Almost classical effects in quantum spin systems

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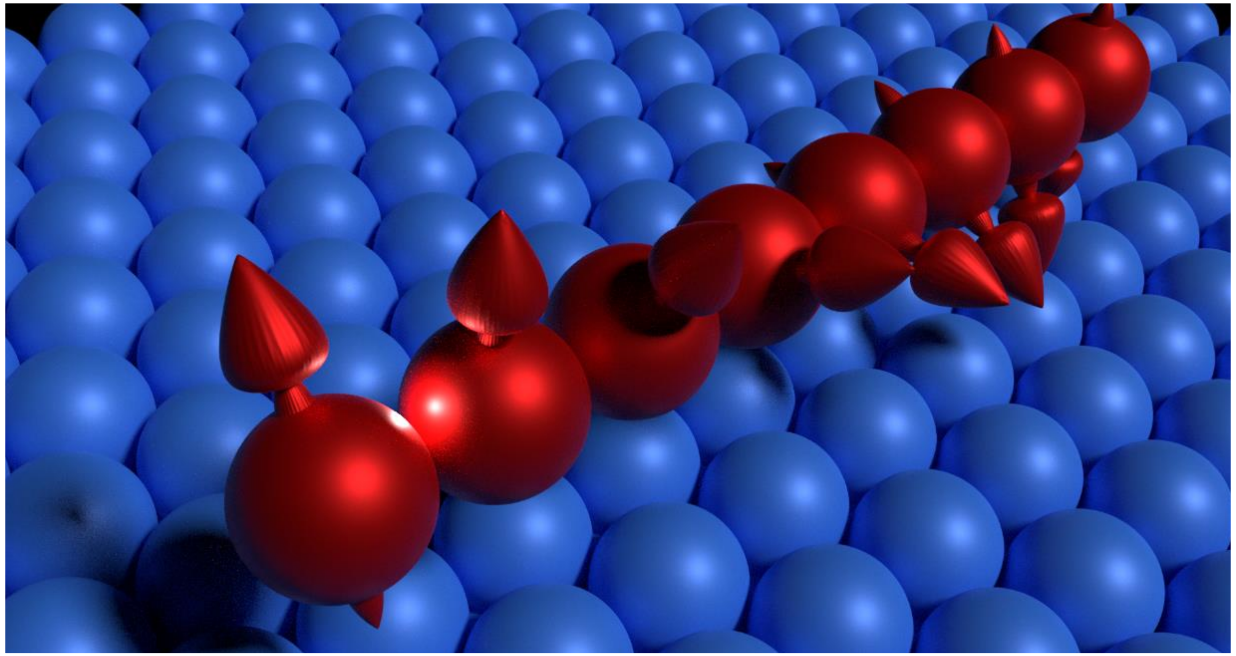
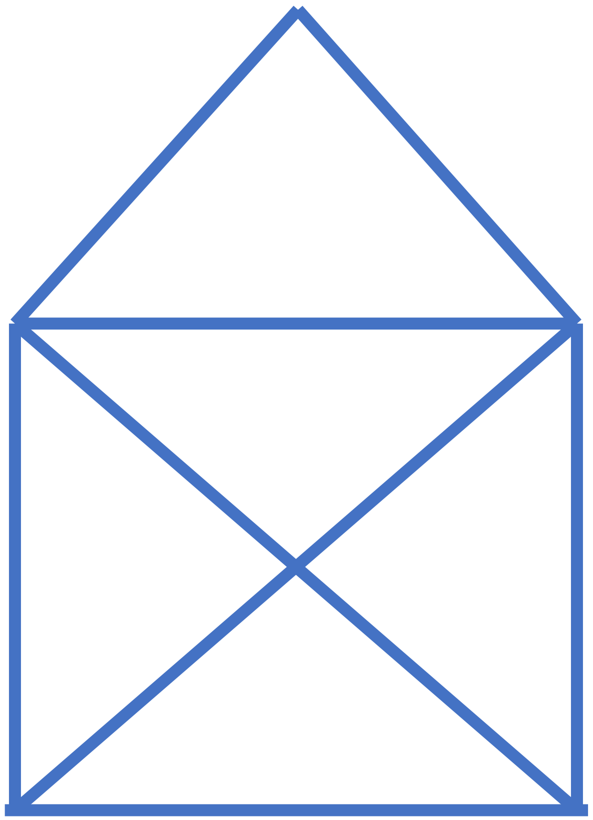
SPICE Workshop Quantum Functionalities of Nanomagnets

June 17, 2025



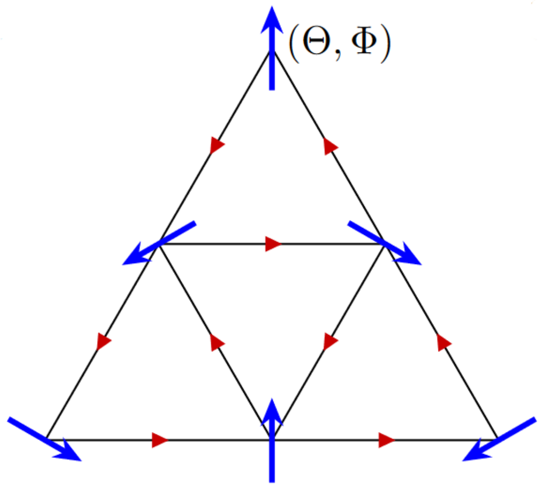
Wake up, brain – Topology

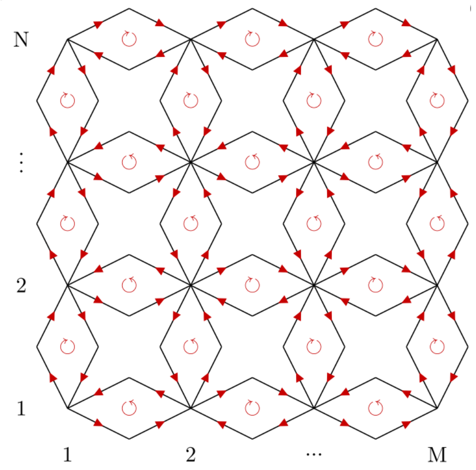
Can you draw the House of Nikolaus?
One continuous stroke, no edge twice,
start from the top.



Outline

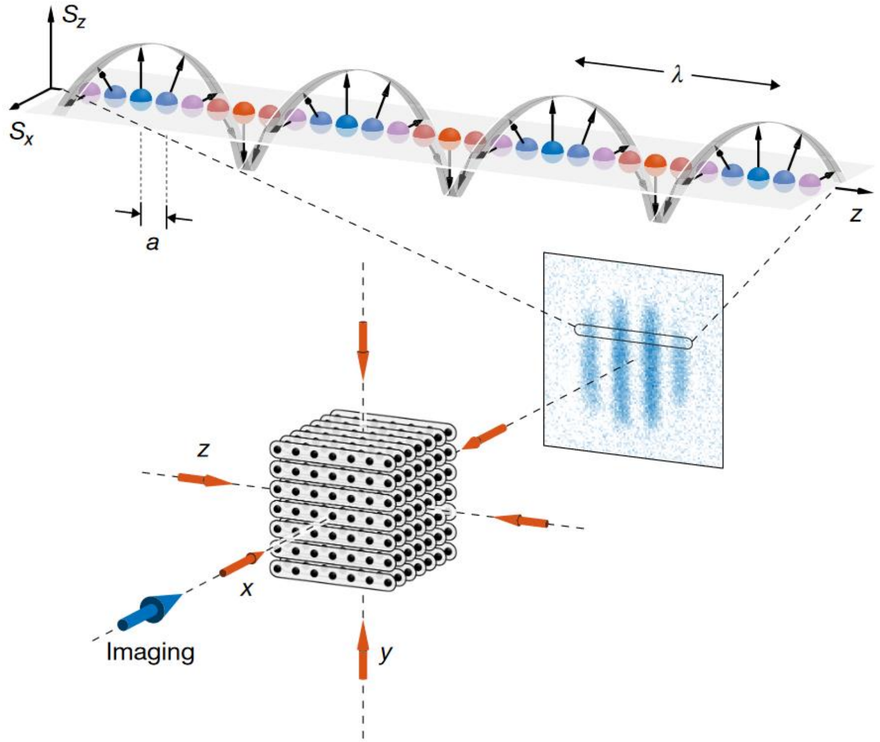
- **Product eigenstate method in easy-plane quantum magnets**
- Large degeneracy in Heisenberg models on a general graph





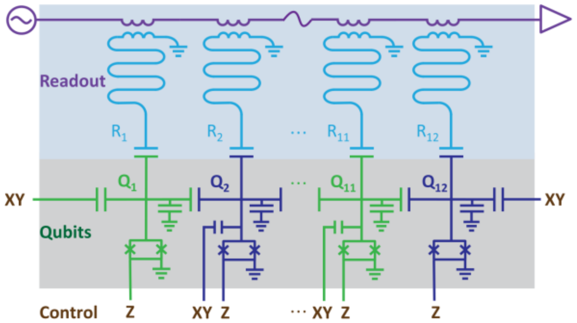
Quantum magnetism - realizations

Cold atoms



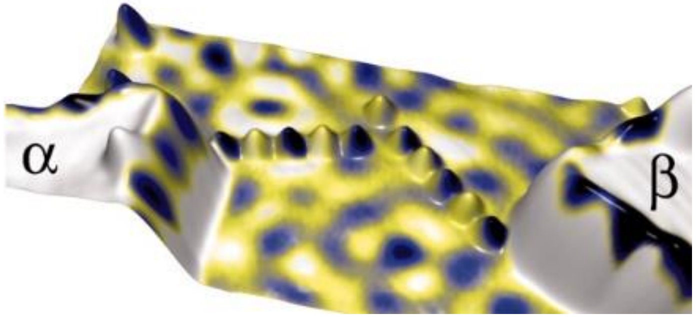
Jepsen [..], Demler & Ketterle
Nature **588**, 403 (2020)

Quantum computers



Gong et al., PRL **112**, 110501 (2019)

Solid states



Khajetoorians, [..], Wiesendanger
Science ***332,*** *1063* (2011)

Hong, [..], Hess
PRL (2024)
Phys. Rev. B **106**, L220406 (2022)

Complete solutions to quantum spin systems

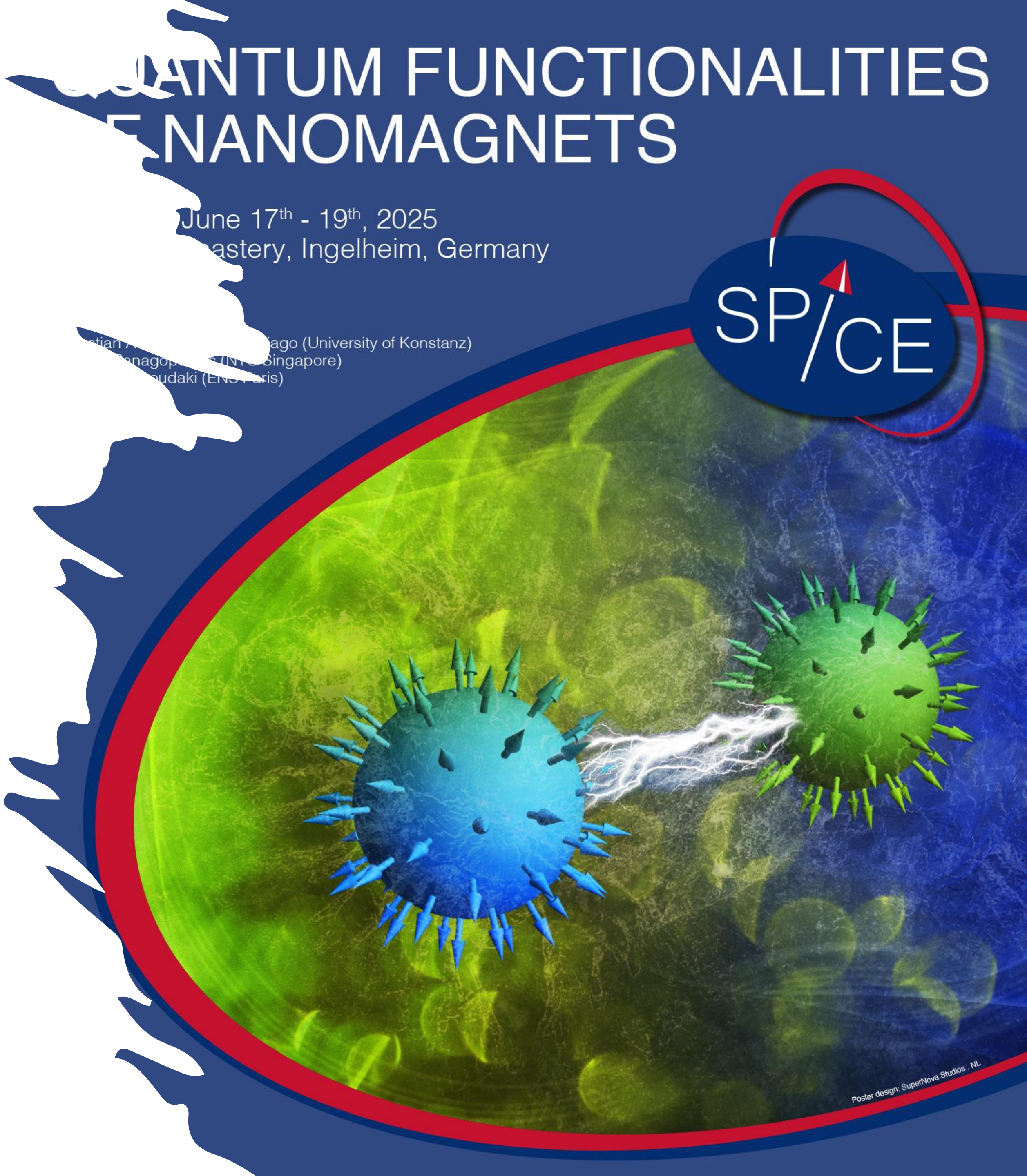
- Integrable systems (Bethe ansatz)
  - Special 1D spin chains [Bethe 1931, Klümper, Göhmann, Frahm, Popkov, ..]
  - Central spin model [Richardson 1963-64, (inhomog.) Bortz, Stoze (2007) (spin S, homog.) Nepomechie 2018]
- Special 2D models
  - Kitaev toric code [Kitaev 2006]
  - Levin-Wen models (string nets) [Levin & Wen 2004]
- Solutions by statistical transmutation
  - Jordan-Wigner transform (spins to fermions) [Jordan & Wigner 1928]
  - Holstein-Primakov transform (spins to bosons) [Holstein & Primakoff 1940]
  - Slave particle methods

Only special systems with complete solution

Motivation

„There are only very few exact eigenstates known for quantum (spin) Hamiltonians“

– Markus Garst, June 17, 2025



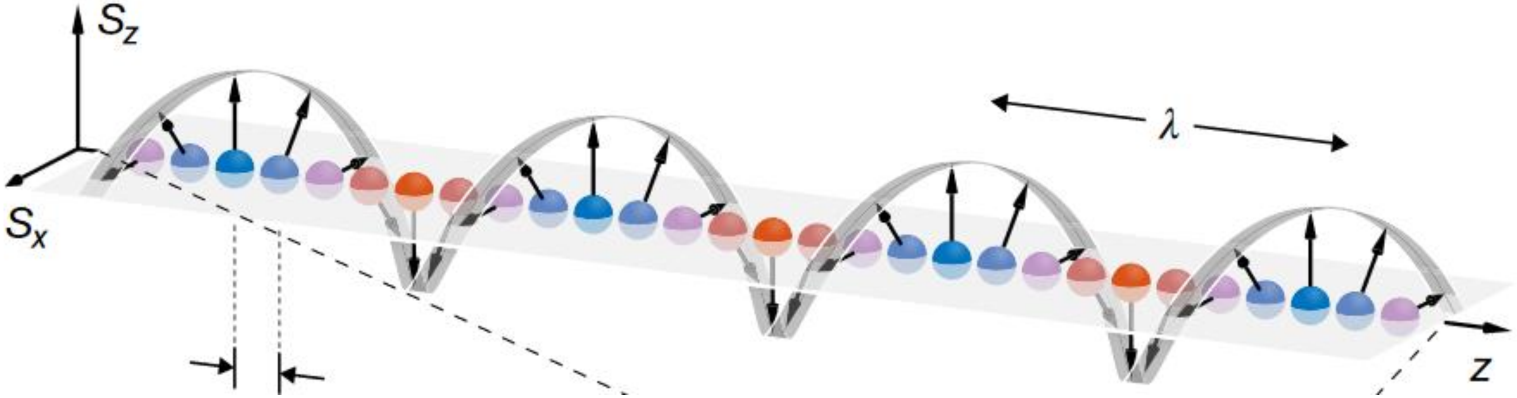
Motivation: Heisenberg model‘s product states

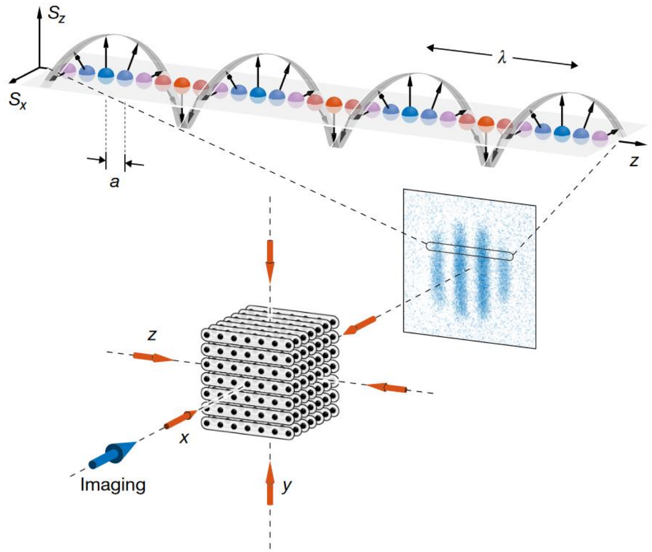
1D Heisenberg model with boundary terms

$$H(\phi) = \sum_{j=2}^{N-2} \left[J(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y) + \Delta S_j^z S_{j+1}^z \right] + J \left[\tilde{\mathbf{S}}_1 \cdot \mathbf{S}_2 + \mathbf{S}_{N-1} \cdot \tilde{\mathbf{S}}_N(\phi) \right].$$

Product eigenstates „Phantom helices

$$|\psi(Q)\rangle = \prod_i \left[\cos(\theta/2) |\uparrow\rangle_i + \sin(\theta/2) \mathrm{e}^{-\mathrm{i}Q z_i} |\downarrow\rangle_i \right]$$





Jepsen [..], Demler & Ketterle
Nature **588**, 403 (2020)

Phantom helices

$$|PH\rangle_1^{\pm} = \frac{1}{\sqrt{2^N}} \bigotimes_{j=2}^{N-1} (|\uparrow\rangle_j - e^{\pm i(j-2)\gamma} |\downarrow\rangle_j)$$

$$|PH\rangle_2^{\pm} = \frac{1}{\sqrt{2^N}} \bigotimes_{j=2}^{N-1} (|\uparrow\rangle_j + e^{\pm ij\gamma} |\downarrow\rangle_j)$$

Boundary phase

$$\phi_{ph} = \pm(N-M)\pi/3 + \pi\delta_{3,M} \mod 2\pi$$

Motivation

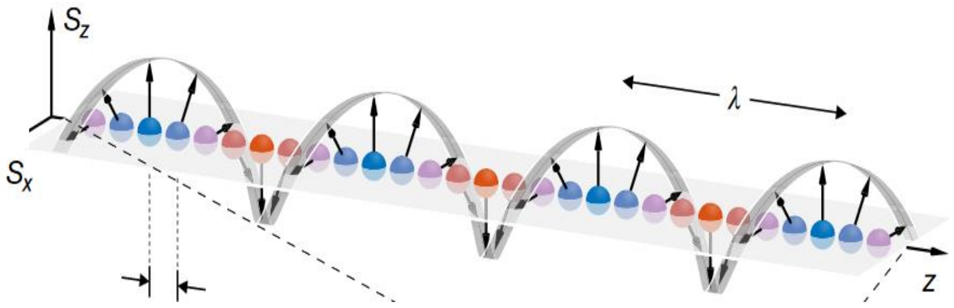
$$|\psi(Q)\rangle = \prod_i \left[\cos(\theta/2)|\uparrow\rangle_i + \sin(\theta/2)\mathrm{e}^{-\mathrm{i}Qz_i}|\downarrow\rangle_i\right]$$

Product eigenstates found in various spin systems

[Cerezo, Rossignoli, … Ríos 2017]
[Pokrovskii, V. L. & Khokhlachev 1975]

1D Phantom helices

[Jepsen .. Ketterle 2022]
[Popkov, Zhang, Göhmann, Klümper … 2020-25]
[Batista & Somma 2012, 2015]

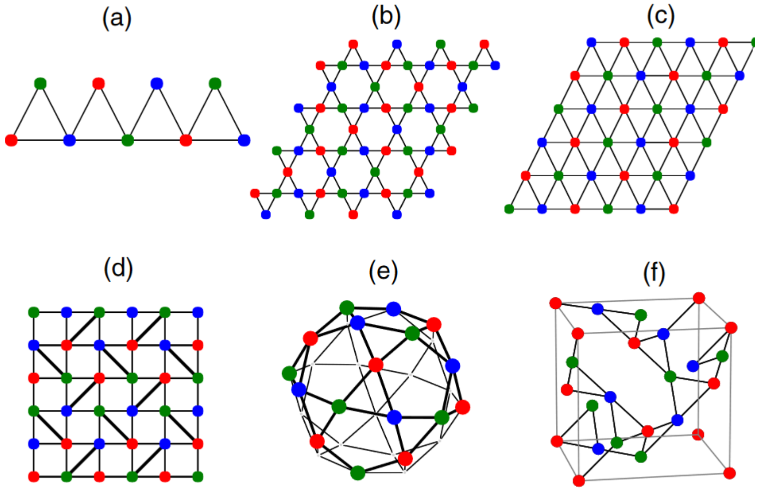


2D Kagome lattice and derivatives

[Changlani .. Fradkin 2018, Fendley 2019]

ND examples

[Jepsen .. Ketterle 2022, Changlani .. Fradkin 2018]



General theory?

All product eigenstates in Heiseberg models from a graphical construction

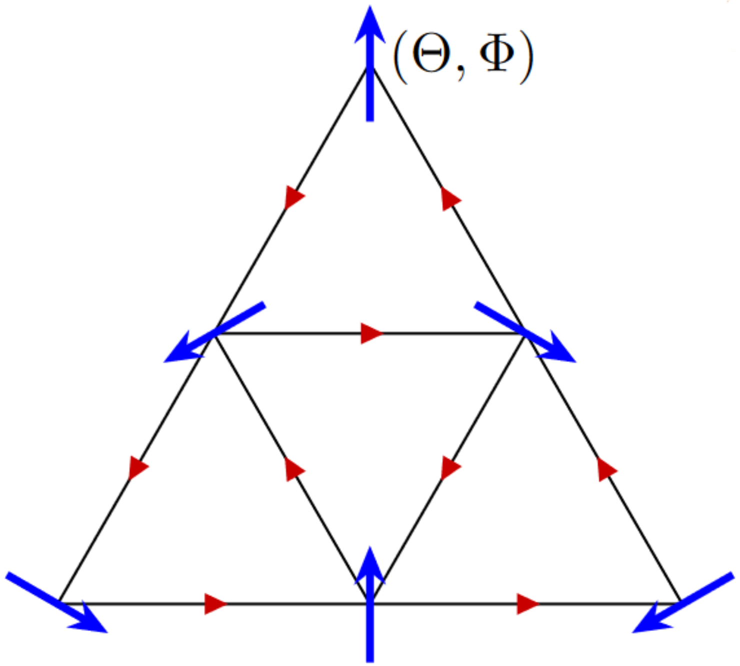
Felix Gerken[1,2,*], Ingo Runkel[3], Christoph Schweigert[3], and Thore Posske[1,2]

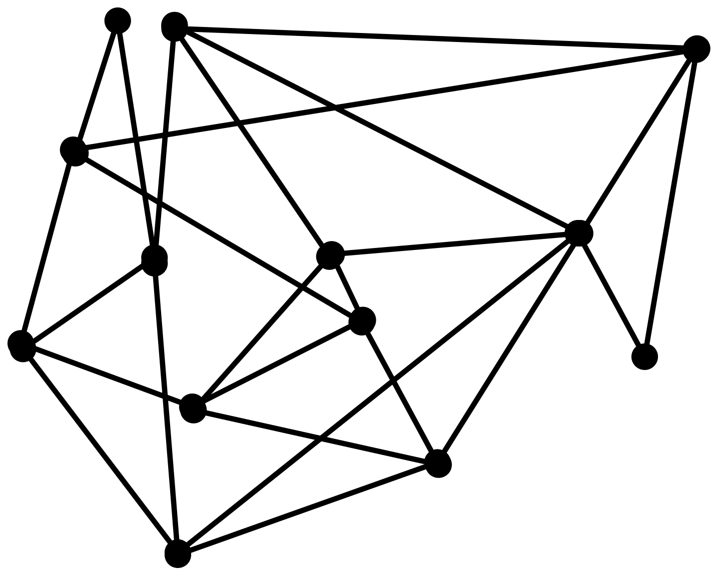
Recently, large degeneracy based on product eigenstates has been found in spin ladders, kagome-like lattices, and motif magnetism, connected to spin liquids, anyonic phases, and quantum scars. We unify these systems by a complete classification of product eigenstates of Heisenberg XXZ Hamiltonians with Dzyaloshinskii-Moriya interaction on general graphs in the form of Kirchhoff rules for spin supercurrents. By this, we construct spin systems with extensive degree of degeneracy linked to exotic condensates which could be studied in atomic gases and quantum spin lattices.

Phys. Rev. Research **7**, L012008 (2025)

Heisenberg model on a general graph

Graph
vertices k
edges $\{k, l\}$





$$H = \sum_{\{k,l\}} J\, S_k^x S_l^x + J\, S_k^y S_l^y + \Delta\, S_k^z S_l^z$$

Easy plane magnetism $|J| < |\Delta|$

Symmetries and spin currents

$$H = \sum_{\{k,l\}} J\, S_k^x S_l^x + J\, S_k^y S_l^y + \Delta\, S_k^z S_l^z \qquad [S^a, S^b] = i\,\epsilon_{abc} S^c$$

U(1) spin rotation **symmetry** around z-axis

$$U = e^{i\theta S_{\mathrm{tot}}^{\mathrm{z}}} \text{ with } \mathrm{S}_{\mathrm{tot}}^{\mathrm{z}} = \sum_j S_j^z$$

Spin-z change

$$\dot{S_j^z} = -i\left[S_j^z, H\right] = i \sum_{\{j,l\}} 2J\left(S_j^x S_l^y - S_j^y S_l^x\right) = i \sum_{\{j,l\}} C_{jl}$$

With the **z-spin current**

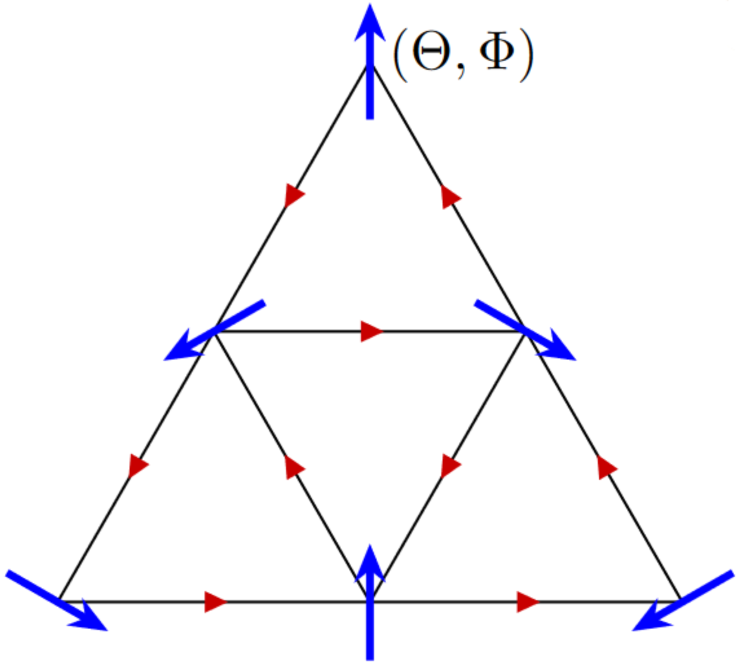
$$C_{jl} = 2J\left(S_j^x S_l^y - S_j^y S_l^x\right)$$

Conservation of spin currents

In eigenstates:

Kirchhoff rule for spin currents

$$\langle \dot{S}_j^z \rangle = \sum_{\{j,l\}} \langle C_{jl} \rangle = 0$$



Product state ansatz

$$H = \sum_{\{k,l\}} J\, S_k^x S_l^x + J\, S_k^y S_l^y + \Delta\, S_k^z S_l^z$$

$$|\Psi(\boldsymbol{\vartheta}, \boldsymbol{\varphi})\rangle = \bigotimes_{i \in V} \left(\cos\left(\frac{\vartheta_i}{2}\right) e^{-\mathrm{i}\frac{\varphi_i}{2}} |\uparrow\rangle_i + \sin\left(\frac{\vartheta_i}{2}\right) e^{\mathrm{i}\frac{\varphi_i}{2}} |\downarrow\rangle_i \right)$$

Coupled trigonometric equations

$$\begin{aligned} \frac{2}{\hbar} U_i^\dagger h_i U_i |\Omega\rangle = & \left(\sin(\vartheta_i)\cos(\varphi_i) B_i^x + \sin(\vartheta_i)\sin(\varphi_i) B_i^y + \cos(\vartheta_i) B_i^z\right) |\Omega\rangle \\ & + \left(\cos(\vartheta_i)\cos(\varphi_i) B_i^x + \cos(\vartheta_i)\sin(\varphi_i) B_i^y - \sin(\vartheta_i) B_i^z - \mathrm{i}\left(\sin(\varphi_i) B_x - \cos(\varphi_i) B_y\right)\right) |i\rangle . \end{aligned}$$

$$\begin{aligned} & \frac{4}{\hbar^2} U_j^\dagger U_i^\dagger h_{ij} U_i U_j |\Omega\rangle \\ = & \left(\cos(\vartheta_i)\cos(\vartheta_j)\Delta + \sin(\vartheta_i)\sin(\vartheta_j)\left(\cos(\varphi_i - \varphi_j) J + \sin(\varphi_i - \varphi_j)\kappa_{ij} D\right)\right) |\Omega\rangle \\ & + \left(\cos(\vartheta_i)\sin(\vartheta_j)\left(\cos(\varphi_i - \varphi_j) J + \sin(\varphi_i - \varphi_j)\kappa_{ij} D\right) - \sin(\vartheta_i)\cos(\vartheta_j)\Delta - \mathrm{i}\sin(\vartheta_j)\left(\sin(\varphi_i - \varphi_j) J - \cos(\varphi_i - \varphi_j)\kappa_{ij} D\right)\right) |i\rangle \\ & + \left(\cos(\vartheta_j)\sin(\vartheta_i)\left(\cos(\varphi_i - \varphi_j) J + \sin(\varphi_i - \varphi_j)\kappa_{ij} D\right) - \sin(\vartheta_j)\cos(\vartheta_i)\Delta + \mathrm{i}\sin(\vartheta_i)\left(\sin(\varphi_i - \varphi_j) J - \cos(\varphi_i - \varphi_j)\kappa_{ij} D\right)\right) |j\rangle \\ & + \left(\sin(\vartheta_i)\sin(\vartheta_j)\Delta + \left(\cos(\vartheta_i)\cos(\vartheta_j) - 1\right)\left(\cos(\varphi_i - \varphi_j) J + \sin(\varphi_i - \varphi_j)\kappa_{ij} D\right)\right) |i,j\rangle \\ & + \mathrm{i}\left(\left(\cos(\vartheta_i) - \cos(\vartheta_j)\right)\left(\sin(\varphi_i - \varphi_j) J - \cos(\varphi_i - \varphi_j)\kappa_{ij} D\right)\right) |i,j\rangle . \end{aligned}$$

$$\begin{aligned} \varepsilon(\boldsymbol{\vartheta}, \boldsymbol{\varphi}) = & \frac{\hbar^2}{4} \sum_{\{i,j\} \in E} \left(\cos(\vartheta_i)\cos(\vartheta_j)\Delta + \sin(\vartheta_i)\sin(\vartheta_j)\left(\cos(\varphi_i - \varphi_j) J + \sin(\varphi_i - \varphi_j)\kappa_{ij} D\right)\right) \\ & + \frac{\hbar}{2} \sum_{i \in V} \left(\sin(\vartheta_i)\cos(\varphi_i) B_i^x + \sin(\vartheta_i)\sin(\varphi_i) B_i^y + \cos(\vartheta_i) B_i^z\right) . \end{aligned}$$

Product state ansatz – results

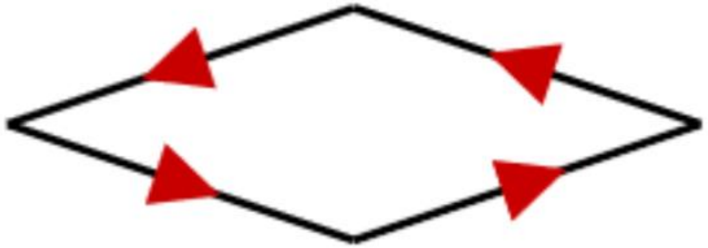
$$|\Psi(\boldsymbol{\vartheta}, \boldsymbol{\varphi})\rangle = \bigotimes_{i \in V} \left(\cos\left(\frac{\vartheta_i}{2}\right) e^{-\mathrm{i}\frac{\varphi_i}{2}} |\uparrow\rangle_i + \sin\left(\frac{\vartheta_i}{2}\right) e^{\mathrm{i}\frac{\varphi_i}{2}} |\downarrow\rangle_i \right)$$

Findings:

- Polar angle constant throughout system $\vartheta_i = \Theta$
- **Adjacent azimuthal angles change by $\pm\boldsymbol{\gamma}$** with $\gamma = \arccos\left(\frac{\Delta}{\mathrm{J}}\right)$
    - Spin current constant magnitude on every edge
- All product eigenstates degenerate $\epsilon = \frac{\hbar^2 \Delta}{4} \times \#edges$

Circuit rule

$$\sum_{loop} \varphi_{j+1} - \varphi_j = 2\,\pi\,n$$

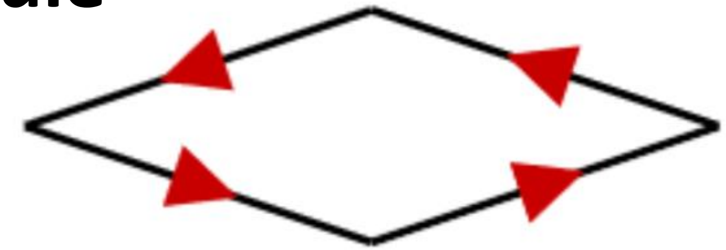


Analogue to quantized flux through plaquettes

Product eigenstates from conserved spin currents

Kirchhoff rule

Circuit rule

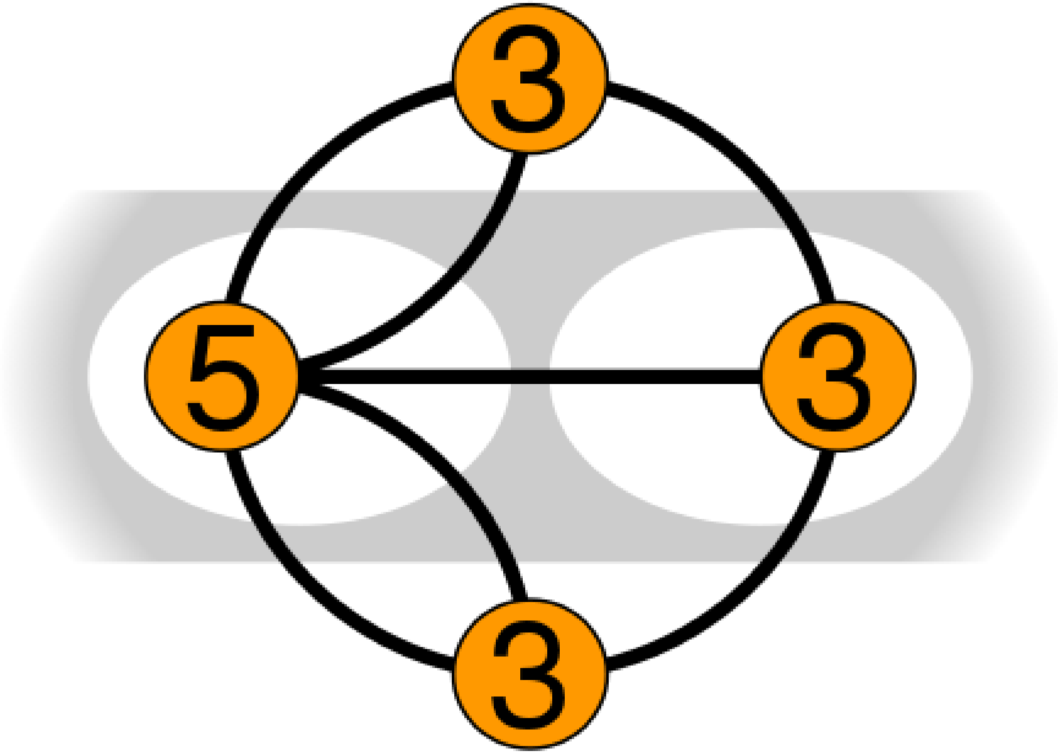


Consistent spin current pattern ⇔ product eigenstate

Complete product eigenstate classification
of XXZ Heisenberg models on graphs

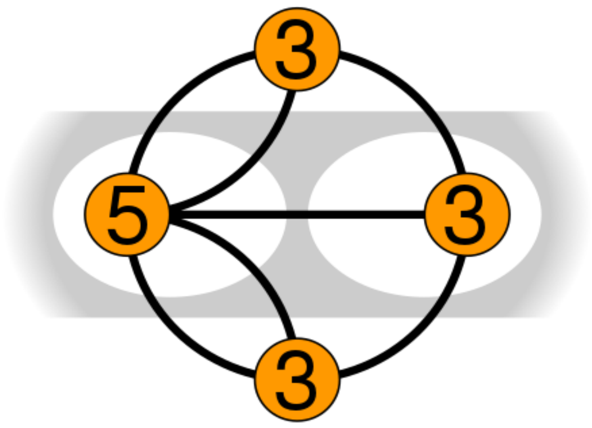
Euler cycles

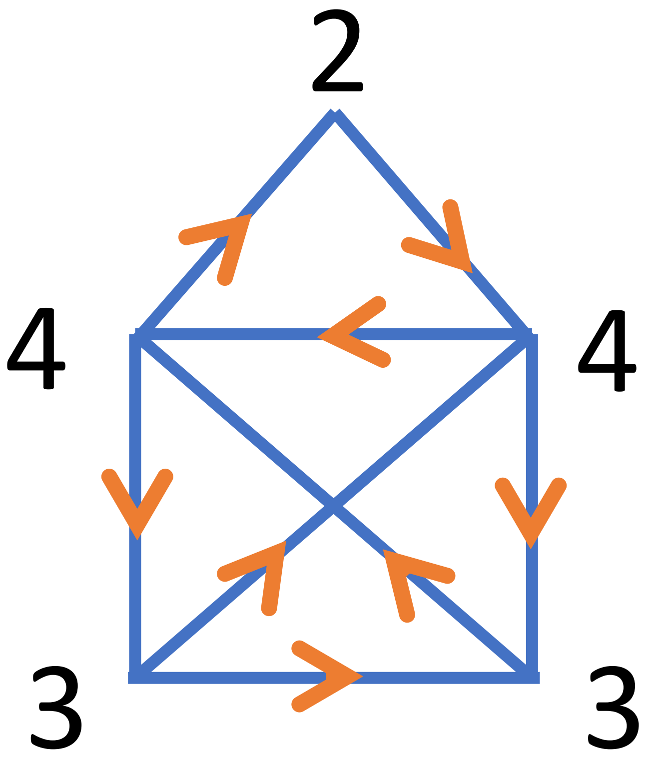
Königsberg bridge problem



Leonhard Euler
Picture: Jakob Emanuel Handmann,
Public domain, via Wikimedia Commons

Euler cycles





Leonhard Euler
Picture: Jakob Emanuel Handmann,
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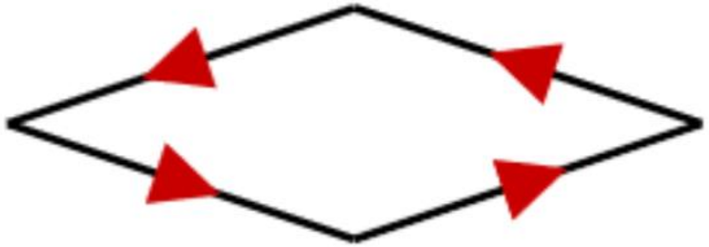
Impossible to draw House of Nikolaus starting from the top

Euler cycles

Kirchhoff rule

$\Leftrightarrow$ Path through graph exists that travels each edge exactly once.

Circuit rule



$\Leftrightarrow$ Choose angle γ appropriate to lattice

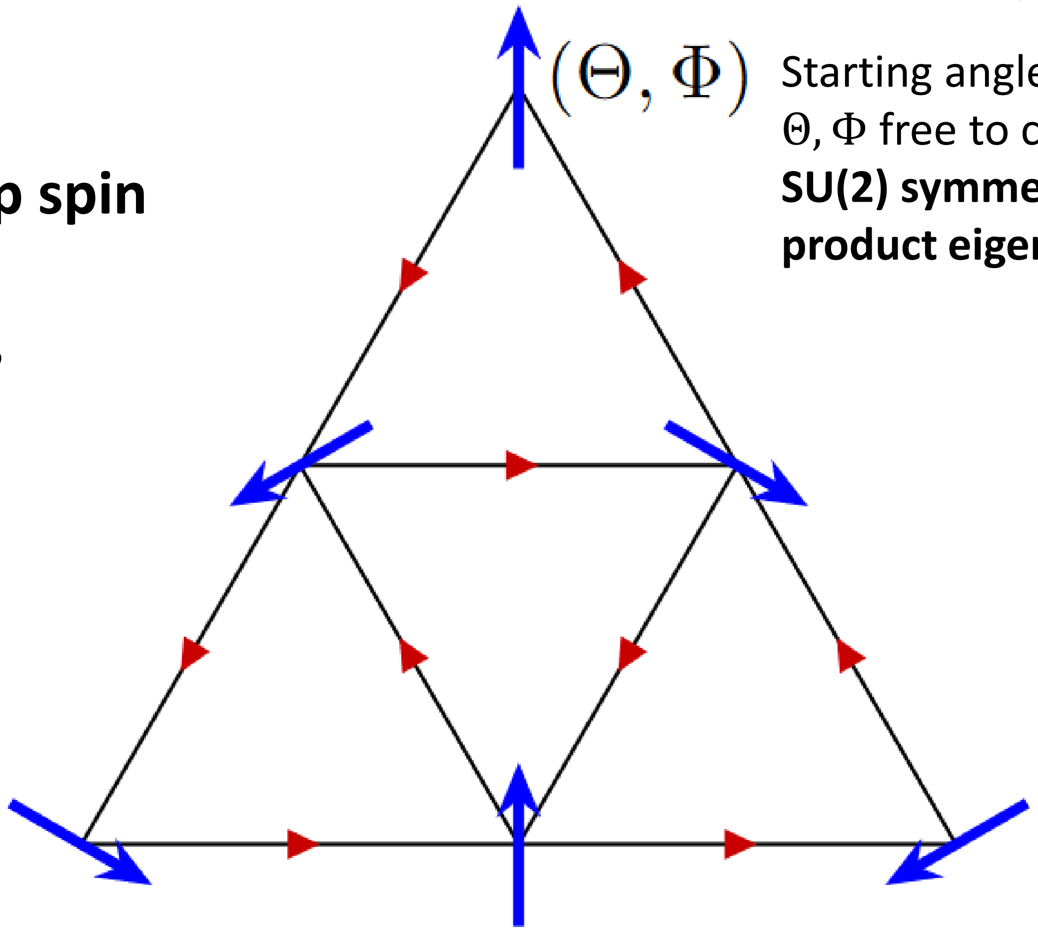
Leonhard Euler
Picture: Jakob Emanuel Handmann,
Public domain, via Wikimedia Commons

Hierholzer, Carl
Mathematische Annalen, ***6*** *(1): 30–32 (1873)*

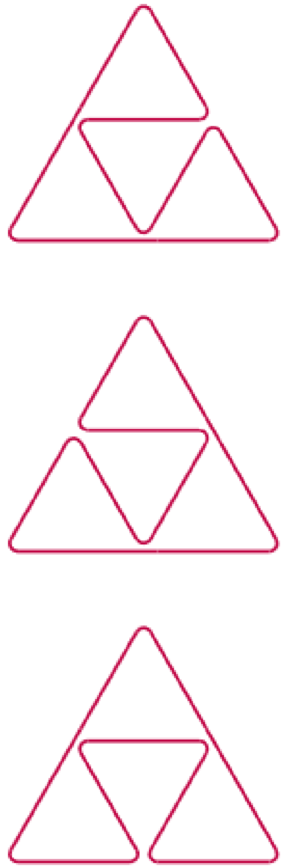
Example product eigenstate and Euler cycles

Wrapped up spin spiral along Euler cycles

$$\gamma = \frac{2\pi}{3}$$



Starting angles
Θ, Φ free to choose
SU(2) symmetry of product eigenspace



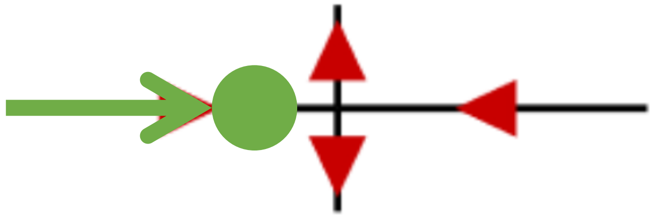
Extended class of magnetic systems

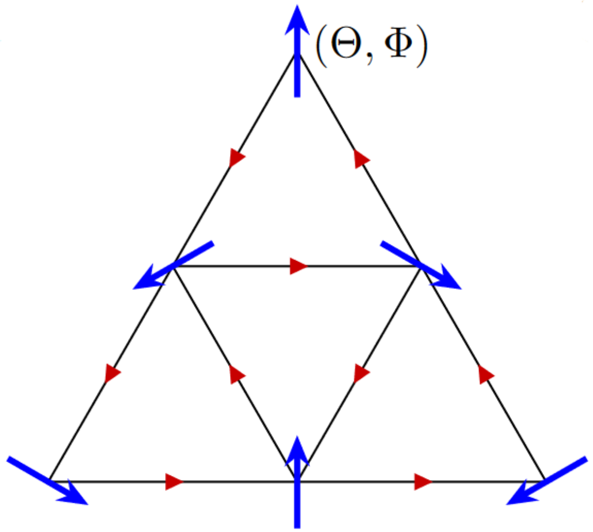
$$H = \sum_{\{k,l\}} J\, S_k^x S_l^x + J\, S_k^y S_l^y + \Delta\, S_k^z S_l^z + D_{k,l}\left(S_k^x S_l^y - S_k^y S_l^x\right) + \sum_k \overrightarrow{B_k} \cdot \overrightarrow{S_k}$$

- Dzyaloshinskii-Moriya interatcion $D_{k,l}\left(S_k^x S_l^y - S_k^y S_l^x\right)$ adds to angle γ of adjacent spins

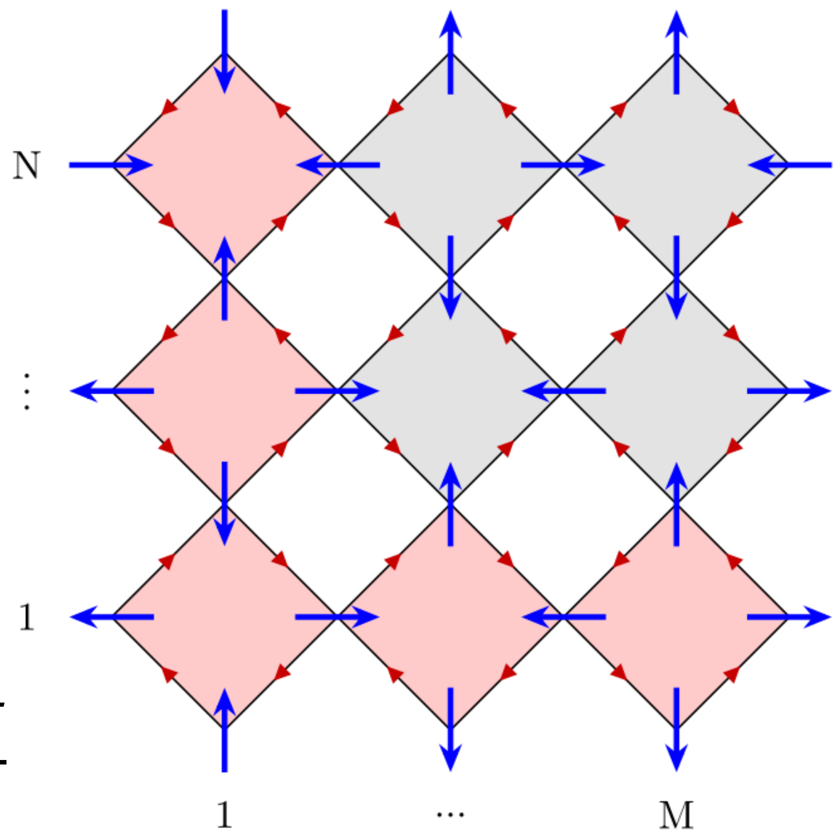
$$\gamma \to \gamma + \mathrm{sgn}\left(D_{k,l}\right) \arg\left(J + i\, D_{k,l}\right)$$

- Local magnetic fields $\sum_k \overrightarrow{B_k} \cdot \overrightarrow{S_k}$ are local sources of spin currents that break rot. symmetry



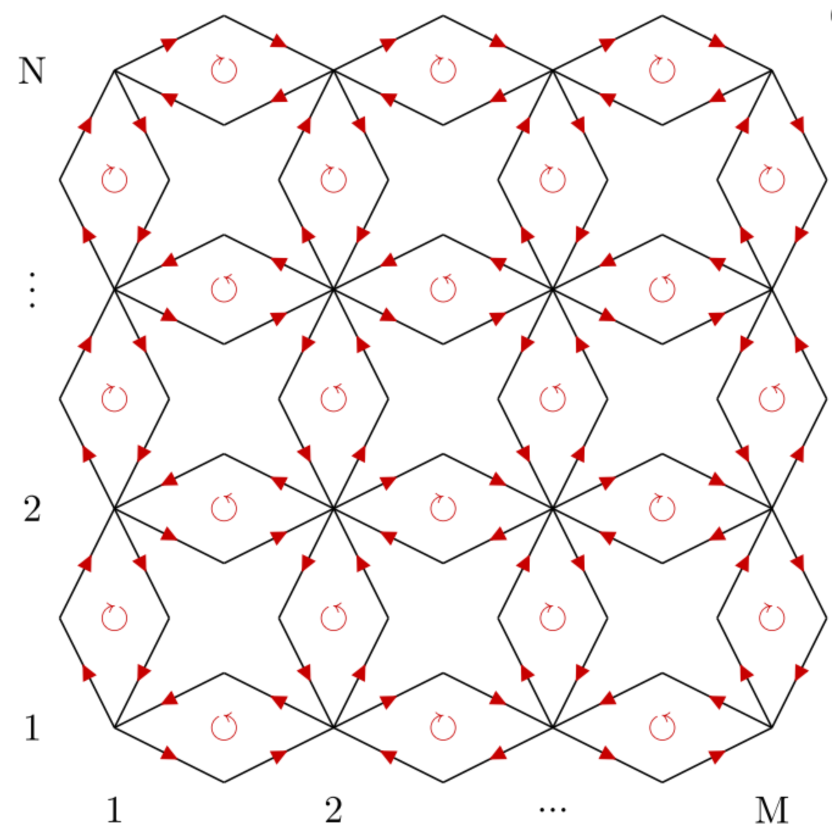


Large degenerace product eigenspaces on special graphs



$\gamma = \frac{\pi}{2}$

Degeneracy $> 2^{M+N+1}$



$\gamma = \frac{\pi}{2}$

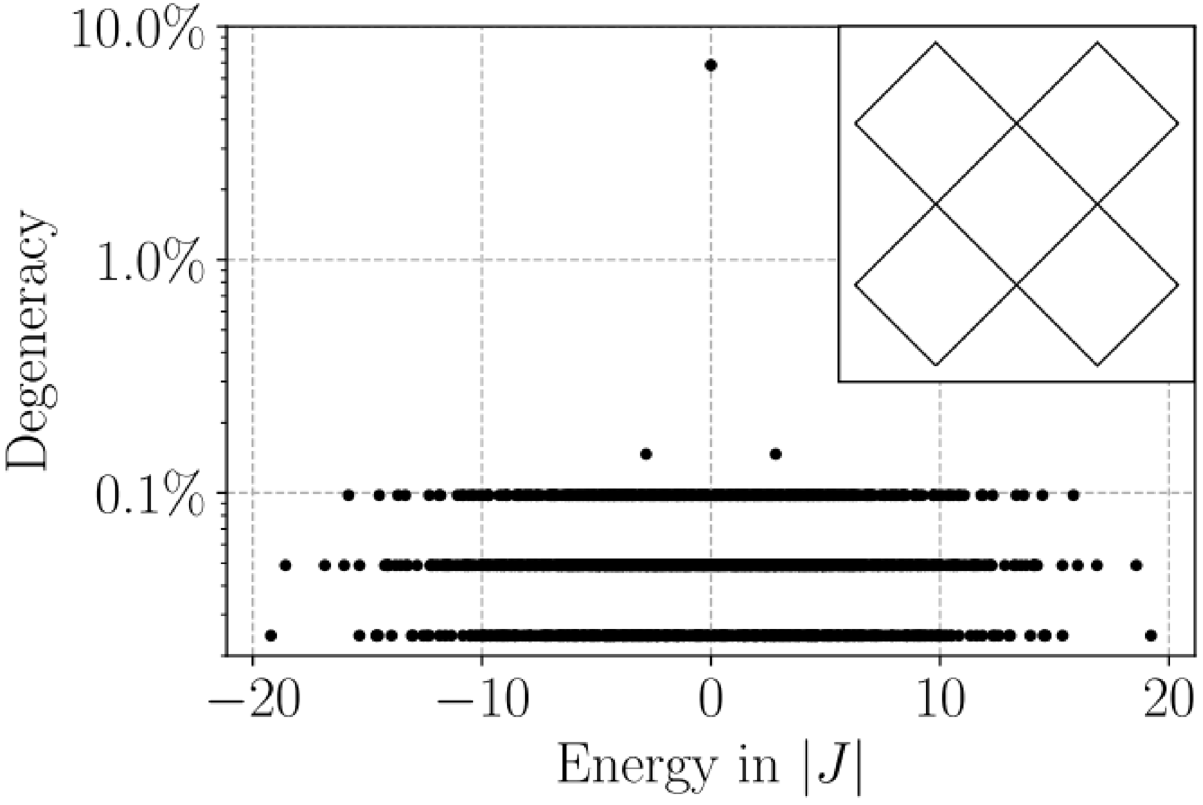
Degeneracy $> 2^{2NM+N+M}$

Mid-spectrum condensate

Macroscopic mid-spectrum degeneracy at energy

$$\epsilon = \frac{\hbar^2 \Delta}{4} \times \#edges$$

Not all degeneracy explained by product states



Hidden anyonic condensate?

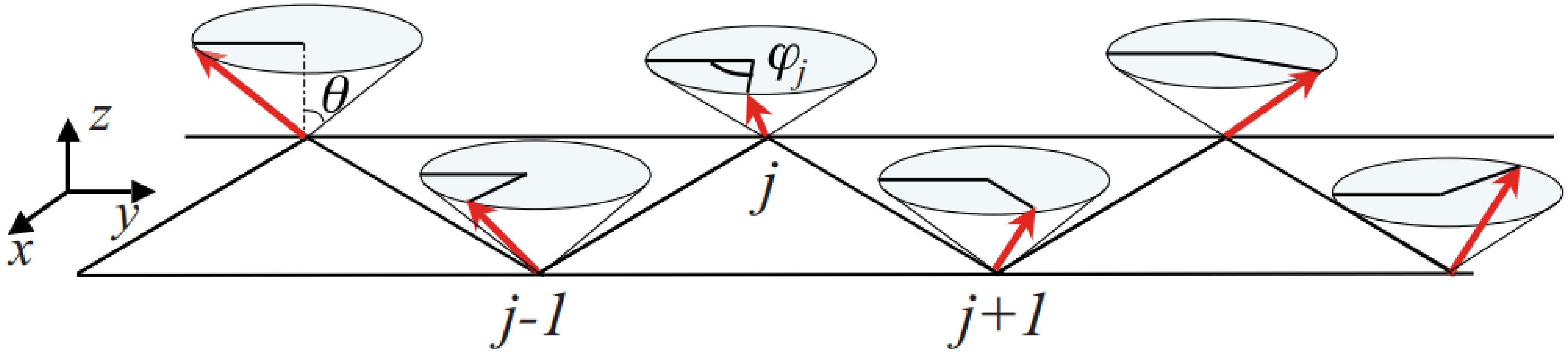
PRL **109**, 227203 (2012) PHYSICAL REVIEW LETTERS week ending 30 NOVEMBER 2012

Condensation of Anyons in Frustrated Quantum Magnets

C. D. Batista* and Rolando D. Somma

Theoretical Division, T-4 and CNLS, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA

(Received 23 June 2012; published 28 November 2012)



Maps spins to anyons

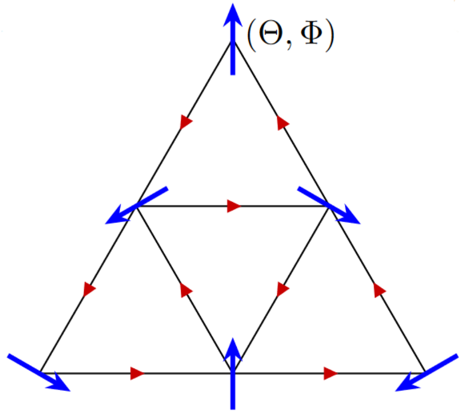
$$a_j^\dagger = \exp\{i\phi \textstyle\sum_{l<j}(s_l^z + 1/2)\} s_j^+$$

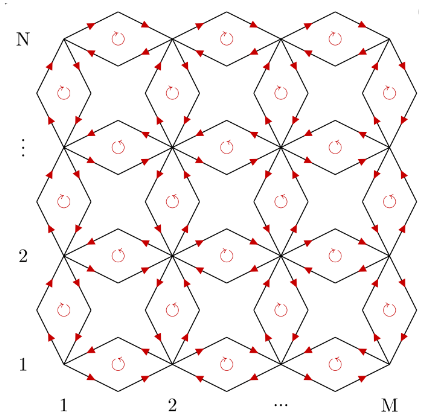
Anyon condensate

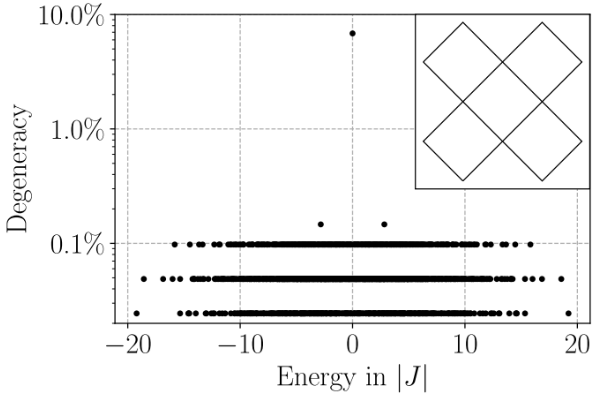
$$|\psi_{n,m}(Q)\rangle = \lim_{\phi \to -4Q} c_\phi (\bar{a}_Q^\dagger)^n (\bar{a}_{-Q}^\dagger)^m |\varnothing\rangle$$

Conclusions and outlook

- **Flexible general theory of product eigenstate in easy-plane quantum magnets**
- **Degenerate spin states at ground state and finite energy**
- New vacua for Bethe ansatz and semi-analytical ansätze
- Pertubative approaches
- Benchmark for numerical methods
- Preparation schemes and **physical properties** of mid-spectrum **condensates**, optical/excitation properties
- Connection to hardcore anyons **beyond product states**

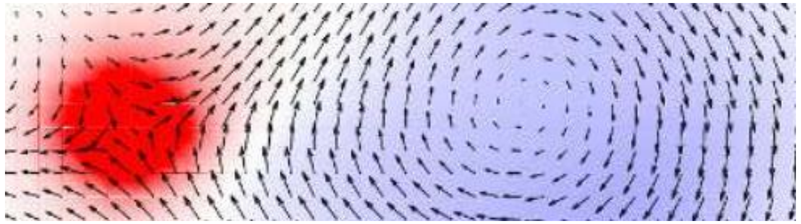






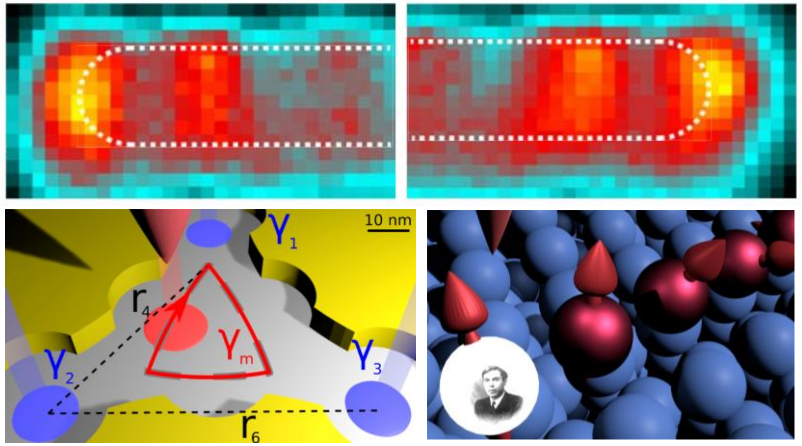
Posske group research

Topological magnetism



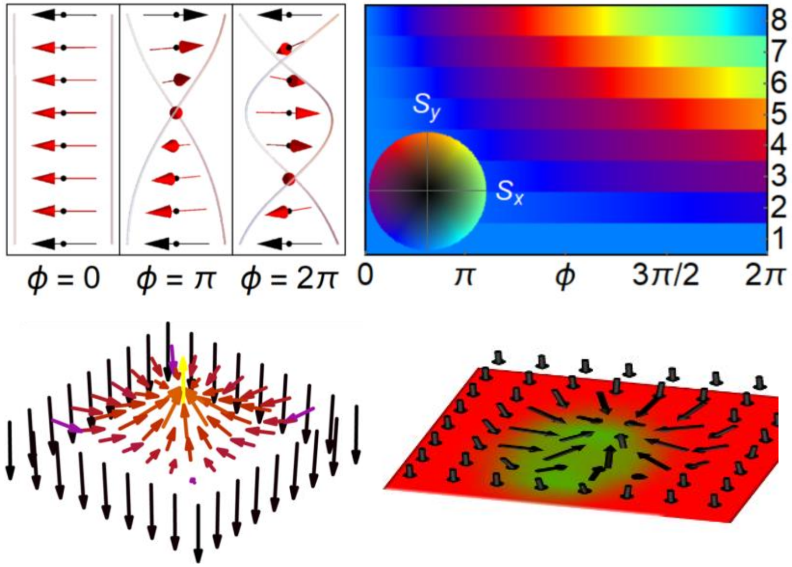
PRL (2017)

Topological phases & Majoranas



Science Adv. (2018), **Nat. Phys.** (2021), **Nat. Nano.** (2022), **Nature** (2023), **US patent** (2024)

Topological quantum magnetism



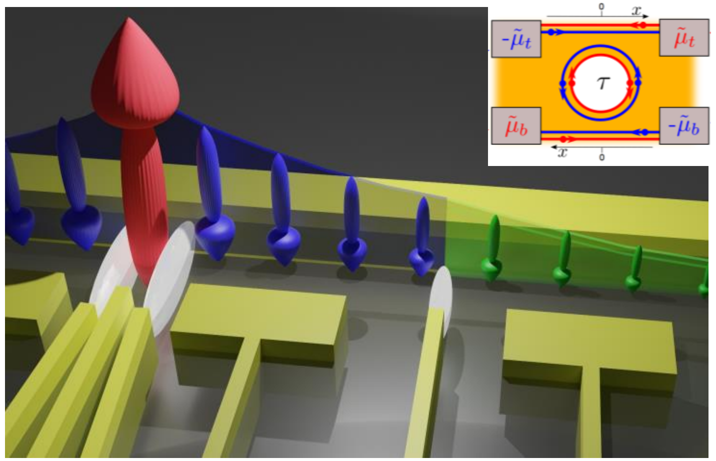
PRL (2019), **DFG** (2020), **ERC** (2023)

Anyons

$$a_p^\dagger a_q^\dagger = e^{i\phi_\eta(p-q)} a_q^\dagger a_p^\dagger,$$
$$a_p a_q^\dagger = e^{-i\phi_\eta(p-q)} a_q^\dagger a_p + \delta(p-q)$$

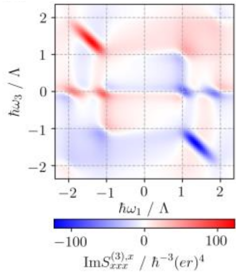
PRL (2021)

Kondo effect



PRL (2013), **PRL** (2015)

Spectroscopy + top. matter



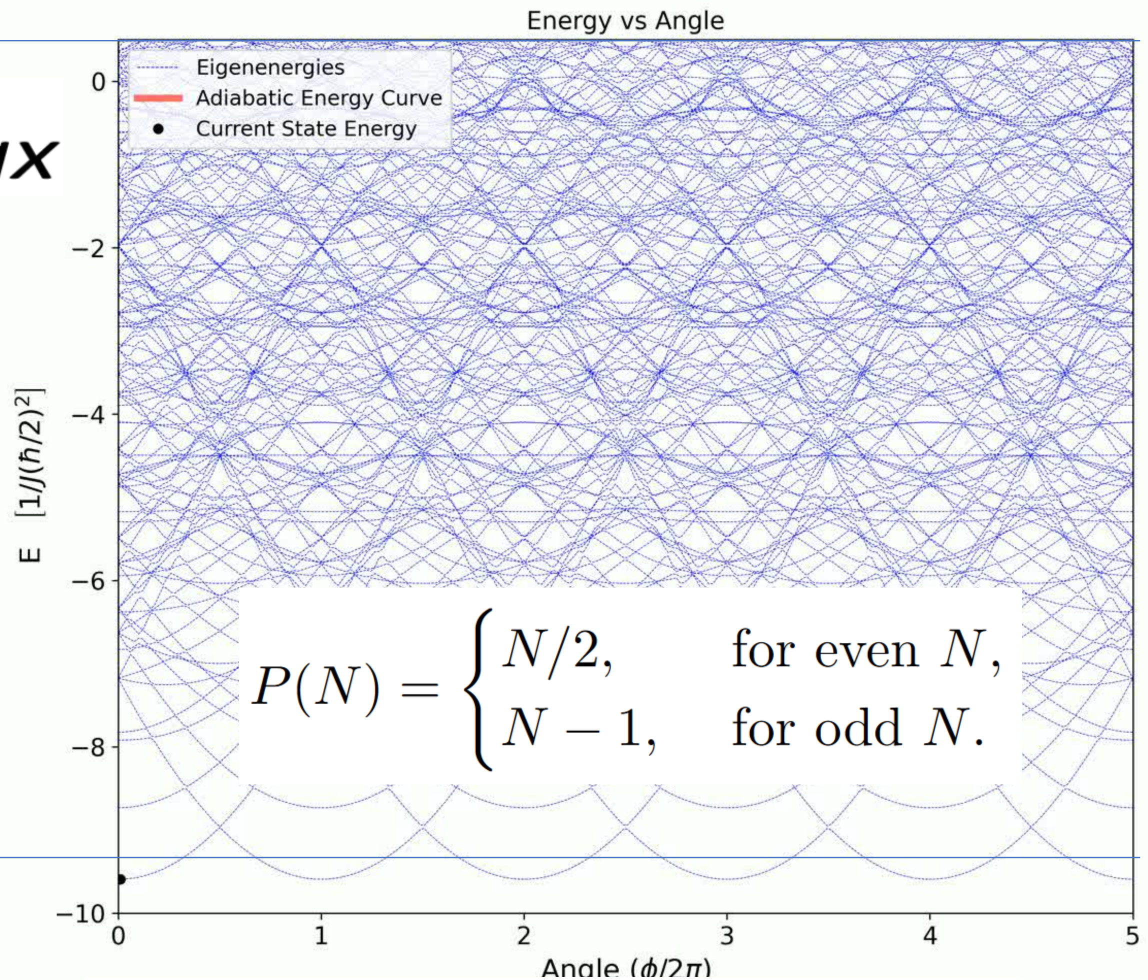
PRL (2022)

Generalized high-periodicity Josephson effect

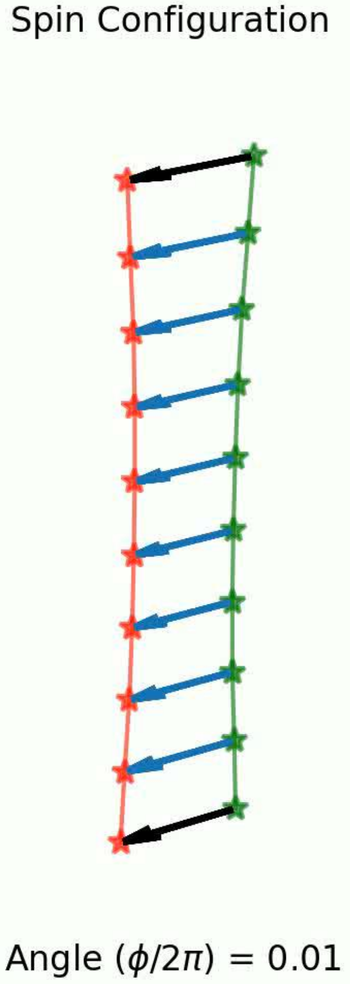
Tripathi, Gerken, Schmitteckert, Thorwart, Trif, Posske, PRR **7**, 013272 (2025)

E_{max}

E_0



$$P(N) = \begin{cases} N/2, & \text{for even } N, \\ N-1, & \text{for odd } N. \end{cases}$$

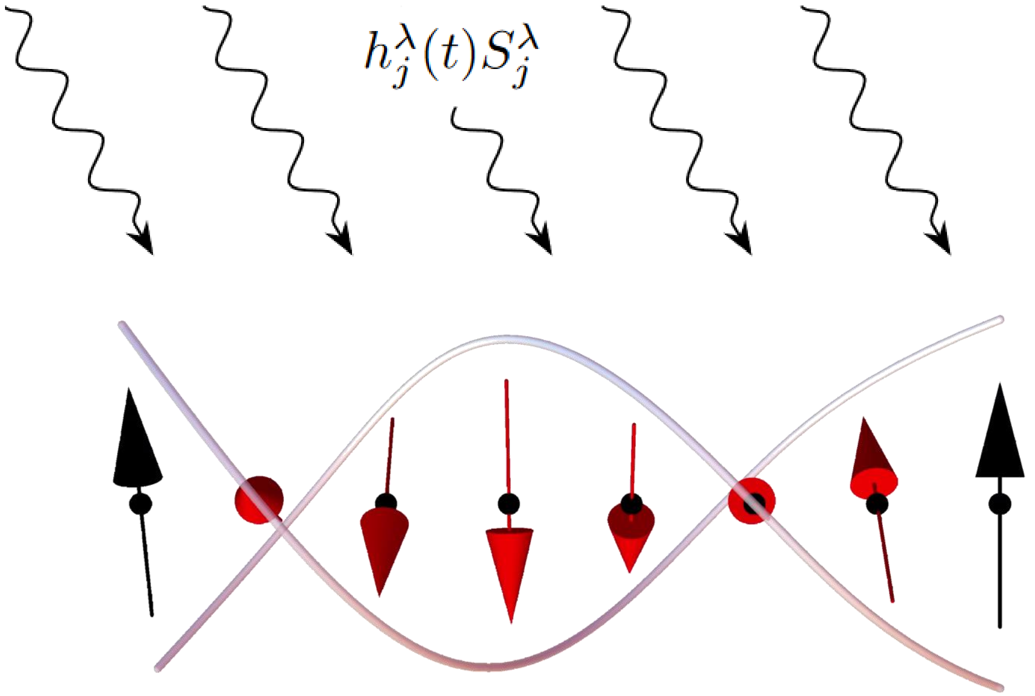


$$H(\phi) = \sum_{j=2}^{N-2} \left[J(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y) + \Delta S_j^z S_{j+1}^z \right] + J \left[\tilde{\mathbf{S}}_1 \cdot \mathbf{S}_2 + \mathbf{S}_{N-1} \cdot \tilde{\mathbf{S}}_N(\phi) \right].$$

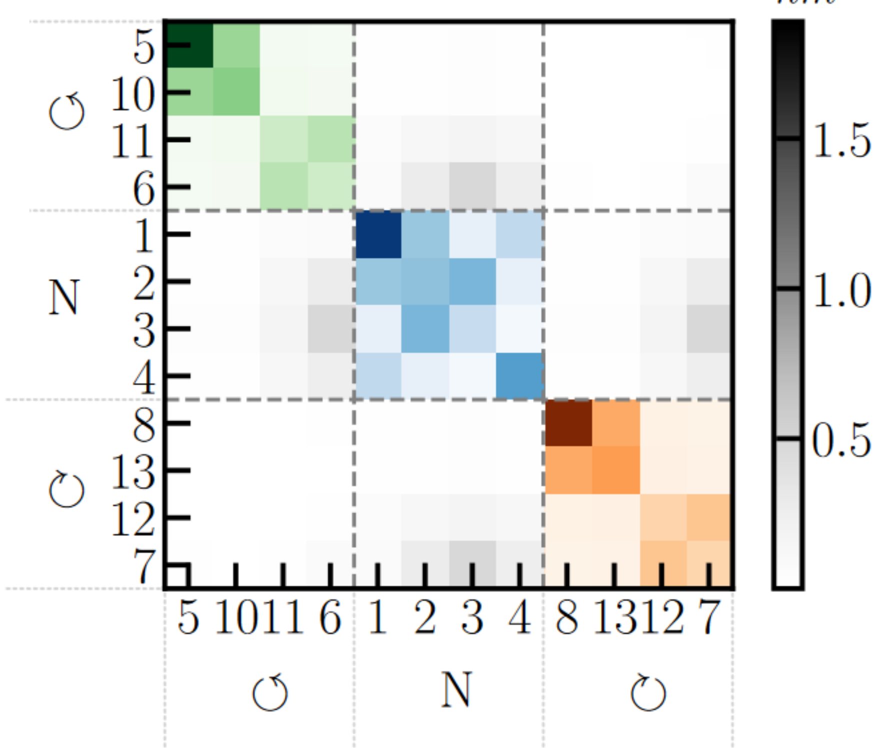
Stability

Kühn, Gerken, Funcke, Hartung, Stornati, Jansen, Posske PRB **107**, 214422 (2023)

$$\Gamma_{nm} \propto \left| \left\langle n \left| \sum h_j^\lambda S_j^\lambda \right| m \right\rangle \right|^2$$

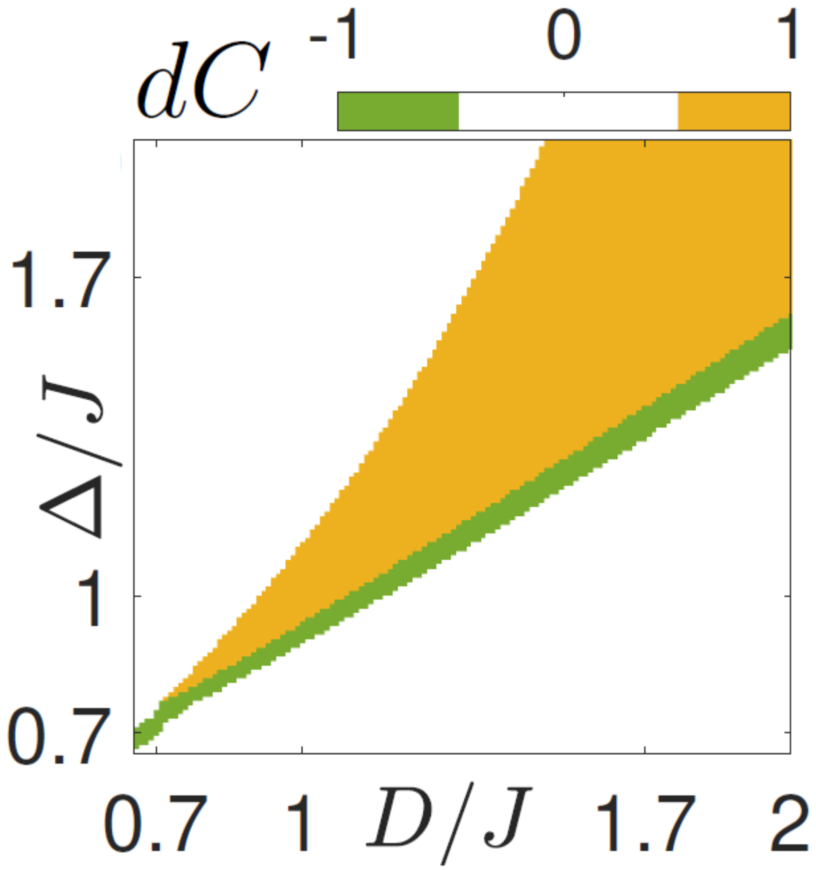


Tunneling amplitude



Creating quantum skyrmions

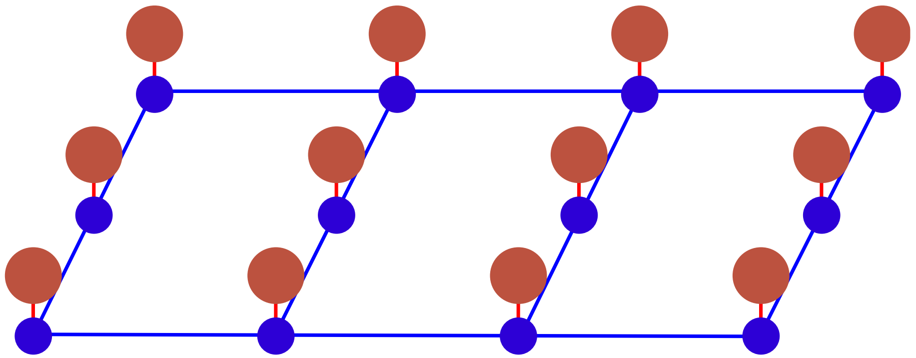
Siegl, Vedmedenko, Stier, Thorwart, Posske PRR **4**, 023111 (2022)



Electronic skyrmions

Peters, Neuhaus-Steinmetz, Posske, PRR **5**, 033180 (2023)

f electrons $\quad U\, f_\uparrow^\dagger f_\uparrow f_\downarrow^\dagger f_\downarrow$

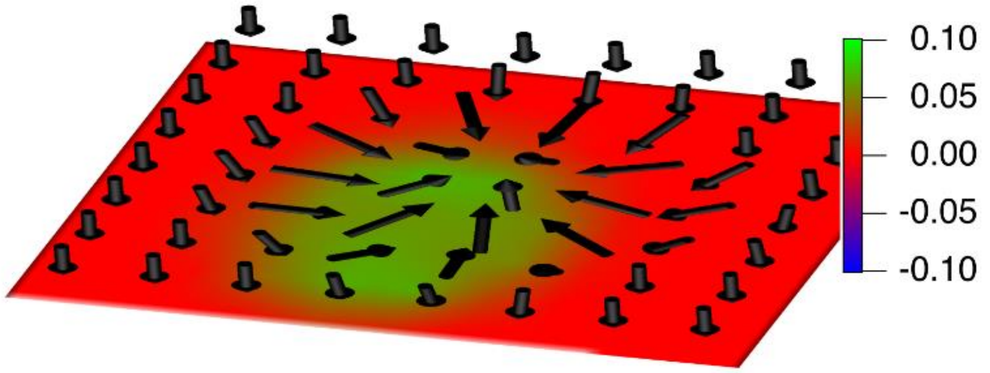


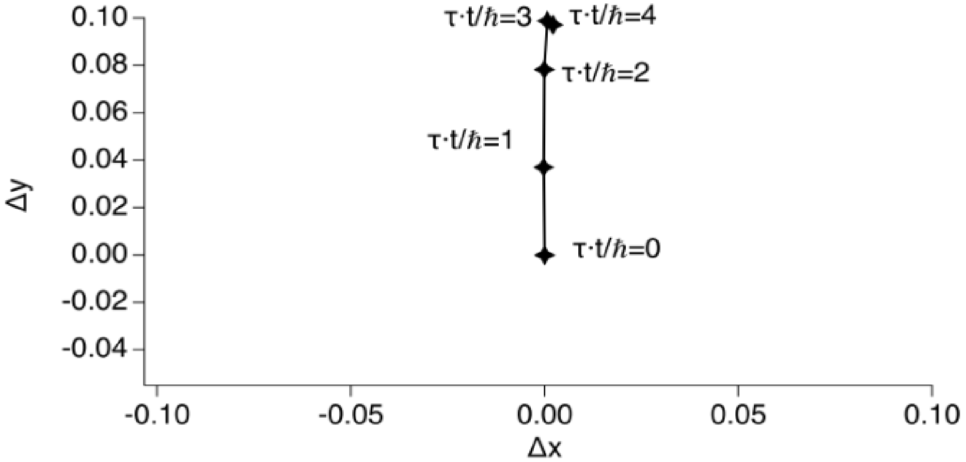
c electrons $\quad t\, c_i^\dagger c_{i+hop} + h.c.$

Spin-orbit coupling

$$\alpha_c \left(f_k^\dagger \sin(k_y)\sigma^x c_k - f_k^\dagger \sin(k_x)\, \sigma^y c_k \right) + h.c.$$

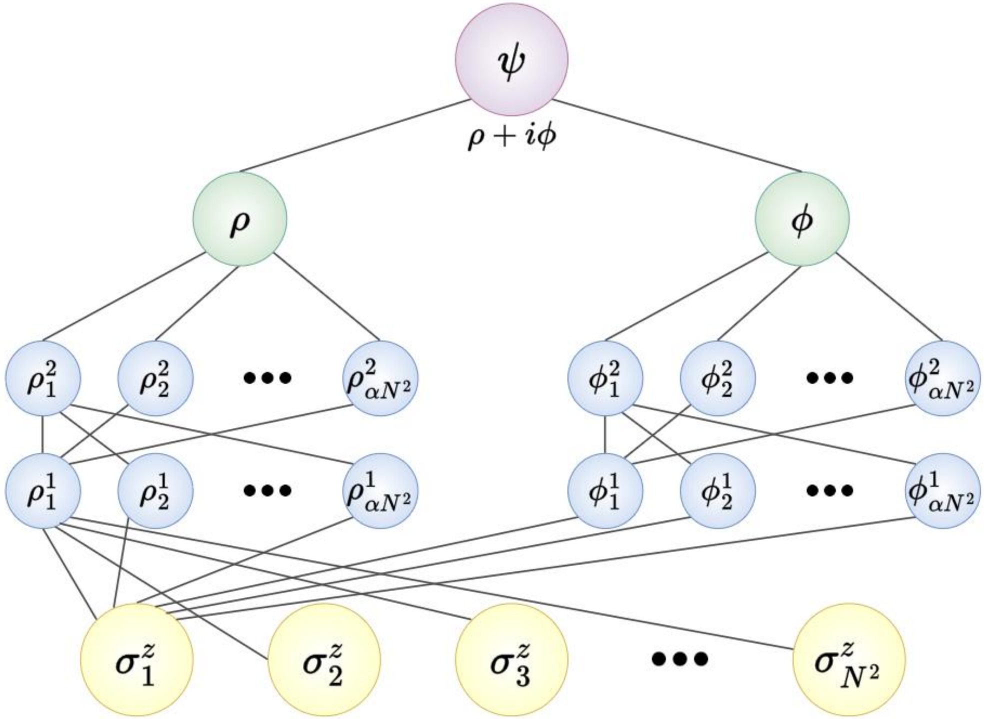
Skyrmion density

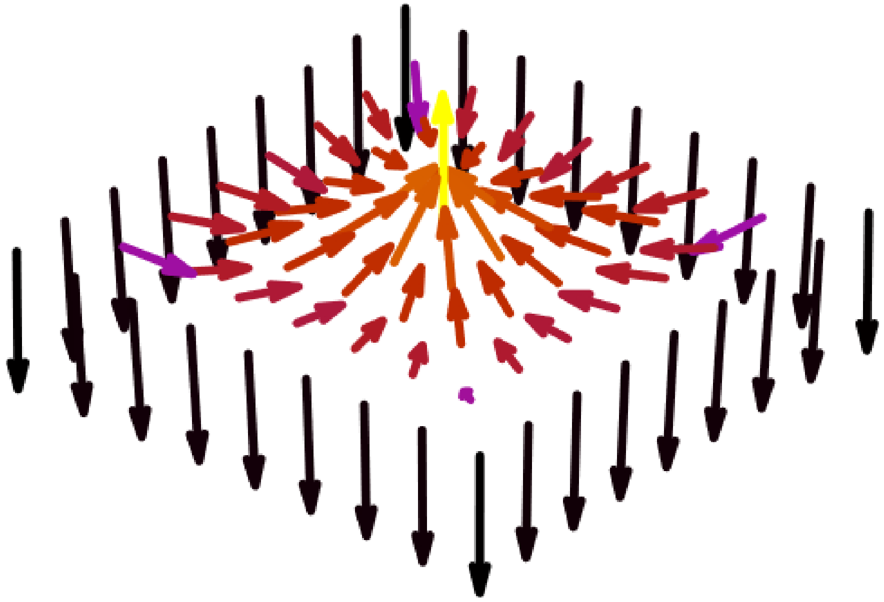




Artificial neural network quantum states

Joshi, Peters, Posske, PRB **110**, 104411 (2024)
Joshi, Peters, Posske, PRB **108**, 094410 (2023)





Thank you

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DFG Deutsche Forschungsgemeinschaft

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