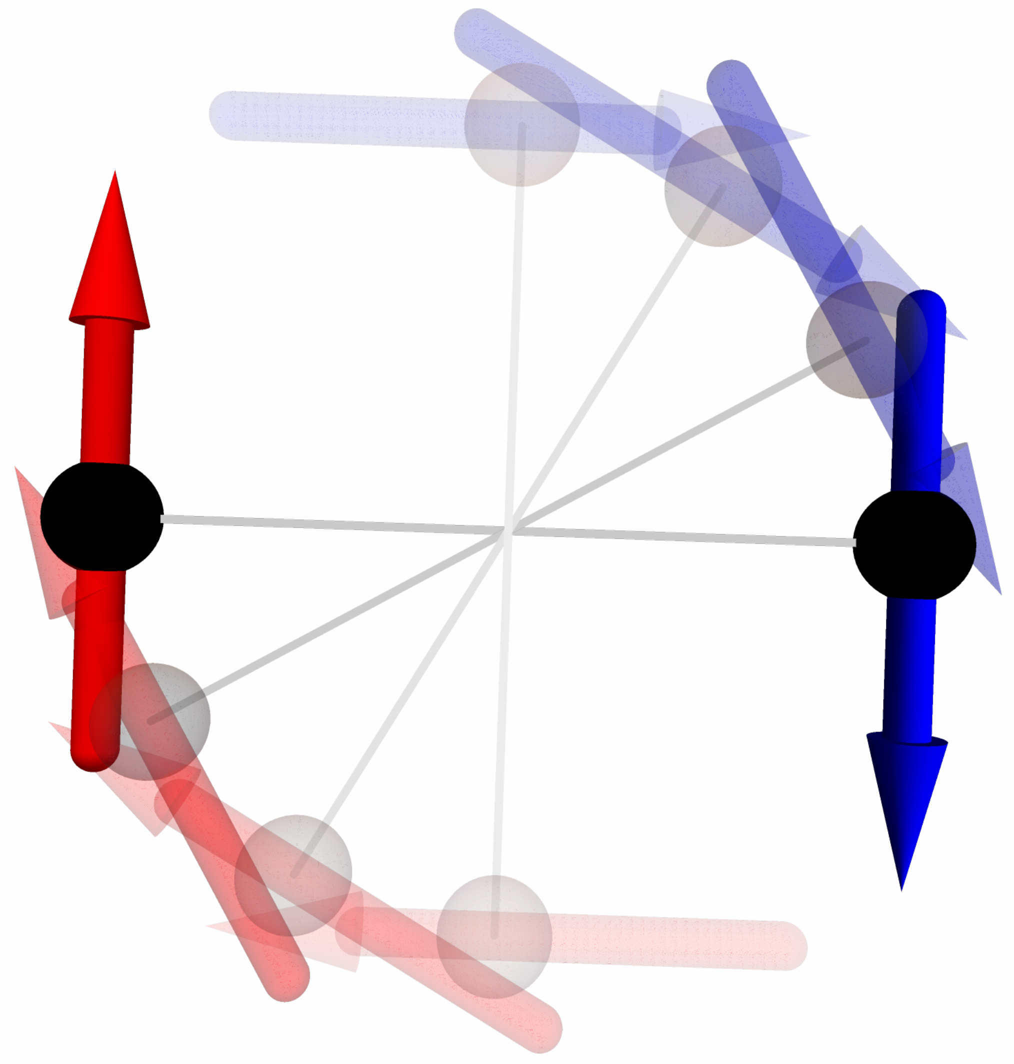
Erwin-Félix Lewy-Bertaut


1968

Magnetic symmetry & spin-orbit coupling

1830 Point groups 1890 Space groups **1960** Magnetic groups 2020

Spin Orbit Coupling (SOC)

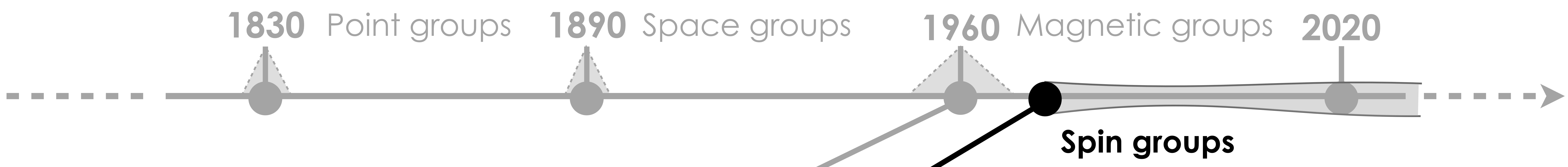


✦ Spins are "locked" to the lattice

✦ Transformation applied to both lattice and spins

✦ Spins are axial, time-reversal (TR) odd vectors

Magnetic symmetry & spin-orbit coupling



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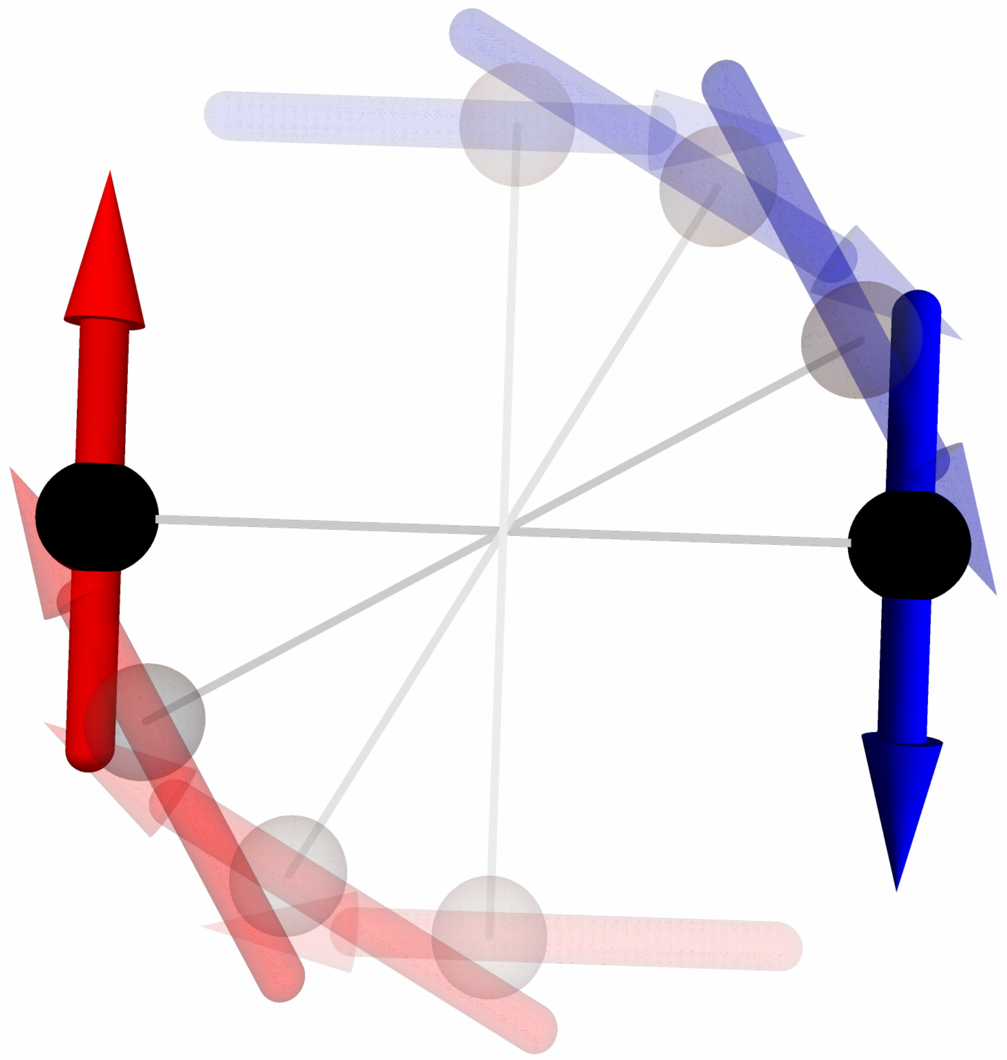

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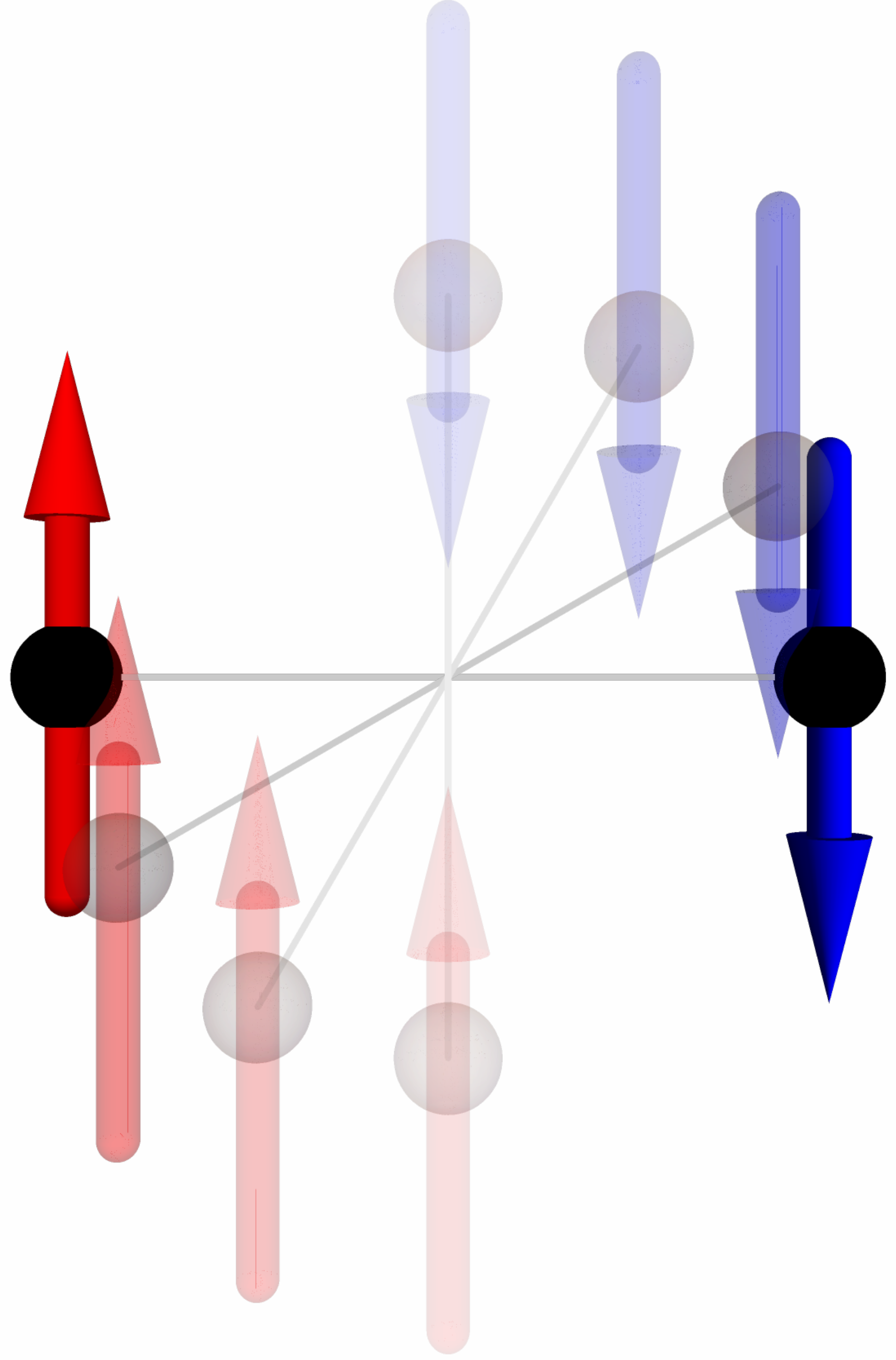
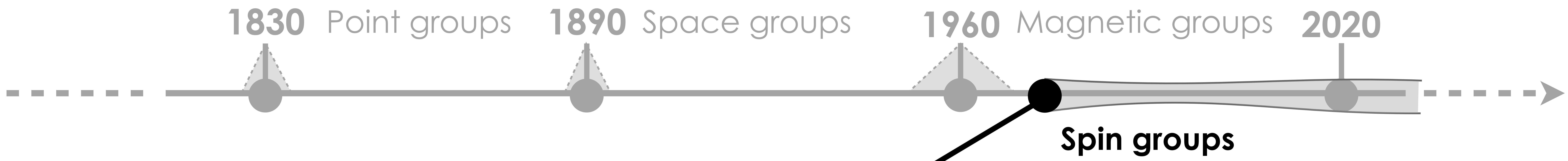
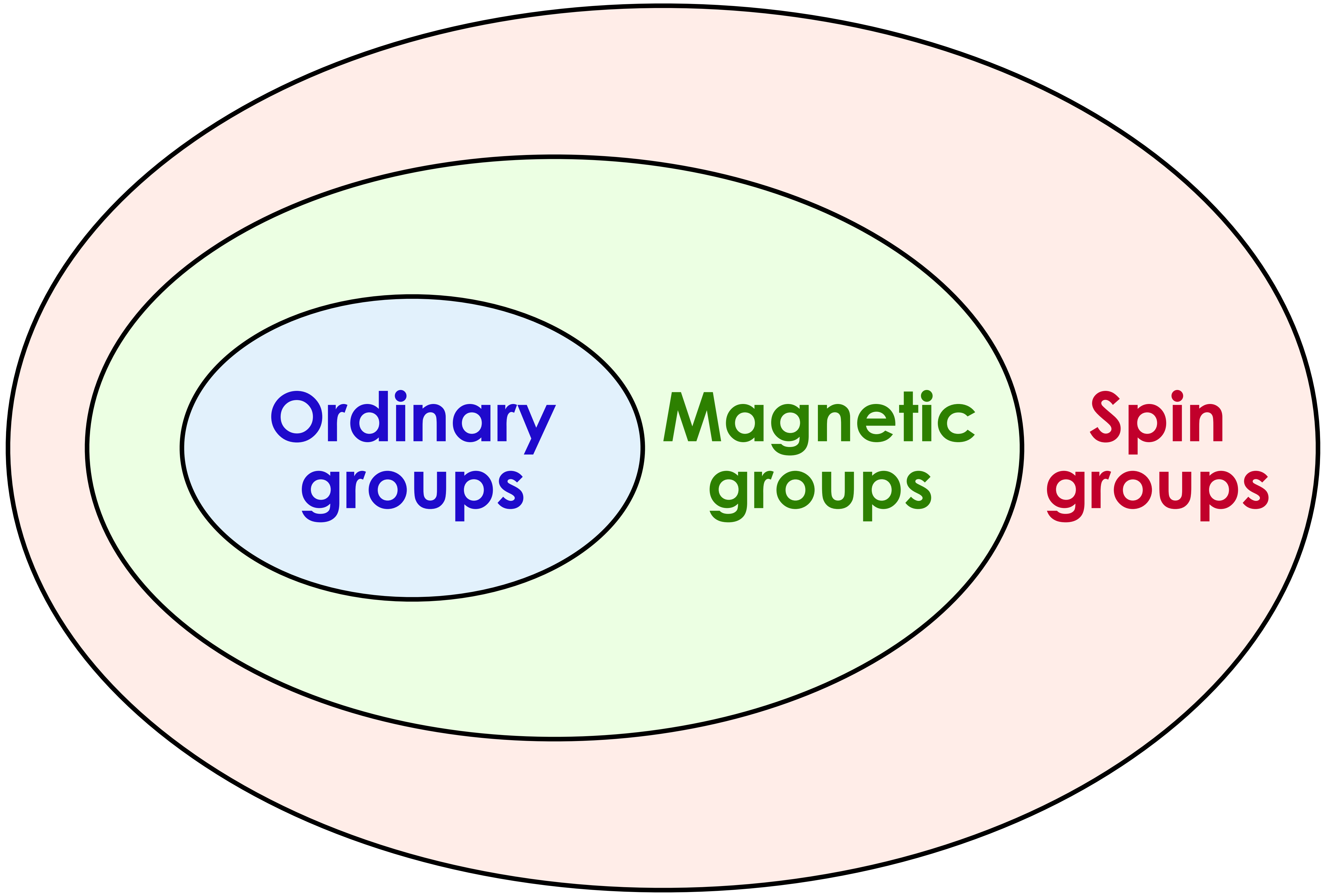
Negligible or No SOC



- Spins (and Hilbert space) decoupled from lattice
- Ex: Heisenberg $\mathscr{H} = \sum_{\langle ij \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$
- Spin-space elements from $\mathbf{O}^s(3) \equiv \mathbf{SO}(3) + \tau\mathbf{SO}(3)$

Magnetic symmetry & spin-orbit coupling





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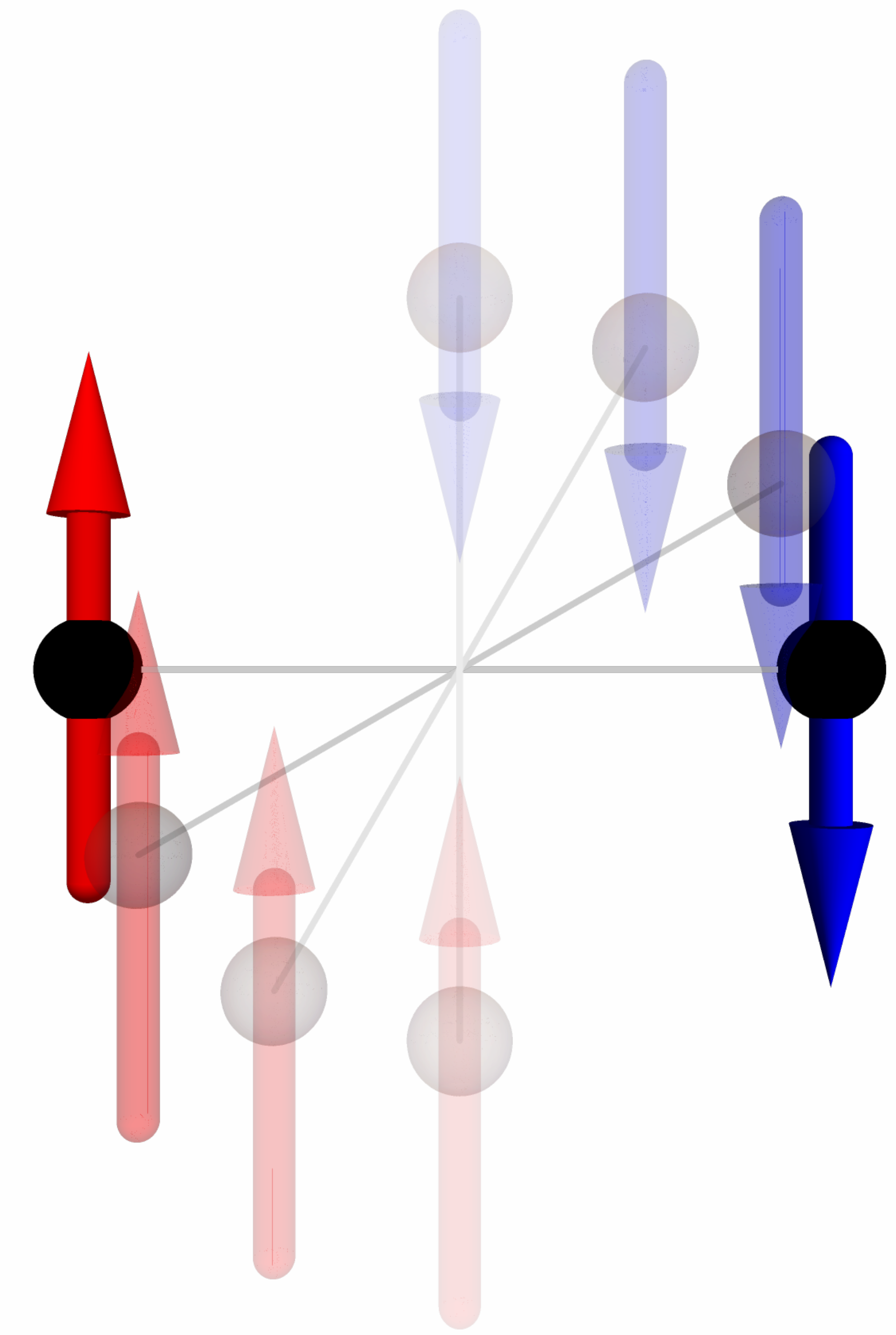

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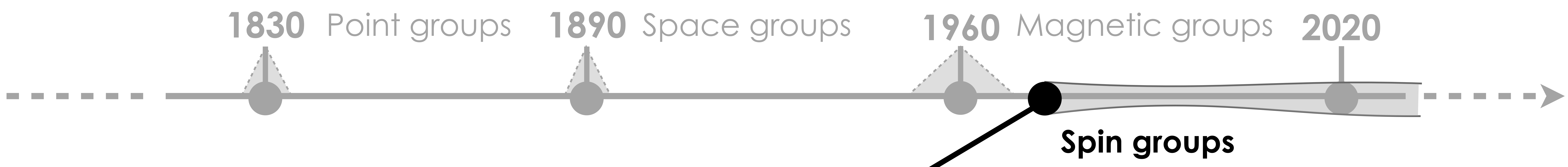

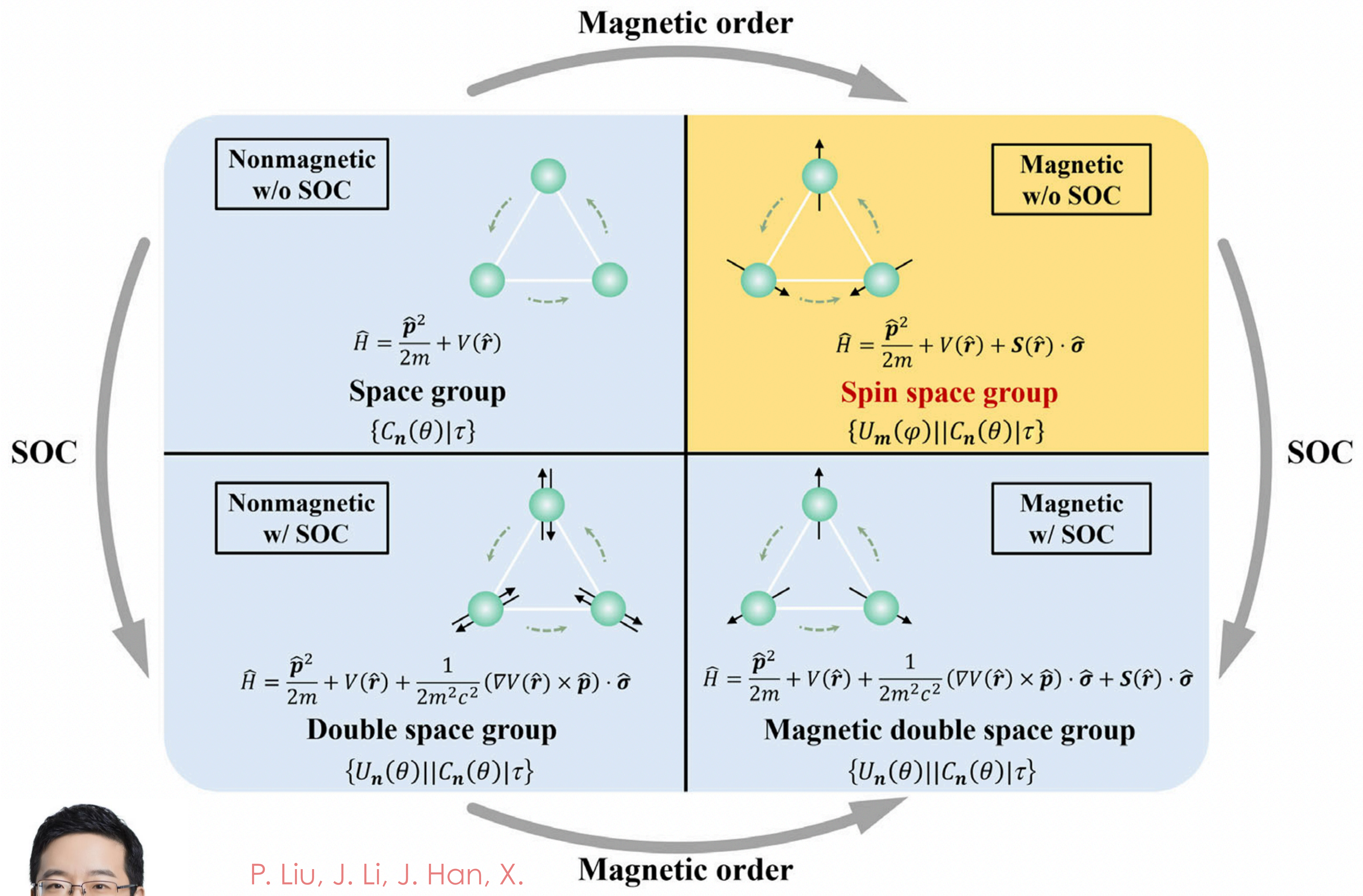

P. Liu, J. Li, J. Han, X. Wan and Q. Liu, Phys. Rev. X 12, 021016 (2022)

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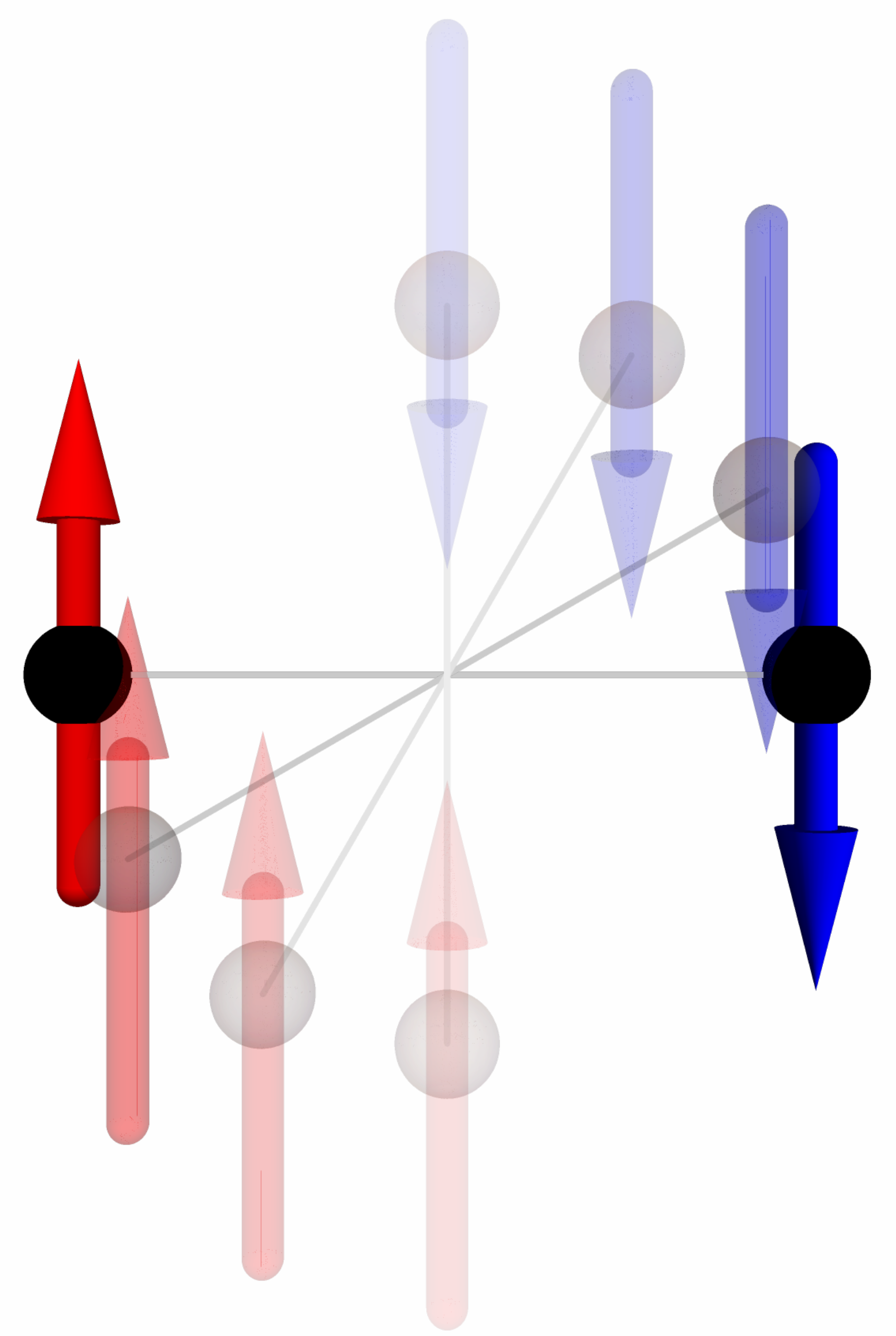

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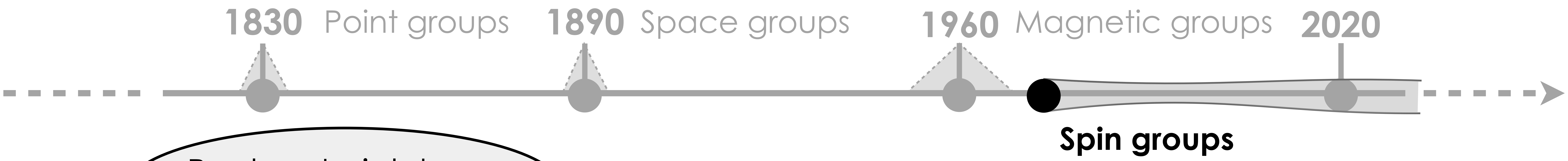
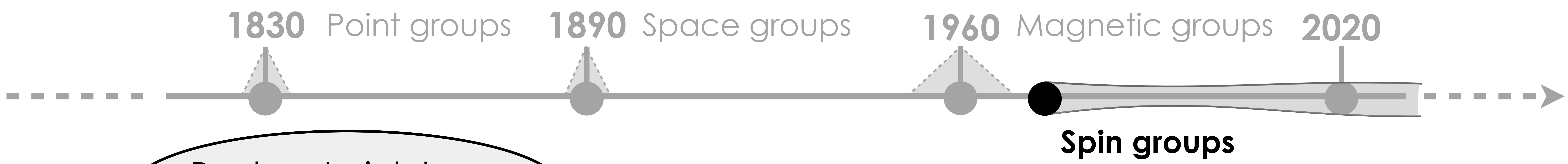
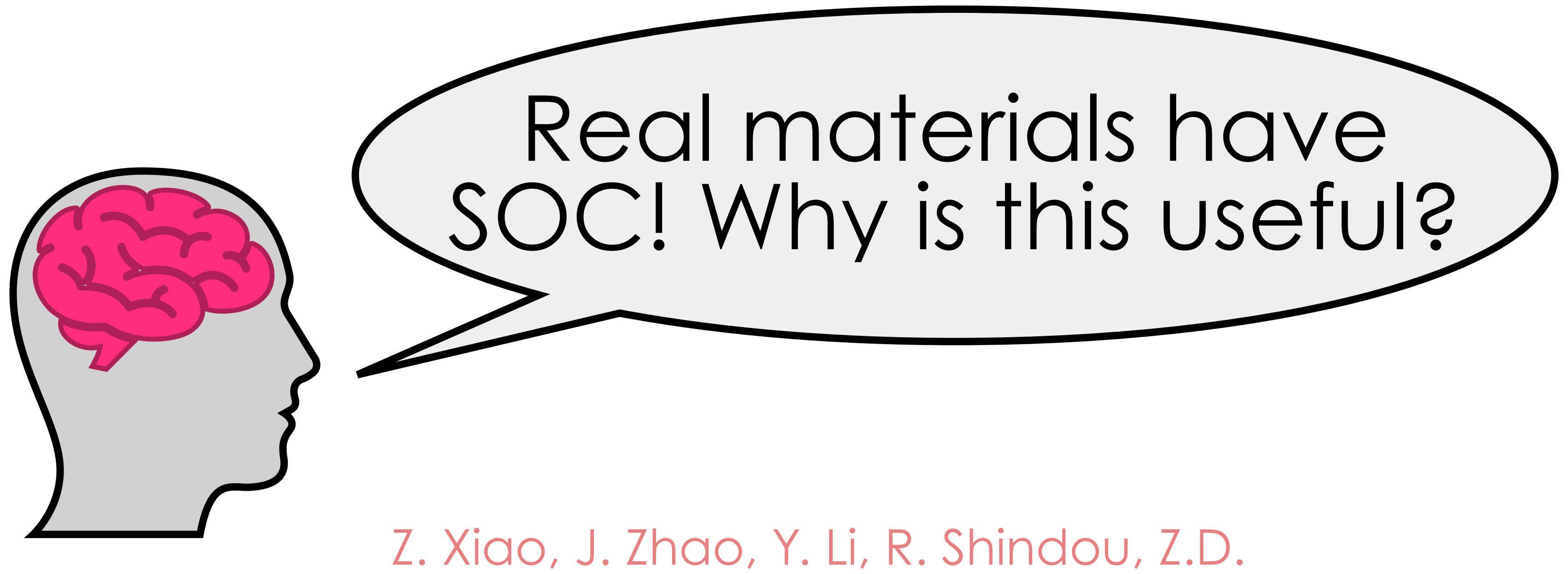
Magnetic symmetry & spin-orbit coupling



Magnetic symmetry & spin-orbit coupling





Z. Xiao, J. Zhao, Y. Li, R. Shindou, Z.D. Song Phys. Rev. X 14, 031037 (2024)

Topological electronic (and magnetic) states

- Exotic nodal-line and -point semimetals
- Exotic otherwise forbidden TIs

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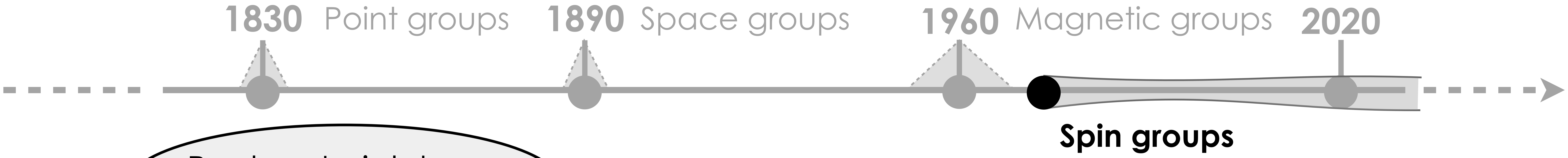
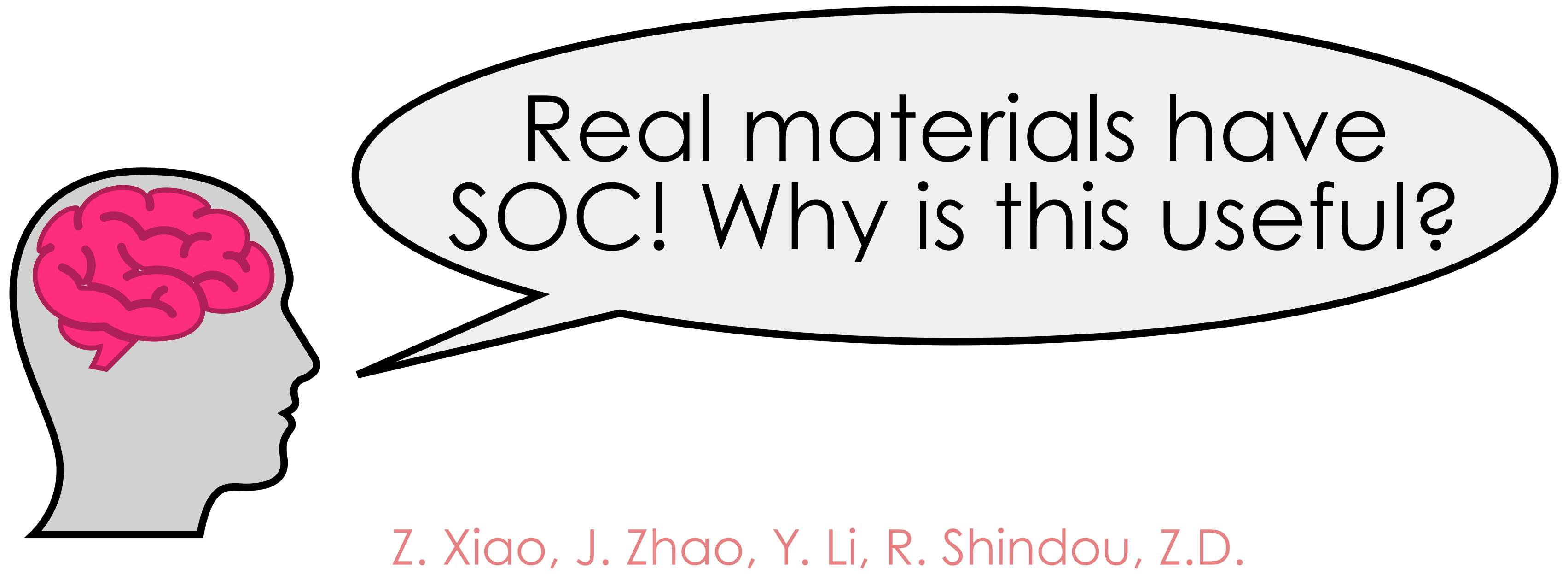
Magnetic symmetry & spin-orbit coupling





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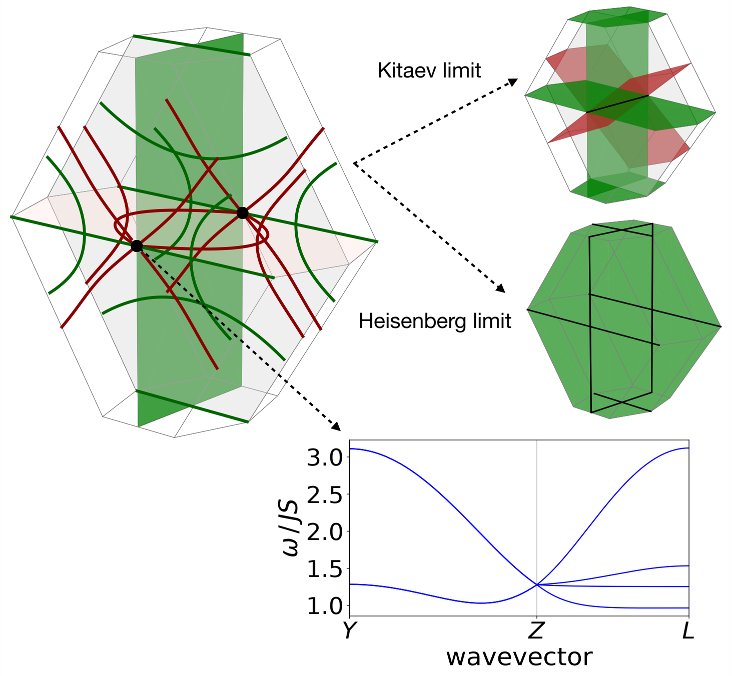
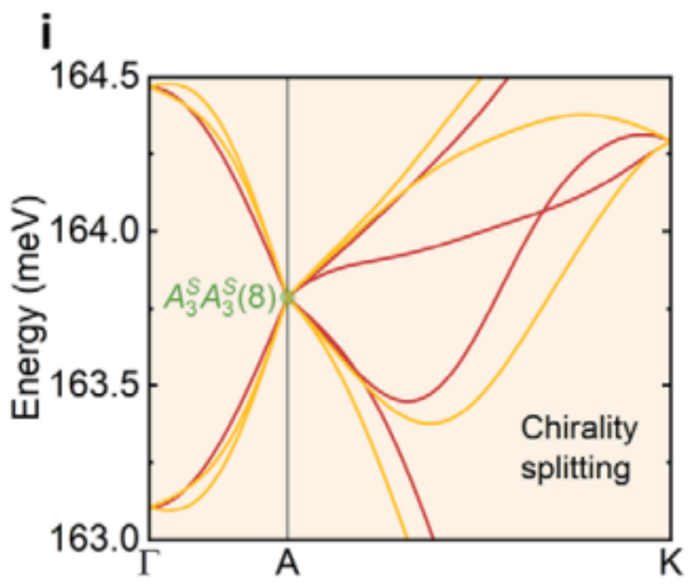
Topological magnons



A. Corticelli, R. Moessner, P. McClarty Phys. Rev. B 105, 064430 (2022)



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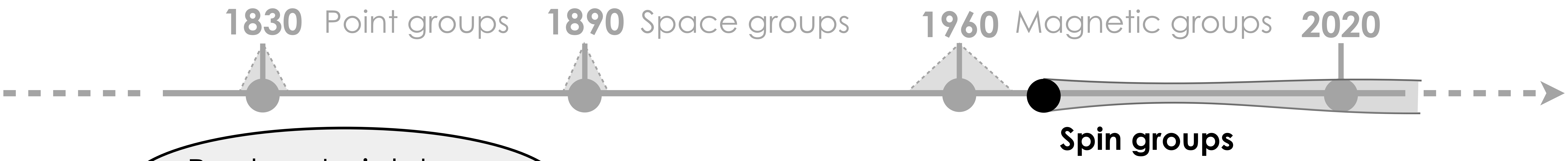

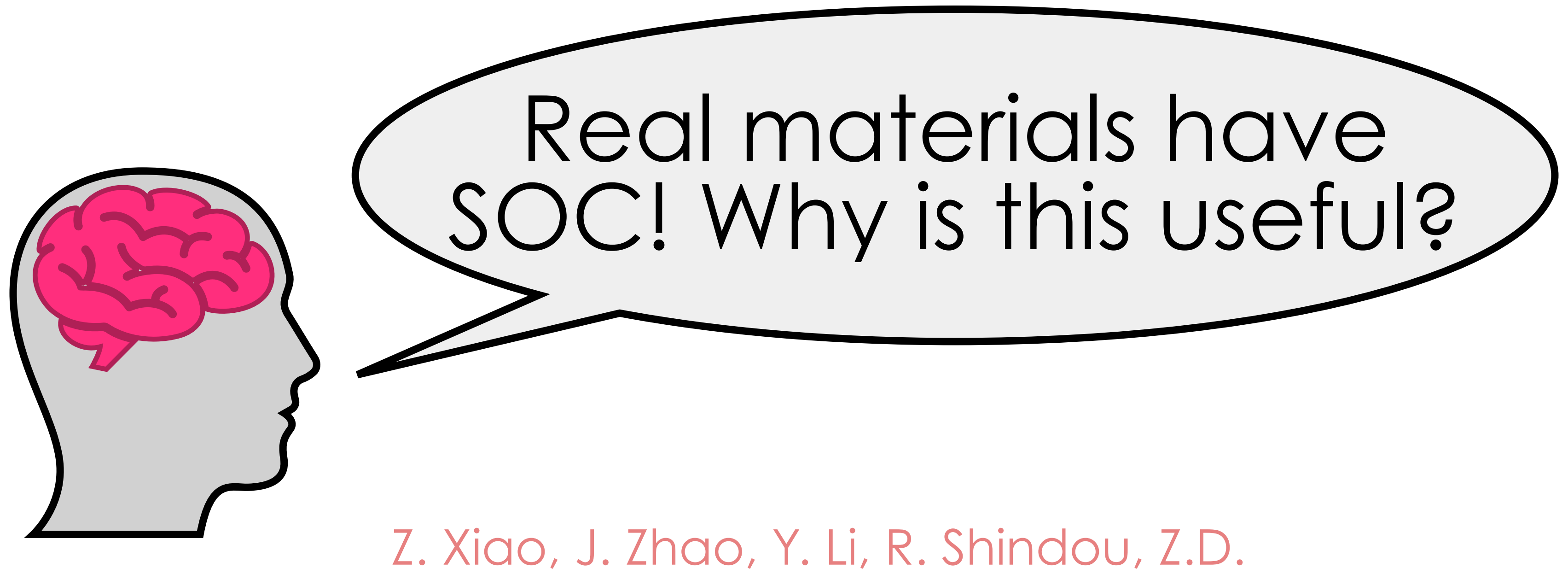

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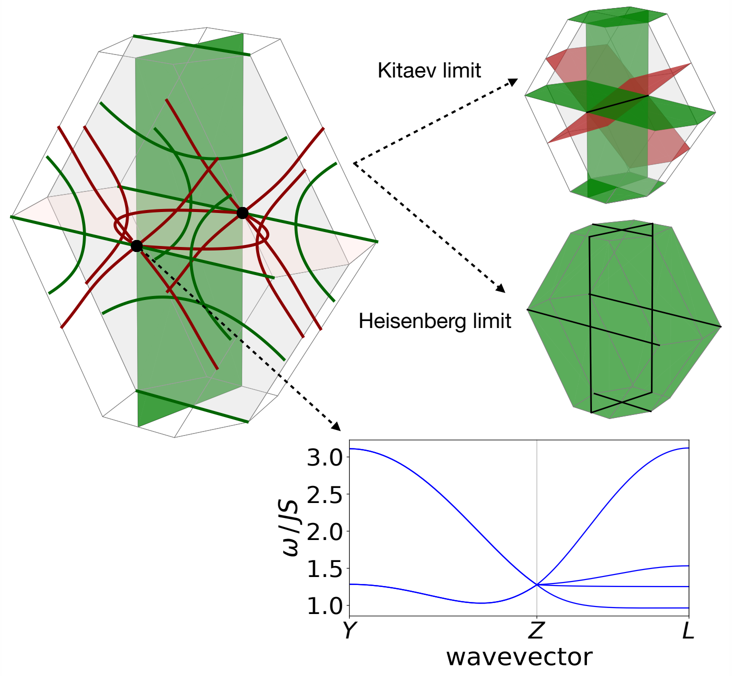

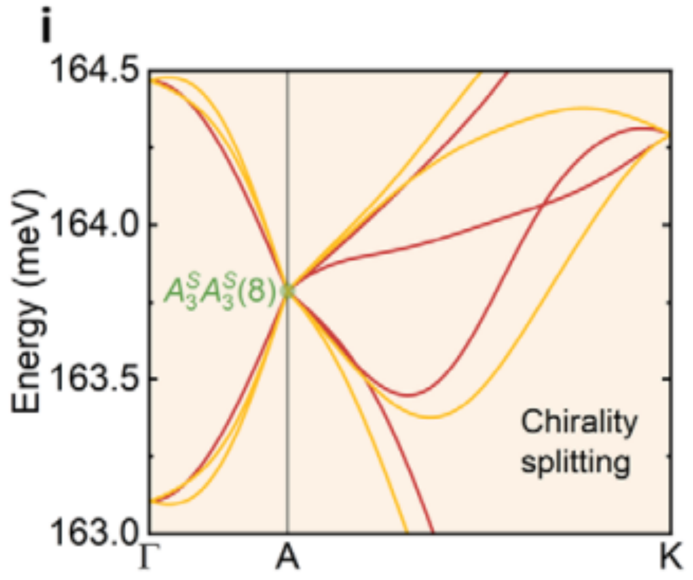

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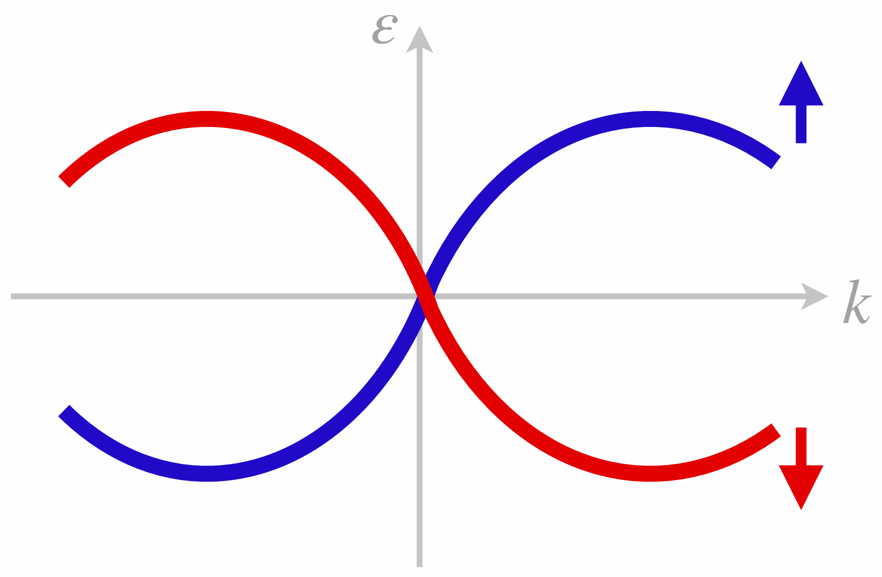
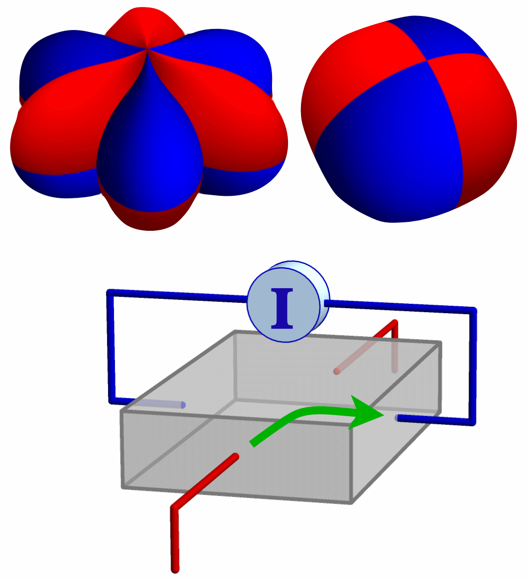
Altermagnets





L. Šmejkal, J. Sinova and T. Jungiwirth, Phys. Rev. X 12, 040501 (2022)

HS, P. McClarty, J. Rau, J. Romhányi, arXiv 2412.18025

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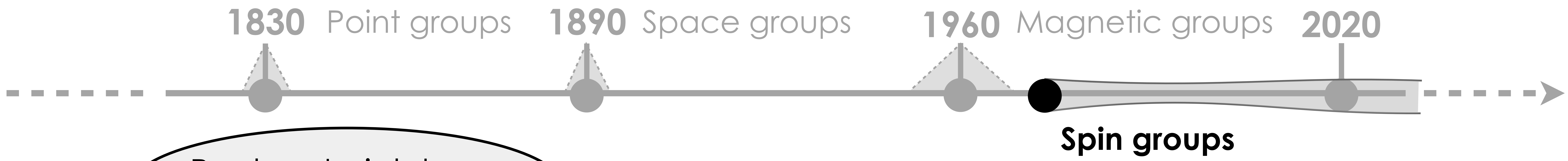

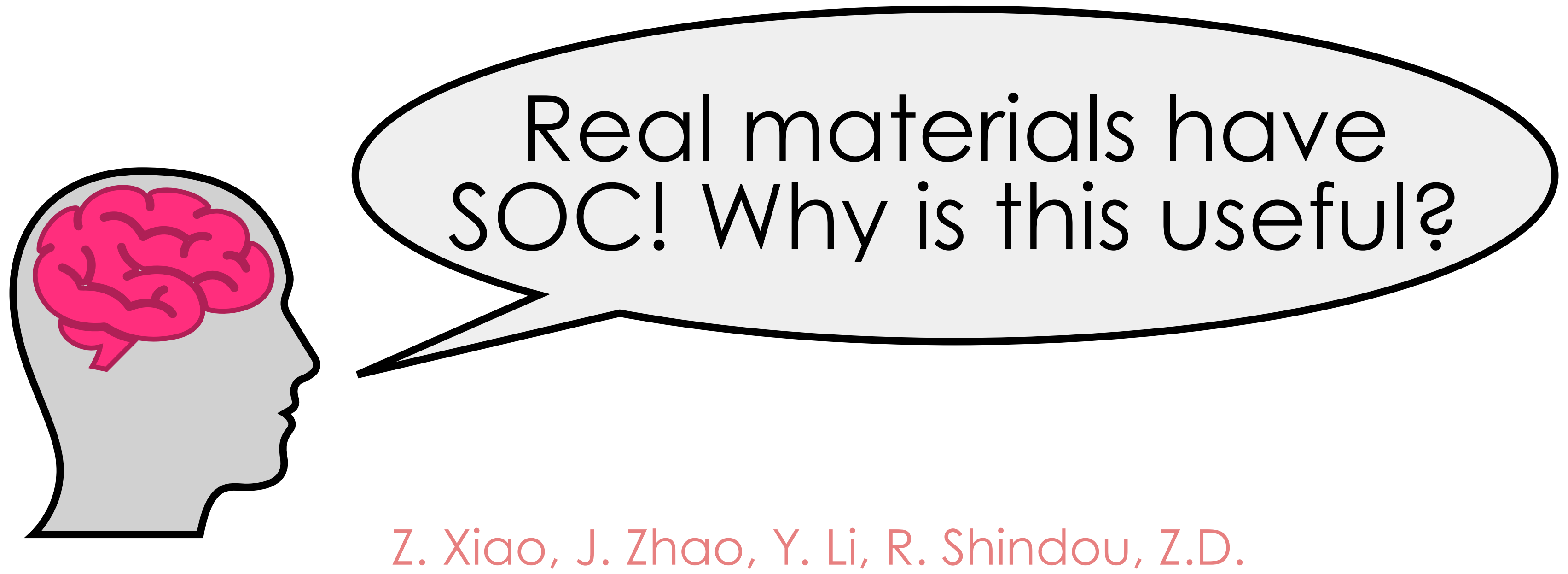

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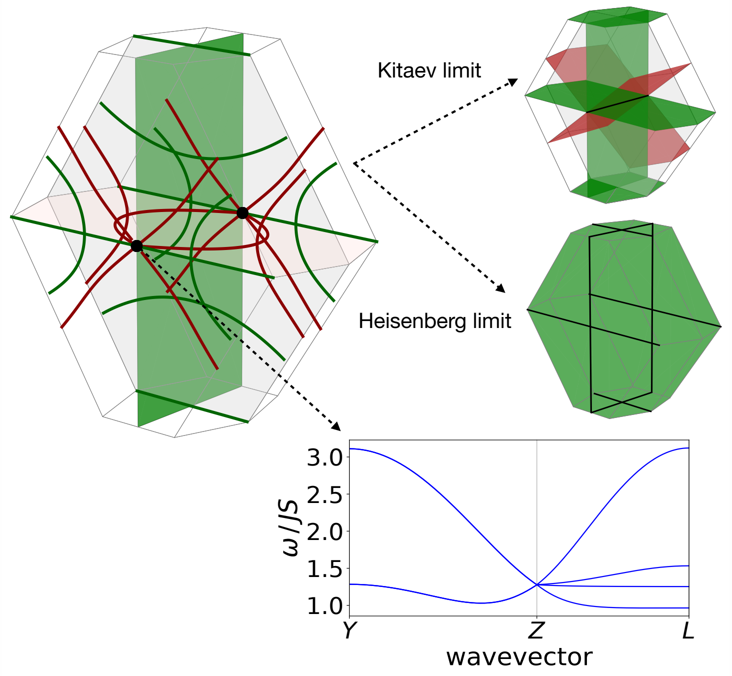

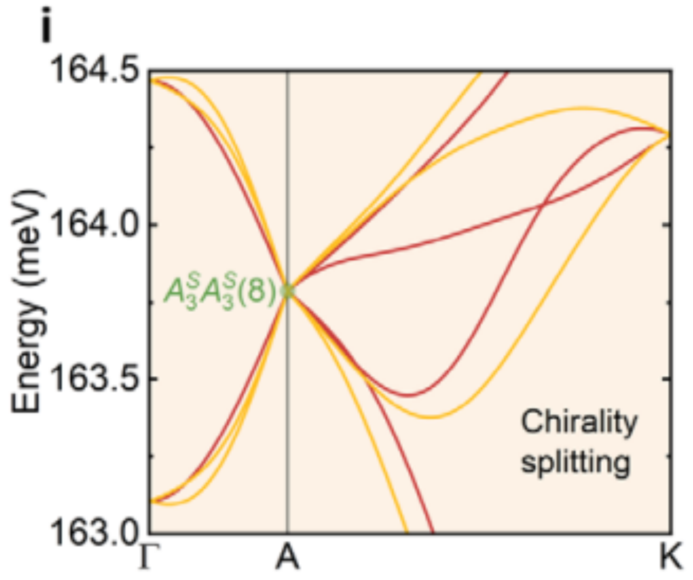

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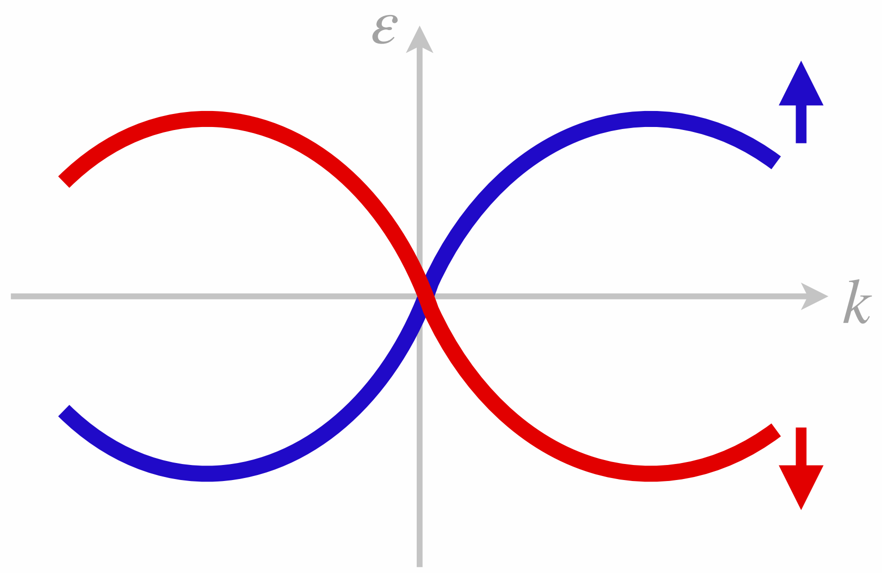

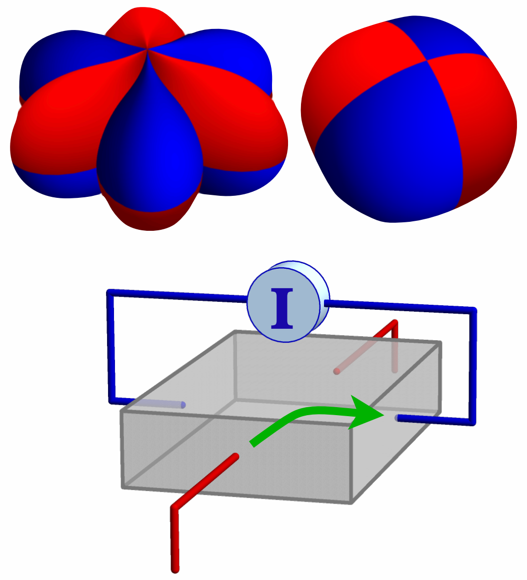

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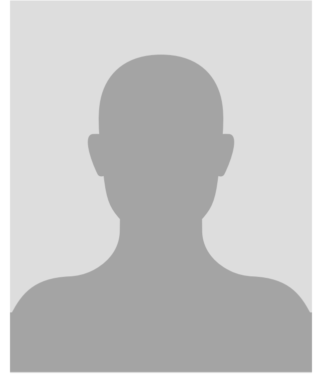
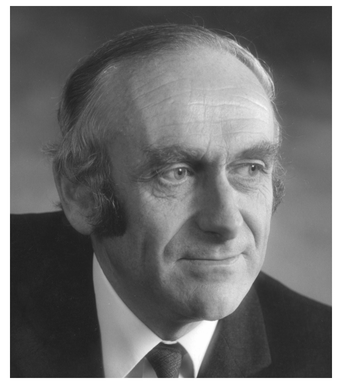
Precise when SOC is negligible, and dictates salient features of systems with weak SOC.

Spin groups

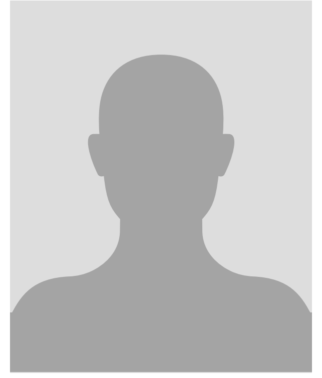


Albrecht Kitz

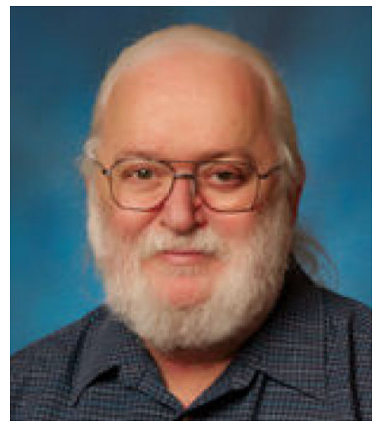
William Brinkman



Roger Elliott



Wladyslaw Opechowski



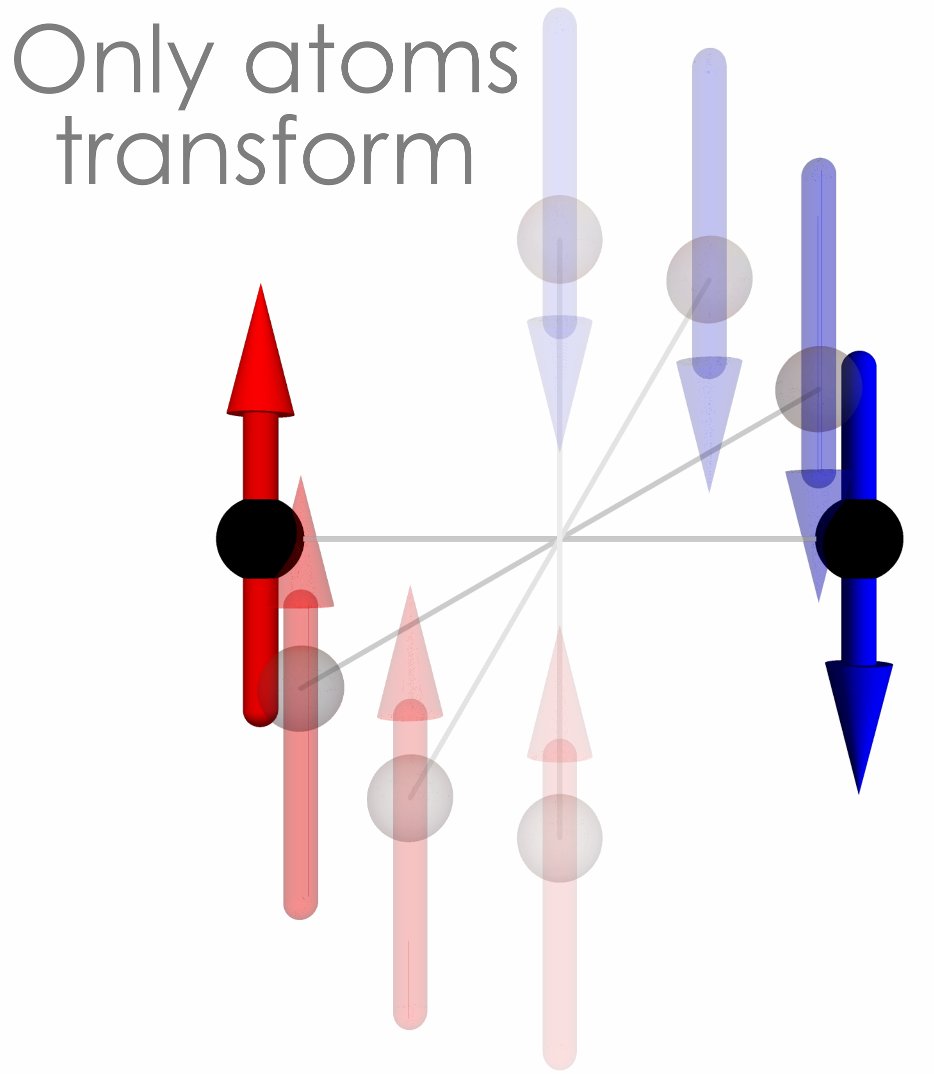
Daniel B. Litvin

1830 Point groups **1890** Space groups **1960** Magnetic groups **2020**

Spin groups

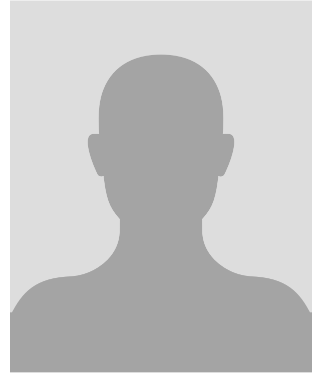
Lattice operation

$$[E||R]$$

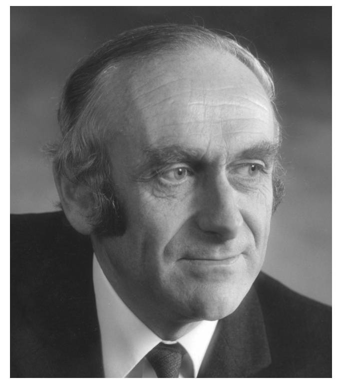


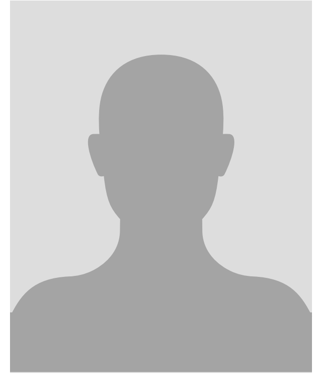
$$\left[E||R\right] M^{\alpha}(\mathbf{r}) = M^{\alpha}(R^{-1}\mathbf{r})$$

Spin groups


Albrecht Kitz


William Brinkman


Roger Elliott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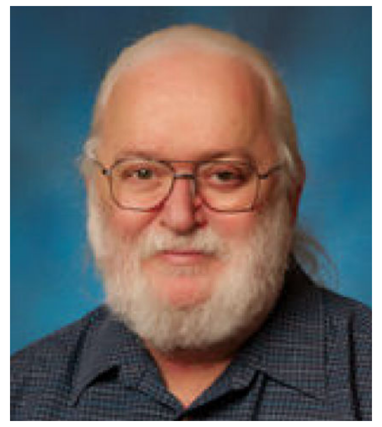
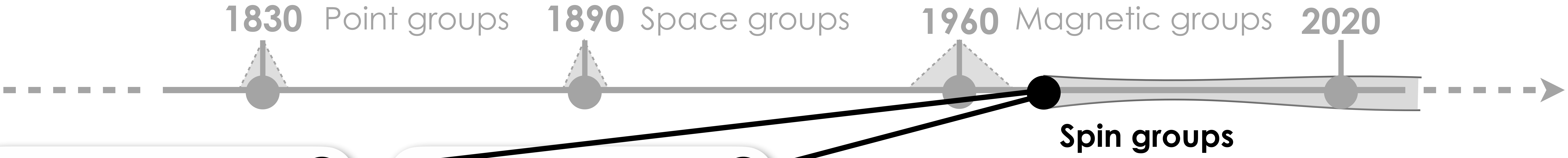
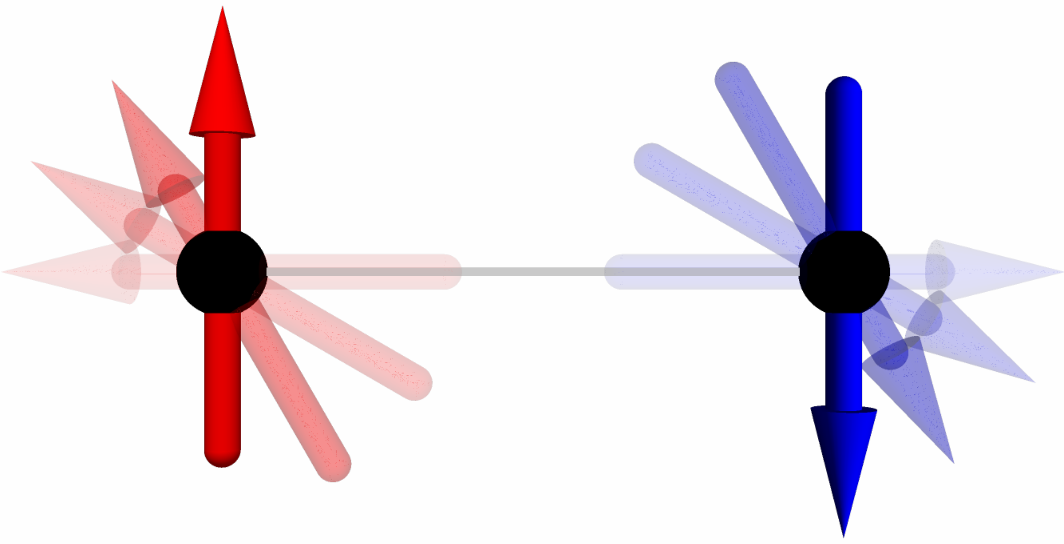

Daniel B. Litvin



Spin operation

$$[S||E]$$

Only spins transform

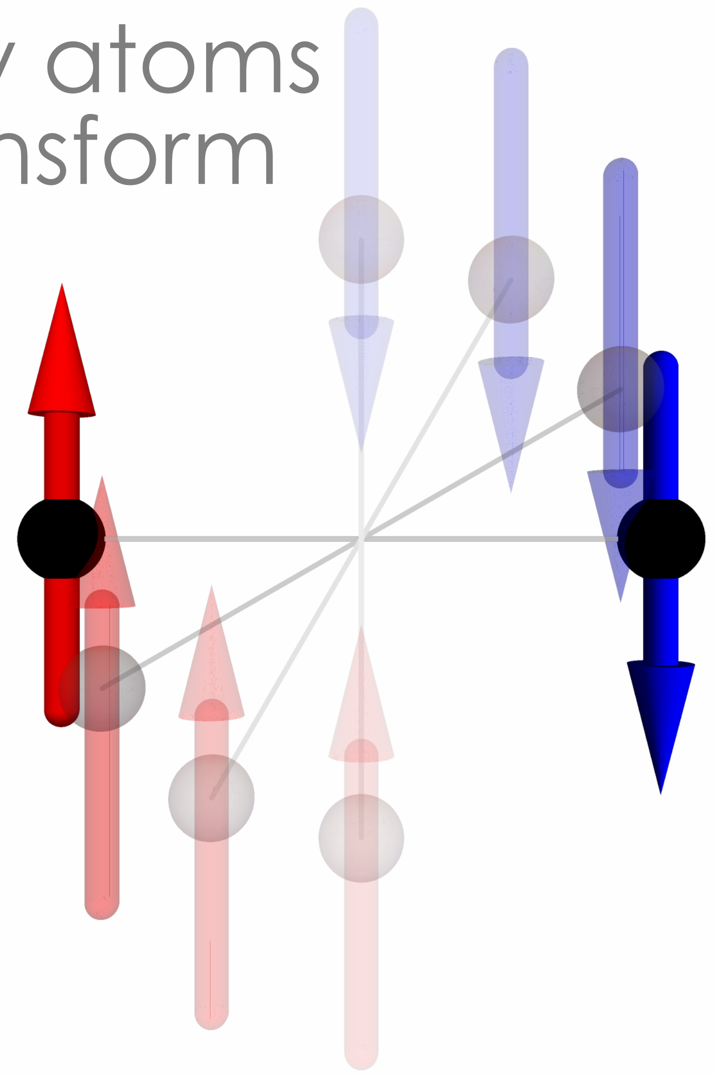


$$[S||E]\, M^{\alpha}(\mathbf{r}) = S_{\alpha\beta} M^{\beta}(\mathbf{r})$$

Lattice operation

$$[E||R]$$

Only atoms transform



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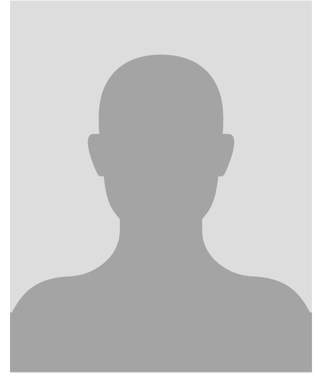
Spin groups


Albrecht Kitz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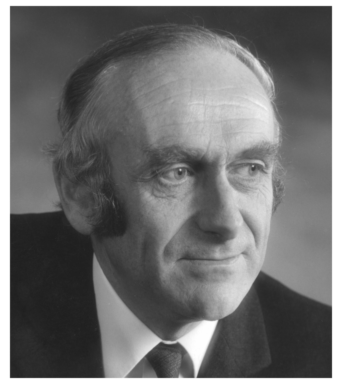
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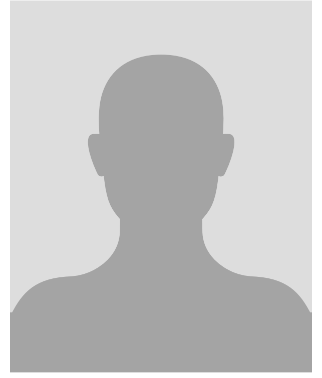
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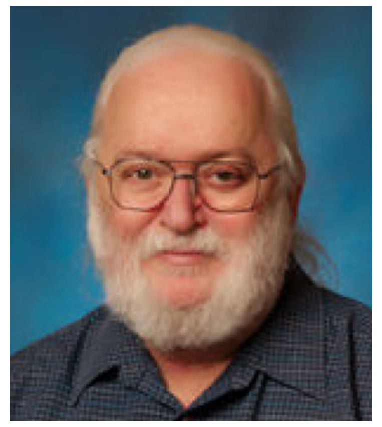
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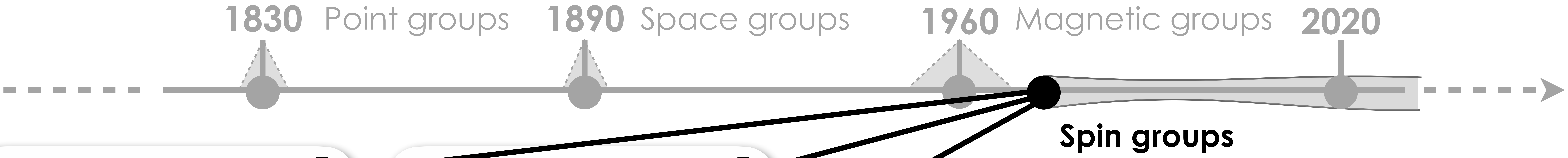
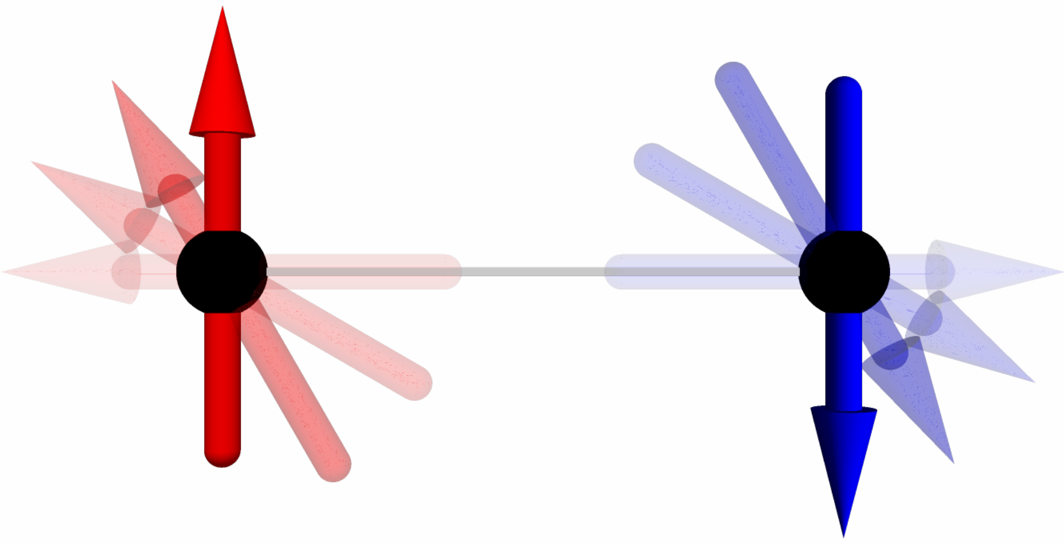


Spin operation

$$[S\|E]$$

Only spins transform

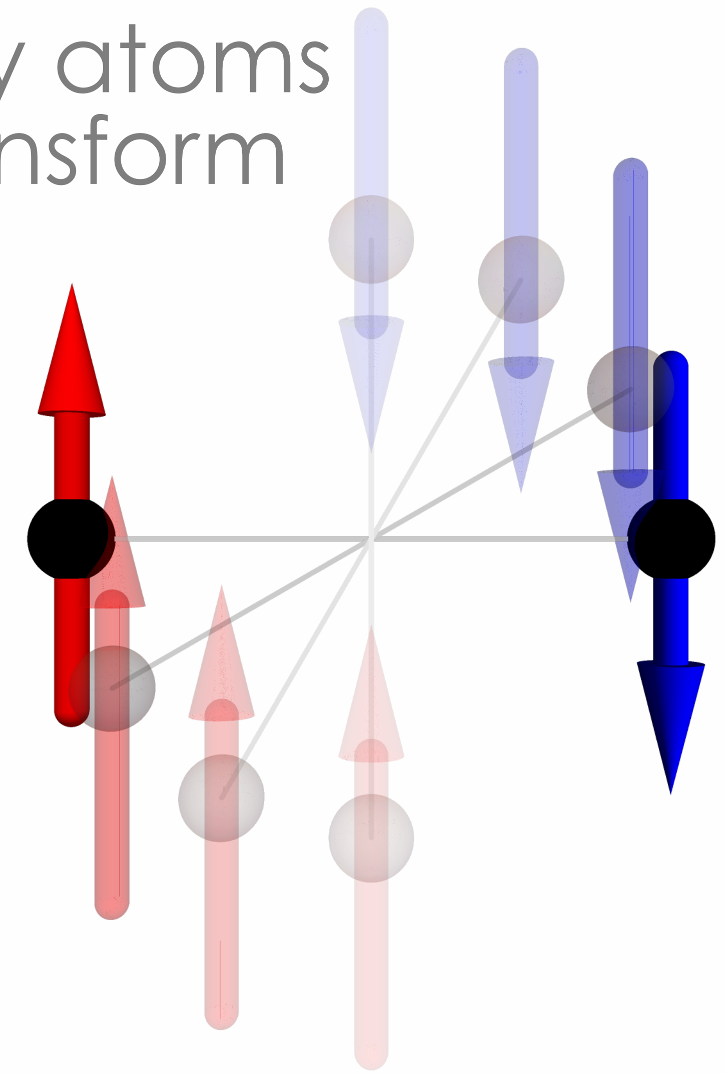


$$[S\|E]\, M^{\alpha}(\mathbf{r}) = S_{\alpha\beta} M^{\beta}(\mathbf{r})$$

Lattice operation

$$[E\|R]$$

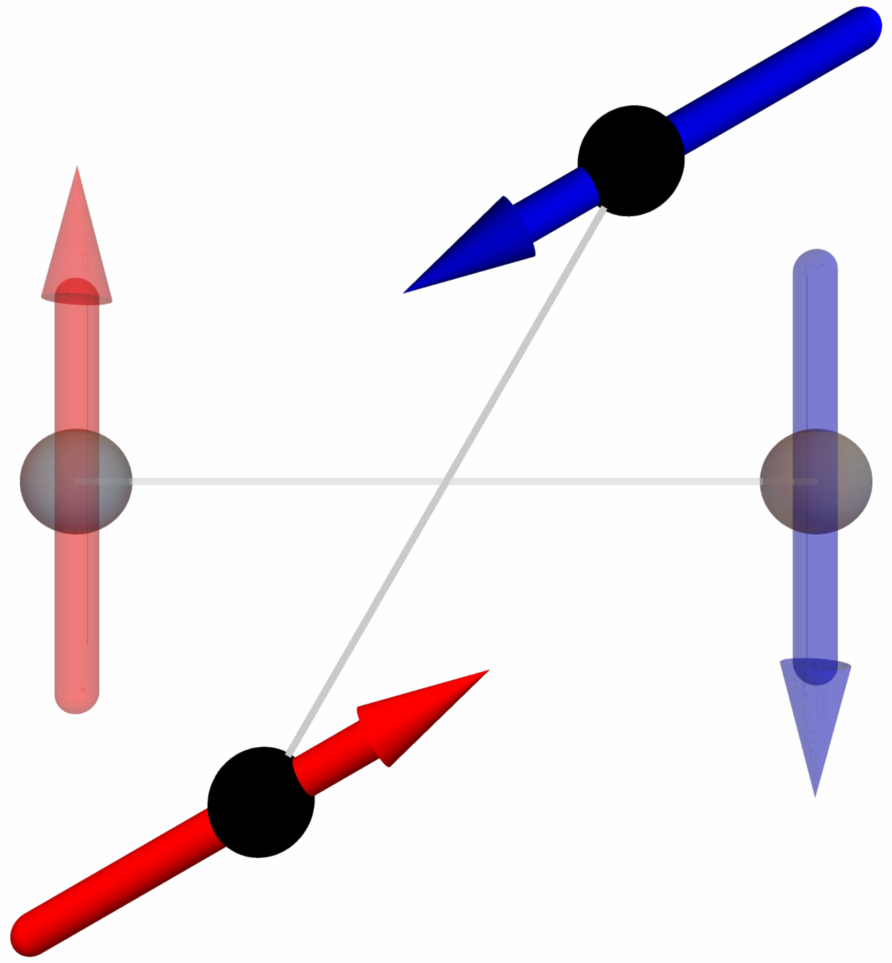
Only atoms transform



$$[E\|R]\, M^{\alpha}(\mathbf{r}) = M^{\alpha}(R^{-1}\mathbf{r})$$

Spin group transformations

$$[S\|R]$$



Generically different operations on spins and lattice

Many more possible groups!

$$[S\|R]\, M^{\alpha}(\mathbf{r}) = S_{\alpha\beta} M^{\beta}(R^{-1}\mathbf{r})$$

Spin groups

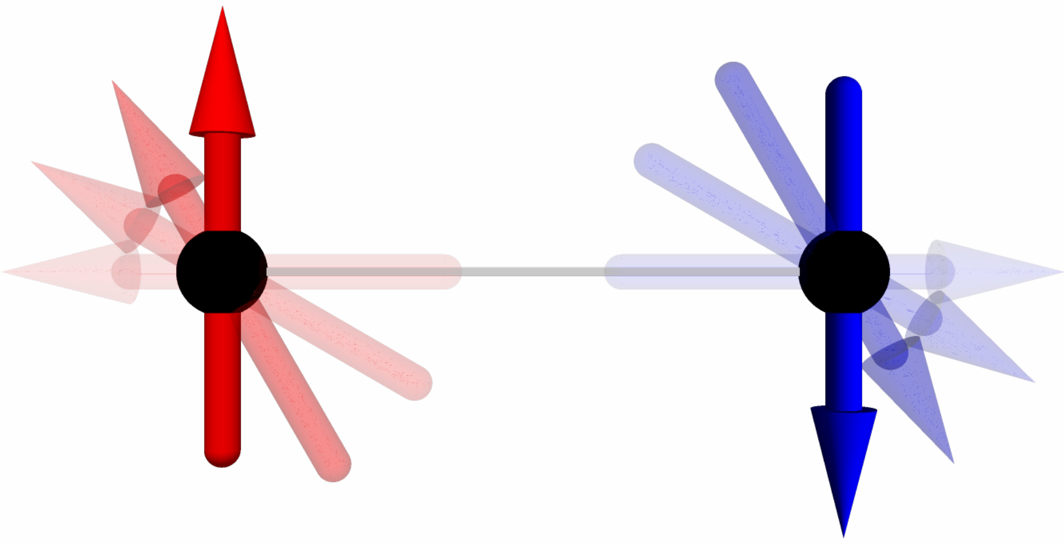
All $[S||R]$ leaving magnetic order invariant: **spin group $\underline{\mathbf{X}}$**

$$\underline{\mathbf{X}} < \mathbf{O}^s(3) \times \mathbf{E}(3)$$

Spin operation

$$[S||E]$$

Only spins transform



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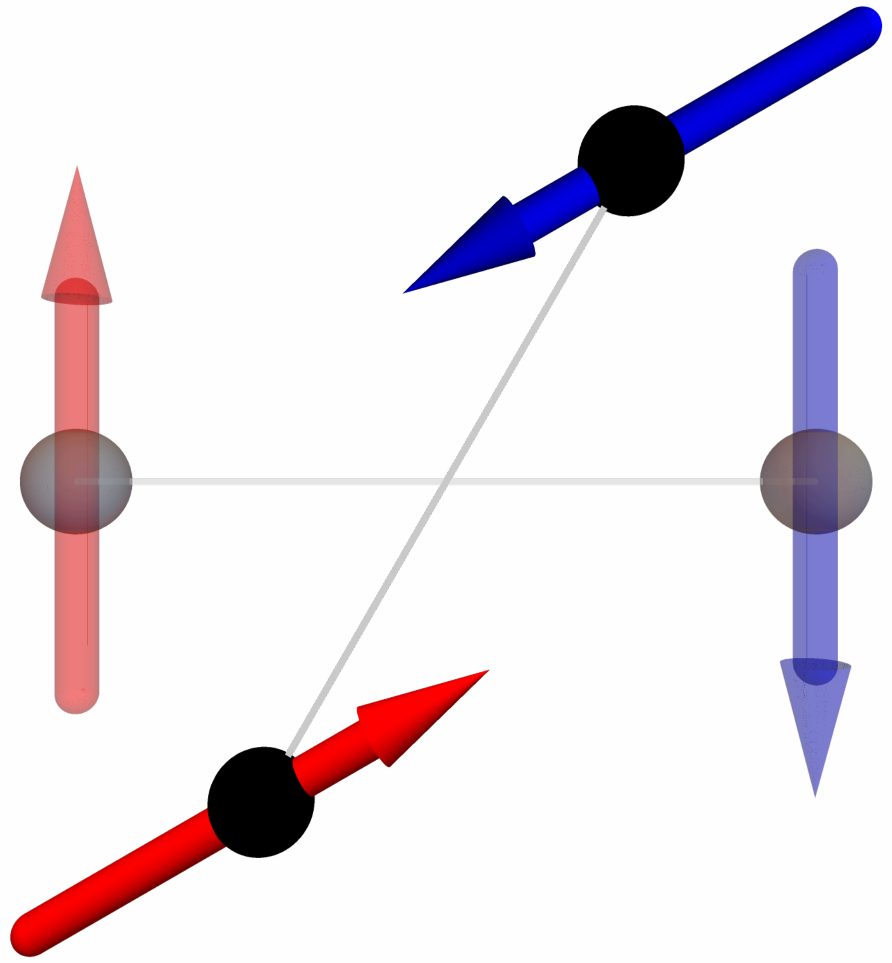

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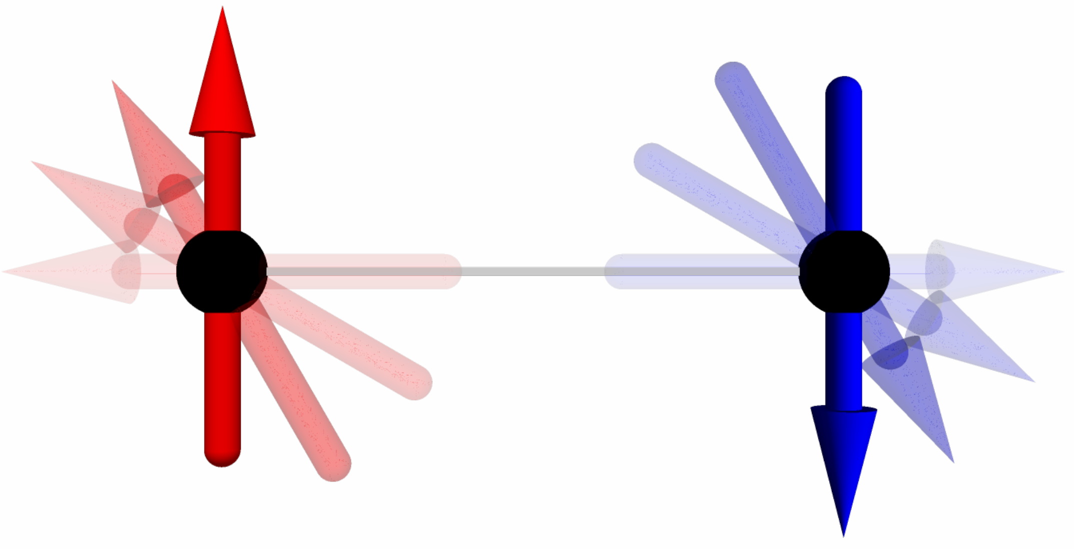
$\mathbf{SO}(3) + \tau\mathbf{SO}(3)$ $\quad$ $\mathbf{T}(3) \rtimes \mathbf{O}(3)$

Spin operation

$$[S||E]$$

Only spins transform



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Spin group transformations

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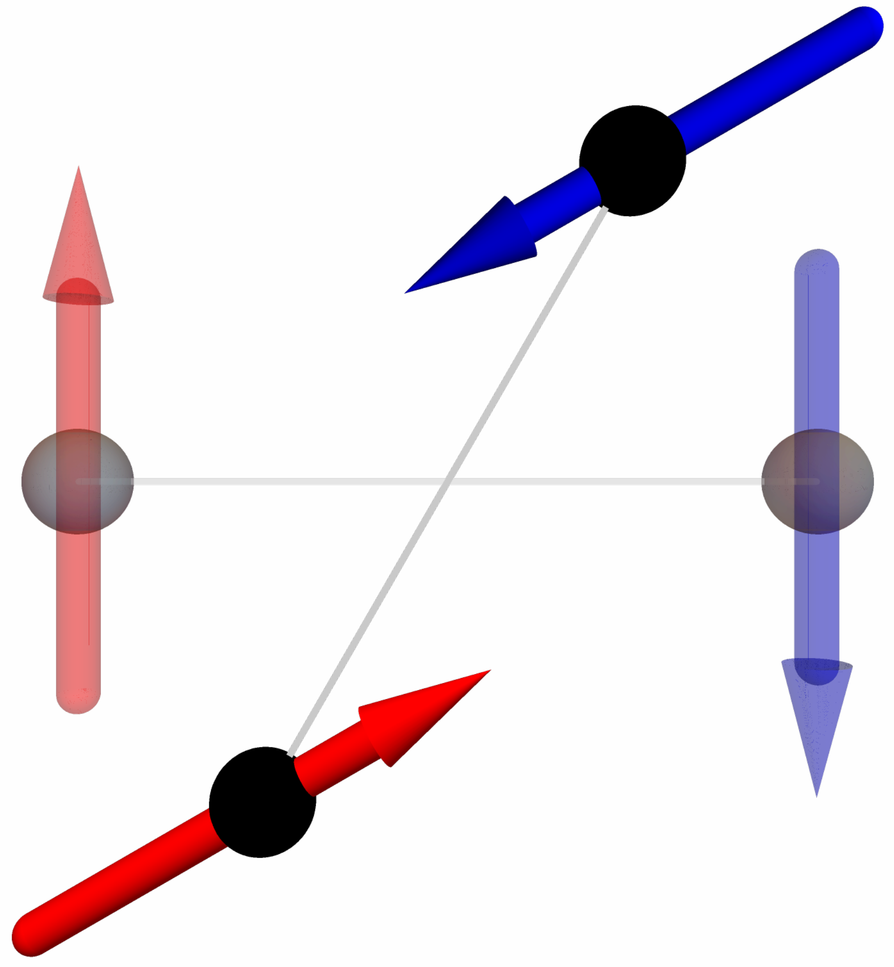

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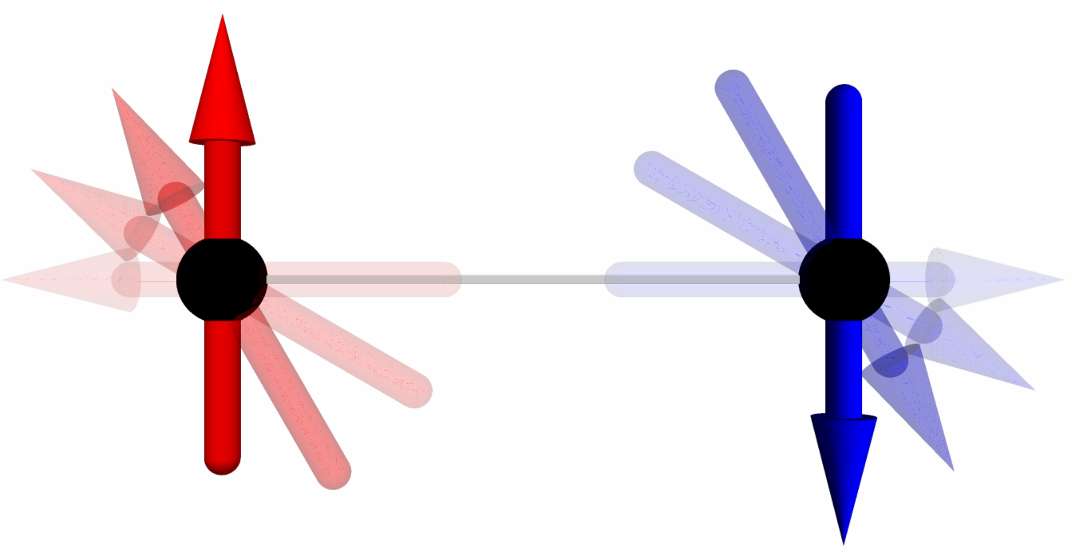


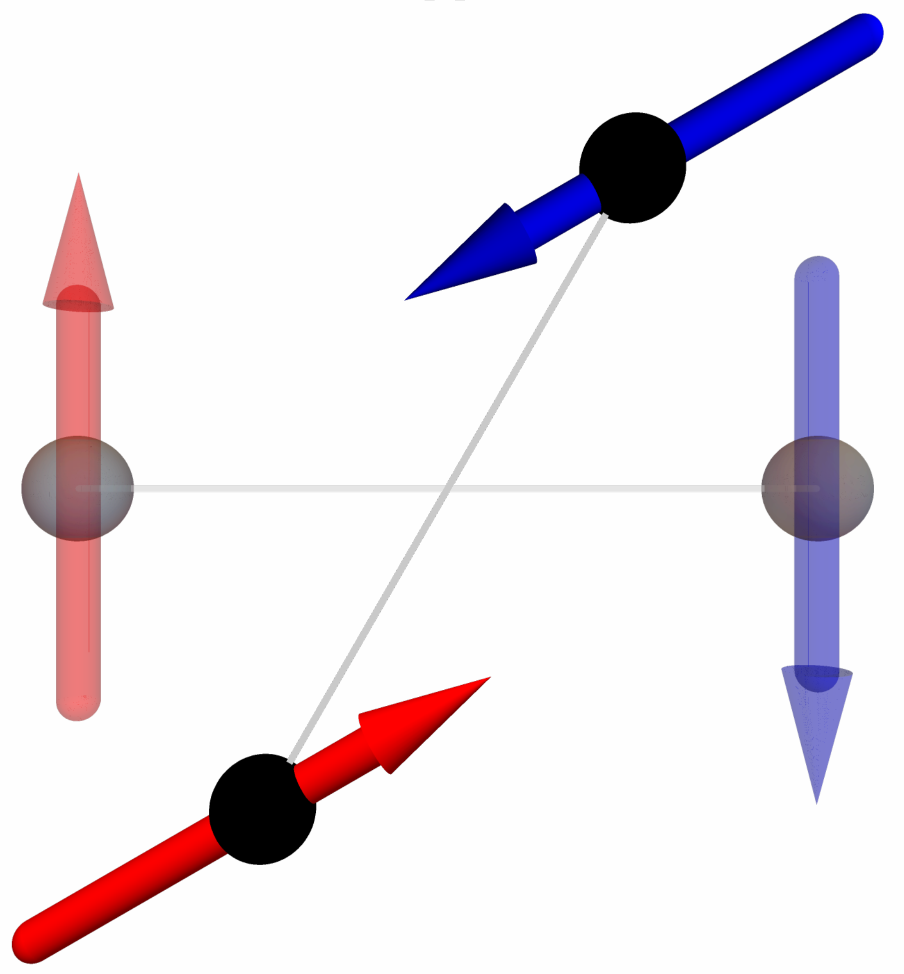
$$[S||E]\, M^\alpha(\mathbf{r}) = S_{\alpha\beta} M^\beta(\mathbf{r})$$

All elements $[S||E]$ form a normal subgroup, the **spin-only group**

$$\underline{\mathbf{b}} = \{\, [S||E] \in \underline{\mathbf{X}} \,\}$$

Spin group transformations

$$[S||R]$$



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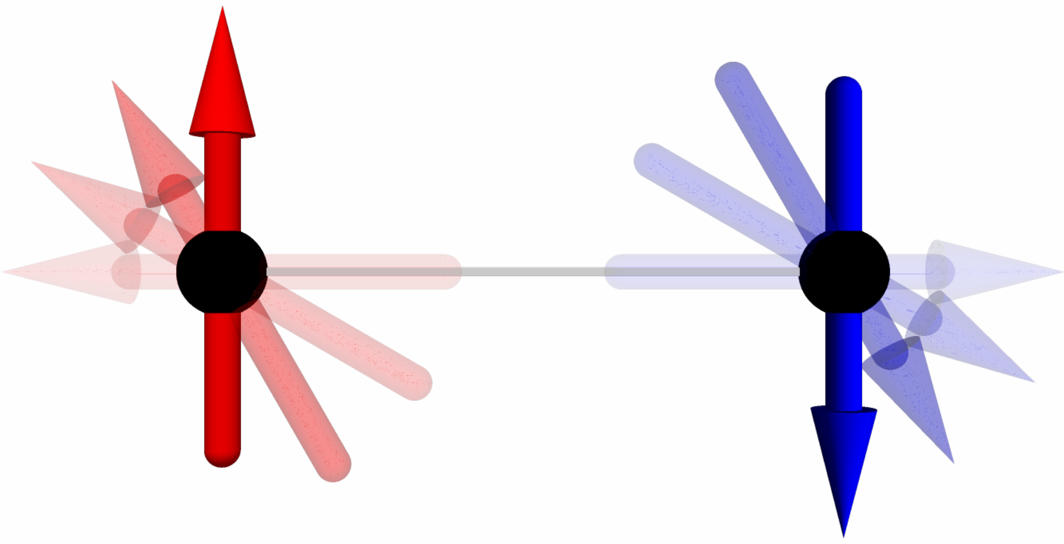
Spin groups

Spin operation

$$[S\|E]$$

Only spins transform



$$[S\|E]\, M^\alpha(\mathbf{r}) = S_{\alpha\beta} M^\beta(\mathbf{r})$$

$$\underline{\mathbf{X}} = \underline{\mathbf{b}} \times \underline{\mathbf{S}}$$

The **non-trivial (NT) spin group $\underline{\mathbf{S}}$** only contains $[S\|R]$ where $R \neq E$

All elements $[S\|E]$ form a normal subgroup, the **spin-only group**

$$\underline{\mathbf{b}} = \{\, [S\|E] \in \underline{\mathbf{X}} \,\}$$

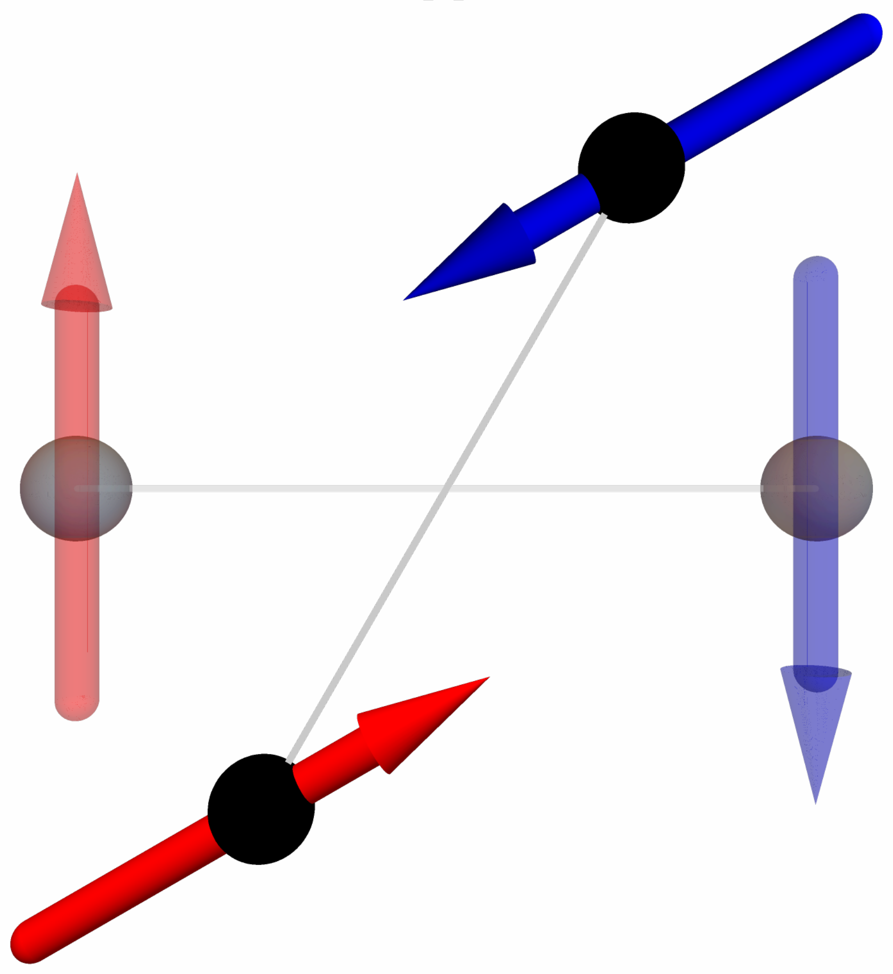
All $[S\|R]$ leaving magnetic order invariant: **spin group $\underline{\mathbf{X}}$**

$$\underline{\mathbf{X}} < \mathbf{O}^s(3) \times \mathbf{E}(3)$$

$\mathbf{SO}(3) + \tau\mathbf{SO}(3)$ $\mathbf{T}(3) \rtimes \mathbf{O}(3)$

Spin group transformations

$$[S\|R]$$



Generically different operations on spins and lattice

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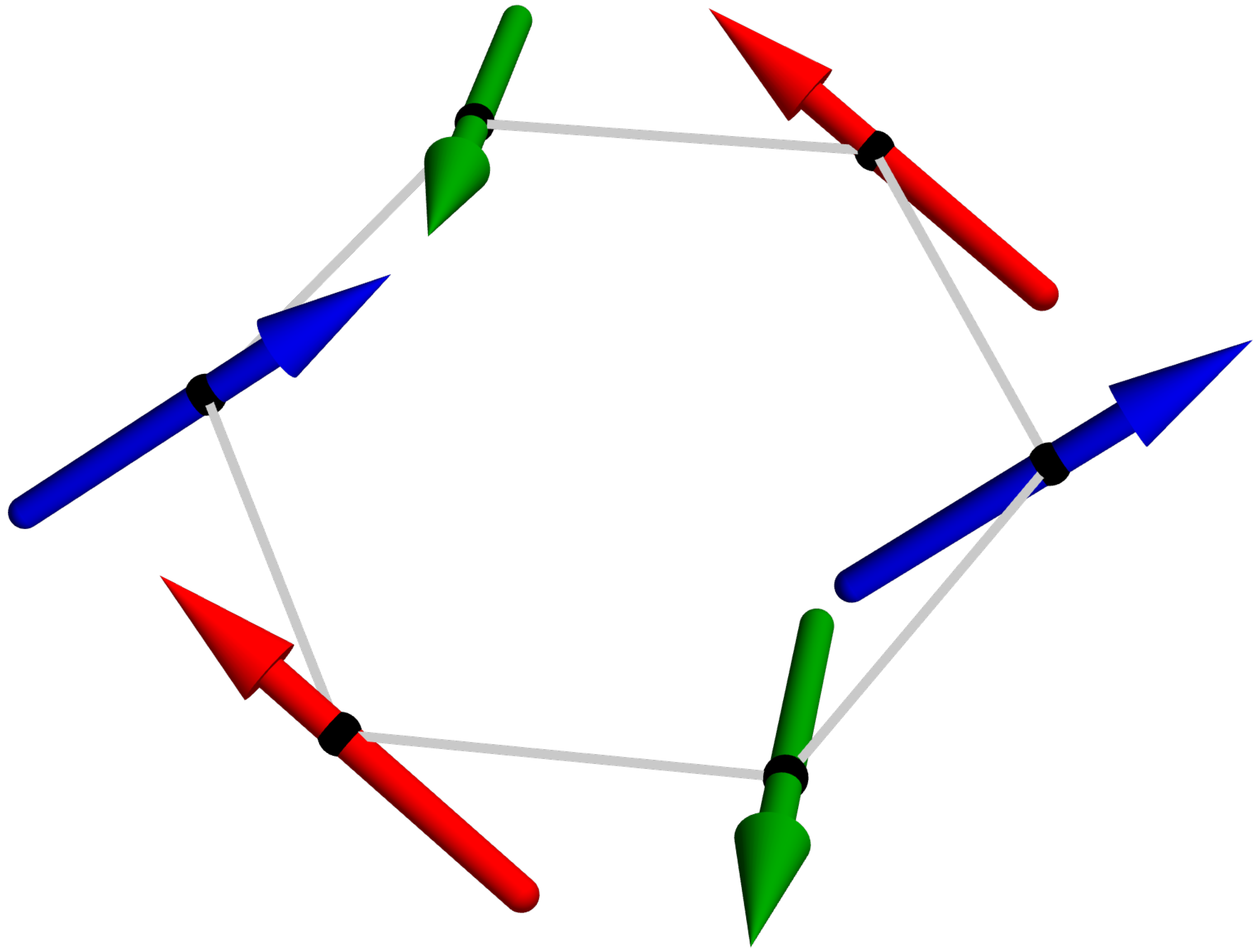
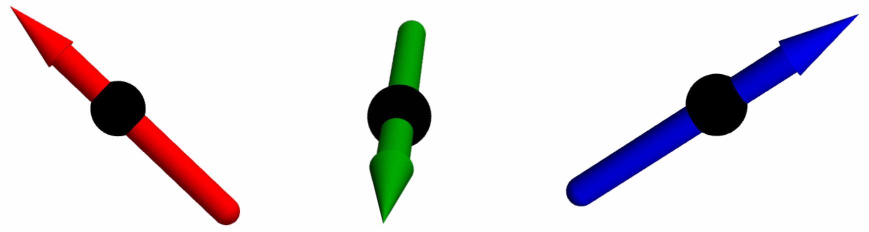
Example

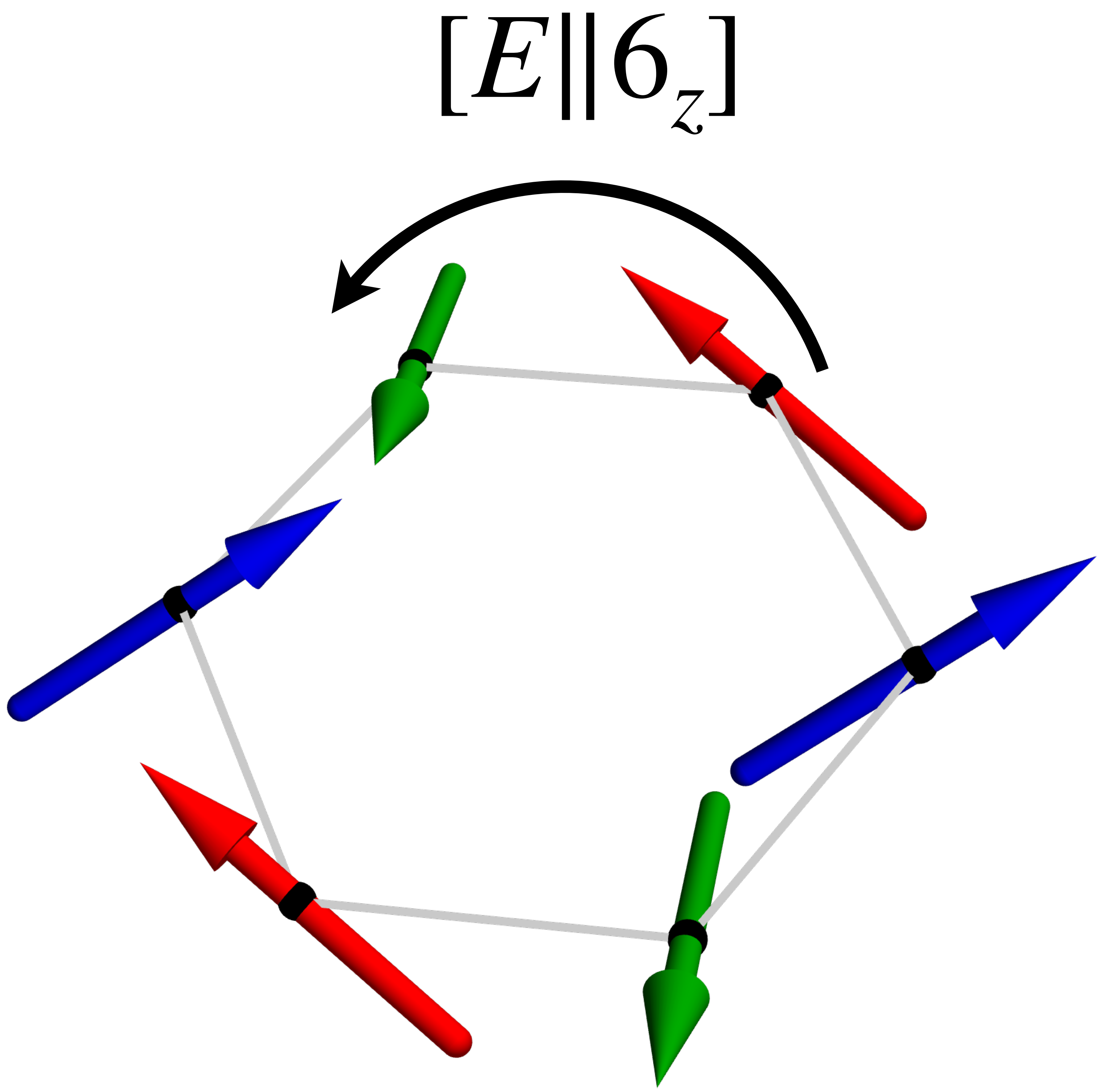


Atoms related by 60° rotation, but spins on neighboring sites are 120° relative to each other

(Colors correspond to possible spin directions)

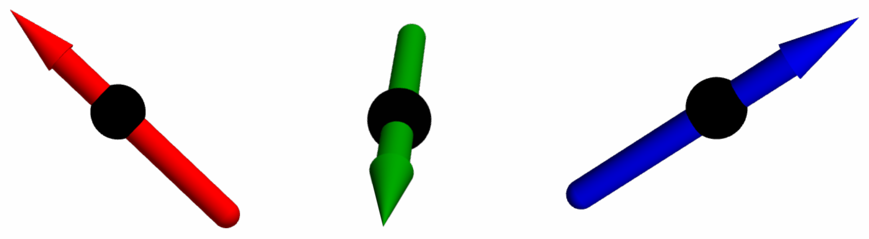


Example

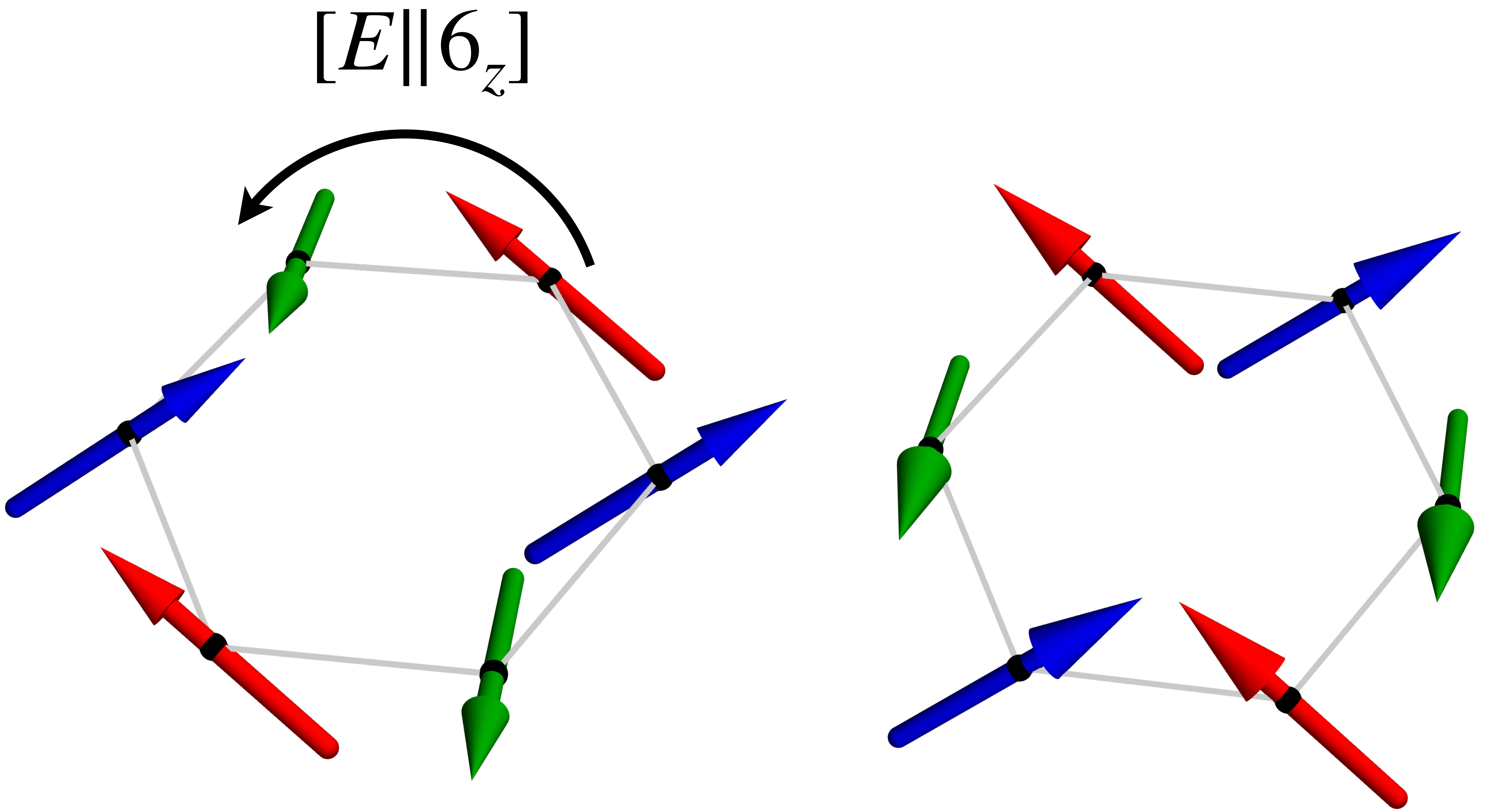


Atoms related by $60°$ rotation, but spins on neighboring sites are $120°$ relative to each other

(Colors correspond to possible spin directions)



Example



Atoms related by $60°$ rotation, but spins on neighboring sites are $120°$ relative to each other

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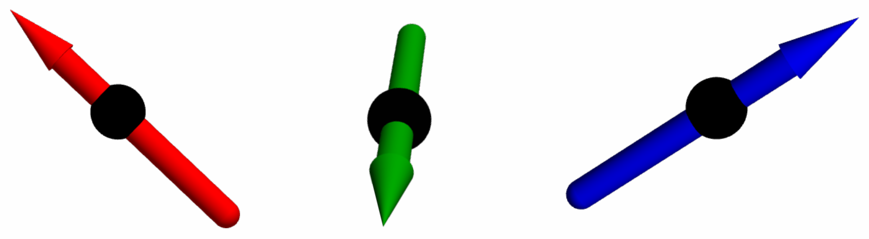
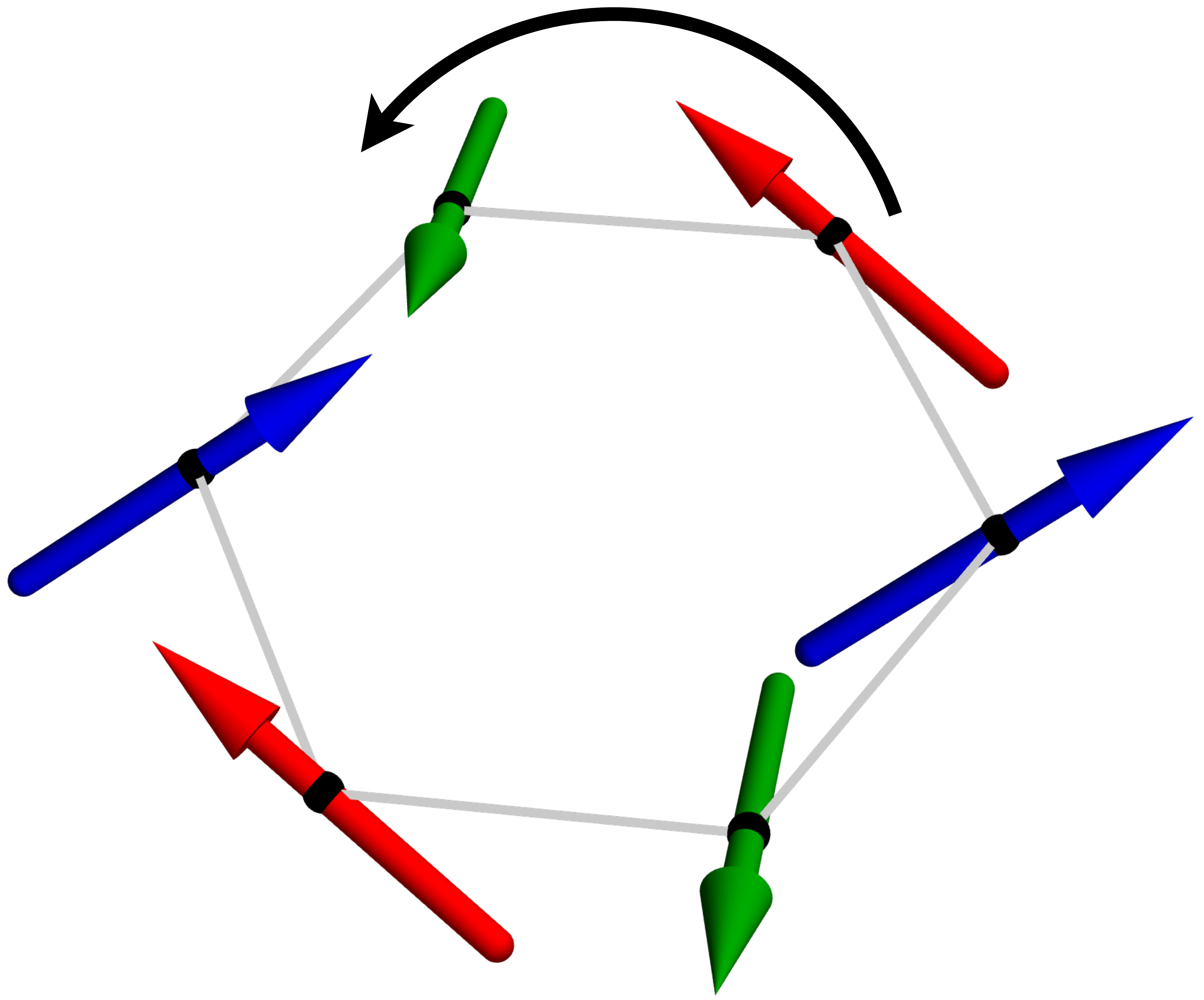
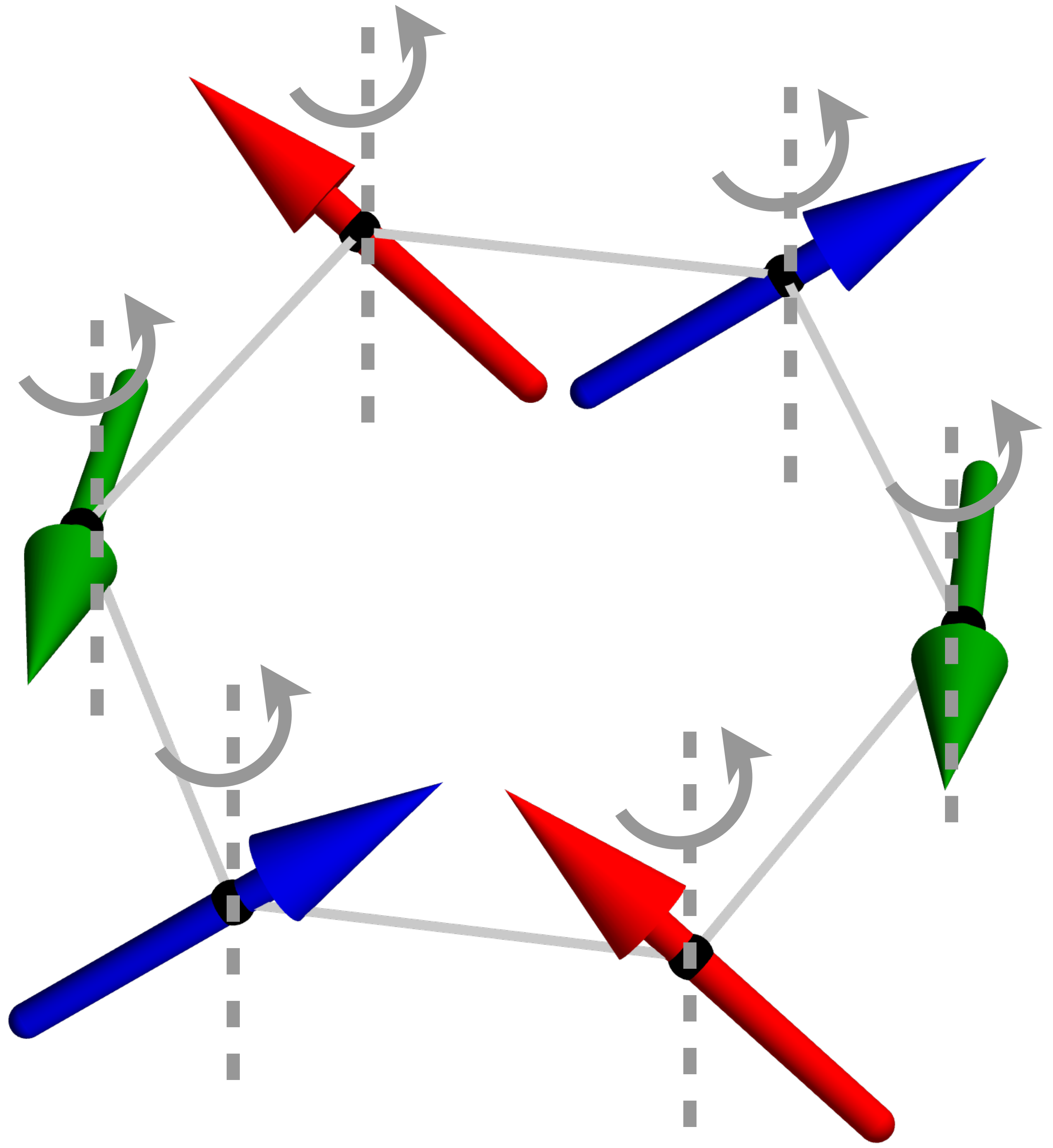


Example

$$[E\|6_z] \quad \bullet \quad [3_z\|E]$$





Atoms related by $60°$ rotation,
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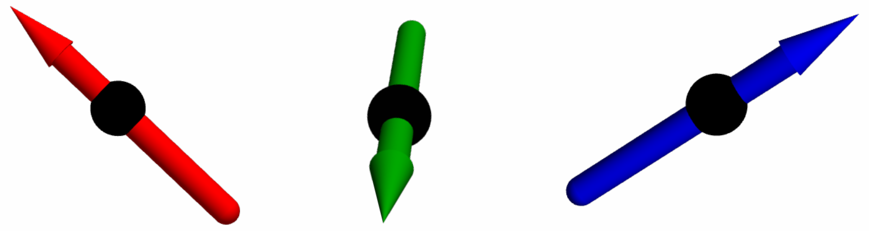
are $120°$ relative to each other

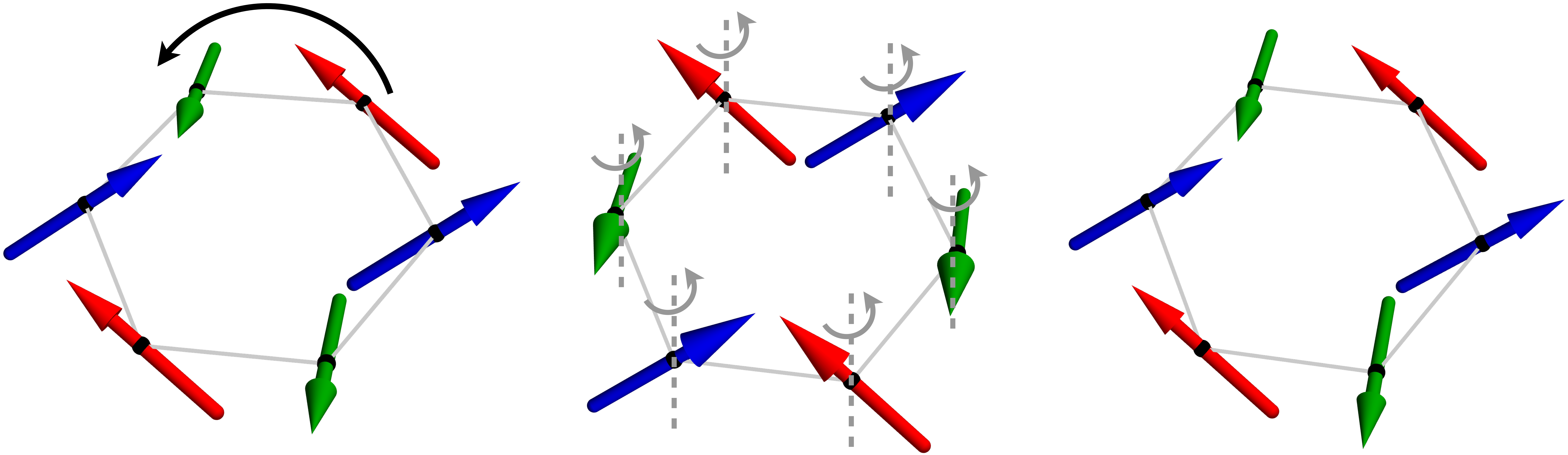
(Colors correspond to possible spin directions)



Example

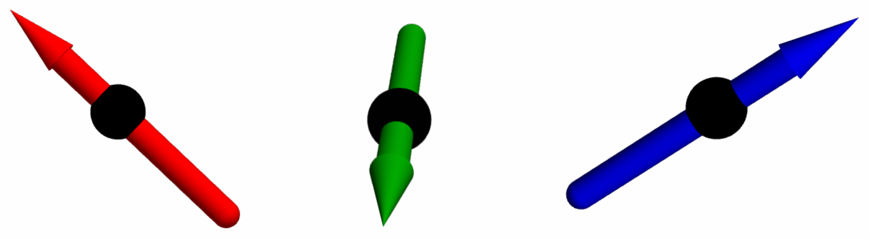
$^{3_z}6/^{1}m^{m_x}m^{m_{xy}}m$

$[E||6_z] \quad \bullet \quad [3_z||E] \quad = \quad [\mathbf{3}_z||\mathbf{6}_z]$



Atoms related by 60° rotation, but spins on neighboring sites are 120° relative to each other

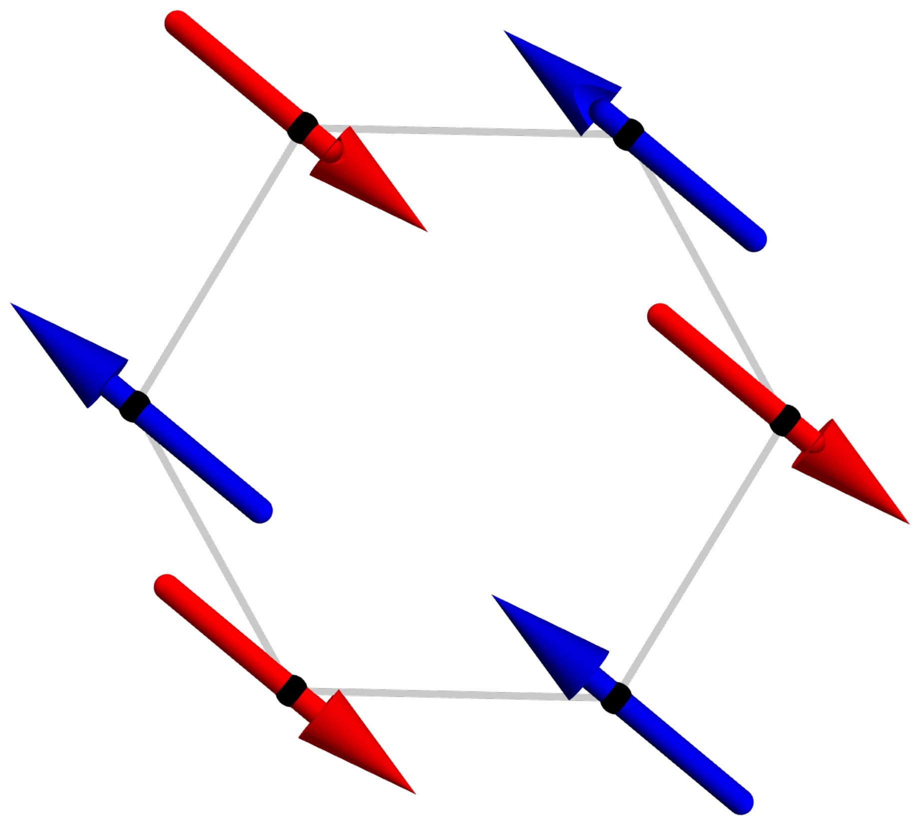
(Colors correspond to possible spin directions)



Spin-only group

depends on mutual spin orientations

Collinear



D.B. Litvin, W. Opechowski, Physica Vol. 76, Iss. 3, (1974)

Spin groups $\underline{\mathbf{X}}$

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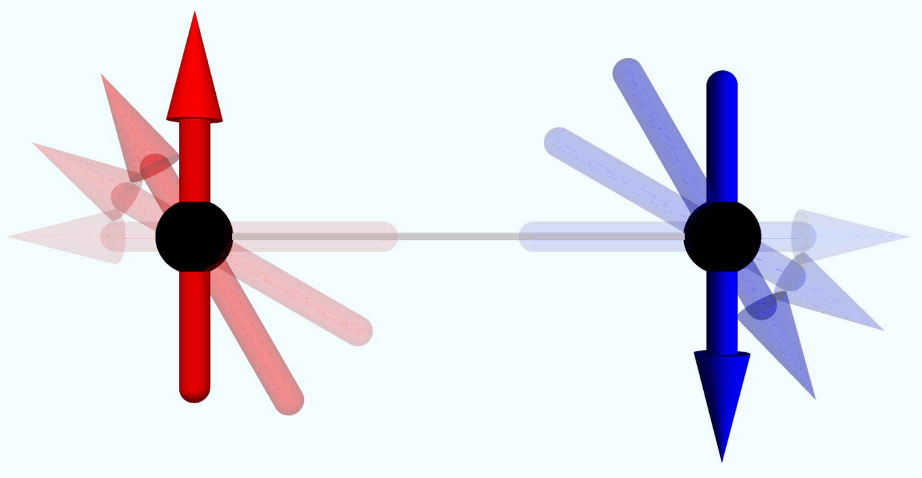
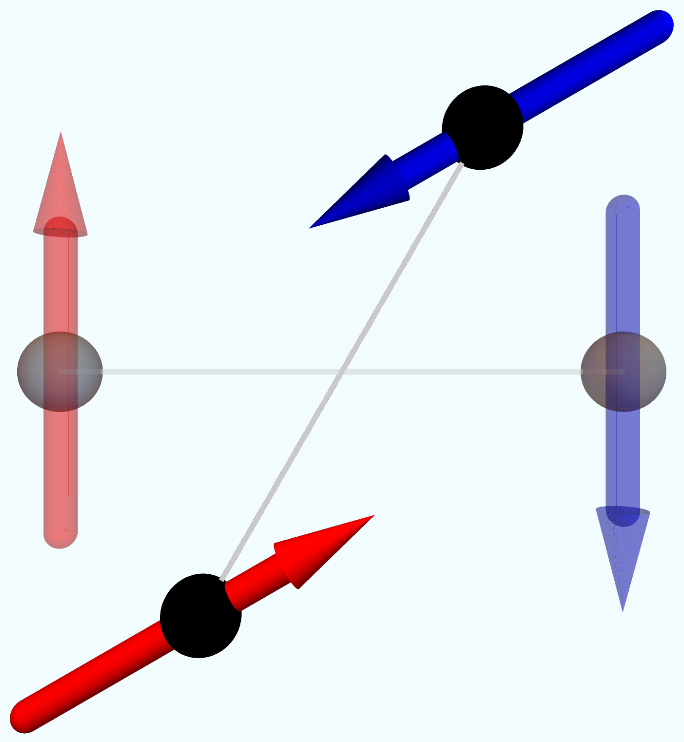


Non-trivial spin group

$[S||R]$ with $R \neq E$



$$[S||R]\, M^{\alpha}(\mathbf{r}) = S_{\alpha\beta} M^{\beta}(R^{-1}\mathbf{r})$$

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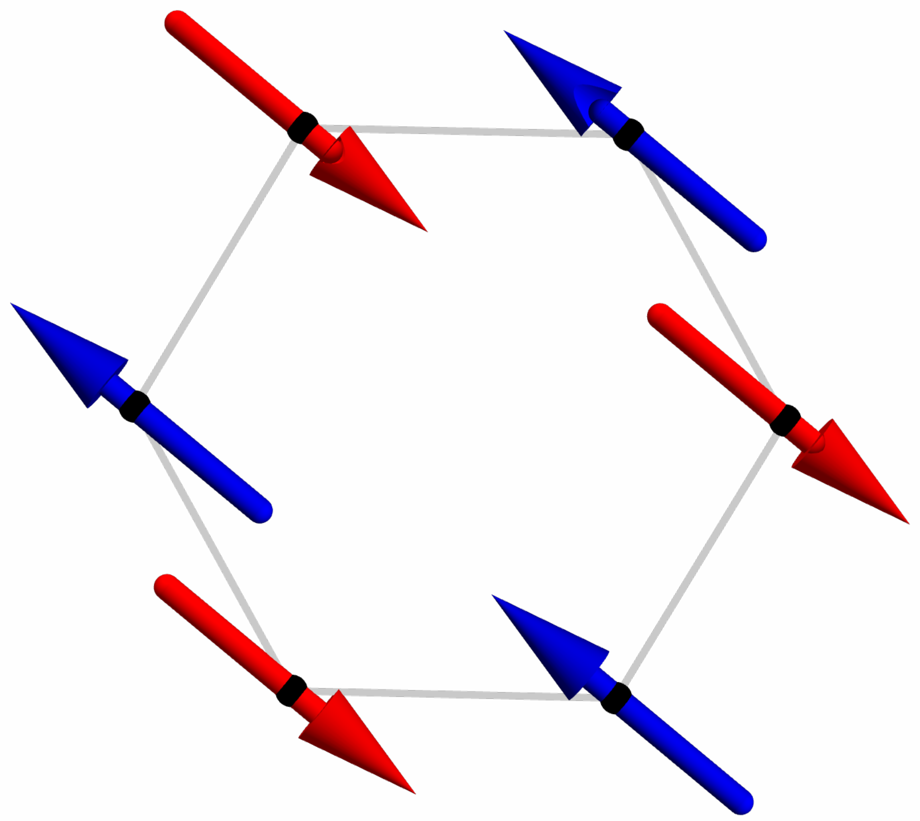
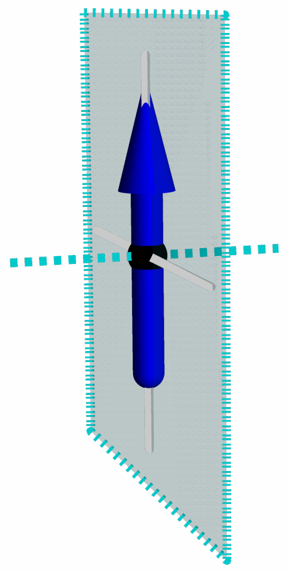


$\underline{\mathbf{b}}^{\infty} \cong \mathbf{SO}(2) \rtimes \{E, \tau 2_{\perp}\}$

$2_{\perp}$: π rotation about axis $\perp$ to spin axis



$\tau 2_{\perp}$ is a mirror containing the spin axis

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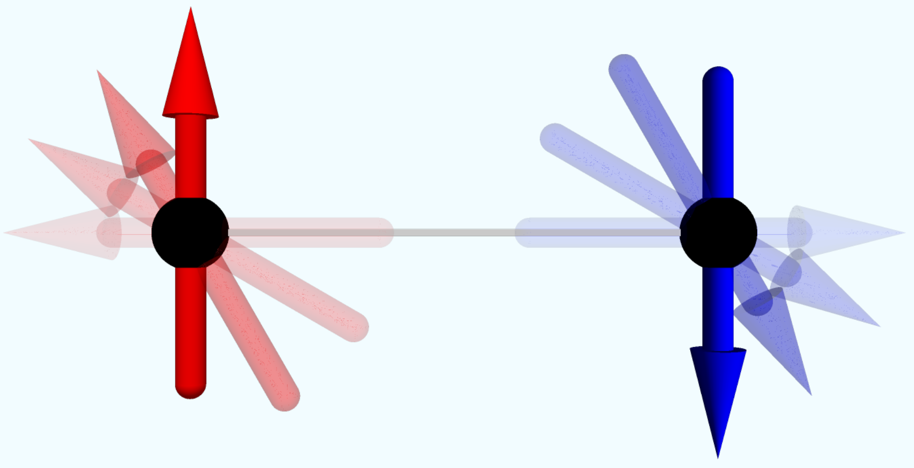

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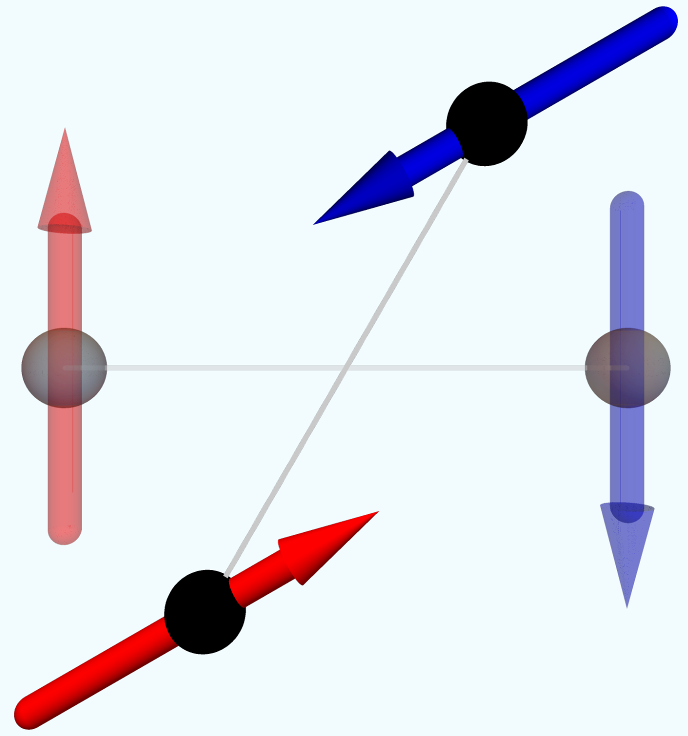

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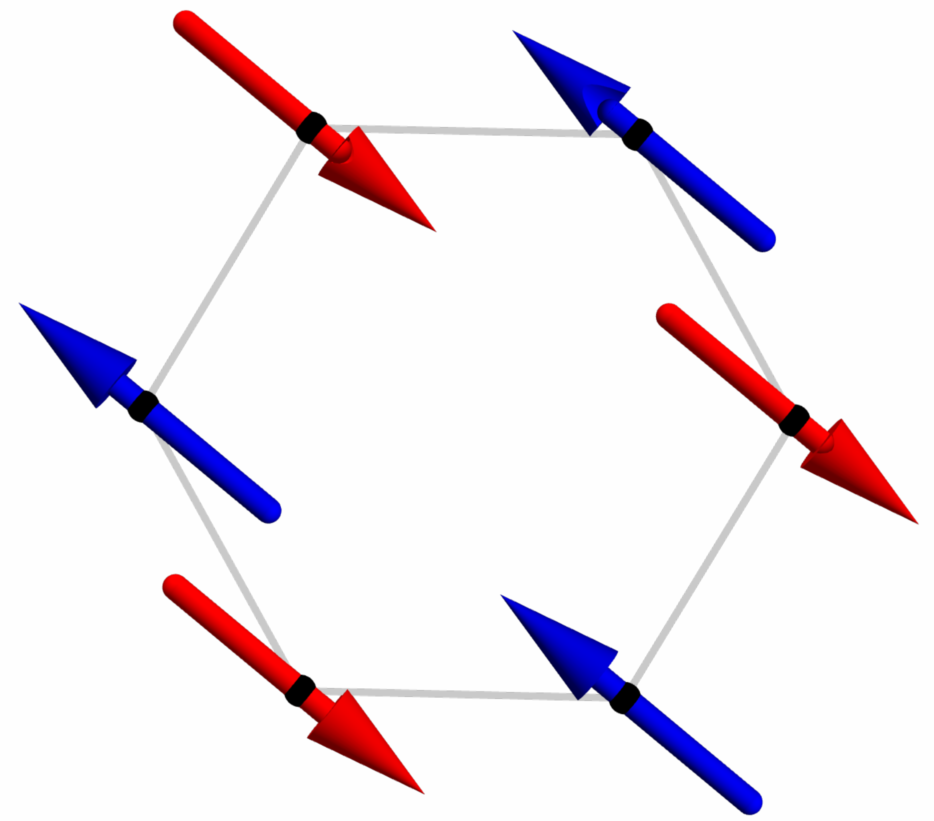

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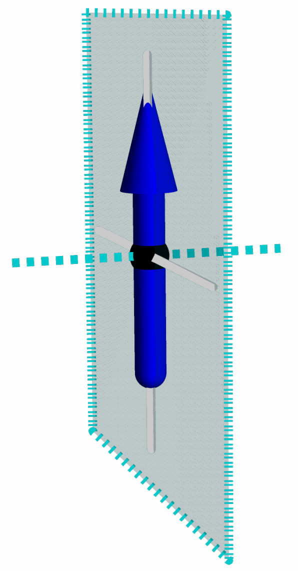

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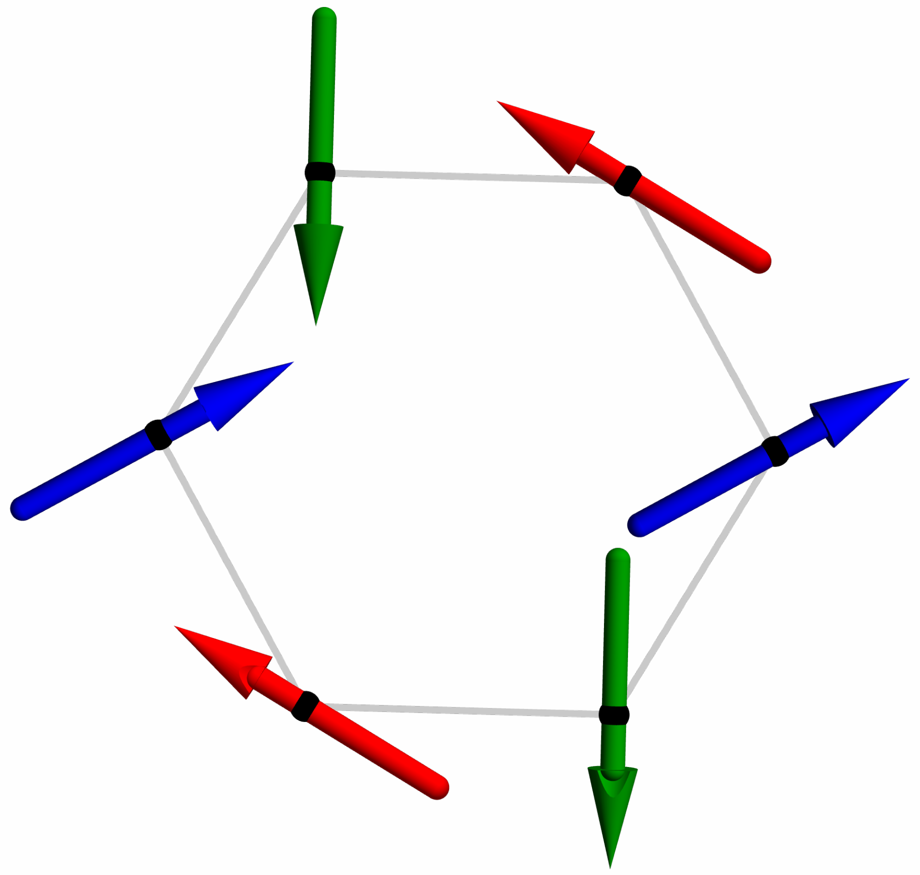

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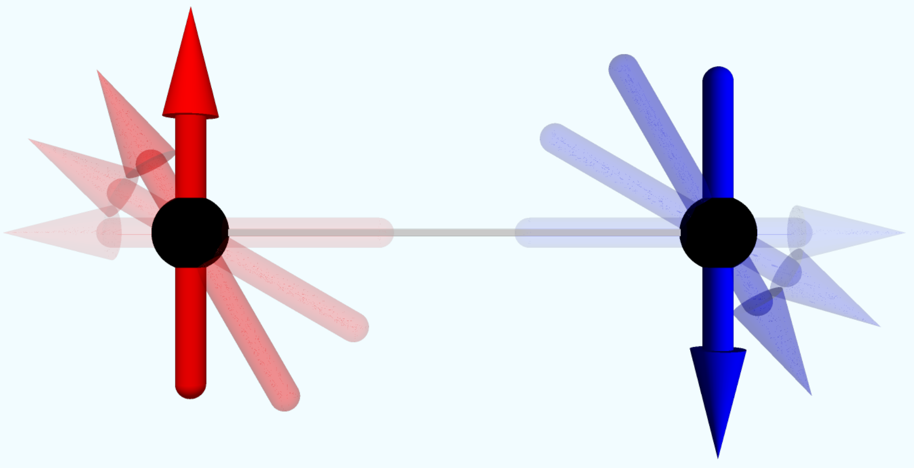

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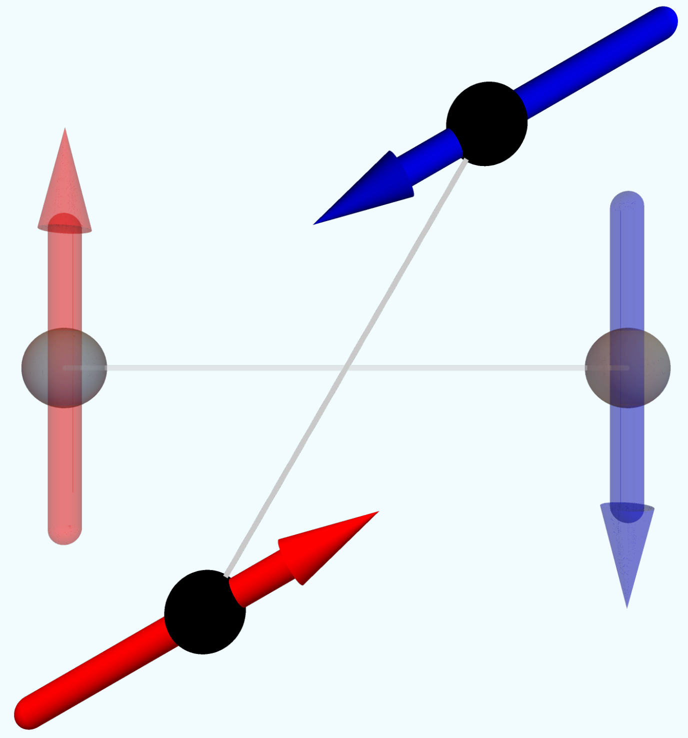

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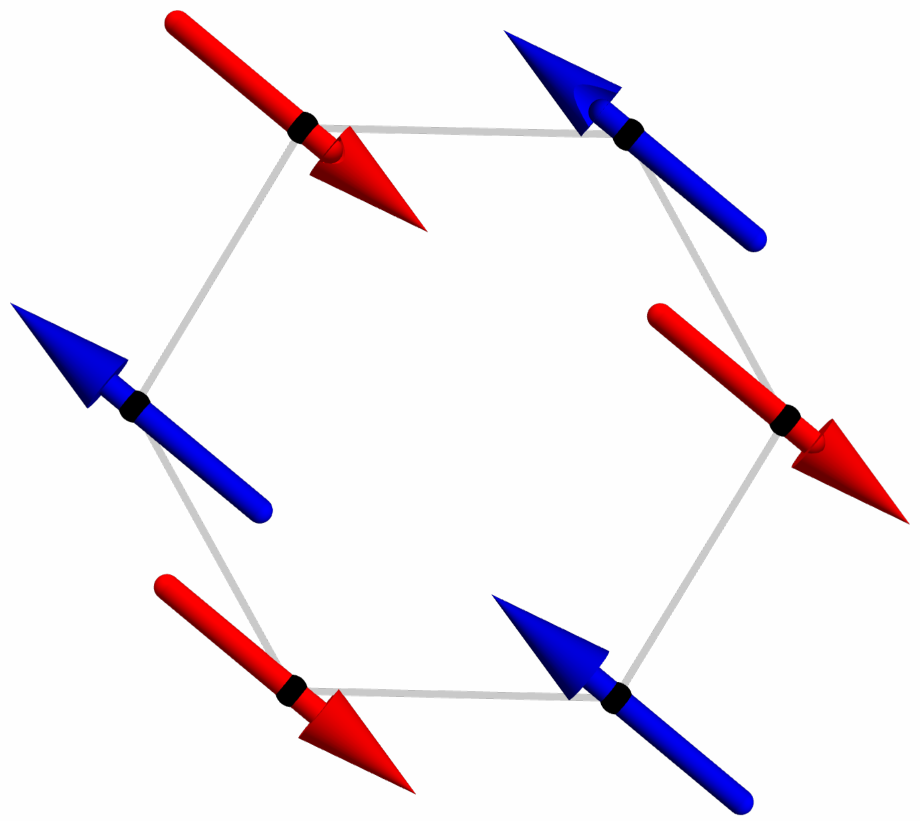

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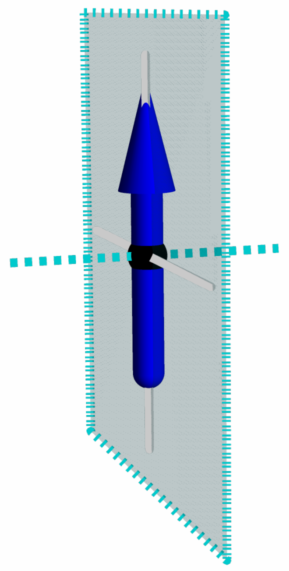

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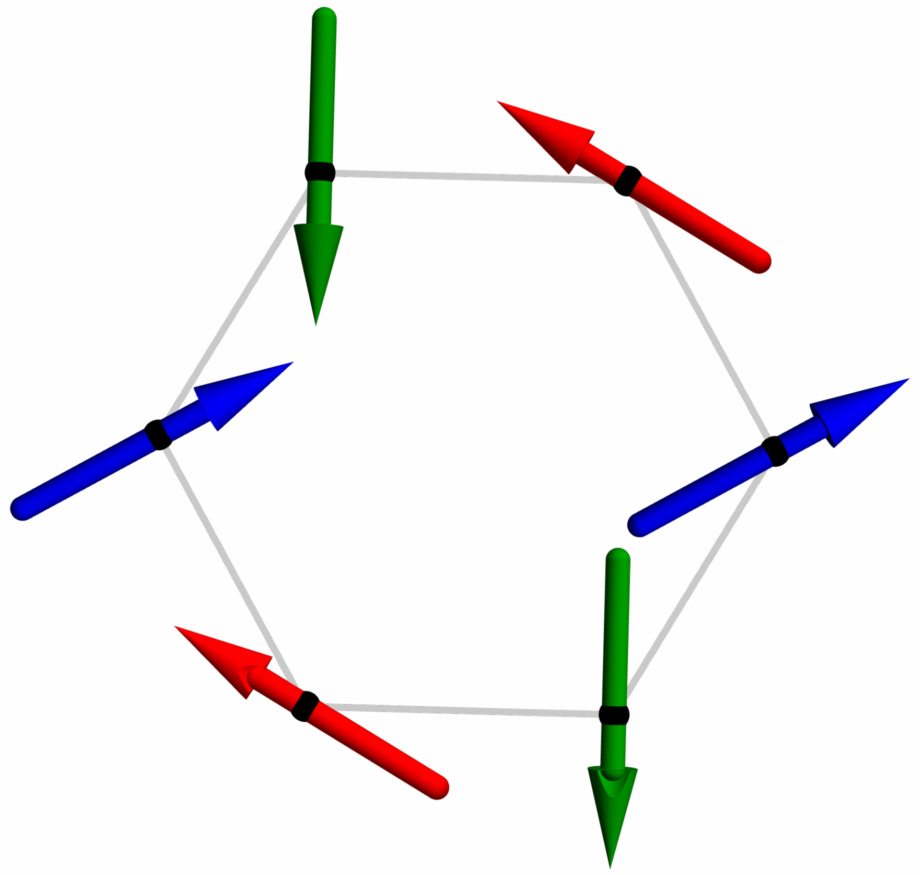

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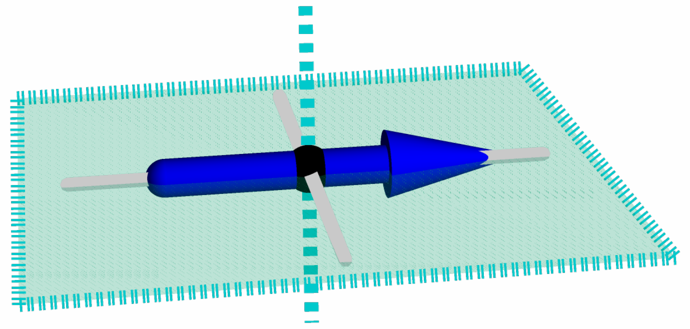

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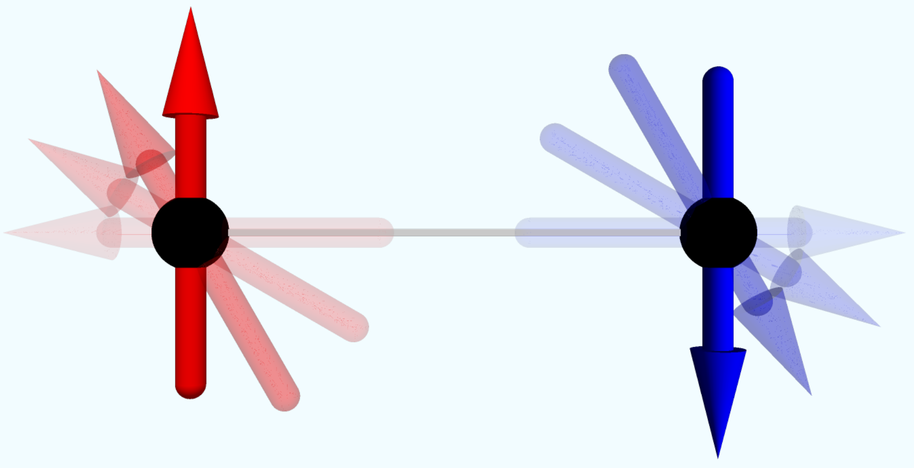

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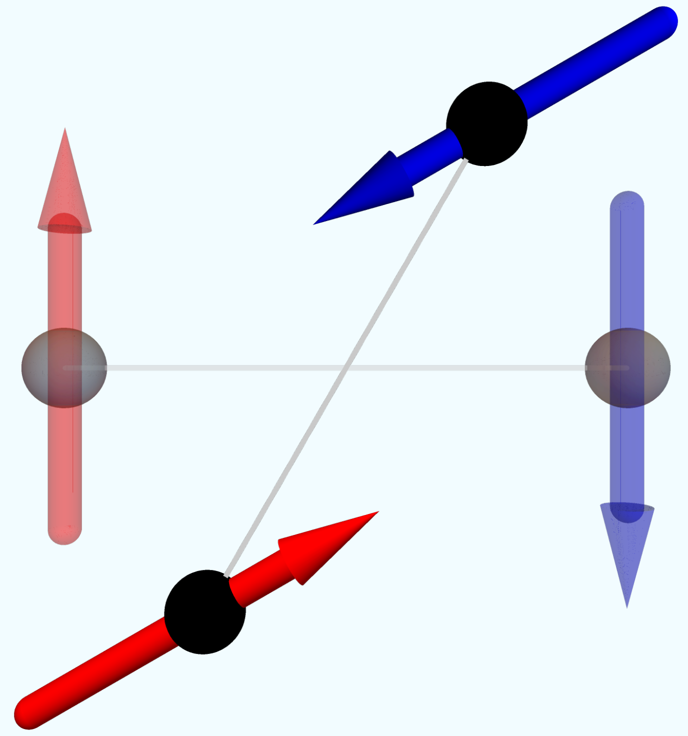

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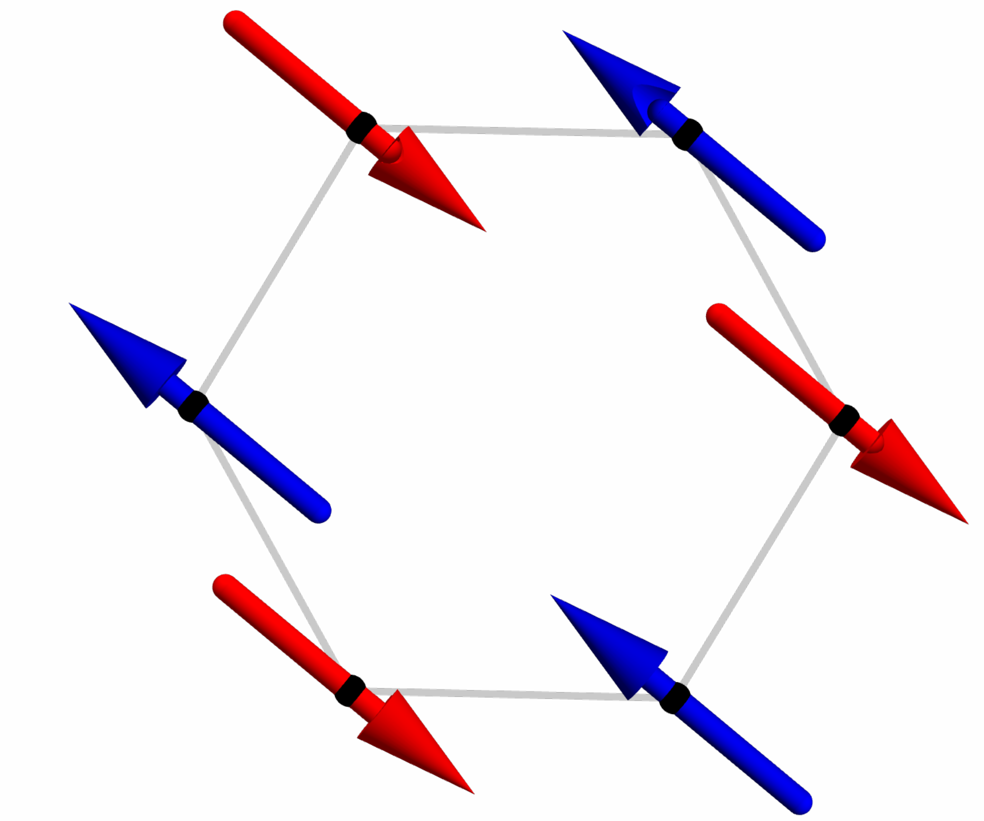

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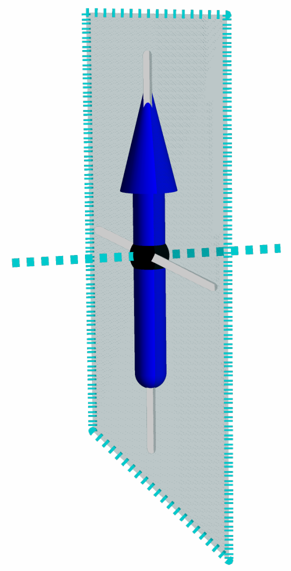

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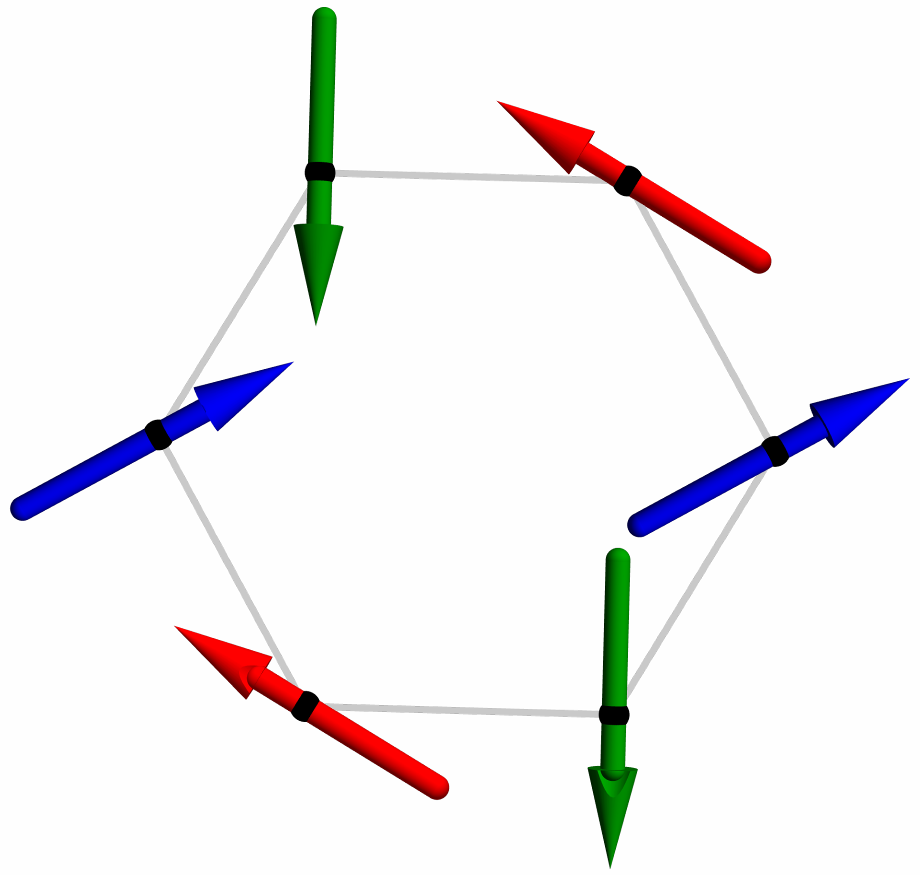

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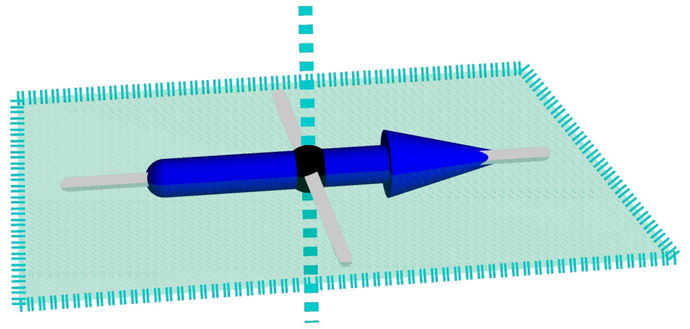

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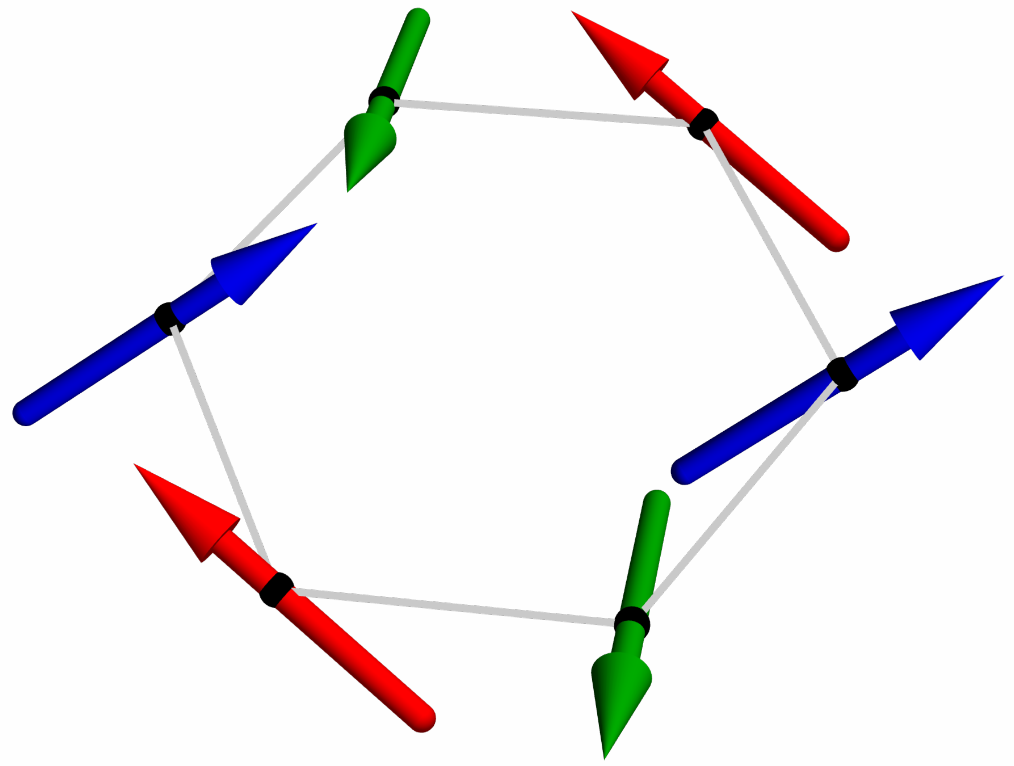

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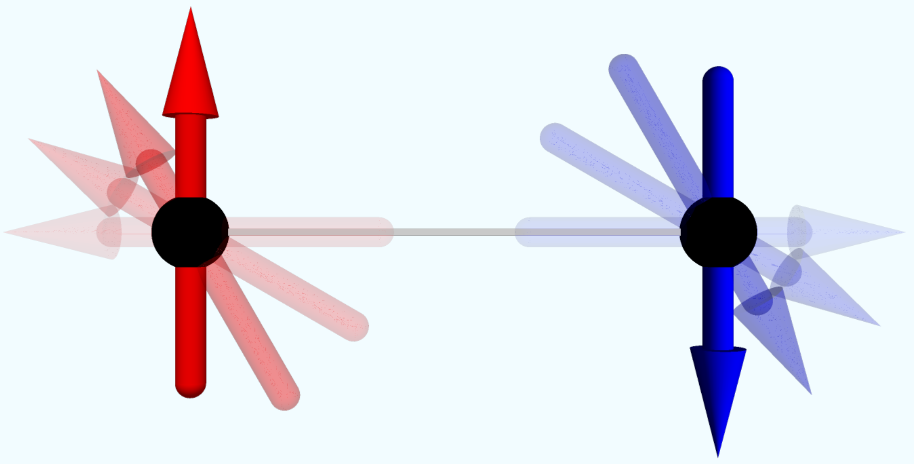

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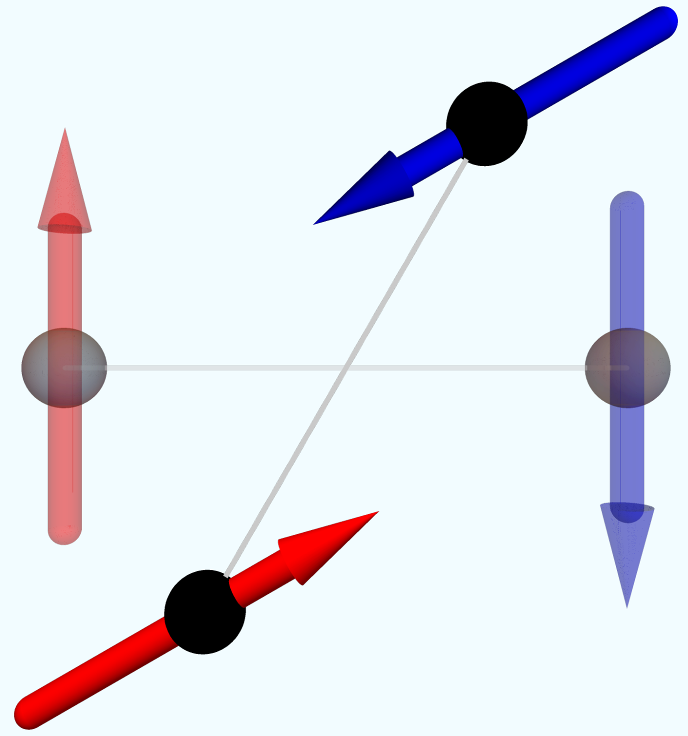

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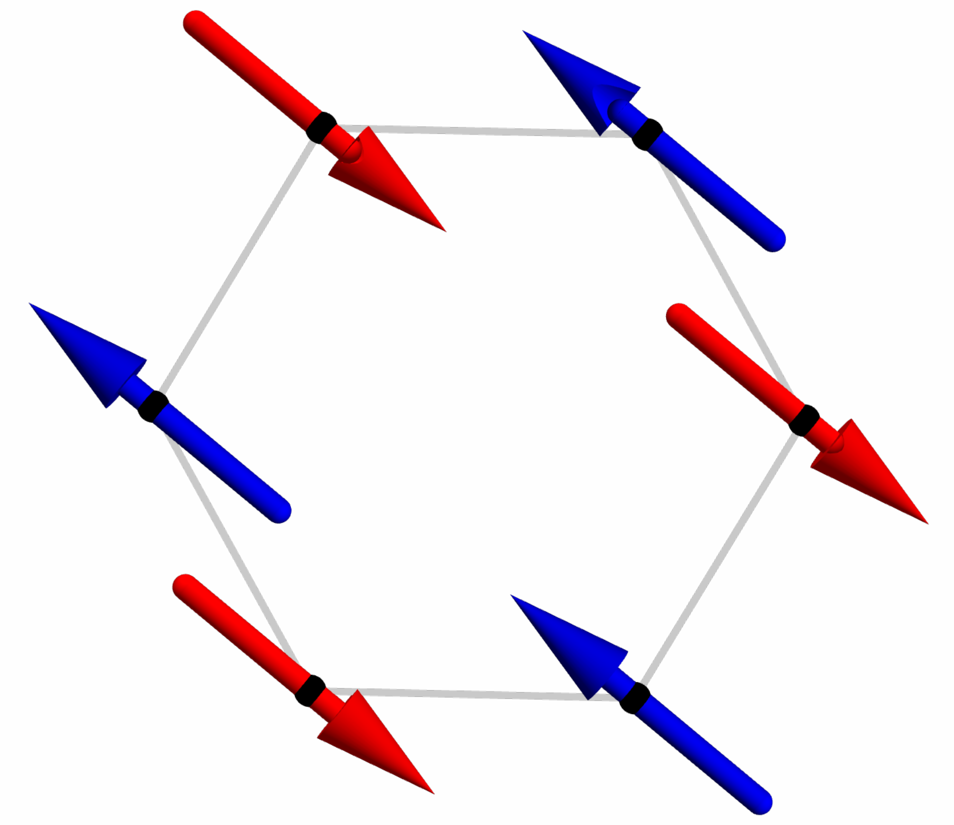

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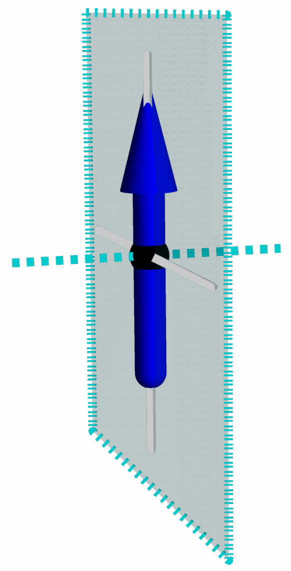

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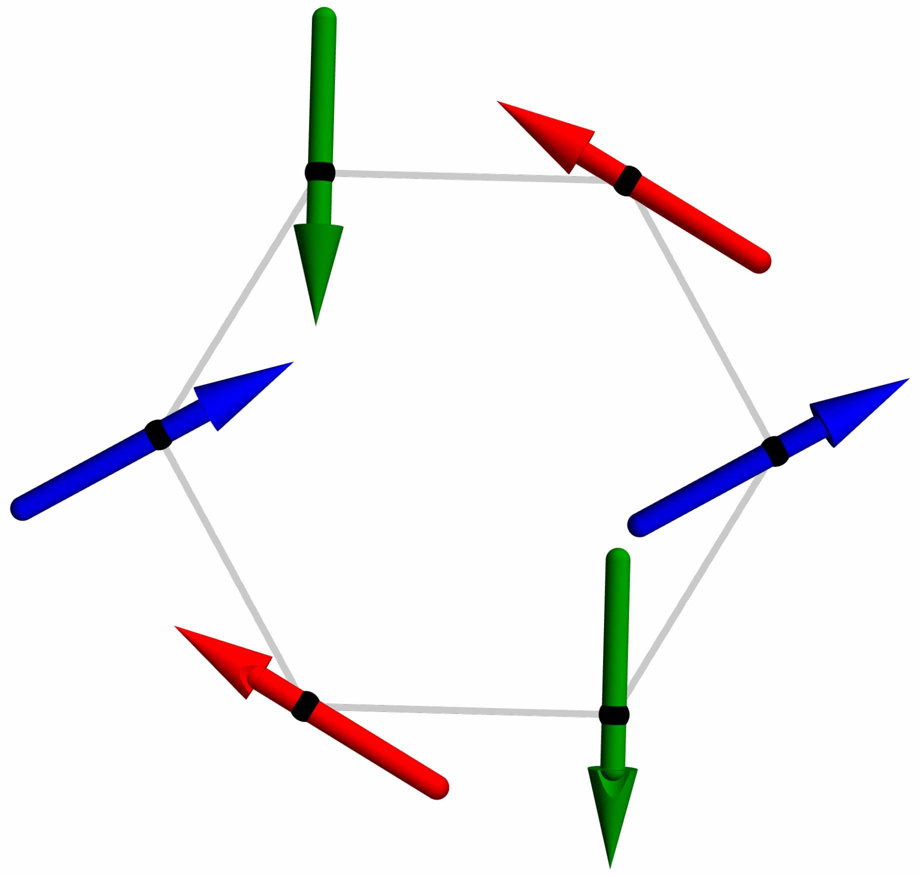

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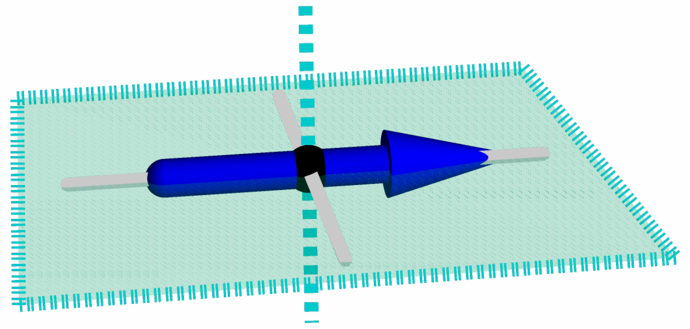

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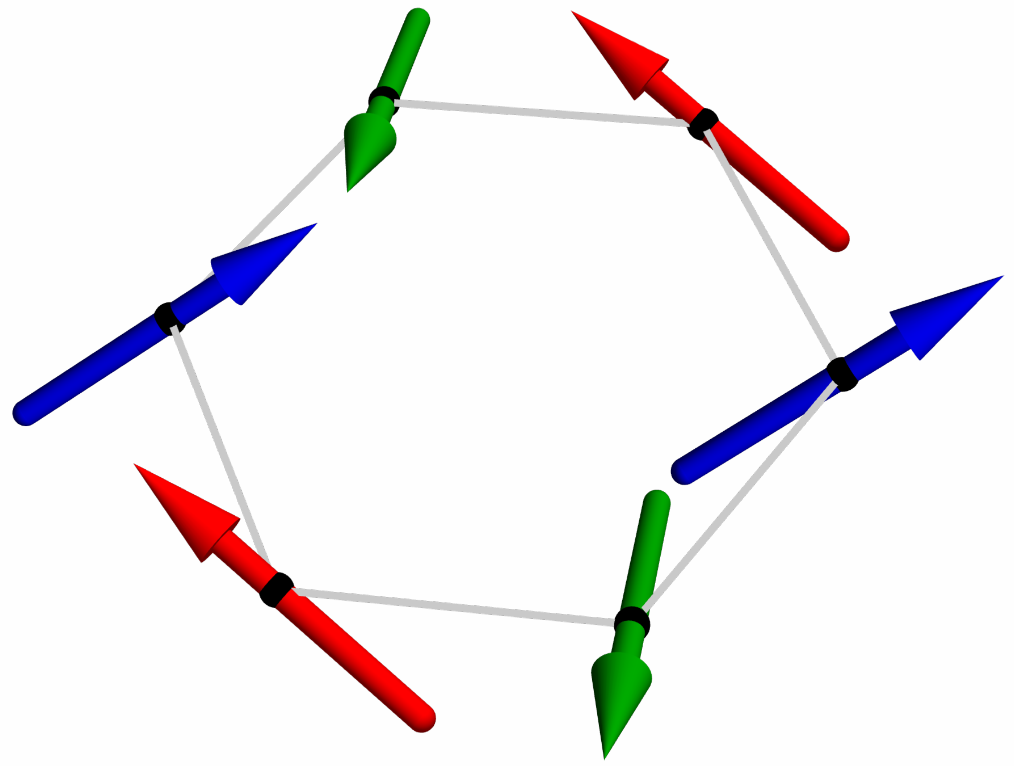

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No global spin operation preserves the spin configuration

D.B. Litvin, W. Opechowski, Physica Vol. 76, Iss. 3, (1974)

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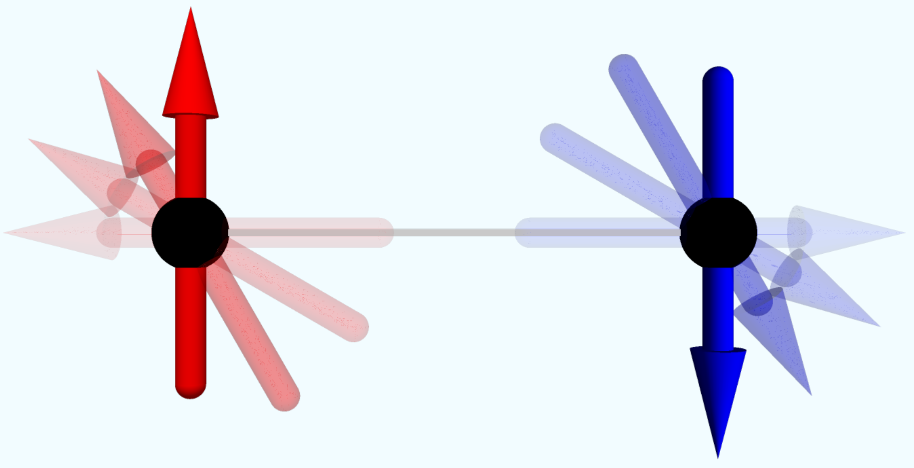

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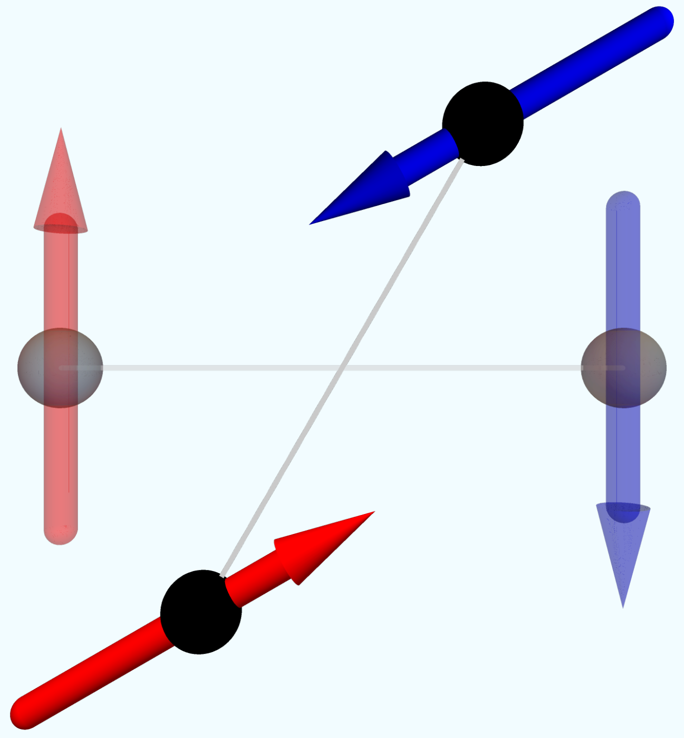

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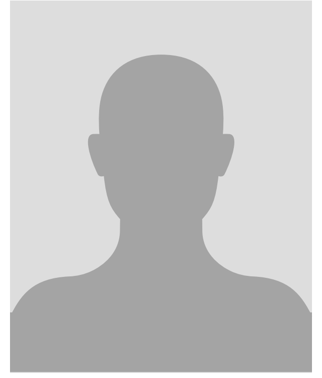
Group derivation algorithm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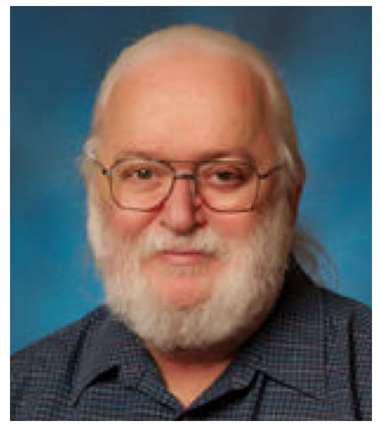
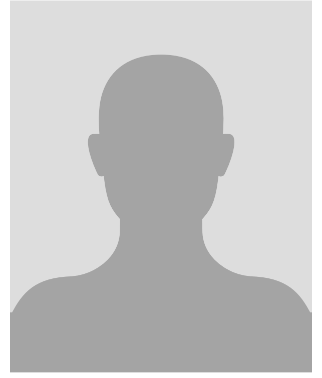

Daniel B. Litvin

Find all NT spin groups

Group derivation algorithm



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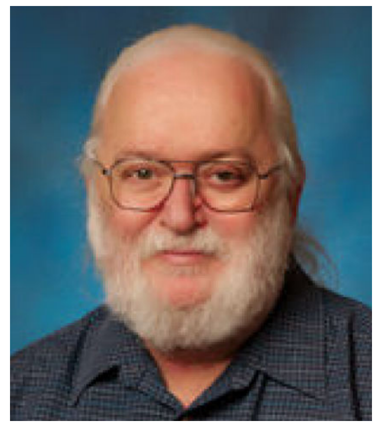

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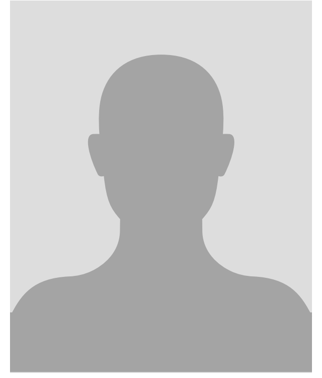
For each ordinary parent group $\underline{\mathrm{G}}$:

Group derivation algorithm



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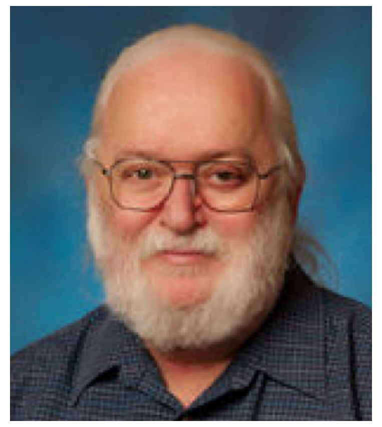

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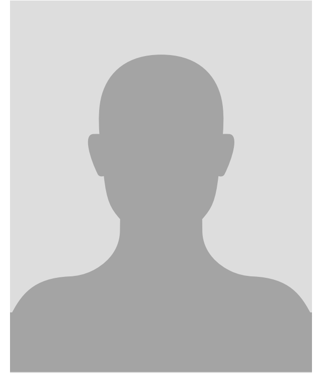
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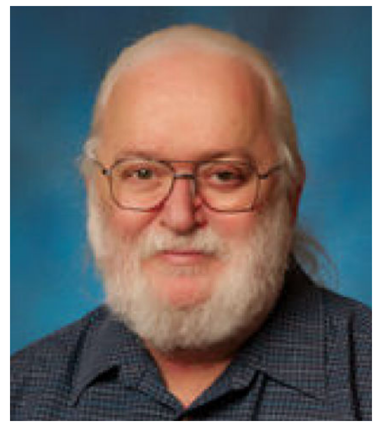
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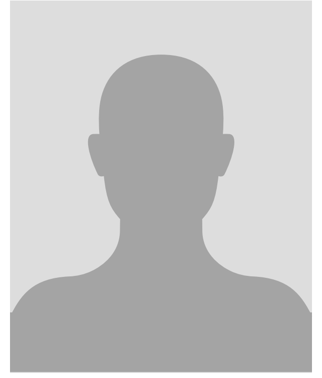

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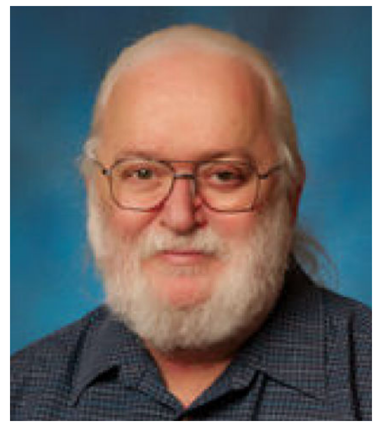

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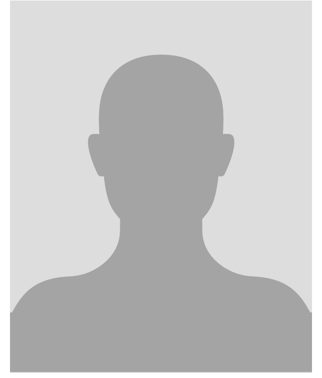

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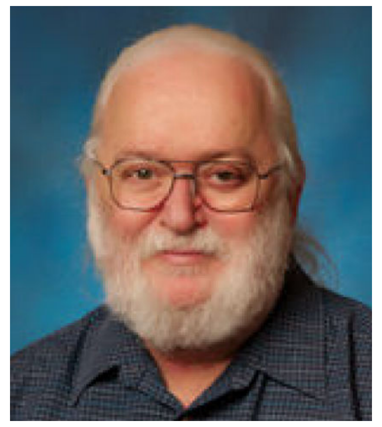

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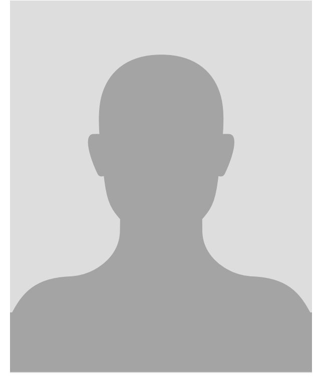

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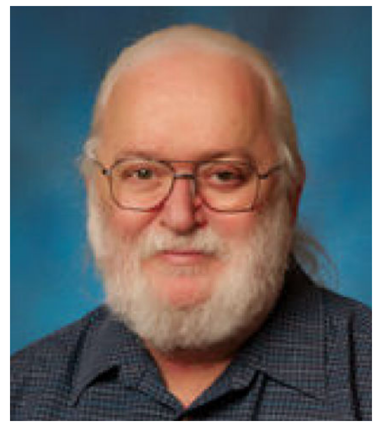

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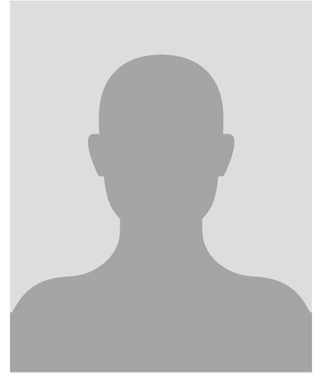

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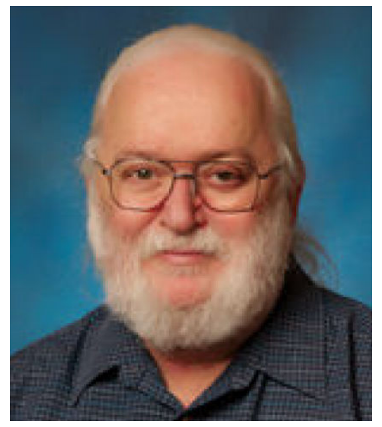

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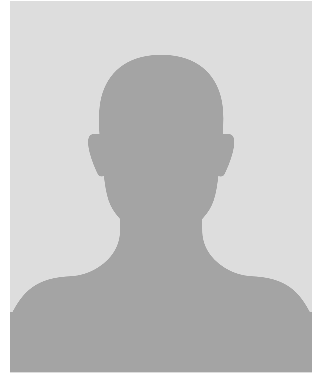
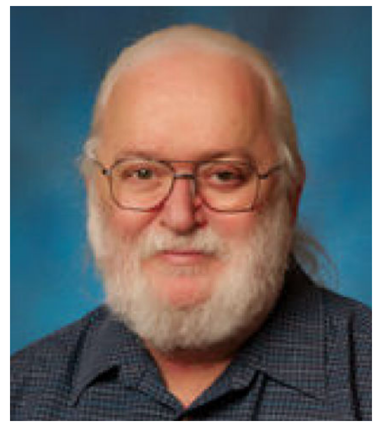
$$\underline{\mathbf{G}} = \underline{\mathbf{g}} + G_2\,\underline{\mathbf{g}} + G_3\,\underline{\mathbf{g}} + G_4\,\underline{\mathbf{g}} + \dots$$

$$D(\underline{\mathbf{G}}) = \{ \square , \square , \square , \square \dots \}$$

Group derivation algorithm



Wladyslaw Opechowski



Daniel B. Litvin

Find all NT spin groups

For each ordinary parent group $\underline{\mathbf{G}}$:

- List all normal subgroups $\underline{\mathbf{g}} \triangleleft \underline{\mathbf{G}}$
- Determine quotient groups $\underline{\mathbf{B}} \cong \underline{\mathbf{G}}/\underline{\mathbf{g}}$
  - *Option 1*: determine explicit isomorphism between $\underline{\mathbf{B}}$ and $\underline{\mathbf{G}}/\underline{\mathbf{g}}$
  - *Option 2*: generate $\mathbf{O}(3)$ representation of $\underline{\mathbf{G}}$ with kernel $\underline{\mathbf{g}}$
- **Form** $\underline{\mathbf{S}} = \underline{\mathbf{g}} + [B_2||G_2]\,\underline{\mathbf{g}} + [B_3||G_3]\,\underline{\mathbf{g}} + \ldots$

$$\underline{\mathbf{G}} = \underline{\mathbf{g}} + G_2\,\underline{\mathbf{g}} + G_3\,\underline{\mathbf{g}} + G_4\,\underline{\mathbf{g}} + \ldots$$

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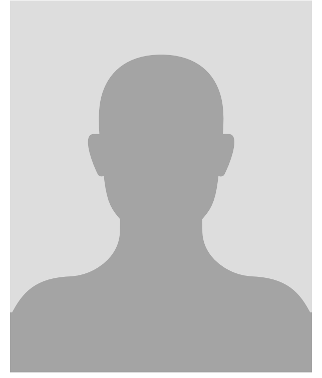
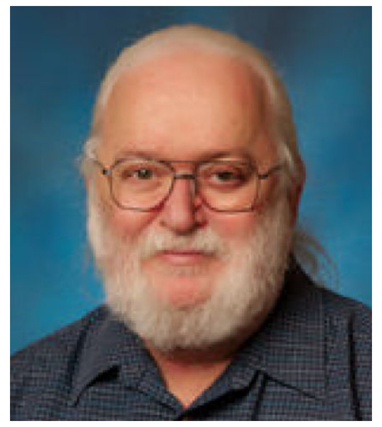
$$D(\underline{\mathbf{G}}) = \{\, \boxed{}\,, \boxed{}\,, \boxed{}\,, \boxed{} \;\ldots \}$$

Group derivation algorithm



Wladyslaw Opechowski



Daniel B. Litvin

Find all NT spin groups → Pair with allowed spin-only groups

For each ordinary parent group $\underline{\mathbf{G}}$:

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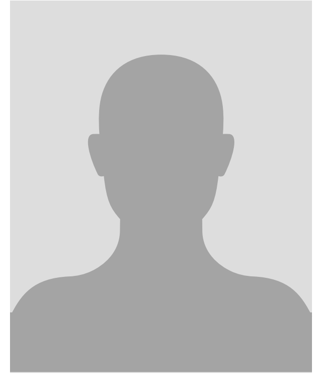
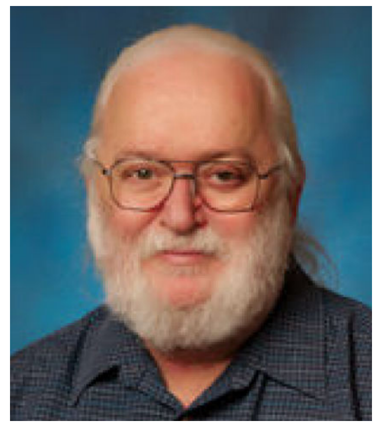
$$\underline{\mathbf{X}} = \underline{\mathbf{b}} \times \underline{\mathbf{S}}$$

Group derivation algorithm



Wladyslaw Opechowski



Daniel B. Litvin

Find all NT spin groups → Pair with allowed spin-only groups → Remove duplicates

For each ordinary parent group $\underline{\mathbf{G}}$:

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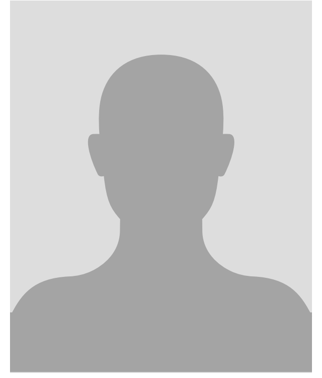
$$\underline{\mathbf{X}} = \underline{\mathbf{b}} \times \underline{\mathbf{S}}$$

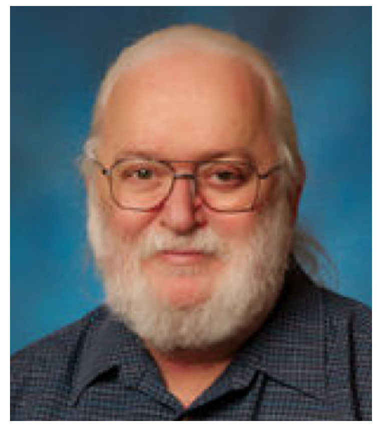
Two spin groups $\underline{\mathbf{X}}$ and $\underline{\mathbf{X}}'$ are **equivalent** if they are conjugate in $\mathbf{O}^s(3) \times \mathbf{E}(3)$

$$\exists y \in \mathbf{O}^s(3) \times \mathbf{E}(3): \quad \underline{\mathbf{X}}' = y\underline{\mathbf{X}}y^{-1}$$

Spin point groups (SPGs)



Wladyslaw Opechowski



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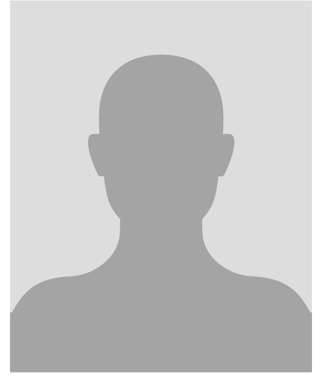
Spin point groups (SPGs)

- 598 non-trivial SPGs
  - 90 are B&W groups.



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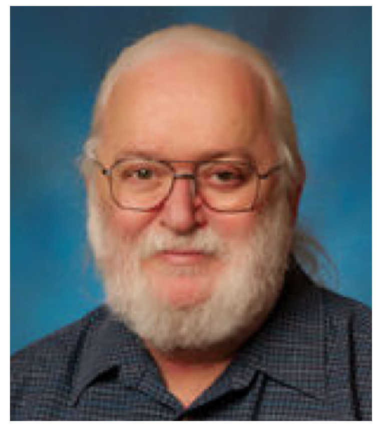

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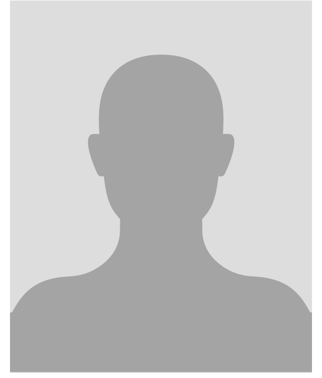
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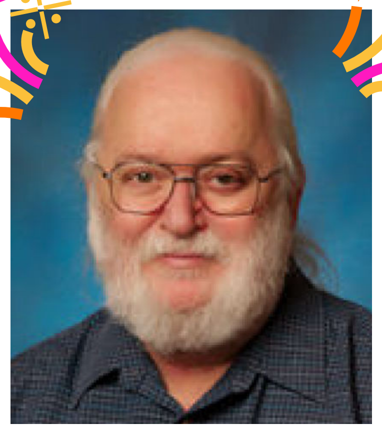
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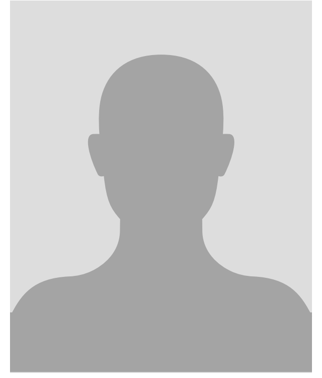

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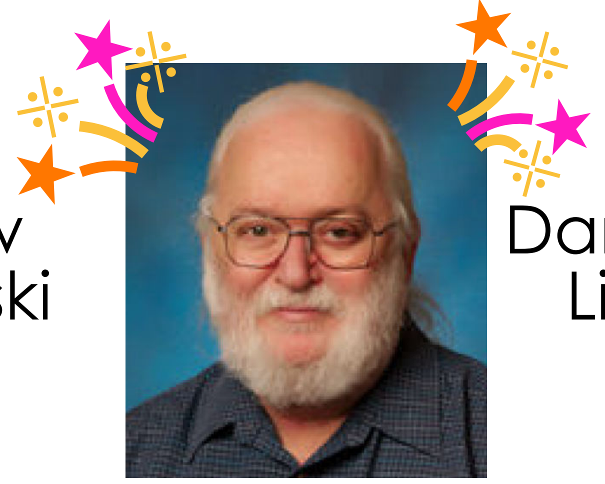

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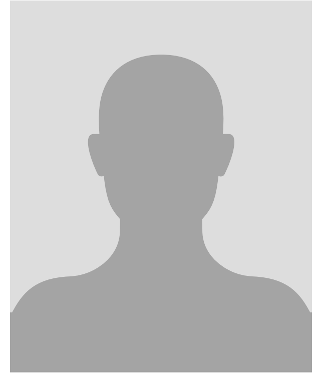

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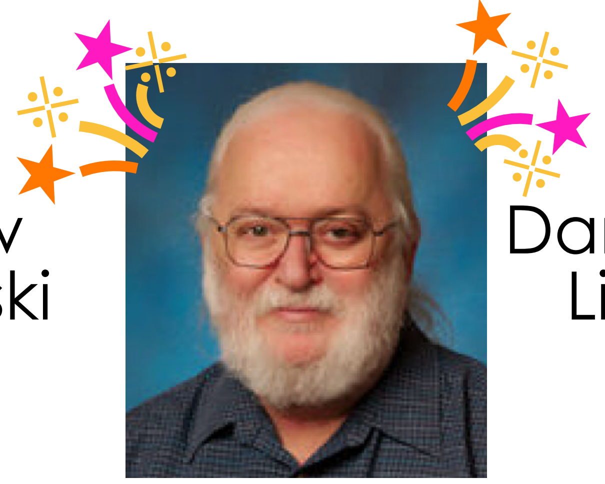

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  - When $\underline{\mathbf{B}} = \mathbf{C}_n$ or $\mathbf{D}_n$

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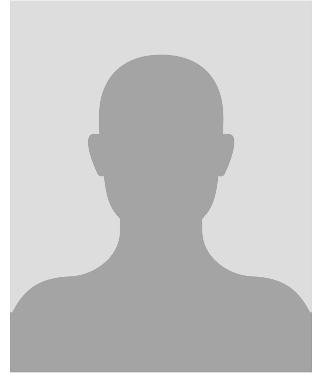
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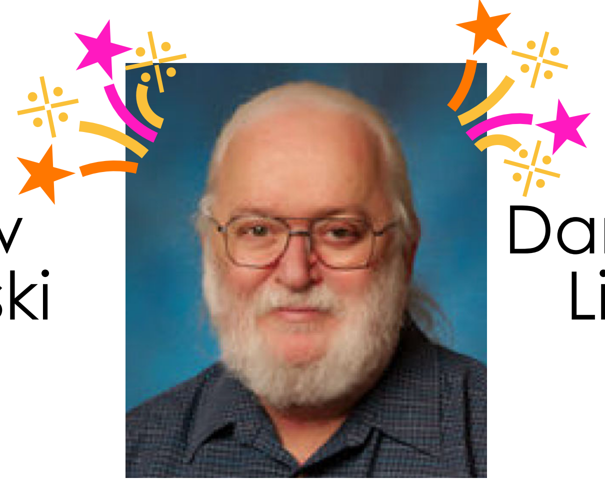
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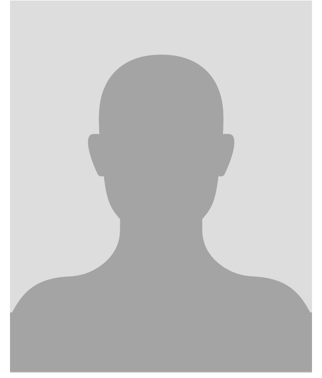

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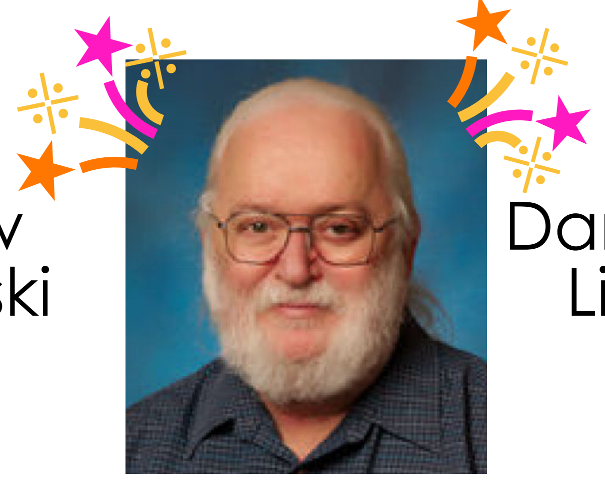

Daniel B. Litvin

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Original Litvin table

130.	422	422	1	$1_4 1_2 1_2$	
131.		4	2	$1_4 2_2 2_2$	
132			m	$1_4 m_2 m_2$	
133.			$\bar{1}$	$1_4 \bar{1}_2 \bar{1}_2$	
134.		222	2	$2_4 1_2 2_2$	
135.			m	$m_4 1_2 m_2$	
136.			$\bar{1}$	$\bar{1}_4 1_2 \bar{1}_2$	
137.		2	222	$2x_4 2y_2 2z_2$	
138.			mm2	$2z_4 mx_2 my_2$	
139.				$mx_4 2z_2 my_2$	
140.			2/m	$2_4 \bar{1}_2 m_2$	
141.				$\bar{1}_4 2_2 m_2$	
142.				$m_4 \bar{1}_2 2_2$	
143.		1	422	$4z_4 2x_2 2xy_2$	422
144.			4mm	$4z_4 mx_2 mxy_m$	42'2'
145.			$\bar{4}$2m	$\bar{4}z_4 2x_2 mxy_2$	4'22'

D. B. Litvin, Acta Crystallographica A 33 (2), 279 (1977)

Paul McClarty

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+ HS

422	130	${}^{1}4^{1}2^{1}2$	422
	131	${}^{1}4^{2}2^{2}2$	422
	132	${}^{1}4^{m}2^{m}2$	$42'2'$
	133	${}^{1}4^{\bar{1}}2^{\bar{1}}2$	$42'2'$
	134	${}^{2}4^{2}2^{1}2$	422
	135	${}^{m}4^{m}2^{1}2$	$4'2'2$
	136	${}^{\bar{1}}4^{\bar{1}}2^{1}2$	$4'2'2$
	137	${}^{2_x}4^{2_z}2^{2_y}2$	422
	138	${}^{2_z}4^{m_x}2^{m_y}2$	$42'2'$
	139	${}^{m_x}4^{2_z}2^{m_y}2$	$4'2'2$
	140	${}^{2}4^{\bar{1}}2^{m}2$	$42'2'$
	141	${}^{\bar{1}}4^{2}2^{m}2$	$4'2'2$
	142	${}^{m}4^{\bar{1}}2^{2}2$	$4'2'2$
	143	${}^{4_z}4^{2_x}2^{2_{xy}}2$	422
	144	${}^{4_z}4^{m_x}2^{m_{xy}}m$	$42'2'$
	145	${}^{\overline{4z}}4^{2_x}2^{m_{xy}}2$	$4'2'2$

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HS, A. Corticelli, A. Guerreiro, J. Romhányi, P. McClarty, SciPost Phys. 18, 109 (2025)

Spin space groups

Depends on unit cell enlargement


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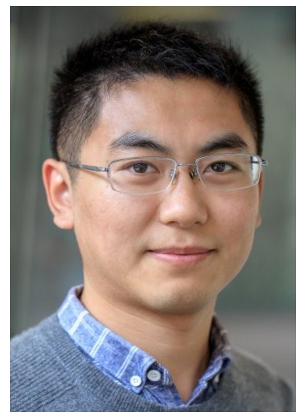
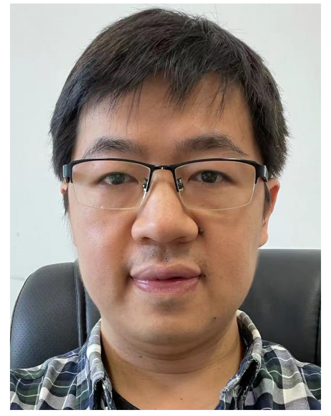

Zhi-Da Song

Zhenyu Xiao
Jianzhou Zhao
Yanqi Li
Ryuichi Shindou

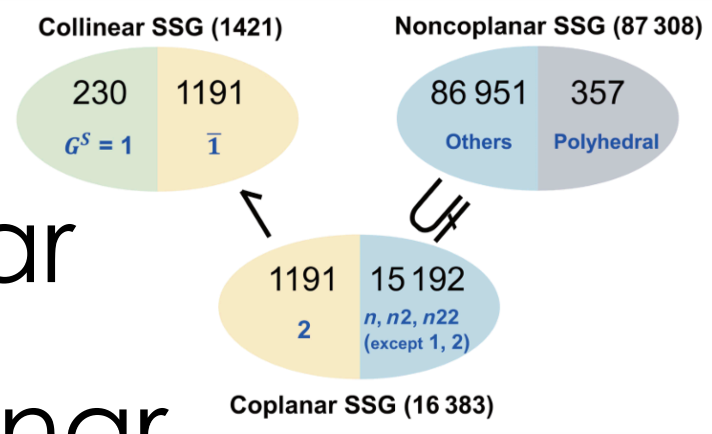

Chen Fang

Yi Jiang
Ziyin Song
Tiannian Zhu
Zhong Fang
Hongming
Weng
Zheng-Xin Liu
Jian Yang

Supercells up to order 8

100612 SSGs

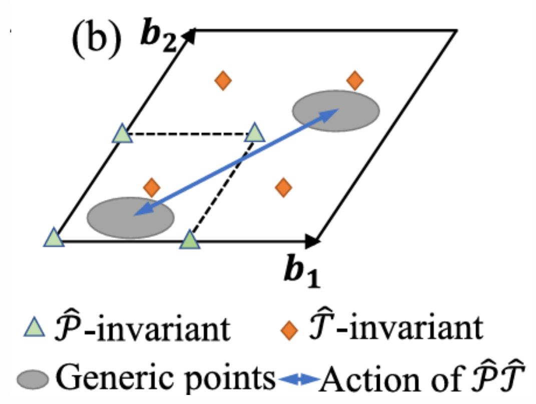
- 1421 collinear
- 16383 coplanar
- 87308 non-coplanar



Families ~ mod. symmetry of the propagation vector

67475 SSGs

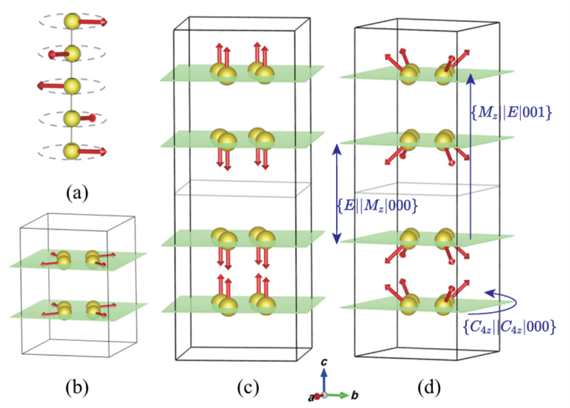
- 1421 collinear
- 9542 coplanar
- 56512 non-coplanar



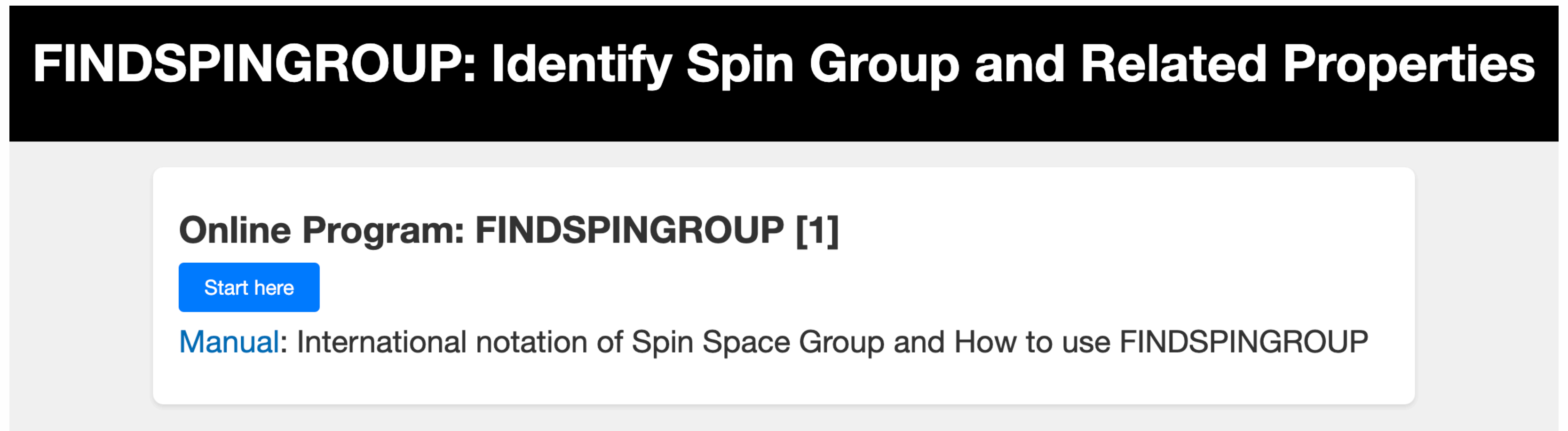
Supercells up to order 12

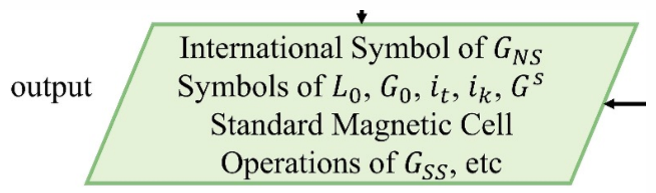
183498 SSGs

- 1421 collinear
- 24788 coplanar
- 157289 non-coplanar



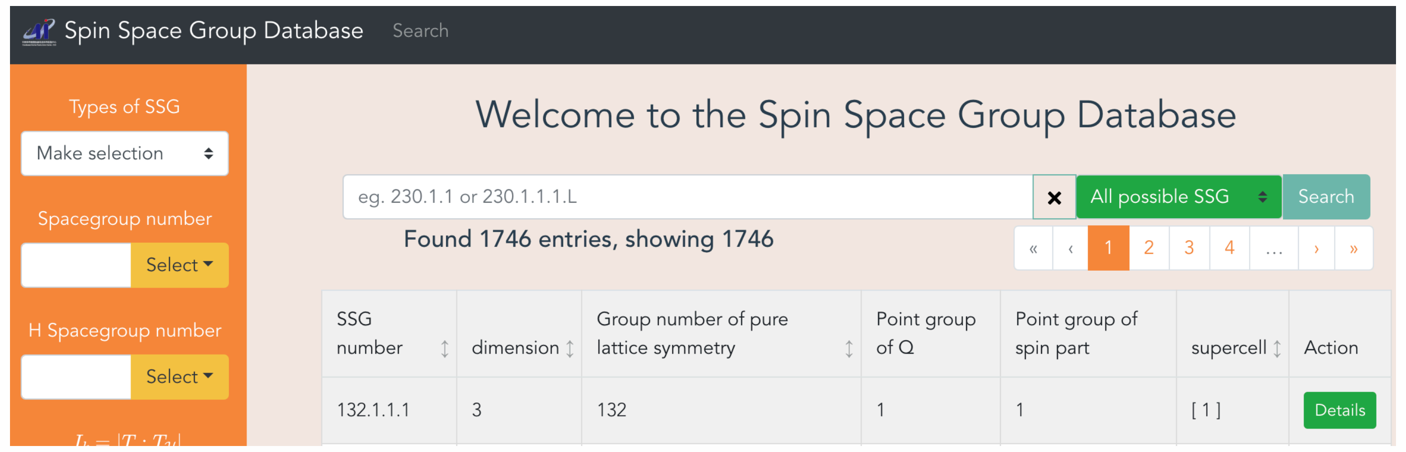
Identify the spin group from .cif, .mcif or .txt





FINDSPINGROUP

Groups & group elements for SSGs:



Search based on parent group, list elements, etc…

Spin Space Group Database

(Co-)representation Theory

SPIN PHENOMENA
INTERDISCIPLINARY CENTER

SP/CE

Hana Schiff

Thursday, May 8th, 2025

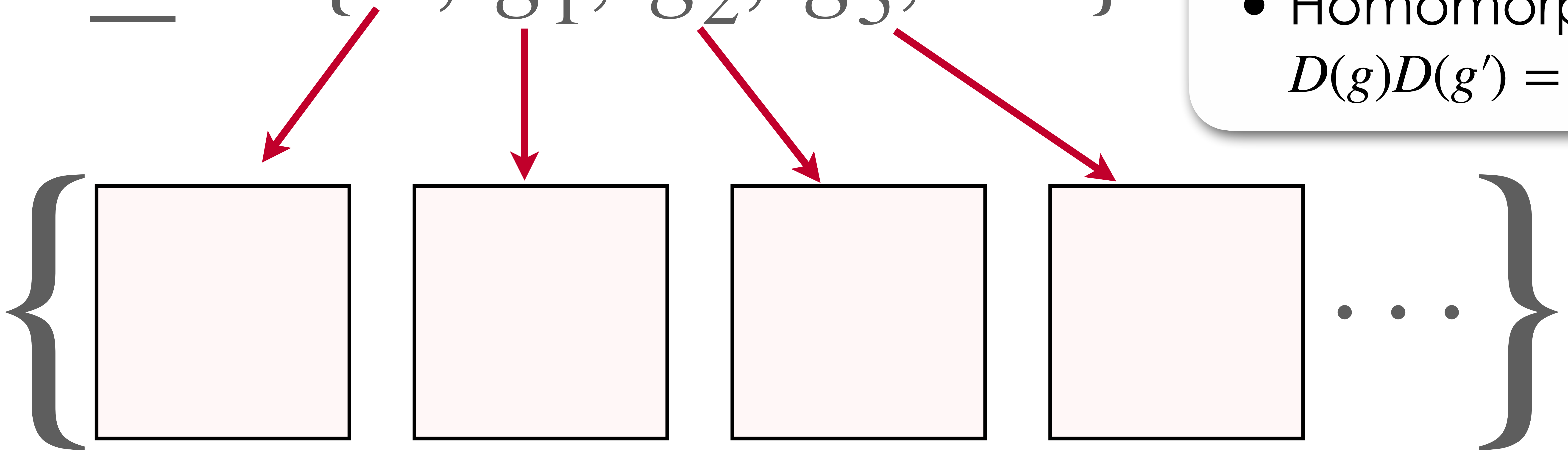
Ordinary representation theory

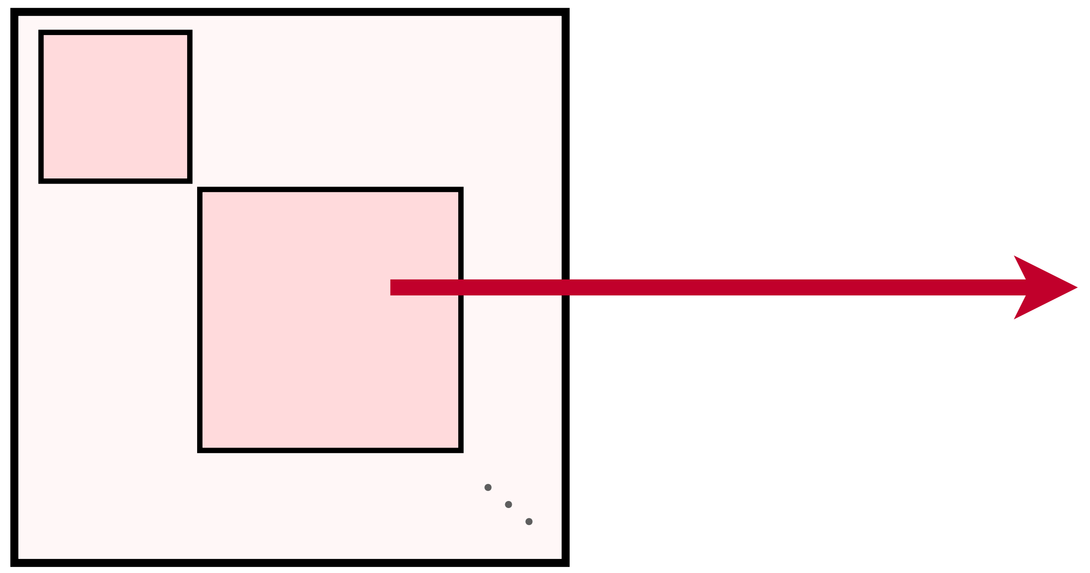
$$\underline{\mathbf{G}} = \{E,\ g_1,\ g_2,\ g_3, \cdots\}$$

- Linear
- Homomorphism

  $D(g)D(g') = D(gg')$

$D(\underline{\mathbf{G}}) =$





Irreducible representation
"building blocks"

Intro to co-representations

Eugene Wigner

Time-reversal (TR) ***must*** *be represented by an* ***antiunitary*** *operator.*

Intro to co-representations

Eugene Wigner

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Antiunitary elements τg : composition of unitary operation g with TR τ

Intro to co-representations

Eugene Wigner

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Antiunitary elements τg : composition of unitary operation g with TR τ

Any group containing antiunitary elements:

$$\underline{\mathbf{G}} = \underline{\mathbf{H}} + a\underline{\mathbf{H}}$$

Where $\underline{\mathbf{H}}$ is unitary, half size of $\underline{\mathbf{G}}$, and $a = \tau g$

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Representations not homomorphisms onto matrices anymore!

If $a = \tau g$ and h is unitary:

$$D(h)D(a) = D(ha)$$
$$D(a)D^{\star}(h) = D(ah)$$

Intro to co-representations


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Can derive a set of irreducible matrices satisfying these properties from the irreps of $\underline{\mathbf{H}}$

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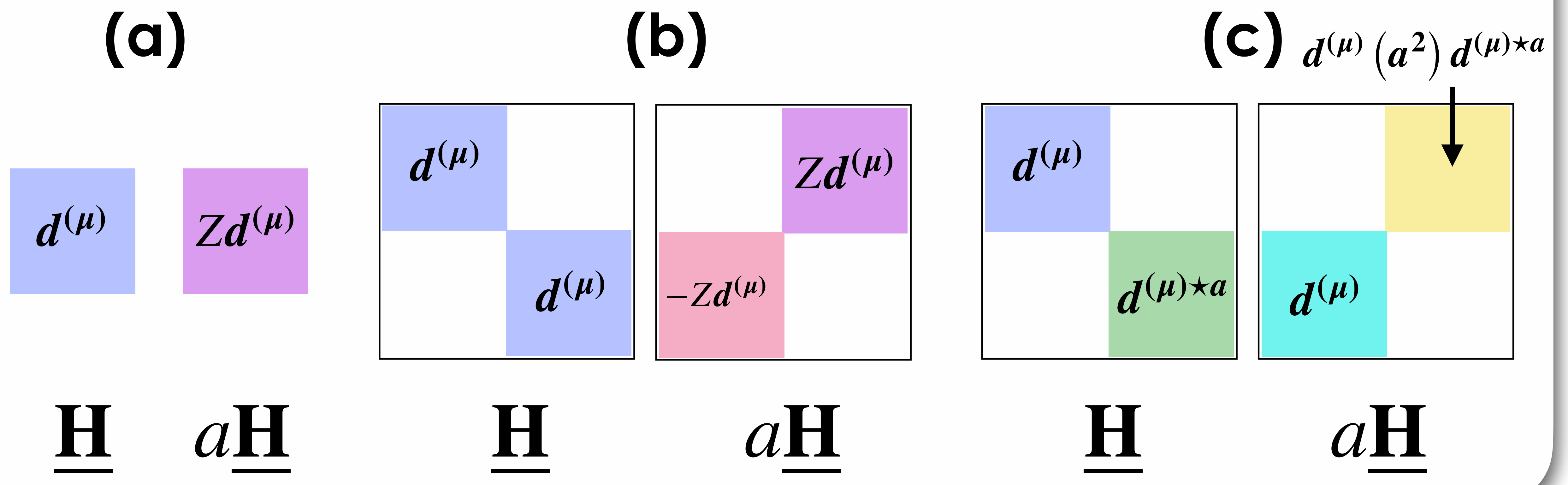
Let $d^{(\mu)}(\underline{\mathbf{H}})$ be an irrep of $\underline{\mathbf{H}}$. Depending on transformation under conjugation by a (and $\star$):



Intro to co-representations

Eugene Wigner

Time-reversal (TR) ***must*** *be represented by an* ***antiunitary*** *operator.*

Antiunitary elements τg : composition of unitary operation g with TR τ

Any group containing antiunitary elements:

$$\underline{\mathbf{G}} = \underline{\mathbf{H}} + a\underline{\mathbf{H}}$$

Where $\underline{\mathbf{H}}$ is unitary, half size of $\underline{\mathbf{G}}$, and $a = \tau g$

Representations not homomorphisms onto matrices anymore!

If $a = \tau g$ and h is unitary:

$$D(h)D(a) = D(ha)$$
$$D(a)D^{\star}(h) = D(ah)$$

Can derive a set of irreducible matrices satisfying these properties from the irreps of $\underline{\mathbf{H}}$

Let $d^{(\mu)}(\underline{\mathbf{H}})$ be an irrep of $\underline{\mathbf{H}}$. Depending on transformation under conjugation by a (and $\star$):

Dimmock indicator:

$$\frac{1}{|\underline{\mathbf{H}}|}\sum_{h\in\underline{\mathbf{H}}}\chi^{(\mu)}\left((ah)^2\right) = \begin{cases} +1 & \mathbf{(a)} \\ -1 & \mathbf{(b)} \\ 0 & \mathbf{(c)} \end{cases}$$

No doubling

Doubling

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(a): No doubling

(b), (c): Doubling

Co-reps: non-trivial & coplanar SPGs

Theorem: non-trivial spin groups $\underline{\mathbf{S}}$ are isomorphic to their parent group $\underline{\mathbf{G}}$.

HS, A. Corticelli, A. Guerreiro, J. Romhányi, P. McClarty, SciPost Phys. 18, 109 (2025)

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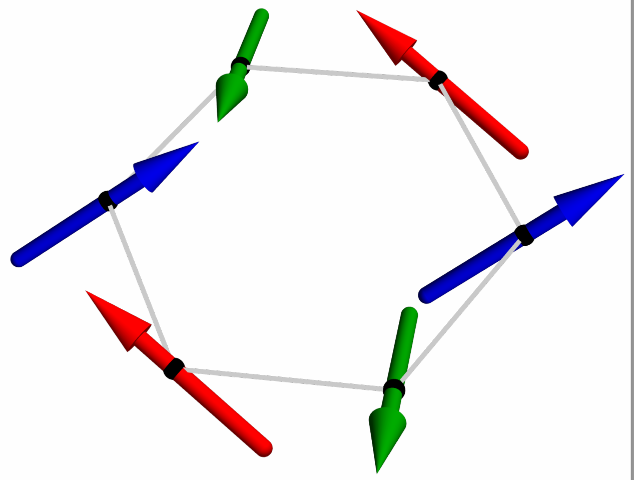

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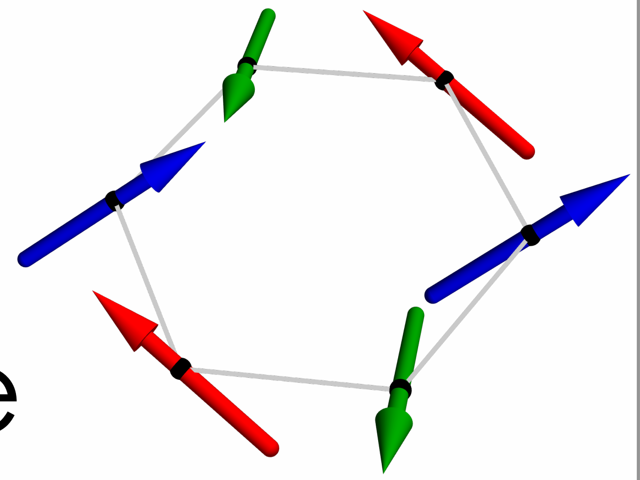

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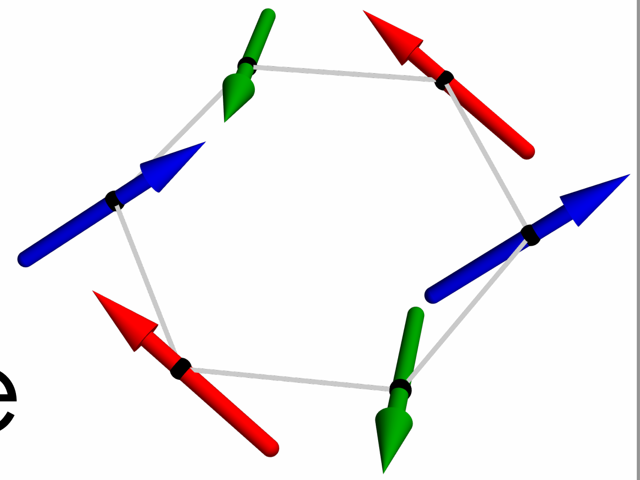

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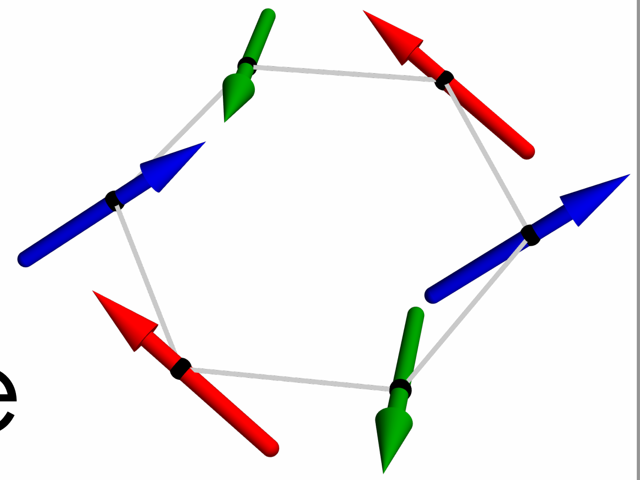

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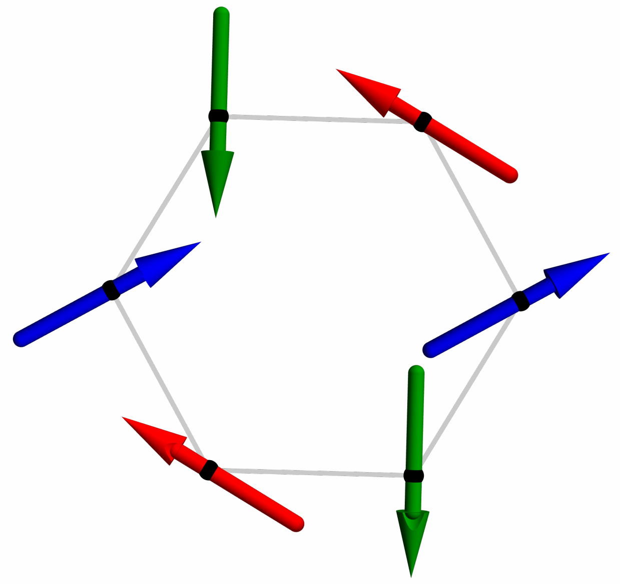

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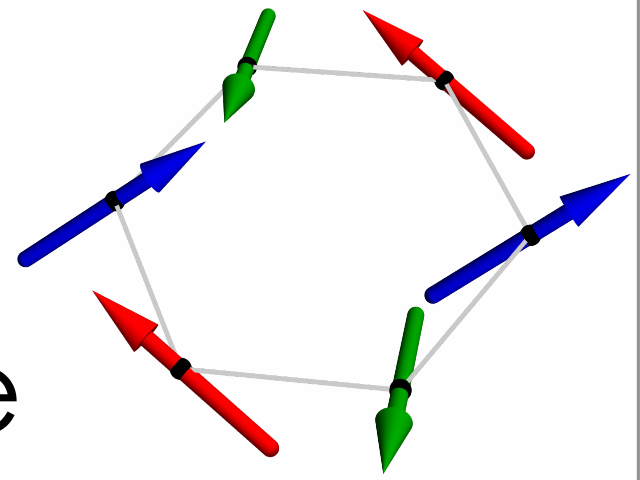

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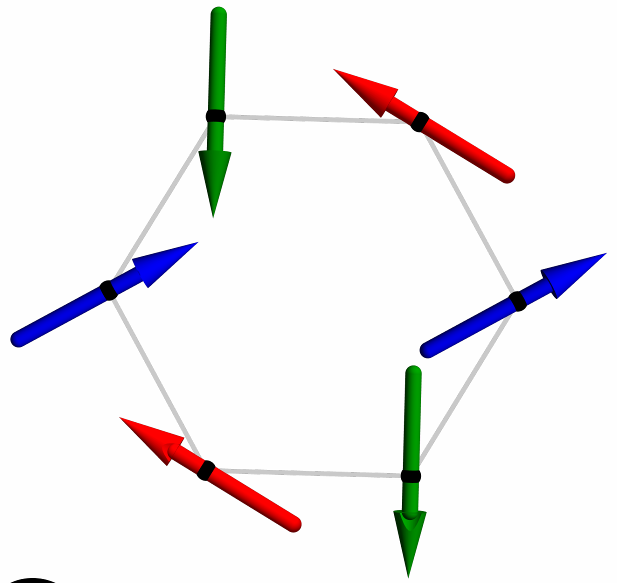

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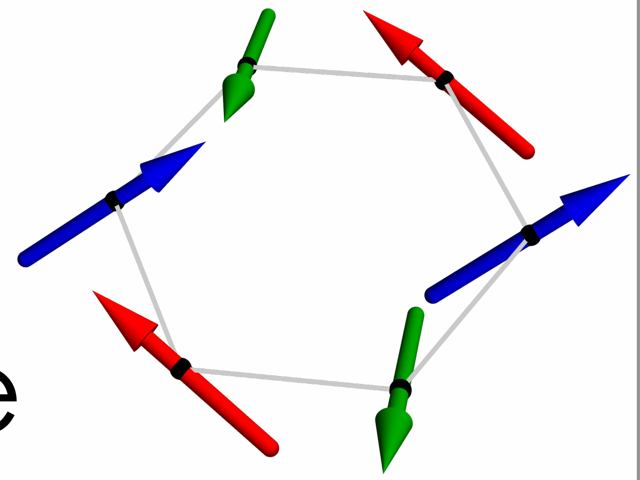

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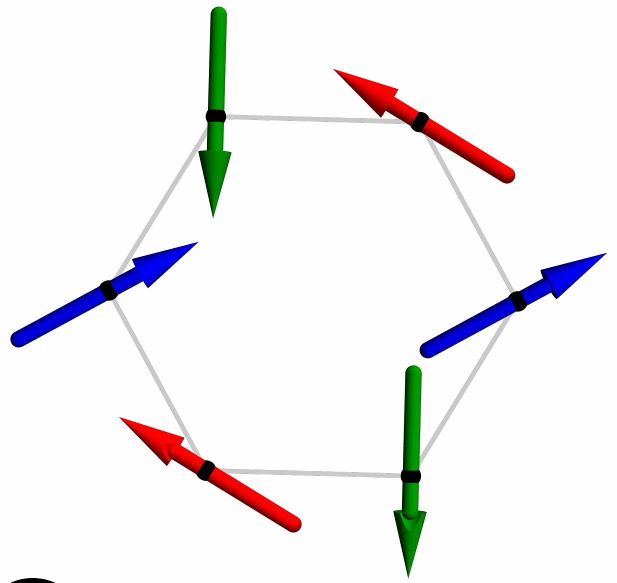

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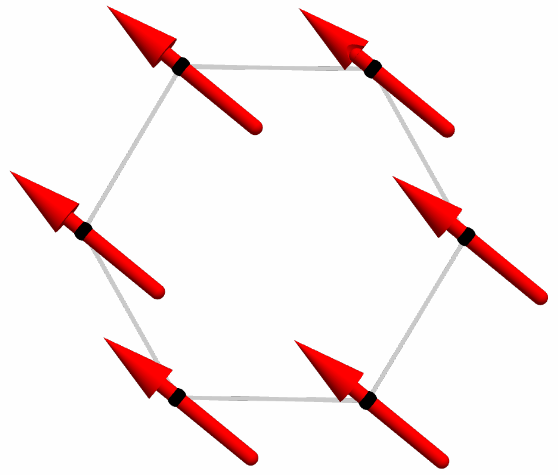
Co-reps: collinear SPGs

$$\underline{\mathbf{X}} = \underline{\mathbf{b}}^{\infty} \times \underline{\mathbf{S}}$$

Recall: $\underline{\mathbf{b}}^{\infty} \cong \mathbf{SO}(2) \rtimes \{E, \tau 2_{\perp}\}$

Unitary $\underline{\mathrm{S}}$:

All elements of $\underline{\mathbf{S}}$ are $[E||g]$: $\underline{\mathbf{S}} = \underline{\mathbf{G}}$



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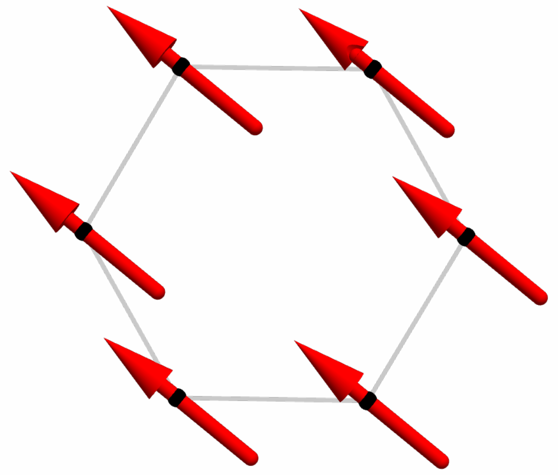

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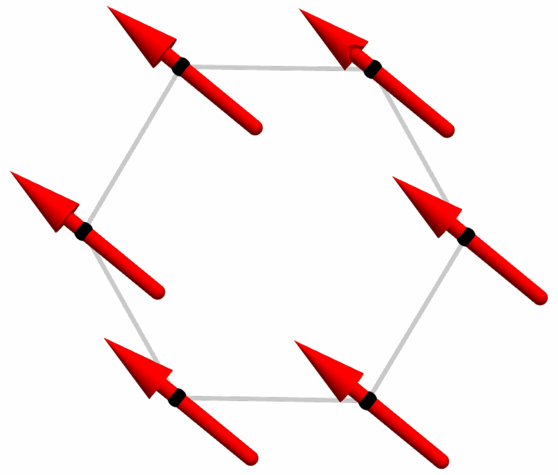

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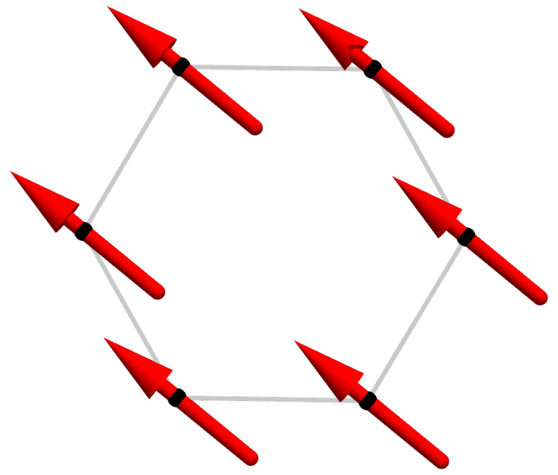

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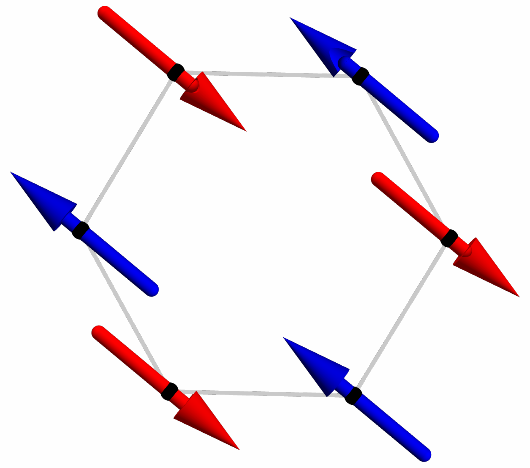

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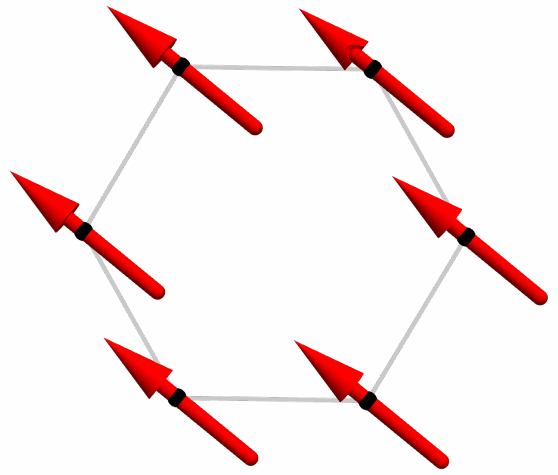

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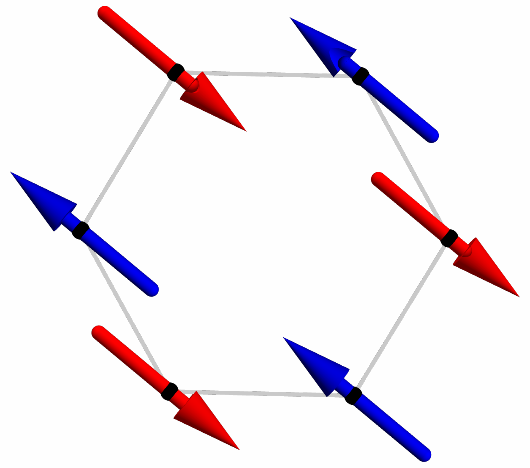

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Irreps of $\underline{\mathbf{X}}_{1/2}$ a priori unknown & new!

Co-irreps are not trivially found from knowledge of irreps $\Delta^{(\nu)}$ of $\underline{\mathbf{G}}$.

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Γ_2	1	-1	1	-1
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Summary of SPG co-representations

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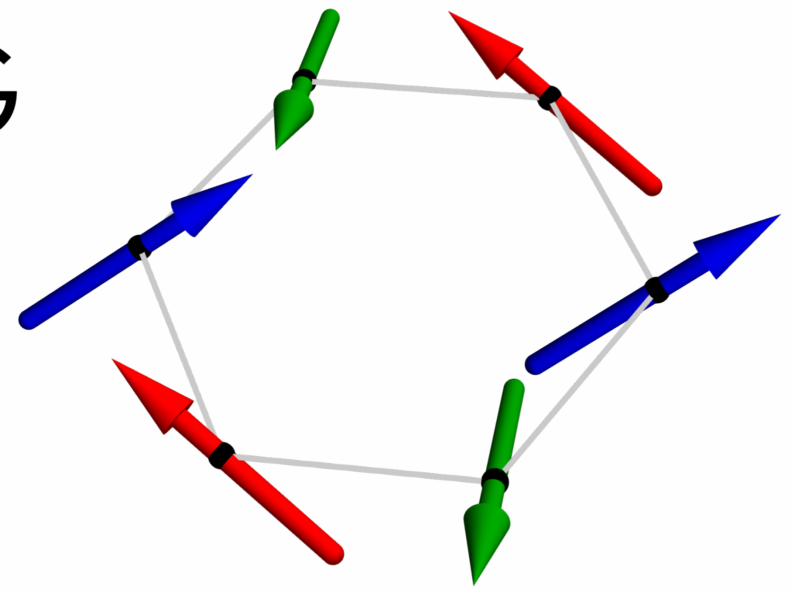

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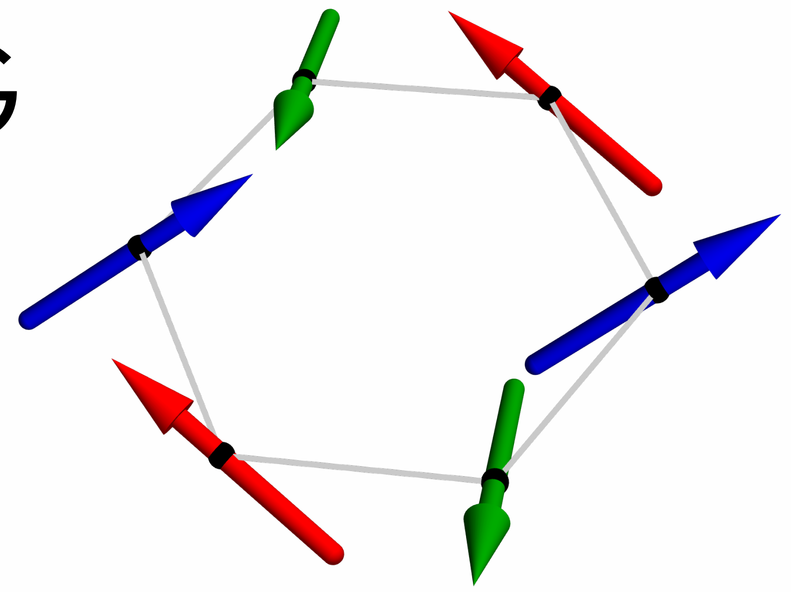

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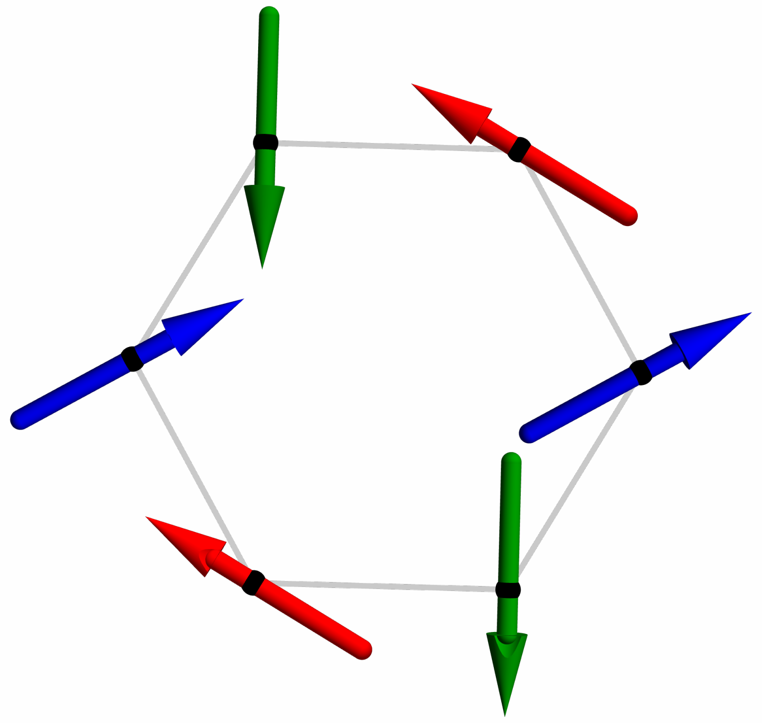

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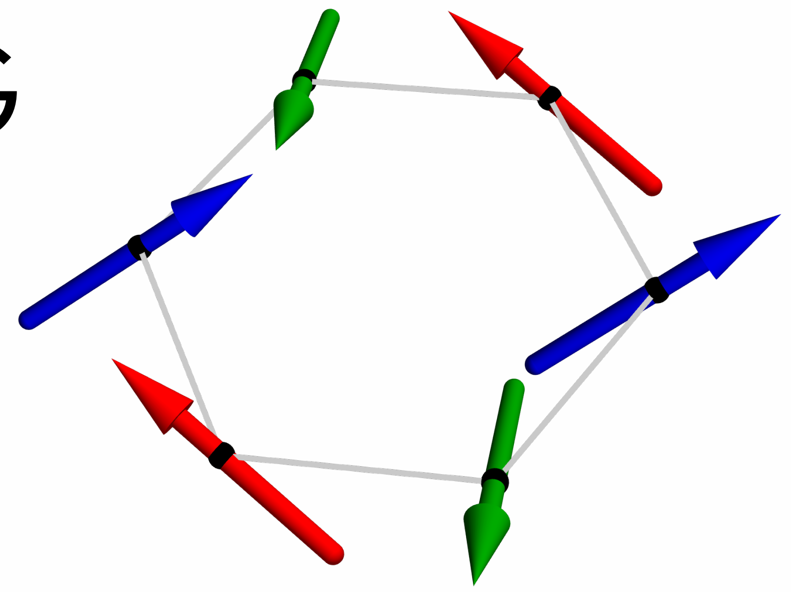

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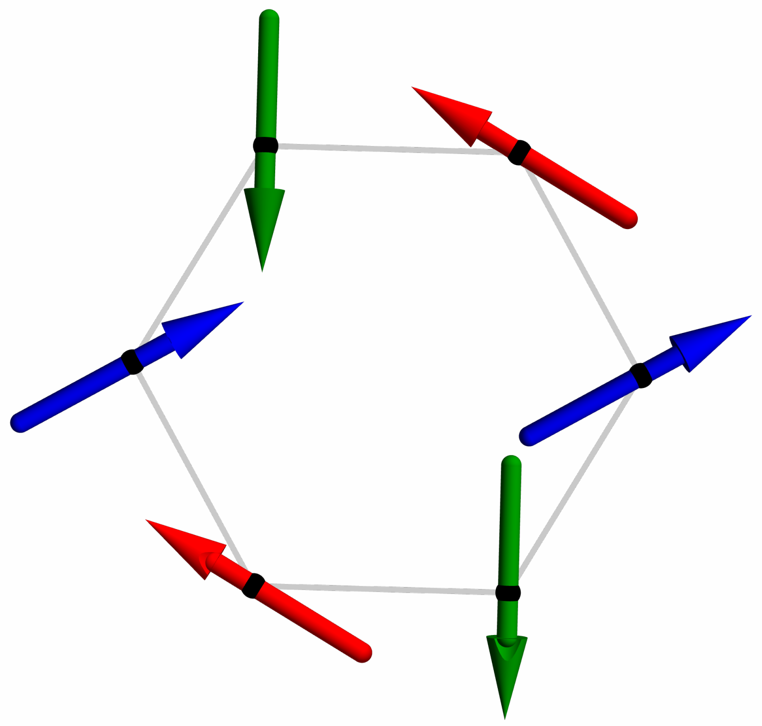
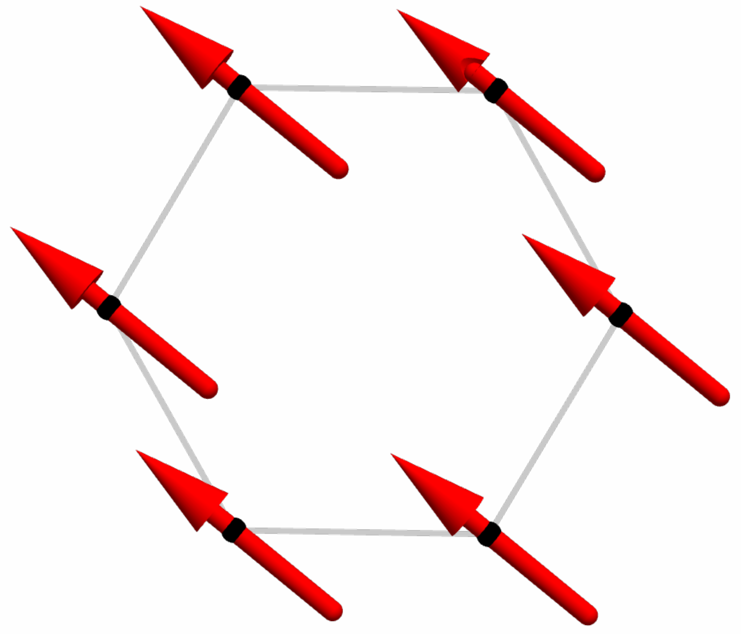


Collinear SPGs

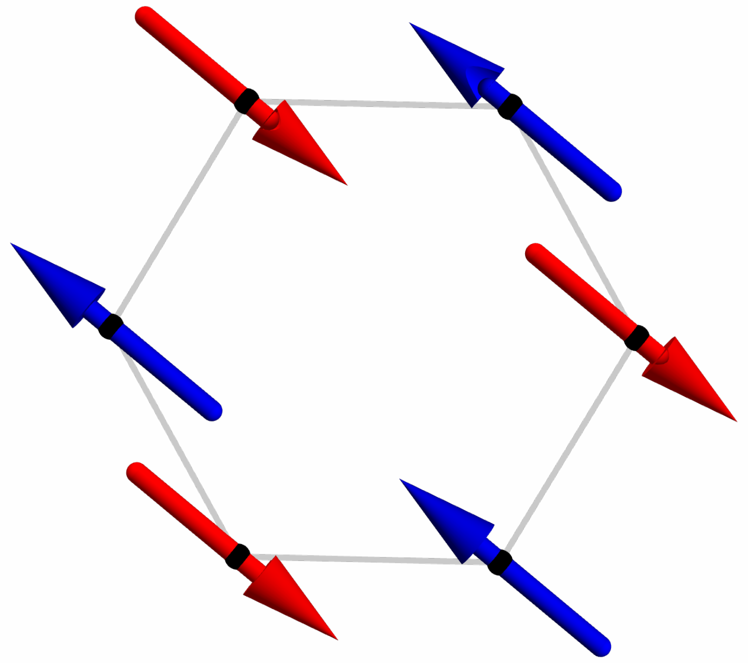
Unitary $\underline{\mathbf{S}}$:

Co-irreps induced from $\mathbf{SO}(2) \times \underline{\mathbf{G}}$, based on reality of $\Delta^{(\nu)}(\underline{\mathbf{G}})$



Antiunitary $\underline{\mathbf{S}}$:

Co-irreps induced from $\underline{\mathbf{X}}_{1/2}$, are **new** and non-trivial!



HS, A. Corticelli, A. Guerreiro, J. Romhányi, P. McClarty, SciPost Phys. 18, 109 (2025)

Do we need the spin groups?

SPIN PHENOMENA
INTERDISCIPLINARY CENTER

Do we need the spin groups?

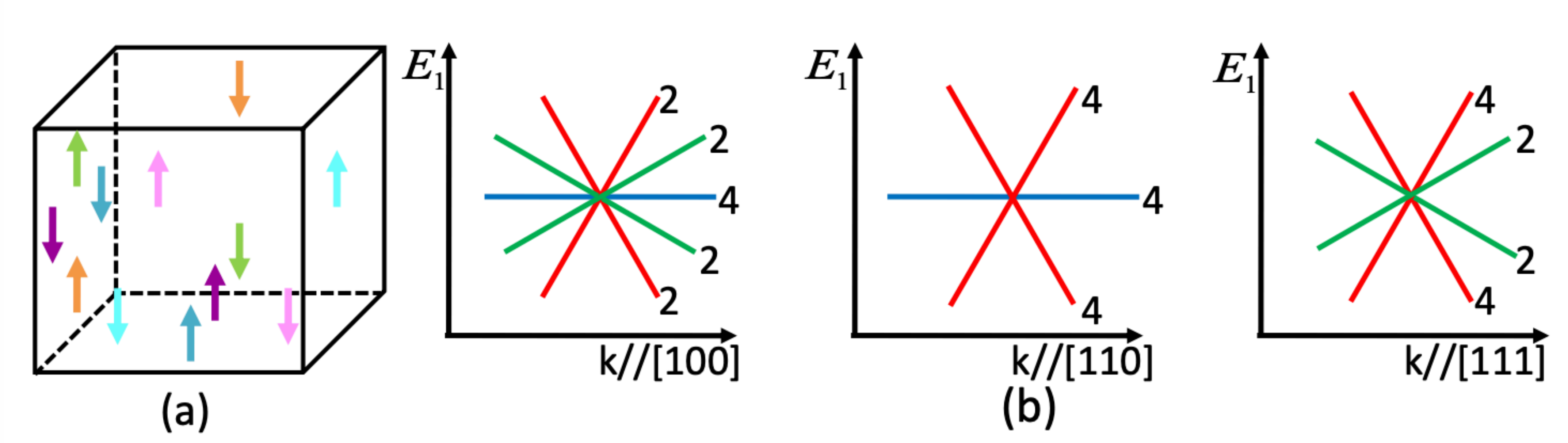
Usually yes, sometimes no!

Band structure degeneracy

✓ Need spin group theory!

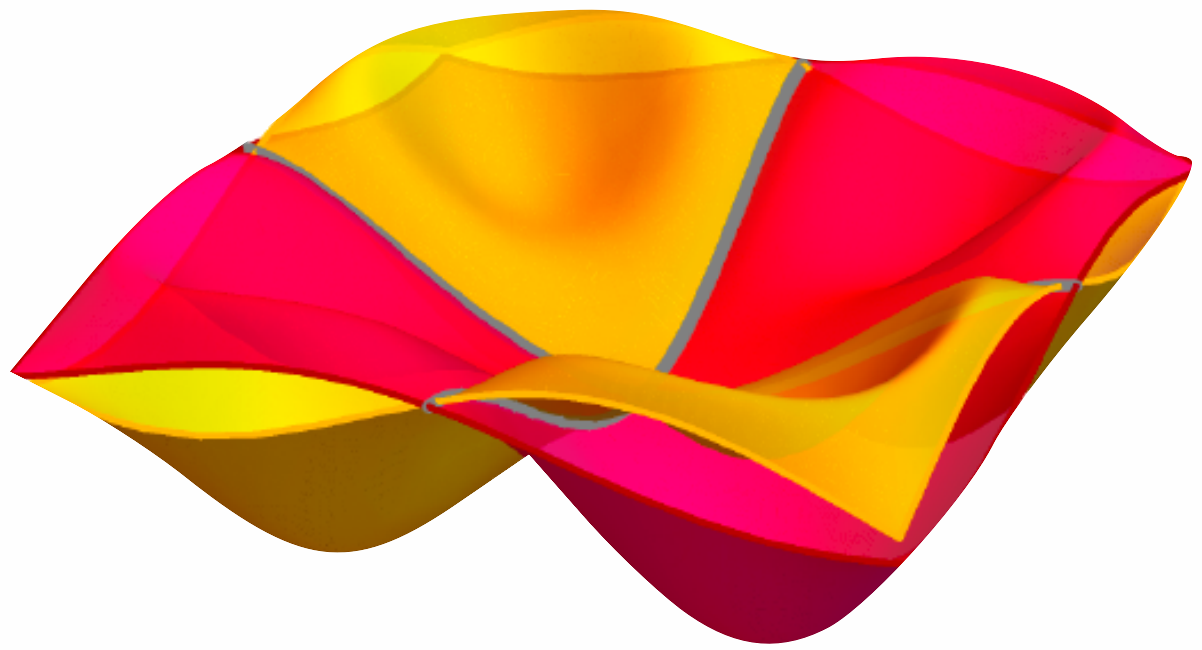
Fermionic band structures

Ex: 12-fold degenerate fermions



J. Yang, Z.X. Liu, C. Fang, Nature Comm. 15 10203 (2024)

Altermagnetic band structures

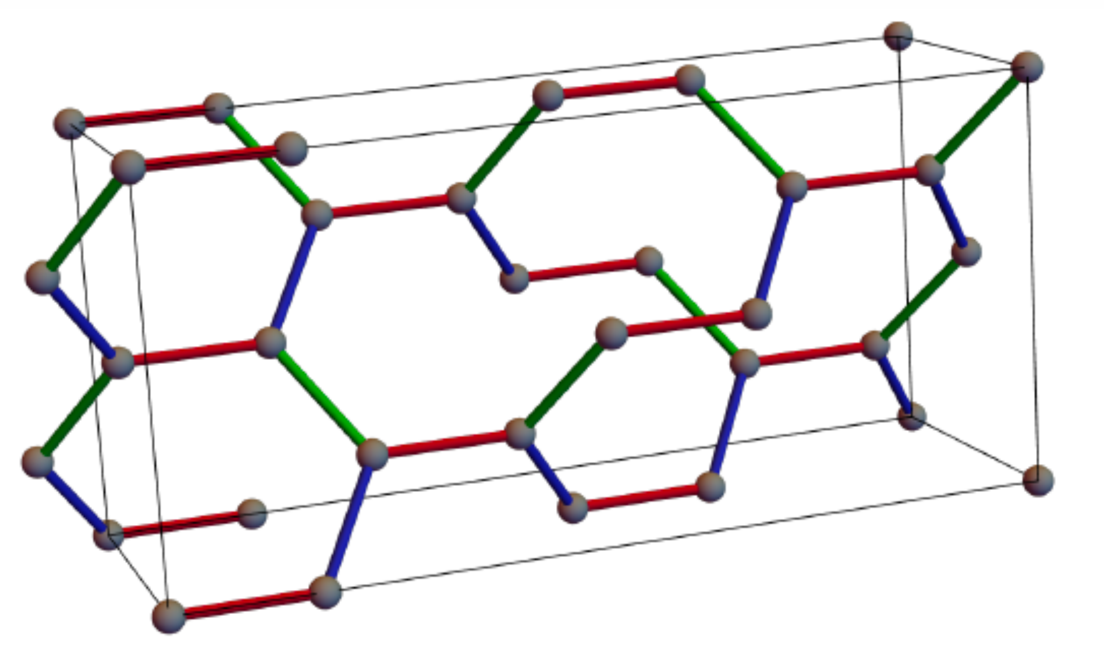


HS, A. Corticelli, A. Guerreiro, J. Romhányi, P. McClarty, SciPost Phys. 18, 109 (2025)

L. Šmejkal, J. Sinova and T. Jungwirth, Phys. Rev. X 12, 040501 (2022)

Magnon band structures

Ex: hyper-honeycomb Heisenberg-Kitaev model

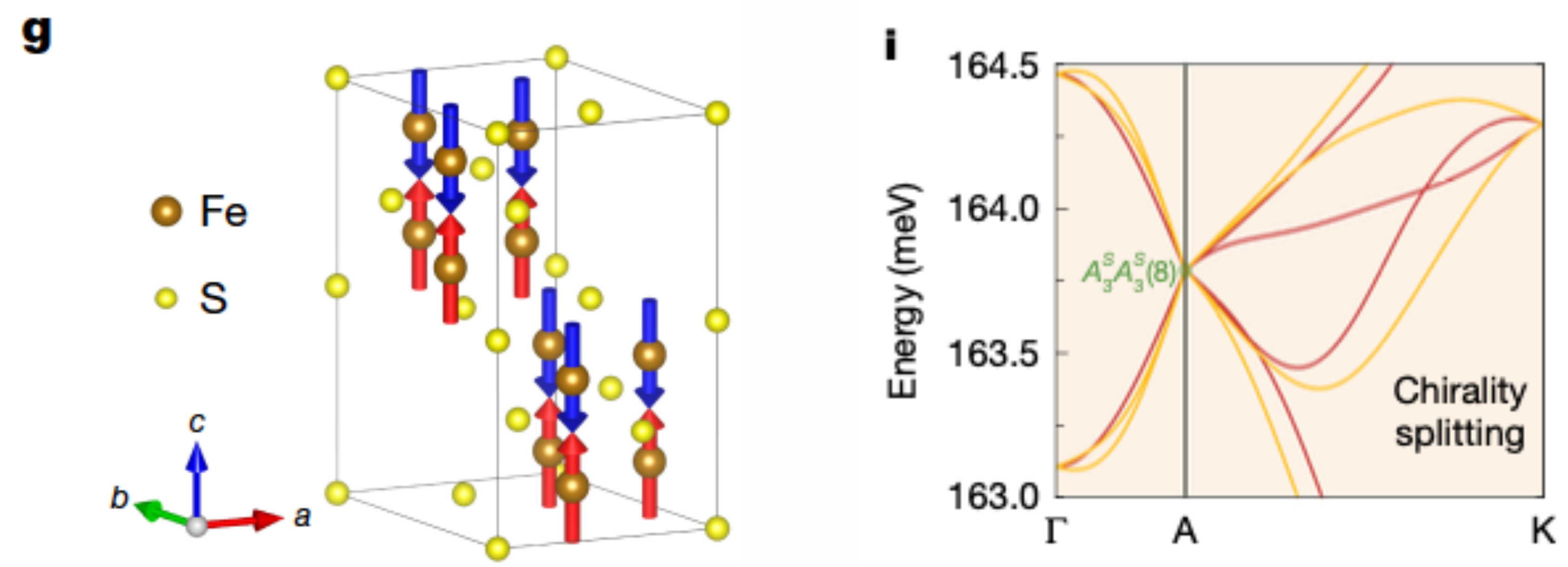


A. Corticelli, R. Moessner, P. McClarty Phys. Rev. B 105, 064430 (2022)

HS, A. Corticelli, A. Guerreiro, J. Romhányi, P. McClarty, SciPost Phys. 18, 109 (2025)

Unconventional magnons

X. Chen, Y. Liu, P. Liu, Y. Yui, J. Ren, J. Li, A. Zhang, Q. Liu, Nature 640, 349-354 (2025)



Can avoid spin group theory!

Landau theory for collinear altermagnets

Equivalent *formulation for collinear systems without spin groups*

Salient *physics from the SO-free limit even for real materials with some SOC*

Background

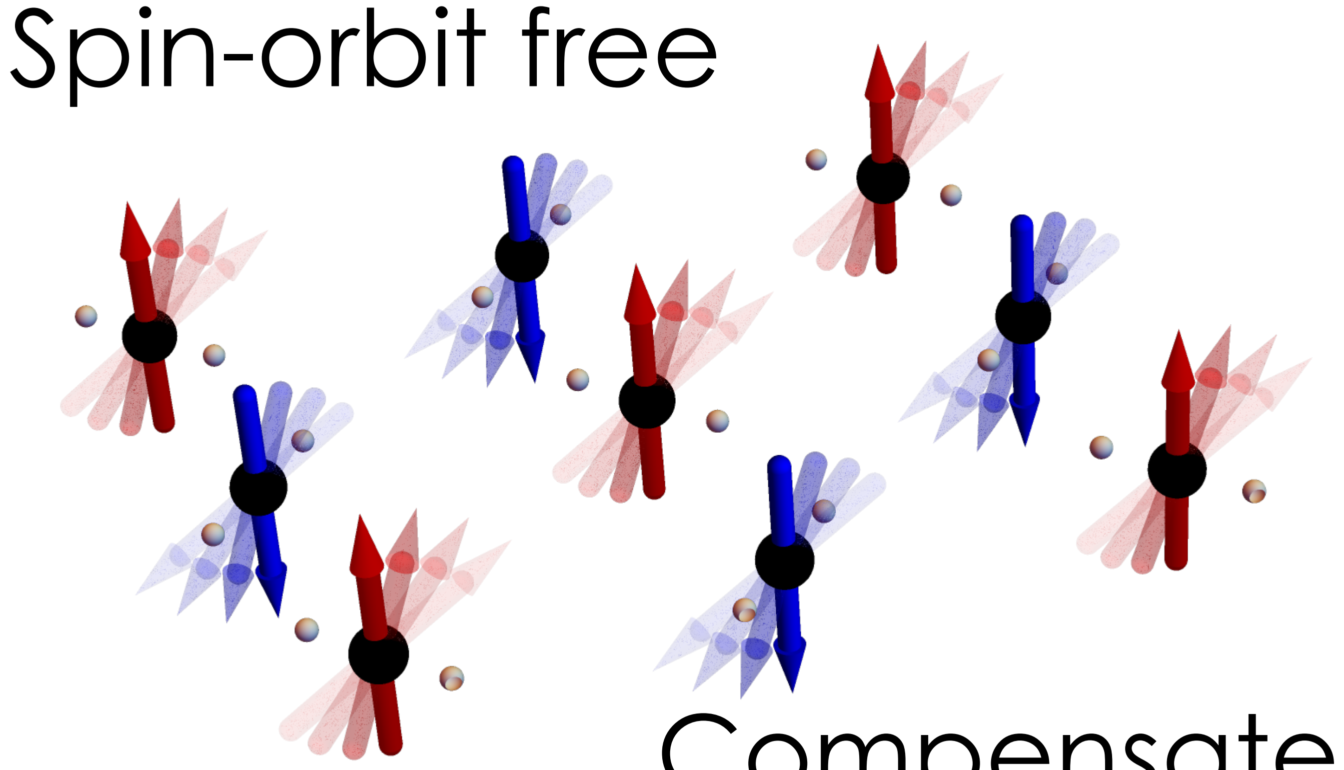
What are collinear altermagnets?

Background

collinear
What are altermagnets?

Ideal Altermagnets

Spin-orbit free



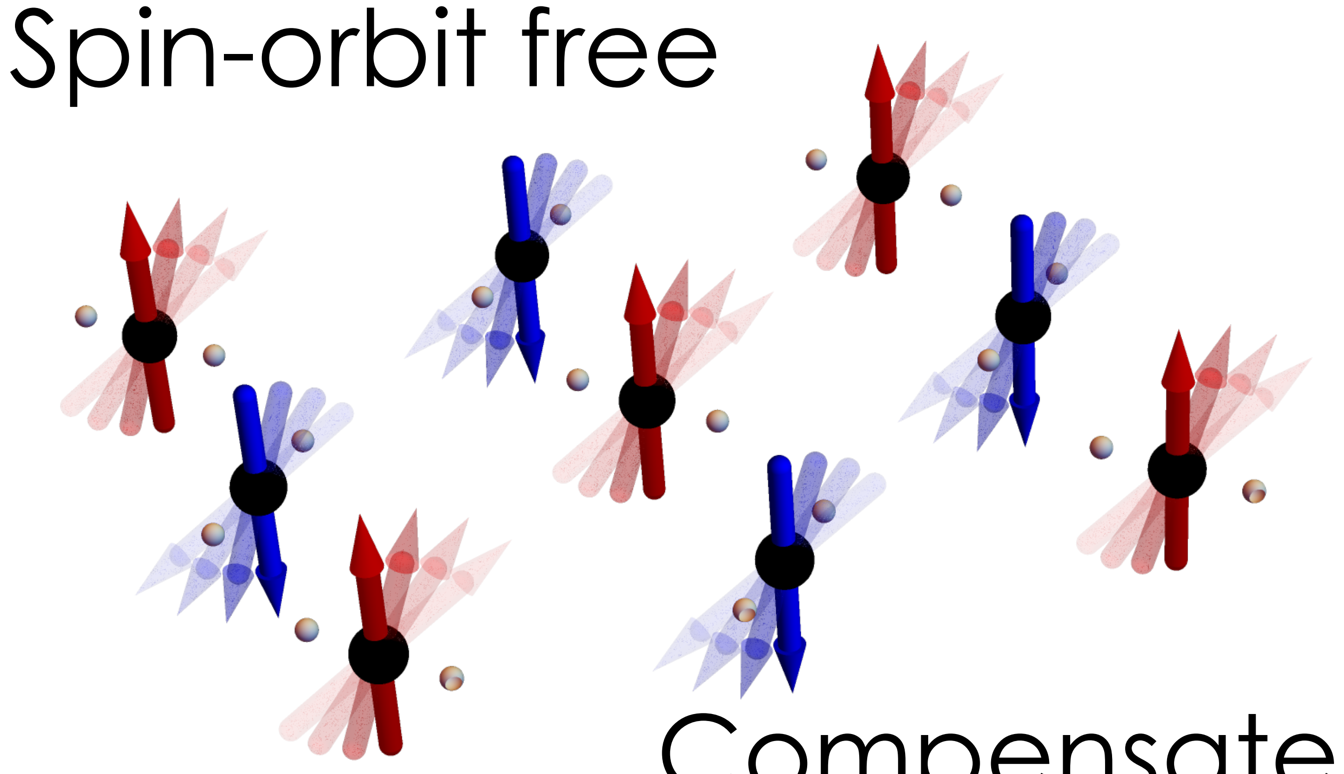
Compensated

Background

What are collinear altermagnets?

Ideal Altermagnets

Spin-orbit free



Compensated

Litvin, Opechowski, Physica Volume 76, Issue 3 (1974)

Decoupled spin & orbital space

Spins and lattice can transform independently

Spin groups

Šmejkal, Sinova, Jungwirth, Phys. Rev. X **12**, 031042 (2022)

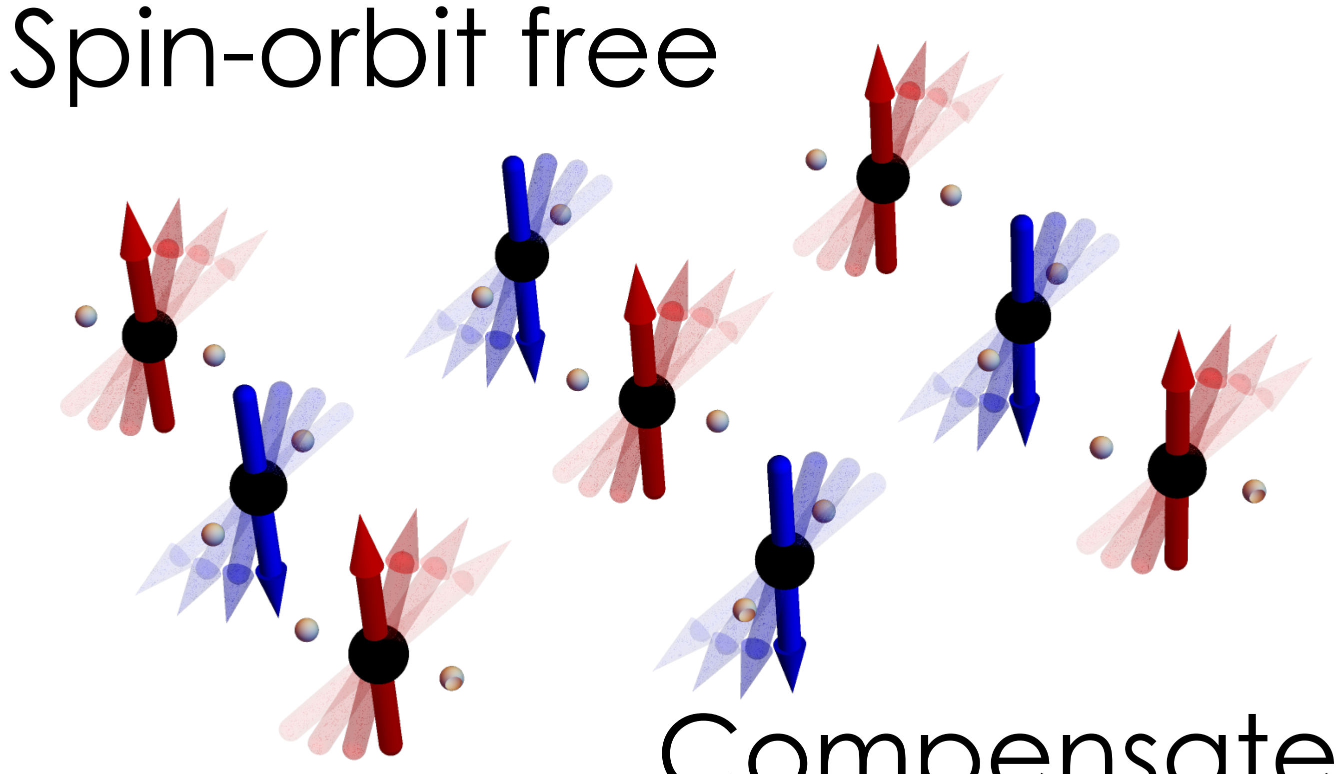
Background

collinear

What are altermagnets?

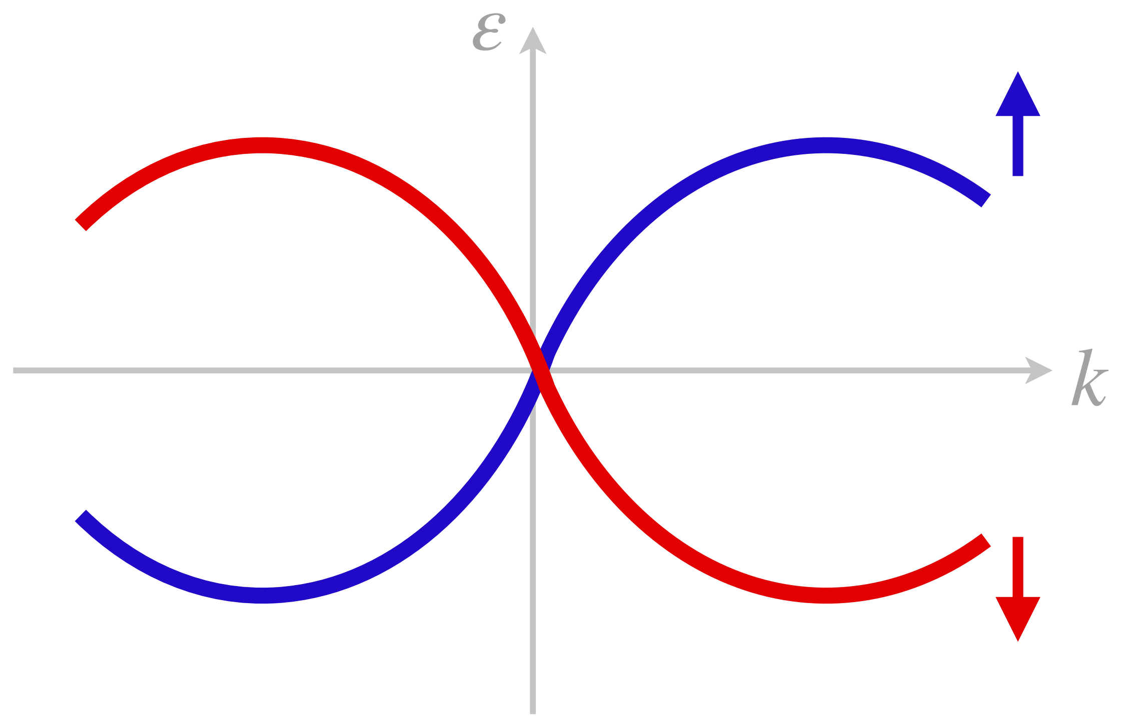
Ideal Altermagnets

Spin-orbit free



Compensated

Alternating spin-splitting



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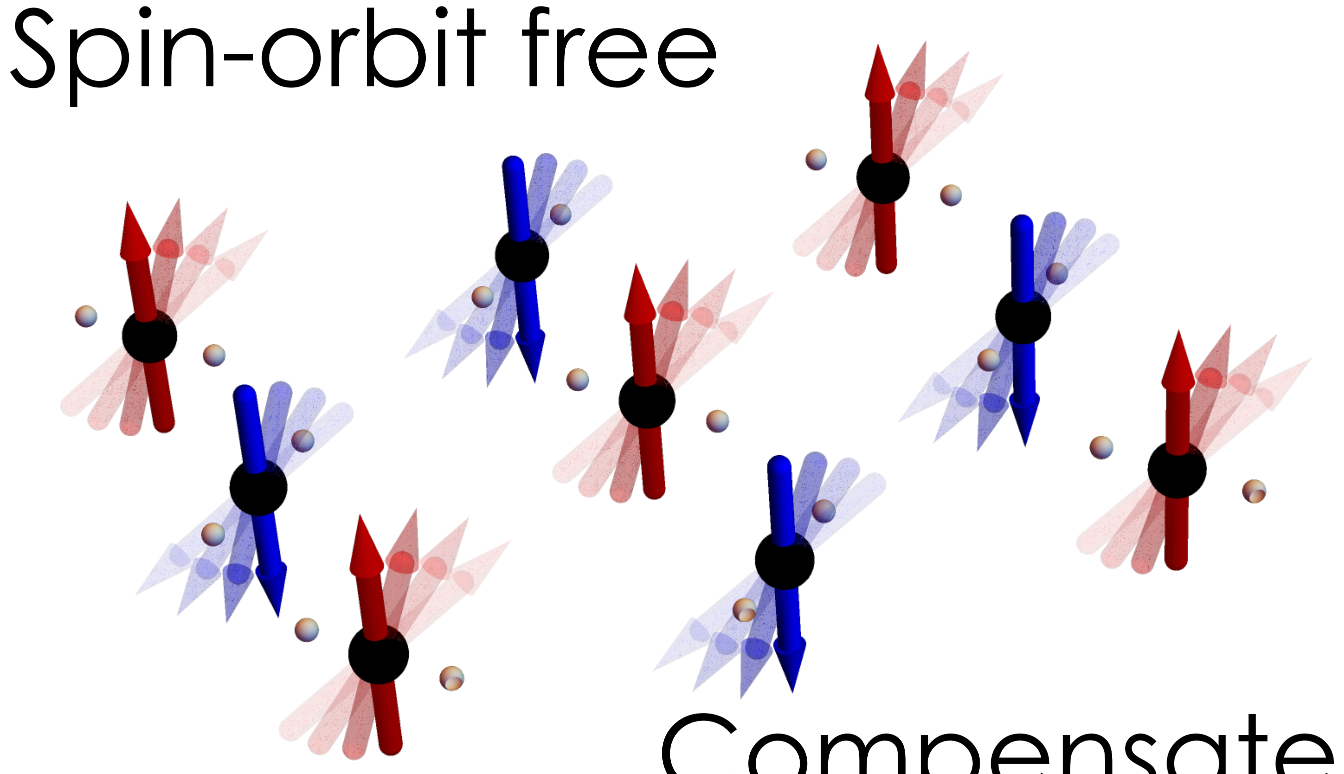

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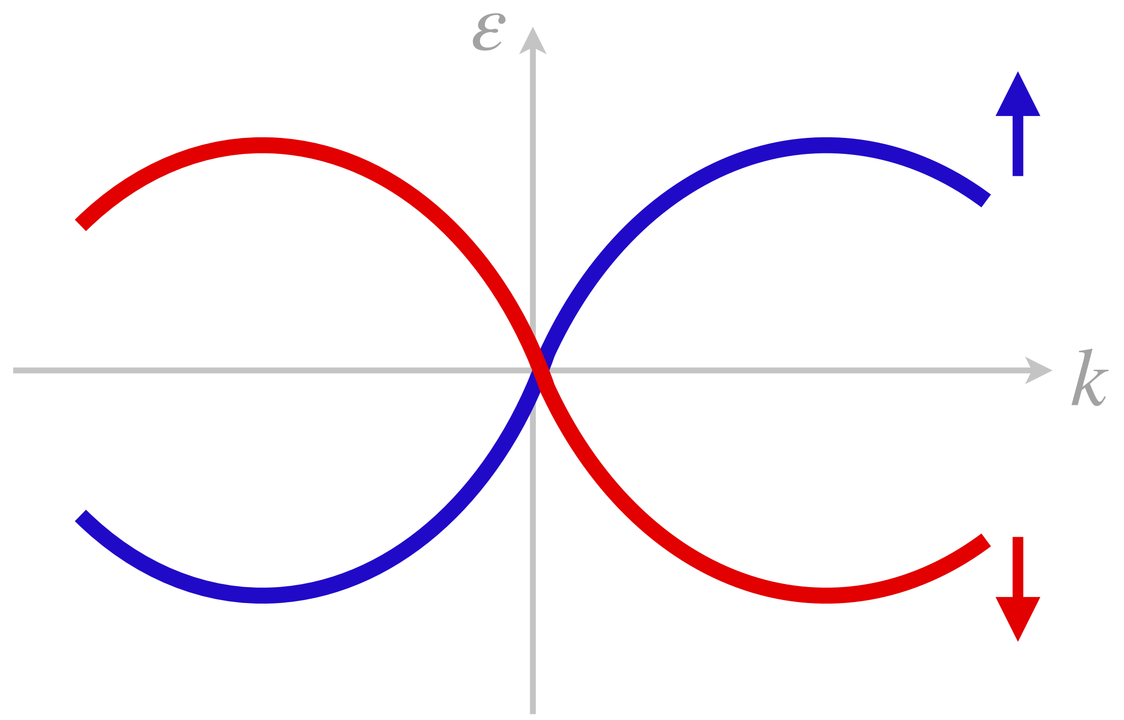

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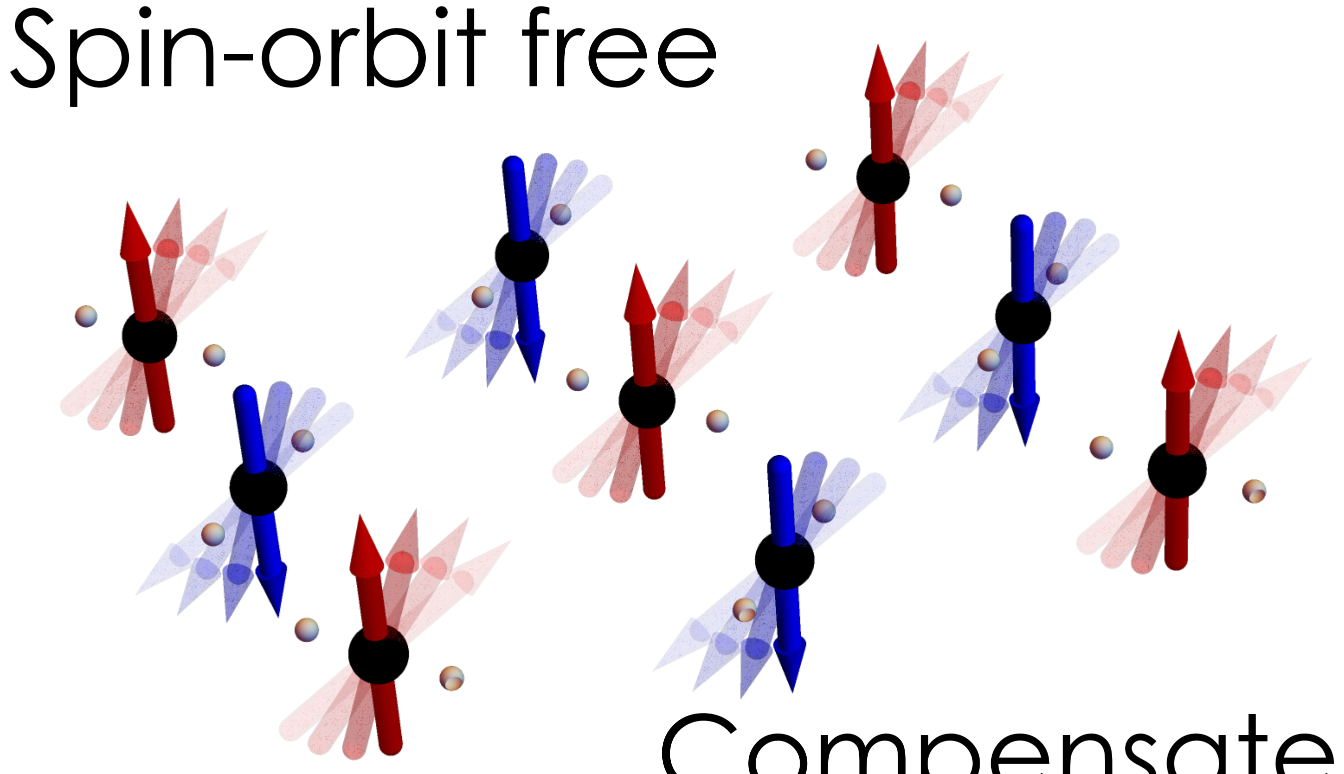

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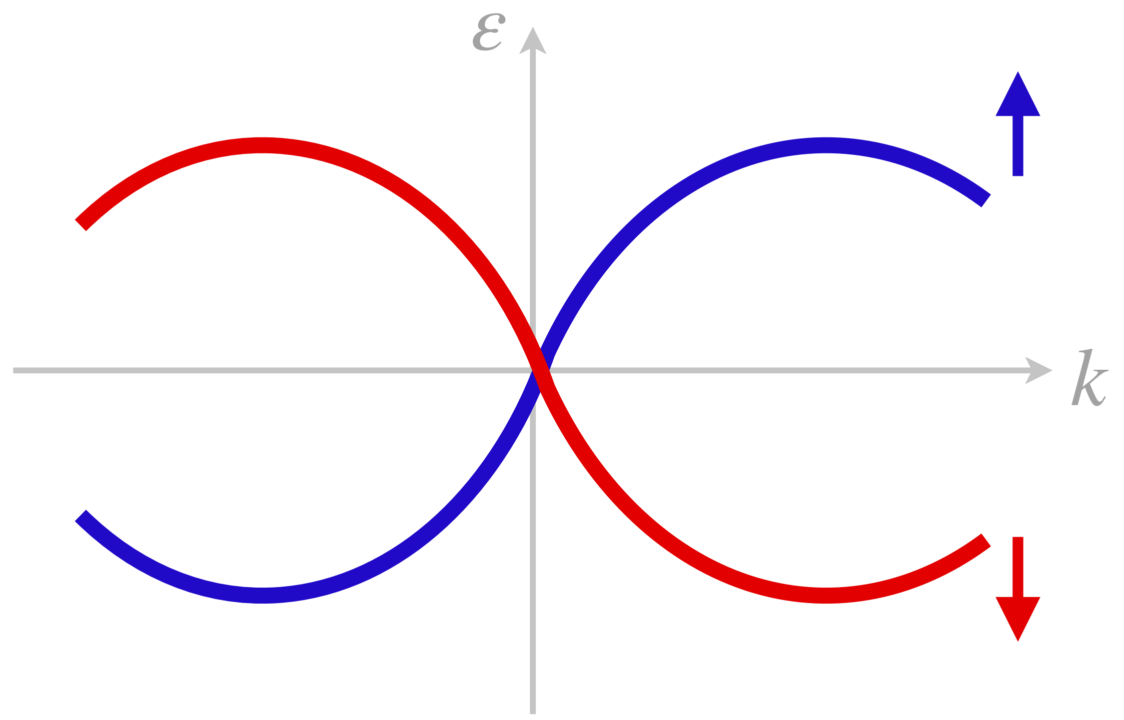

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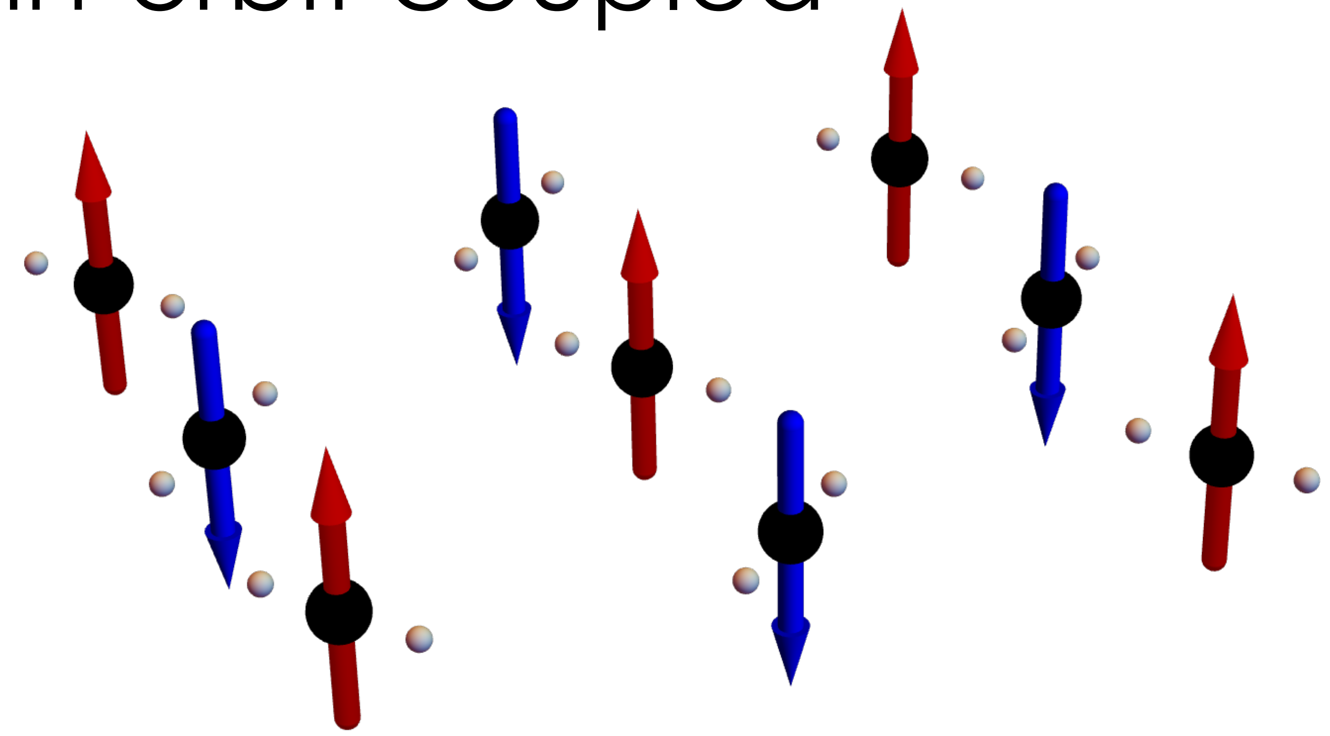
Real Altermagnets

Spin-orbit coupled

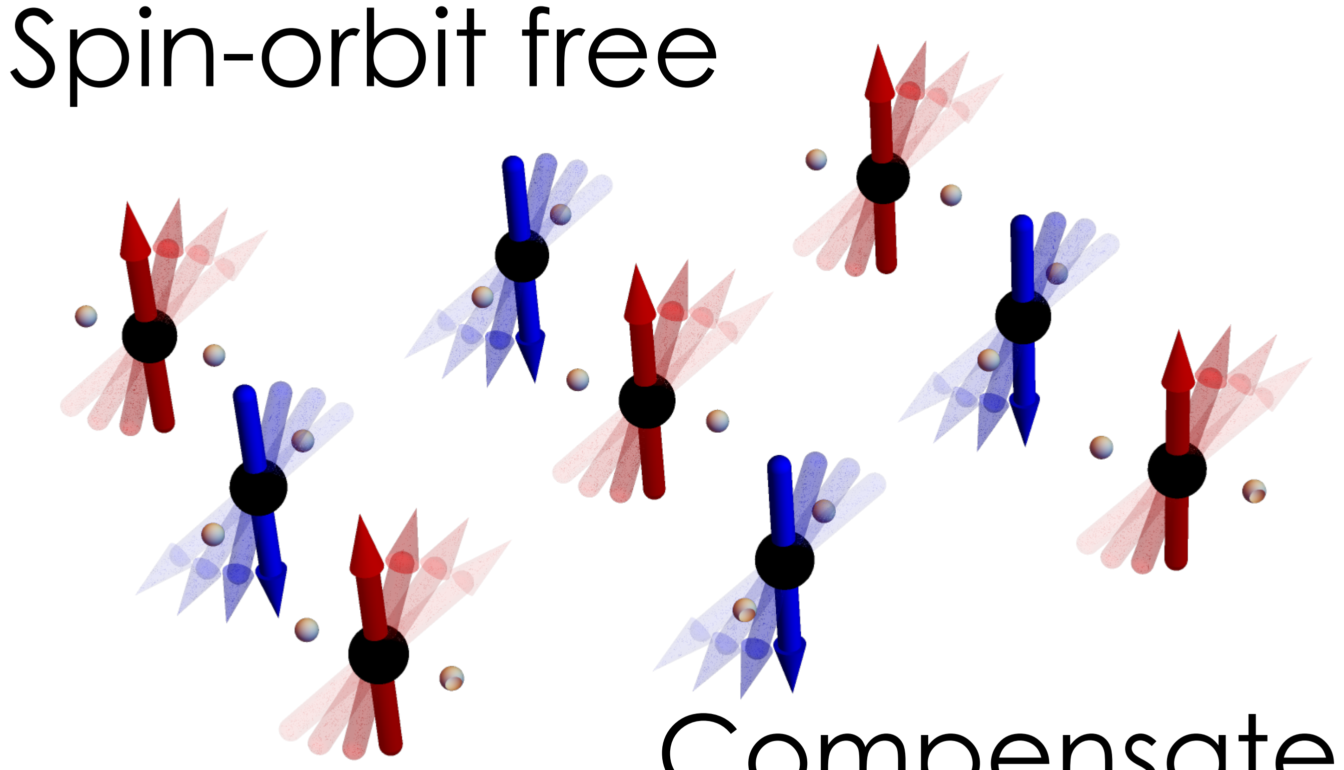


Background

What are collinear altermagnets?

Ideal Altermagnets

Spin-orbit free



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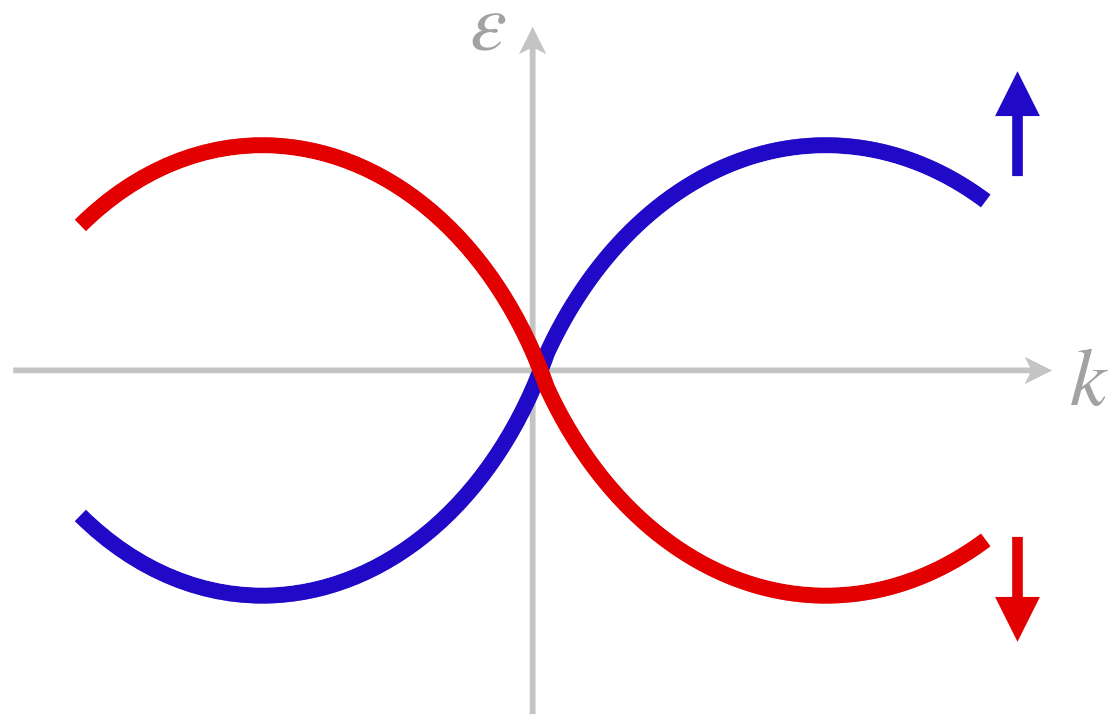

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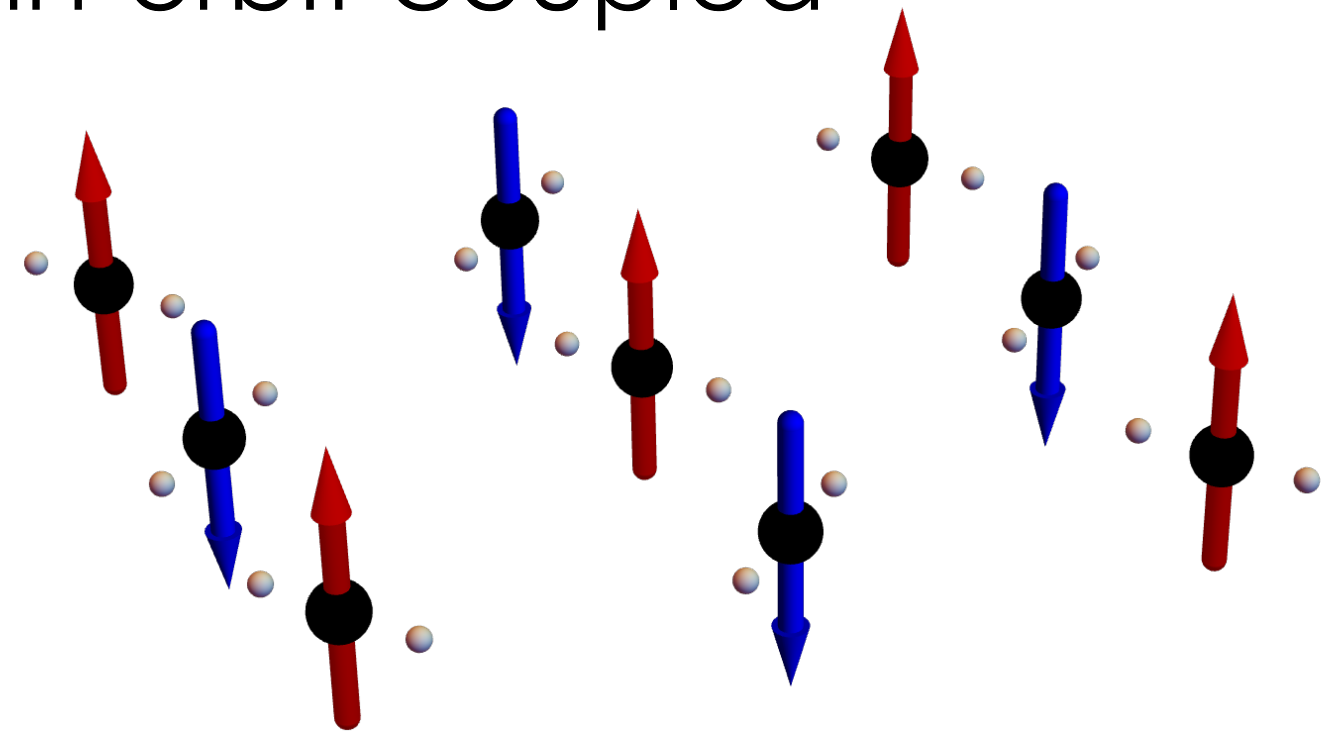


Locked spin & orbital space

Spins transform with lattice

Magnetic groups

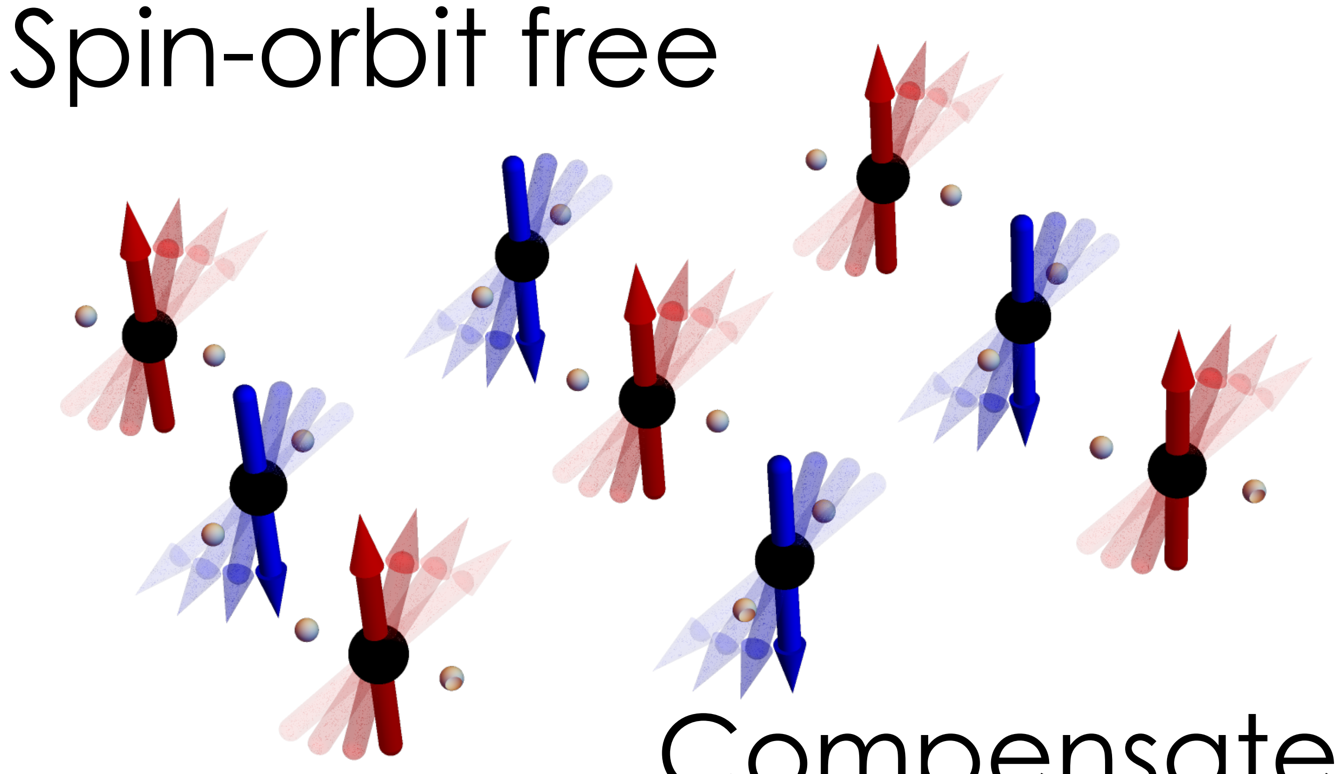
Šmejkal, Sinova, Jungwirth, Phys. Rev. X **12**, 031042 (2022)

Background

What are collinear altermagnets?

Ideal Altermagnets

Spin-orbit free



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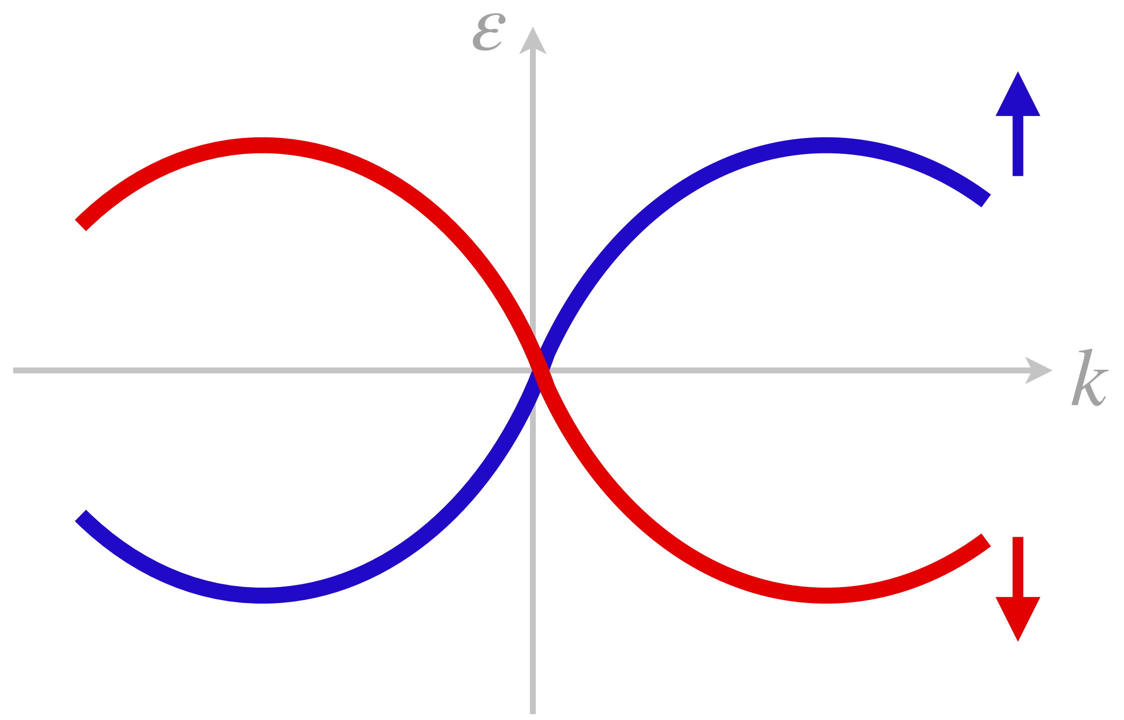

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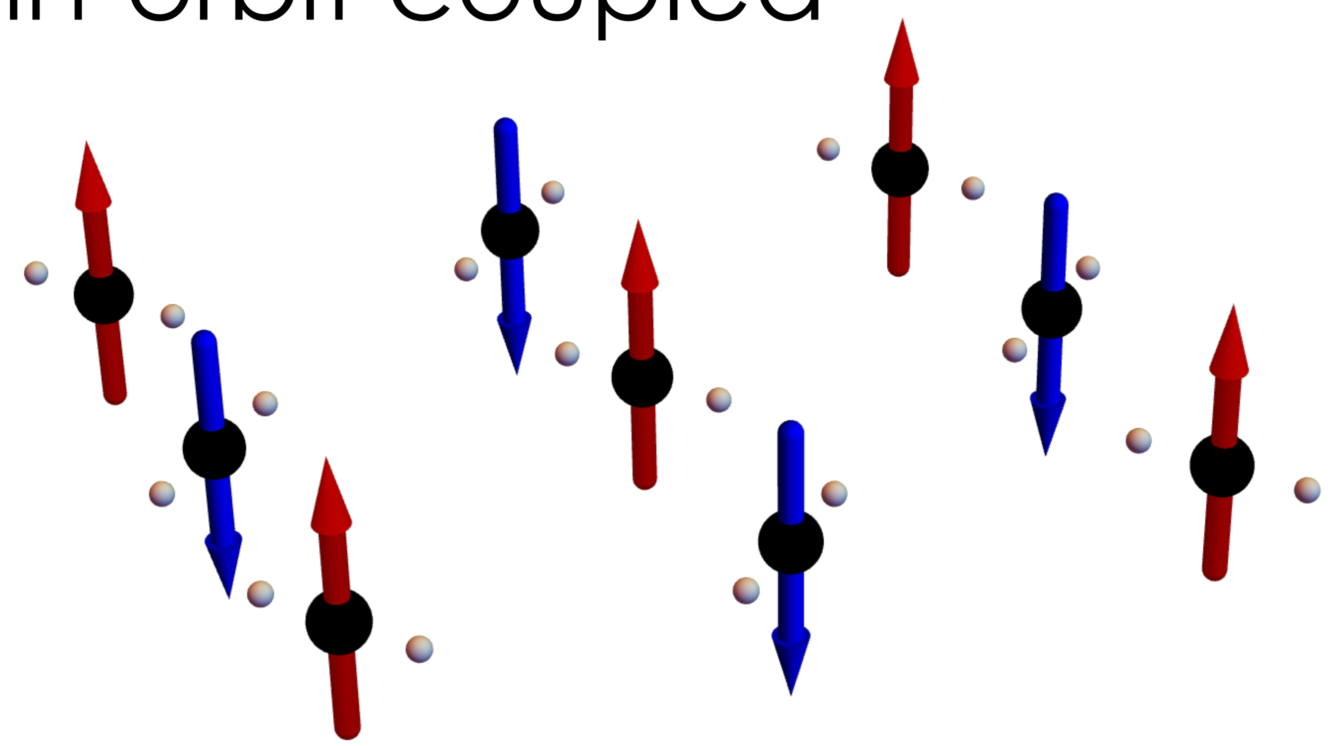
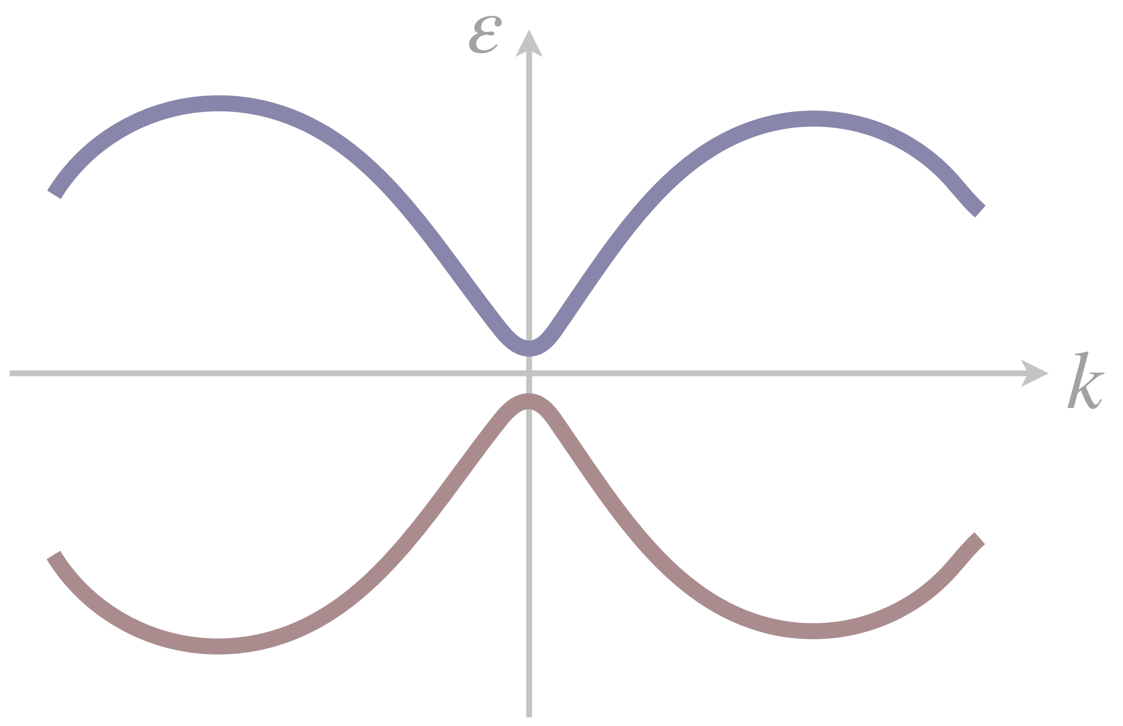


…but SOC is small!



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Ideal Altermagnets

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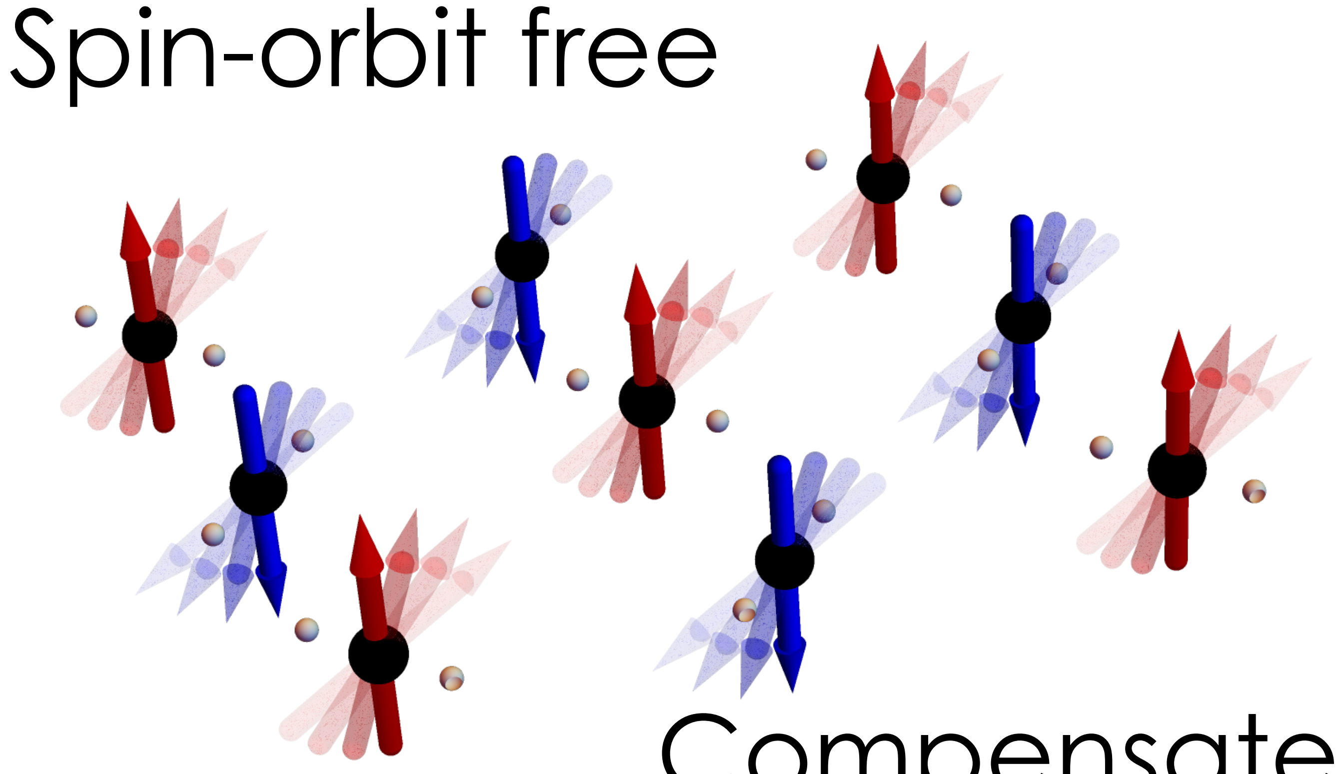

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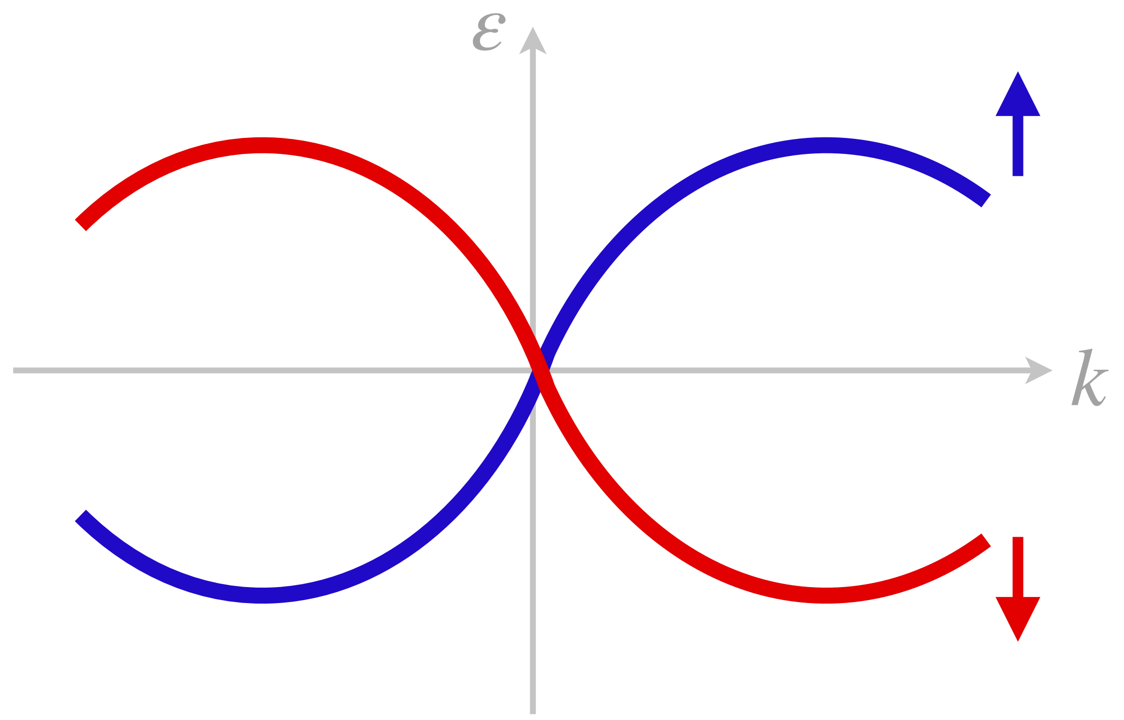

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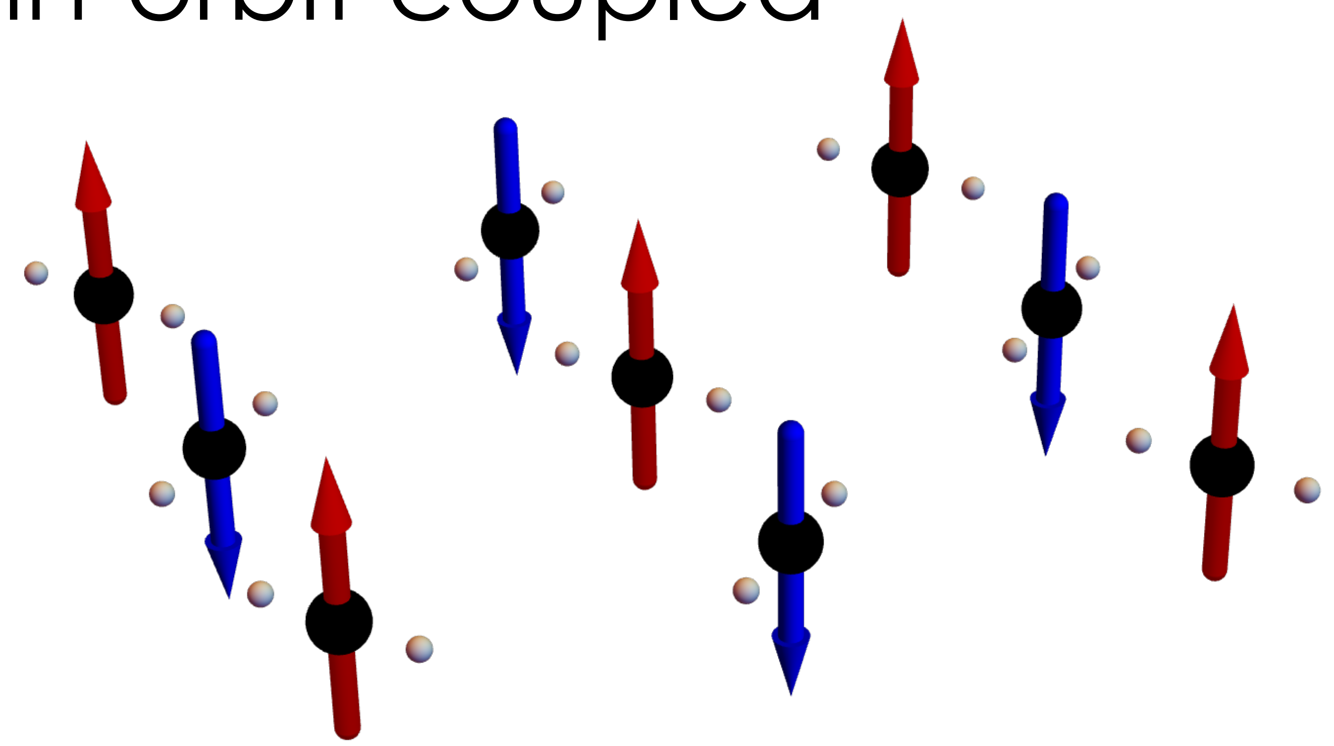

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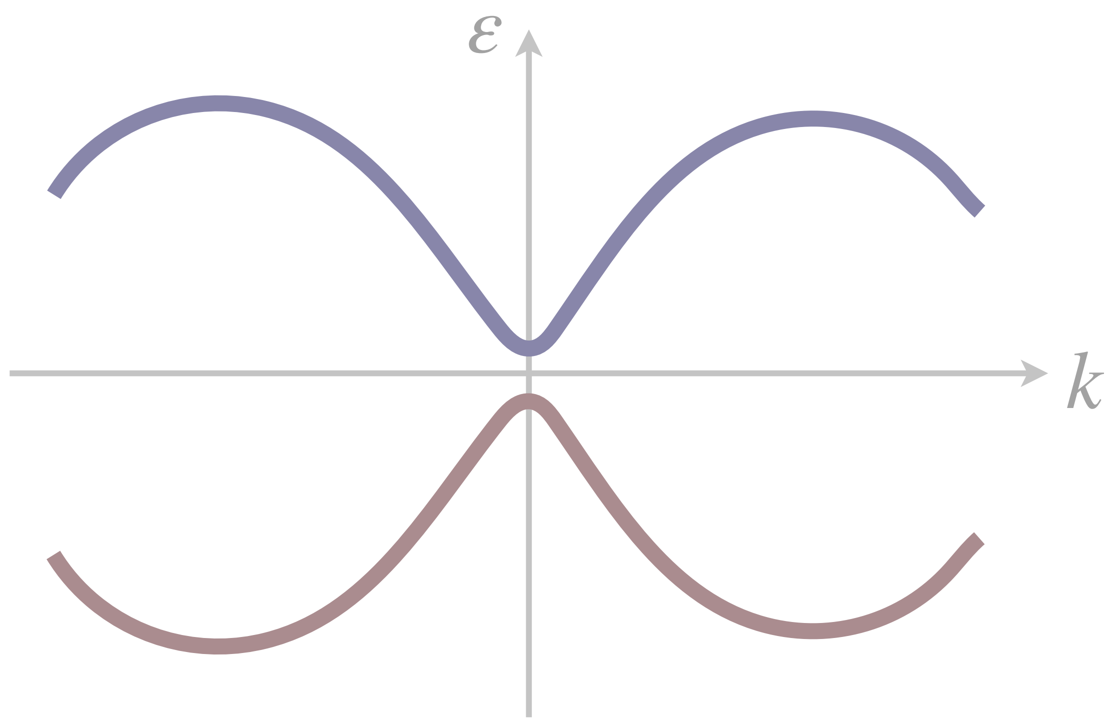


Locked spin & orbital space

Spins transform with lattice

Magnetic groups

SOC < Splitting

Dominant features from SO-free limit

Presence of SOC changes symmetry

Structures

Where to place magnetic ions?

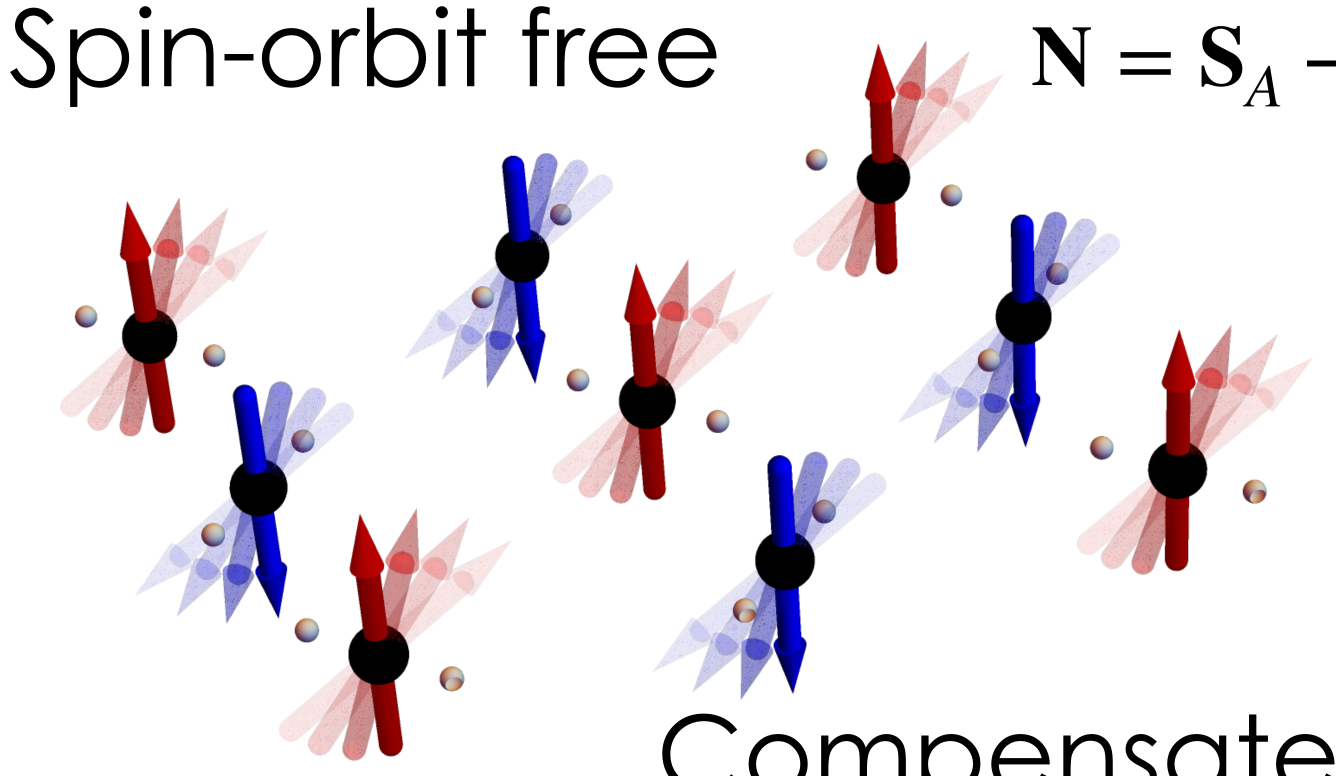
Structures

Where to place magnetic ions?

Ideal Altermagnets

Spin-orbit free

$\mathbf{N} = \mathbf{S}_A - \mathbf{S}_B$



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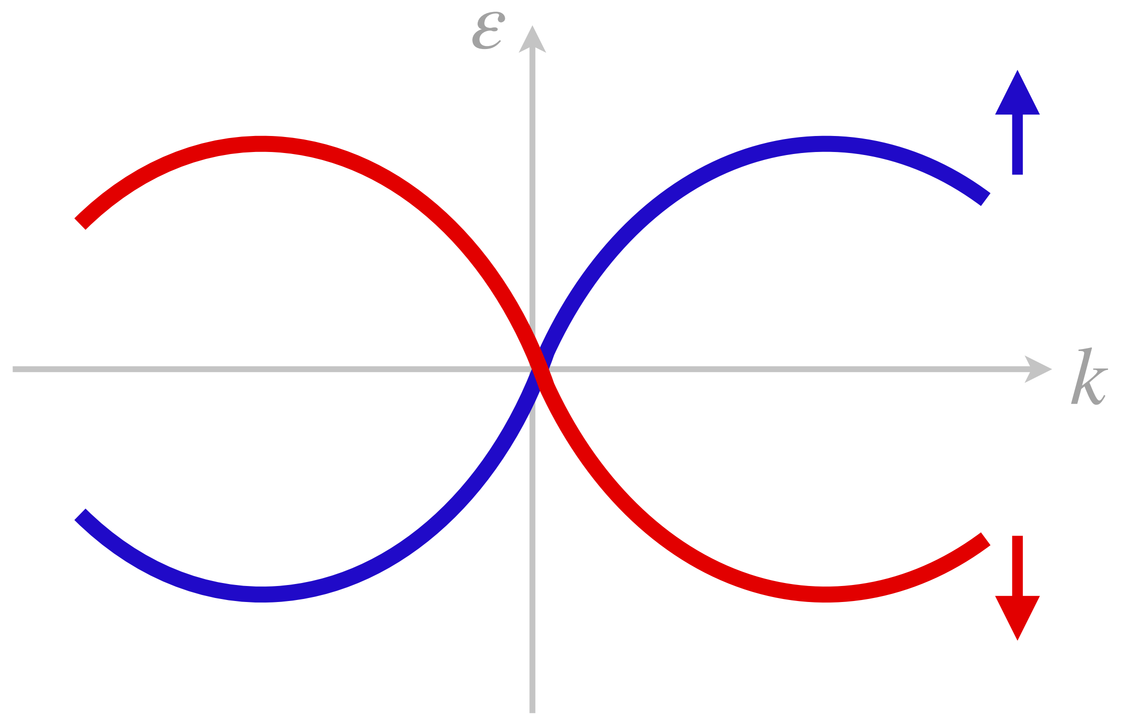
Alternating spin-splitting



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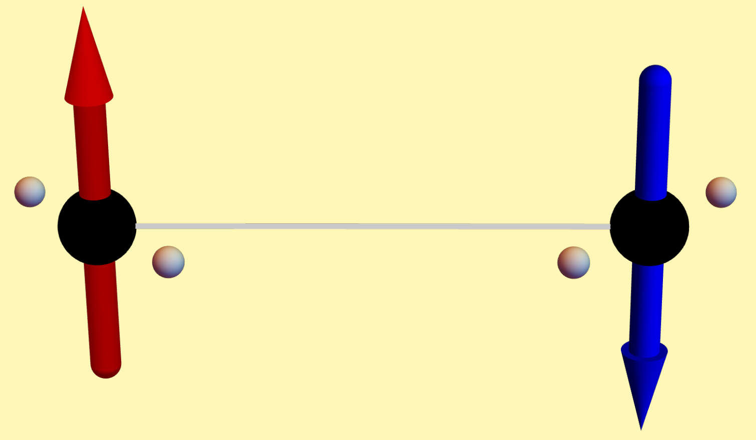


Translation & inversion don't connect ↑ / ↓

Structures

Where to place magnetic ions?

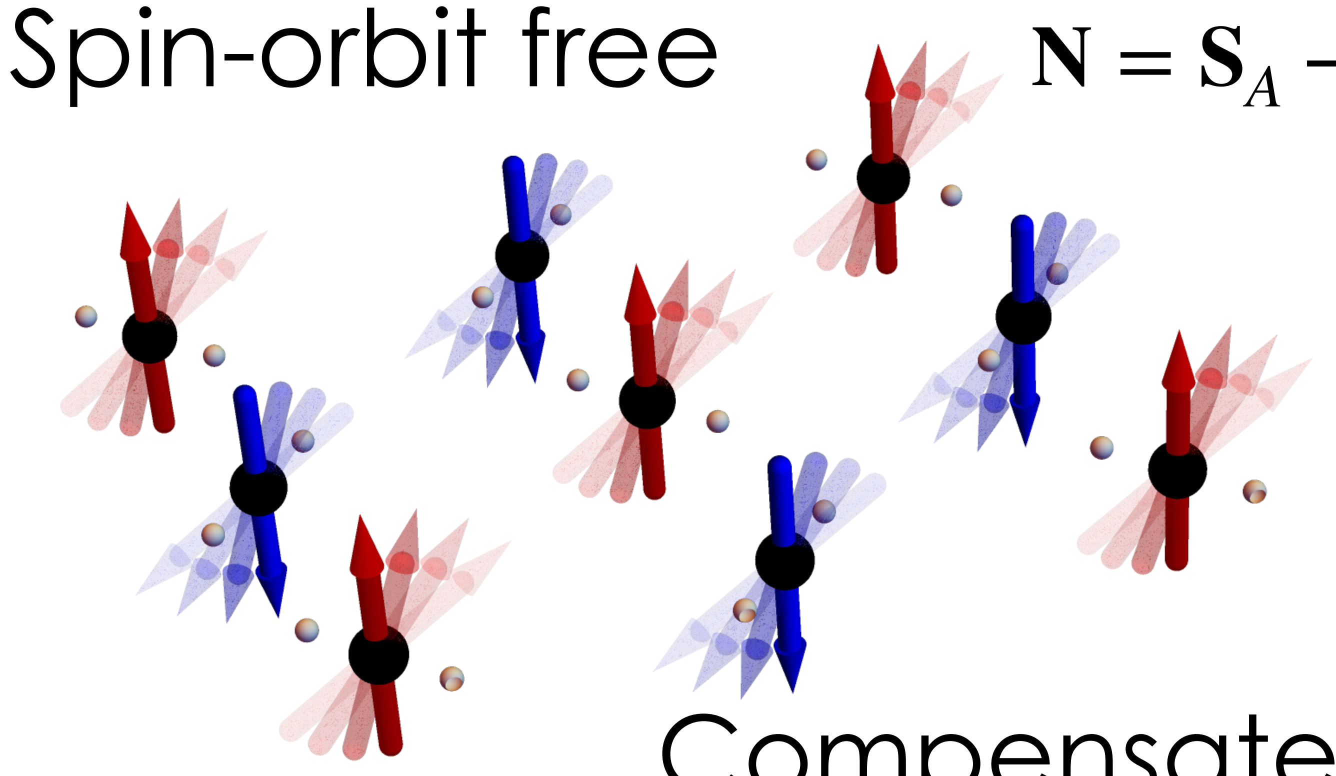
Under space group operations

Preserves ↑ and ↓ $\mathbf{N} \to \mathbf{N}$ or **Swaps ↑ and ↓** $\mathbf{N} \to -\mathbf{N}$

We focus on case where translations act trivially!

Ideal Altermagnets

Spin-orbit free $\mathbf{N} = \mathbf{S}_A - \mathbf{S}_B$



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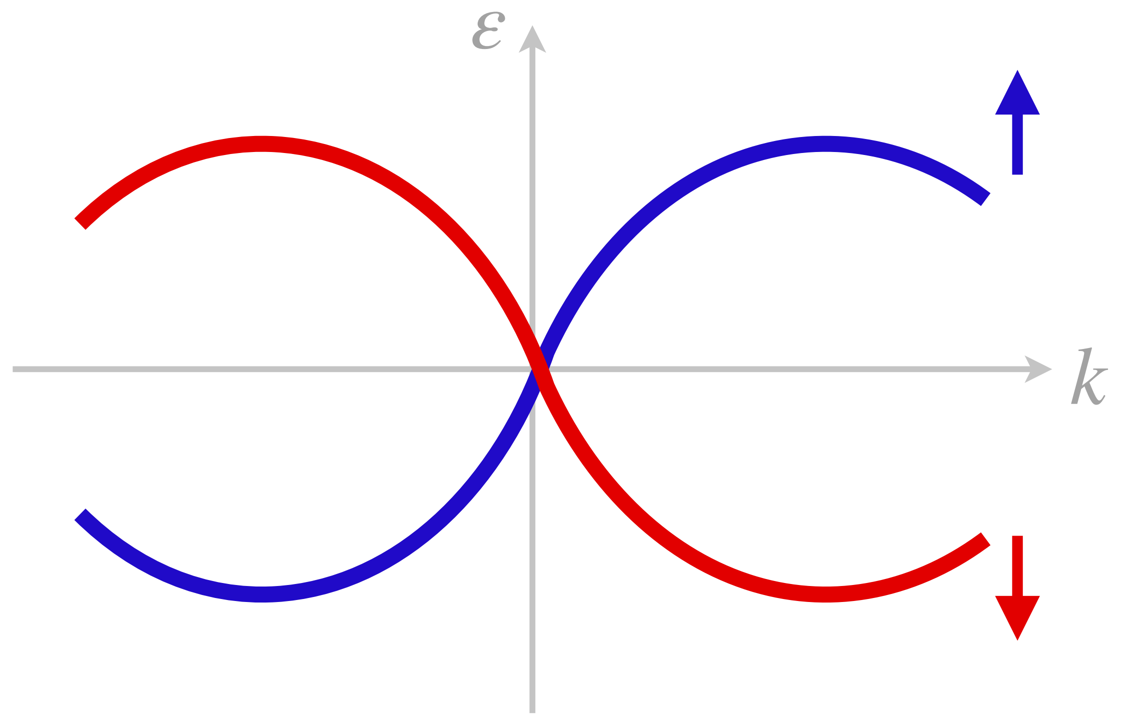

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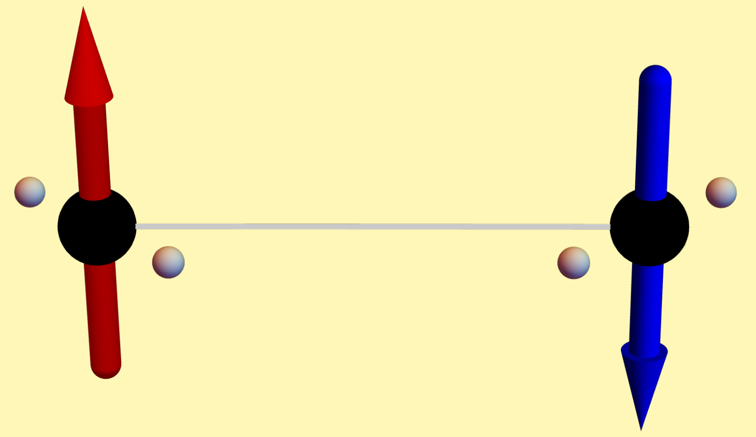

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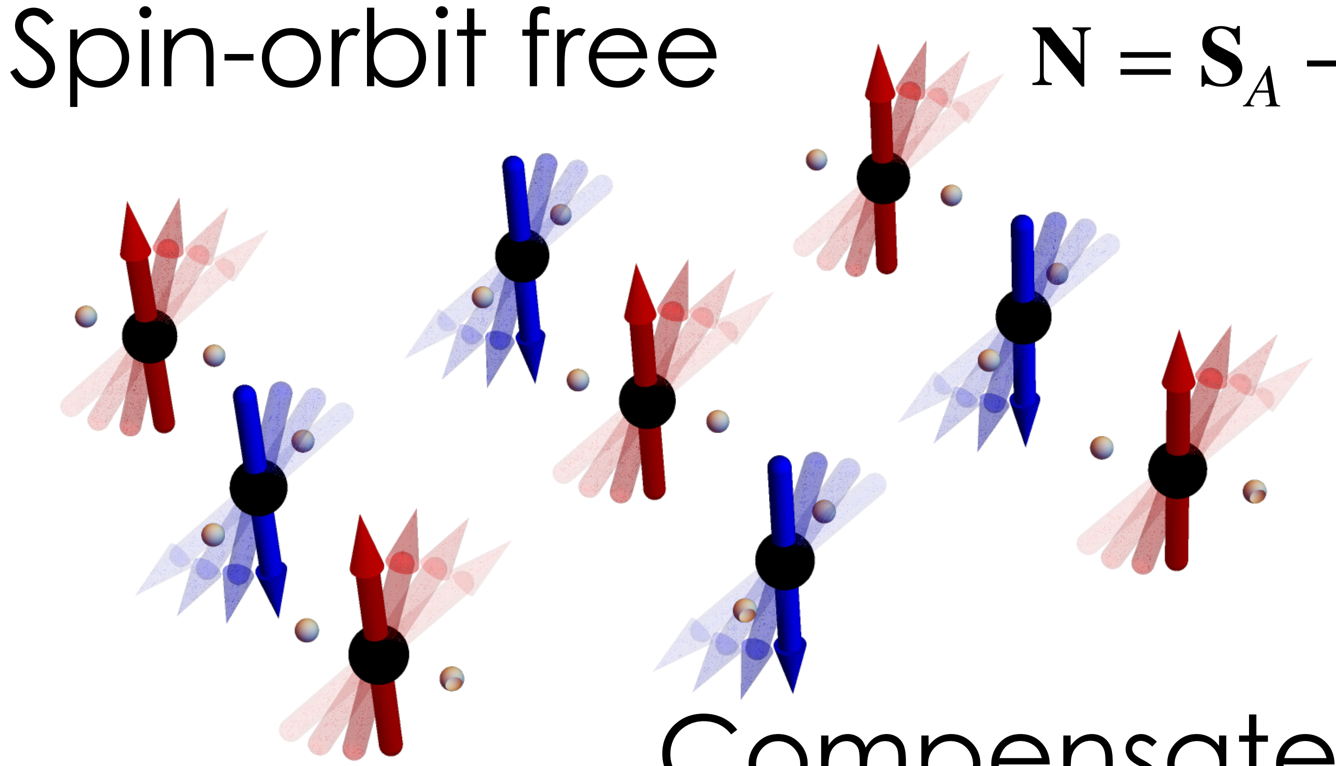

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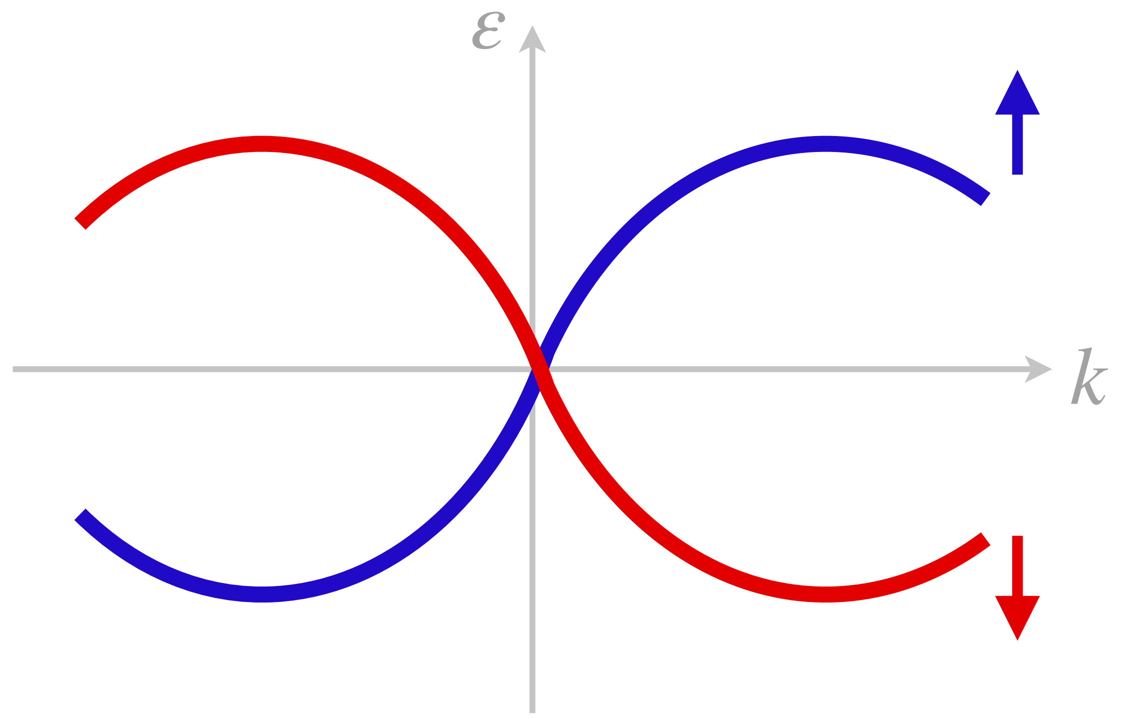

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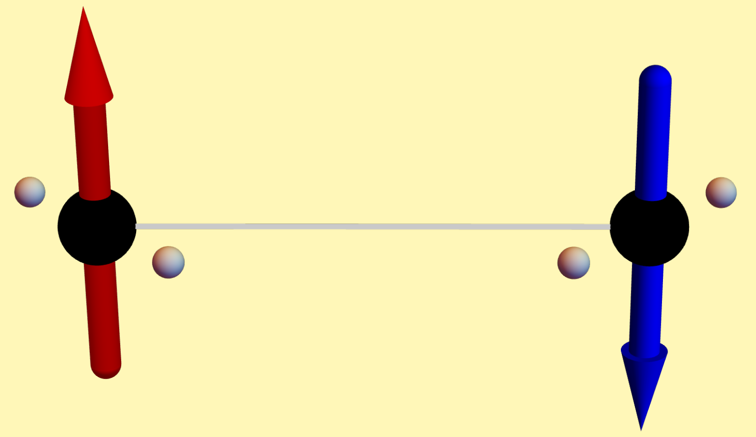

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Altermagnetic if:

$\mathbf{N}$ transforms as a 1D, $\mathbb{R}$ inversion even irrep of the crystal point group: $\Gamma_{\mathbf{N}}$

Structures

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Ideal Altermagnets

Spin-orbit free

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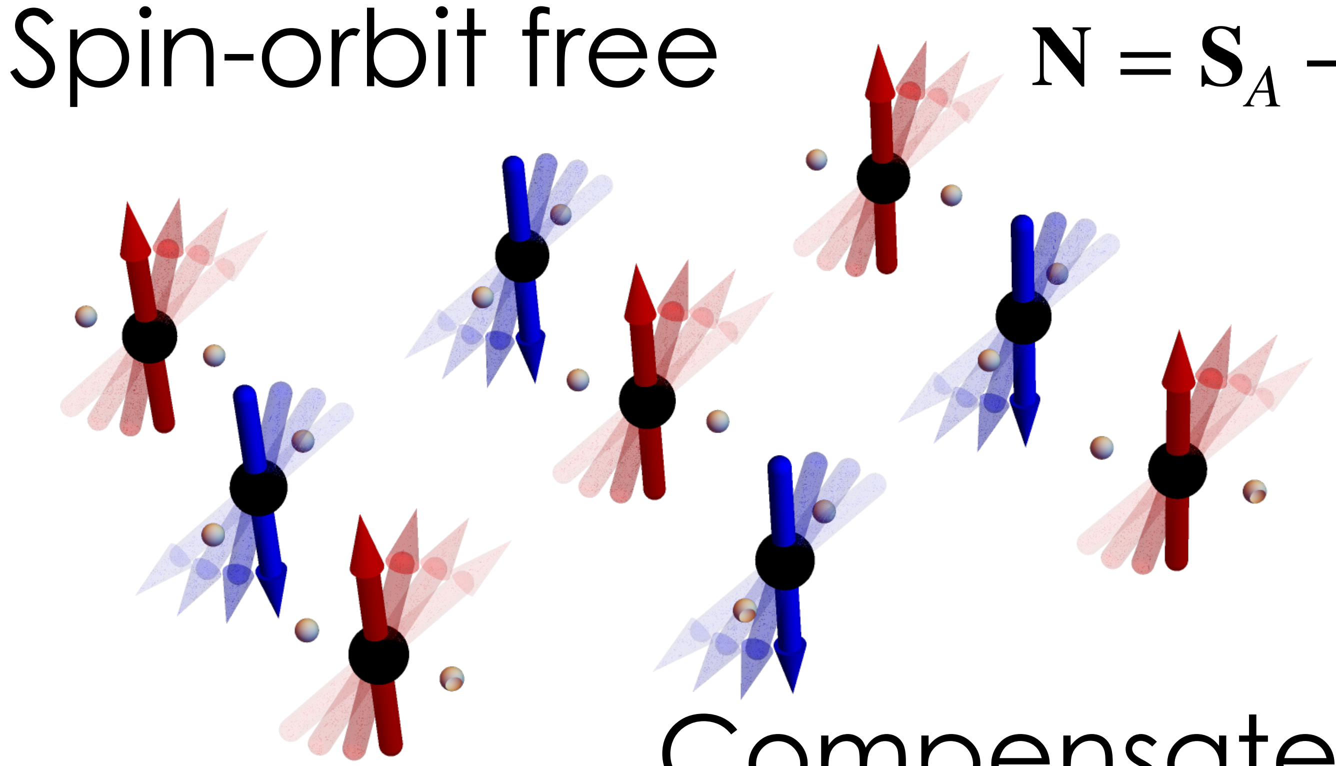

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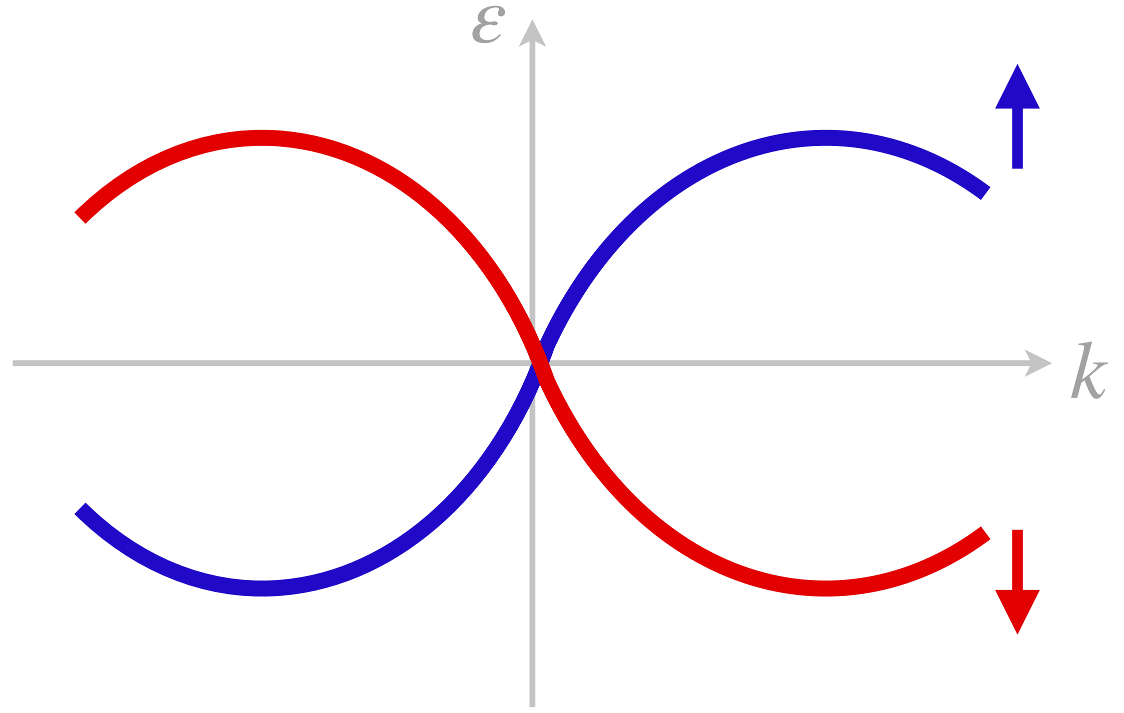

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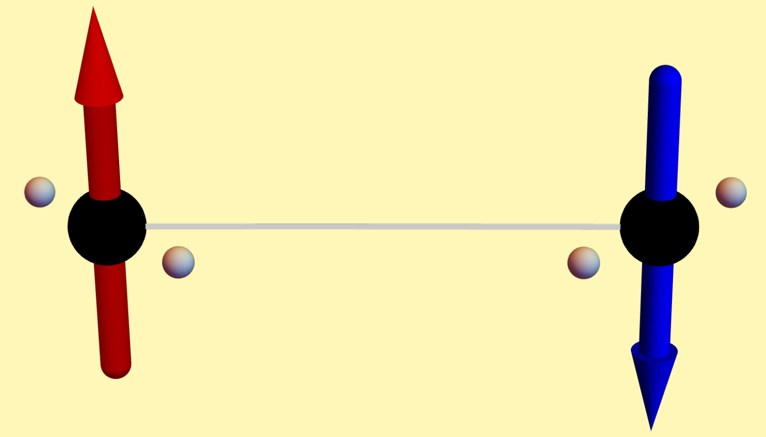

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Any crystal structure defined by Wyckoff position(s) in space group

Structures

Where to place magnetic ions?

Ideal Altermagnets

Spin-orbit free

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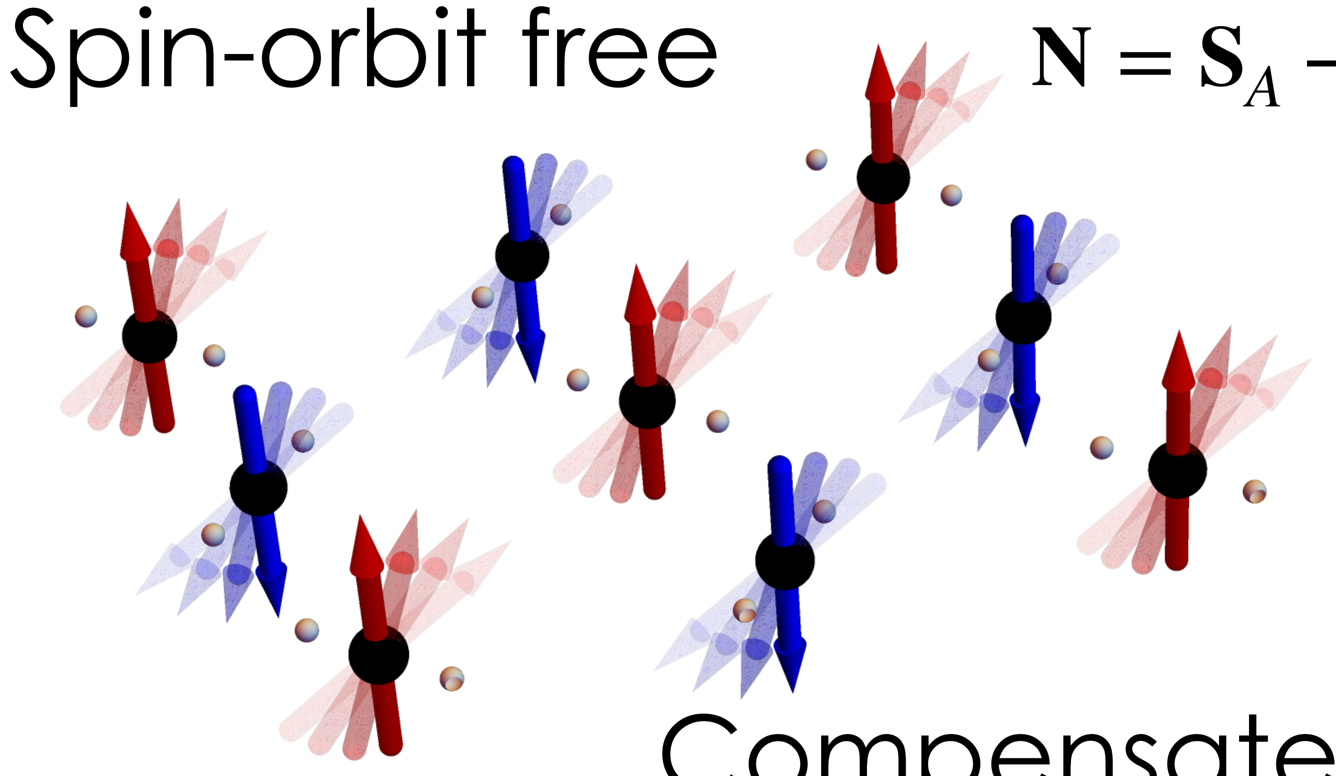

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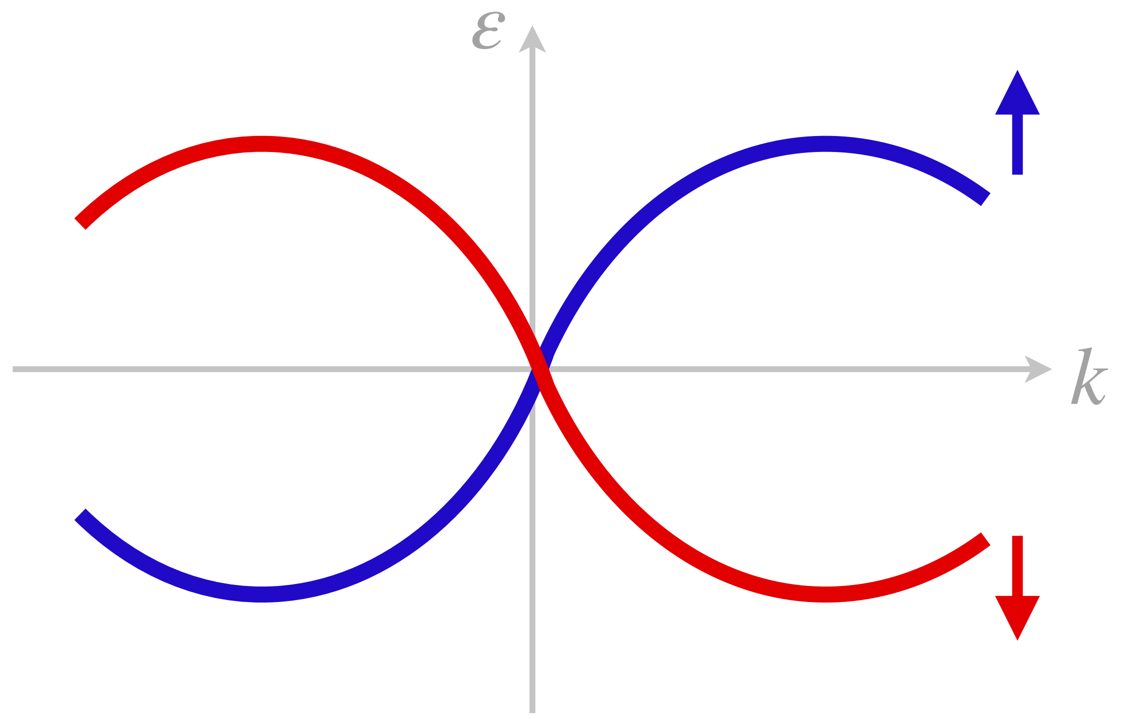

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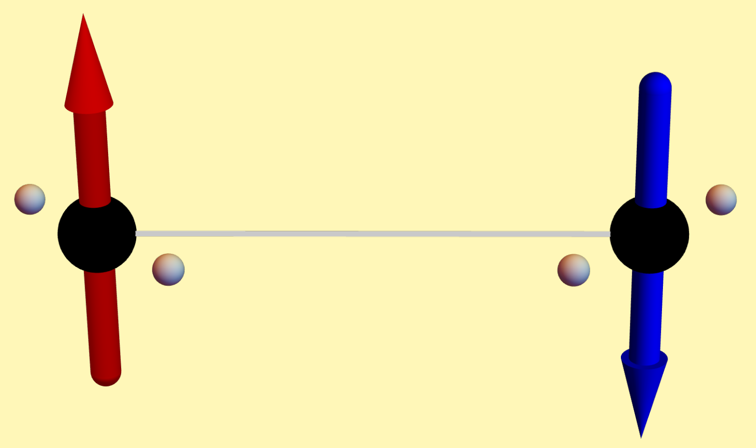

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Any crystal structure defined by Wyckoff position(s) in space group

For all Wyckoff positions:
Form *permutation representation,* check if it contains 1D, $\mathbb{R}$ inversion even irrep of crystal point group

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Where to place magnetic ions?

Ideal Altermagnets

Spin-orbit free

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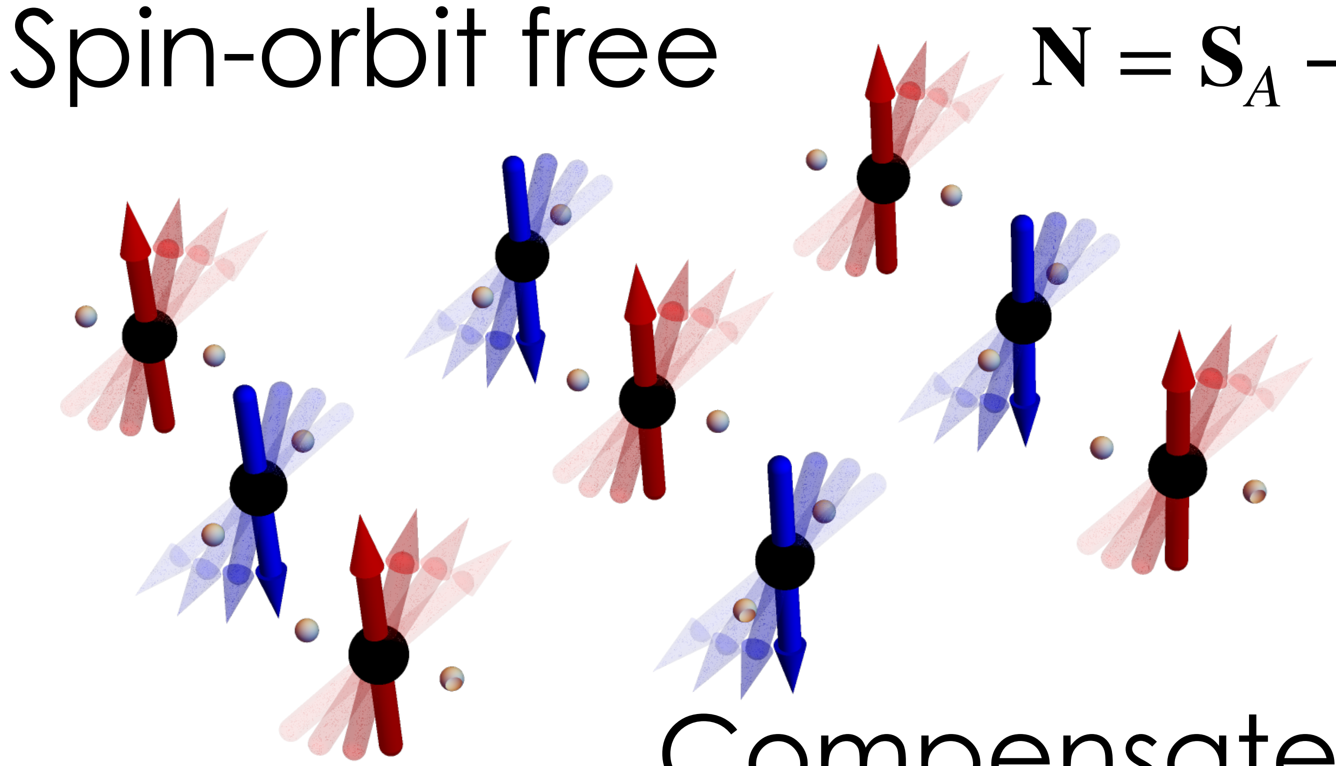

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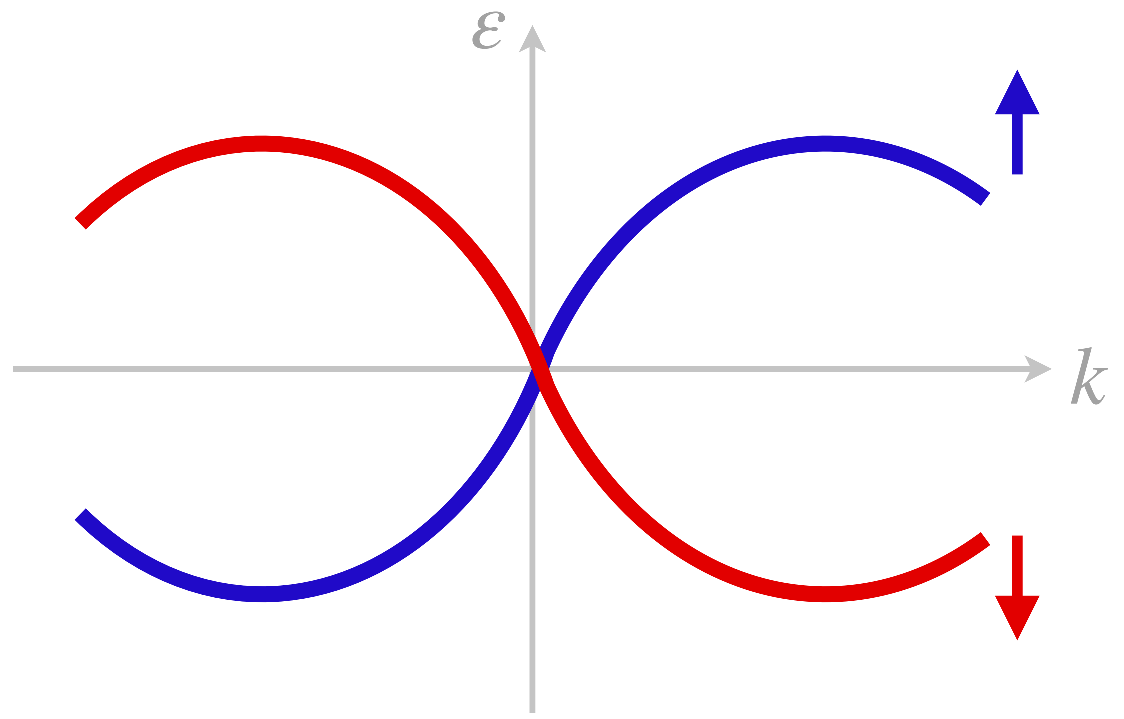

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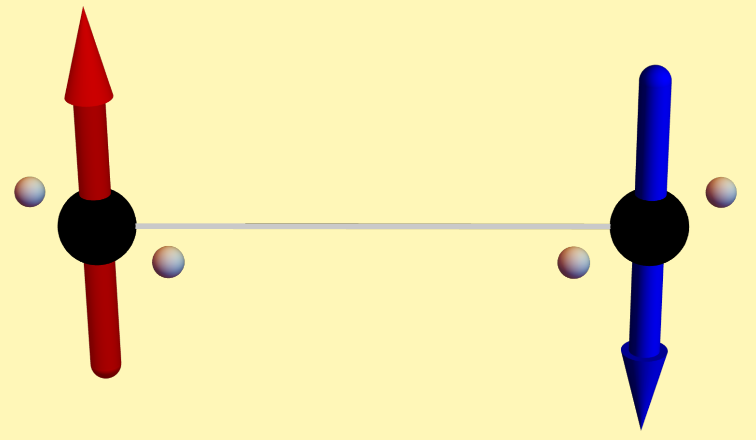


Translation & inversion don't connect ↑ / ↓

PG	SG	WP	$\Gamma_{\mathbf{N}}$
422	94	$\{8g, 4d, 4c\}$	$\{B_1, A_2, B_2\}$
		$\{4f, 4e, 2b, 2a\}$	$\{B_2\}$
	95	$\{8d\}$	$\{B_1, A_2, B_2\}$
		$\{4b, 4a\}$	$\{B_1\}$
		$\{4c\}$	$\{B_2\}$
	96	$\{8b\}$	$\{B_1, A_2, B_2\}$
		$\{4a\}$	$\{B_2\}$
	97	$\{16k, 8j, 8i, 8f, 4e, 4d\}$	$\{B_1, A_2, B_2\}$
		$\{8h, 4c\}$	$\{B_1\}$
		$\{8g, 2b\}$	$\{B_2\}$
	98	$\{16g, 8f, 8c\}$	$\{B_1, A_2, B_2\}$
		$\{8e, 8d, 4b, 4a\}$	$\{B_2\}$
4mm	99	$\{8g\}$	$\{B_1, B_2, A_2\}$
		$\{4f, 4e, 2c\}$	$\{B_1\}$
		$\{4d\}$	$\{B_2\}$
	100	$\{8d\}$	$\{B_1, B_2, A_2\}$
		$\{2a\}$	$\{A_2\}$
		$\{4c, 2b\}$	$\{B_2\}$
	101	$\{8e, 4c\}$	$\{B_1, B_2, A_2\}$
		$\{4d, 2b, 2a\}$	$\{B_2\}$
	102	$\{8d, 4b\}$	$\{B_1, B_2, A_2\}$
		$\{4c, 2a\}$	$\{B_2\}$
	103	$\{8d, 4c\}$	$\{B_1, B_2, A_2\}$
		$\{2b, 2a\}$	$\{A_2\}$
	104	$\{8c, 4b\}$	$\{B_1, B_2, A_2\}$
		$\{2a\}$	$\{A_2\}$
	105	$\{8f\}$	$\{B_1, B_2, A_2\}$
		$\{4e, 4d, 2c, 2b, 2a\}$	$\{B_1\}$
	106	$\{8c, 4b, 4a\}$	$\{B_1, B_2, A_2\}$
	107	$\{16e, 8d, 4b\}$	$\{B_1, B_2, A_2\}$

210/230 space groups

1197/1731 Wyckoff positions

PG	SG	WP	$\Gamma_{\mathbf{N}}$
$\mathbf{\bar{4}m2}$	115	$\{8l\}$	$\{B_1, B_2, A_2\}$
		$\{4i, 4h\}$	$\{A_2\}$
		$\{4k, 4j, 2g, 2f, 2e\}$	$\{B_1\}$
	116	$\{8j, 4i, 4h, 4g\}$	$\{B_1, B_2, A_2\}$
		$\{4f, 4e, 2b, 2a\}$	$\{A_2\}$
		$\{2d, 2c\}$	$\{B_2\}$
	117	$\{8i, 4f, 4e\}$	$\{B_1, B_2, A_2\}$
		$\{4h, 4g, 2d, 2c\}$	$\{A_2\}$
		$\{2b, 2a\}$	$\{B_2\}$
	118	$\{8i, 4h, 4e\}$	$\{B_1, B_2, A_2\}$
		$\{4g, 4f, 2d, 2c\}$	$\{A_2\}$
		$\{2b, 2a\}$	$\{B_2\}$
	119	$\{16j, 8i, 8h, 4f, 4e, 2d, 2c\}$	$\{B_1, B_2, A_2\}$
		$\{8g, 2b\}$	$\{A_2\}$
	120	$\{16i, 8g, 8f, 8e, 4c, 4a\}$	$\{B_1, B_2, A_2\}$
		$\{8h, 4d\}$	$\{A_2\}$
		$\{4b\}$	$\{B_2\}$
	121	$\{16j, 8h, 8g, 4d\}$	$\{B_1, B_2, A_2\}$
		$\{8f, 4c\}$	$\{B_1\}$
		$\{8i, 4e, 2b\}$	$\{B_2\}$
	122	$\{16e, 8d, 8c, 4b\}$	$\{B_1, B_2, A_2\}$
		$\{4a\}$	$\{A_2\}$
4/mmm	123	$\{16u, 8q, 8p\}$	$\{B_{1g}, A_{2g}, B_{2g}\}$
		$\{8t, 8s, 4o, 4n, 4m, 4l, 4i, 2f, 2e\}$	$\{B_{1g}\}$
		$\{8r, 4k, 4j\}$	$\{B_{2g}\}$
	124	$\{16n, 8m, 8i, 4e\}$	$\{B_{1g}, A_{2g}, B_{2g}\}$
		$\{4h, 4g, 2d, 2b\}$	$\{A_{2g}\}$
		$\{8l, 8k, 4f\}$	$\{B_{1g}\}$
		$\{8j\}$	$\{B_{2g}\}$
	125	$\{16n\}$	$\{B_{1g}, A_{2g}, B_{2g}\}$

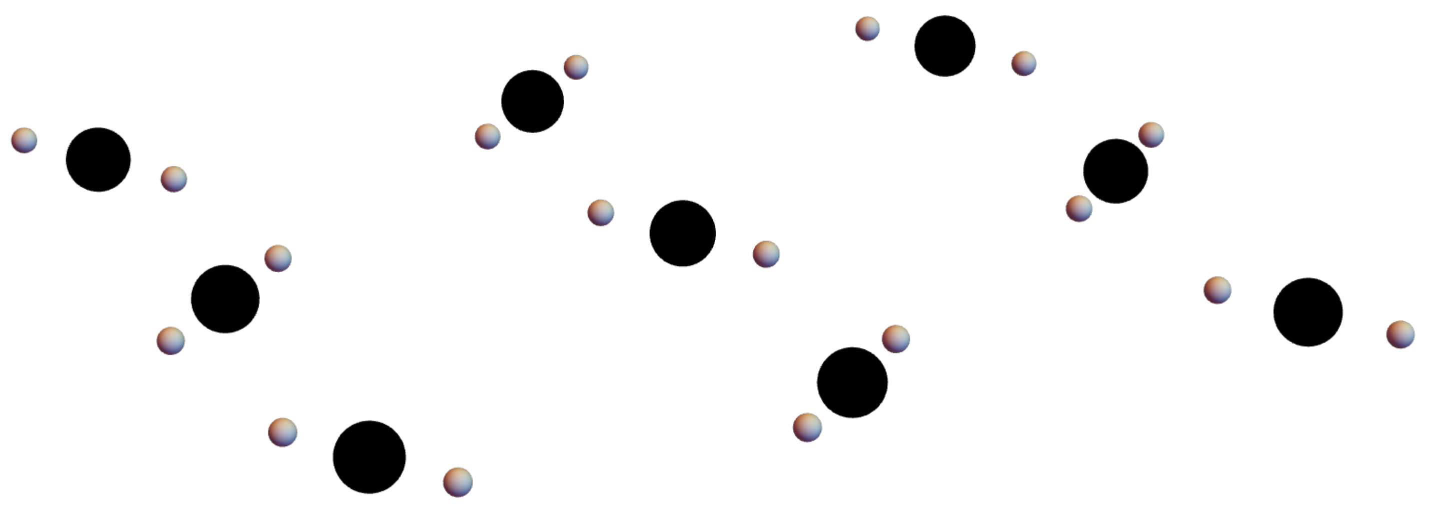
HS, McClarty, Rau, Romhányi, arXiv:2412.18025, (2024)

etc…

check if it contains 1D, ℝ inversion even irrep of crystal point group

Evading spin groups

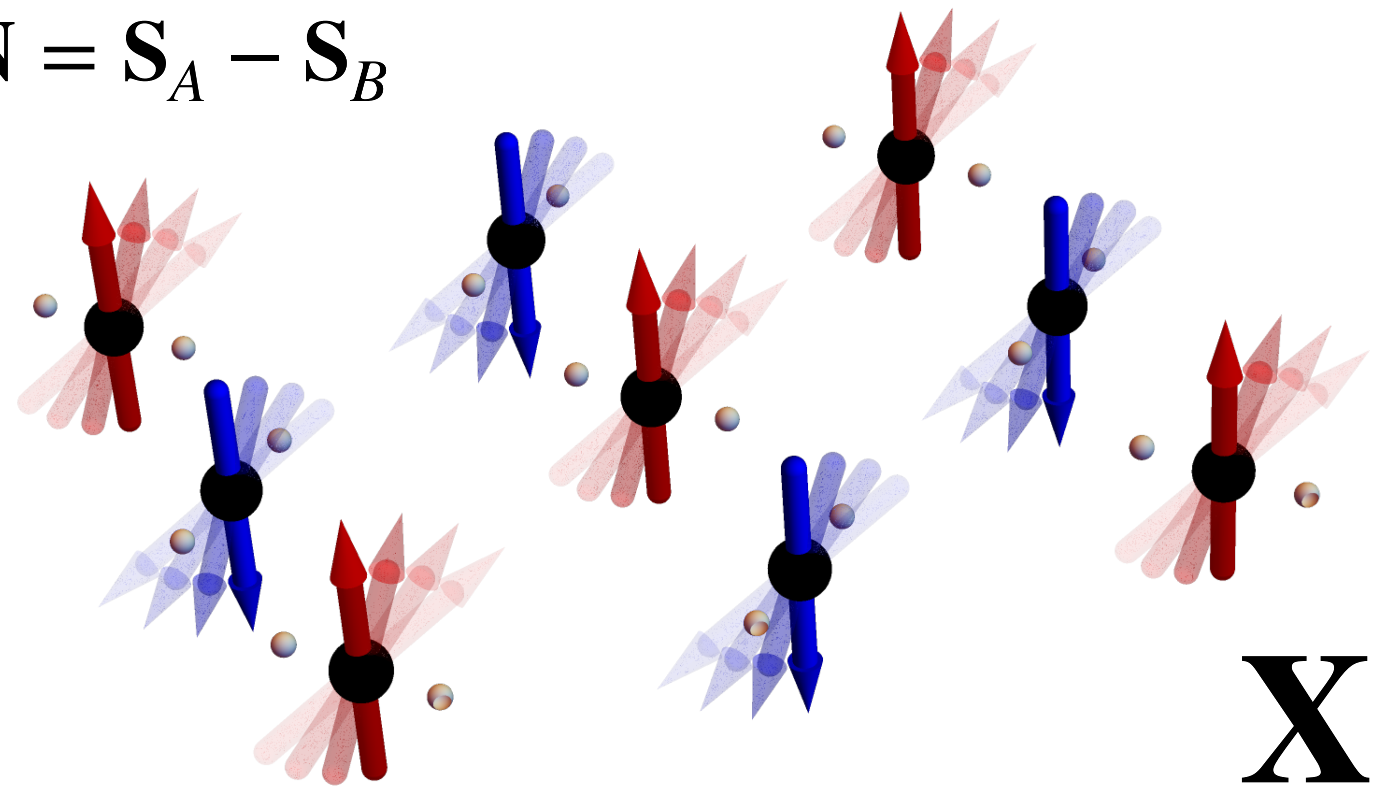
High symmetry phase



$\mathbf{G}^{\text{SO-free}}$

Low symmetry phase

$\mathbf{N} = \mathbf{S}_A - \mathbf{S}_B$



$\mathbf{X}$

SO-free paramagnetic group

$$\mathbf{G}^{\text{SO-free}} = \mathbf{O}^s(3) \times \mathbf{G}^{\text{space}}$$

Global spin rotations & time-reversal

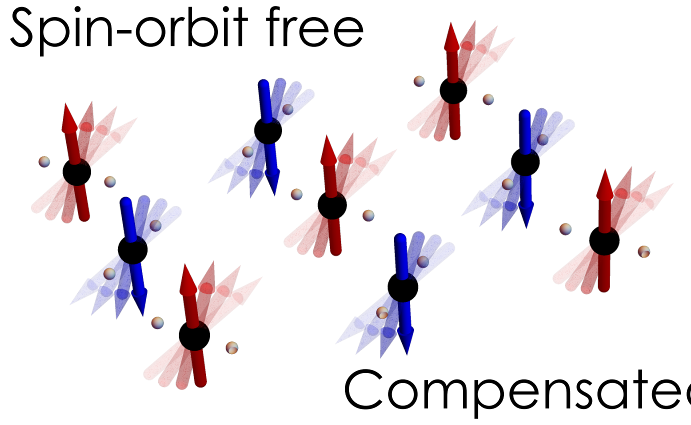
Crystal space group

Irrep Γ under which $\mathbf{N}$ transforms is non-trivial

If crystalline & magnetic unit cells coincide: can use **point group** of $\mathbf{G}^{\text{space}}$

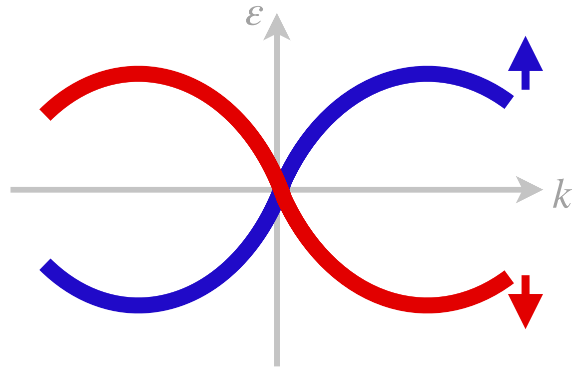
Ideal Altermagnets

Spin-orbit free



Compensated

Alternating spin-splitting



Spin group

If crystalline & magnetic unit cells coincide:

only need collinear ***spin point groups***

Correspondence

Collinear spin point groups ⟷ 1D ℝ irrep of an ordinary point group

Trade-off

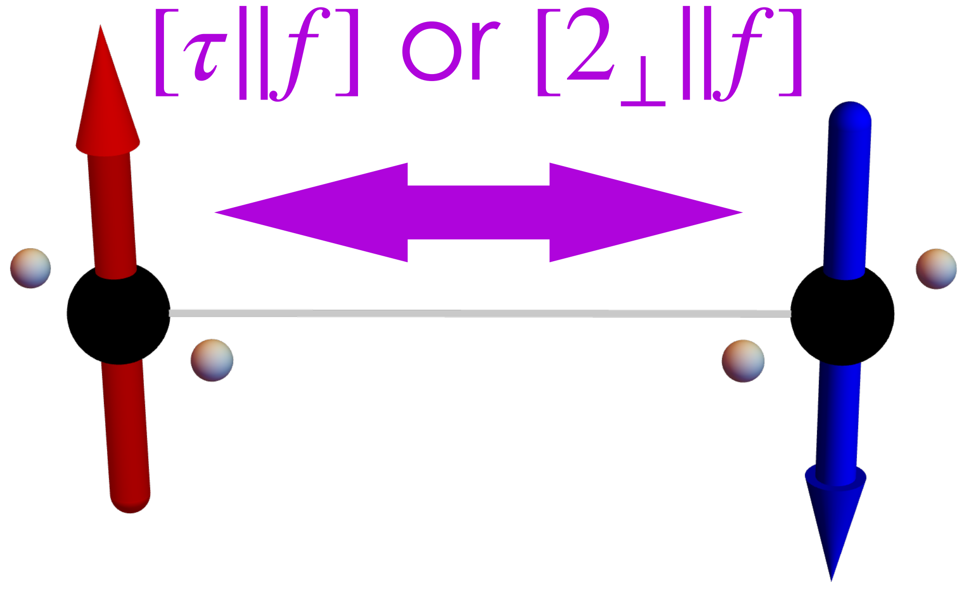
Break into ***non-trivial irrep*** of ***ordinary*** point group

HS, McClarty, Rau, Romhányi, arXiv:2412.18025, (2024)

SPG $\leftrightarrow$ PG Irrep Correspondence

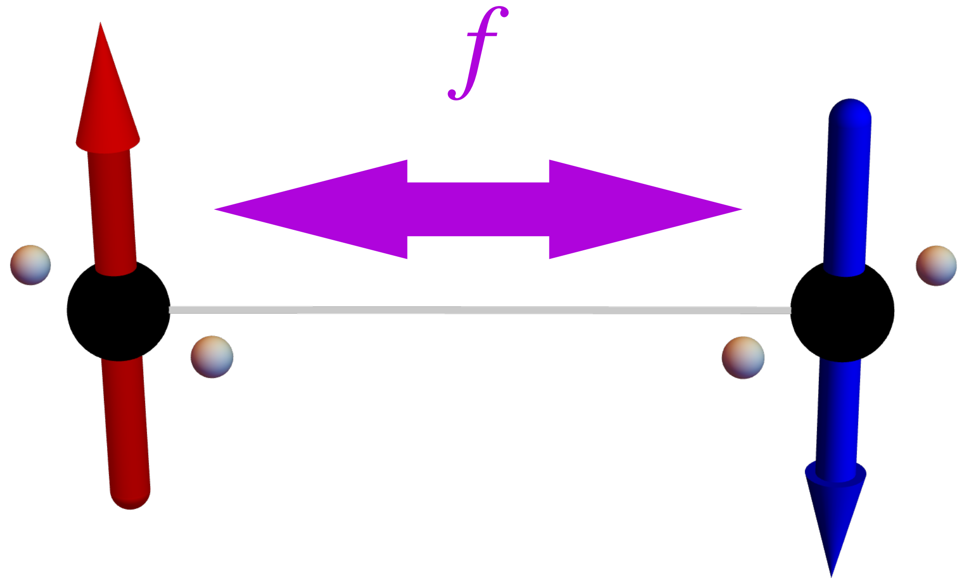
What is the correspondence between point group irreps and (non-trivial) spin point groups, that allows us to avoid using the spin groups entirely?

Non-trivial Spin Point Group



Trivial Irrep:
f composed with spin-only
τ or $2_\perp$ represented by +1

Point Group



Non-trivial Irrep:
f represented by -1

Each 1D, $\mathbb{R}$, inversion-even irrep of a point group corresponds to one of the possible altermagnetic (non-trivial) spin point groups.

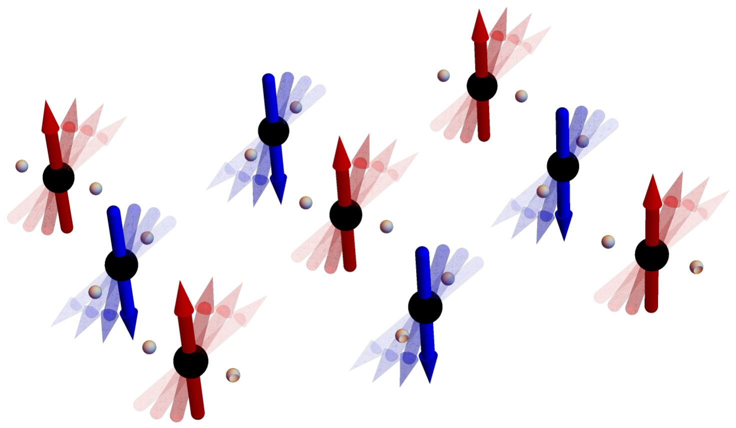
Spin-orbit free Landau Theory

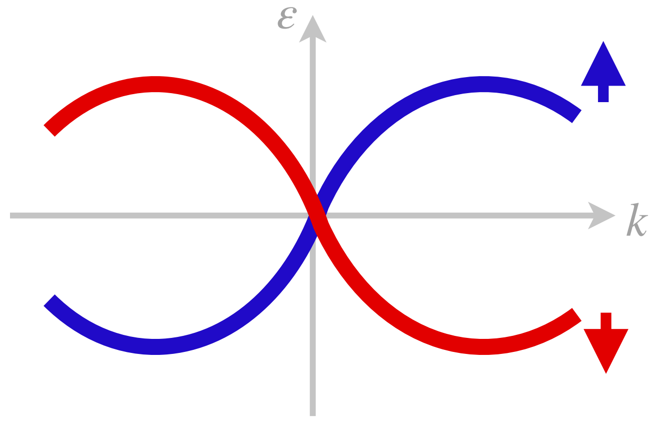
Spin-orbit free Landau Theory

Ideal Altermagnets

Spin-orbit free & compensated

Alternating spin-splitting





Order parameter: $\mathbf{N} = \mathbf{S}_A - \mathbf{S}_B$

(transforms as $\Gamma_{\mathbf{N}}$ in crystal point group)

Multipolar pseudo-primary order parameter

$$\int d^3r\,[r_{\mu_1}\ldots r_{\mu_n}]\,\mathbf{m}(\mathbf{r})$$

Transforms as $[V^n]$ in crystal point group

McClarty, Rau, Phys. Rev. Lett. 132, 176702 (2024)

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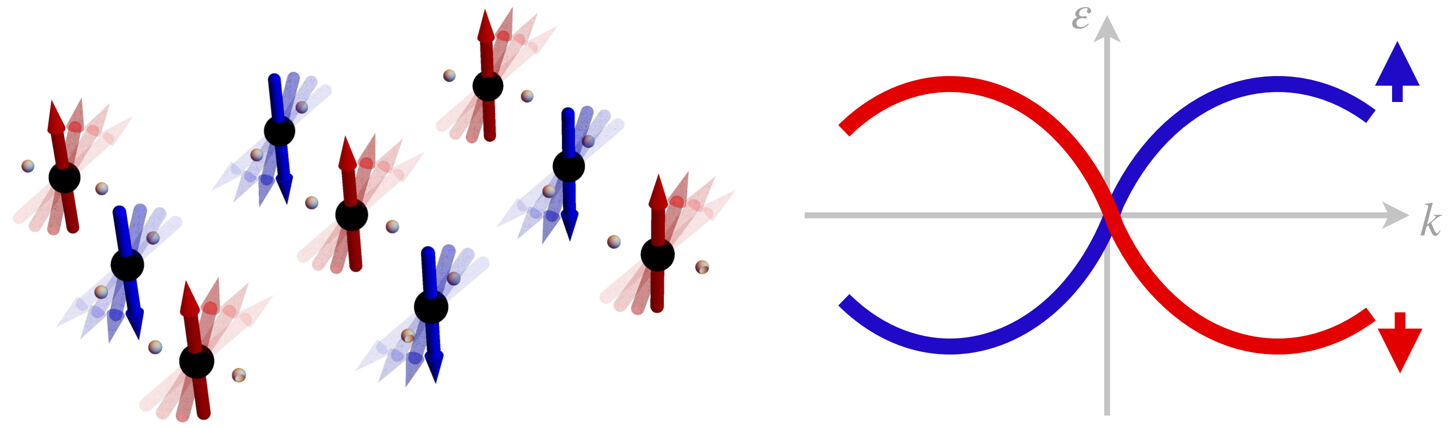

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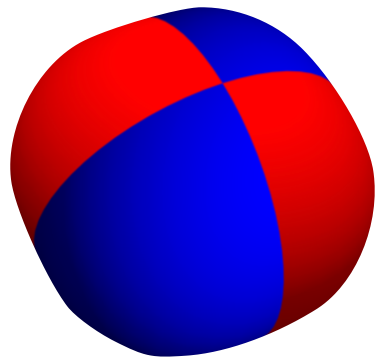
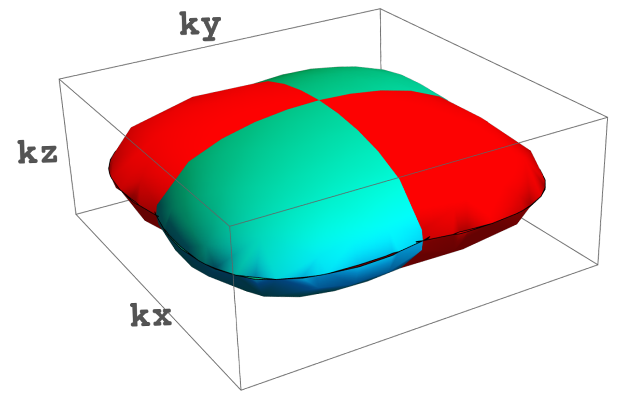
Hayami, Yanagi, Kusunose J. Phys. Soc. Jpn. 88, 123702 (2019)

Spin splitting

Multipole

$\int d^3r\, xy\, \mathbf{m}(\mathbf{r})$ $\longleftrightarrow$ Fermi-surface $k_x k_y\, \mathbf{s}$





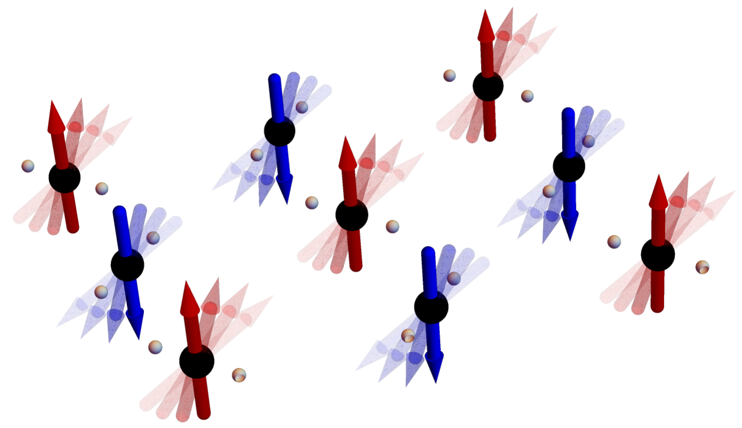
Lowest order multipole determines spin splitting

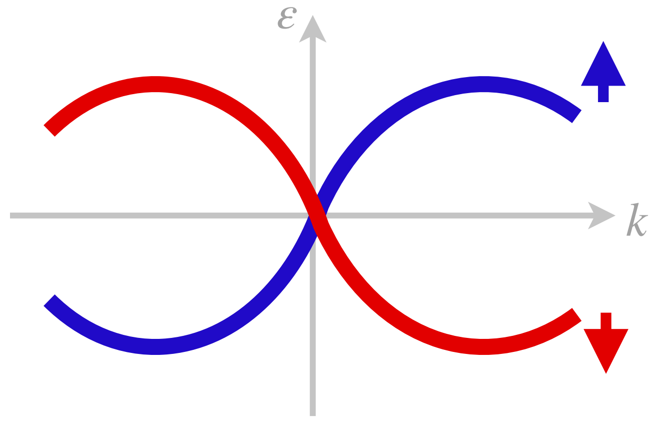
Spin-orbit free Landau Theory

Ideal Altermagnets

Spin-orbit free & compensated

Alternating spin-splitting





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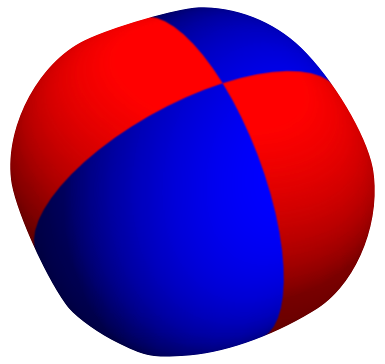
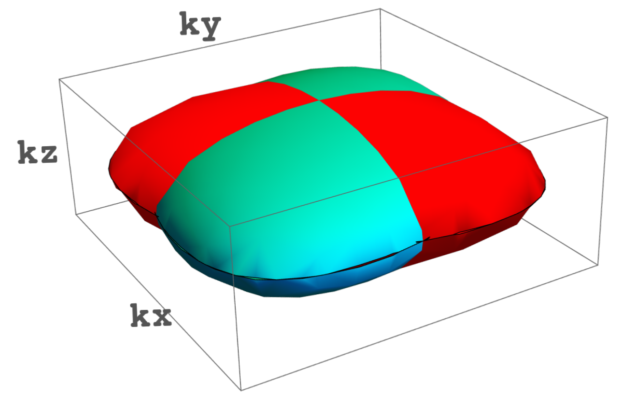
Fermi-surface

$$k_x k_y\,\mathbf{s}$$





Lowest order multipole determines spin splitting

Coupling to N

Condition for linear coupling between multipole and $\mathbf{N}$:

$$\Gamma_{\mathbf{N}} \in [V^n]$$

$$F \sim \mathbf{N} \cdot \int d^3r\,xy\,\mathbf{m}(\mathbf{r})$$

Allowed term in the free energy

McClarty, Rau, Phys. Rev. Lett. 132, 176702 (2024)

HS, McClarty, Rau, Romhányi, arXiv:2412.18025, (2024)

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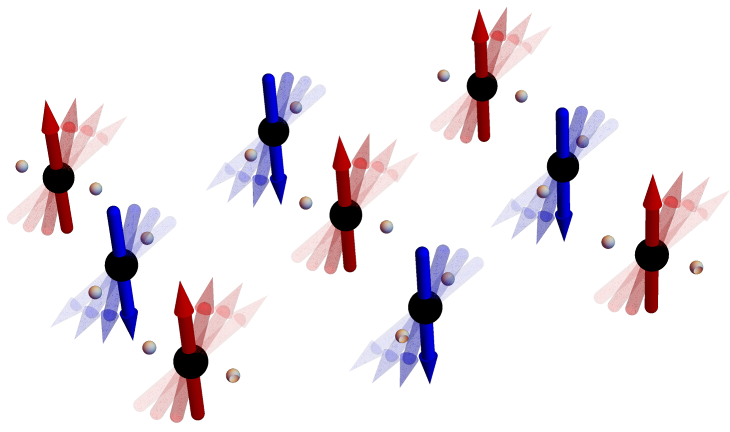
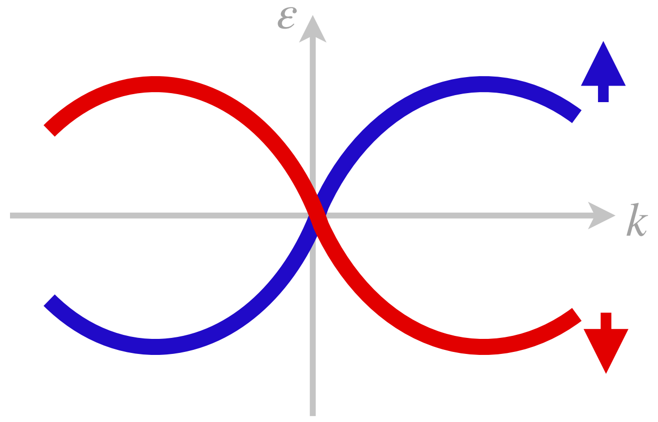


Alternating spin-splitting



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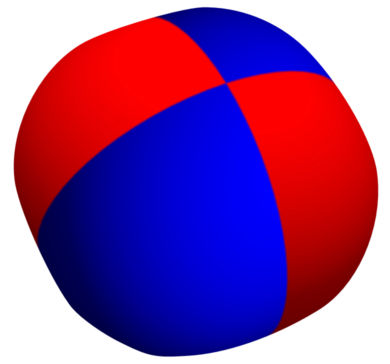

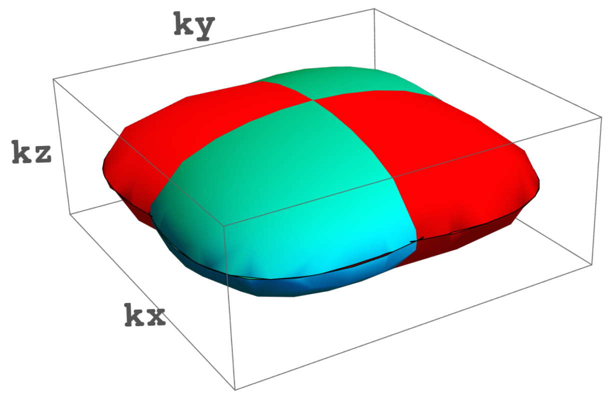

Lowest order multipole determines spin splitting

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$$\Gamma_{\mathbf{N}} \in [V^n]$$

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Allowed term in the free energy

Classify

Multipoles fully specify the the SO-free Landau theory

For each 1D, $\mathbb{R}$ irrep of a crystal point group (i.e. for each $\Gamma_{\mathbf{N}}$) find lowest order n multipole

Find $n = 1, 2, 3, 4, 6$

Find explicit multipole component(s)
(by finding invariant polynomial of order n)

Just 54 possible SO-free Landau theories for collinear altermagnets
(whose chemical and magnetic unit cells coincide)

McClarty, Rau, Phys. Rev. Lett. 132, 176702 (2024)

HS, McClarty, Rau, Romhányi, arXiv:2412.18025, (2024)

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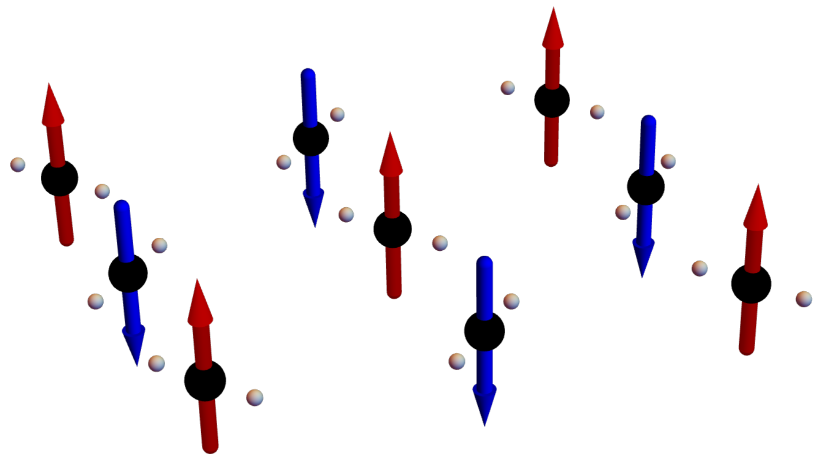
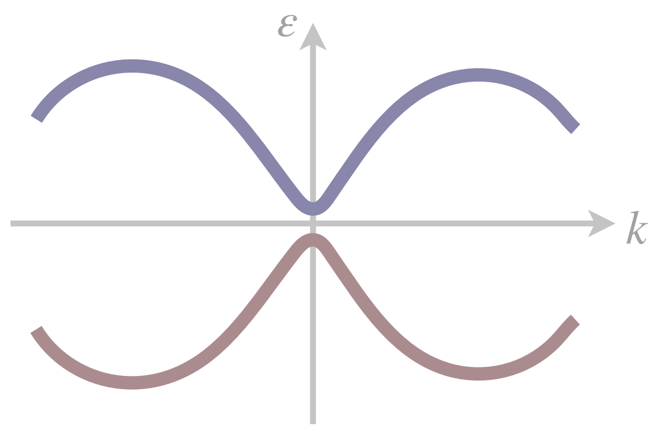
Real Altermagnets

Spin-orbit coupled ...but SOC is small!





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Locked spin & orbital space

Spins transform with lattice

Magnetic groups

SOC < Splitting

Dominant features from SO-free limit

Presence of SOC changes symmetry

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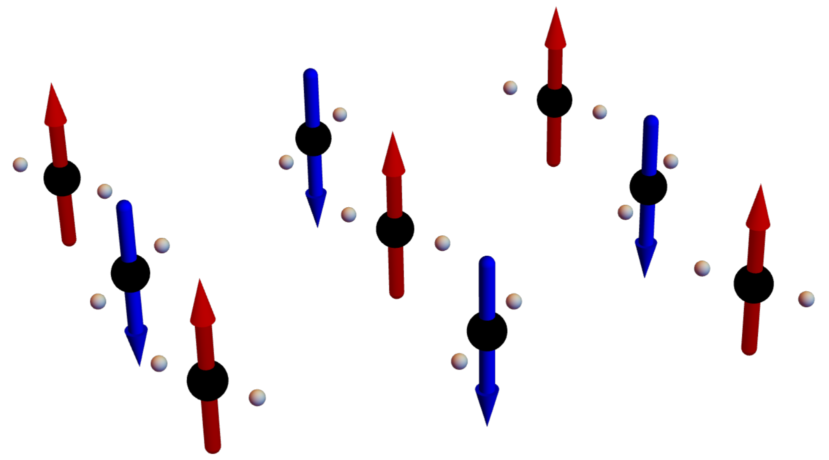

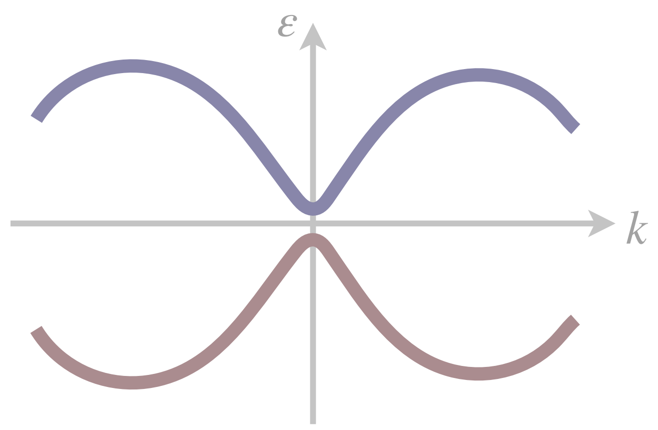

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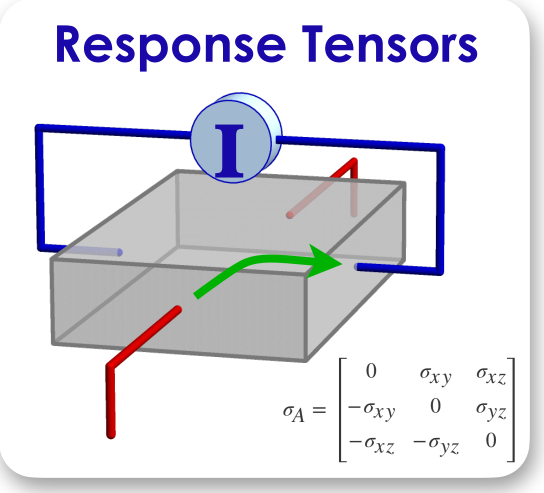
ξ: tensor for physical property of interest (e.g. Hall conductivity, magnetoresistance…)

Γ_ξ SOC paramagnetic group rep. (point group + time-reversal)



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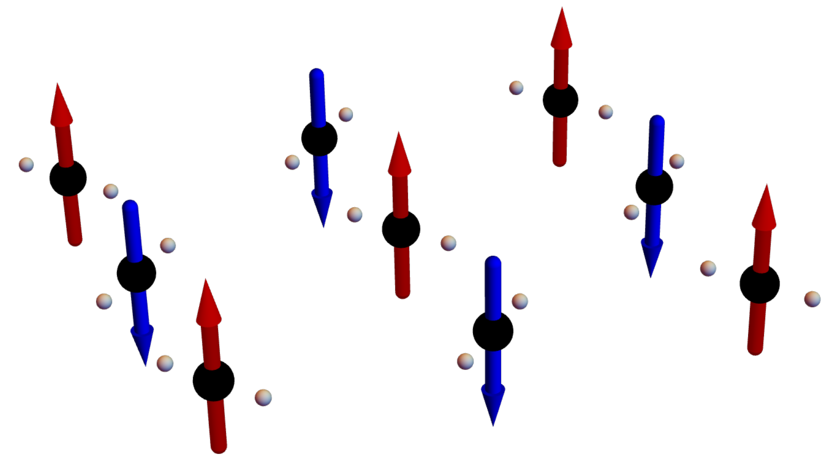

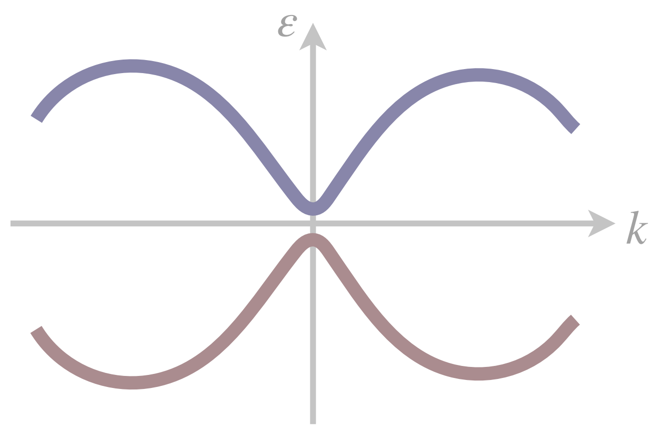

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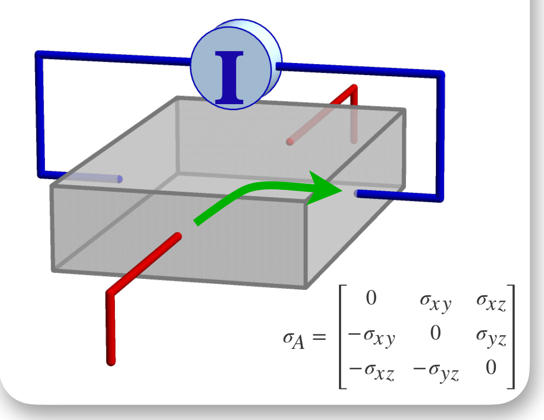
Response Tensors



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Requires that

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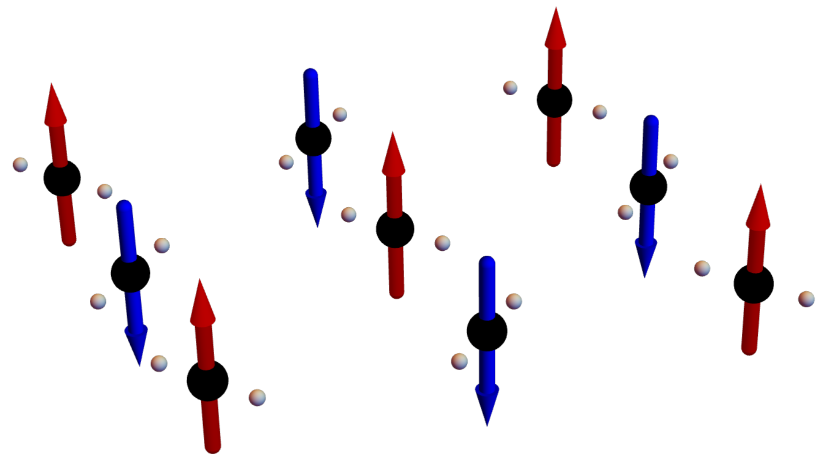

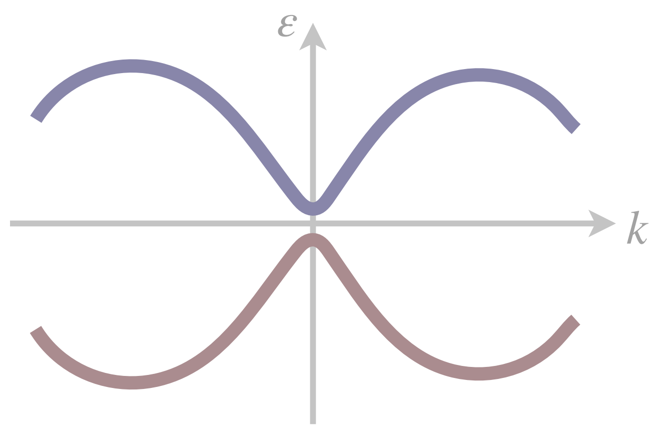

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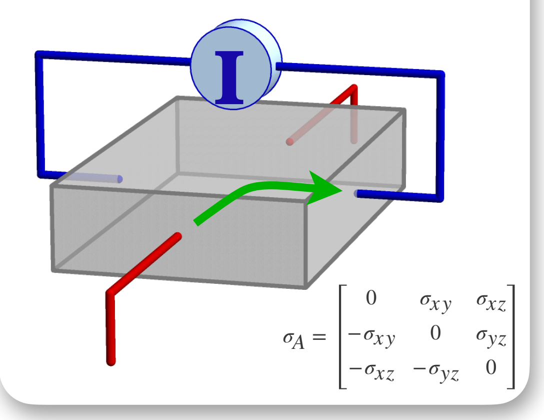

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When is this **guaranteed?**

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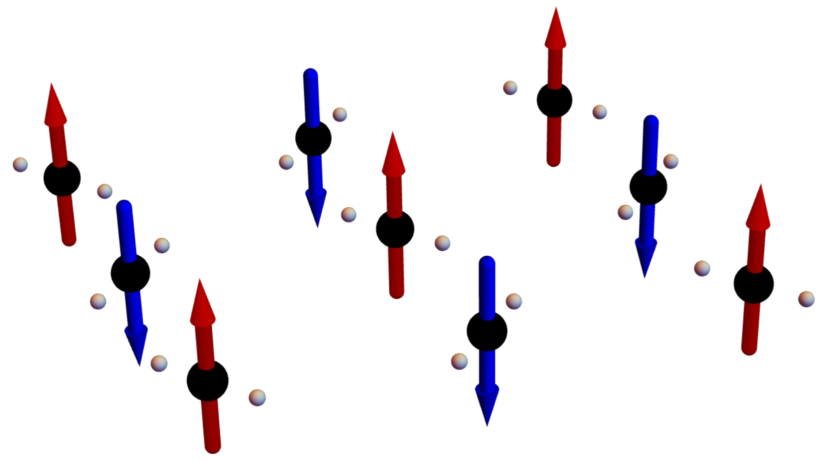

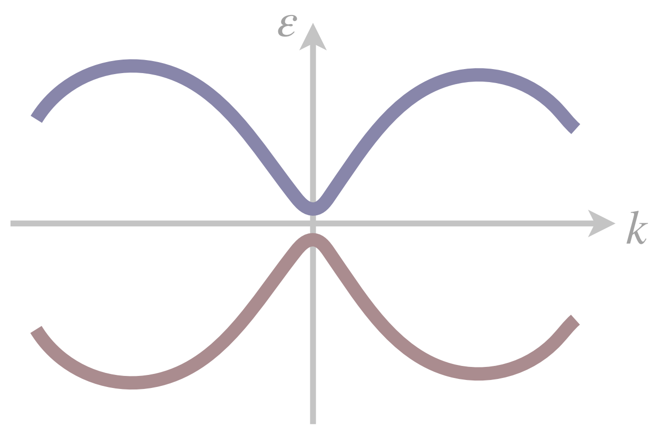

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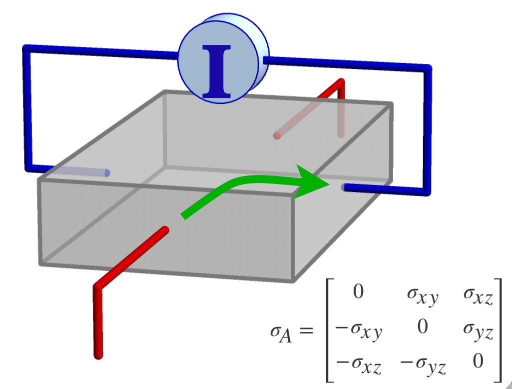

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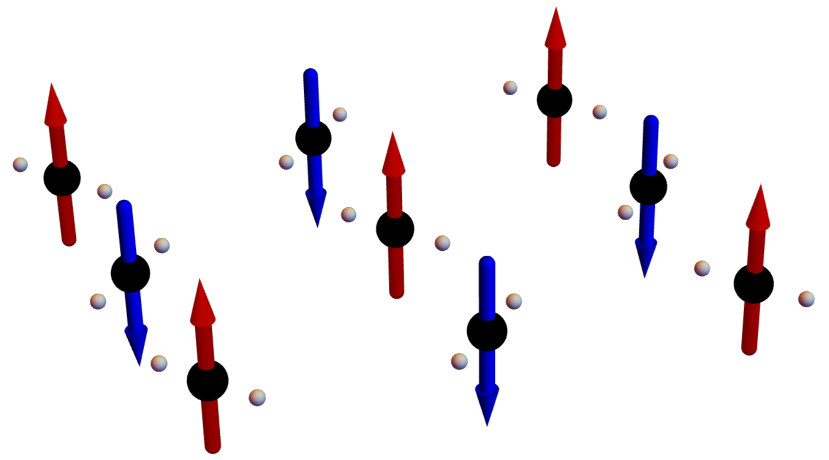

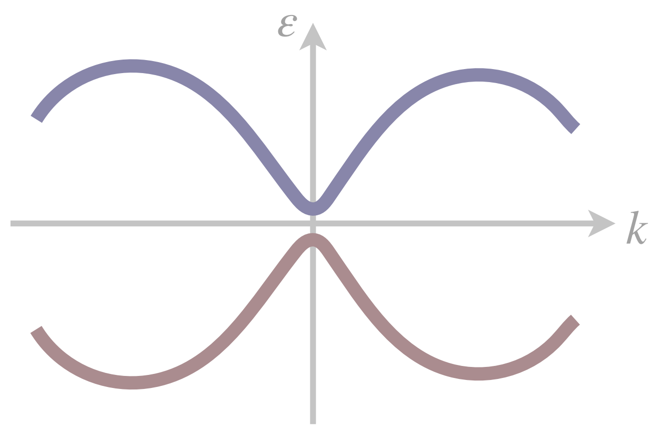

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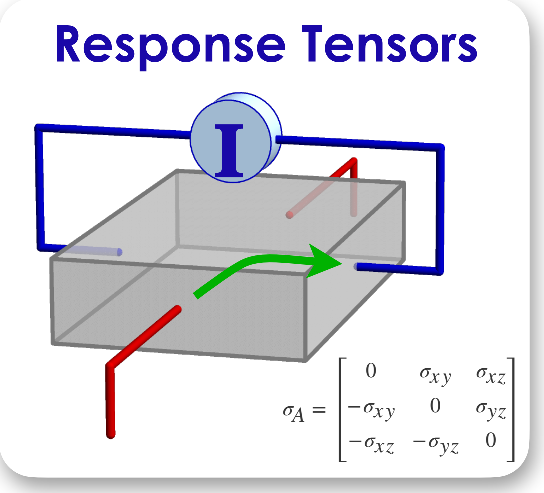

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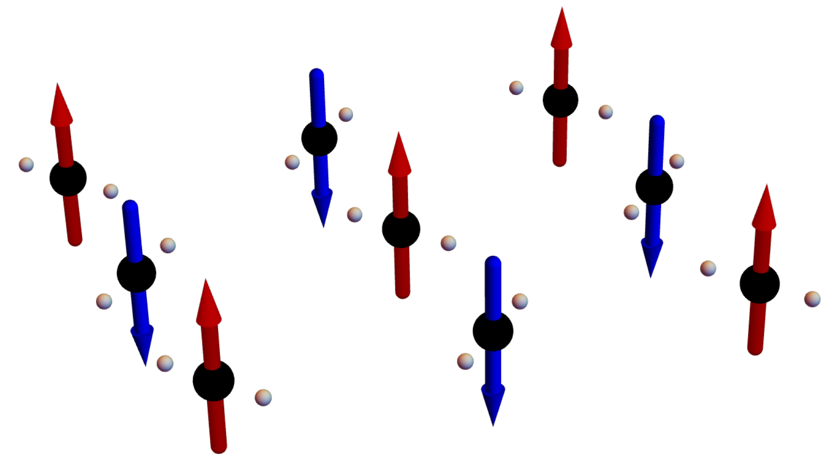

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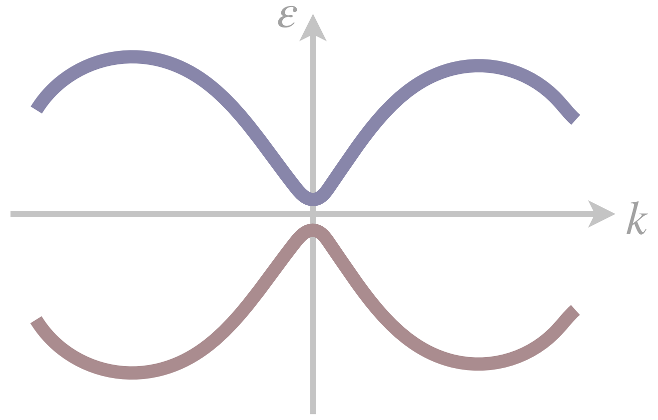

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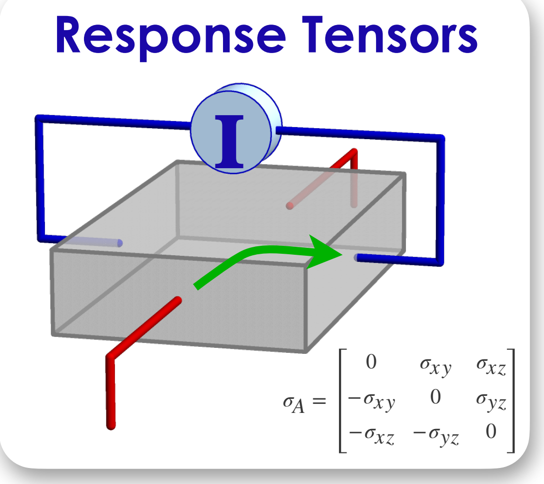

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Requires that

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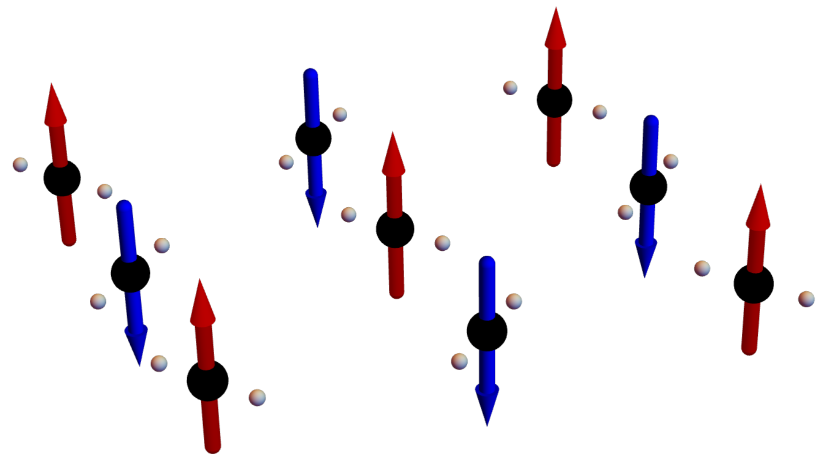
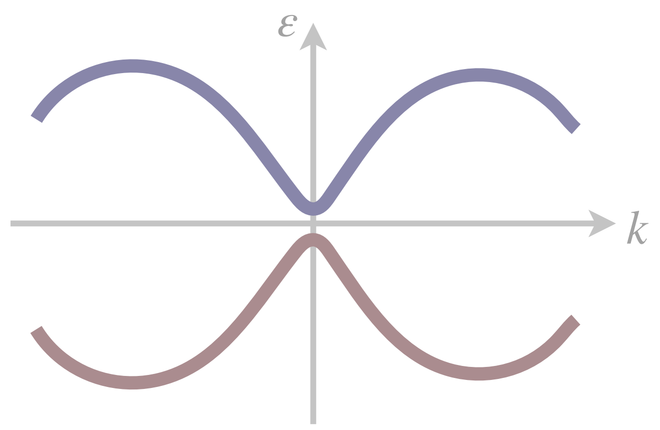
Spin-orbit coupled ...but SOC is small!





Order parameter: $\mathbf{N} = \mathbf{S}_A - \mathbf{S}_B$

(transforms as $\Gamma_{\mathbf{N}} \otimes aeV$)

Multipolar pseudo-primary order parameter

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Magnetic groups

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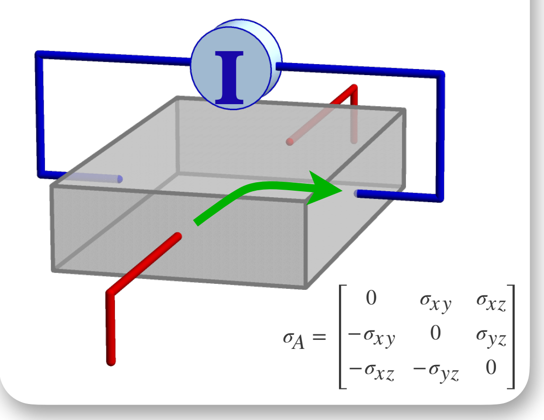

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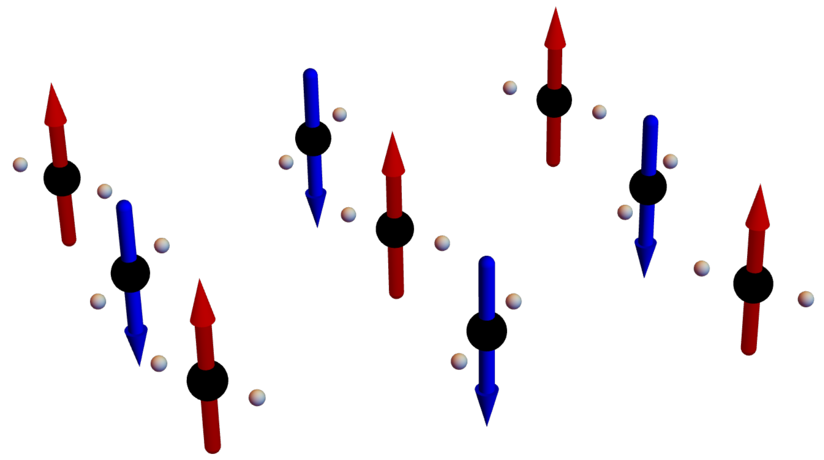

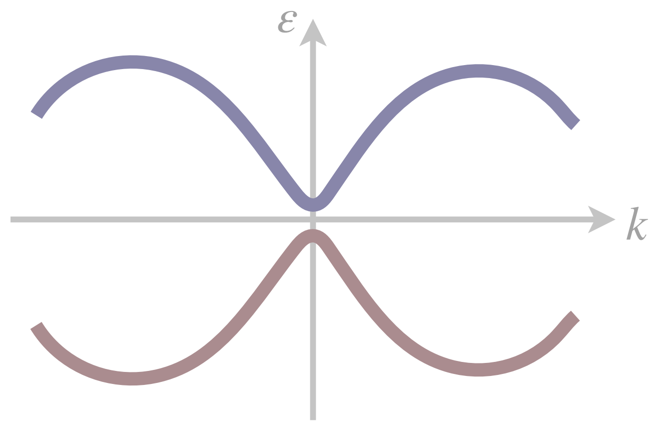

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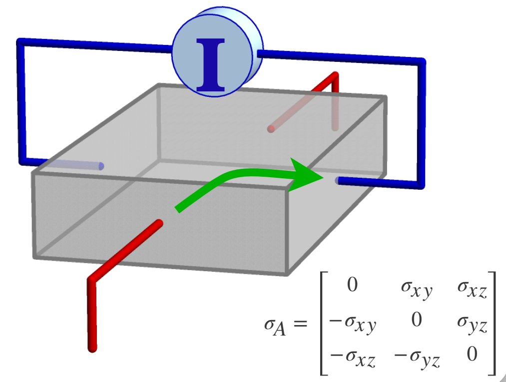

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Coupling classes

Just **4 classes** of observable properties ξ according to n

Multipole rank (n)	Representation Γ_ξ
1	$aV,\ aeV^2$
2	aeV
3	aeV^2
4, 6	$aeV[V^2]$

Inherently altermagnetic!
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Spin-orbit coupled Landau Theory

Γ_ξ	n	Quantity (ξ)	Defining equation	Tensor part
aV	1	Polar Toroidal Moment T_i	-	Full
		Pyrotoroidic tensor r_i	$T_i = r_i \Delta T$	Full
aeV	2	Magnetization M_i	-	Full
		Electric conductivity σ_{ij}	$J_i = \sigma_{ij} E_j$	$\sigma^A_\alpha = \frac{1}{2}\varepsilon_{\alpha ij}\sigma^A_{ij}$
		Soret thermodiffusion tensor s_{ij}	$J_i = s_{ij}(\nabla T)_j$	$s^A_\alpha = \frac{1}{2}\varepsilon_{\alpha ij} s^A_{ij}$
		Thermal conductivity κ_{ij}	$q_i = \kappa_{ij}(\nabla T)_j$	$\kappa^A_\alpha = \frac{1}{2}\varepsilon_{\alpha ij}\kappa^A_{ij}$
		Peltier tensor π_{ij}	$q_i = \pi_{ij} J_j$	$\tilde{S}_\alpha = \frac{1}{2}\varepsilon_{\alpha ij}(\pi^A_{ij} + \beta^A_{ij})$
		Seebeck tensor β_{ij}	$E_i = \beta_{ij}(\nabla T)_j$	
		Spontaneous Faraday effect F_{ij}	-	$F_\alpha = \frac{1}{2}\varepsilon_{\alpha ij} F_{ij}$
aeV^2	1 & 3	Magnetoelectric tensor α_{ij}	$M_i = \alpha_{ij} E_j$	Full
$aeV[V^2]$	4 & 6	Piezomagnetic tensor Λ_{ijk}	$M_i = \Lambda_{ijk}\Sigma_{jk}$	Full
		Second order magnetoelectric tensor α_{ijk}	$M_i = \alpha_{ijk} E_j E_k$	Full
		Magneto-optic Kerr effect z^S_{ijk}	$\varepsilon_{ij} = i z^S_{ijl} H_l$	Full
		Quadratic magneto-optic Kerr effect iC^A_{ijkl}	$\varepsilon_{ij} = C^A_{ijkl} H_k H_l$	$C_{\alpha kl} = \frac{1}{2}\varepsilon_{\alpha ij} C^A_{ijkl}$
		Magnetoresistance R_{ijk}	$E_i = R_{ijk} J_j H_k$	$R^S_{ijk} = \frac{1}{2}(R_{ijk} + R_{jik})$
		Righi-Leduc magnetorhermal tensor Q_{ijk}	$q_i = Q_{ijk}(\nabla T)_j H_k$	$Q^S_{ijk} = \frac{1}{2}(Q_{ijk} + Q_{jik})$
		Ettinghausen tensor M_{ijk}	$q_i = M_{ijk} J_j H_k$	$S_{ijk} = \frac{1}{2}(M^S_{ijk} + N^S_{ijk})$
		Nernst tensor N_{ijk}	$E_i = N_{ijk}(\nabla T)_j H_k$	
		Magnetic resistance tensor T_{ijkl}	$E_i = T_{ijkl} J_j H_k H_l$	$T^A_{\alpha kl} = \frac{1}{2}\varepsilon_{\alpha ij} T^A_{ijkl}$
		Magneto-heat-conductivity tensor $\mathcal{S}_{ijkl}$	$q_i = \mathcal{S}_{ijkl}(\nabla T)_j H_k H_l$	$\mathcal{S}^A_{\alpha kl} = \frac{1}{2}\varepsilon_{\alpha ij}\mathcal{S}^A_{ijkl}$
		Piezoresistivity tensor Π_{ijkl}	$\Delta\rho_{ij} = \Pi_{ijkl}\Sigma_{kl}$	$\Pi^A_{\alpha kl} = \frac{1}{2}\varepsilon_{\alpha ij}\Pi^A_{ijkl}$
		Magneto–Seebek tensor α_{ijkl}	$E_i = \alpha_{ijkl}(\nabla T)_j H_k H_l$	$\tilde{A} = \frac{1}{4}\varepsilon_{\alpha ij}(\alpha^A_{ijkl} - P^A_{ijkl})$
		Magneto-Peltier tensor P_{ijkl}	$q_i = P_{ijkl} H_k H_l H_j$	

operty tensors

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gnetoresistance…)

etic group rep.
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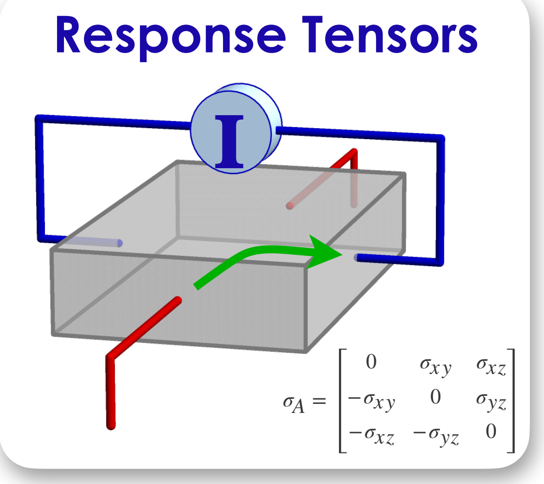

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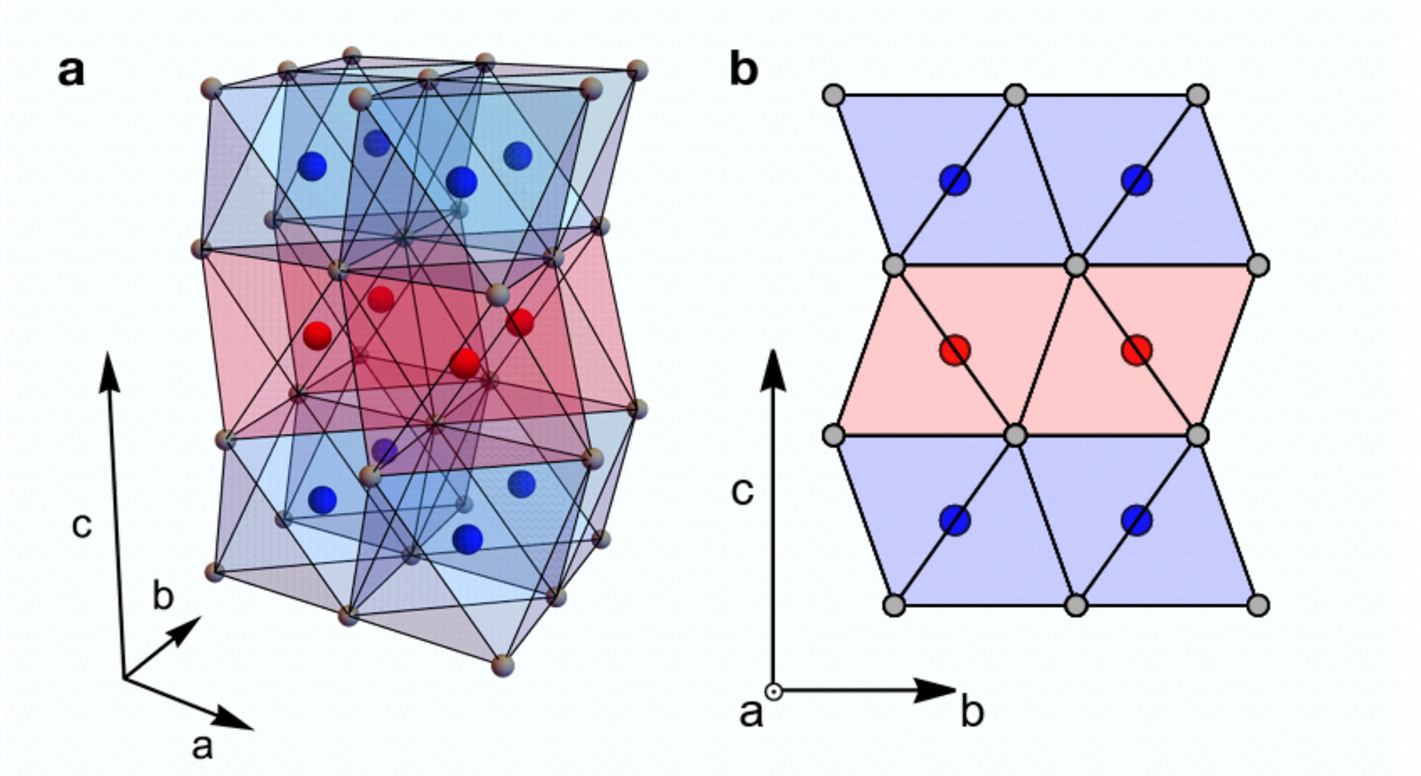

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Point group: $6/mmm$

Irrep: $\Gamma_{\mathbf{N}} = B_{2g}$

Krempasky et al. Nature volume 626, pages 517–522 (2024)

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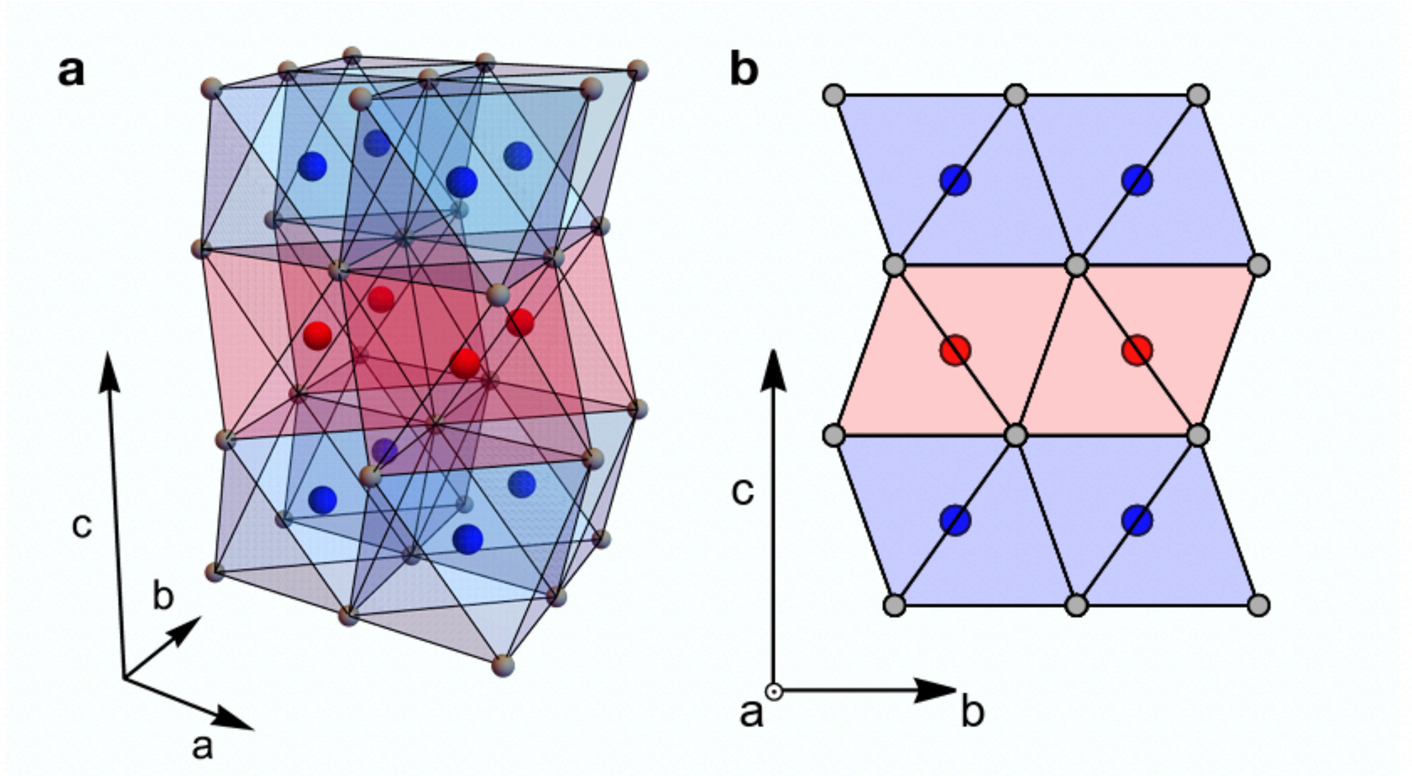

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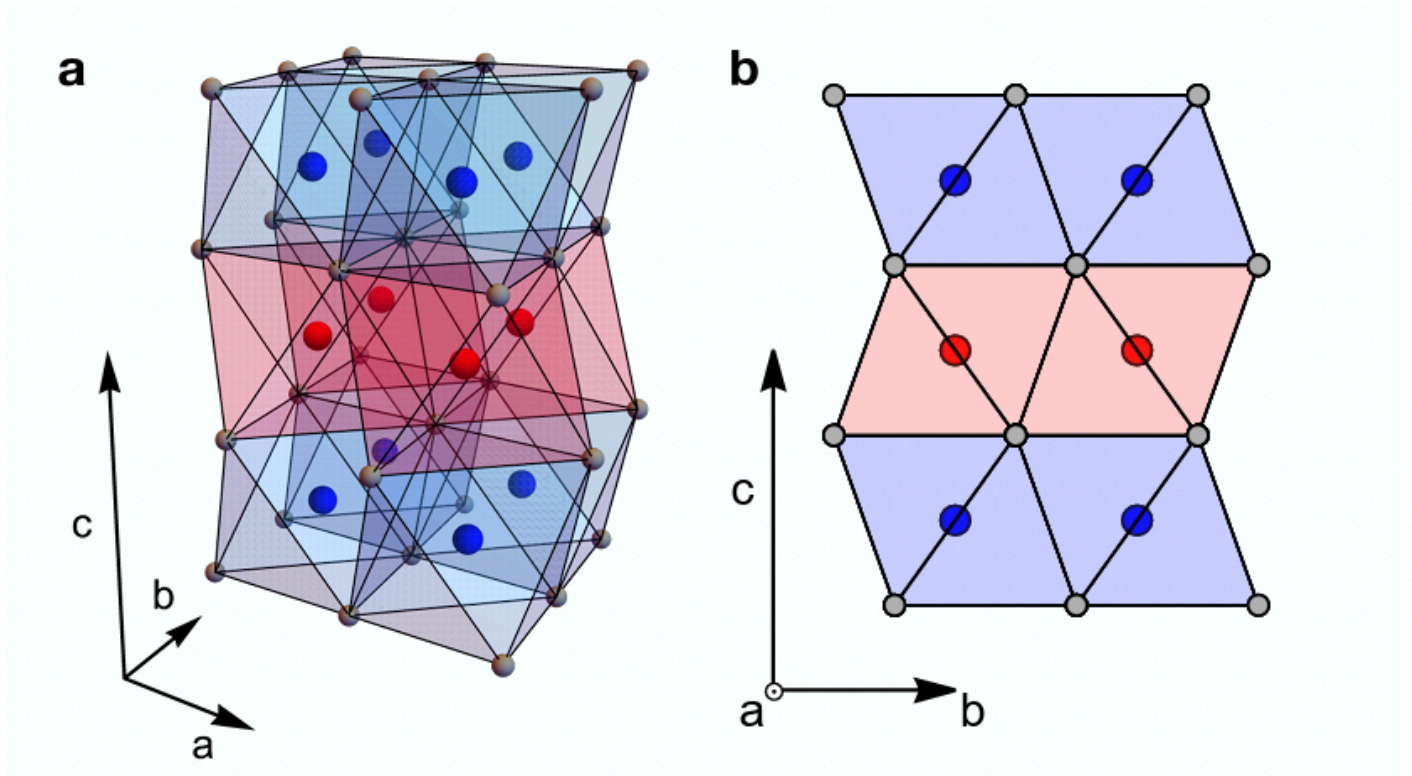

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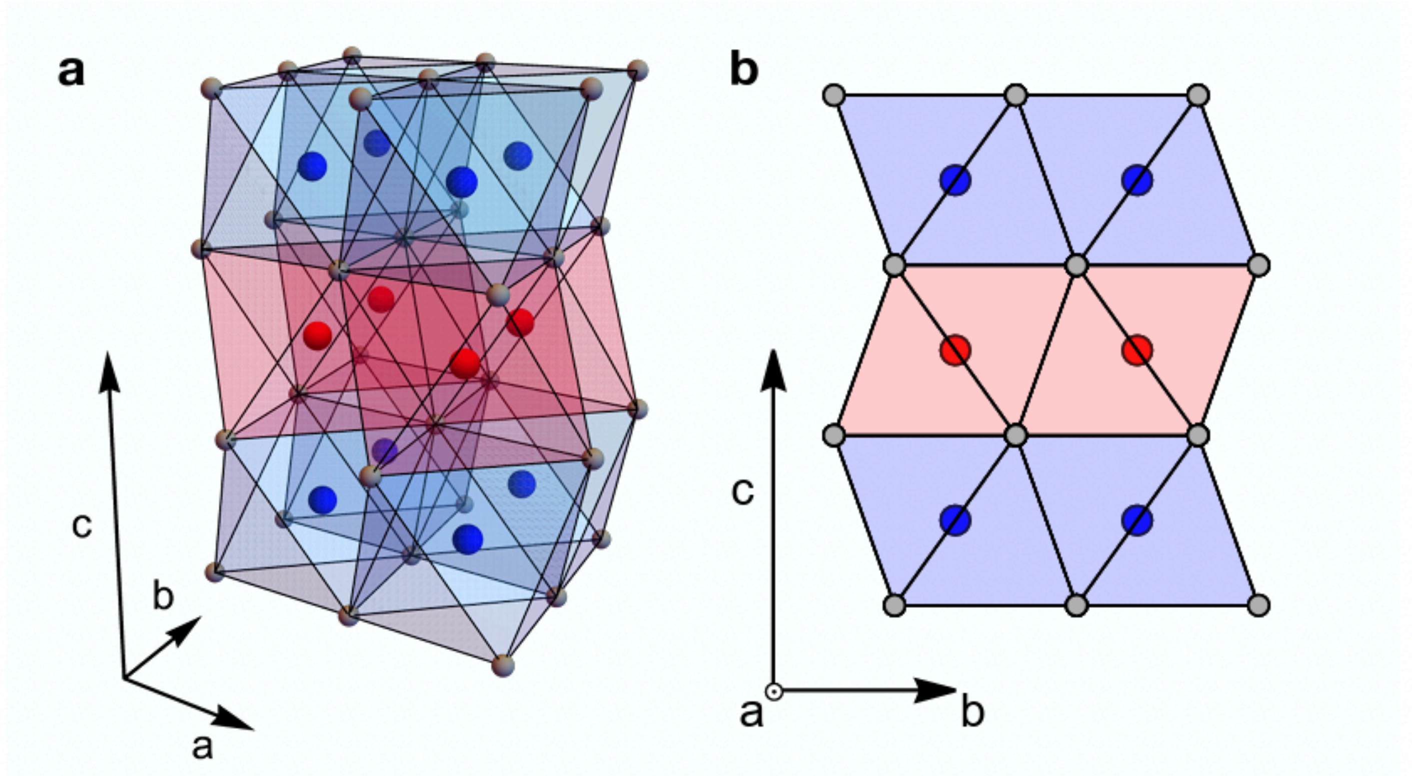

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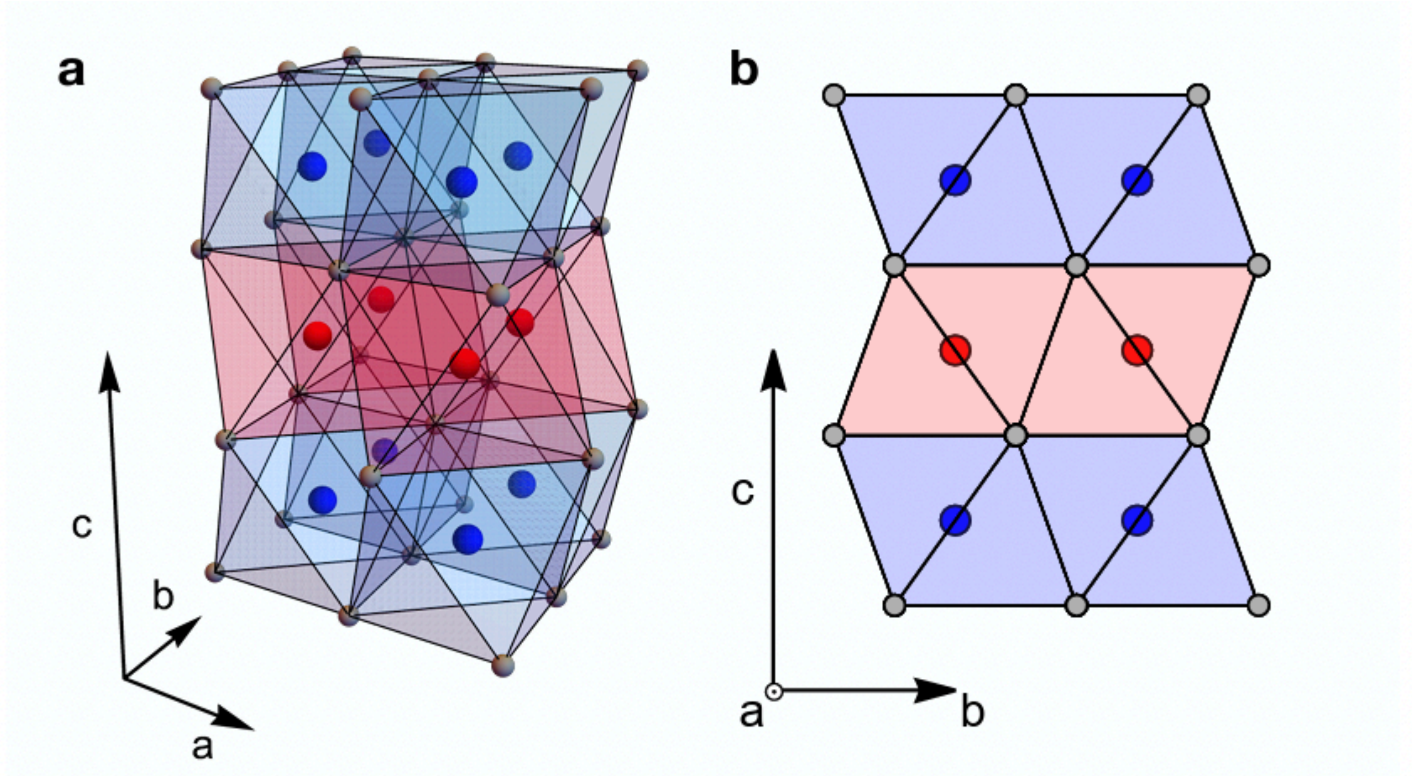

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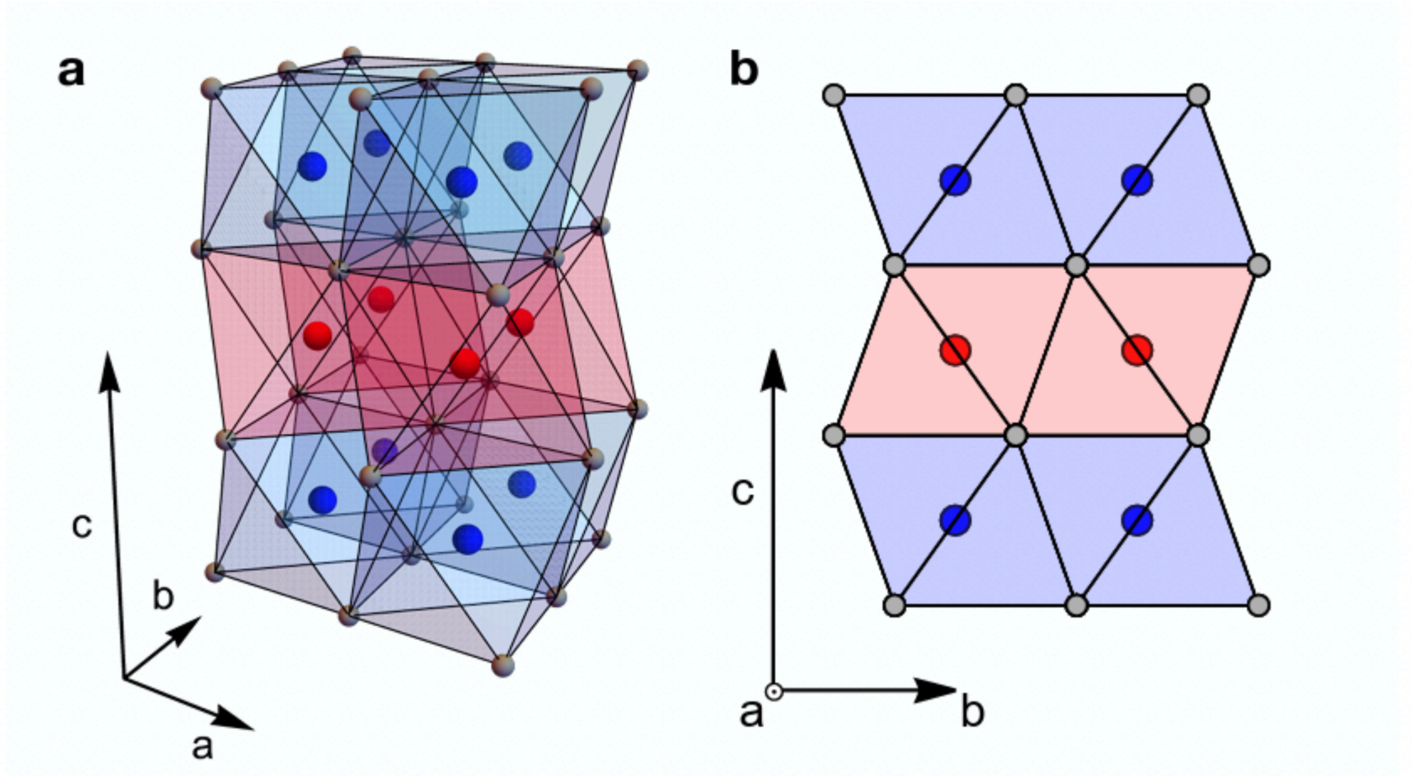

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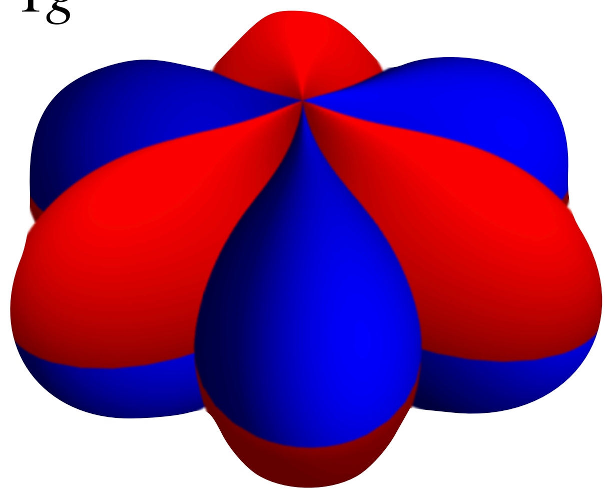

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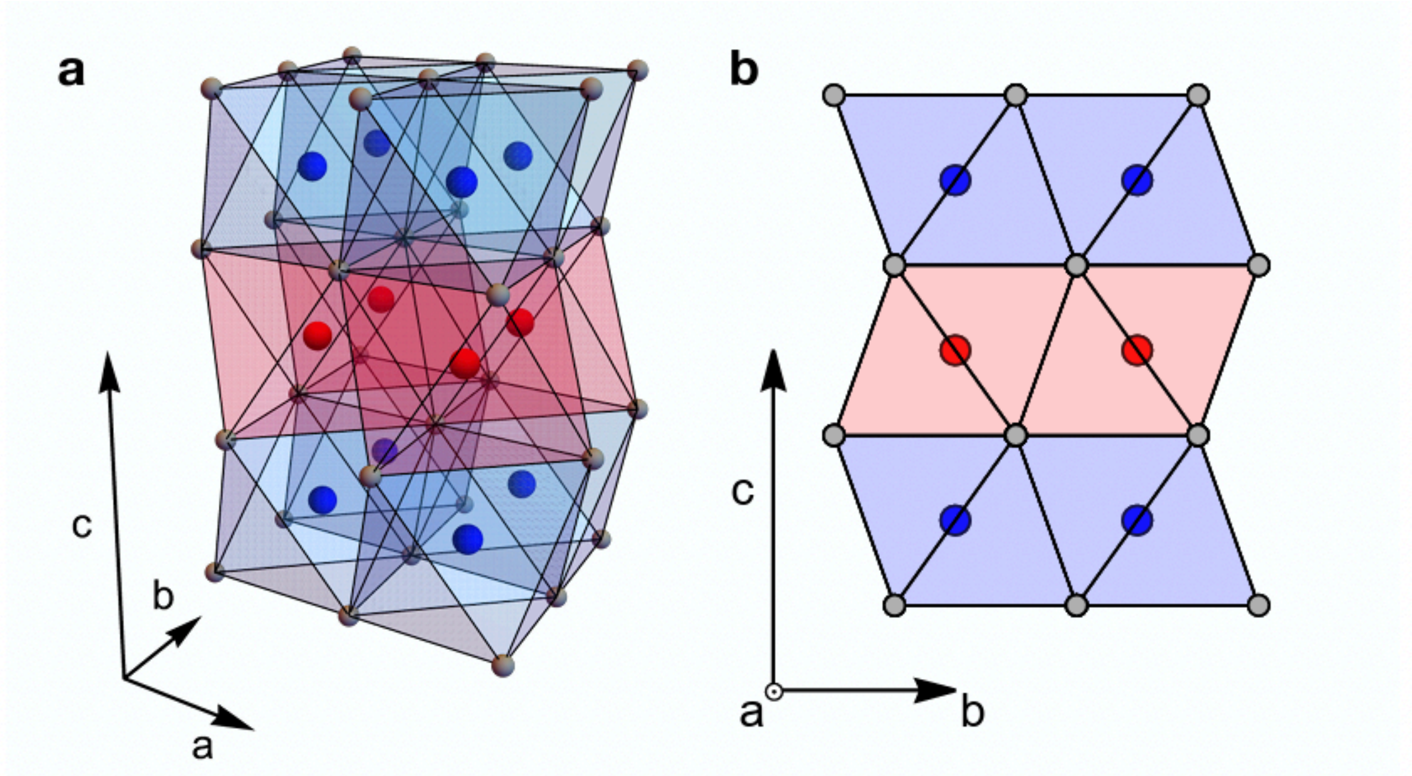

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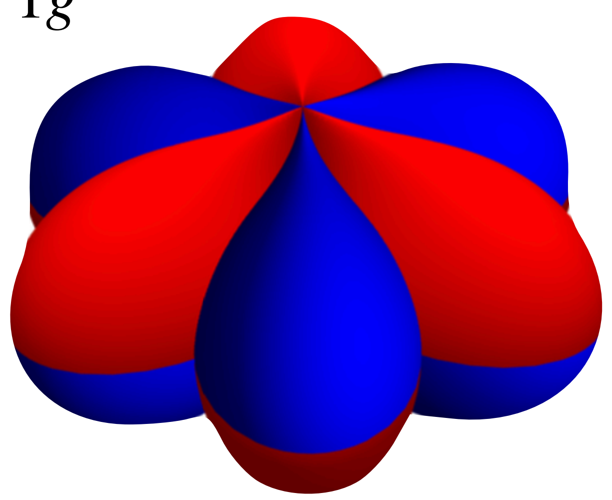

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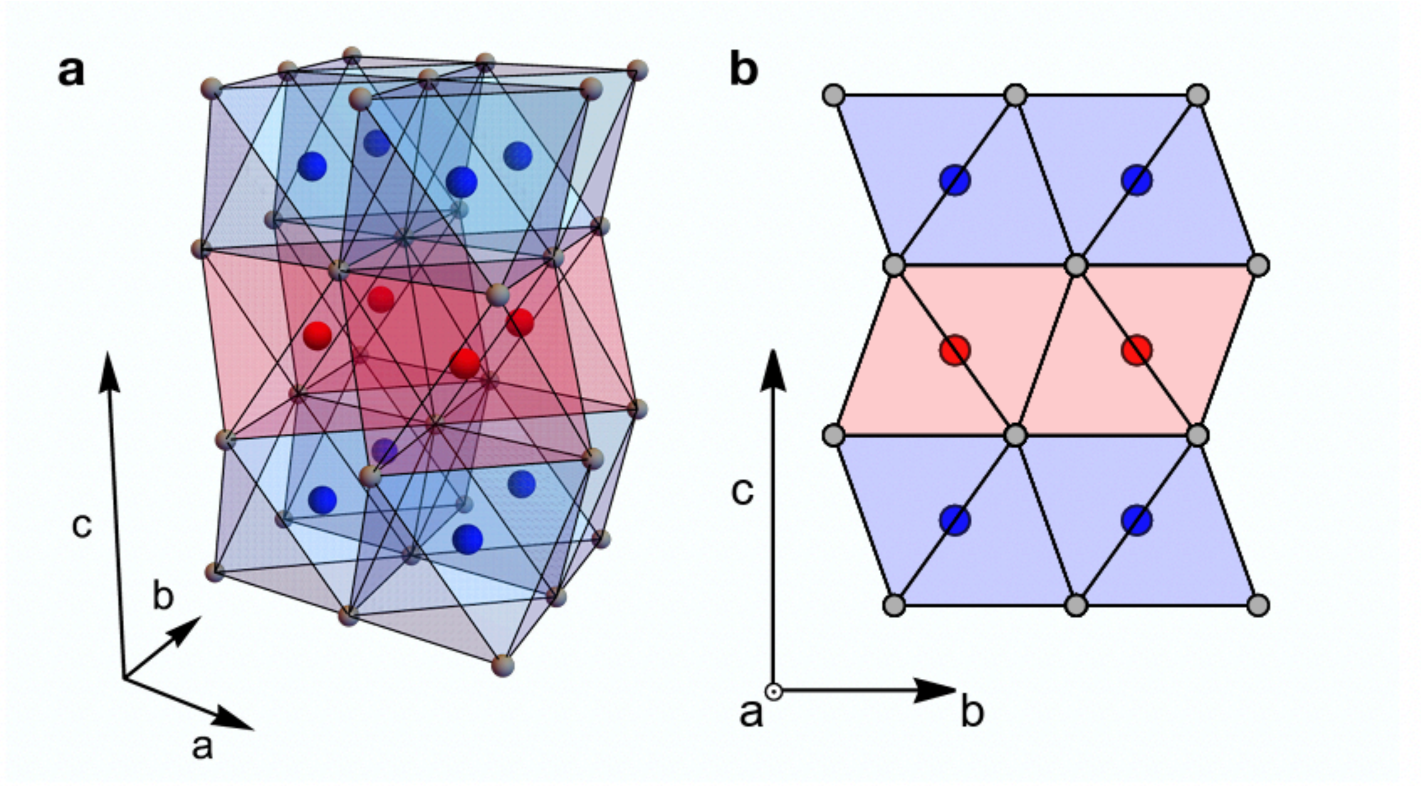

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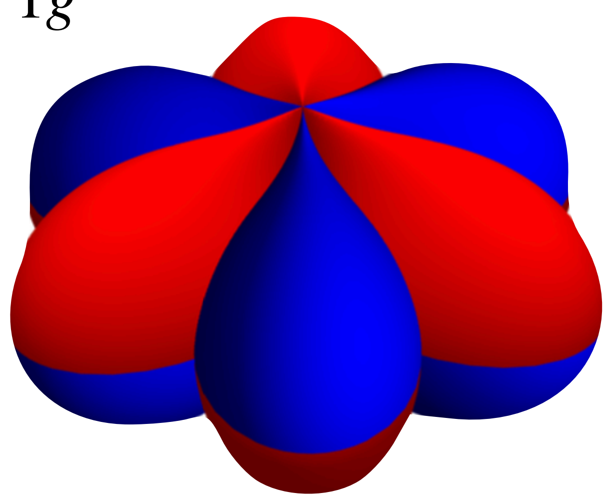

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TABLE IV. Transformation properties of the order parameter $\mathbf{N}$ in MnTe and the part of the magnetoresistance that is symmetric in the first two indices.

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E_{2g}	$\begin{pmatrix} N_x \\ N_y \end{pmatrix}$	$\begin{pmatrix} 2R^S_{xyz} \\ R^S_{xxz} - R^S_{yyz} \end{pmatrix}$ $\begin{pmatrix} R^S_{yzx} + R^S_{xzy} \\ R^S_{xzx} - R^S_{yzy} \end{pmatrix}$

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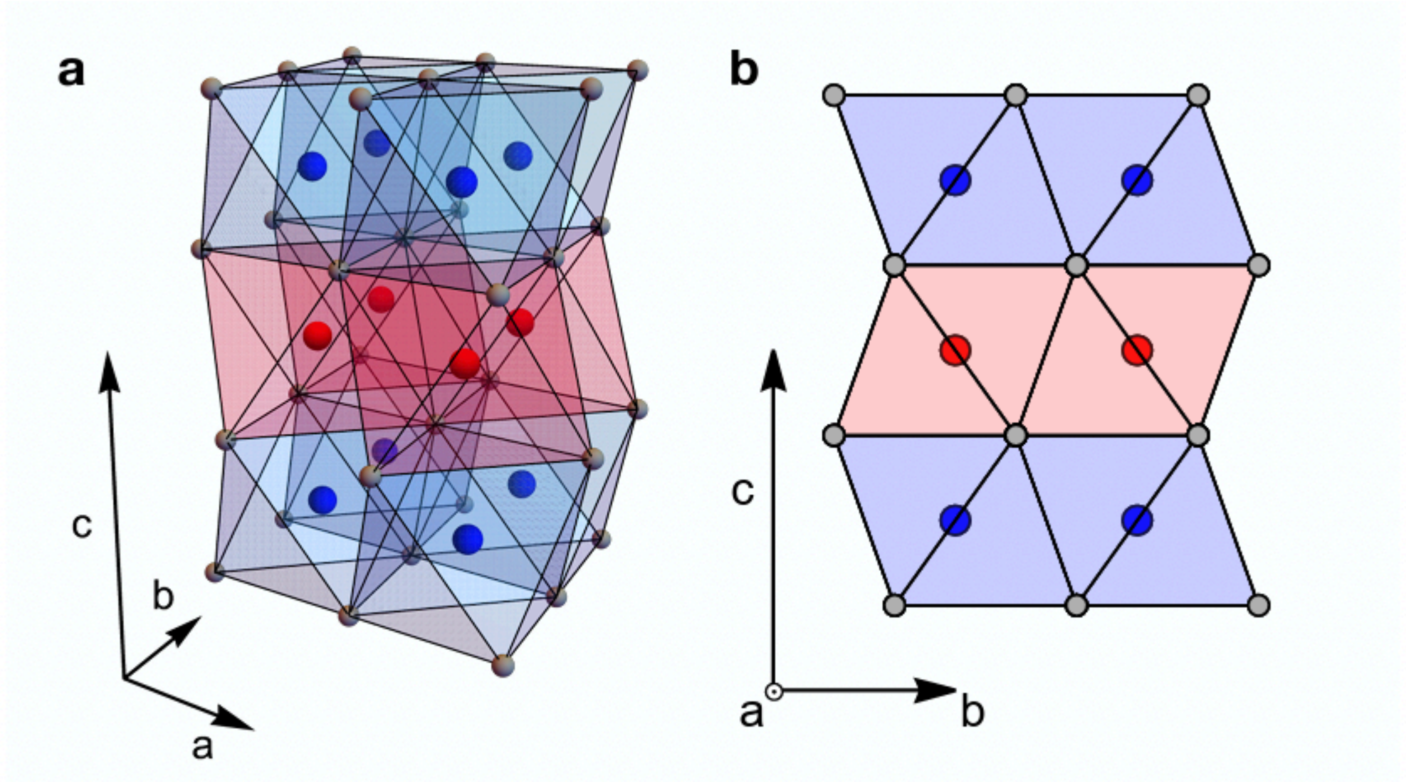

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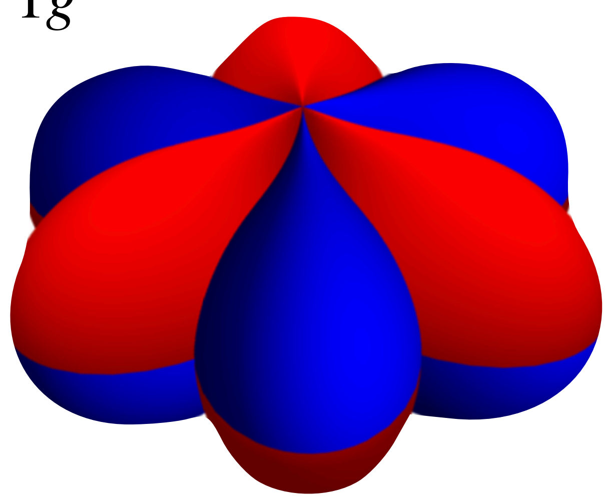

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Néel components transform as

$$\{N_x, N_y\} \sim E_{2g} \qquad N_z \sim B_{1g}$$

Observed experimentally!

Collinear AM Landau theory summary

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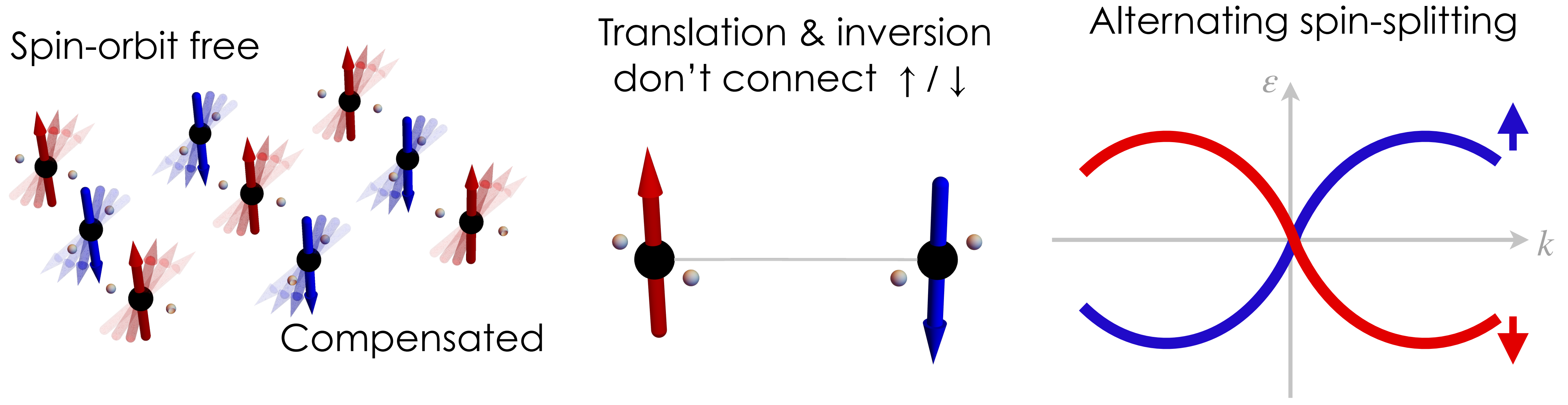

Altermagnetism **defined** by features inherited from ideal SO-free limit

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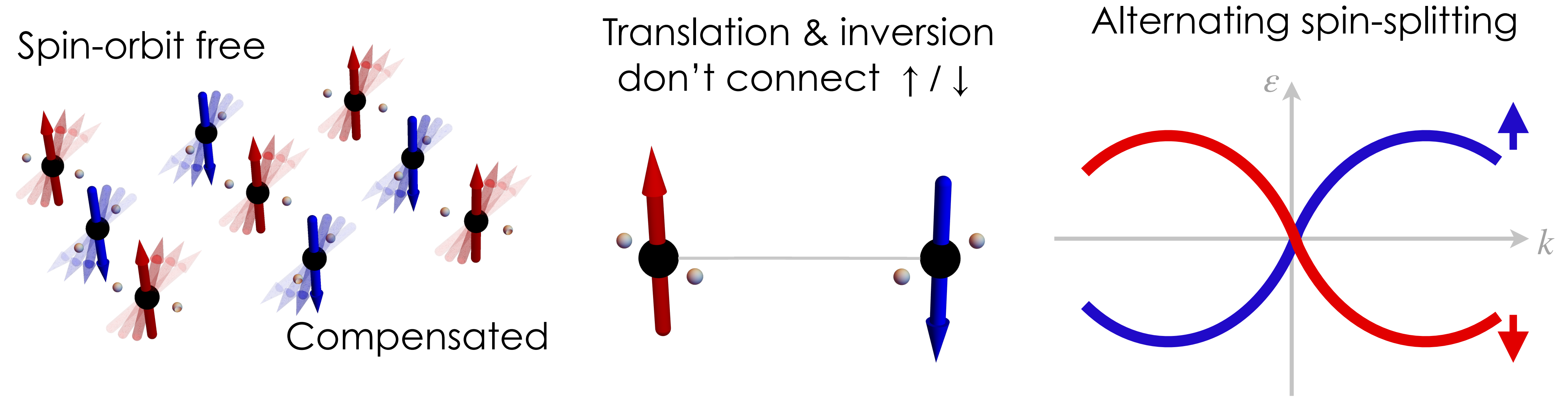

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Derive *all* SO-free Landau theories

- ✦ All structures (space groups WP) compatible with AM

TABLES!

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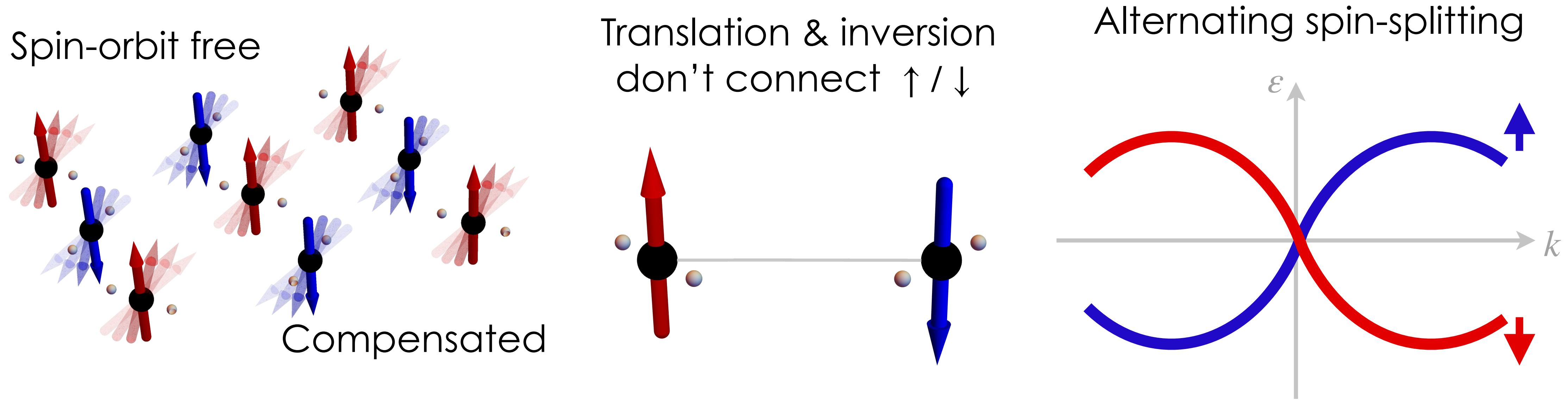

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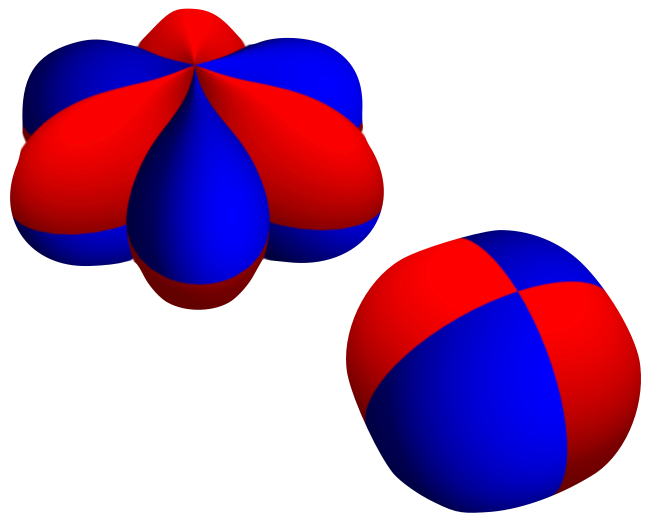

Multipole rank (n)	Representation Γ_ξ
1	$aV,\ aeV^2$
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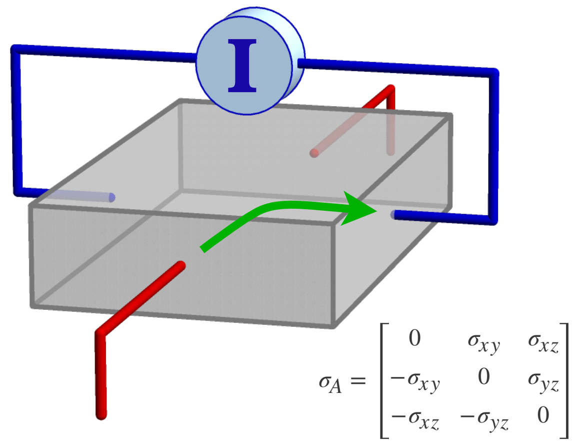
4, 6	$aeV[V^2]$



Guaranteed (allowed) effects with SOC **determined** by SO-free multipoles

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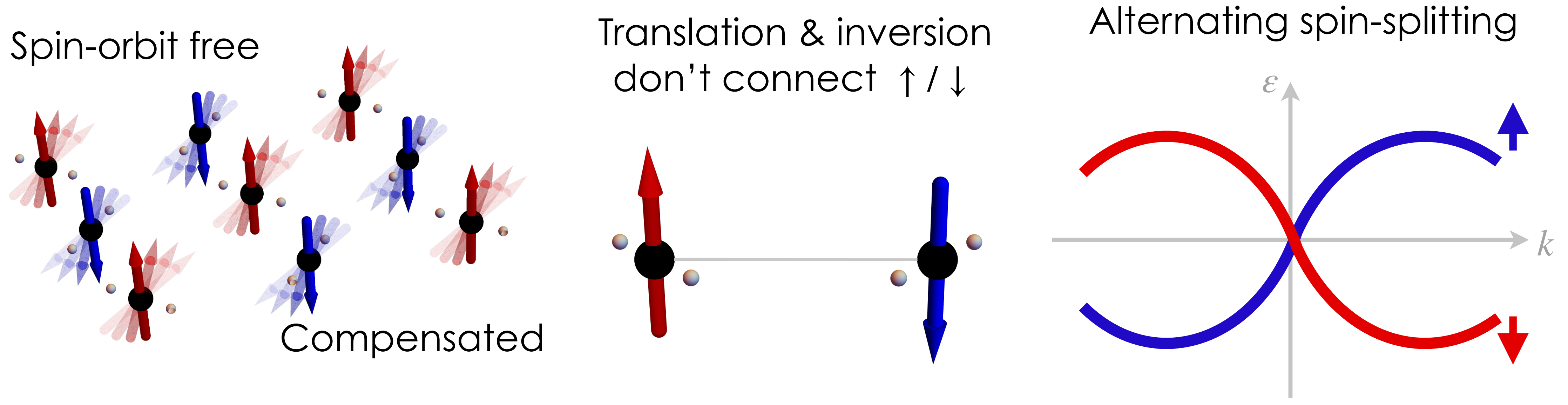

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Bridge SO-free & SOC limits

- ✦ SO-free multipole gives insight into symmetry with SOC
- ✦ Classify response tensors by symmetry type & multipole order

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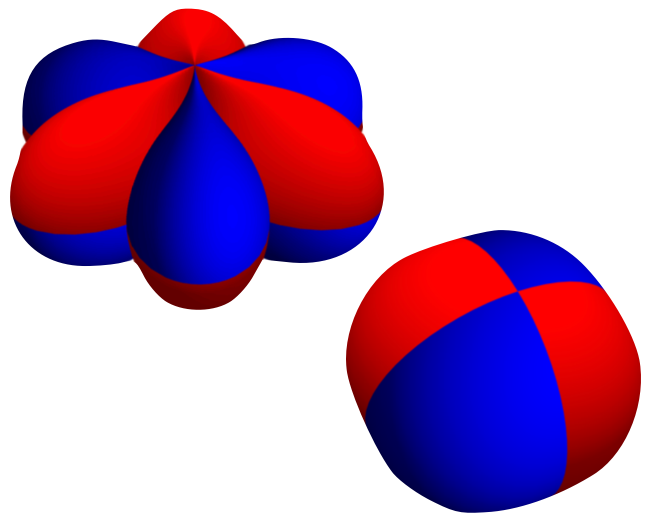

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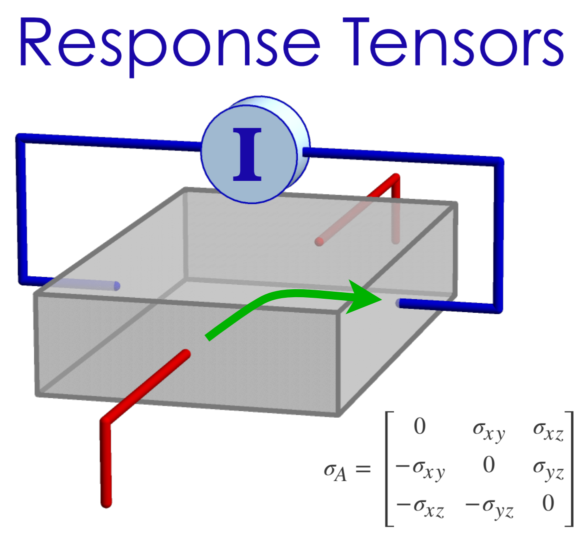

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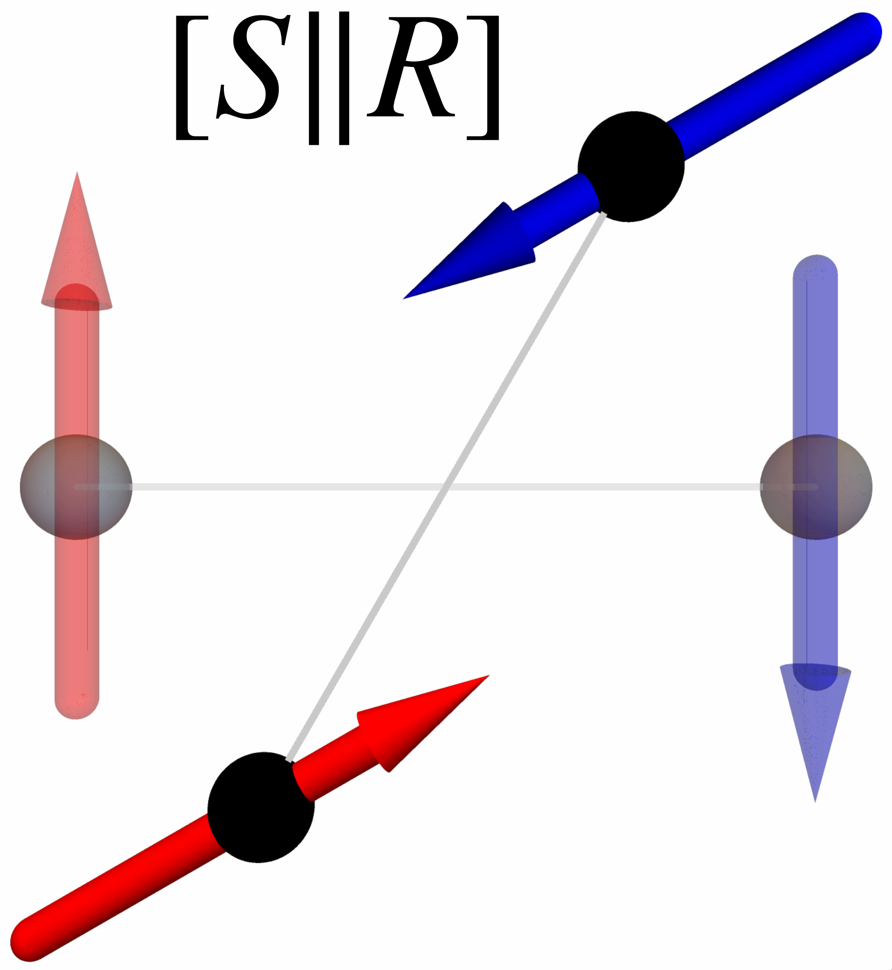

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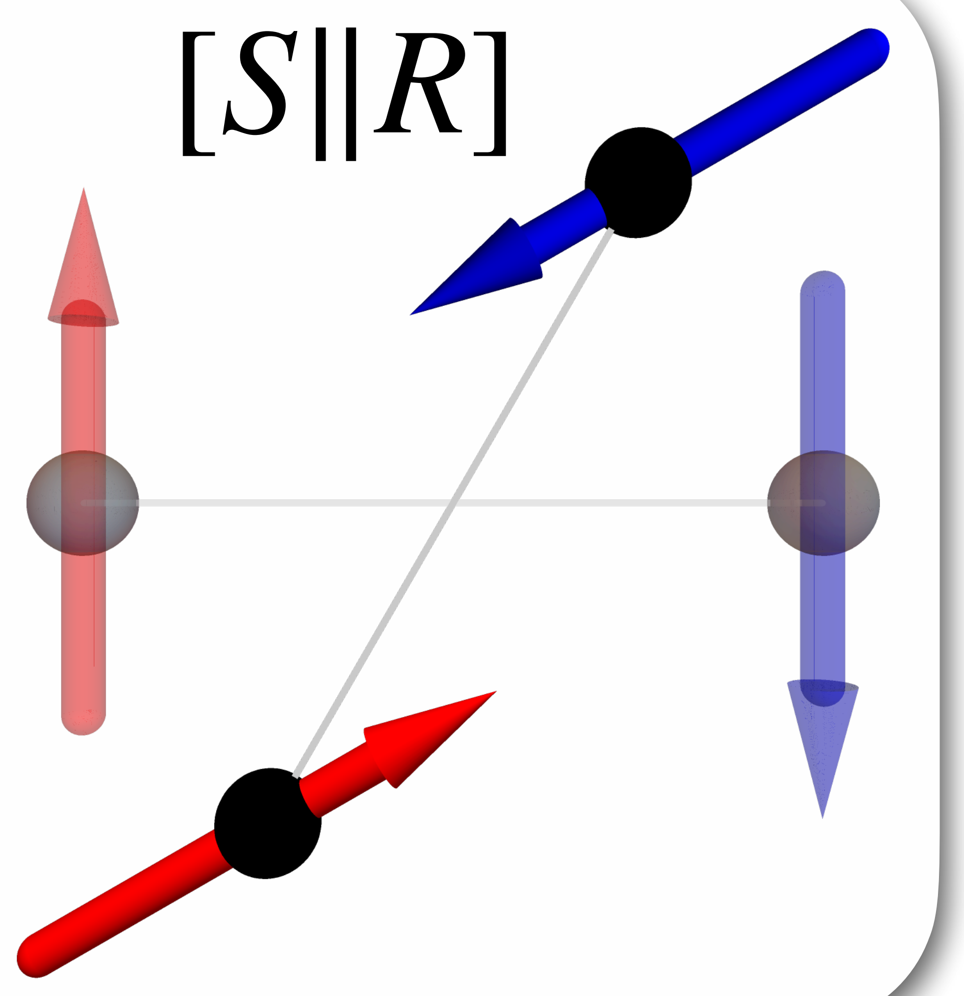

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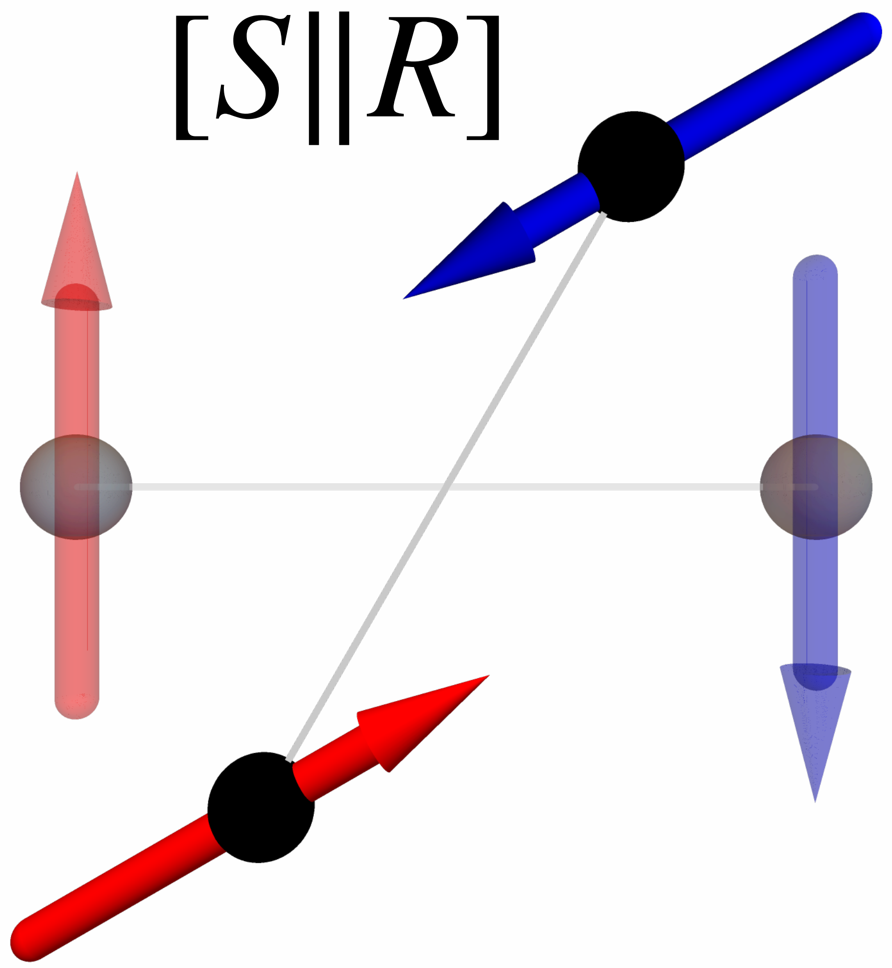

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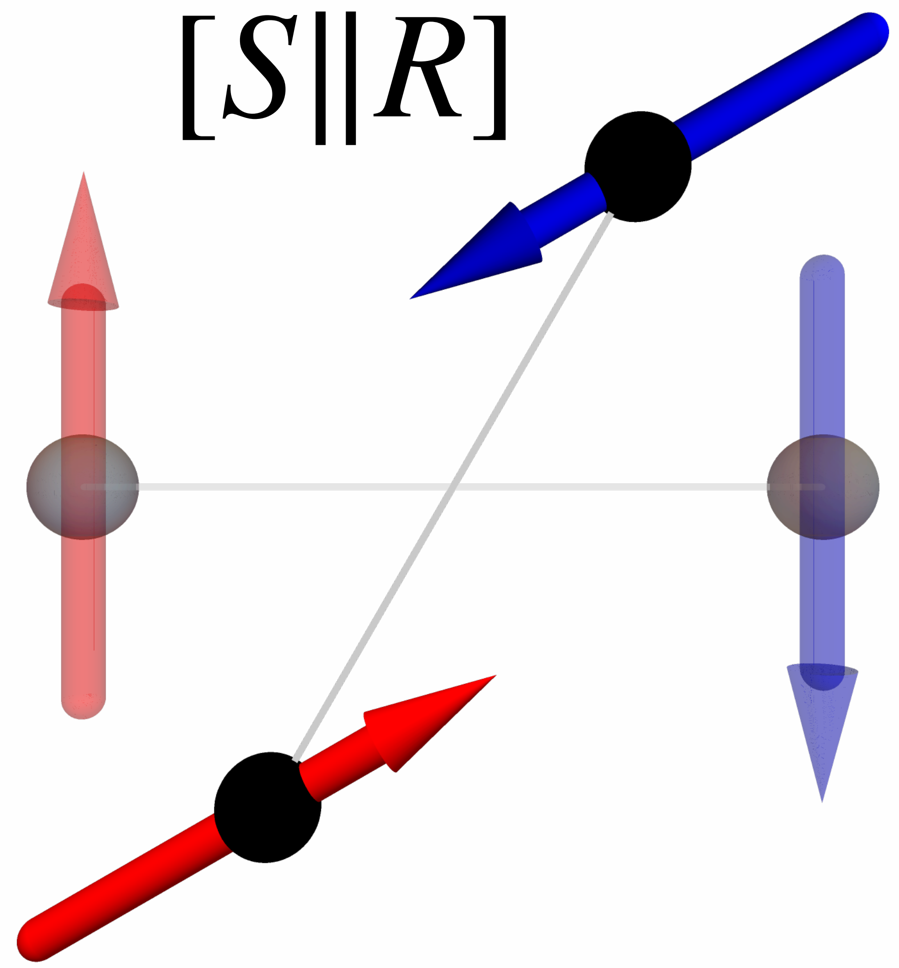

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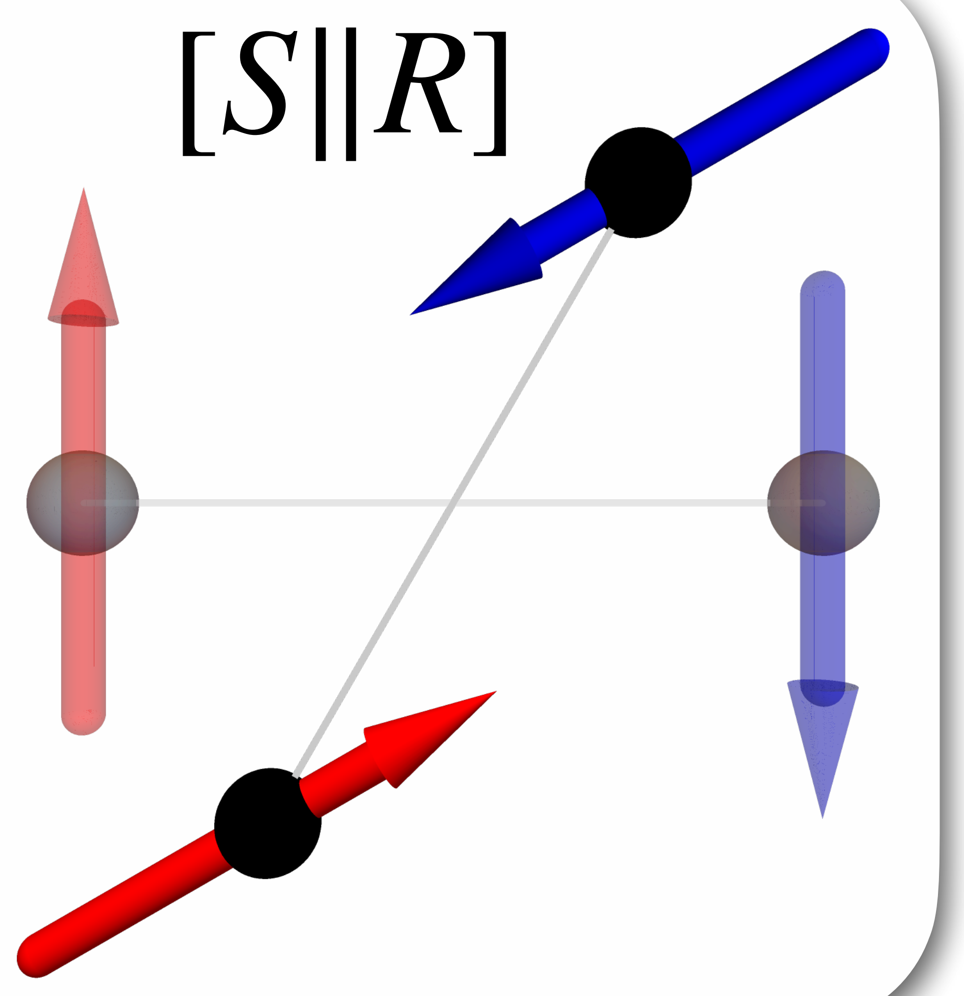

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Whenever band structure degeneracy plays a crucial role in the phenomenon we're studying, we **cannot avoid using spin group theory** in the limit of negligible SOC.

For collinear systems especially, we **can avoid directly using spin groups,** taking advantage of an **equivalent** approach via non-trivial irreps of the paramagnetic group.

Thanks!

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