

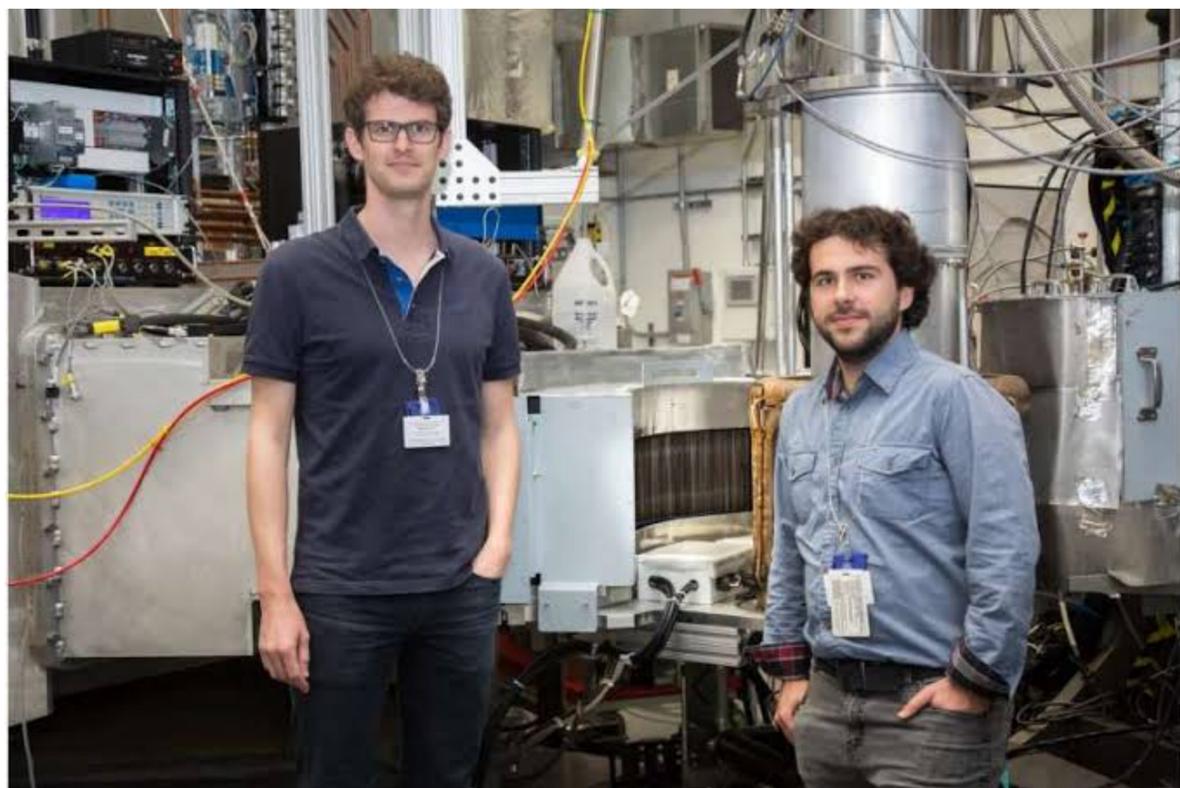
# Designing Light in an Artificial Universe

Han Yan

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**Nat. Phys.** 14, 711–715 (2018) R Sibille, N Gauthier, **HY** *et al*  
**npj Quantum Materials** 7 (1), 51 A Bhardwaj, S Zhang, **HY** *et al*  
**PRB** 106 (9), 094425, B Gao, T Chen, **HY** *et al*  
**Nat. Phys.** 21, 83-88 (2025) V Porée, **HY** *et al*  
**arXiv:2312.11641** A Sanders, **HY**, C Castelnovo, AH Nevidomskyy  
arXiv:2402.08723, arXiv:2305.08261

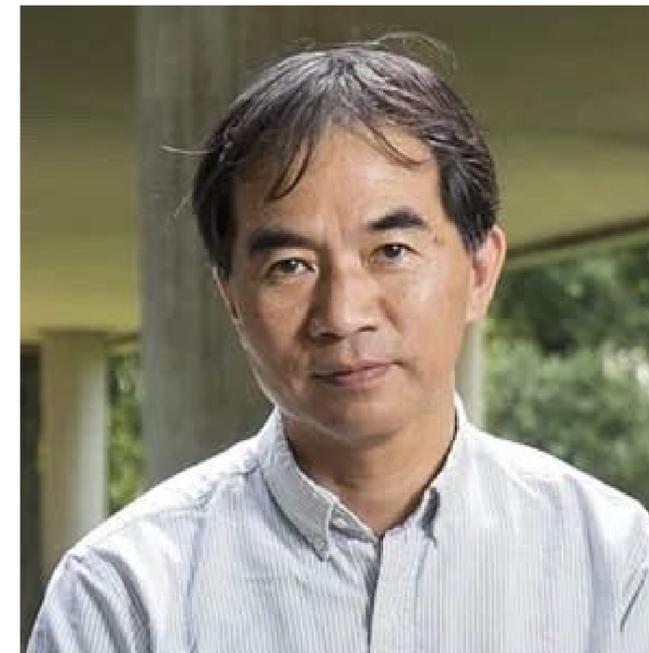
Quantum Geometry and Transport of Collective Excitations in (Non-)Magnetic Insulators  
Ingelheim, May 7 2025



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Paul Scherrer Institute



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Andriy Nevidomskyy  
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Victor Porée  
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Alaric Sanders  
Cambridge

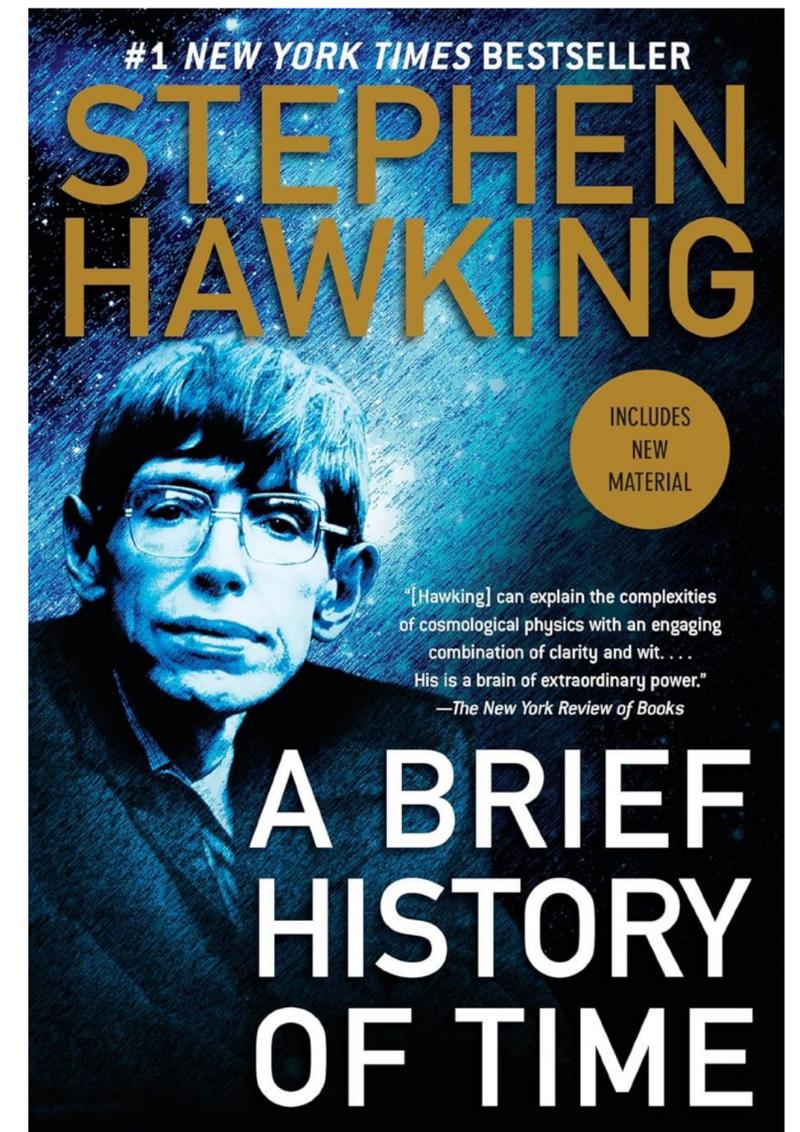


Claudio Castelnovo  
Cambridge

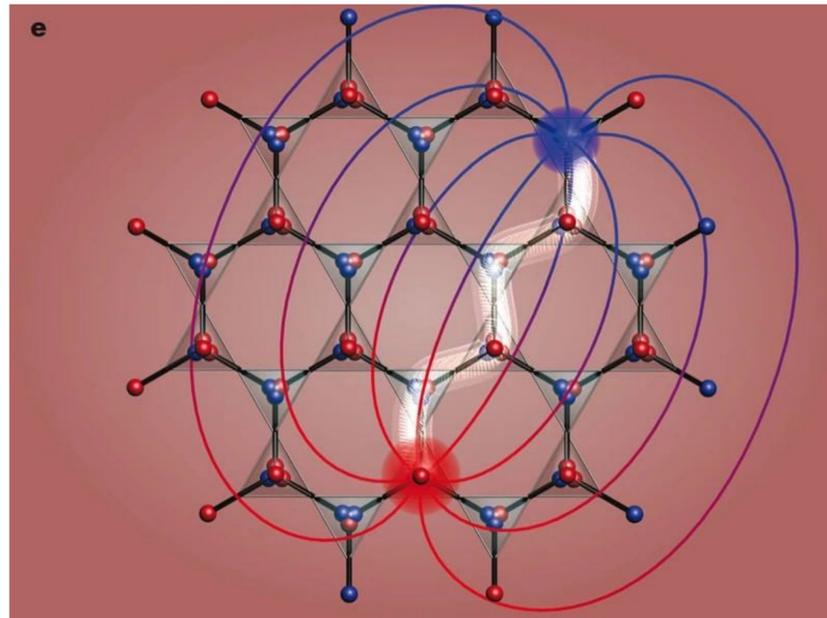
# What if we can change the law of nature?

The laws of science, as we know them at present, contain many fundamental numbers, like the size of the electric charge of the electron and the ratio of the masses of the proton and the electron. ... The remarkable fact is that the values of these numbers seem to have been very finely adjusted to make possible the development of life.

*A Brief History of Time*, Stephen Hawking

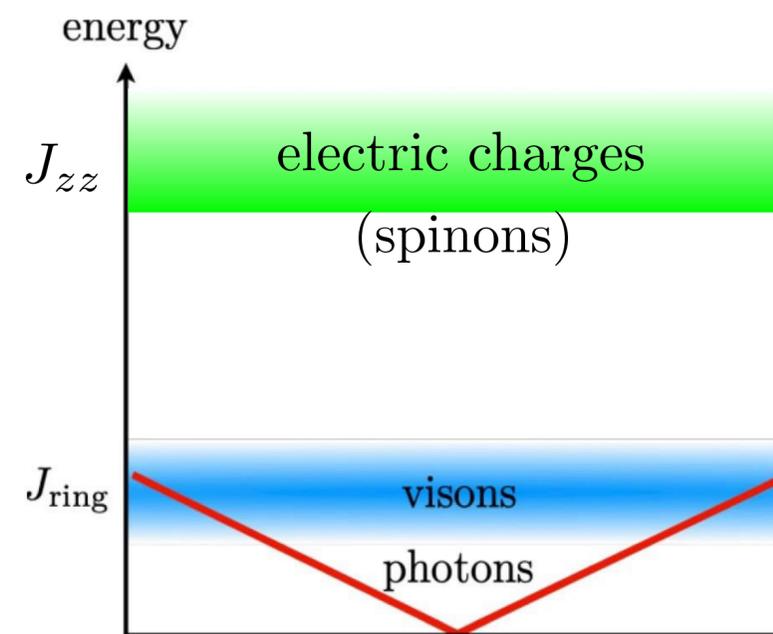


# Quantum Spin Ice: A universe in a grain of sand

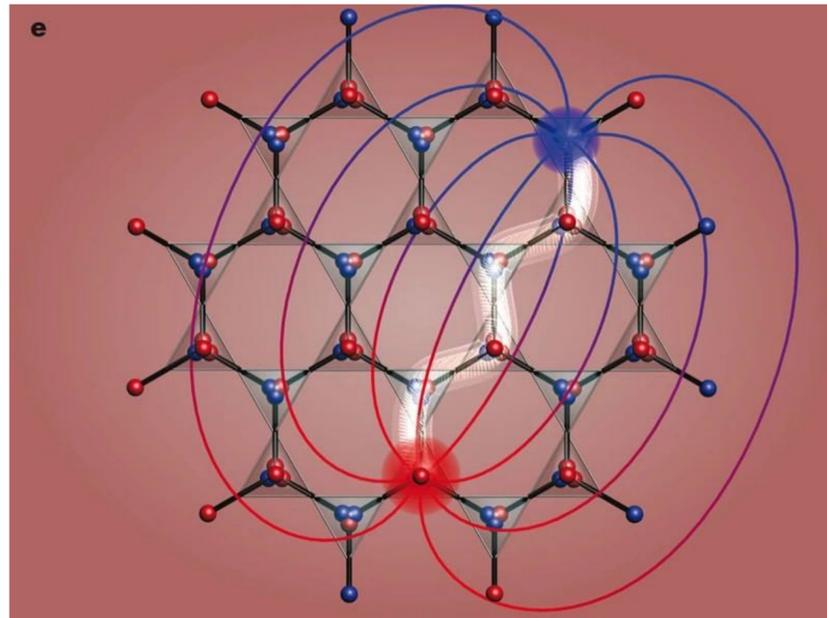


Effective theory: **3D U(1) Quantum Electrodynamics (QED)**

1. Ground state: gapless topological quantum liquid state
2. Gapless excitations: **photons**
3. Gapped excitations: electric **charges** and magnetic **monopoles**



# Quantum Spin Ice: A universe in a grain of sand



Effective theory: **3D U(1) Quantum Electrodynamics (QED)**

1. Ground state: gapless topological quantum liquid state
2. Gapless excitations: **photons**
3. Gapped excitations: electric **charges** and magnetic **monopoles**

Our result: proposal to **experimentally tune the quantum dynamics** of lattice QED of dipolar-octupolar quantum spin ice

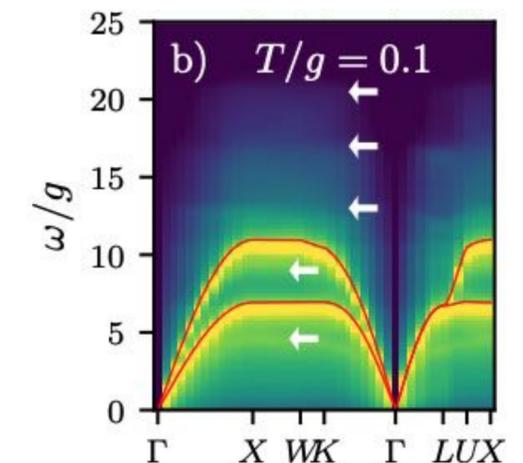
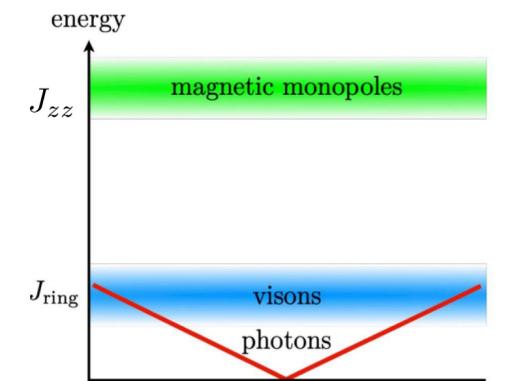
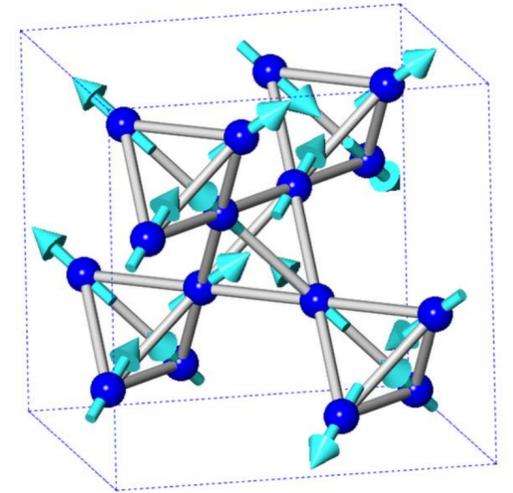
1. New lattice QED **phases** and **phase transitions**.
2. Photon dynamics: tune the emergent **speed of light**, **birefringence**, **non-linear photon coupling**.

# Outline

1. Classical spin ice as electrostatics

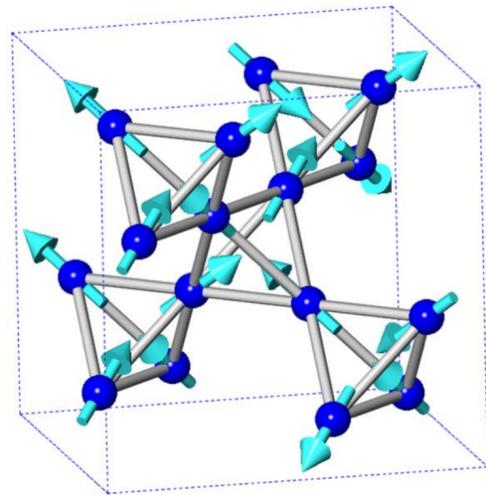
2. Quantum spin ice as emergent Quantum Electrodynamics (eQED)

3. Controlling eQED dynamics in dipolar-octupolar quantum spin ice



# Classical Spin Ice

Classical Spin Ice  
on the pyrochlore lattice

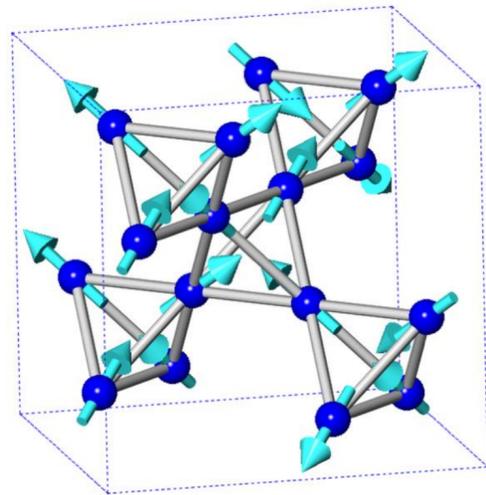


$$\mathcal{H} = \sum_{ij} S_i^z S_j^z$$

$z$  is local basis of in/out tetrahedron

# Classical Spin Ice

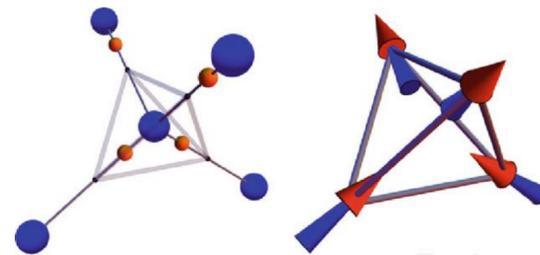
Classical Spin Ice  
on the pyrochlore lattice



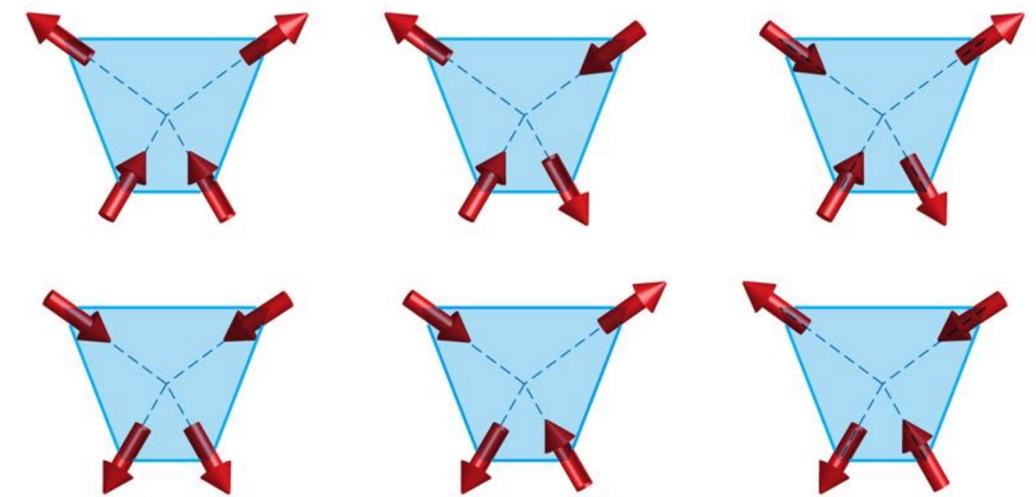
$$\mathcal{H} = \sum_{ij} S_i^z S_j^z$$

$\mathbf{z}$  is local basis of in/out tetrahedron

single tetrahedron  
ground state:  
2-in-2-out (**ice-rule**)



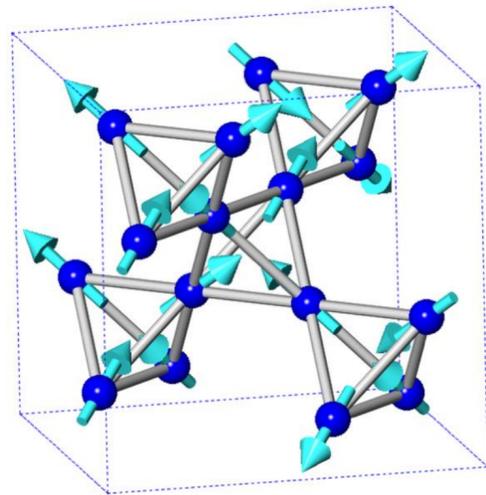
similar hydrogen  
arrangement in ice



$$\nabla \cdot \mathbf{E} = 0$$

# Classical Spin Ice

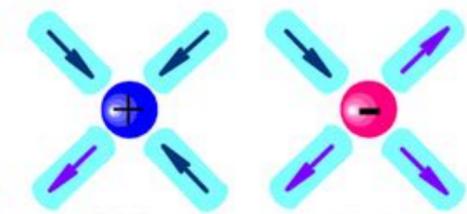
Classical Spin Ice  
on the pyrochlore lattice



$$\mathcal{H} = \sum_{ij} S_i^z S_j^z$$

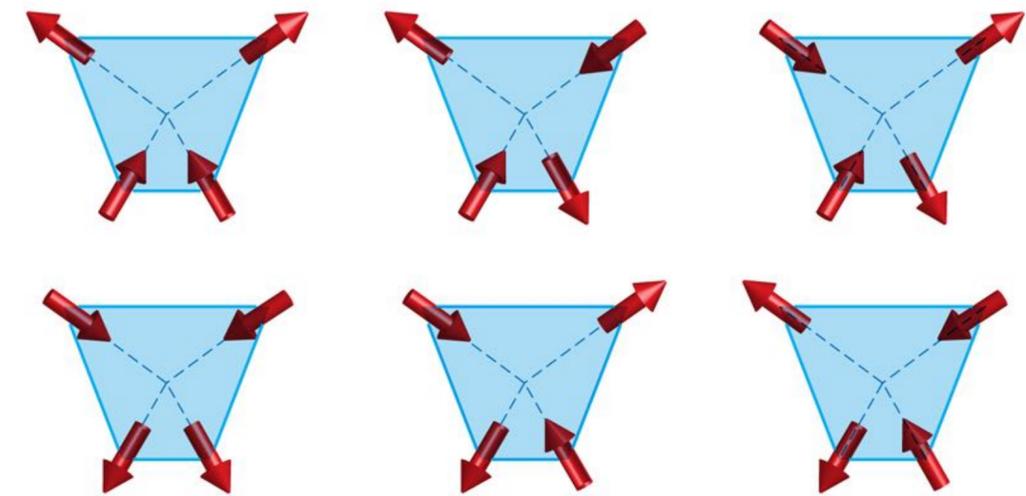
$\mathbf{z}$  is local basis of in/out tetrahedron

single tetrahedron  
excited state:  
3-in-1-out / 3-out-1-in

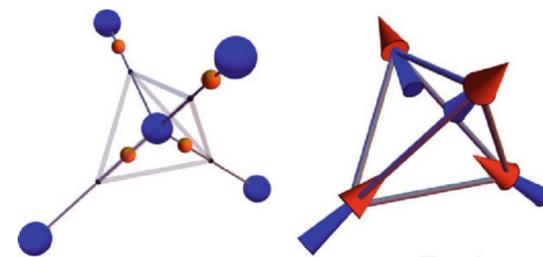


$$\nabla \cdot \mathbf{E} = \pm 1$$

single tetrahedron  
ground state:  
2-in-2-out (**ice-rule**)



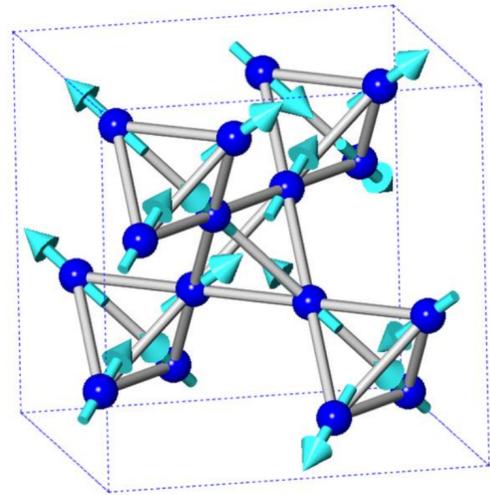
$$\nabla \cdot \mathbf{E} = 0$$



similar hydrogen  
arrangement in ice

# Classical Spin Ice

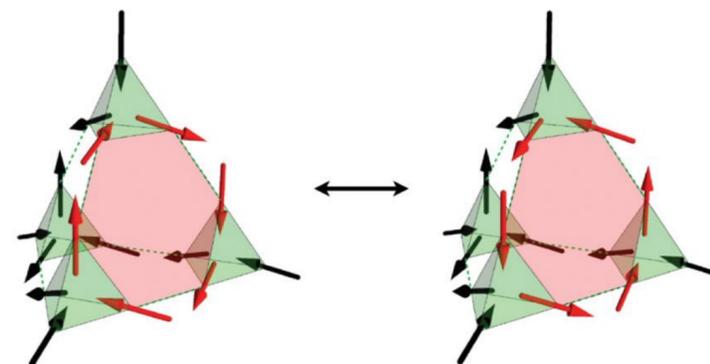
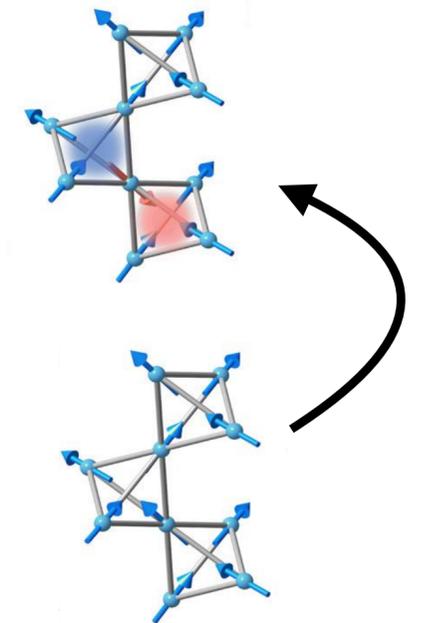
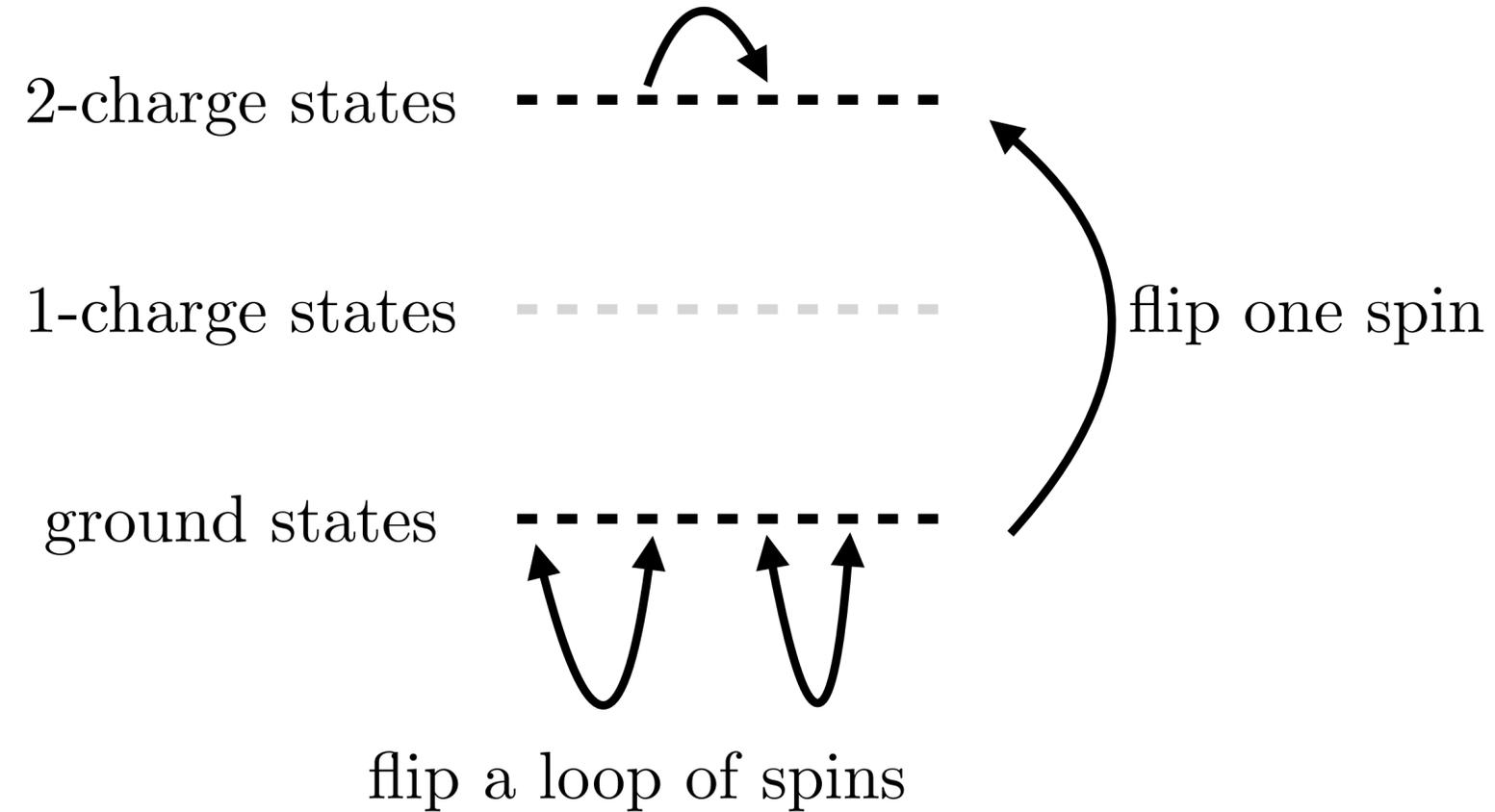
Classical Spin Ice



$$\mathcal{H} = \sum_{ij} S_i^z S_j^z$$

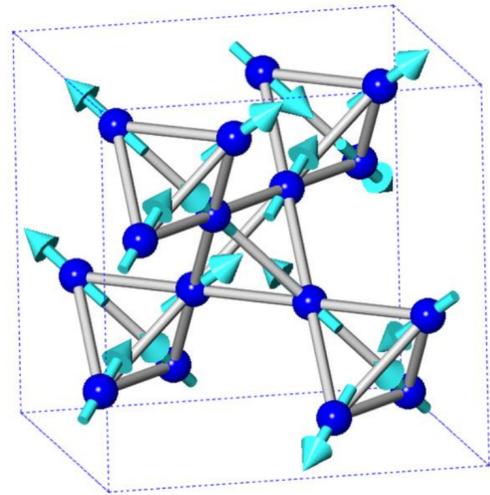
Ground state: 2-in-2-out

$$\nabla \cdot \mathbf{E} = 0$$



# Classical Spin Ice

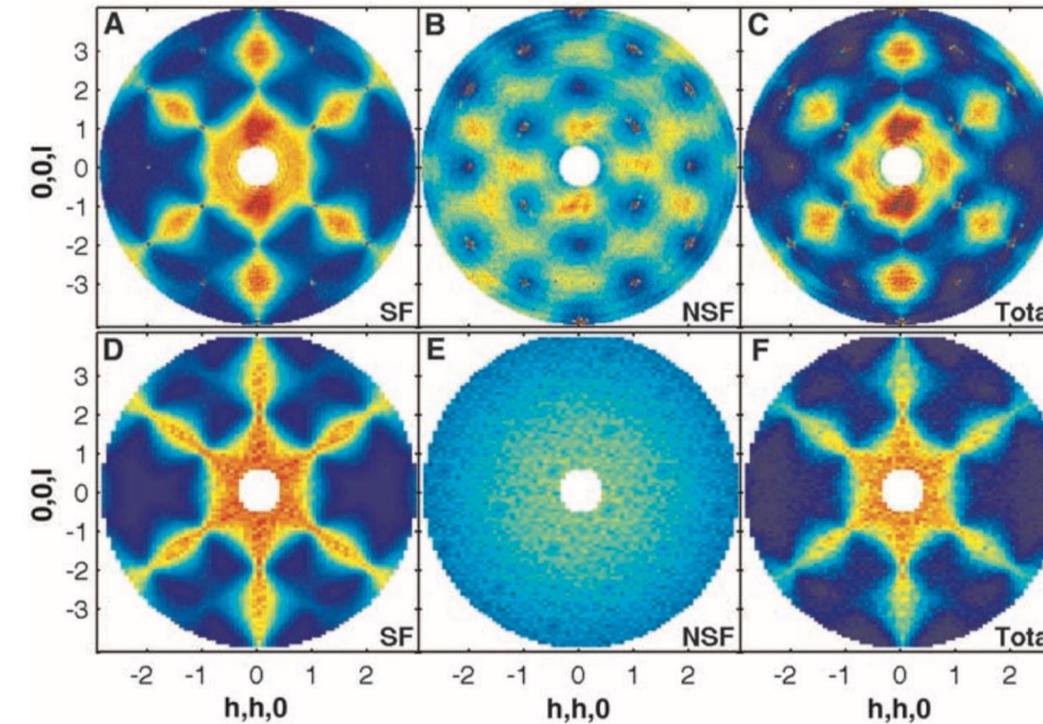
Classical Spin Ice



$$\mathcal{H} = \sum_{ij} S_i^z S_j^z$$

Ground state: 2-in-2-out

$$\nabla \cdot \mathbf{E} = 0$$



**Magnetic Coulomb Phase in the Spin Ice  $\text{Ho}_2\text{Ti}_2\text{O}_7$**

T. Fennell,<sup>1\*</sup> P. P. Deen,<sup>1</sup> A. R. Wildes,<sup>1</sup> K. Schmalzl,<sup>2</sup> D. Prabhakaran,<sup>3</sup> A. T. Boothroyd,<sup>3</sup> R. J. Aldus,<sup>4</sup> D. F. McMorrow,<sup>4</sup> S. T. Bramwell<sup>4</sup>

Holmium Titanium

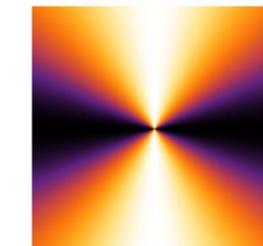
*Science* **326**, 415 (2009)

neutron scattering measures spin-spin correlation in momentum space

$$\nabla \cdot \mathbf{E} = 0 \rightarrow \mathbf{q} \cdot \mathbf{E} = 0$$

$$\langle E_i(\mathbf{q}) E_j(-\mathbf{q}) \rangle \propto \delta_{ij} - q_i q_j / q^2$$

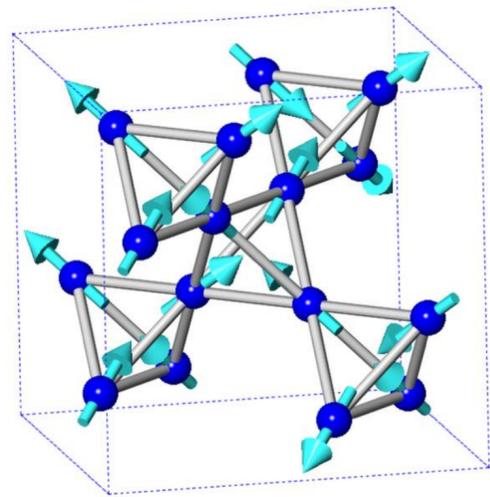
$$\langle E_x(\mathbf{q}) E_x(-\mathbf{q}) \rangle \sim q_y^2 / q^2$$



“pinch point”

# Classical Spin Ice

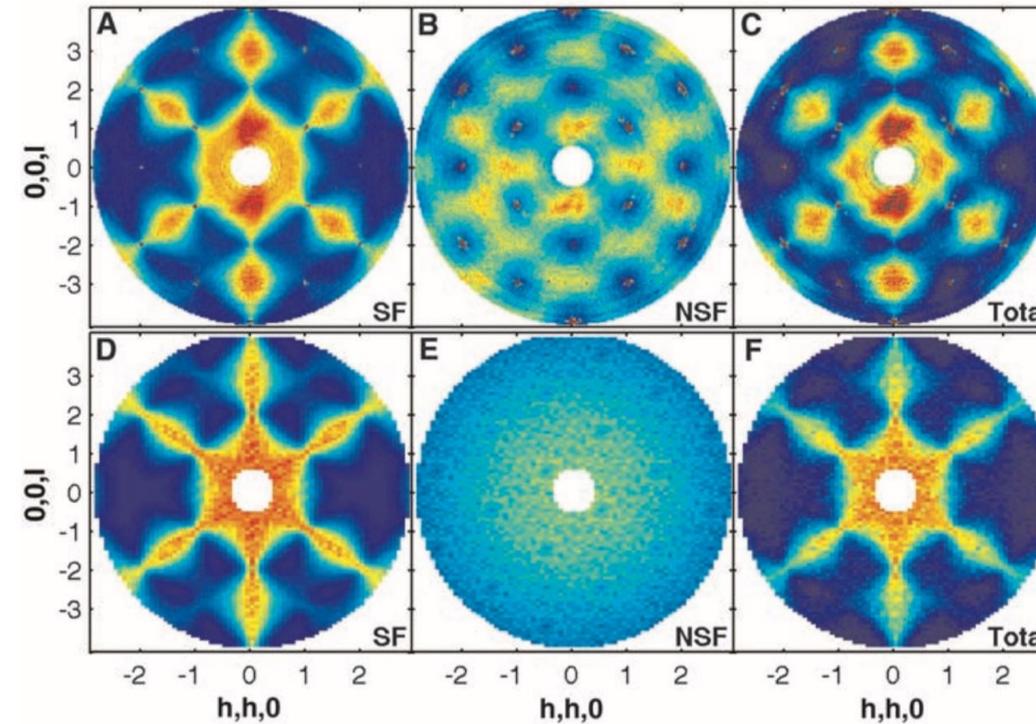
Classical Spin Ice



$$\mathcal{H} = \sum_{ij} S_i^z S_j^z$$

Ground state: 2-in-2-out

$$\nabla \cdot \mathbf{E} = 0$$

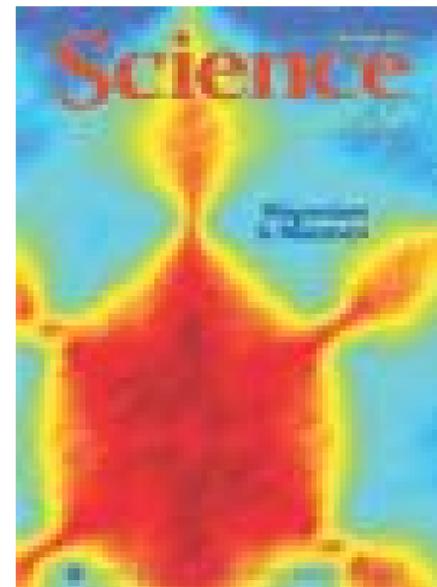


## Magnetic Coulomb Phase in the Spin Ice $\text{Ho}_2\text{Ti}_2\text{O}_7$

T. Fennell,<sup>1\*</sup> P. P. Deen,<sup>1</sup> A. R. Wildes,<sup>1</sup> K. Schmalzl,<sup>2</sup> D. Prabhakaran,<sup>3</sup> A. T. Boothroyd,<sup>3</sup> R. J. Aldus,<sup>4</sup> D. F. McMorrow,<sup>4</sup> S. T. Bramwell<sup>4</sup>

Holmium Titanium

*Science* **326**, 415 (2009)



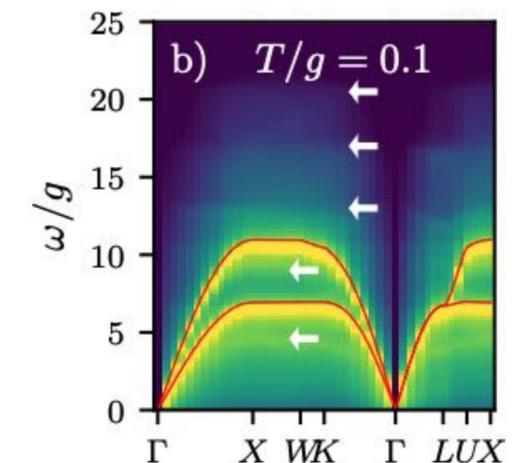
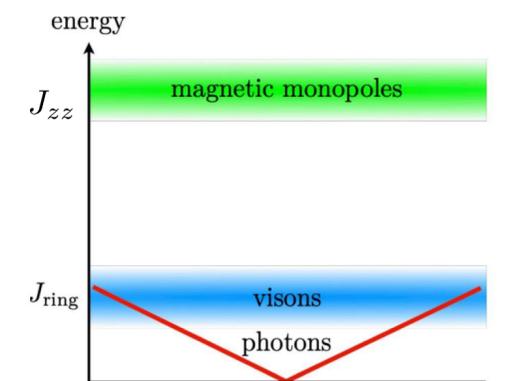
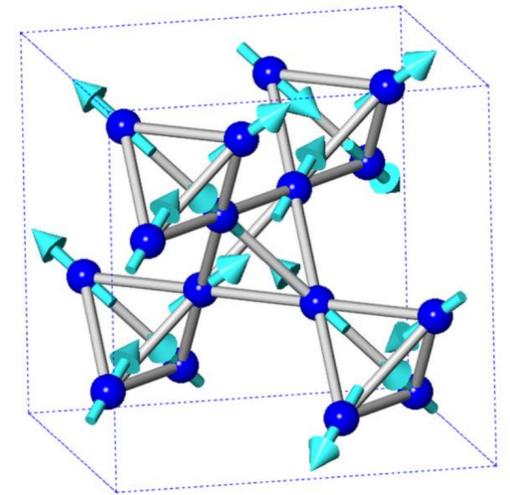
“artificial magnetic monopoles”

# Outline

1. Classical spin ice as electrostatics

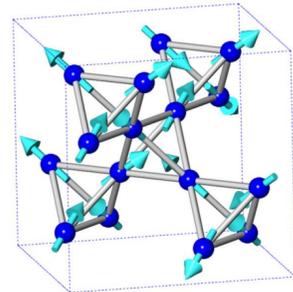
2. Quantum spin ice as emergent Quantum Electrodynamics (eQED)

3. Controlling eQED dynamics in dipolar-octupolar quantum spin ice



# Emergent QED in Quantum Spin Ice

Classical Spin Ice



$$\mathcal{H} = \sum_{ij} S_i^z S_j^z$$

Ground state: 2-in-2-out  $\nabla \cdot \mathbf{E} = 0$

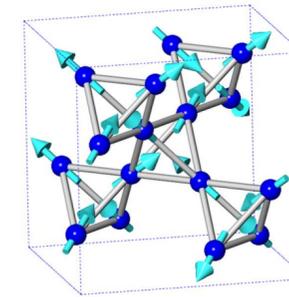
$$H = (\nabla \cdot \mathbf{E})^2 \text{ or } |\phi|^2$$

2 charge states -----

1 charge states - - - - -

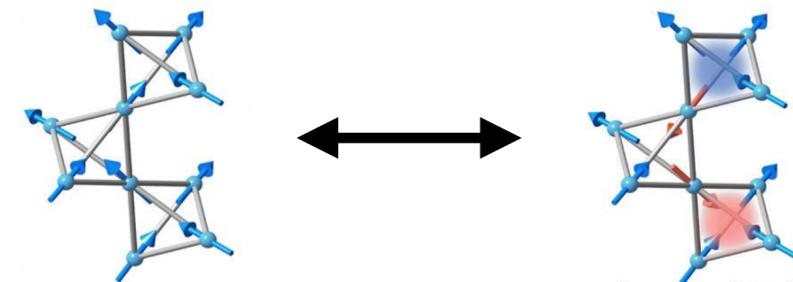
ground states -----

Quantum Spin Ice



quantum dynamics

$$H = \sum_{\langle ij \rangle} \{ J_{zz} \mathbf{S}_i^z \mathbf{S}_j^z - J_{\pm} (\mathbf{S}_i^+ \mathbf{S}_j^- + \mathbf{S}_i^- \mathbf{S}_j^+) \}$$

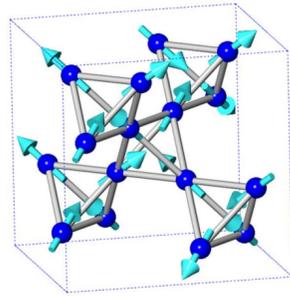


Effective model (roughly speaking)

$$H = J_{zz} |\phi|^2 + J_{\pm} \phi_i e^{iA_{ij}} \phi_j^{\dagger} + \text{h.c.}$$

# Emergent QED in Quantum Spin Ice

Classical Spin Ice



$$\mathcal{H} = \sum_{ij} S_i^z S_j^z$$

Ground state: 2-in-2-out  $\nabla \cdot \mathbf{E} = 0$

$$H = (\nabla \cdot \mathbf{E})^2 \text{ or } |\phi|^2$$

2 charge states -----

1 charge states - - - - -

ground states -----

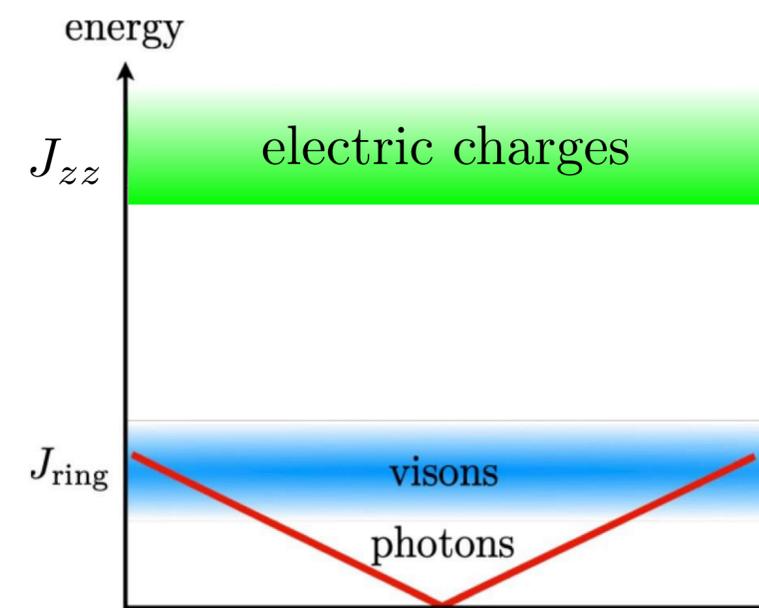
Quantum Spin Ice

$$H = \sum_{\langle ij \rangle} \{ J_{zz} S_i^z S_j^z - J_{\pm} (S_i^+ S_j^- + S_i^- S_j^+) \}$$

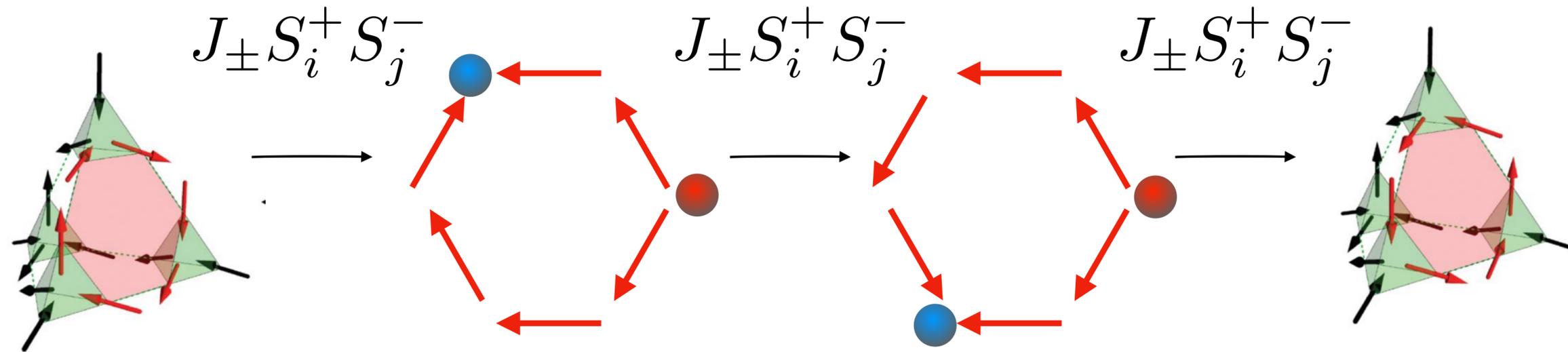
quantum dynamics

Effective model (roughly speaking)

$$H = J_{zz} |\phi|^2 + J_{\pm} \phi_i e^{iA_{ij}} \phi_j^{\dagger} + \text{h.c.}$$

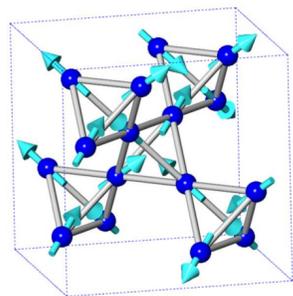


# Emergent QED in Quantum Spin Ice



$$J_{ring} \sum_{\square} (S_1^+ S_2^- S_3^+ S_4^- S_5^+ S_6^- + \text{H.c.}) \sim J_{ring} \cos(b) \quad J_{ring} = 3J_{\pm}^3 / (2J_{zz}^2)$$

# Emergent QED in Quantum Spin Ice



$$H = (\nabla \cdot \mathbf{E})^2 \text{ or } |\phi|^2$$

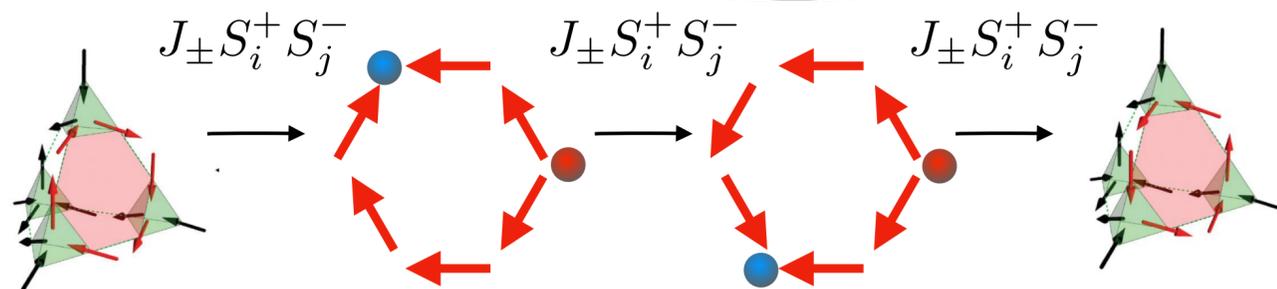
2 charge states -----

1 charge states - - - - -

ground states -----

$$\cos(b) \sim b^2 + \dots$$

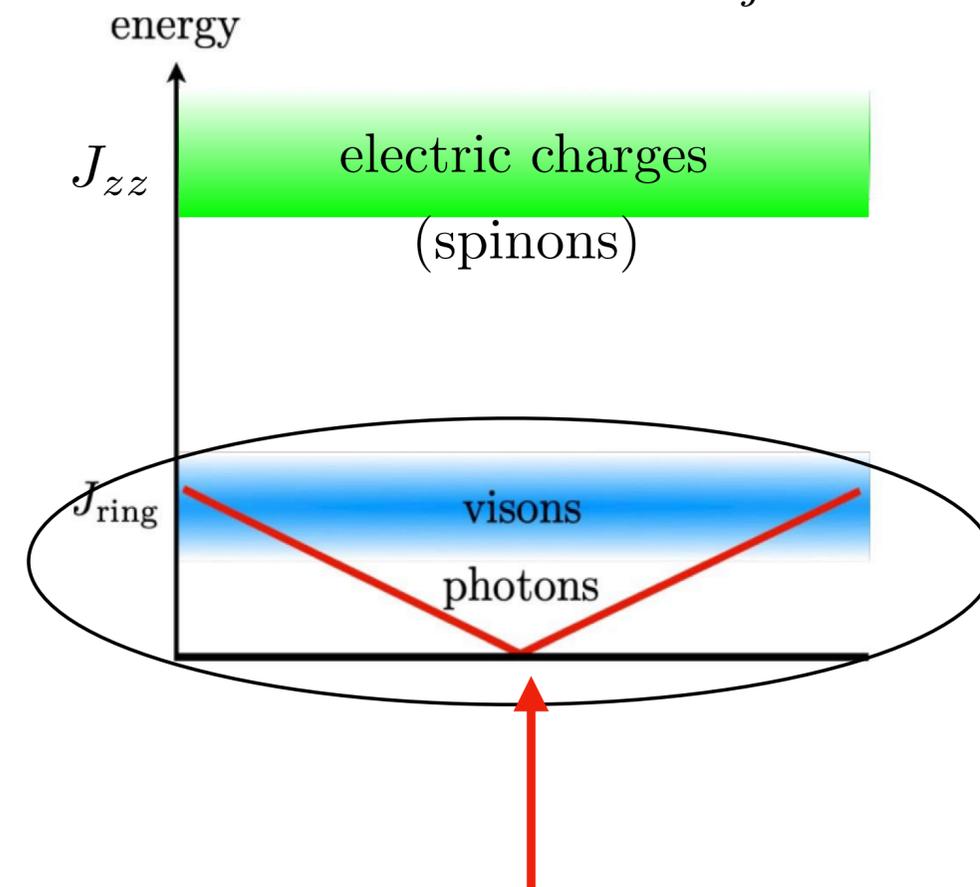
$$H \sim E^2 + b^2$$



$$J_{ring} \sum_{\square} (S_1^+ S_2^- S_3^+ S_4^- S_5^+ S_6^- + \text{H.c.}) \sim J_{ring} \cos(b)$$

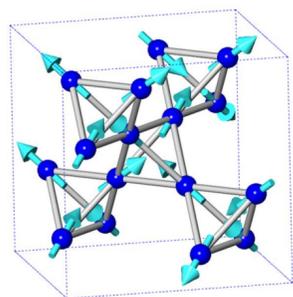
Effective model (roughly speaking)

$$H = J_{zz} |\phi|^2 + J_{\pm} \phi_i e^{iA_{ij}} \phi_j^{\dagger} + \text{h.c.}$$



QSI g.s.: superposition of ice-states

# Emergent QED in Quantum Spin Ice



$$H = (\nabla \cdot \mathbf{E})^2 \text{ or } |\phi|^2$$

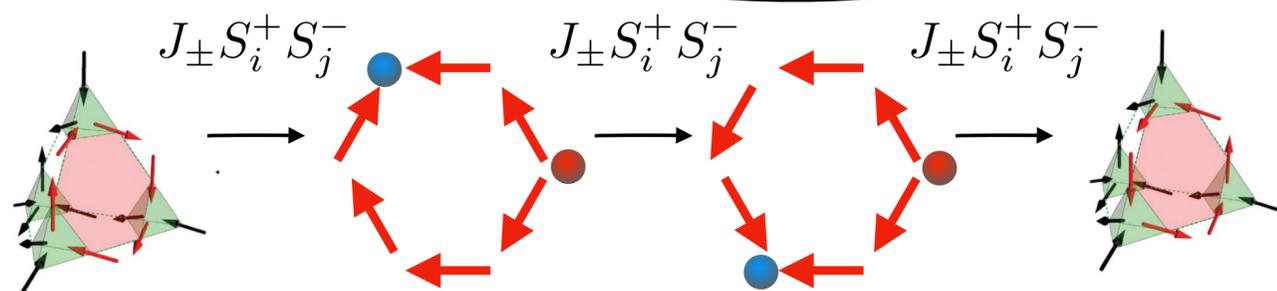
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ground states -----

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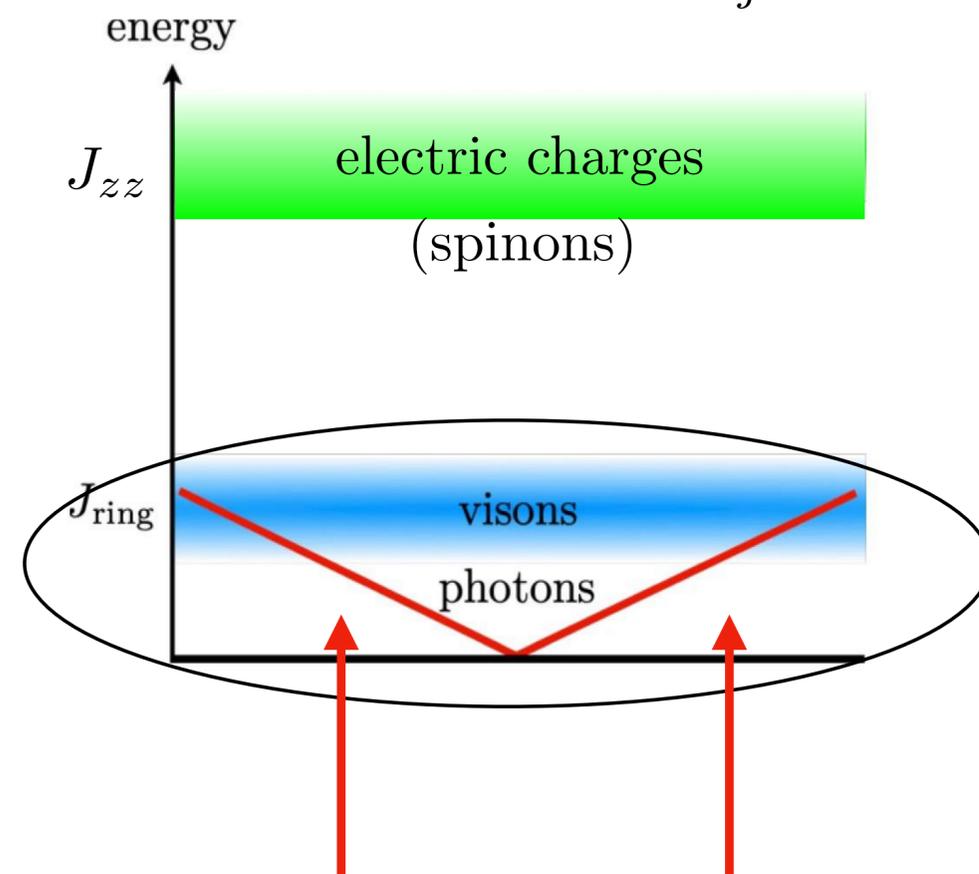
$$H \sim E^2 + b^2$$



$$J_{ring} \sum_{\square} (S_1^+ S_2^- S_3^+ S_4^- S_5^+ S_6^- + \text{H.c.}) \sim J_{ring} \cos(b)$$

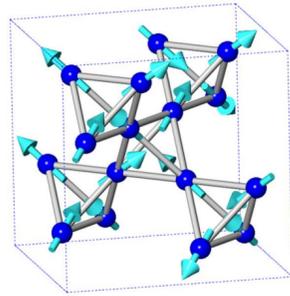
Effective model (roughly speaking)

$$H = J_{zz} |\phi|^2 + J_{\pm} \phi_i e^{iA_{ij}} \phi_j^{\dagger} + \text{h.c.}$$



photons: also superposition of ice-states, with finite momentum

# Emergent QED in Quantum Spin Ice



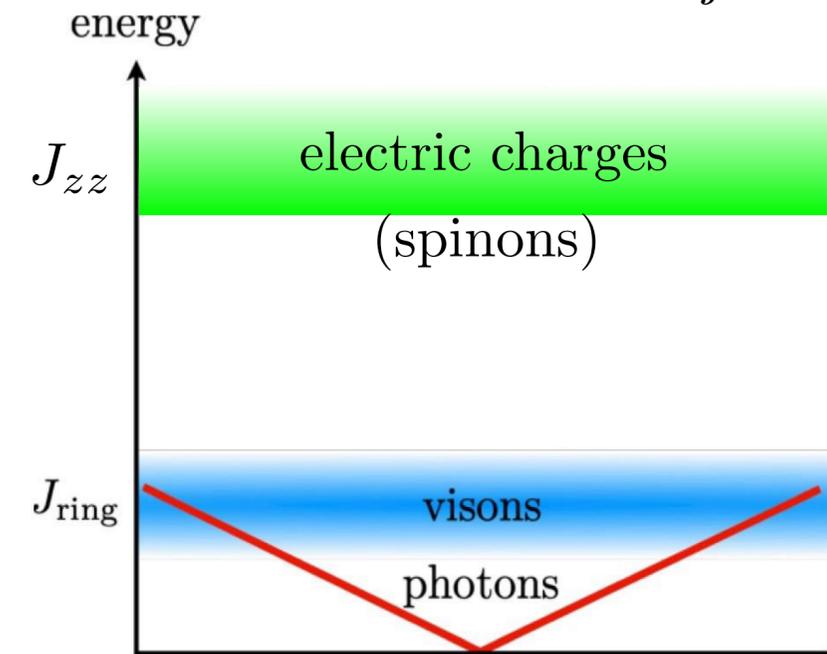
$$H = (\nabla \cdot \mathbf{E})^2 \text{ or } |\phi|^2$$

2 charge states -----  
 1 charge states - - - - -  
 ground states -----

$$J_{\pm} \phi_i e^{iA_{ij}} \phi_j^{\dagger} + \text{h.c.}$$

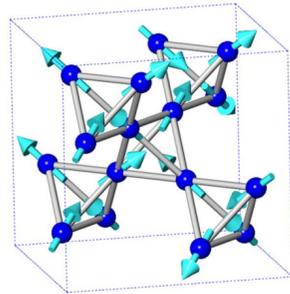
Effective model (roughly speaking)

$$H = J_{zz} |\phi|^2 + J_{\pm} \phi_i e^{iA_{ij}} \phi_j^{\dagger} + \text{h.c.}$$



hopping of charges on a gauge field background

# Emergent QED in Quantum Spin Ice



$$H = (\nabla \cdot \mathbf{E})^2 \text{ or } |\phi|^2$$

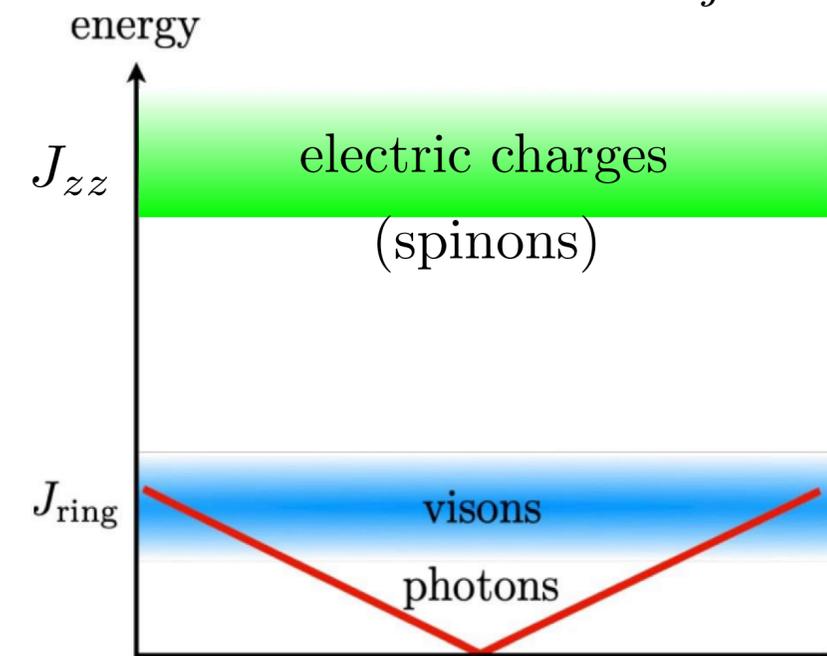
2 charge states -----

1 charge states - - - - -

ground states -----

Effective model (roughly speaking)

$$H = J_{zz} |\phi|^2 + J_{\pm} \phi_i e^{iA_{ij}} \phi_j^{\dagger} + \text{h.c.}$$



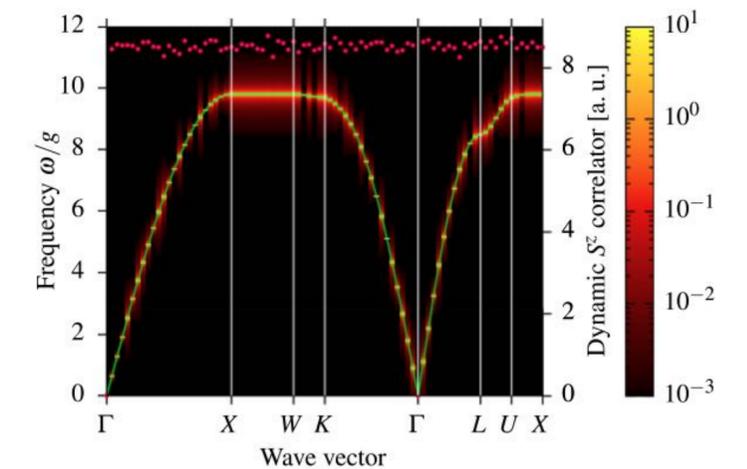
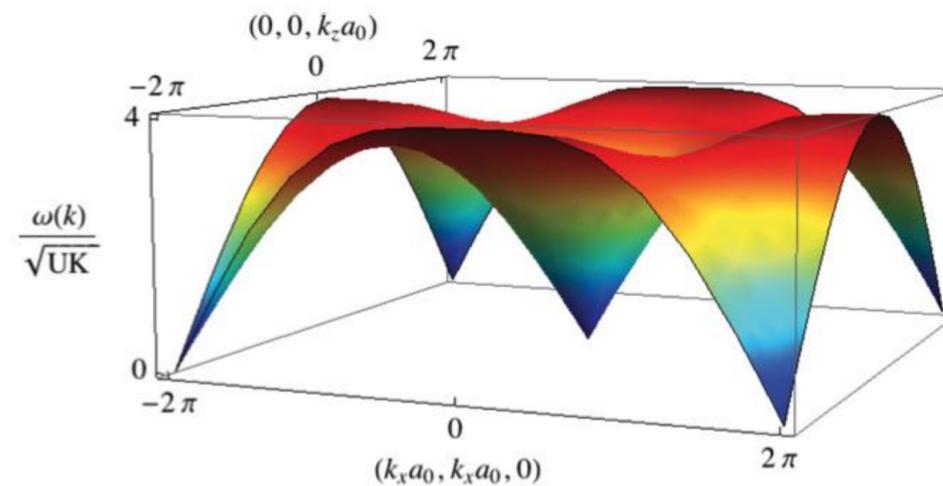
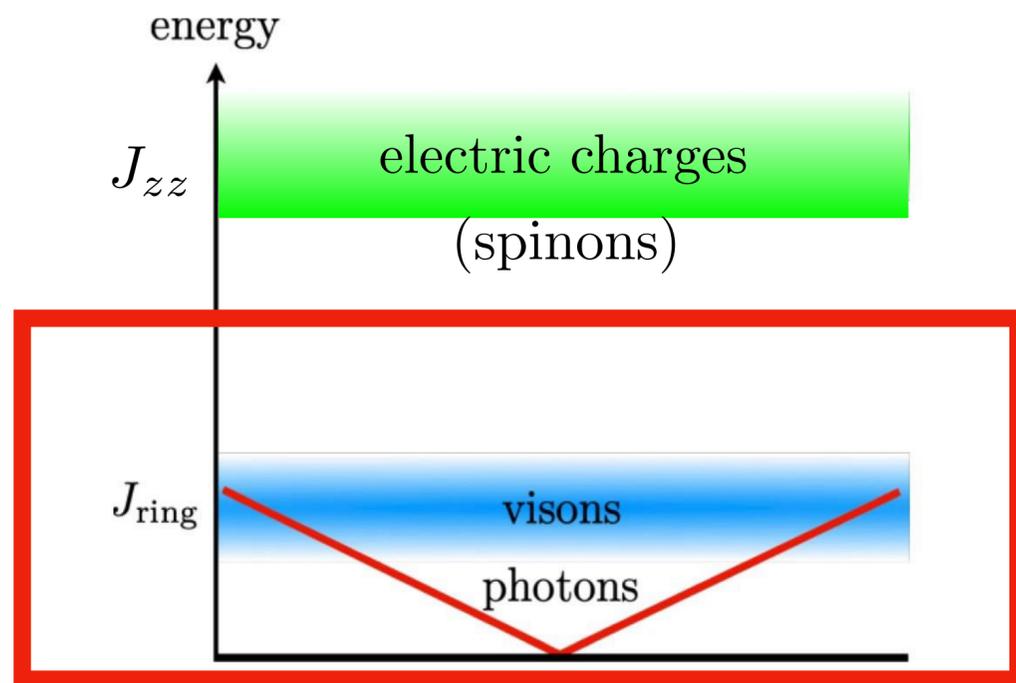
overall picture: **QSI is a lattice emergent QED** (eQED) with photons, electric charges, and magnetic monopoles. All these components have their dynamics and interactions.

# Emergent QED in Quantum Spin Ice

It is generally impossible to solve this model exactly. So mean-field theory and numerics are used to identify the spectrum.

Effective model (roughly speaking)

$$H = J_{zz}|\phi|^2 + J_{\pm}\phi_i e^{iA_{ij}} \phi_j^{\dagger} + \text{h.c.}$$



## Seeing the light: Experimental signatures of emergent electromagnetism in a quantum spin ice

Owen Benton,<sup>1</sup> Olga Sikora,<sup>1,2</sup> and Nic Shannon<sup>1,2,3</sup>

<sup>1</sup>*H. H. Wills Physics Laboratory, University of Bristol, Tyndall Avenue, Bristol BS8 1TL, United Kingdom*

<sup>2</sup>*Okinawa Institute of Science and Technology, 12-22 Suzaki, Uruma, Okinawa, 904-2234, Japan*

<sup>3</sup>*Clarendon Laboratory, University of Oxford, Parks Road, Oxford OX1 3PU, United Kingdom*

(Received 5 April 2012; published 30 August 2012)

## Seeing beyond the light: Vison and photon electrodynamics in quantum spin ice

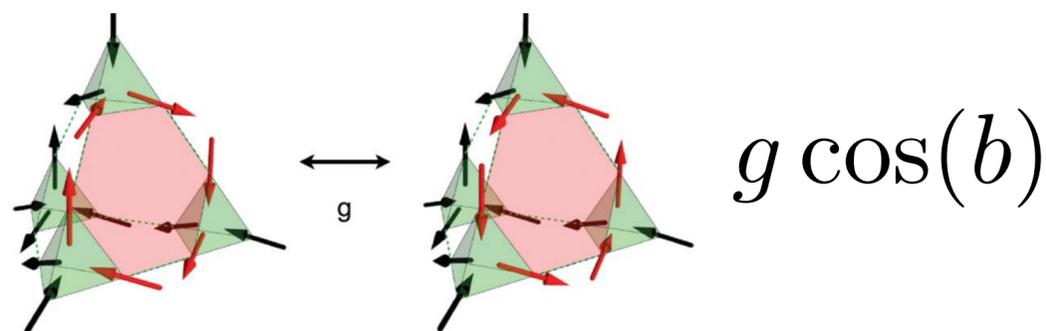
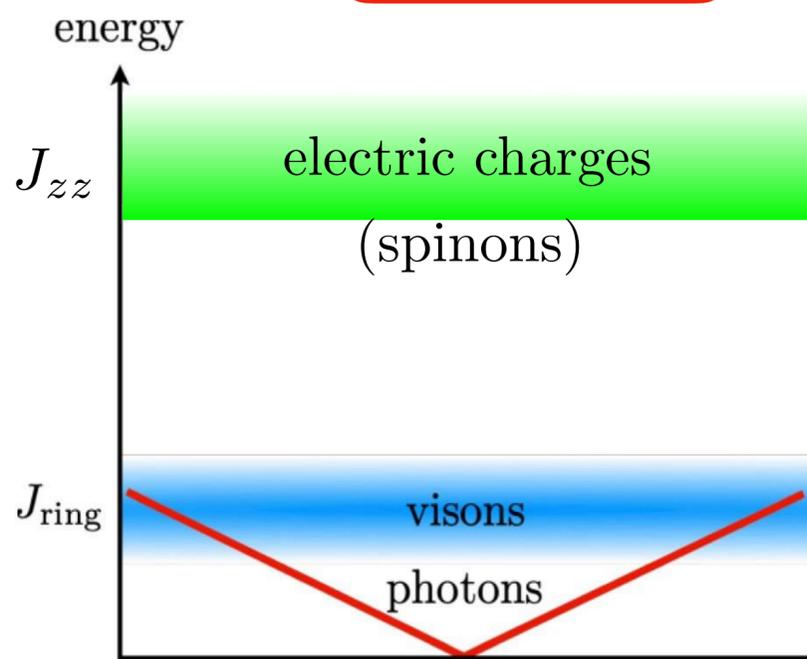
Attila Szabó and Claudio Castelnovo

*TCM Group, Cavendish Laboratory, University of Cambridge, Cambridge CB3 0HE, United Kingdom*

# Emergent QED in Quantum Spin Ice

Effective model (roughly speaking)

$$H = J_{zz}|\phi|^2 + \boxed{J_{\pm}\phi_i e^{iA_{ij}}\phi_j^{\dagger}} + \text{h.c.}$$



It is generally impossible to solve this model exactly. So mean-field theory and numerics are used to identify the spectrum.



$$H \sim -\cos(b)$$

background  $b = 0$

$$H \sim +\cos(b)$$

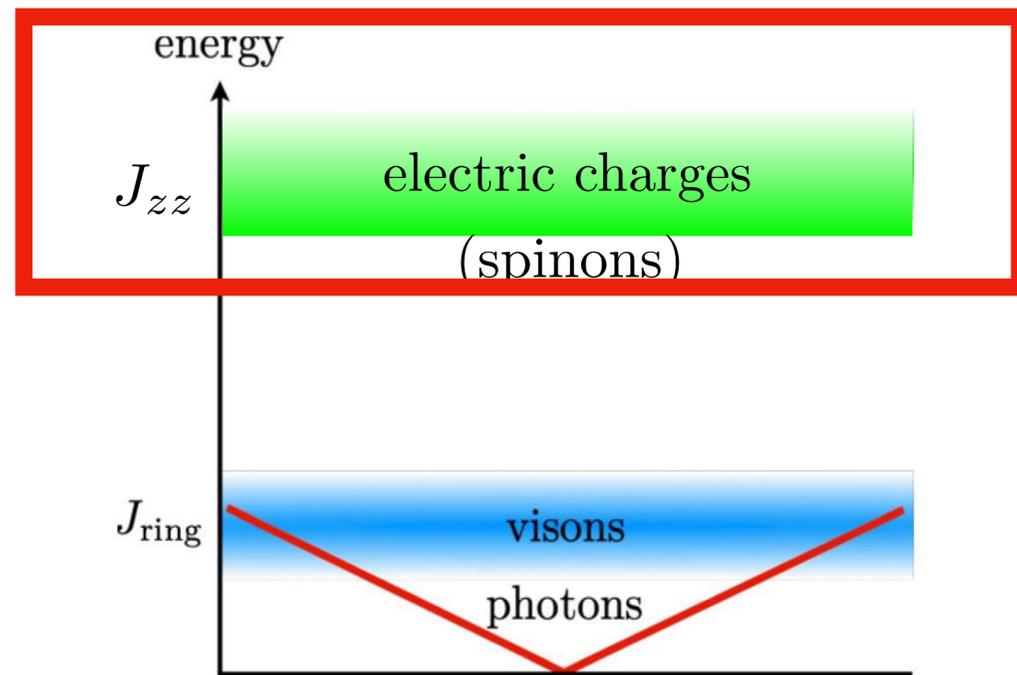
background  $b = \pi$

**2 different phases of lattice eQED**

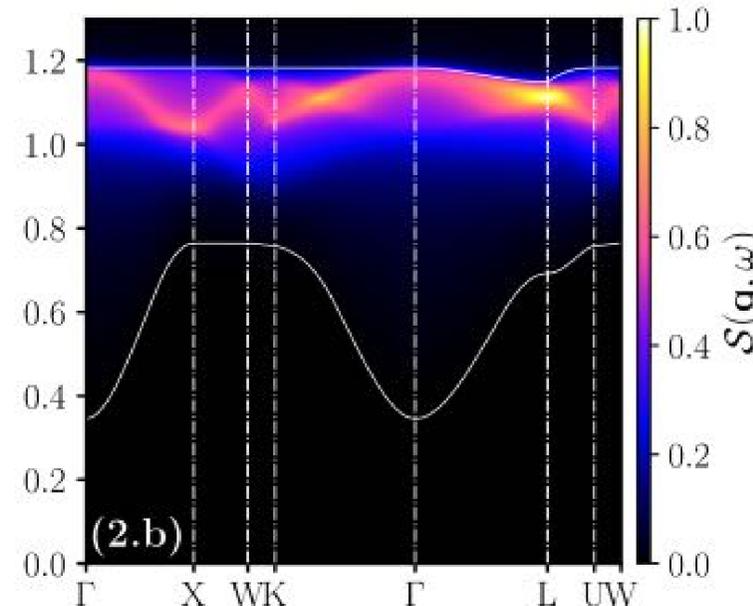
# Emergent QED in Quantum Spin Ice

Effective model (roughly speaking)

$$H = J_{zz}|\phi|^2 + J_{\pm}\phi_i e^{iA_{ij}}\phi_j^{\dagger} + \text{h.c.}$$

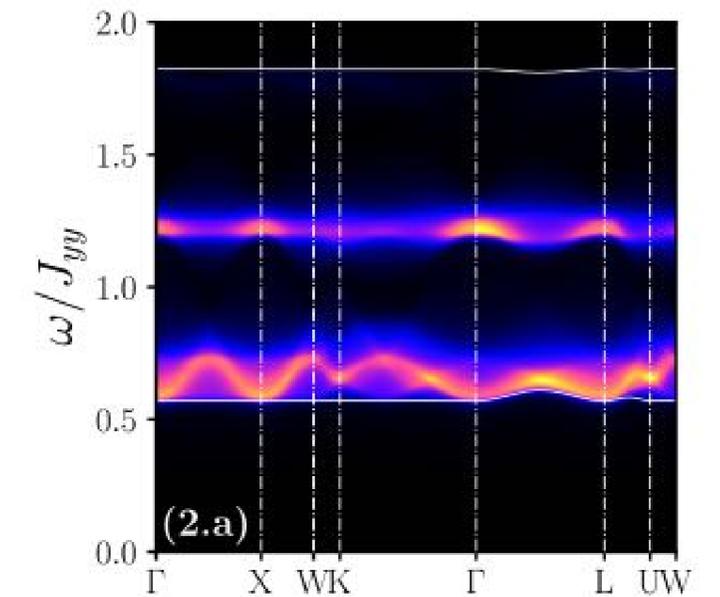


It is generally impossible to solve this model exactly. So mean-field theory and numerics are used to identify the spectrum.



$$H \sim -\cos(b)$$

background  $b = 0$



$$H \sim +\cos(b)$$

background  $b = \pi$

Gang Chen & collaborators:

PRR 5, 033169 PRR 2, 013066 PRB 96, 195127

PRB 96, 085136 PRB 95, 041106 PRL 112, 167203

**Spectroscopic signatures of fractionalization in octupolar quantum spin ice**

Félix Desrochers<sup>1,\*</sup> and Yong Baek Kim<sup>1,†</sup>

<sup>1</sup>Department of Physics, University of Toronto, Toronto, Ontario M5S 1A7, Canada

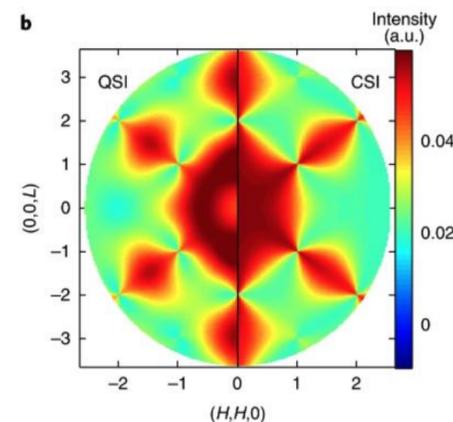
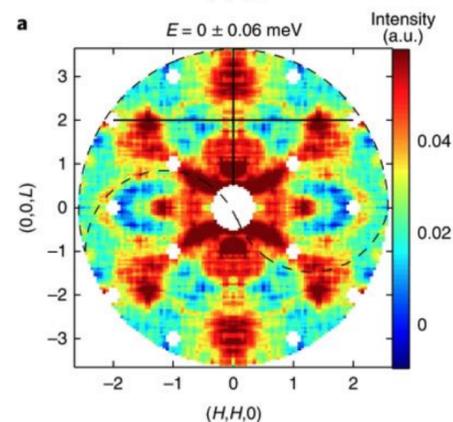
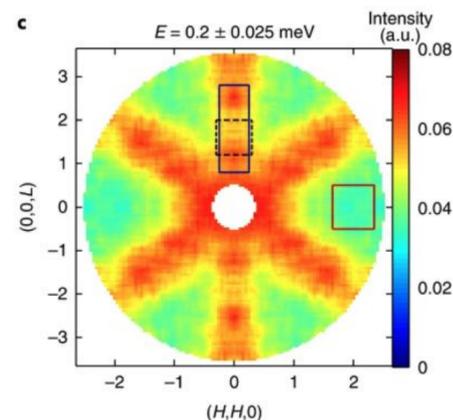
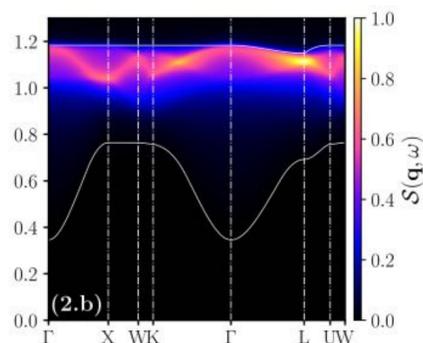
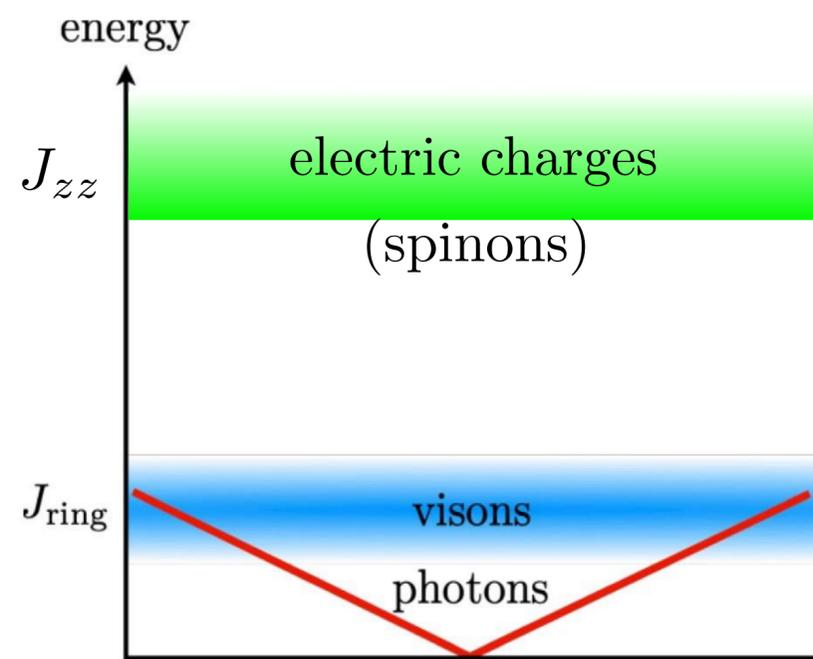
(Dated: January 16, 2023)

# Emergent QED in Quantum Spin Ice

The spectrum can be measured by neutron scattering experiments.

Effective model (roughly speaking)

$$H = J_{zz}|\phi|^2 + J_{\pm}\phi_i e^{iA_{ij}}\phi_j^{\dagger} + \text{h.c.}$$



exp

theory

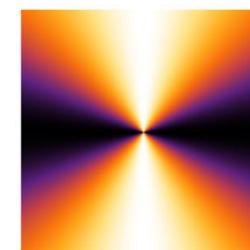
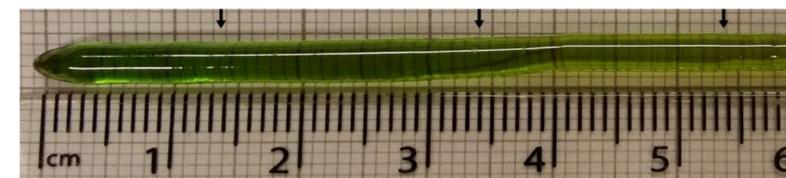
$$\langle E_x(\mathbf{q}) E_x(-\mathbf{q}) \rangle \sim q_y^2 / q^2$$

Praseodymium-Hafnium

**Experimental signatures of emergent quantum electrodynamics in  $\text{Pr}_2\text{Hf}_2\text{O}_7$**

[Romain Sibille](#) , [Nicolas Gauthier](#), [Han Yan](#), [Monica Ciomaga Hatnean](#), [Jacques Ollivier](#), [Barry Winn](#), [Uwe Filges](#), [Geetha Balakrishnan](#), [Michel Kenzelmann](#), [Nic Shannon](#) & [Tom Fennell](#)

Nature Physics 14, 711–715

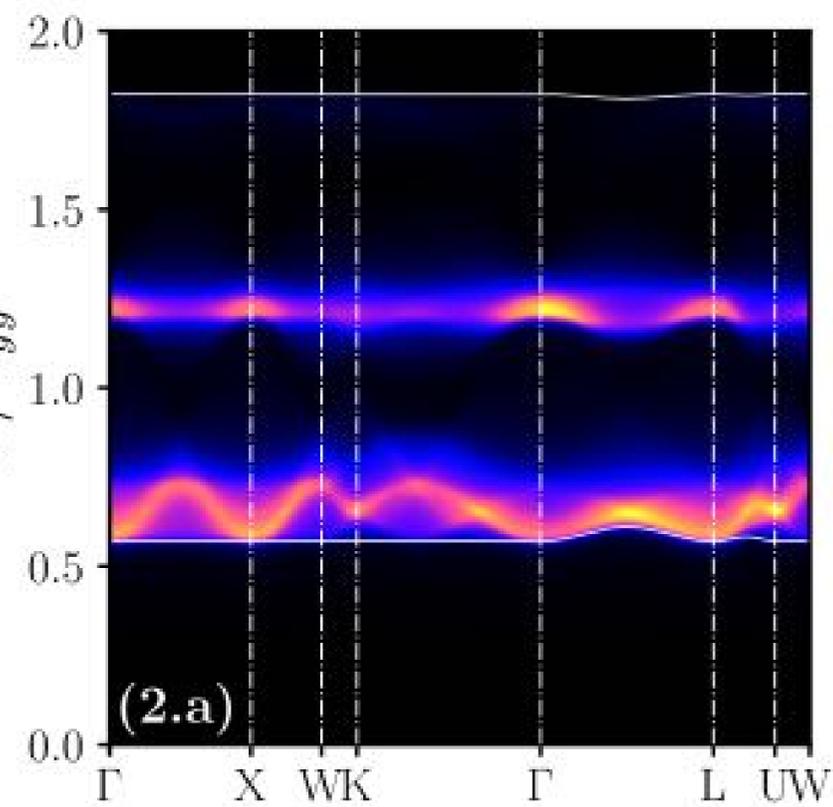
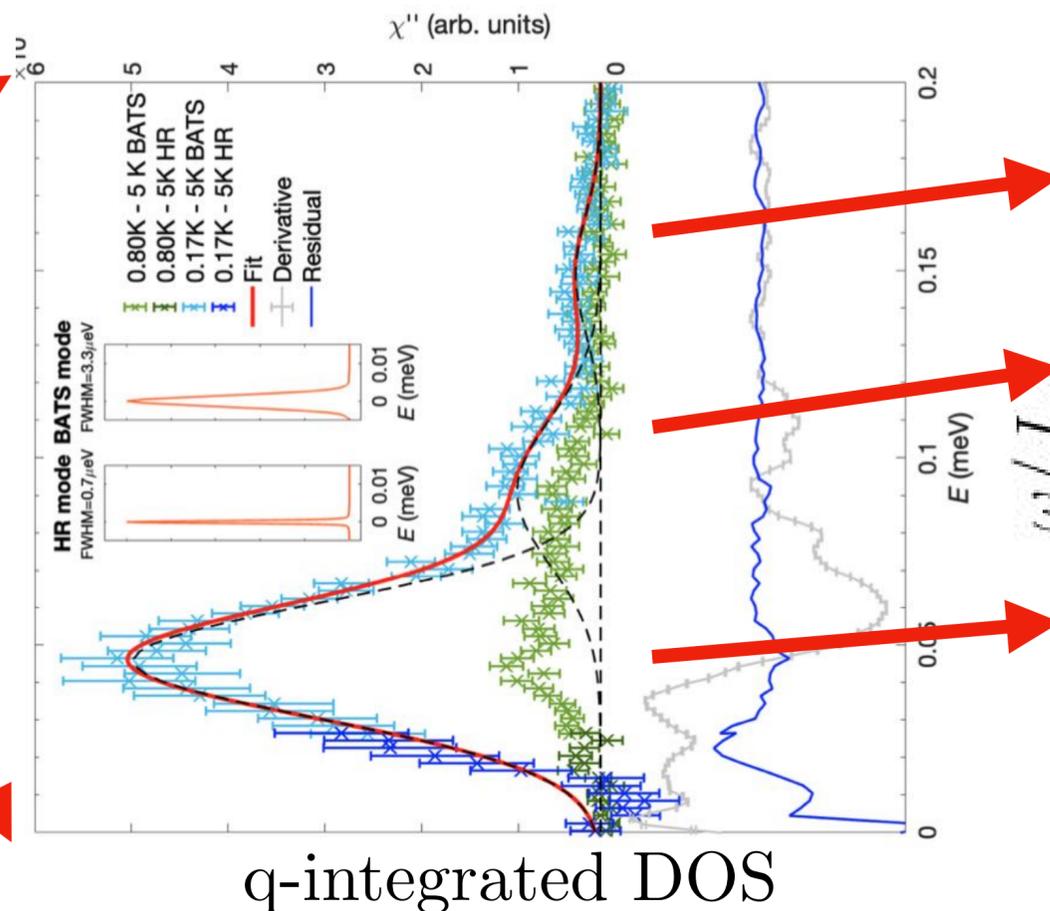
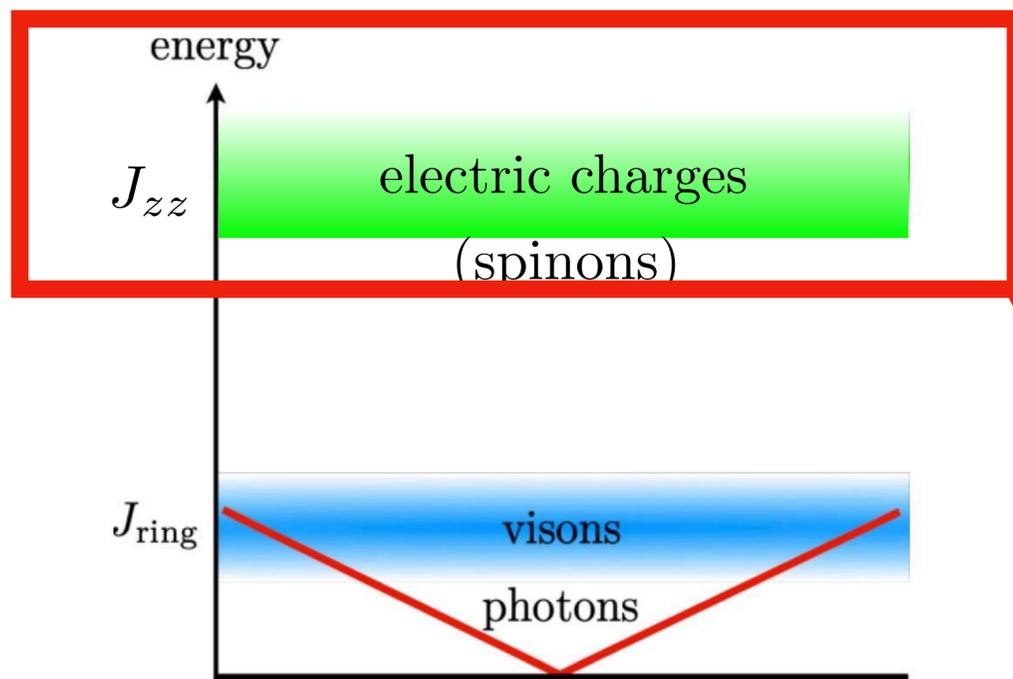


# Emergent QED in Quantum Spin Ice

The spectrum can be measured by neutron scattering experiments.  $\text{Ce}_2\text{Sn}_2\text{O}_7$  Cerium Stannum

Effective model (roughly speaking)

$$H = J_{zz}|\phi|^2 + J_{\pm}\phi_i e^{iA_{ij}}\phi_j^{\dagger} + \text{h.c.}$$



$$H \sim + \cos(b)$$

background  $b = \text{Pi}$

arXiv:2304.05452, to appear on Nat. Phys.

V Porée, **HY**, ..., Romain Sibille

# Emergent QED in Quantum Spin Ice

As a lattice QED, it is interesting for probing QED physics not accessible from fundamental particles

superluminal Cherenkov radiation effect:  
charges propagating faster than light!

large fine-structure constant

## Spectroscopy of Spinons in Coulomb Quantum Spin Liquids

Siddhardh C. Morampudi<sup>1</sup>, Frank Wilczek<sup>2,3,4,5,6</sup> and Chris R. Laumann<sup>1</sup>

<sup>1</sup>Department of Physics, Boston University, Boston, Massachusetts 02215, USA

<sup>2</sup>Center for Theoretical Physics, MIT, Cambridge Massachusetts 02139, USA

<sup>3</sup>T. D. Lee Institute, Shanghai 200240, China

<sup>4</sup>Wilczek Quantum Center, Department of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, China

<sup>5</sup>Department of Physics, Stockholm University, Stockholm 114 19, Sweden

<sup>6</sup>Department of Physics and Origins Project, Arizona State University, Tempe Arizona 25287, USA

## Emergent Fine Structure Constant of Quantum Spin Ice Is Large

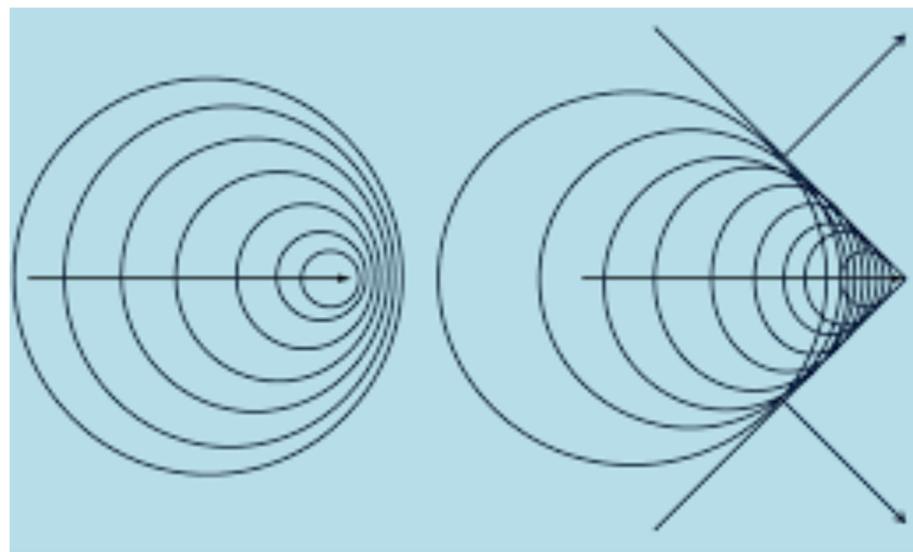
Salvatore D. Pace<sup>1,2</sup>, Siddhardh C. Morampudi<sup>3</sup>, Roderich Moessner<sup>4</sup> and Chris R. Laumann<sup>2</sup>

<sup>1</sup>TCM Group, Cavendish Laboratory, University of Cambridge, Cambridge CB3 0HE, United Kingdom

<sup>2</sup>Department of Physics, Boston University, Boston, Massachusetts 02215, USA

<sup>3</sup>Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

<sup>4</sup>Max-Planck-Institut für Physik komplexer Systeme, 01187 Dresden, Germany



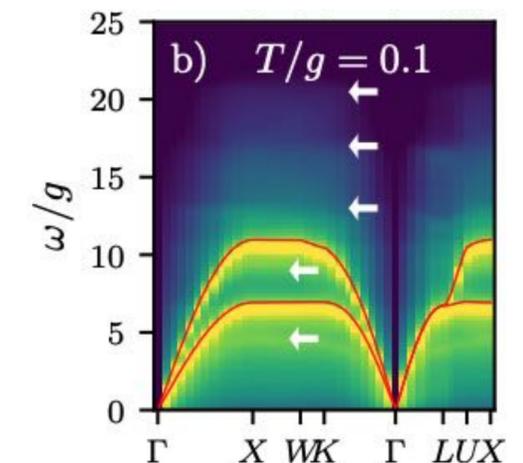
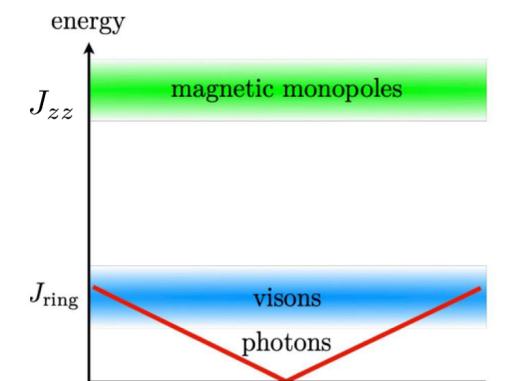
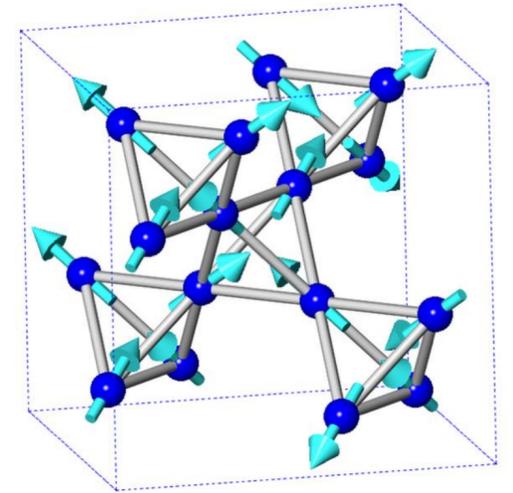
	Candidate QSI material	Vacuum QED
$\alpha$	1/10	1/137
$c$	1 ms <sup>-1</sup>	3.0 × 10 <sup>8</sup> ms <sup>-1</sup>
$e$	10 <sup>-4</sup> √eV nm	1.2 √eV nm
$m$	10 <sup>-3</sup> √eV nm	82.2 √eV nm

# Outline

1. Classical spin ice as electrostatics

2. Quantum spin ice as emergent Quantum Electrodynamics (eQED)

3. Controlling eQED dynamics in dipolar-octupolar quantum spin ice



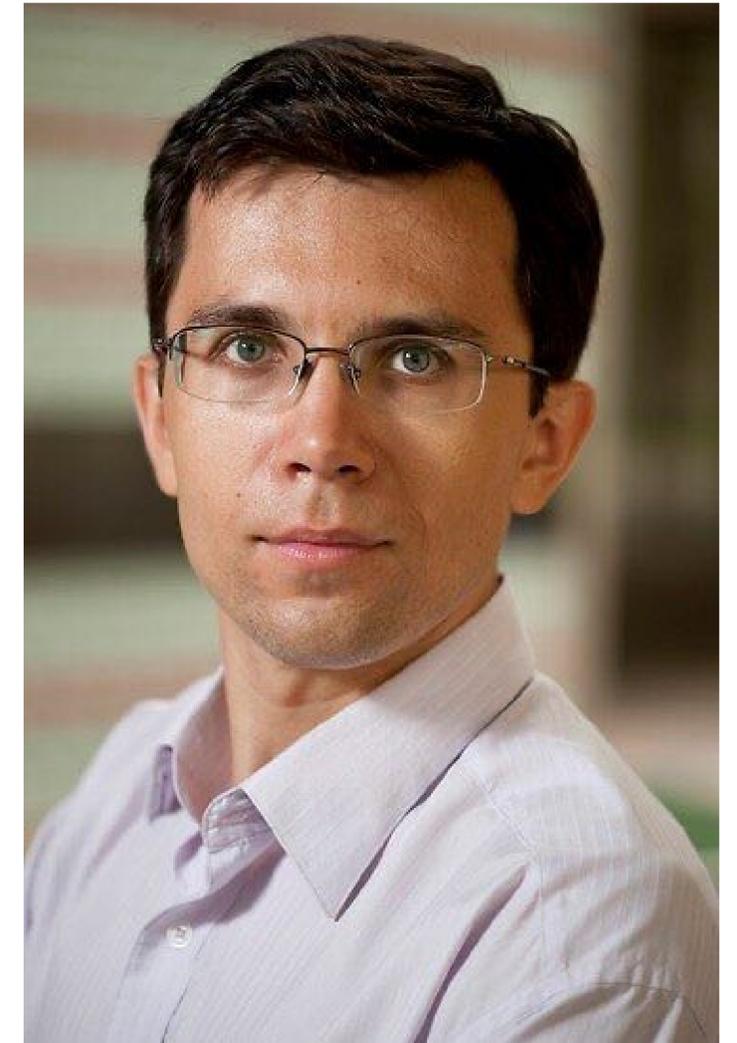
# Control the dynamics of eQED?



Alaric Sanders  
Cambridge



Claudio Castelnovo  
Cambridge



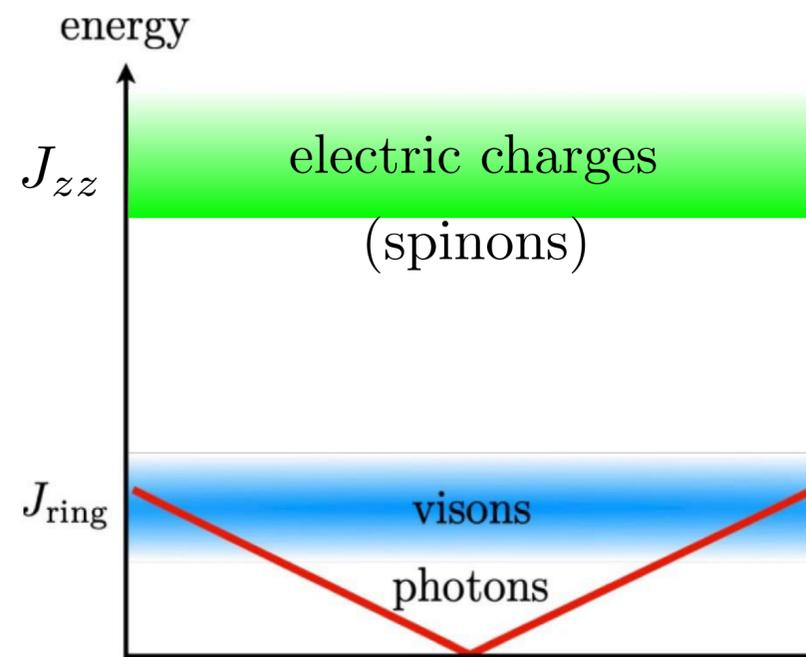
Andriy H. Nevidomskyy  
Rice

arXiv:2312.11641, *Experimentally tunable QED in dipolar-octupolar quantum spin ice*

# Control the dynamics of eQED?

Effective model (roughly speaking)

$$H = J_{zz}|\phi|^2 + J_{\pm}\phi_i e^{iA_{ij}}\phi_j^{\dagger} + \text{h.c.}$$



One material, one version of eQED

Generally very difficult to have accurate control of its dynamics!

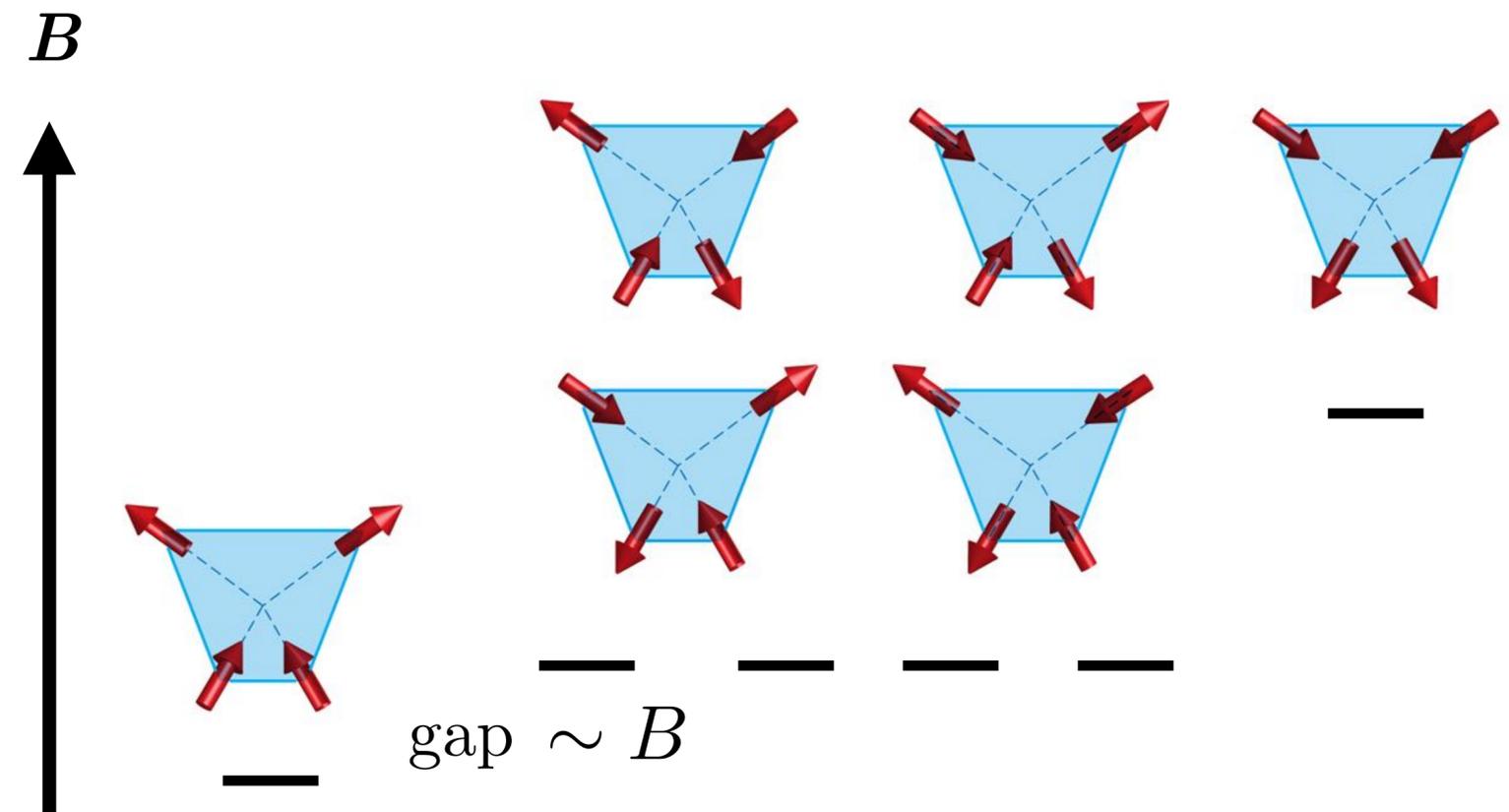
# Control the dynamics of eQED?

Conventional spin 1/2s on pyrochlore lattice:

$$H = \sum_{\langle ij \rangle} \{ J_{zz} \mathbf{S}_i^z \mathbf{S}_j^z - J_{\pm} (\mathbf{S}_i^+ \mathbf{S}_j^- + \mathbf{S}_i^- \mathbf{S}_j^+) \} - g \sum_i \mathbf{B} \cdot \mathbf{S}_i + (\text{other spin-spin terms})$$



classical limit  $H \sim \sum J_z S_i^z S_j^z$



apply magnetic field

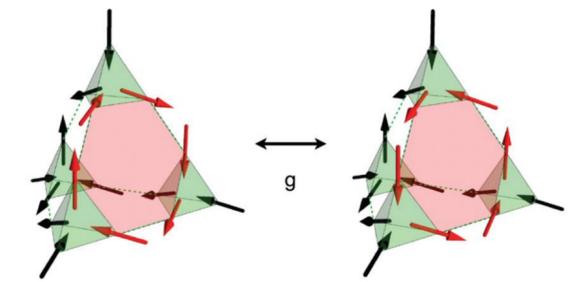
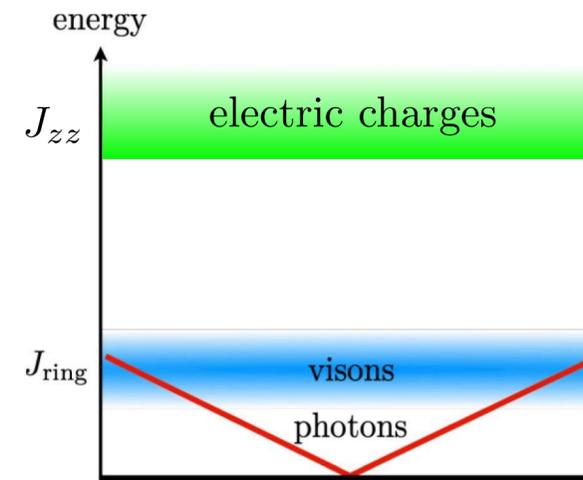
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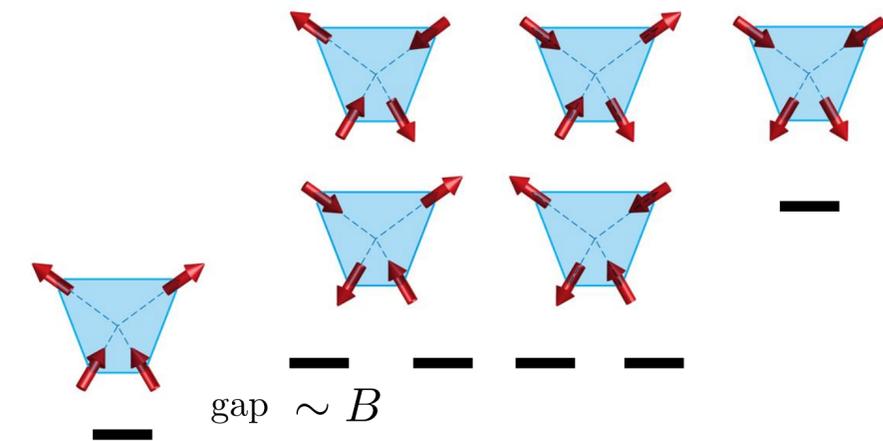


classical limit  $H \sim \sum J_z S_i^z S_j^z$



$$J_{\text{ring}} = 3J_{\pm}^3 / (2J_{zz}^2)$$

drives system to QSI



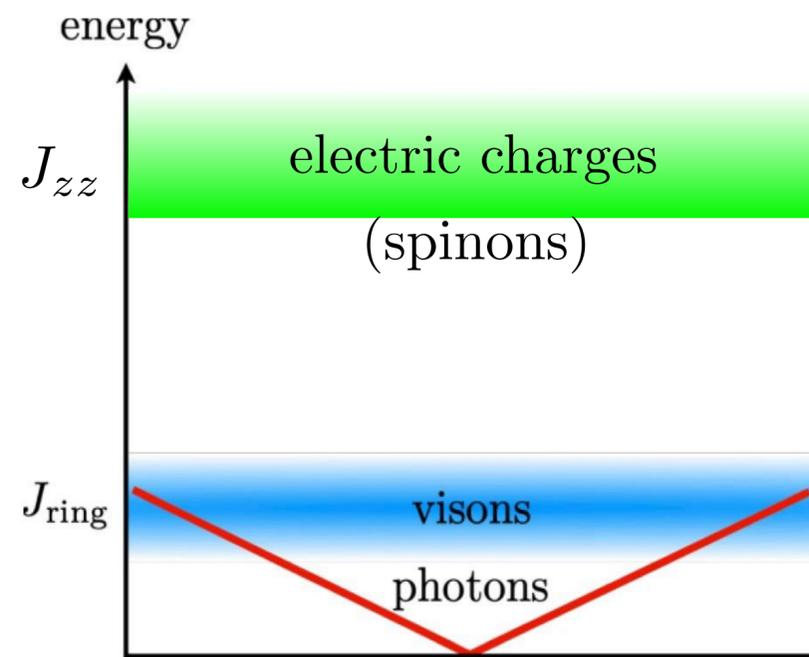
apply magnetic field

drives system to magnetic order

# Control the dynamics of eQED?

Effective model (roughly speaking)

$$H = J_{zz}|\phi|^2 + J_{\pm}\phi_i e^{iA_{ij}}\phi_j^{\dagger} + \text{h.c.}$$



Given a material, the interactions are fixed:  
one material, one eQED

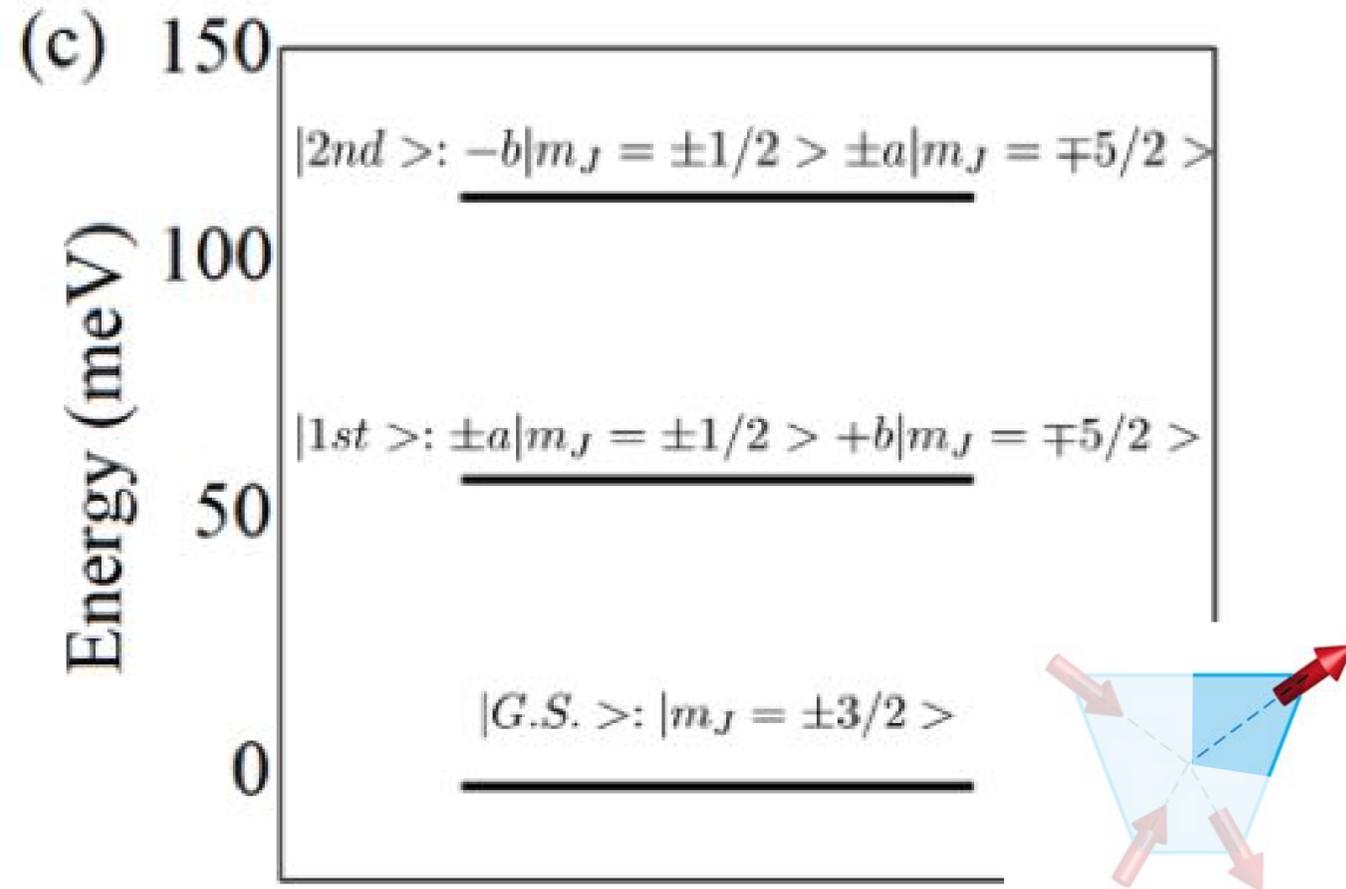
Our work: can we also control the dynamics of the eQED in QSI, so we can probe different regimes of QED in one compound?

It is possible in a class of materials called  
Dipolar-Octupolar QSI!

arXiv:2312.11641 A Sanders, HY, C Castelnovo, AH Nevidomskyy

# Dipolar-Octuplar QSI

electron orbitals on each single site



Quantum Spin Ices and Topological Phases from Dipolar-Octupolar Doublets on the Pyrochlore Lattice

Yi-Ping Huang, Gang Chen, and Michael Hermele  
Phys. Rev. Lett. **112**, 167203 – Published 25 April 2014

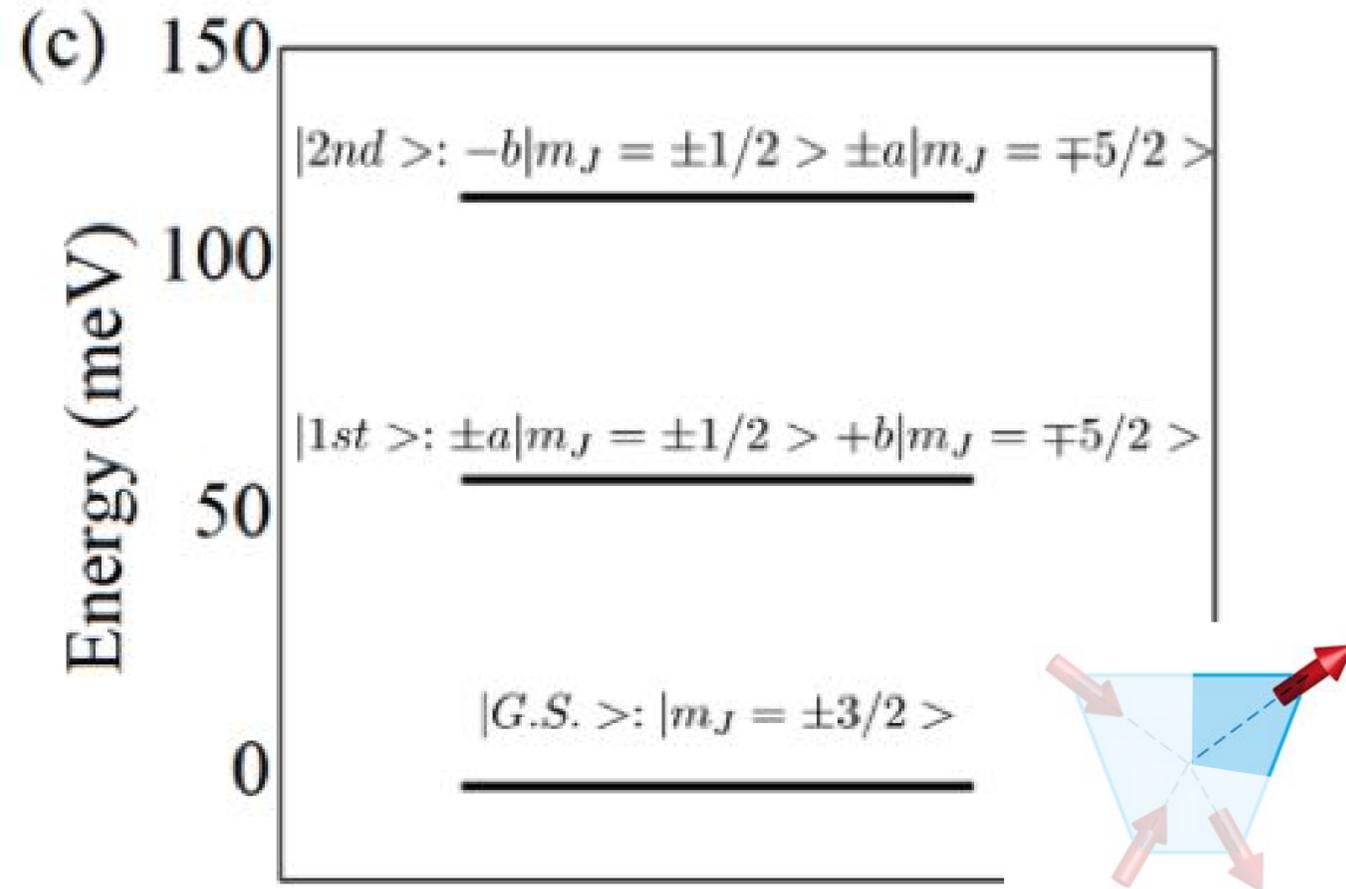
sketch of the idea: spin comes from the d electron orbitals.

lowest two states are two of the  $j_z = 3/2$  states instead of  $1/2$

the pseudo spin- $1/2$  operators act differently under symmetry

# Dipolar-Octuplar QSI

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$S^y$  has the symmetry of an octupole

$S^y$  does not couple to applied magnetic field

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# Dipolar-Octuplar QSI

$S^y$  has the symmetry of an octupole

$S^y$  does not couple to applied magnetic field

DO-pyrochlore lattice spin Hamiltonian

$$\mathcal{H} = \sum_{\langle ij \rangle} \left[ \sum_{\alpha=x,y,z} J_{\alpha} S_i^{\alpha} S_j^{\alpha} + J_{xz} (S_i^x S_j^z + S_i^y S_j^x) \right] - \sum_i (S_i^z g_z + S_i^x g_x) \hat{z}_i \cdot \mathbf{B}$$

2 charge states -----

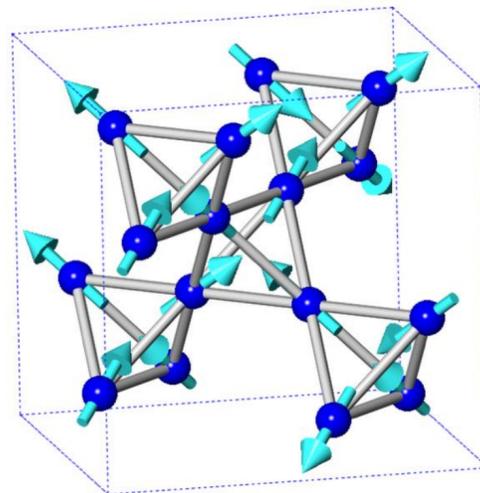
1 charge states -----

ground states -----

classical limit  $H \sim \sum J_y S_i^y S_j^y$

conventional pyrochlore lattice spin Hamiltonian

$$\mathcal{H} = \sum_{\langle ij \rangle} \left[ \sum_{\alpha=x,y,z} J_{\alpha} S_i^{\alpha} S_j^{\alpha} + J_{xz} (S_i^x S_j^z + S_i^y S_j^x) \right] - g \sum_i \mathbf{B} \cdot \mathbf{S}_i + (\text{other spin-spin terms})$$



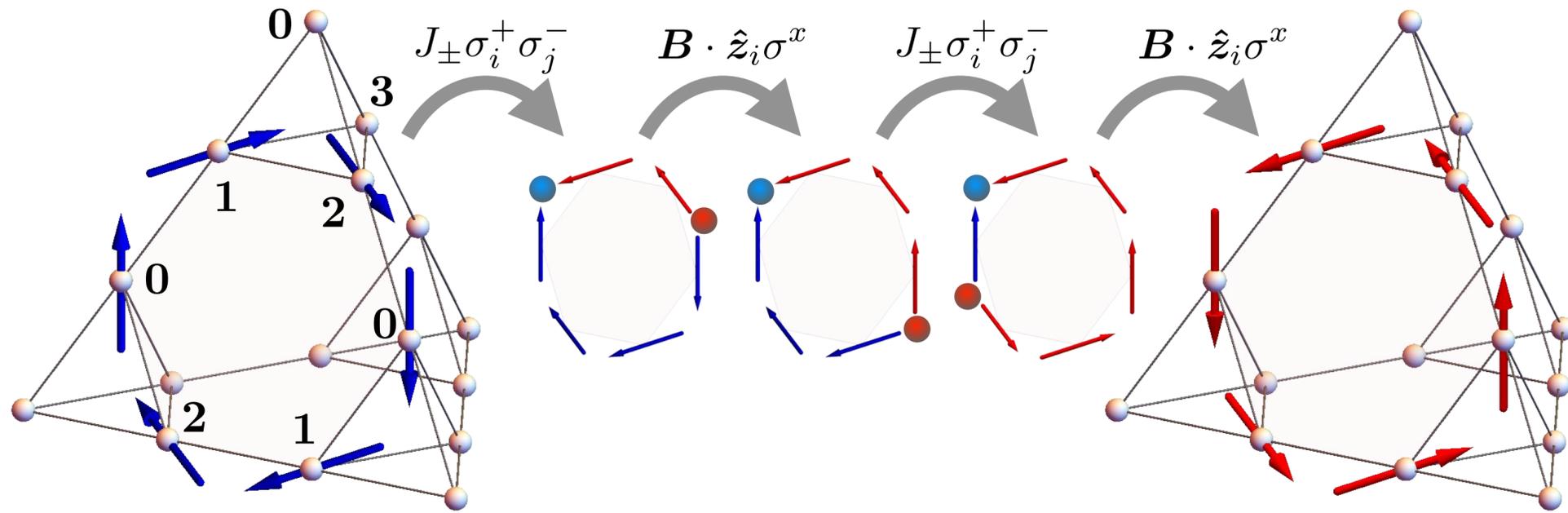
# Emergent QED in DO-QSI

DO-pyrochlore lattice spin Hamiltonian

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$$\mathcal{H}_I = \sum_{\langle ij \rangle} J_y \sigma_i^z \sigma_j^z, \quad (2)$$

$$\mathcal{H}' = \sum_{\langle ij \rangle} \frac{J_{\pm}}{2} (\sigma_i^+ \sigma_j^- + \sigma_i^- \sigma_j^+) - \sum_i \frac{\hat{z}_i \cdot \mathbf{B}}{2} (\sigma_i^+ + \sigma_i^-). \quad (3)$$



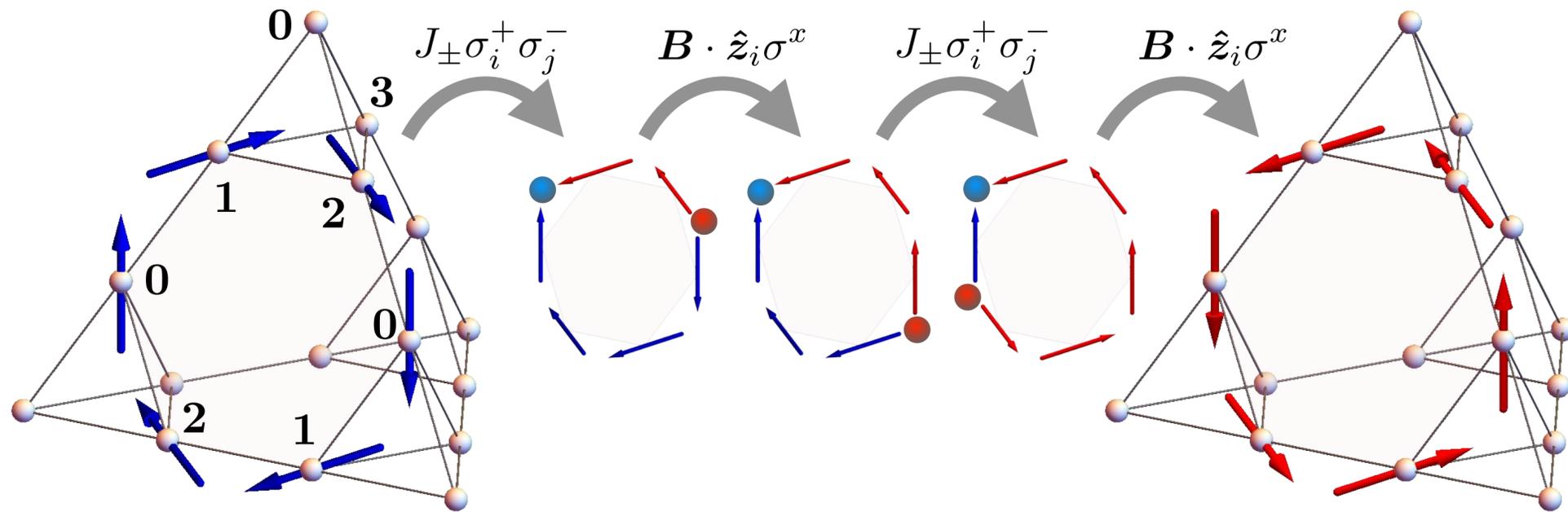
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$$\begin{aligned} \mathcal{H}_{\text{eff}} &= (1 - \mathcal{P}) \left[ -\mathcal{H}' \frac{\mathcal{P}}{\mathcal{H}_I} \mathcal{H}' + \mathcal{H}' \frac{\mathcal{P}}{\mathcal{H}_I} \mathcal{H}' \frac{\mathcal{P}}{\mathcal{H}_I} \mathcal{H}' \right. \\ &\quad \left. + \dots \right] (1 - \mathcal{P}), \\ &= \sum_{\mathbf{R} \in \{\text{all } \square\}} g(\mathbf{R}) \cos(b) \end{aligned}$$

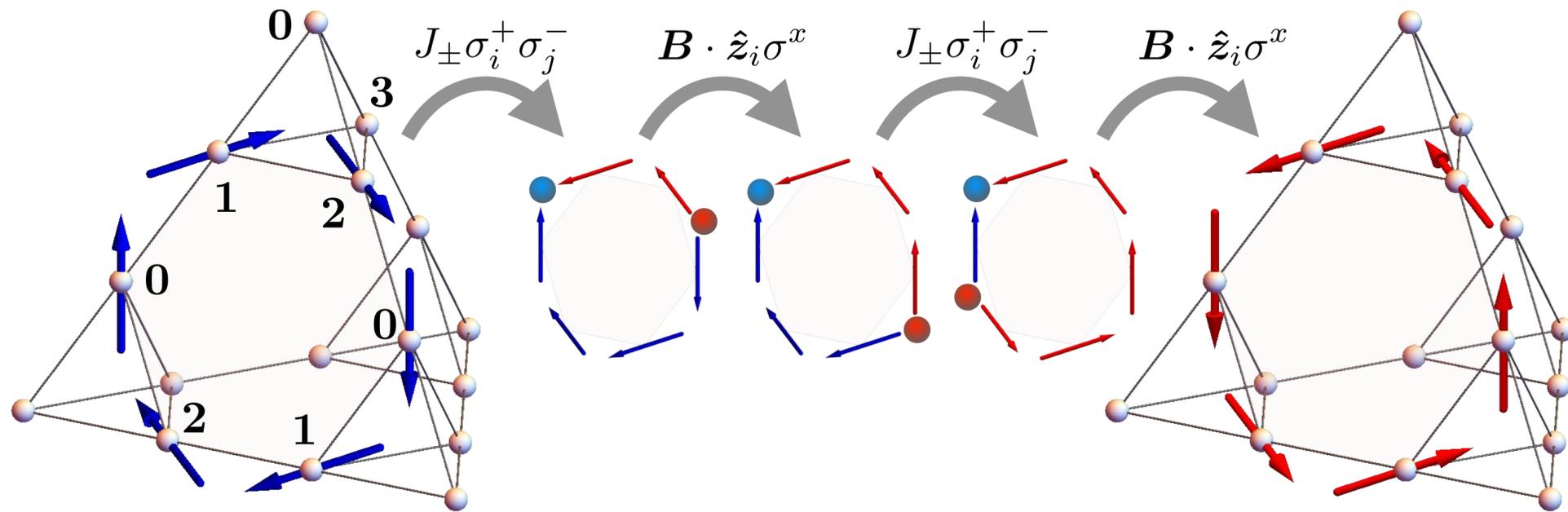
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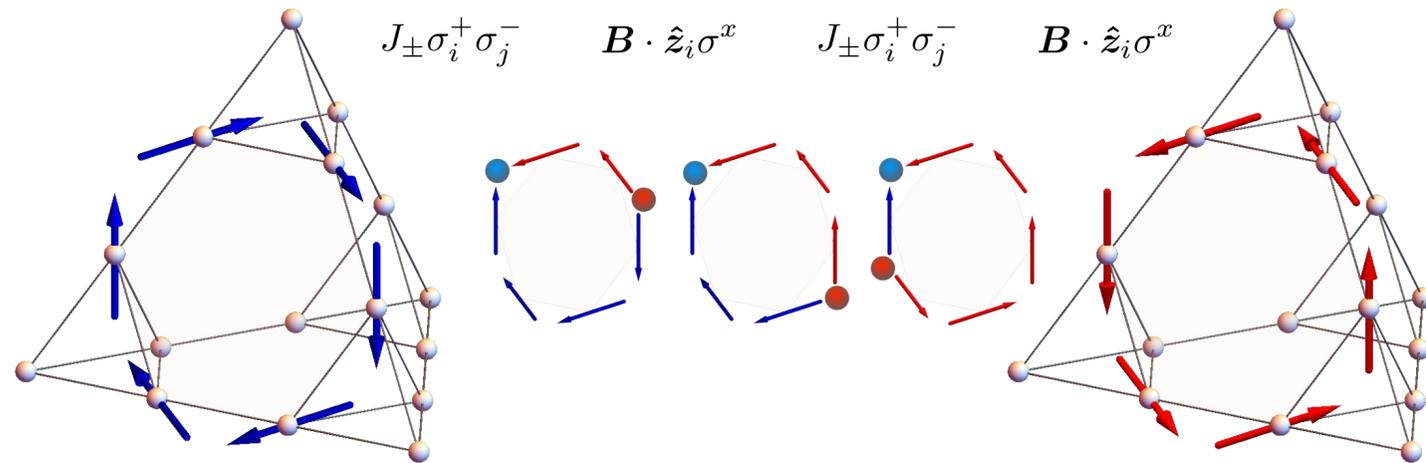
$$\mathcal{H}_{\text{eff}} = (1 - \mathcal{P}) \left[ -\mathcal{H}' \frac{\mathcal{P}}{\mathcal{H}_I} \mathcal{H}' + \mathcal{H}' \frac{\mathcal{P}}{\mathcal{H}_I} \mathcal{H}' \frac{\mathcal{P}}{\mathcal{H}_I} \mathcal{H}' + \dots \right] (1 - \mathcal{P}),$$

$$= \sum_{\mathbf{R} \in \{\text{all } \square\}} g(\mathbf{R}) \cos(b)$$

$$g_p = \frac{3J_{\pm}^3}{2J_y^2} + \frac{5\sqrt{3}J_{\pm}^2}{12J_y^3} (\hat{z}_p \cdot \mathbf{B})^2.$$

p=0,1,2,3 for 4 types of rings

# Emergent QED in DO-QSI

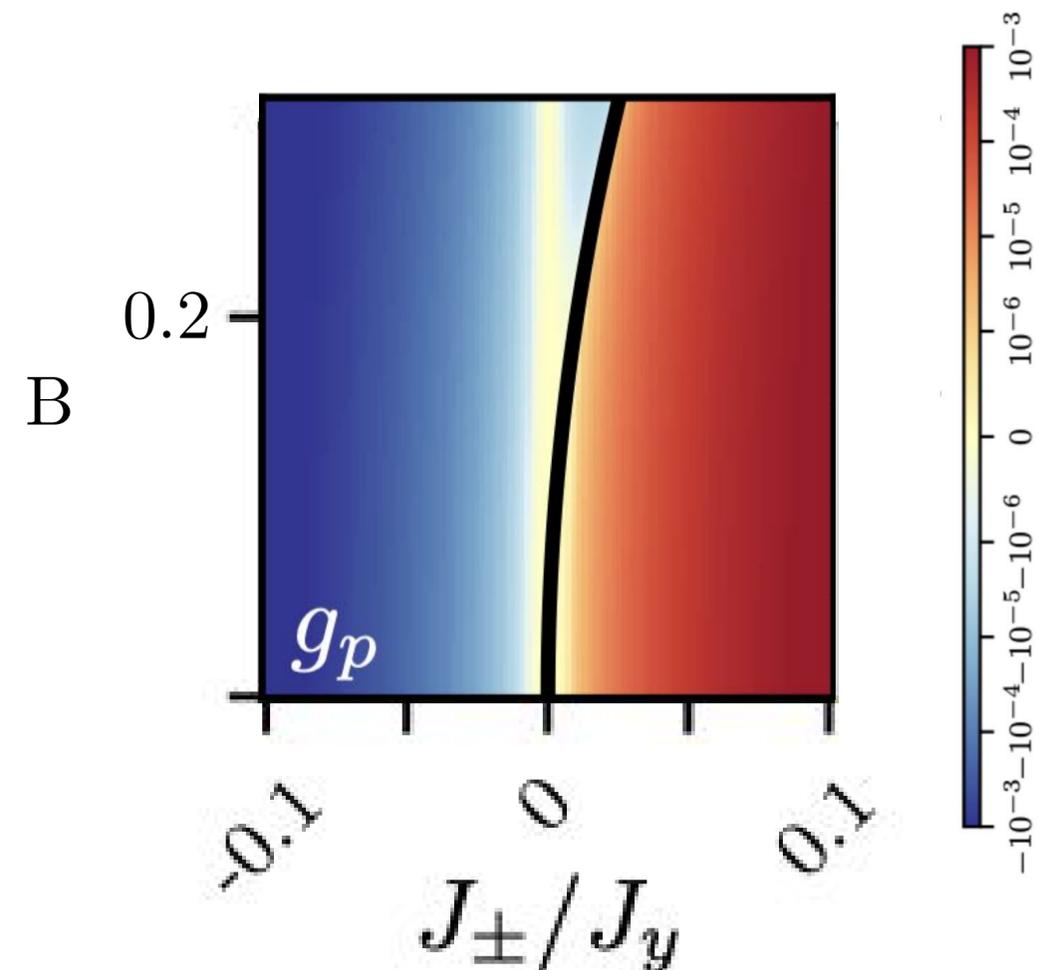


$$\mathbf{B} \parallel [100] : g_p = \frac{3J_{\pm}^3}{2J_y^2} + \frac{5B^2 J_{\pm}^2}{12J_y^3}.$$

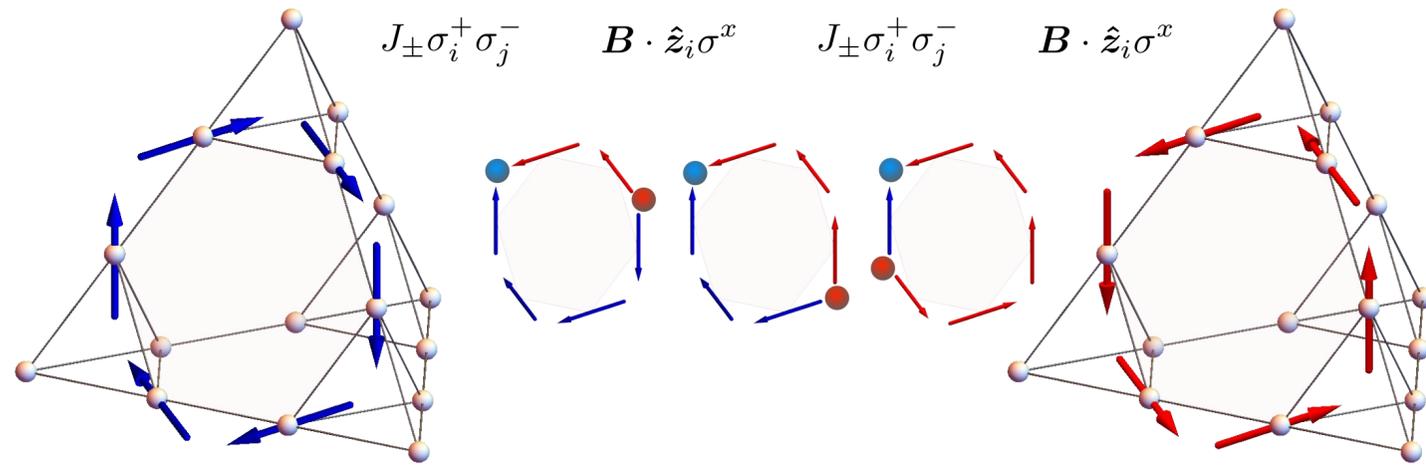
four rings have the same dynamics

$$\mathcal{H}_{\text{eff}} = \sum_{R \in \{\text{all } \bigcirc\}} g(\mathbf{R}) \cos(b)$$

$$g_p = \frac{3J_{\pm}^3}{2J_y^2} + \frac{5\sqrt{3}J_{\pm}^2}{12J_y^3} (\hat{z}_p \cdot \mathbf{B})^2.$$



# Emergent QED in DO-QSI



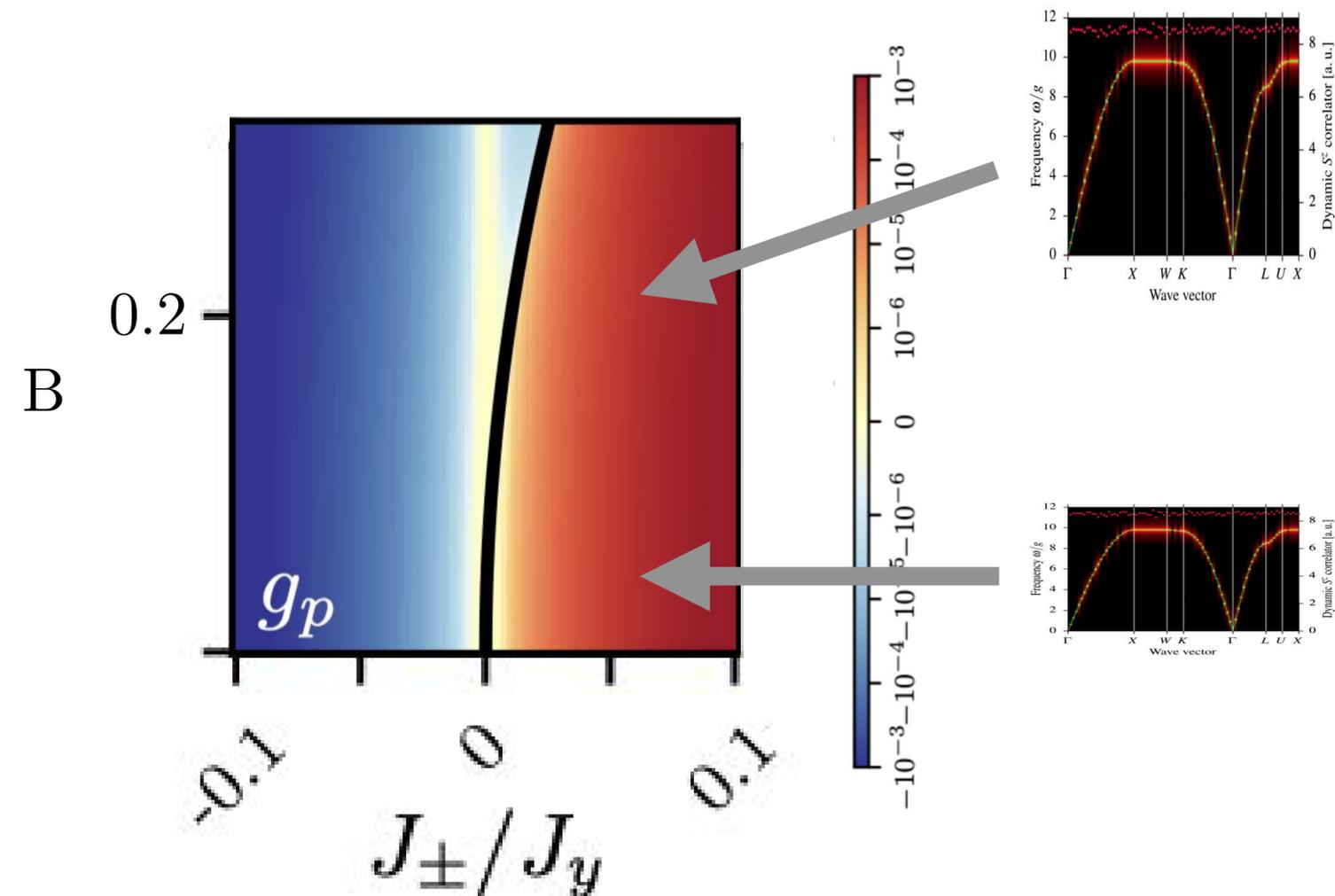
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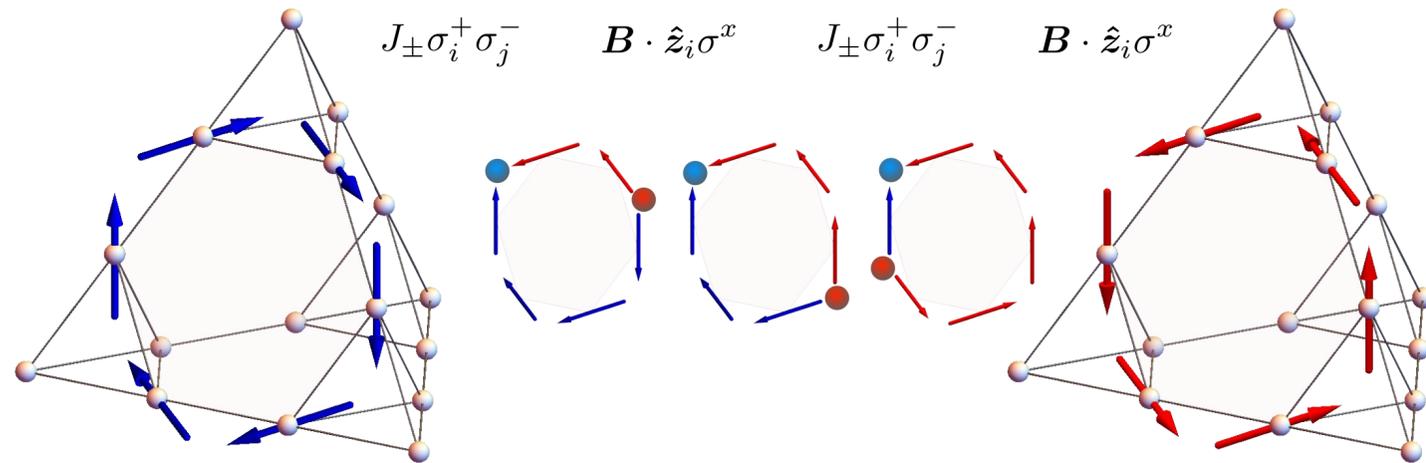
$$\mathcal{H}_{\text{eff}} = \sum_{\mathbf{R} \in \{\text{all O}\}} g(\mathbf{R}) \cos(b)$$

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1. We can tune the speed of light!



# Emergent QED in DO-QSI



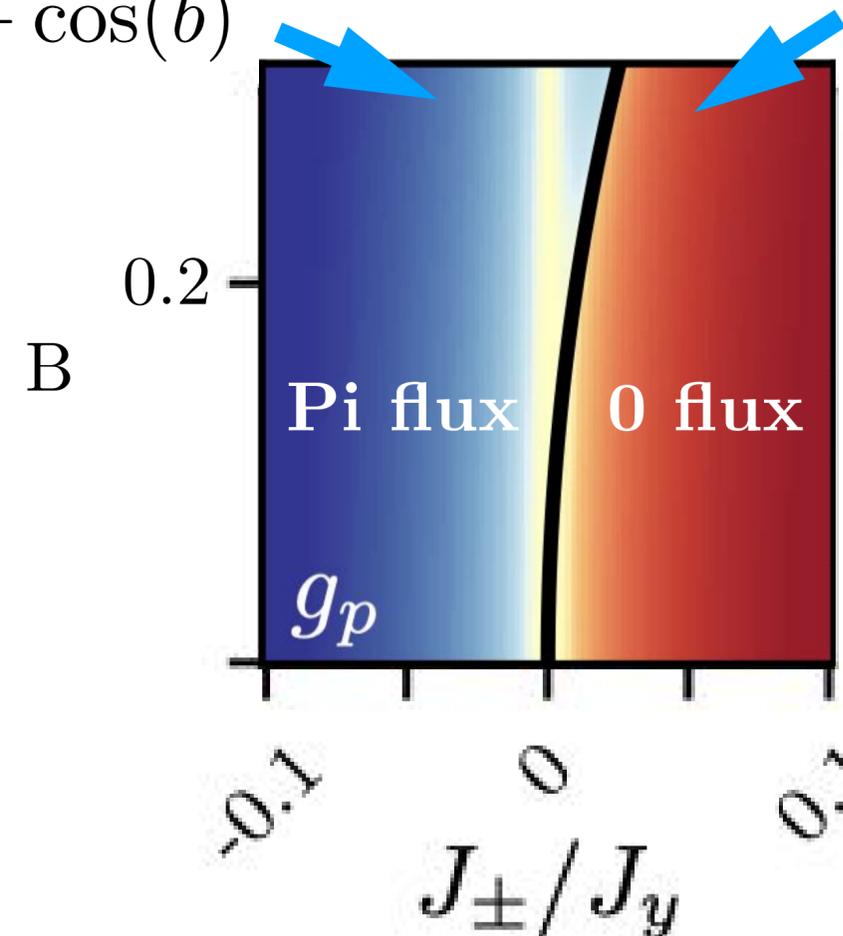
$$\mathbf{B} \parallel [100] : g_p = \frac{3J_{\pm}^3}{2J_y^2} + \frac{5B^2 J_{\pm}^2}{12J_y^3}.$$

four rings have the same dynamics

$$H \sim -\cos(b) \qquad H \sim +\cos(b)$$

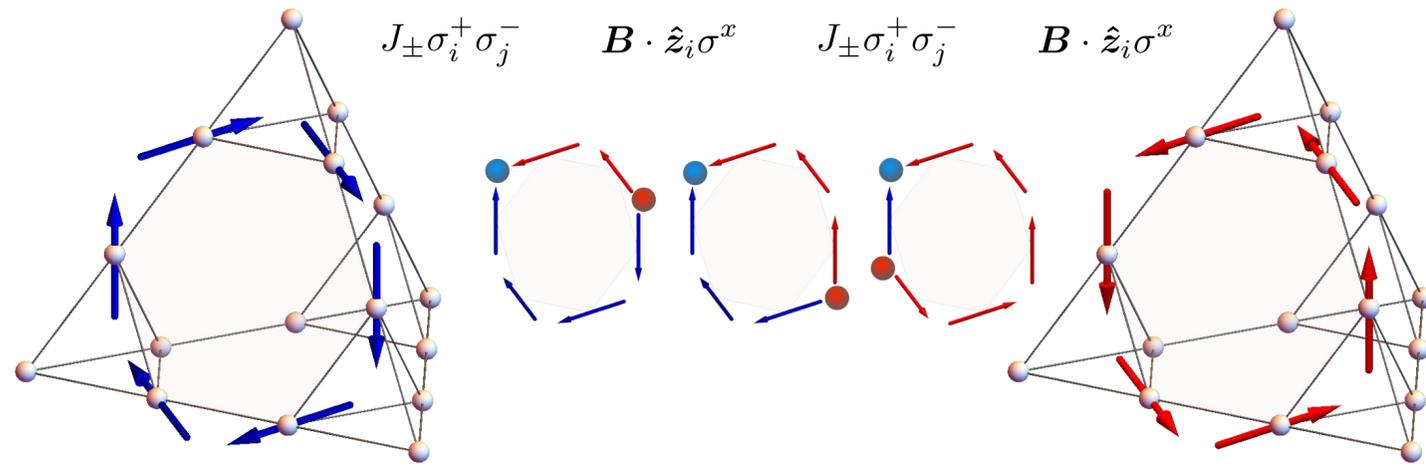
$$\mathcal{H}_{\text{eff}} = \sum_{\mathbf{R} \in \{\text{all O}\}} g(\mathbf{R}) \cos(b)$$

$$g_p = \frac{3J_{\pm}^3}{2J_y^2} + \frac{5\sqrt{3}J_{\pm}^2}{12J_y^3} (\hat{z}_p \cdot \mathbf{B})^2.$$



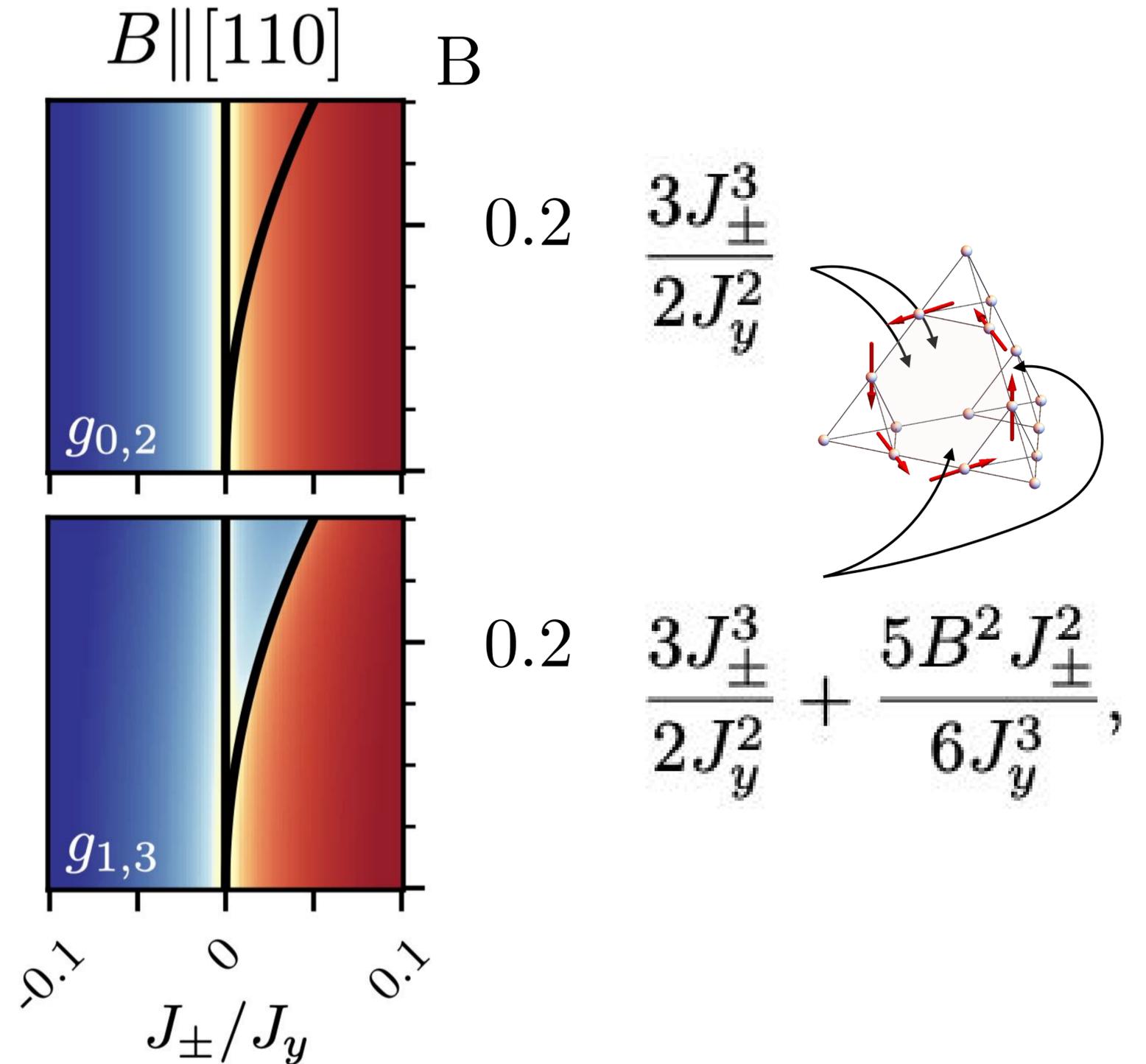
1. We can tune the speed of light.
2. We can also induce quantum phase transition.

# Emergent QED in DO-QSI

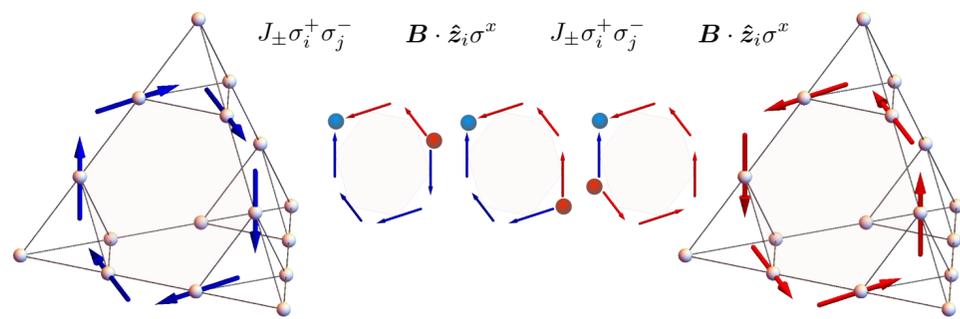


$$\mathcal{H}_{\text{eff}} = \sum_{\mathbf{R} \in \{\text{all O}\}} g(\mathbf{R}) \cos(b)$$

$$g_p = \frac{3J_{\pm}^3}{2J_y^2} + \frac{5\sqrt{3}J_{\pm}^2}{12J_y^3} (\hat{\mathbf{z}}_p \cdot \mathbf{B})^2.$$



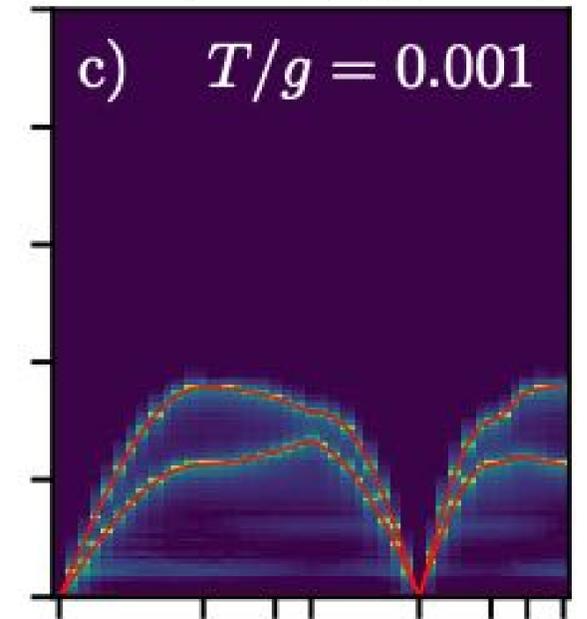
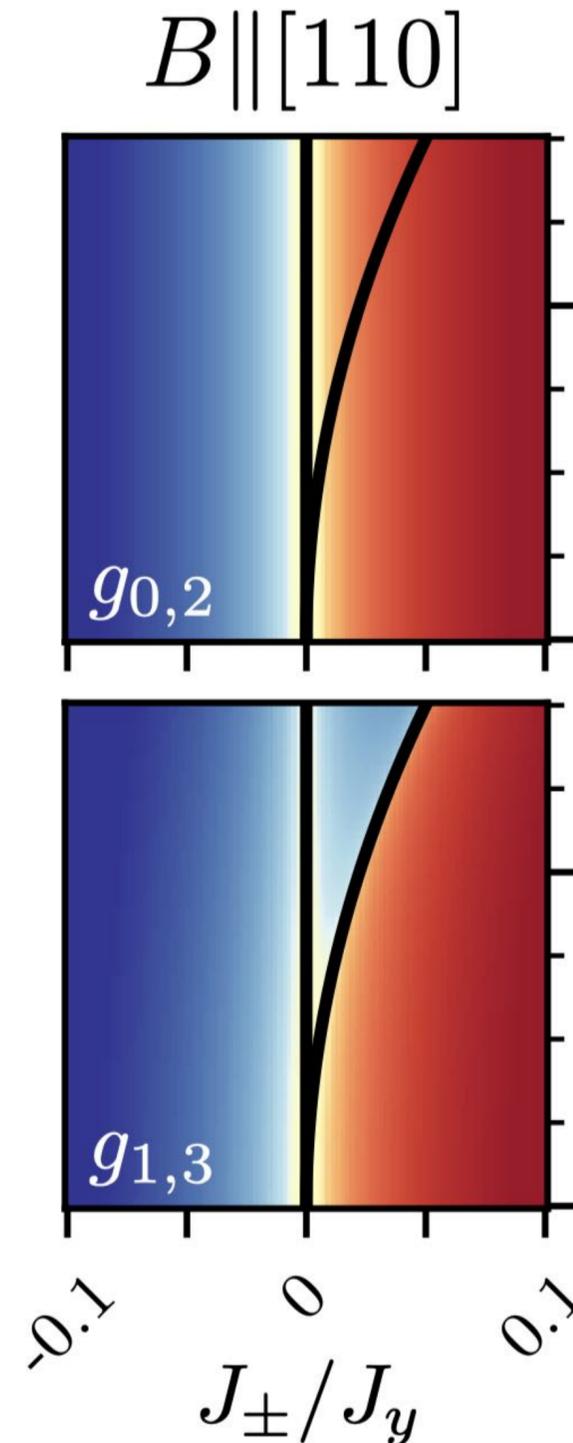
# Emergent QED in DO-QSI



$$\mathcal{H}_{\text{eff}} = \sum_{\mathbf{R} \in \{\text{all } \odot\}} g(\mathbf{R}) \cos(b)$$

$$g_p = \frac{3J_{\pm}^3}{2J_y^2} + \frac{5\sqrt{3}J_{\pm}^2}{12J_y^3} (\hat{\mathbf{z}}_p \cdot \mathbf{B})^2.$$

3. We can realize new emergent birefringence.



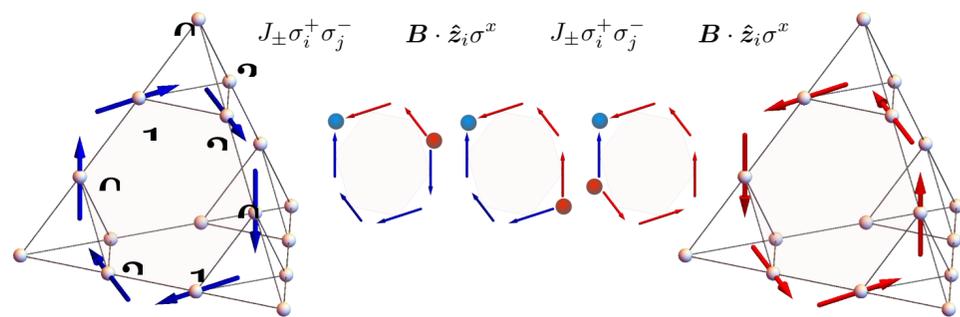
photon dispersion:

**birefringence:** two photon polarizations have different speeds of light and refraction rates.



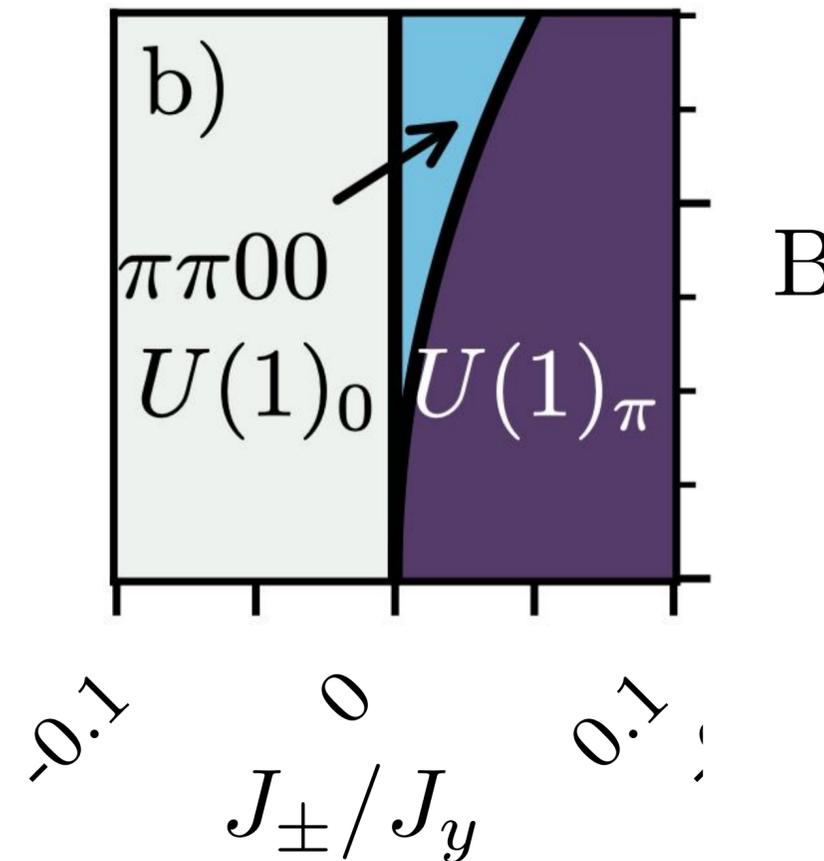
# Emergent QED in DO-QSI

$B \parallel [110]$



$$\mathcal{H}_{\text{eff}} = \sum_{\mathbf{R} \in \{\text{all } \odot\}} g(\mathbf{R}) \cos(b)$$

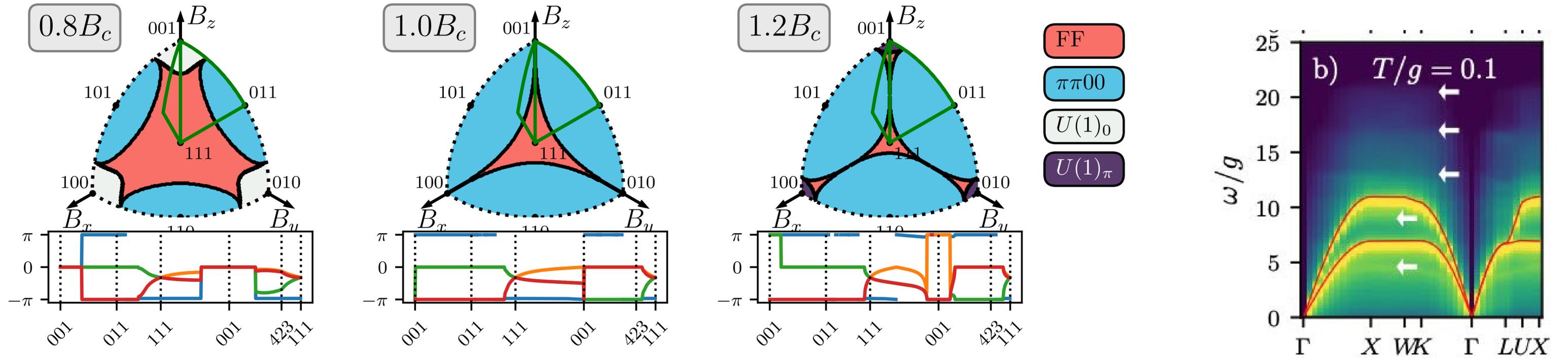
$$g_p = \frac{3J_{\pm}^3}{2J_y^2} + \frac{5\sqrt{3}J_{\pm}^2}{12J_y^3} (\hat{\mathbf{z}}_p \cdot \mathbf{B})^2.$$



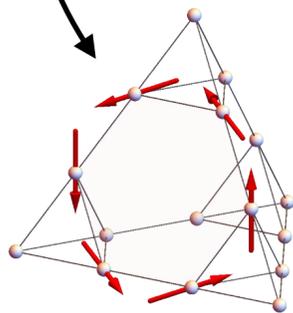
A new eQED phase of different gauge background

3. We can realize new emergent birefringence.
4. We have new quantum liquid phase(s)

# Emergent QED in DO-QSI



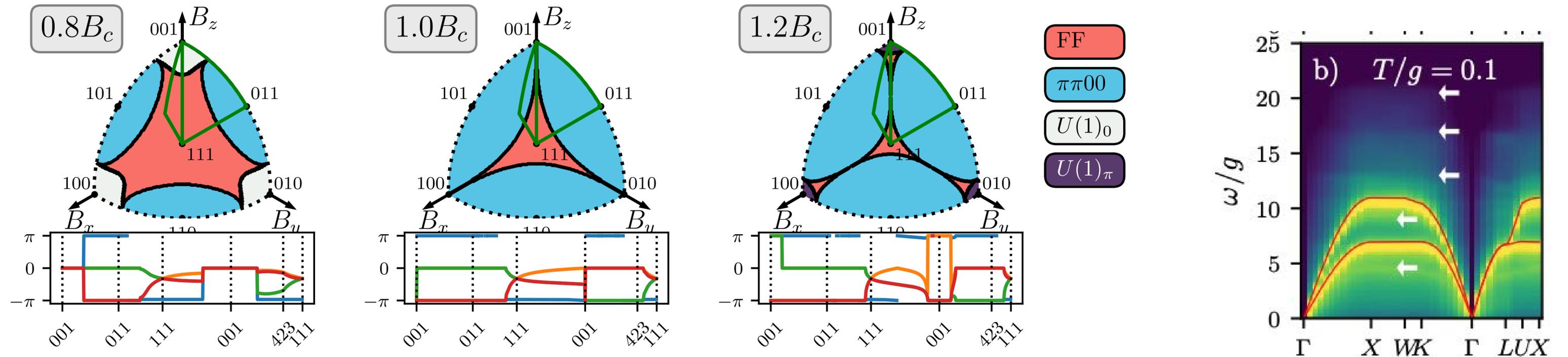
background b-flux on different hexagons



Possible three-photon couplings

C. L. Basham and P. K. Kabir  
 Phys. Rev. D **15**, 3388 – Published 1 June 1977

# Emergent QED in DO-QSI

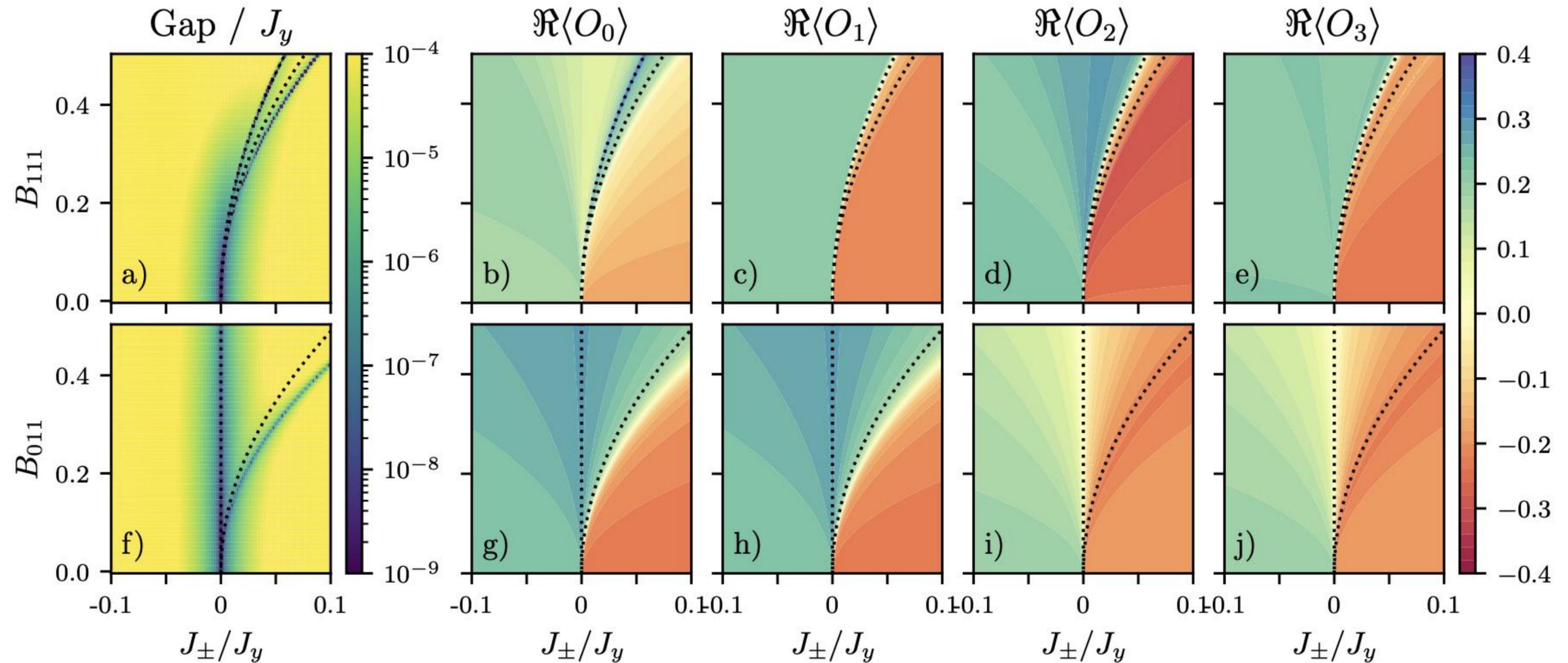


1. We can tune the speed of light.
2. We can also induce quantum phase transition.
3. We have new quantum liquid phase(s)
4. We can realize new emergent birefringence.
5. We can realize non-linear photon coupling of  $b^3$  in frustrated flux phase.

Possible three-photon couplings

C. L. Basham and P. K. Kabir  
 Phys. Rev. D **15**, 3388 – Published 1 June 1977

# Exact diagonalizations



ED Expectation values of ring-flip operators on a 72-site cluster subject to (a-e) 111 and (f-j) 011 magnetic fields.

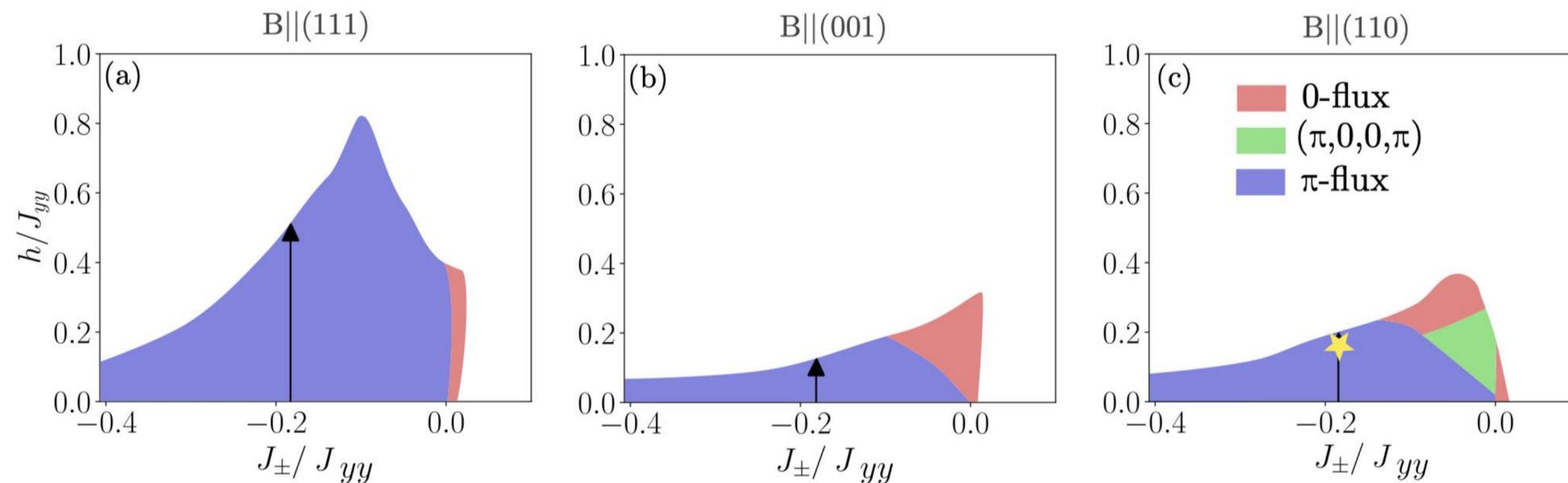
Black dotted lines indicate the semiclassical phase boundaries

# Emergent QED in DO-QSI

$$\mathcal{H} = \sum_{\langle ij \rangle} \left[ \sum_{\alpha=x,y,z} J_{\alpha} S_i^{\alpha} S_j^{\alpha} + J_{xz} (S_i^x S_j^z + S_i^y S_j^x) \right] - \sum_i (S_i^z g_z + S_i^x g_x) \hat{z}_i \cdot \mathbf{B}$$

	$J_y$	$J_x$	$J_z$	$J_{xz}$
Ce <sub>2</sub> Sn <sub>2</sub> O <sub>7</sub>	0.069	0.001	0.001	0
Ce <sub>2</sub> Hf <sub>2</sub> O <sub>7</sub>	0.044	0.011	0.016	-0.002
Ce <sub>2</sub> Zr <sub>2</sub> O <sub>7</sub>	0.062	0.063	0.011	0

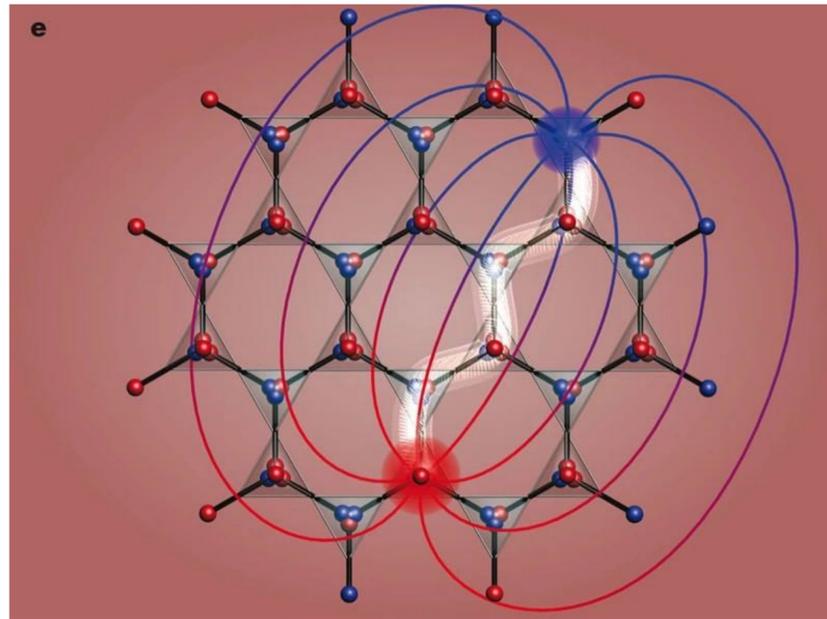
Cerium Tin/Stannum Hafnium Zirconium  
 Various works by Romain Sibille, Bruce Gaulin, Pengcheng Dai



arXiv: 2502.14067, Z Zhou, Y-B Kim

According to GMFT, such exotic phase transitions may not happen to current materials.

# Quantum Spin Ice: A universe in a grain of sand



Effective theory: **3D U(1) Quantum Electrodynamics**

1. Ground state: gapless topological quantum liquid state
2. Gapless excitations: **photons**
3. Gapped excitations: electric **charges** and magnetic **monopoles**

Our result: proposal to **experimentally tune the quantum dynamics** of lattice QED of dipolar-octupolar quantum spin ice

1. New lattice QED **phases** and **phase transitions**.
2. Photon dynamics: tune the emergent **speed of light**, **birefringence**, **non-linear photon coupling**.



## [OIST Symposium] Generalized Symmetries in Quantum Matter - Registration

Registration for the symposium (ONLY necessary if you have not previously been contacted by OIST/TSVP)

"**Generalized Symmetries in Quantum Matter**" organized by Daniel Bulmash, Abhinav Prem, Masahito Yamazaki, Han Yan. The **symposium is scheduled to take place from June 16-21** at the Okinawa Institute of Science and Technology Graduate University (OIST).  
Symposium website: <https://www.oist.jp/events/tsvp-symposium-aspects-generalized-symmetries>

This symposium is part of the **TSVP Thematic Program** "Generalized Symmetries in Quantum Matter". The program will run from May 12 to July 5, 2025.

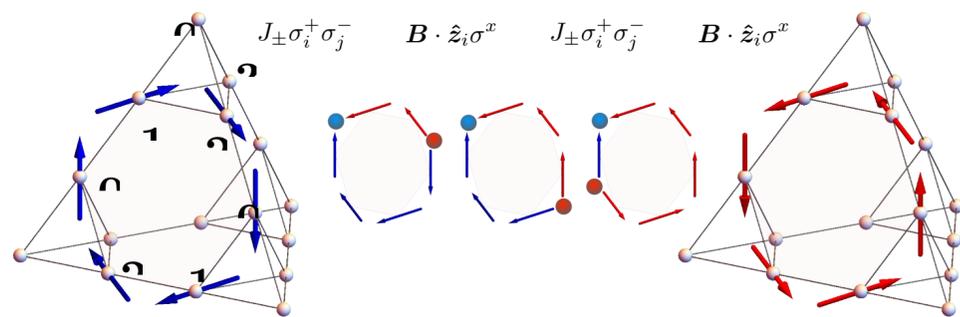
Thematic Program website: <https://www.oist.jp/visiting-program/tp25qm>

<https://groups.oist.jp/tsvp/tp25qm>

Back-up slides

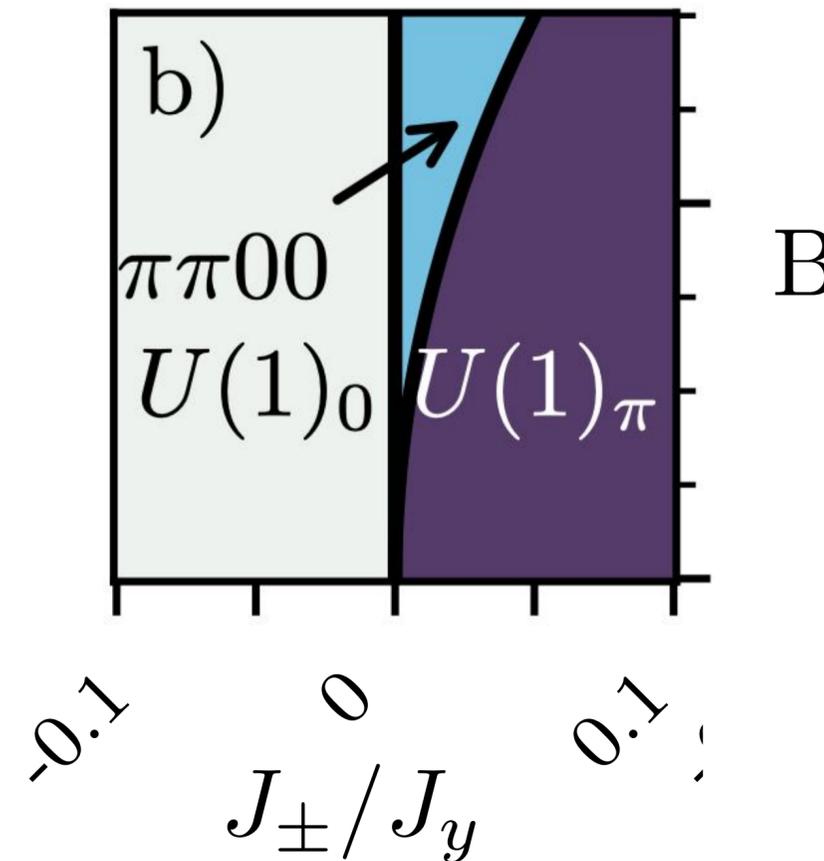
# Emergent QED in DO-QSI

$B \parallel [110]$



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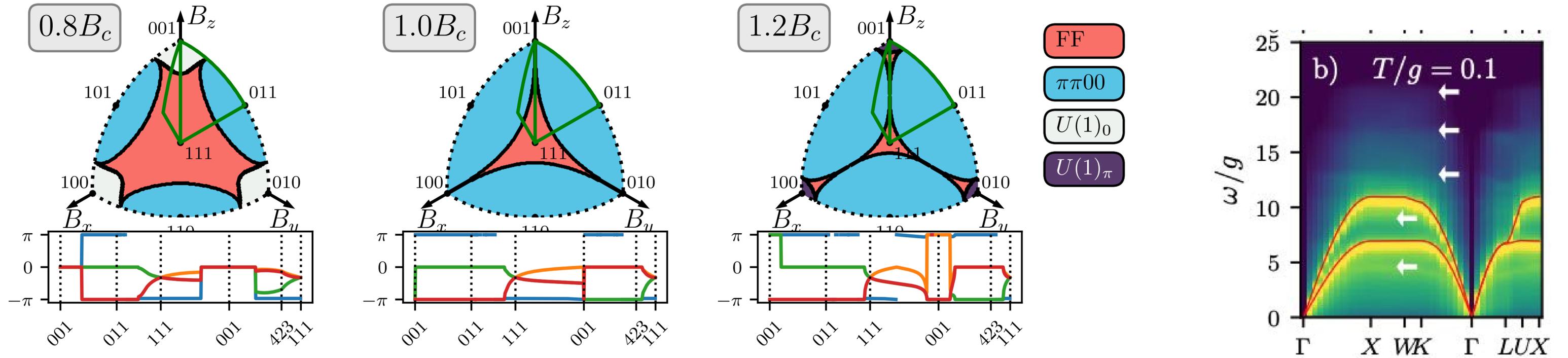
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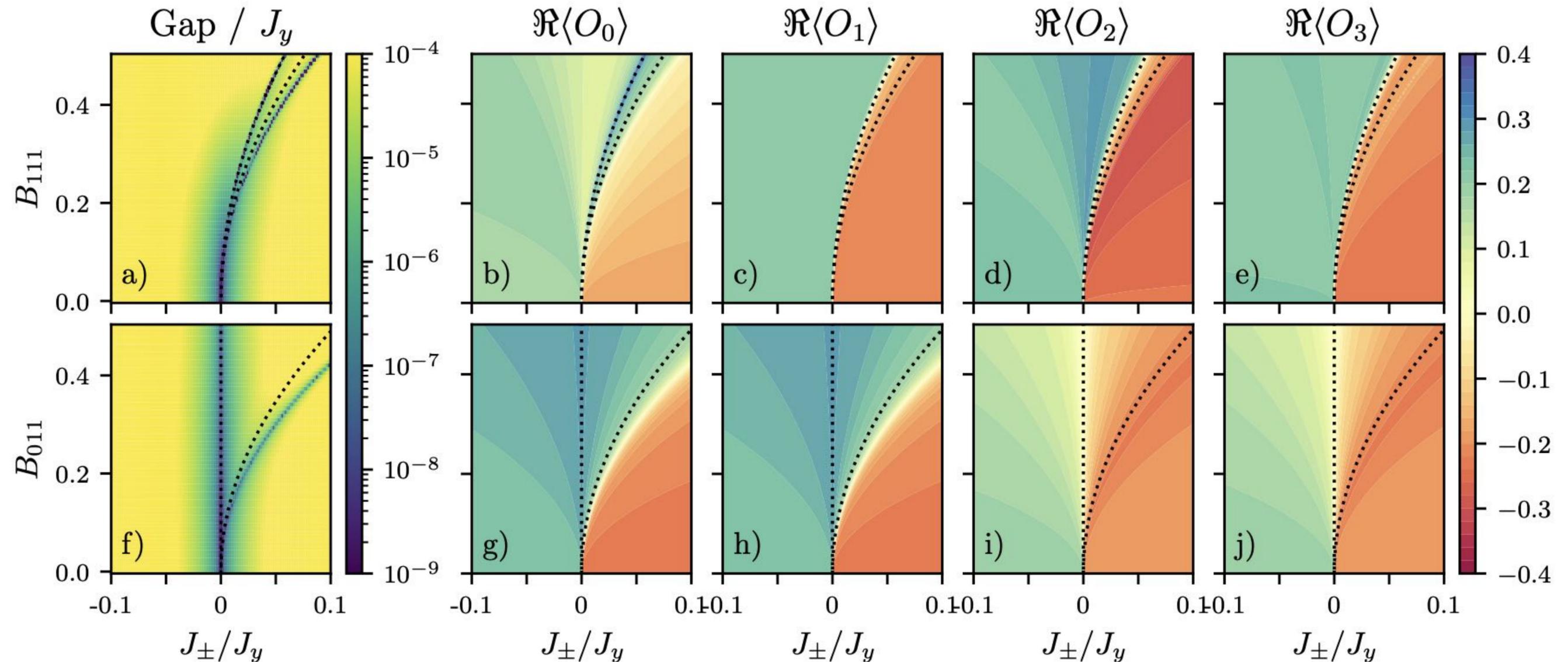
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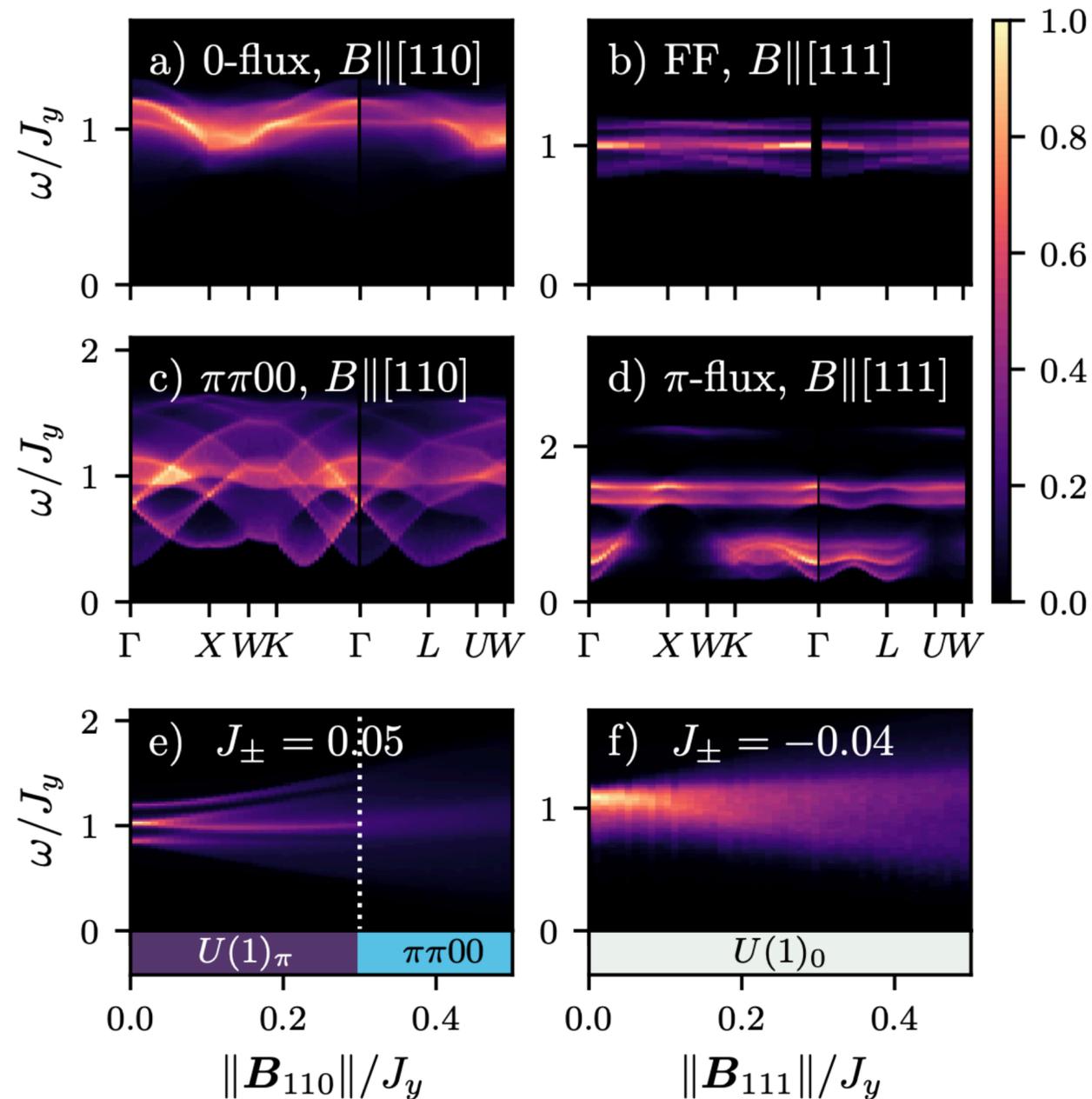
# Exact diagonalizations



ED Expectation values of ring-flip operators on a 72-site cluster subject to (a-e) 111 and (f-j) 011 magnetic fields.

Black dotted lines indicate the semiclassical phase boundaries

# Spinon Signatures



The spinon spectrum is much easier to be measured in current neutron scattering experiments.

Different phases have very different features.

Also ref. F Desrochers, Y-B Kim PRL, 132(6), 066502 (2024)  
Z Zhou, F Desrochers, Y-B Kim PhysRevB.110.174441