Magnetic skyrmions in the quantum limit

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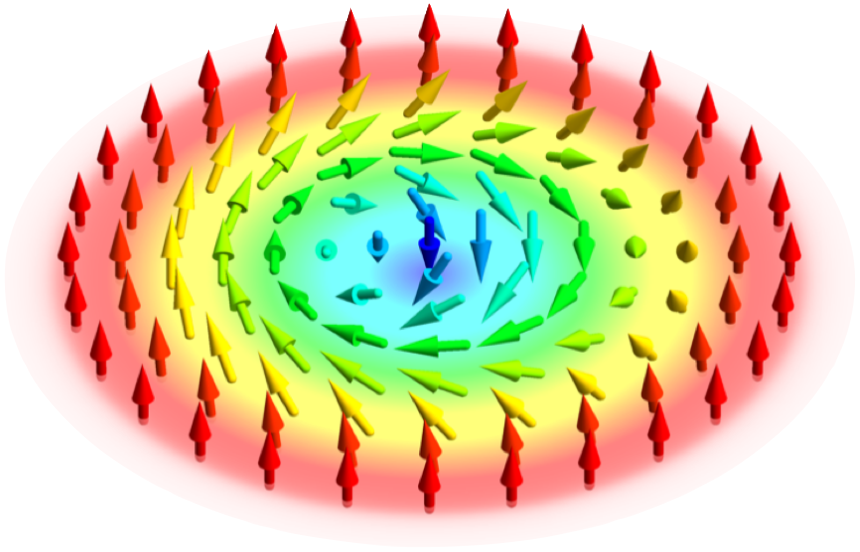


[1] A. Petrović, C. Psaroudaki, P. Fischer, MG, C. Panagopoulos, RMP in press, arXiv:2410.11427
[2] S Sorn, J Schmalian, MG, arXiv:2412.16284

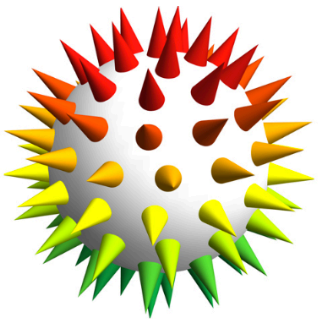
Outline

- magnetic skyrmions — classical topological field configurations
- magnetic skyrmions in quantum magnetism
- semiclassical quantization of skyrmions:

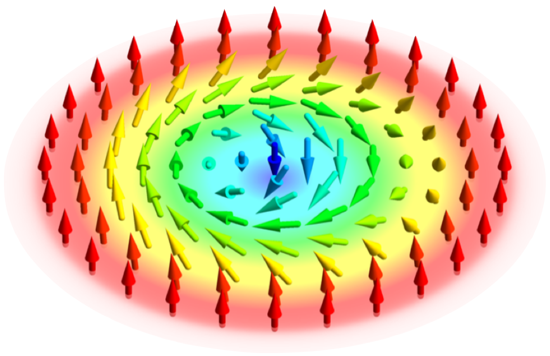
  conservation of linear momentum, dipole conservation law & skyrmions as fracton matter
- topological dipole conservation law for quantum skyrmions

Magnetic skyrmions — classical topological field configurations

topologically non-trivial mapping from sphere *S*² to 2d plane: $\pi_2(S^2) = \mathbb{Z}$



Heisenberg magnet:
order parameter
manifold: *S*²



2d plane

topological skyrmion density

$$\rho_{\mathrm{top}} = \frac{1}{4\pi}\hat{n}(\partial_x \hat{n} \times \partial_y \hat{n})$$

unit vector field $\hat{n}(\vec{r})$

winding number:

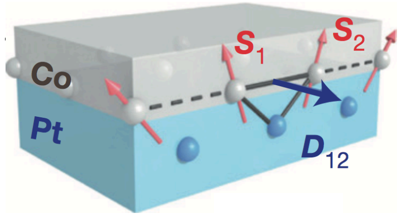
$$Q = \int d^2x \rho_{\mathrm{top}} = -1$$

Magnetic skyrmions — materials

intensively studied over the past 15 years….

in magnetic multilayers:

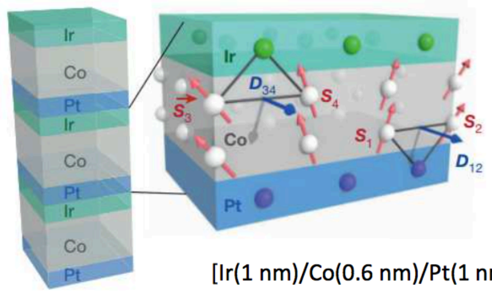
stabilized by interfacial Dzyaloshinskii-Moriya interaction



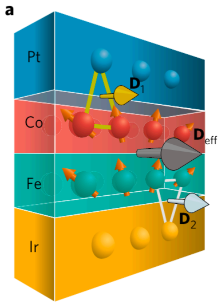
Fert & Levy, PRL (1980)

even at room temperature in designed Ir/Co/Fe/Pt multilayers

C. Moreau-Luchaire *et al.*, Nat. Nanotech. **11**, 444 (2016)



$[Ir(1\,nm)/Co(0.6\,nm)/Pt(1\,nm)]_{10}$



A. Soumyanarayanan *et al,*
B. Nature Mat 16, 898 (2017)

in bulk magnetic crystals:

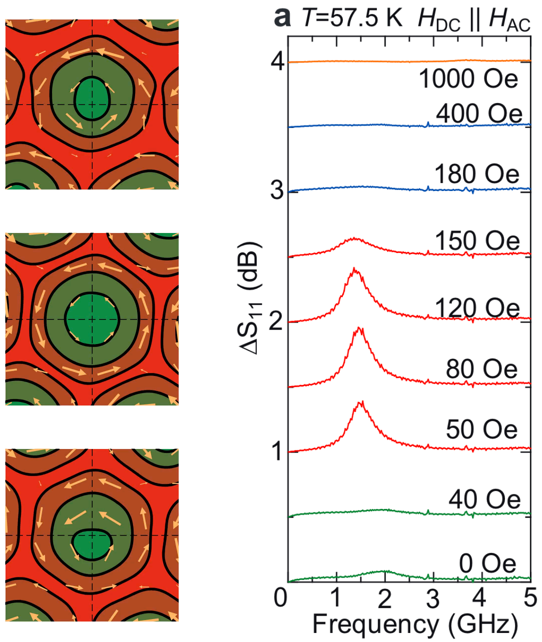
stabilized by Dzyaloshinskii-Moriya interaction due to lack of inversion symmetry

Material	Crys. Class	SG	Skyrm. Type	T_c	λ (nm)	Trans.	References
MnSi	*T*	$P2_13$	Bloch	30 K	18	Metal	S. Mühlbauer *et al.*, Science **323**, 915 (2009)
FeGe	*T*	$P2_13$	Bloch	279 K	70	Metal	X.Z. Yu *et al.*, Nat. Mater. **10**, 106 (2010)
$Fe_{1-x}Co_xSi$	*T*	$P2_13$	Bloch	< 36 K	40-230	Semi-cond.	W. Münzer *et al.*, PRB **81**, 041203(R) (2010) X.Z. Yu *et al.*, Nature **465**, 901 (2010)
$Mn_{1-x}Fe_xSi$	*T*	$P2_13$	Bloch	< 17 K	10-12	Metal	S.V. Grigoriev *et al.*, PRB **79**, 144417 (2009)
$Mn_{1-x}Fe_xGe$	*T*	$P2_13$	Bloch	< 220 K	5–220	Metal	K. Shibata *et al.*, Nat. Nanotech. **8**, 723–728 (2013)
Cu_2OSeO_3	*T*	$P2_13$	Bloch	58 K	60	Insulator	S. Seki *et al.*, Science **336**, 198 (2012) T. Adams *et al.*, PRL **108**, 237204 (2012)
$Co_xZn_yMn_z$	O	$P4_132$ / $P4_332$	Bloch	150 K – 500 K	120 – 200	Metal	Y. Tokunaga *et al.*, Nat. Commun. **6**, 7638 (2015)
$(Fe,Co)_2Mo_3N$	O	$P4_132$ / $P4_332$	Bloch	< 36 K	110	Metal	W. Li *et al.*, Phys. Rev. B **93**, 060409(R) (2016)
GaV_4S_8	C_{3v}	*R*3m	Néel	13 K	17	Semicond/ Insulator	I. Kézsmárki *et al.*, Nat. Mater. **14**, 1116 (2015)

Magnetic skyrmions —dynamics

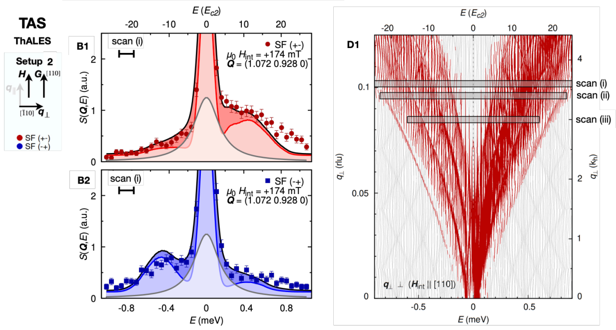
intensively studied over the past 15 years…. can all be understood on the semiclassical level

magnetic resonance spectroscopy



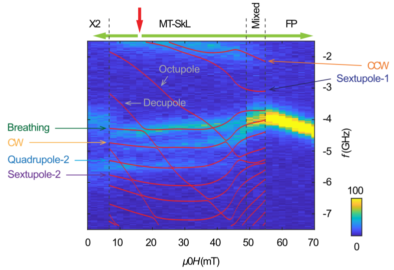
Mochizuki Phys. Rev. Lett. **108**, 017601 (2012)
Onose *et al.* Phys. Rev. Lett. **109**, 037603 (2012)

inelastic neutron scattering



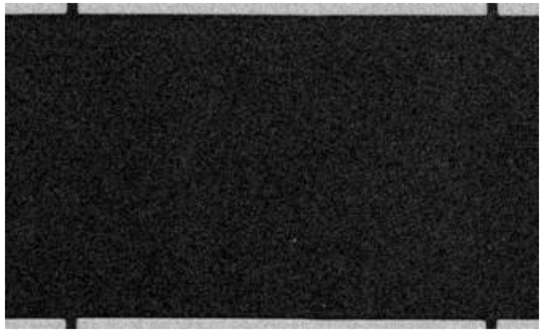
Weber *et al.* Science **375**, 1025 (2022)

Brillouin light scattering



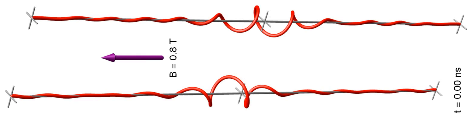
Che et al, arXiv:2404.14314

skyrmion Hall effect



Jiang et al. Nat. Phys. (2017)

non-linear dynamics of strings



V. Kravchuk et al. PRB 102, 220408 (2020)

What are the signatures of magnetic skyrmions in the quantum limit?

Magnetic skyrmions in quantum magnetism

classical skyrmion configurations are not quantum eigenstates of the Hamiltonian

(similar to the Néel state of antiferromagnetic order)

⇨ expect substantial corrections due to fluctuations!

full quantum problem: brute force numerical studies of small system sizes

challenge: linear system size L > linear size of the skyrmion r_{sk} >> lattice spacing a

for 2d:

$$\sqrt{N} \sim L/a > r_{sk}/a \gg 1$$

exact diagonalization for S=1/2

Lohani et al. 2019	N=31
Sotnikov et al. 2021,2023	N=19
Siegl et al 2022	N=16

density matrix renormalization group for S=1/2

usually only access to groundstates

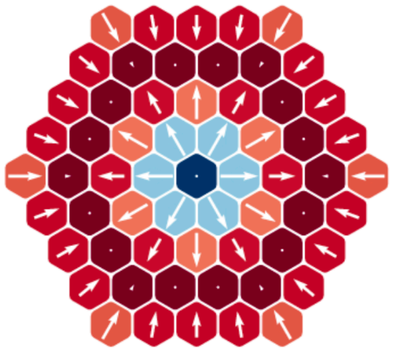
Haller et al. 2022, … N ~ 500

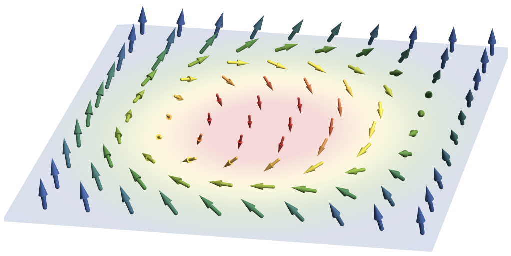
Magnetic skyrmions in quantum magnetism

How to detect skyrmionic content from a spin wave eigenfunction?

Evaluation of expectation value of local spin operator $\langle \mathbf{S}(\mathbf{r}) \rangle$ might work or not:

quantum skyrmion stabilized by DMI
with open boundary conditions:

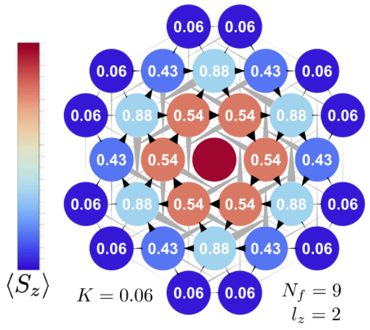


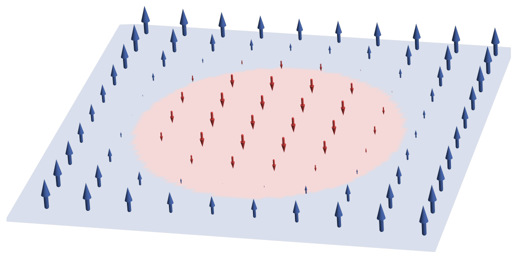


Haller et al. 2022

magnitude renormalized but otherwise recognizable

quantum skyrmion stabilized by frustration
with fixed boundary conditions:





Lohani et al. 2019

internal degrees of freedom wash out
skyrmionic structures; only encoded in correlations!

Winding number for quantum skyrmions?

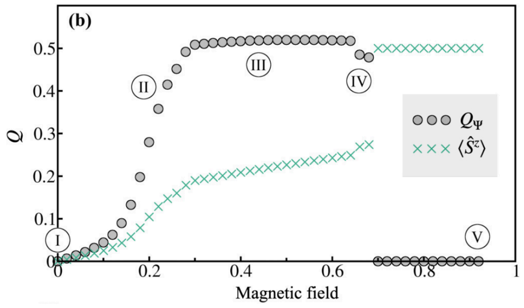
- classical skyrmion winding number in the continuum $Q = \int d^2x \rho_{\text{top}}$ with $\rho_{\text{top}} = \frac{1}{4\pi} \hat{n}(\partial_x \hat{n} \times \partial_y \hat{n})$

- classical skyrmion winding number on the discrete lattice (sum over triangulations) Berg and Lüscher (1981)

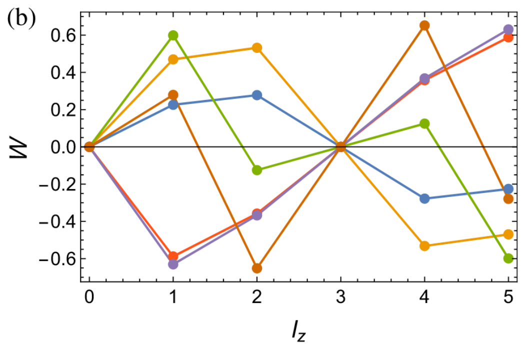
$$W = \frac{1}{2\pi} \sum_{\langle ijk \rangle} \arctan \frac{\frac{1}{S^3}\langle \hat{\mathbf{S}}_i \cdot (\hat{\mathbf{S}}_j \times \hat{\mathbf{S}}_k)\rangle}{1 + \frac{1}{S^2}(\langle \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j\rangle + \langle \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_k\rangle + \langle \hat{\mathbf{S}}_j \cdot \hat{\mathbf{S}}_k\rangle)}$$

- scalar spin chirality $\chi_{ijk} = \mathbf{S}_i(\mathbf{S}_j \times \mathbf{S}_k)$ winding: $Q_\Psi = \frac{1}{\pi S^3} \sum_{\langle ijk \rangle} \langle \chi_{ijk} \rangle$

for quantum skyrmions these winding numbers are not quantized and are only indicators:



Sotnikov et al. 2021



Lohani et al. 2019

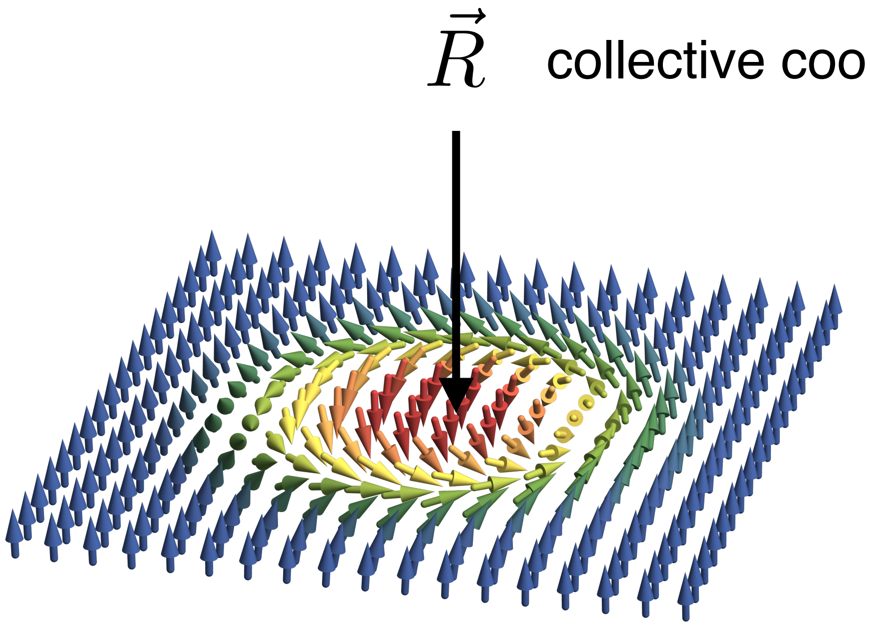
How to quantize magnetic skyrmions on the semiclassical level?

Semiclassical quantization of magnetic skyrmions

Idea: at low energies treat the skyrmion as a point particle with position $\vec{R}$ and momentum $\overrightarrow{P}$ and replace their Poisson bracket with a commutator

Ochoa & Tserkovnyak, Int. J. Mod. Phys. B **33**, 1930005 (2019)

But what is the momentum $\overrightarrow{P}$ of a skyrmion?



Thiele (1976), Haldane (1986), Volovik (1987)
Papanikolaou & Tomaras (1991)
Yan, Kamra, Cao, Bauer (2013), Tchernyshyov (2015), …

Conservation law for the topological density

topological density

$$\rho_{\mathrm{top}} = \frac{1}{4\pi}\hat{n}(\partial_x \hat{n} \times \partial_y \hat{n})$$

geometric conservation law
for the unit vector field

$$\partial_t \rho_{\mathrm{top}} + \partial_k j_{\mathrm{top},k} = 0$$

topological current

$$j_{\mathrm{top},k} = \frac{\epsilon_{ki}}{4\pi}\hat{n}(\partial_i \hat{n} \times \partial_t \hat{n})$$

Classical field theory for magnetism

Landau-Lifshitz equation:

$$\partial_t \hat{n} = \frac{1}{s} \hat{n} \times \frac{\delta \mathcal{W}}{\delta \hat{n}} = \{\hat{n}, \mathcal{W}\}$$

spin density s

with some energy functional $\mathcal{W} = \mathcal{W}[\hat{n}, \partial_i \hat{n}, ...]$ details not important here!

Poisson bracket

$$\{\hat{n}_\alpha(\vec{x}), \hat{n}_\beta(\vec{x}')\} = -\frac{1}{s} \epsilon_{\alpha\beta\gamma} \hat{n}_\gamma(\vec{x}) \delta(\vec{x} - \vec{x}')$$

Here: no extrinsic Gilbert damping, no other degrees of freedom!

Conservation law for linear momentum

Landau-Lifshitz equation:

$$\partial_t \hat{n} = \frac{1}{s} \hat{n} \times \frac{\delta \mathcal{W}}{\delta \hat{n}}$$

mutliplying with $\partial_i \hat{n} (\hat{n} \times$ yields

$$\partial_i \vec{n} (\vec{n} \times \partial_t \vec{n}) = -\frac{1}{s} \partial_i \vec{n} \frac{\delta \mathcal{W}}{\delta \vec{n}}$$

for a translationally invariant theory $\mathcal{W} = \int_V d\vec{x}\, \mathcal{H}(\vec{n}, \partial_i \vec{n})$ we have $\partial_i \vec{n} \frac{\delta \mathcal{W}}{\delta \vec{n}} = \partial_j \sigma_{ji}$

with the stress tensor

$$\sigma_{ji} = \delta_{ji} \mathcal{H} - \frac{\partial \mathcal{H}}{\partial \partial_j \vec{n}} \cdot \partial_i \vec{n}$$

topological current related to
divergence of stress tensor

$$4\pi s \epsilon_{ki} j_{\mathrm{top},k} = \partial_j \sigma_{ji}$$

However: this is not the form of a conservation law?!

Conservation law for linear momentum

$$4\pi s \epsilon_{ki} j_{\mathrm{top},k} = \partial_j \sigma_{ji}$$

This is not the form of a conservation law?!

combining it with the conservation of topological charge

$$j_{\mathrm{top},i} = \partial_j (x_i j_{\mathrm{top},j}) - x_i \partial_j j_{\mathrm{top},j}$$
$$= \partial_j (x_i j_{\mathrm{top},j}) + x_i \partial_t \rho_{\mathrm{top}},$$

$$\Rightarrow \quad \partial_t p_i + \partial_j \Pi_{ji} = 0$$

This is a canonical conservation law!

with the linear momentum and its current

$$p_i = 4\pi s\, \epsilon_{ki} x_k \rho_{\mathrm{top}},$$
$$\Pi_{ji} = -\sigma_{ji} + 4\pi s\, \epsilon_{ki} x_k j_{\mathrm{top},j}.$$

Linear momentum of a skyrmion

Total linear momentum of a magnetic system:

$$P_i = \int d^2x p_i = 4\pi s \epsilon_{ki} \int d^2x x_k \rho_{\mathrm{top}}$$

determined by the dipole moment of topological charge!

For a closed system: linear momentum = dipole moment of a skyrmion is conserved!

Area-preserving diffeomorphisms

Poisson bracket $$\{\hat{n}_\alpha(\vec{x}), \hat{n}_\beta(\vec{x}')\} = -\frac{1}{s}\epsilon_{\alpha\beta\gamma}\hat{n}_\gamma(\vec{x})\delta(\vec{x}-\vec{x}')$$

consider Poisson bracket for the topological density $$\rho_{\rm top} = \frac{1}{4\pi}\hat{n}(\partial_x \hat{n} \times \partial_y \hat{n})$$

$$\{\rho_{\rm top}(\vec{x}), \hat{n}(\vec{x}')\} = \frac{1}{4\pi s}\epsilon_{ij}\partial_j\delta(\vec{x}-\vec{x}')\partial_i\hat{n}(\vec{x})$$

topological density is the generator of area-preserving diffeomorphisms:

$$\int d^2x'\,\lambda(\vec{x}')\{4\pi s\,\rho_{\rm top}(\vec{x}'), \hat{n}(\vec{x})\} = -\xi_i(\vec{x})\partial_i\hat{n}(\vec{x})$$

generates local transformations $\xi_i(\vec{x}) = \epsilon_{ij}\partial_j\lambda(\vec{x})$ that are area-preserving as $\partial_i\xi_i(\vec{x}) = 0$

Y.-H. Du, U. Mehta, D. X. Nguyen, and D T. Son, SciPost Phys **12,** 050, (2022)

Topological dipole conservation law

applying the generator of area-preserving diffeomorphisms to an energy functional possessing translational invariance

$$\int d^2x \lambda(\vec{x})\{4\pi s\, \rho_{\rm top}(\vec{x}), \mathcal{W}\} = -\int d^2x \epsilon_{ki}\partial_i\lambda(\vec{x})\partial_j\sigma_{jk}(\vec{x})$$

with the stress tensor $\sigma_{jk}(\vec{x})$

variation wrt $\lambda(\vec{x})$ yields the Noether conservation law (using $\partial_t\rho_{\rm top} = \{\rho_{\rm top}, \mathcal{W}\}$)

$$\partial_t\rho_{\rm top} + \frac{1}{4\pi s}\epsilon_{ik}\partial_i\partial_j\sigma_{jk} = 0$$

Papanicolaou & Tomaras, Nuclear Physics B **360,** 425 (1991)

Topological dipole conservation law

$$\partial_t \rho_{\text{top}} + \frac{1}{4\pi s}\epsilon_{ik}\partial_i\partial_j\sigma_{jk} = 0$$

integration with benign boundary conditions yields the conservation of

topological charge $\partial_t Q = 0$

topological dipole $\partial_t D_i = 0$

⇨ magnetic textures behave as **fracton matter**

"… class of models where quasiparticles are strictly immobile or display restricted mobility …"

wealth of exotic phenomena:
anomalous hydrodynamics, ergodicity breaking and anomalous thermalization processes, emergence of high-rank tensor gauge fields

to be studied in skyrmion materials! Filling the "experimental void"

for a review see Gromov & Radzihovsky, *Colloquium: Fracton Matter*, Rev. Mod. Phys. **96**, 011001 (2024)

Topological dipole as a collective coordinate

collective coordinate $R_i = \frac{D_i}{Q} = \frac{\int d^2x\, x_i \rho_{\rm top}}{\int d^2x\, \rho_{\rm top}}$

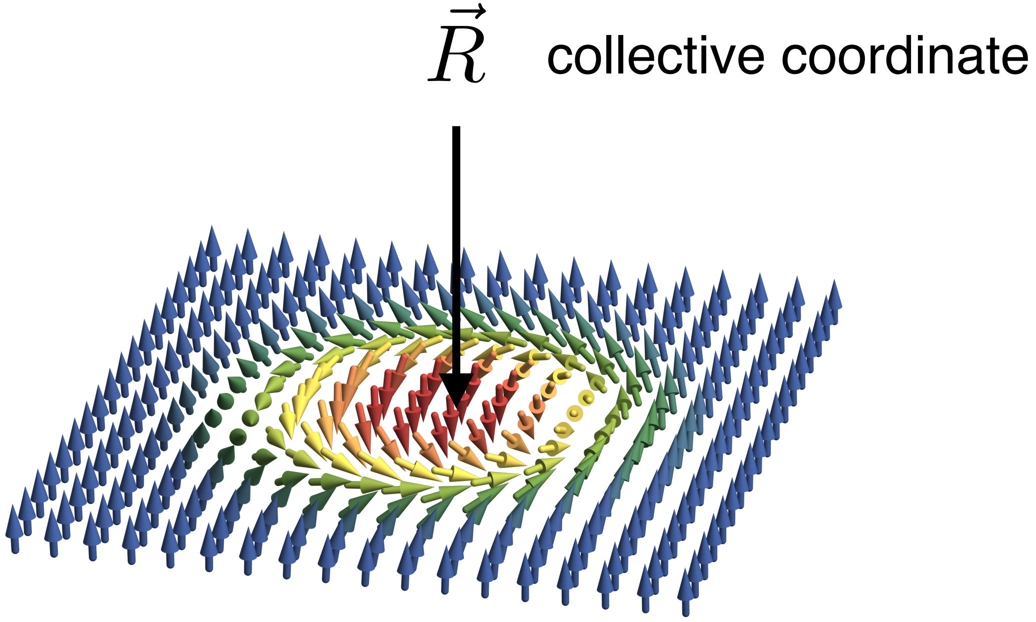
topological dipole conservation law enforces

$$\partial_t \vec{P} = 4\pi s(\hat{z} \times \partial_t \vec{D}) = \vec{G} \times \partial_t \vec{R} = 0$$

massless Thiele equation!

gyrocoupling vector $\vec{G} = 4\pi s Q \hat{e}_z$

Skyrmion does not move — fracton!



Poisson algebra and semiclassical quantization

consider Poisson bracket for the topological density $$\rho_{\rm top} = \frac{1}{4\pi}\hat{n}(\partial_x \hat{n} \times \partial_y \hat{n})$$

$$\{\rho_{\rm top}(\vec{x}), \rho_{\rm top}(\vec{x}')\} = \frac{1}{4\pi s}\epsilon_{ij}\partial_j \delta(\vec{x} - \vec{x}')\partial_i \rho_{\rm top}(\vec{x})$$

Girvin-MacDonald-Platzman algebra

also fulfilled by the electron density in the lowest Landau level

multiplying by positions and integrating

$$\{D_i, D_j\} = \frac{Q}{4\pi s}\epsilon_{ij}$$

components of the topological dipole possess non-vanishing Poisson bracket for a finite skyrmion charge $Q \neq 0$

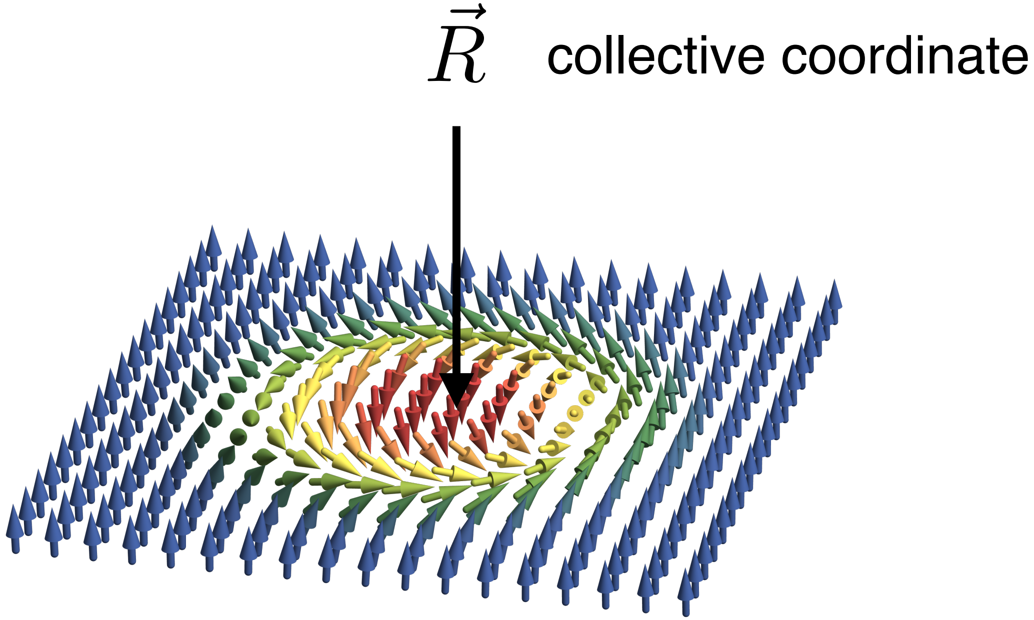
Semiclassical quantization of magnetic skyrmions

collective coordinate $$R_i = \frac{D_i}{Q} = \frac{\int d^2x\, x_i \rho_{\text{top}}}{\int d^2x\, \rho_{\text{top}}}$$

Poisson bracket $$\{R_i, R_j\} = \frac{1}{4\pi s Q}\epsilon_{ij}$$

quantization $$[R_i, R_j] = \frac{i\hbar}{4\pi s Q}\epsilon_{ij}$$

effective theory $$L = 2\pi s Q \epsilon_{kj} R_j \partial_t R_k - V(\vec{R})$$



Ochoa & Tserkovnyak, Int. J. Mod. Phys. B **33**, 1930005 (2019)

M. Stone, Phys. Rev. B **53,** 16573 (1996)

Is this algebraic structure robust with respect to quantum fluctuations?

Topological dipole of quantum skyrmions

claim: topological dipole conservation law also holds on the quantum level — no anomalies!

$$\partial_t \rho_{\text{top}} + \frac{1}{4\pi s} \epsilon_{ik} \partial_i \partial_j \sigma_{jk} = 0$$

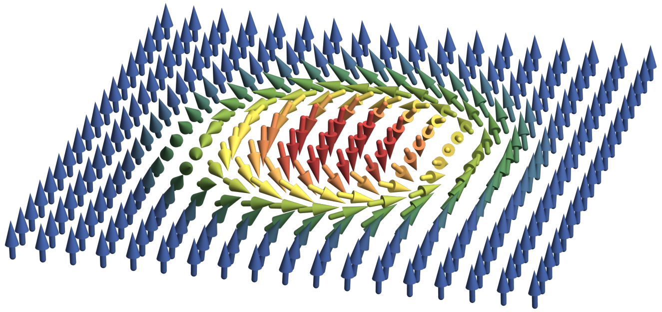
for details see arXiv:2412.16284

in the following: only lowest order quantum corrections to the classical theory

Classical field configuration

consider a topologically non-trivial classical field configuration

with a classical skyrmion collective coordinate



$$\hat{n}_{cl}(\vec{x}) \equiv e_3(\vec{x})$$

$$R_{\mathrm{cl},i} = \frac{D_{\mathrm{cl},i}}{Q_{\mathrm{cl}}} = \frac{\int d^2x\, x_i \rho_{\mathrm{top,cl}}}{\int d^2x\, \rho_{\mathrm{top,cl}}}$$

introduce local orthogonal Dreibein $\hat{e}_i(\vec{x})\hat{e}_j(\vec{x}) = \delta_{ij}$

with $\hat{e}_1(\vec{x}) \times \hat{e}_2(\vec{x}) = \hat{e}_3(\vec{x})$ and $\hat{e}_\pm = \frac{1}{\sqrt{2}}(\hat{e}_1 \pm i\hat{e}_2)$

Holstein-Primakoff representation of the unit vector field:

$$\hat{n} = \hat{e}_3\left(1 - \frac{\hbar}{s}\psi^*\psi\right) + \sqrt{1 - \frac{1}{2}\frac{\hbar}{s}\psi^*\psi}\sqrt{\frac{\hbar}{s}}(\hat{e}_-\psi^* + \hat{e}_+\psi)$$

with complex spin wave field ψ

Quantum corrections to the topological charge density

Expansion of the topological charge density in lowest order in $\hbar/s$ (after some algebra…)

$$\rho_{\rm top} = \rho_{\rm top,cl} + \sqrt{\frac{\hbar}{s}}\frac{i}{4\pi}\epsilon_{ij}\partial_j\left(\psi\hat{e}_+\partial_i\hat{e}_3 - \psi^*\hat{e}_-\partial_i\hat{e}_3\right) + \frac{\hbar}{s}\frac{1}{8\pi}\epsilon_{ij}\partial_j\Big(-i\psi^*\partial_i\psi + i\psi\partial_i\psi^* + 2(\hat{e}_1\partial_i\hat{e}_2)\psi^*\psi\Big) + \ \ldots$$

Integration yields:

$$Q = Q_{\rm cl}$$

no quantum correction to the topological charge — integer!

$$D_i = D_{{\rm cl},i} + \frac{i\epsilon_{ij}}{4\pi}\sqrt{\frac{\hbar}{s}}\int d^2x\,\vec{\Phi}^\dagger_{o,j}\tau^z\vec{\Psi} + \frac{\epsilon_{ij}}{4\pi s}P_{{\rm mag},j}$$

… but to the dipole

Quantum correction to the topological dipole

$$D_i = D_{\mathrm{cl},i} + \frac{i\epsilon_{ij}}{4\pi}\sqrt{\frac{\hbar}{s}}\int d^2x\,\vec{\Phi}^\dagger_{o,j}\tau^z\vec{\Psi} + \frac{\epsilon_{ij}}{4\pi s}P_{\mathrm{mag},j}$$

with the spinor $\vec{\Psi}^T = (\psi, \psi^*)$

where

$$\vec{\Phi}_{o,i} = \begin{pmatrix} \hat{e}_-\partial_i\hat{e}_3 \\ \hat{e}_+\partial_i\hat{e}_3 \end{pmatrix}$$

translational zero mode

$$P_{\mathrm{mag},j} = \frac{\hbar}{2}\int d^2x\,\vec{\Psi}^\dagger(-i\tau^z\partial_j + \hat{e}_1\partial_j\hat{e}_2)\vec{\Psi}$$

gauge-invariant linear momentum of the spin wave

spin-connection =
effective vector potential

Quantum correction to the collective coordinate

topological dipole as a collective coordinate $R_i = D_i/Q$

$$R_i = R_{\mathrm{cl},i} + \frac{i\epsilon_{ij}}{4\pi Q}\sqrt{\frac{\hbar}{s}}\int d^2x\,\vec{\Phi}^\dagger_{o,j}\tau^z\vec{\Psi} + \frac{\epsilon_{ij}}{4\pi s Q}P_{\mathrm{mag},j}$$

trivial shift that can be absorbed into the classical coordinate

$$\Rightarrow \quad R_i = R_{\mathrm{cl},i} + R_{\mathrm{mag},i}$$

with the quantum correction due to spin waves

$$R_{\mathrm{mag},i} = \frac{\epsilon_{ij}}{4\pi s Q}P_{\mathrm{mag},j}$$

Collective coordinate of the skyrmion

collective coordinate defined in terms of the topological dipole $R_i = D_i/Q$ conserved

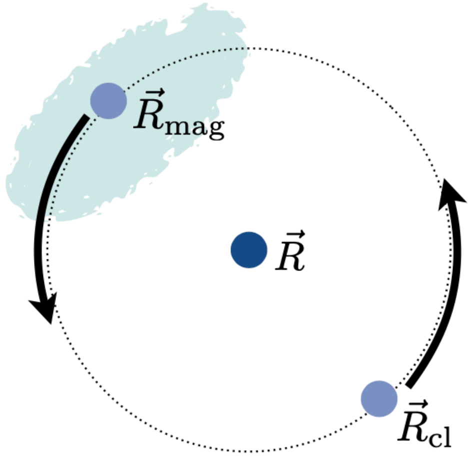
$$\partial_t R_i = 0$$ skyrmion immobile — fracton

decomposes however into a classical and a quantum part

$$R_i = R_{\mathrm{cl},i} + R_{\mathrm{mag},i}$$

previous work computed a finite skyrmion mass — but for $\vec{R}_{\mathrm{cl}}$

C. Psaroudaki, S. Hoffman, J. Klinovaja, and D. Loss
Quantum dynamics of skyrmions in chiral magnets, Phys. Rev. X **7** 041045 (2017)



resulting cyclotron motion exactly compensated
by the quantum magnon cloud!

Acknowledgements

Sopheak Sorn

Jörg Schmalian

[1] A. Petrović, C. Psaroudaki, P. Fischer, MG, C. Panagopoulos, RMP in press, arXiv:2410.11427
[2] S Sorn, J Schmalian, MG, arXiv:2412.16284

Summary

- topological charge density $\rho_{\text{top}} = \frac{1}{4\pi}\hat{n}(\partial_x \hat{n} \times \partial_y \hat{n})$ obeys closed Girvin-MacDonald-Platzman algebra
- close correspondence with quantum Hall systems remains to be explored
- ρ_{top} generates area-preserving diffeomorphisms and obeys dipole conservation law
- magnetic skyrmion textures are fractons with restricted mobility — platform to study fracton phenomena
- topological dipole of a single skyrmion conserved
- dipole serves as a collective coordinate — skyrmion mass is zero

Thank you!