

Chiral phonons

Thermal Hall effects and vacuum fluctuations

Benedetta Flebus
(Boston College)



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(UT Austin)

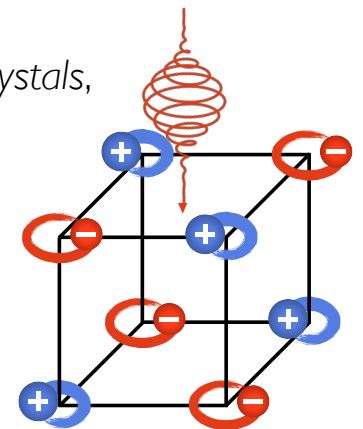


R. Yi



V. Williams

- R. Yi, V. Williams and B. Flebus, *Angular Momentum Fluctuations in the Phonon Vacuum of Symmetric Crystals*, arXiv (2025).
- B. Flebus, and A. H. MacDonald, *The phonon Hall viscosity of ionic crystals*, PRL 131, 236301 (2023).
- B. Flebus, and A. H. MacDonald, *Charged defects and phonon Hall effects in ionic crystals*, PRB 105, L220301 (2022).



Which type of 'chiral' phonons?

Acoustic phonons, 2d

$$\sum_j D_{ij}(\mathbf{k}, \omega) \xi_j = \omega^2 \xi_i$$

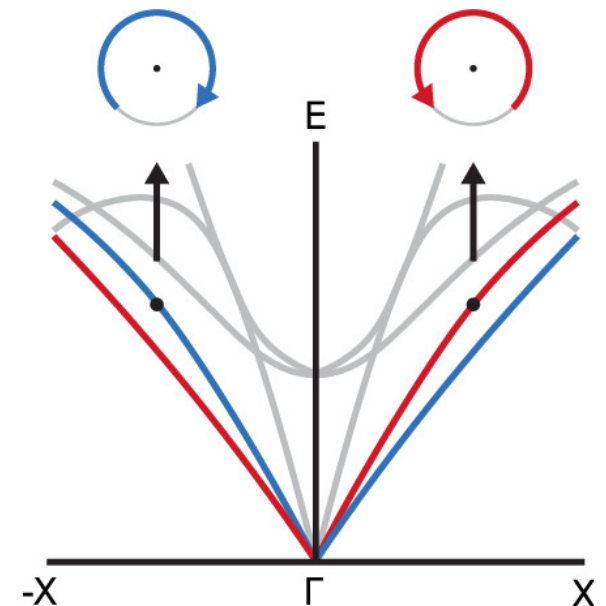
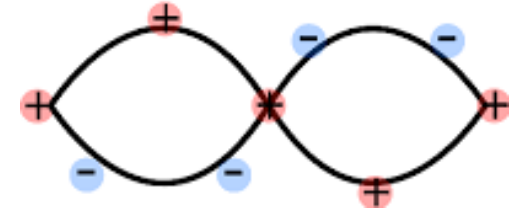
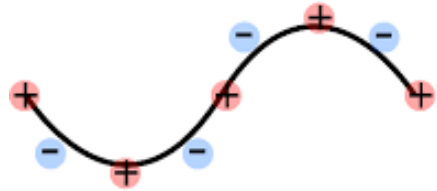
where

$$D_{ij}(\mathbf{k}, \omega) = \underbrace{c k^2 \delta_{ij} + \alpha k_i k_j}_{\text{'Usual dispersion'}} + \lambda(k_i \chi_j + k_j \chi_i) + i\eta\omega \epsilon_{ij}$$

'Usual dispersion'

- $\lambda(k_i \chi_j + k_j \chi_i)$ enabled by inversion symmetry breaking

Structural chirality
(Not in this talk)



TRS-broken phonons

$$\sum_j D_{ij}(\mathbf{k}, \omega) \xi_j = \omega^2 \xi_i$$

$$D_{ij}(\mathbf{k}, \omega) = c k^2 \delta_{ij} + \alpha k_i k_j + i\eta\omega \epsilon_{ij}$$

- $i\eta\omega\epsilon_{ij}$ enabled by time-reversal symmetry breaking



- Finite phonon angular momentum (intrinsic chirality)
- Finite thermal Hall effects (Onsager)

-
- Giant thermal Hall conductivity observed in several dielectric oxides
 - Mechanisms underlying phonon-driven thermal Hall effect
 - Do these thermal Hall responses imply an intrinsic phonon chirality?

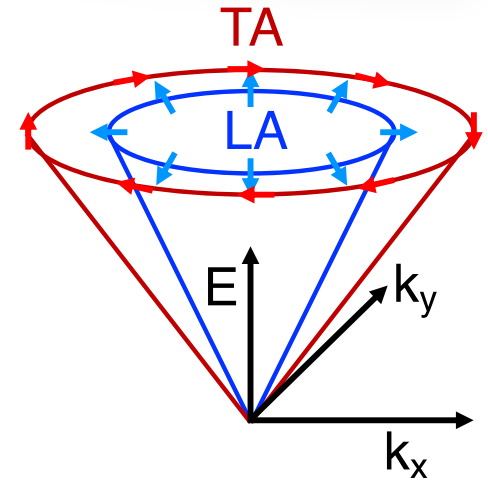
The 'simplest' phonons

$$D_{ij}(\mathbf{k}) = c k^2 \delta_{ij} + \alpha k_i k_j$$

$$\rightarrow \omega_{LA}(k) = \sqrt{c + \alpha} k, \quad \omega_{TA}(k) = \sqrt{c} k$$

● *PT*-symmetry implies :

No individual or net phonon angular momentum



Park and Yang, NanoLett (2020)

-
- However, there are **finite** angular momentum fluctuations
 - They survive at $T=0$: novel, purely collective, type of vacuum noise
 - How can we experimentally access these vacuum effects?

R. Yi, V. Williams and B. Flebus, arXiv (2025)

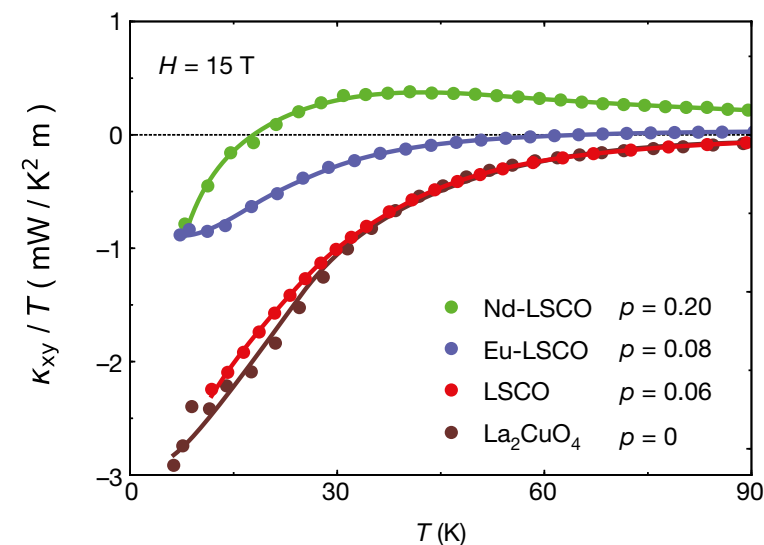
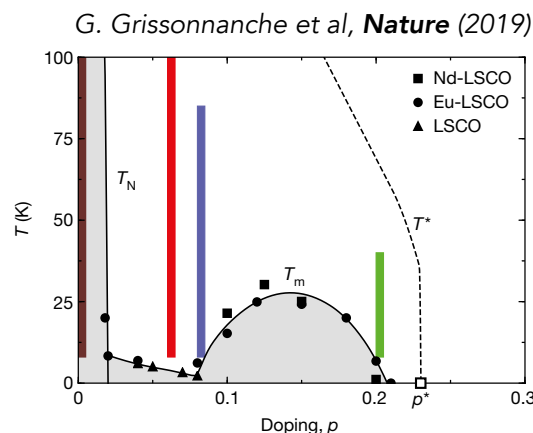
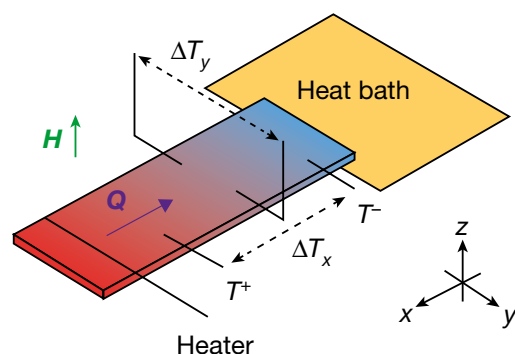
Giant Phonon thermal Hall (inception)

LETTER

<https://doi.org/10.1038/s41586-019-1375-0>

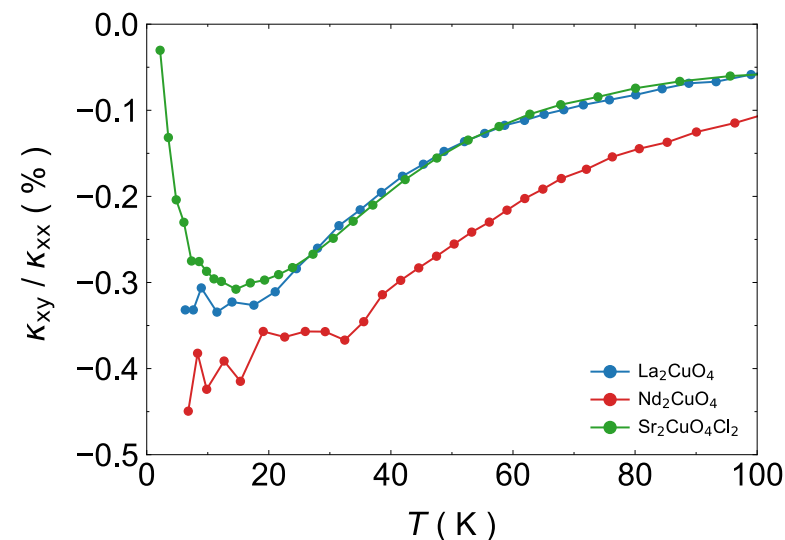
Giant thermal Hall conductivity in the pseudogap phase of cuprate superconductors

G. Grissonnanche^{1*}, A. Legros^{1,2}, S. Badoux¹, E. Lefrançois¹, V. Zlatko¹, M. Lizaire¹, F. Laliberté¹, A. Gourgout¹, J.-S. Zhou³, S. Pyon^{4,5}, T. Takayama^{4,6}, H. Takagi^{4,6,7,8}, S. Ono⁹, N. Doiron-Leyraud¹ & L. Taillefer^{1,10*}

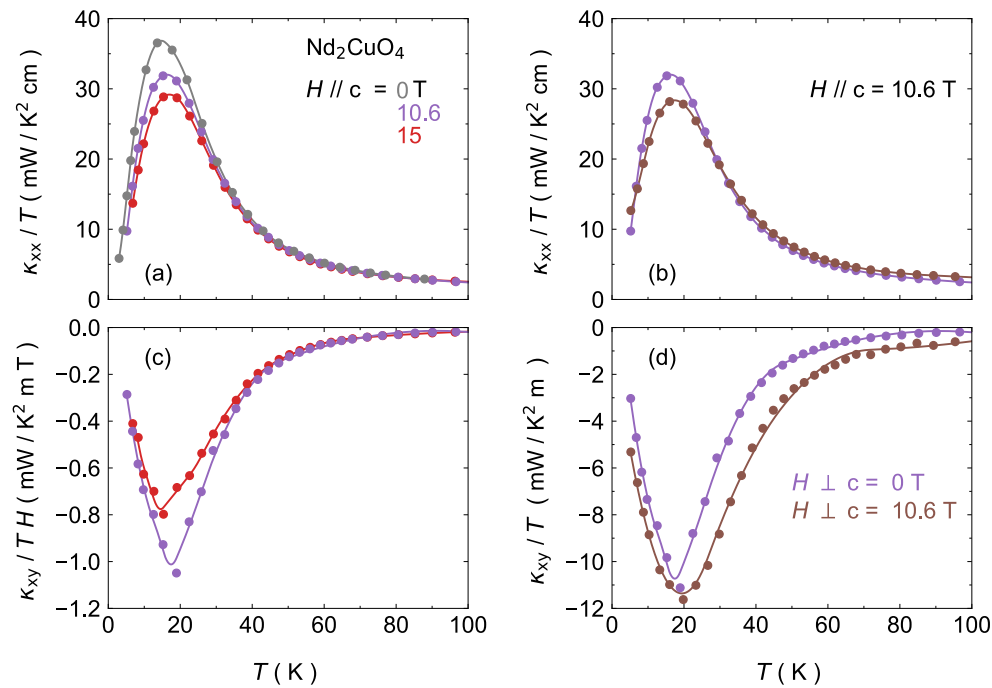


Giant thermal Hall signal appears in the pseudo-gap phase and reaches its maximum value in the Mott **insulating phase**

Electrons are not the main heat carriers

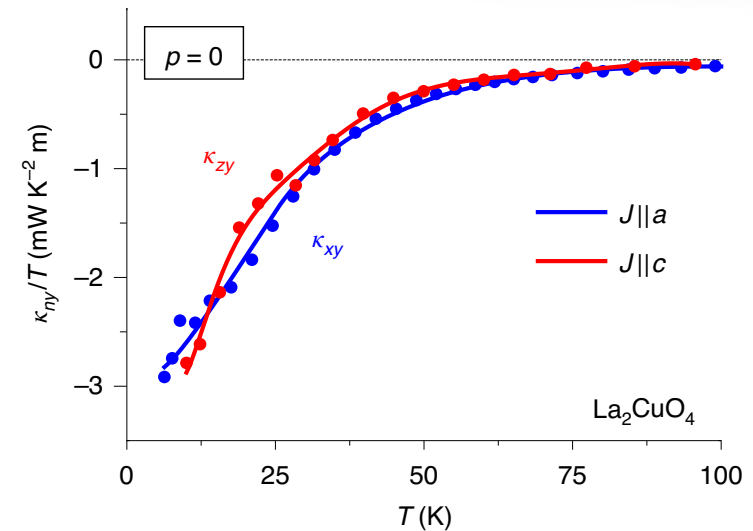


Magnons?



Strong in-plane magnetic field
modifies magnetic order

*Data show a very weak dependence of
the signal on the magnetic order*



Magnon spectrum in La_2CuO_4
is very anisotropic (quasi-2d)

C. Hess et al., PRL (2003)

*The thermal Hall conductivity
is almost isotropic,
like the phonon spectrum.*

PHYSICAL REVIEW LETTERS **124**, 105901 (2020)

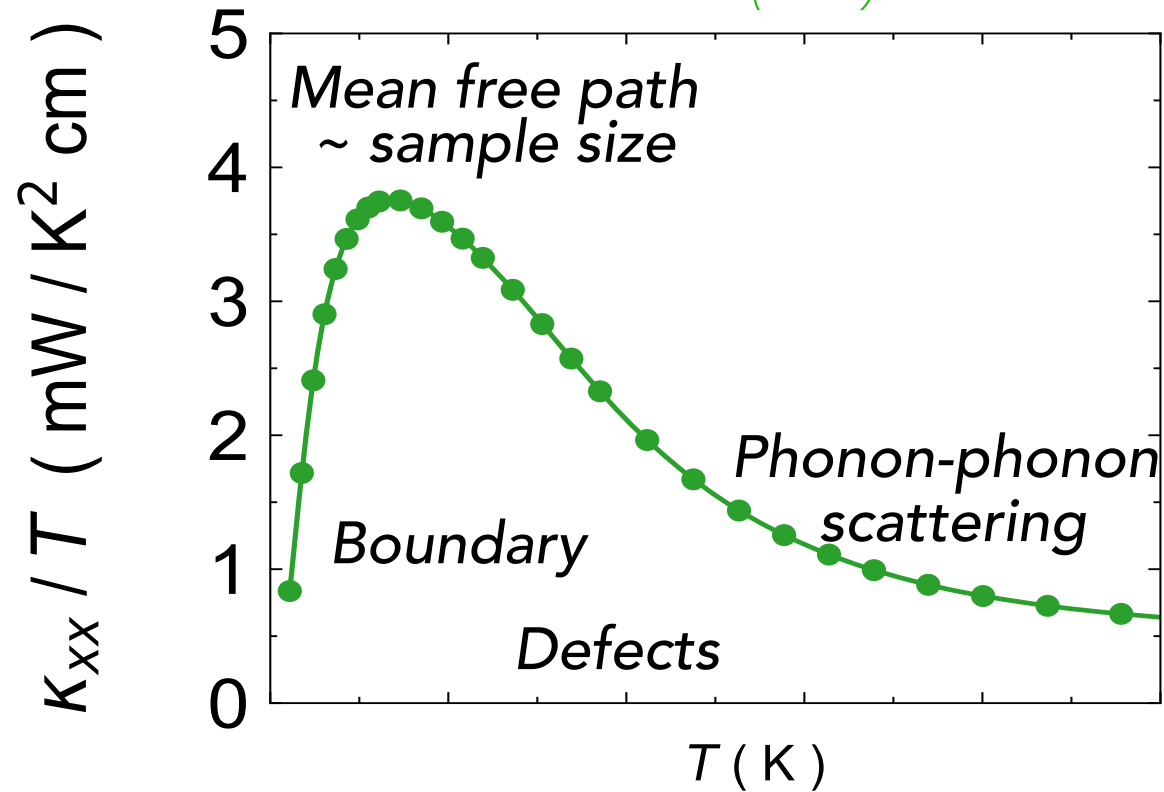
Phonon Thermal Hall Effect in Strontium Titanate

Xiaokang Li,^{1,2} Benoît Fauqué,³ Zengwei Zhu,² and Kamran Behnia^{1,4}

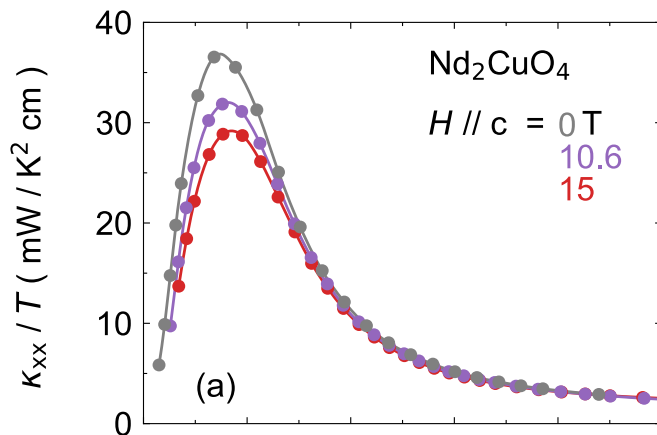
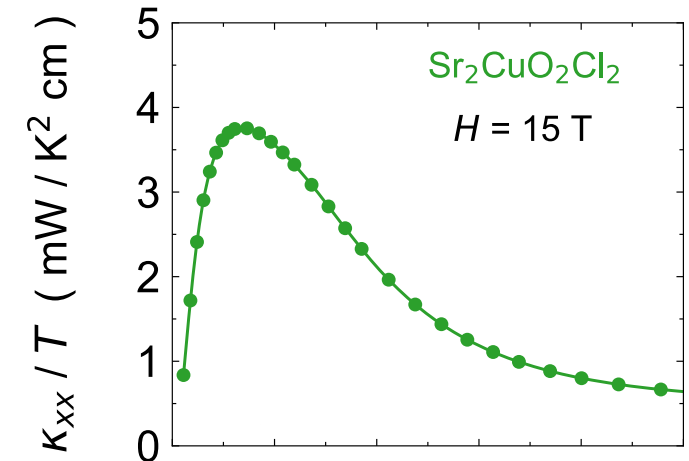
*Giant thermal Hall in systems
with no magnetic order*

Phonons as the leading heat carriers

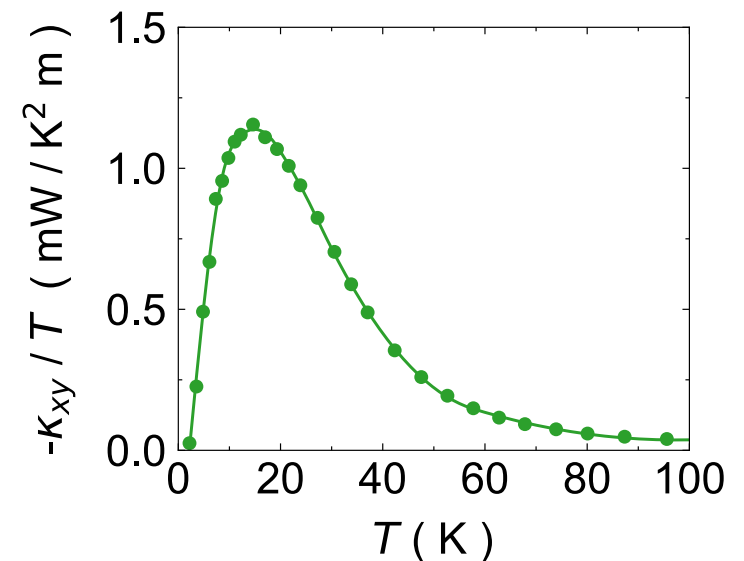
P. G. Klemens (1951)



SIMILAR TEMPERATURE DEPENDENCE



Longitudinal conductivity due to phonons



Giant thermal Hall effects in dielectric oxides

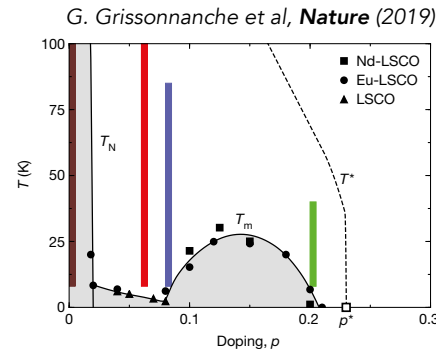
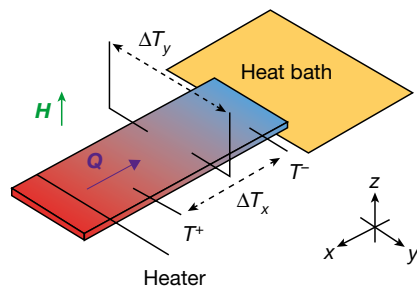
Insulating phase of high- T_c superconductors

LETTER

<https://doi.org/10.1038/s41586-019-1375-0>

Giant thermal Hall conductivity in the pseudogap phase of cuprate superconductors

G. Grissonnanche^{1*}, A. Legros^{1,2}, S. Radoux¹, E. Lefrançois¹, V. Zaitko¹, M. Lizaire¹, F. Laliberté¹, A. Gourgout¹, J.-S. Zhou¹, S. Pyon^{4,5}, T. Takayama^{4,6}, H. Takagi^{4,6,7,8}, S. Ono⁹, N. Doiron-Leyraud¹ & L. Taillefer^{1,10*}



PNAS

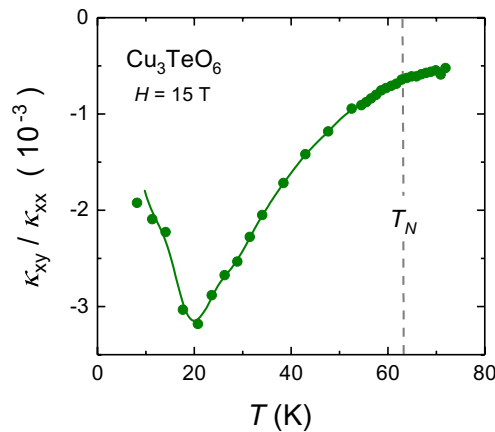
RESEARCH ARTICLE | PHYSICS

OPEN ACCESS



Large phonon thermal Hall conductivity in the antiferromagnetic insulator Cu_3TeO_6

Lu Chen^{1*}, Marie-Eve Boulanger¹, Zhi-Cheng Wang¹, Fazel Tafti¹, and Louis Taillefer^{1,2*}



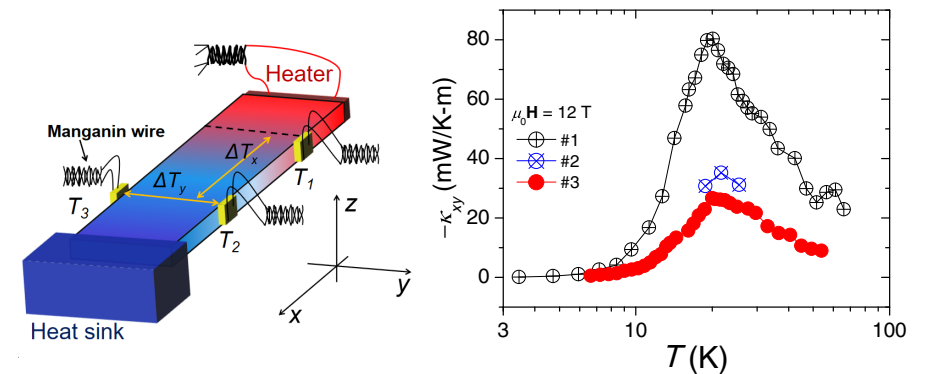
Insulating AFM

Non-magnetic dielectric oxide

PHYSICAL REVIEW LETTERS **124**, 105901 (2020)

Phonon Thermal Hall Effect in Strontium Titanate

Xiaokang Li,^{1,2} Benoît Fauqué,³ Zengwei Zhu,² and Kamran Behnia^{1,4}



Material	κ_{xy} [mW/Km]	κ_{xx} [W/Km][10^{-3}]	κ_{xy}/κ_{xx}	T, H [K], [T]
Cu_3TeO_6	- 1000	330	- 3.0	20, 15
$\text{Fe}_2\text{Mo}_3\text{O}_8$ [4]	12	2.5	4.8	65, 14
$\text{Tb}_2\text{Ti}_2\text{O}_7$ [19]	1.2	0.27	4.4	15, 12
$\text{Y}_2\text{Ti}_2\text{O}_7$ [19, 20]	0	18	0	15, 8
La_2CuO_4 [5]	- 38	12	- 3.2	20, 15
$\text{Sr}_2\text{CuO}_2\text{Cl}_2$ [6]	- 21	7	- 3.0	20, 15
Nd_2CuO_4 [6]	- 200	56	- 3.6	20, 15
SrTiO_3 [8]	- 80	36	- 2.2	20, 12
KTaO_3 [8]	2	32	0.06	30, 12
RuCl_3 [21]	8	15.5	0.5	20, 15

Breaking time-reversal symmetry

Low- T thermal Hall effects $\longrightarrow$ Acoustic phonons

What allows acoustic phonons to couple to the magnetic field?

Time-reversal-symmetry-breaking (TRSB) mechanisms :

- **TRSB embedded in the phonon medium - *intrinsic***
(i.e., Hamiltonian coupling term)

Coupling to a degree of freedom that couples to the magnetic field :
electronic states, polarization, magnetization, ...

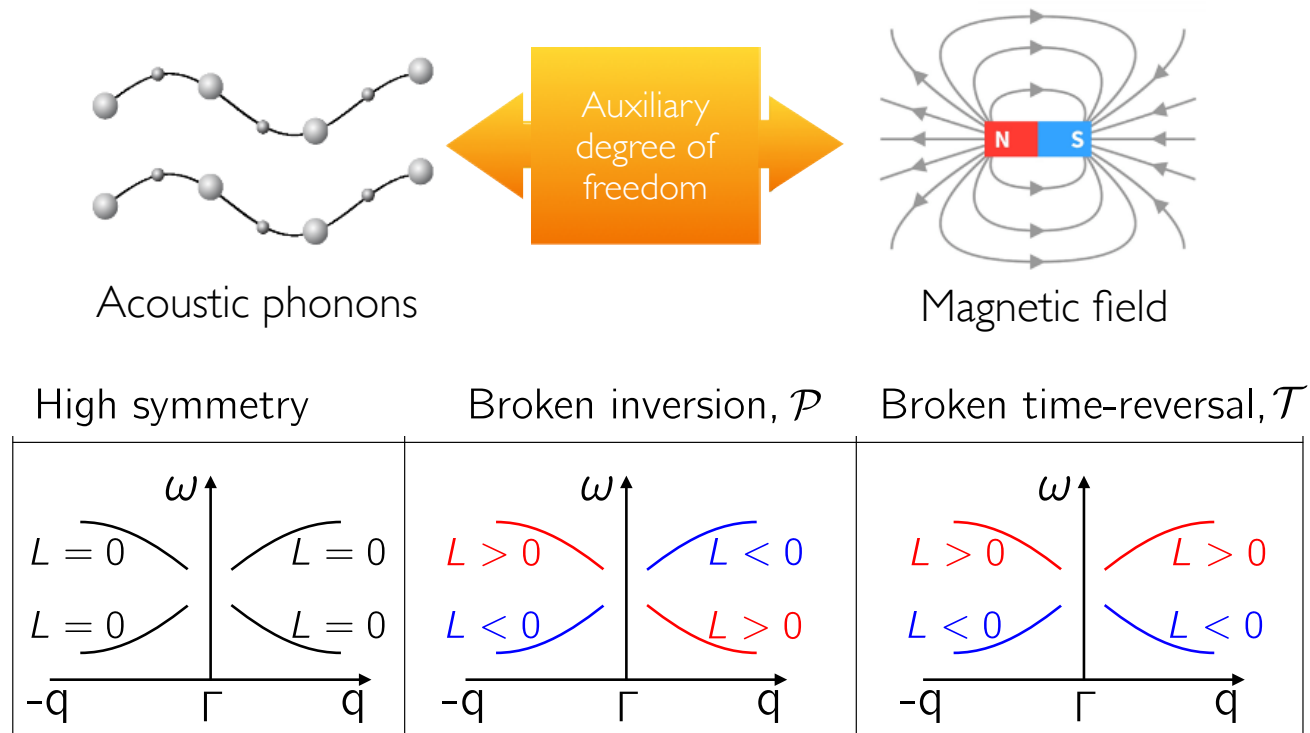
Chen et al., PRL (2020); Barkeshli et al., PRB (2012); Tügel et al., PRB (2017); Shapourian et al., PRB (2015); Ye et al., PRR (2020)...

- **TRSB embedded in the scatterers - *extrinsic***
(i.e., skew scattering rates in a transport theory)

Phonon scattering on time-reversal-symmetry-breaking impurities

Flebus et al., PRB (2022); Sun et al., (2022); Mangeolle et al. (2023), ...

Intrinsic TRSB mechanisms



Coh, PRB (2023)

- Acoustic phonons acquire a net angular momentum L
- Acoustic phonons acquire a dissipationless Hall viscosity η_H *Barkeshli et al. PRB (2012)*

$$S_H = \int d^2x dt \left[-\frac{\eta_H}{2} (\nabla^2 u_{ax} \dot{u}_{ay} - \nabla^2 u_{ay} \dot{u}_{ax}) \right]$$

$$\begin{aligned} \omega^2 u_{ax} &= \dots + i\eta_H \omega k_x^2 u_{ay} \\ \omega^2 u_{ay} &= \dots - i\eta_H \omega k_x^2 u_{ax} \end{aligned}$$

Sources of intrinsic mechanisms

Magnons

PRL **112**, 085503 (2014)

PHYSICAL REVIEW LETTERS

week ending
28 FEBRUARY 2014

Angular Momentum of Phonons and the Einstein–de Haas Effect

Lifa Zhang¹ and Qian Niu^{1,2}

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²International Center for Quantum Materials, Peking University, Beijing 100871, China

(Received 16 August 2013; revised manuscript received 13 November 2013; published 27 February 2014)

Spinons

PHYSICAL REVIEW RESEARCH **2**, 033283 (2020)

Unquantized thermal Hall effect in quantum spin liquids with spinon Fermi surfaces

Yanting Teng[✉], Yunchao Zhang[✉], Rhine Samajdar[✉], Mathias S. Scheurer[✉], and Subir Sachdev[✉]

Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

Electric polarization

PHYSICAL REVIEW LETTERS **124**, 167601 (2020)

Enhanced Thermal Hall Effect in Nearly Ferroelectric Insulators

Jing-Yuan Chen[✉],¹ Steven A. Kivelson[✉],² and Xiao-Qi Sun^{2,3}

¹Stanford Institute for Theoretical Physics, Stanford University, Stanford, California 94305, USA

²Department of Physics, Stanford University, Stanford, California 94305, USA

³Stanford Center for Topological Quantum Physics, Stanford University, Stanford, California 94305, USA

Quantized anomalous Hall states

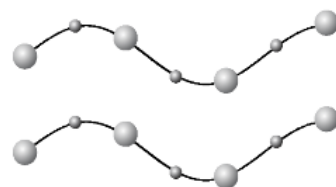
PHYSICAL REVIEW B **85**, 245107 (2012)

Dissipationless phonon Hall viscosity

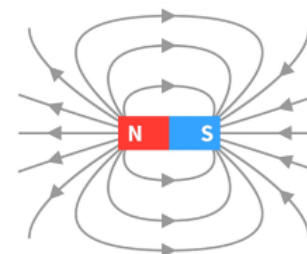
Maissam Barkeshli, Suk Bum Chung, and Xiao-Liang Qi

Department of Physics, Stanford University, Stanford, California 94305, USA

Can also ‘boring’ ionic insulators with no magnetism, ferroelectricity or non trivial electronic topology have a finite Hall viscosity?



Acoustic phonons



Magnetic field

Ionic crystals

Equation of motion for the lattice displacements

$$\omega^2 u_{ni} = \sum_{m,j} D_{ij}(mn, \mathbf{k}) u_{mj}$$

Dynamical matrix

- **In/out of phase motion** : eigenvectors of D at the Γ point

q_-, m_-



q_+, m_+



In-phase phonon modes

Acoustic phonon at the Γ point

q_-, m_-



q_+, m_+



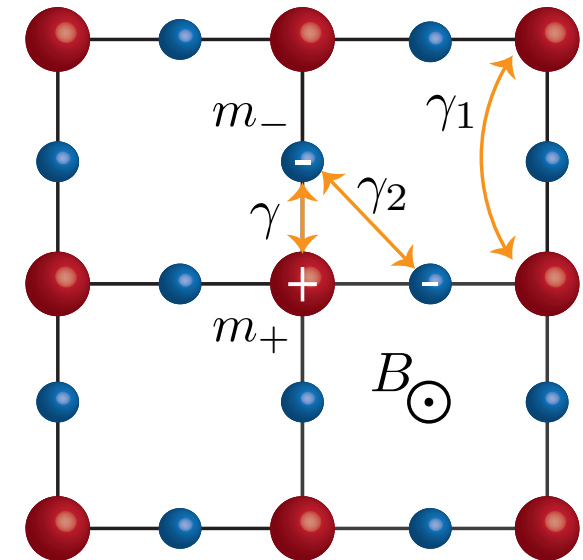
Out-of-phase phonon modes

Optical phonon at the Γ point

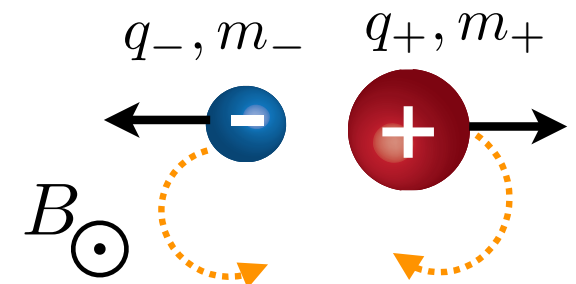
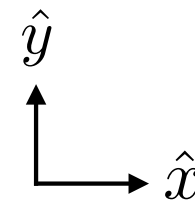
- Elastic interactions $\propto f(\gamma, \gamma_1, \gamma_2) k^2$ couple in-phase and out-of-phase modes

- **Out-of-phase motion of cations and anions** :
Lorentz force does not cancel

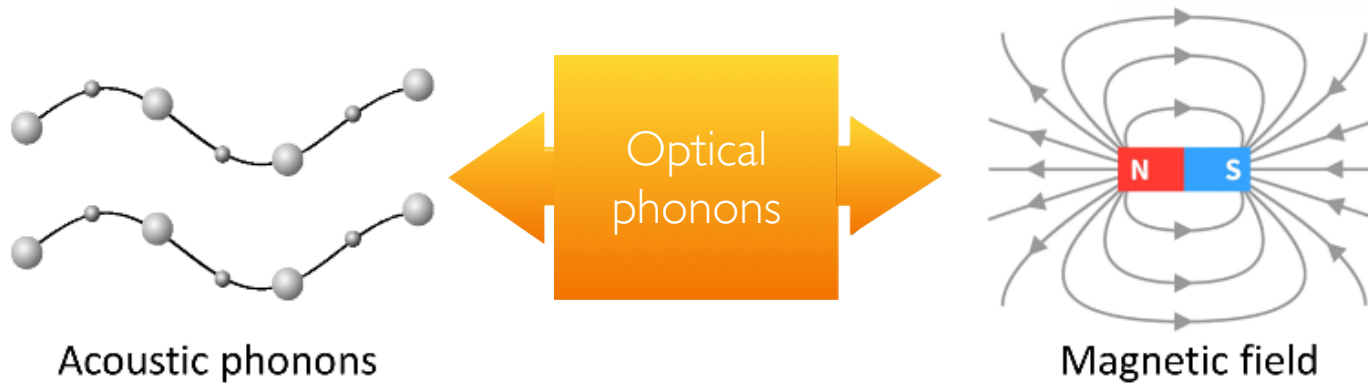
Bipartite ionic crystal lattice



B. Flebus & A.H. MacDonald, PRL (2022)



TRSB mechanism



Effective coupling of the acoustic phonons to the magnetic field

Lorentz force $\times$ Elastic force (wave-vector dependent)

Finite dissipationless Hall viscosity = finite phonon angular momentum

- Equation of motion for the lattice displacements

$$\omega^2 u_{ni} = \sum_{m,j} D_{ij}(mn, \mathbf{k}) u_{mj} + \sum_j i\omega B_{qn} u_{nj} \epsilon_{ij},$$

→ Integrate out out-of-phase modes (gapped)



Effective theory for long-wavelength acoustic modes

Intrinsic mechanism

Effective action for the
long-wavelength acoustic phonons

$$\omega^2 \left[1 + \frac{\omega_c^2}{\omega_o^2} \right] u_x = c_l^2 k^2 u_x + \frac{1}{2} \frac{i\eta\omega k^2}{m_+ + m_-} u_y ,$$

$$\omega^2 \left[1 + \frac{\omega_c^2}{\omega_o^2} \right] u_y = c_t^2 k^2 u_y - \frac{1}{2} \frac{i\eta\omega k^2}{m_+ + m_-} u_x .$$

Dissipationless Hall viscosity

$$\gamma \gg \gamma_{1(2)} , \quad m_+ \gg m_-$$

$$\eta \sim ZeB \sim 10^{-17} \text{ Kg/s}$$

$$B = 15 \text{ T}, \quad Z = 2$$

Due to coupling to electrons:

$$\eta \sim 10^{-23} - 10^{-17} \text{ Kg/s}$$

Barkeshli et al. PRB (2012)

- Intrinsic Hall viscosity + scattering on point-like impurities;
Rayleigh regime, weak scattering

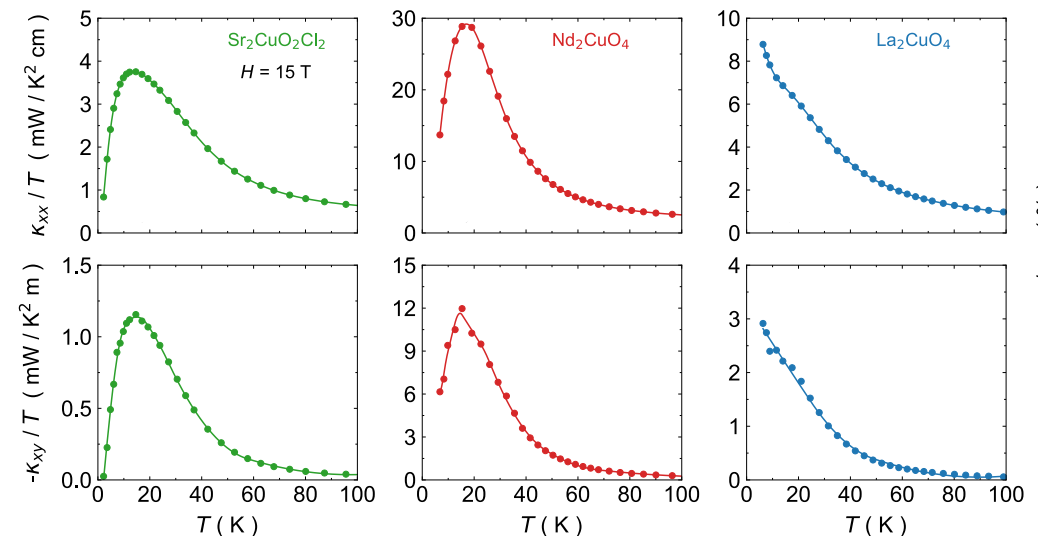
$$\kappa_{xx} \sim T^{-1}$$

$$\kappa_{xy} \sim T^5$$

**Inconsistent with
experiments?**

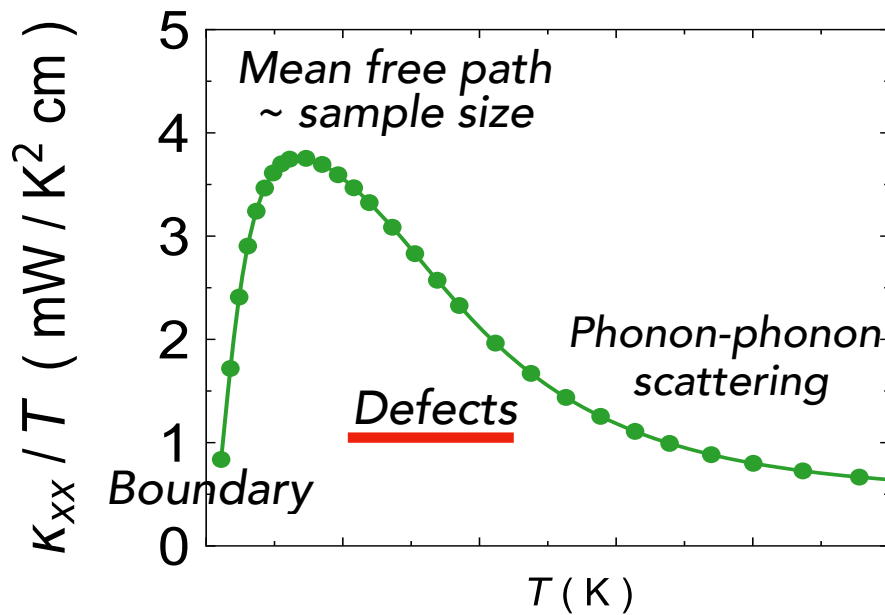
Guo et al., PRB (2021)

More recently : giant thermal Hall in non-ionic crystals



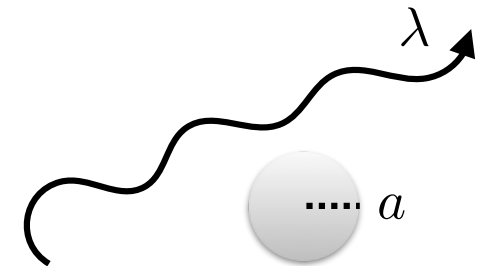
Extrinsic mechanism

Phonon thermal Hall → TRSB embedded in the scatterers



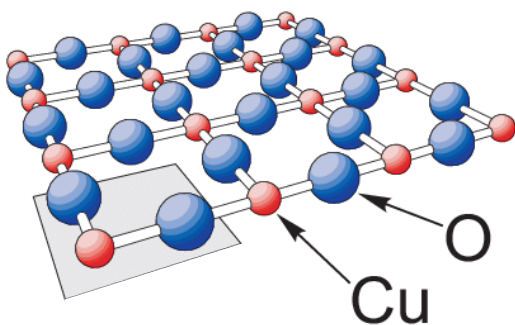
Low Temperature

Only long-wavelength phonons are excited



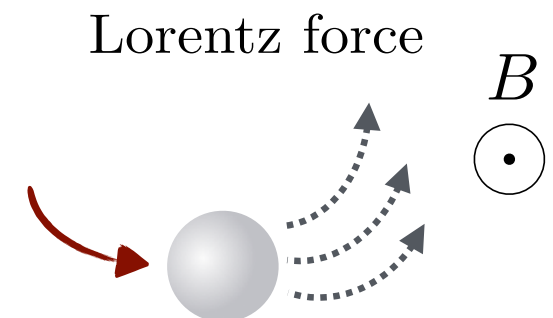
Defects modelled as a spherical region with a specific strain and mass density

$\lambda \gg a \rightarrow$ Rayleigh scattering



Oxygen vacancies :
common defect in oxides

*Imperfect local screening
of the charge defects*



Transport theory

Boltzmann equation for phonon transport driven by a temperature gradient ∇T

$$\omega_{\mathbf{k}} \frac{\partial \omega_{\mathbf{k}}}{\partial \mathbf{k}} \cdot \left(\frac{\nabla T}{T} \right) \left(- \frac{\partial f_{\mathbf{k}}^{(0)}}{\partial \omega_{\mathbf{k}}} \right) = \sum_{\mathbf{q}} (W_{\mathbf{q} \rightarrow \mathbf{k}} g_{\mathbf{q}} - W_{\mathbf{k} \rightarrow \mathbf{q}} g_{\mathbf{k}})$$

Scattering rate

$$W_{\mathbf{k} \rightarrow \mathbf{q}} = \frac{1}{V} \tau_{\omega}^{-1} v_{\omega}^{-1} [1 - \Lambda_{\omega} \hat{\mathbf{n}}_B \cdot (\hat{\mathbf{k}} \times \hat{\mathbf{q}})] \delta(\omega - c q)$$

Skew phonon scattering

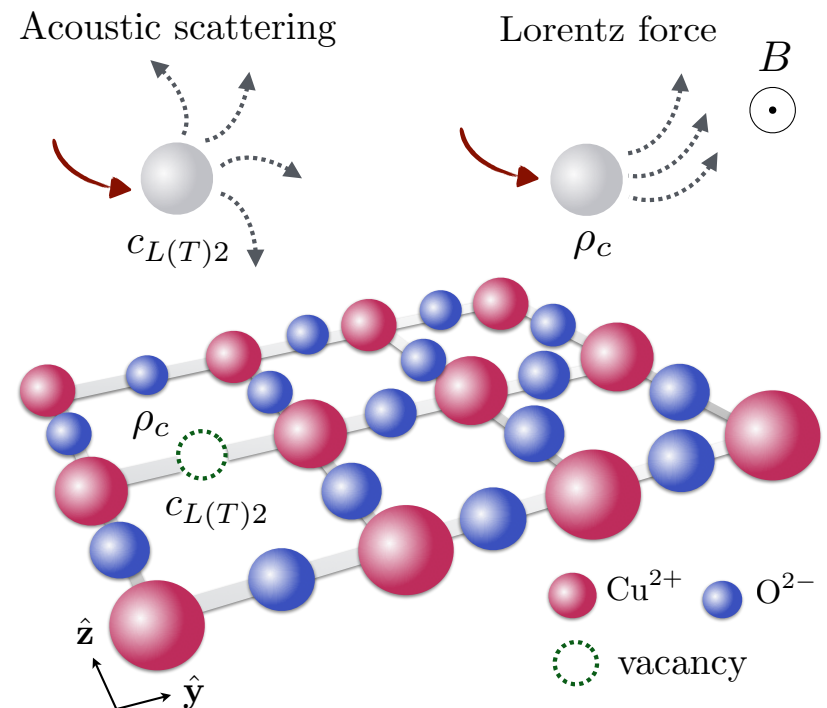
Longitudinal thermal conductivity

$$\kappa_{xx} = \frac{T^3}{2\pi^2 c} \int \tau_{\omega} \frac{x^4 e^x}{(e^x - 1)^2}$$

Thermal Hall conductivity

$$\kappa_H = \frac{k_B^4 T^3}{2\pi^2 \hbar^3 c} \int dx \frac{\Lambda_{\omega} \tau_{\omega}}{3} \frac{x^4 e^x}{(e^x - 1)^2}$$

→ Scattering theory needed



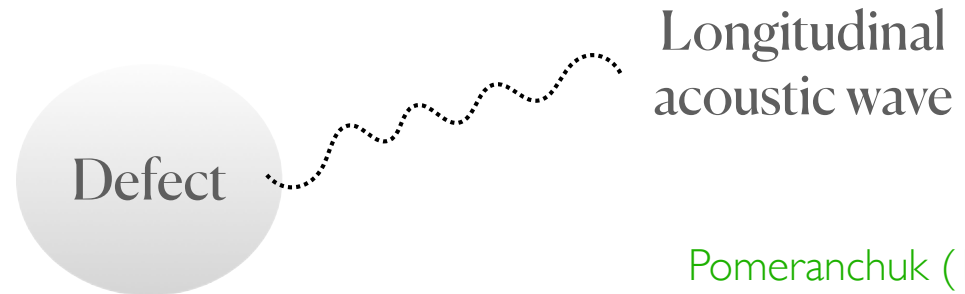
Model

Upon scattering on an oxygen vacancy,
long-wavelength phonons experience two perturbations :

- Acoustic scattering

We account for viscosity
inside the defect

(*Kelvin-Voigt model*)



Pomeranchuk (1941)

M. K. Hinders (1990)

Wave equation in spherical coordinates for a longitudinal incident wave

- Lorentz potential $A_{ij}^L = i\omega\omega_c \left(\hat{i} \times \hat{j} \right) \cdot \hat{n}_B$

ρ_c charge density ρ_2 mass density

Field B direction

Cyclotron frequency of phonons inside the charge defect: $\omega_c = B\rho_c/\rho_2$

We consider random impurities with no spatial correlation among them

Scattering rate

Fermi Golden rule :

$$W(\vec{k}', L, \vec{k}, L) \propto \left| 4\pi c_L^2 \frac{i\text{Re}C_1^{(0)} - \text{Im}C_1^{(0)}}{k} - i4\pi a^3 \omega \omega_c (\hat{k} \times \hat{k}') \cdot \hat{n}_B \right|^2$$

Acoustic scattering

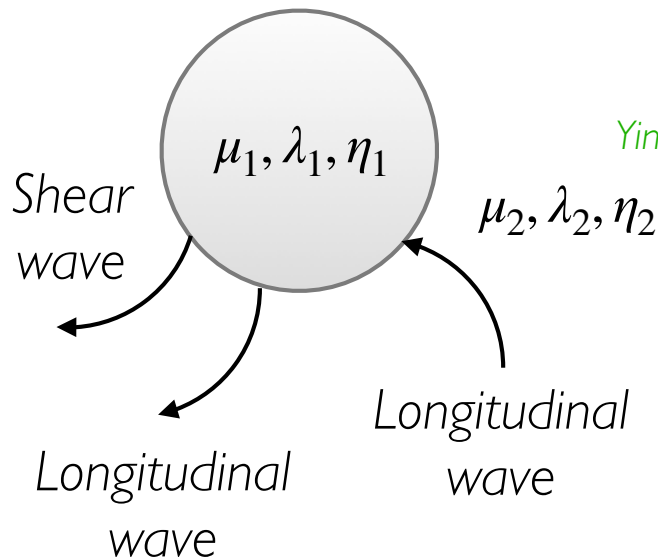
(Solve wave equation
& continuity conditions
in spherical coordinates)

*Ying et al., JAP (1956); Pao et al., JAP (1963);
M. K. Hinders, PRA (1991) ...*

Lorentz force

(By itself $\propto B^2$)

Kelvin-Voigt model



$\text{Im}C_1^{(0)} \propto$ Elastic scattering

$\text{Re}C_1^{(0)} \propto$ Wave attenuation $\propto \eta_1, \eta_2$

Skew scattering $\propto B$

Interference between Lorentz force and inelastic scattering contributions

Results

$$\Lambda_{\omega} \simeq \frac{\omega_c \eta_1 \left(\frac{4\mu_1}{\lambda_1} + \frac{6\mu_2}{\lambda_1} + \frac{9\lambda_2}{\lambda_1} - 4 \right) - 5\omega_c \eta_2 \left(1 + \frac{2\mu_1}{\lambda_1} \right)}{\lambda_1 \left[\left(1 - \frac{\lambda_2}{\lambda_1} \right) + \frac{2}{3} \left(\frac{\mu_1}{\lambda_1} - \frac{\mu_2}{\lambda_1} \right) \right]^2}$$

- cyclotron frequency for a single oxygen defect ω_c
- frequency of longitudinal phonons at 20 K ω_L

$$\frac{\omega_c}{\omega_L} \sim 10^{-7}$$

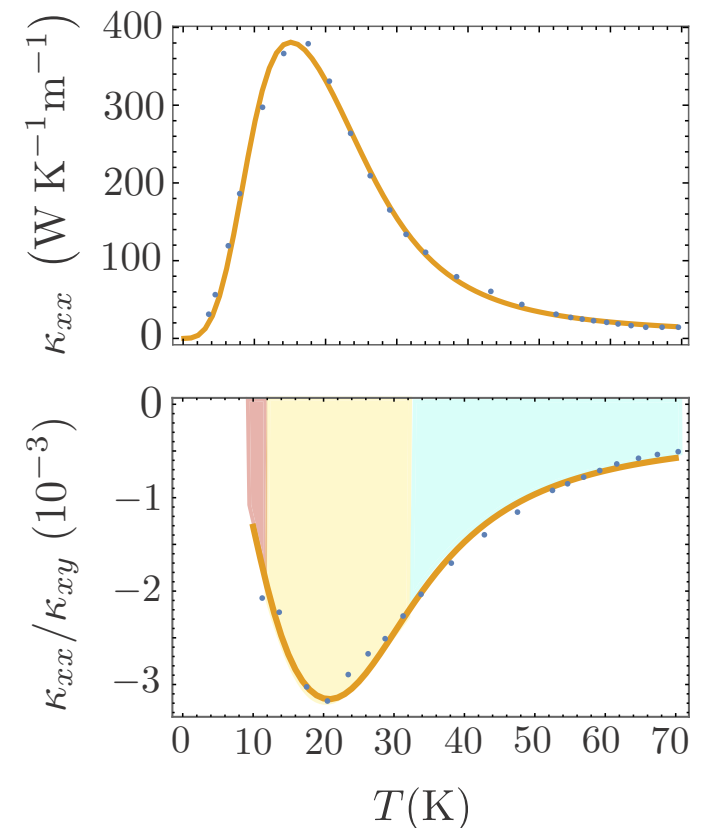
Assuming a change of 1% for
the elastic constants inside
and outside the defect

$$\frac{\kappa_{xy}}{\kappa_{xx}} \sim 10^{-3}$$

observed order of magnitude

Fit of the data for Cu_3TeO_6

Chen et al., PNAS (2022)



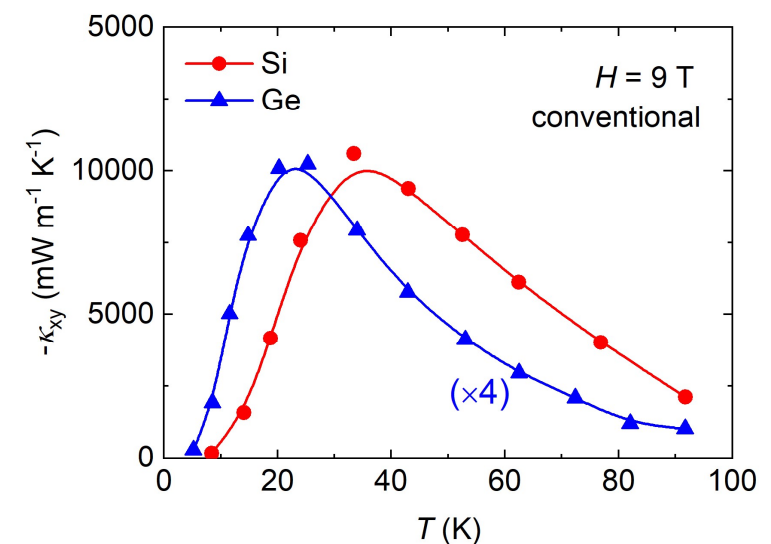
Summary (Part I)

- Finite phonon Hall viscosity in ionic crystals without coupling to degrees of freedom other than phononic
- The observed giant thermal Hall conductivity might be explained by *Rayleigh scattering of phonons on charged impurities*
- This mechanism requires imperfect local screening of the charged defects and acoustic wave attenuation

Discovery of universal phonon thermal Hall effect in crystals

Xiaobo Jin^{1*}, Xu Zhang^{1*}, Wenbo Wan¹, Hanru Wang¹, Yihan Jiao¹ and Shiyan Li^{1,2,3}

... Still an open puzzle :
Universal mechanism (or not)?



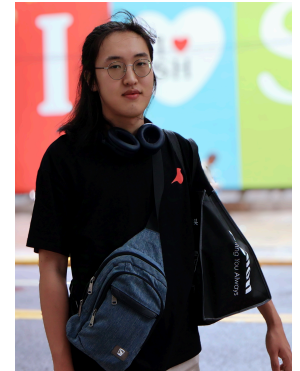
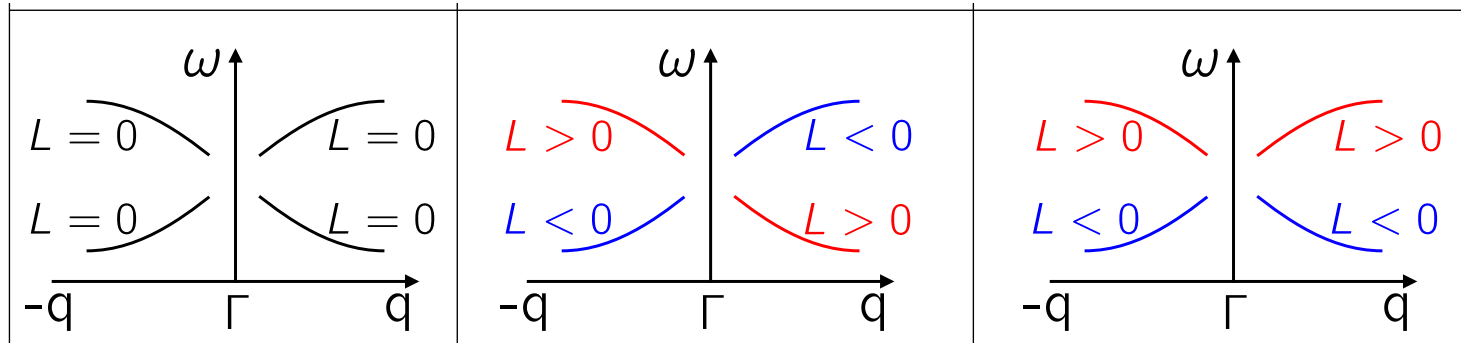
Phonon angular momentum fluctuations

$$\langle [J_z^{ph}(t), J_z^{ph}(0)] \rangle$$

High symmetry

Broken inversion, $\mathcal{P}$

Broken time-reversal, $\mathcal{T}$



Rule Yi

Minimal model

- 2d crystal with two *non-degenerate* phonon modes
- Time- and inversion-symmetry preserving
- No external drive (e.g., temperature gradients)
- *Vacuum state*

Angular momentum operator

$$\hat{H} = \hbar \sum_k \omega_k (a_k^\dagger a_k + \frac{1}{2}), \quad k = (\mathbf{k}, \sigma)$$

*L. Zhang and Q. Niu,
PRL (2014)*

$$\hat{J}_z^{\text{ph}} = \hat{J}_{\text{nc}} + \frac{\hbar}{2} \sum_{\mathbf{k}, \sigma, \sigma'} \left(\underbrace{\left[\sqrt{\frac{\omega_{\mathbf{k}, \sigma}}{\omega_{\mathbf{k}, \sigma'}}} - \sqrt{\frac{\omega_{\mathbf{k}, \sigma'}}{\omega_{\mathbf{k}, \sigma}}} \right]}_{\text{Fast oscillating}} \varepsilon_{\mathbf{k}, \sigma}^\dagger M \varepsilon_{-\mathbf{k}, \sigma'}^* a_{\mathbf{k}, \sigma}^\dagger a_{-\mathbf{k}, \sigma'}^\dagger + \text{h.c.} \right)$$

- Stationary contributions :

$$\hat{J}_{\text{nc}} = \frac{\hbar}{2} \sum_{\mathbf{k}, \sigma, \sigma'} \sqrt{\frac{\omega_{\mathbf{k}, \sigma'}}{\omega_{\mathbf{k}, \sigma}}} \left[\varepsilon_{\mathbf{k}, \sigma}^\dagger M \varepsilon_{\mathbf{k}, \sigma'} a_{\mathbf{k}, \sigma}^\dagger a_{\mathbf{k}, \sigma'} + \varepsilon_{\mathbf{k}, \sigma}^T M \varepsilon_{\mathbf{k}, \sigma'}^* a_{\mathbf{k}, \sigma} a_{\mathbf{k}, \sigma'}^\dagger \right]$$

$$\langle \hat{J}_z^{\text{ph}} \rangle = \sum_k l_k \left[f(\beta \omega_k) + \frac{1}{2} \right], \quad l_k \equiv \hbar \varepsilon_{\mathbf{k}, \sigma}^\dagger M \varepsilon_{\mathbf{k}, \sigma}$$

- TRS : $\langle \hat{J}_z^{\text{ph}} \rangle = 0$

- PT-symmetry : $l_k = 0$

Vacuum angular momentum fluctuations

Angular momentum susceptibility :

$$\chi(\omega) = -\frac{i}{\hbar} \int_0^\infty dt e^{i\omega t} \langle 0 | [\hat{J}_z^{\text{ph}}(t), \hat{J}_z^{\text{ph}}(0)] | 0 \rangle \quad \text{zero-point quantum fluctuations}$$

- Number non-conserving terms $J_z^{\text{ph}} \propto a_{\mathbf{k},\sigma}^\dagger a_{-\mathbf{k},\sigma'}^\dagger$ yield

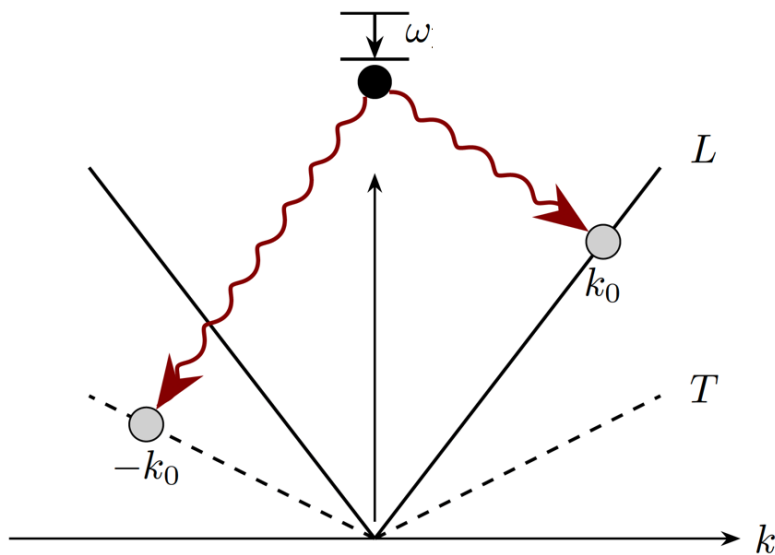
$$\chi(\omega) = \frac{\hbar}{4} \sum_{\mathbf{k}, \sigma \neq \sigma'} \left| \varepsilon_{\mathbf{k},\sigma}^T M \varepsilon_{-\mathbf{k},\sigma'} \right|^2 \underbrace{\left(\frac{\omega_{\mathbf{k},\sigma}}{\omega_{\mathbf{k},\sigma'}} + \frac{\omega_{\mathbf{k},\sigma'}}{\omega_{\mathbf{k},\sigma}} - 2 \right)} \frac{\omega_{\mathbf{k},\sigma} + \omega_{\mathbf{k},\sigma'}}{\omega^2 - (\omega_{\mathbf{k},\sigma} + \omega_{\mathbf{k},\sigma'})^2}$$

- No fluctuations if the modes are degenerate, i.e., $\omega_{\mathbf{k},\sigma} = \omega_{\mathbf{k},\sigma'}$

(If not, they can be combined in circularly polarized modes that are also eigenmodes of $\hat{H}$)

Vacuum angular momentum fluctuations

$$\chi(\omega) = \frac{\hbar}{4} \sum_{\mathbf{k}, \sigma \neq \sigma'} \left| \epsilon_{\mathbf{k},\sigma}^T M \epsilon_{-\mathbf{k},\sigma'} \right|^2 \left(\frac{\omega_{\mathbf{k},\sigma}}{\omega_{\mathbf{k},\sigma'}} + \frac{\omega_{\mathbf{k},\sigma'}}{\omega_{\mathbf{k},\sigma}} - 2 \right) \frac{\omega_{\mathbf{k},\sigma} + \omega_{\mathbf{k},\sigma'}}{\omega^2 - (\omega_{\mathbf{k},\sigma} + \omega_{\mathbf{k},\sigma'})^2}$$



Example : two-level system coupled to angular momentum fluctuations

Spontaneous emission
 $\Rightarrow$ pair of phonons with opposite momenta $\mathbf{k}$ and orthogonal linear polarization

$$\omega = \omega_{\mathbf{k},\sigma} + \omega_{\mathbf{k},\sigma'}$$

Can the phonon pair form a superposition state associated with a finite $\hat{J}_z^{\text{ph}}(t)$?

$$[\hat{H}, \hat{J}_z^{\text{ph}}] \propto (\omega_{\mathbf{k},\sigma} - \omega_{\mathbf{k},\sigma'})$$

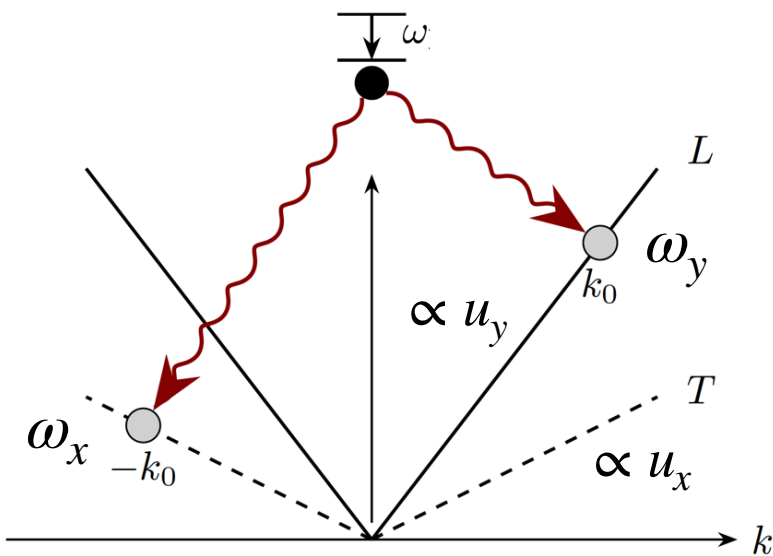
A classical analogy

Generation of a superposition of two classical waves $u_x(t)$ and $u_y(t)$

Only superposition allowed by constraints of phonon-pair emission

$$\mathbf{u}(t) = e^{-i\omega_x t} (\hat{\mathbf{x}} + \hat{\mathbf{y}} e^{-i\delta\omega t})$$

$$\delta\omega = \omega_y - \omega_x > 0$$



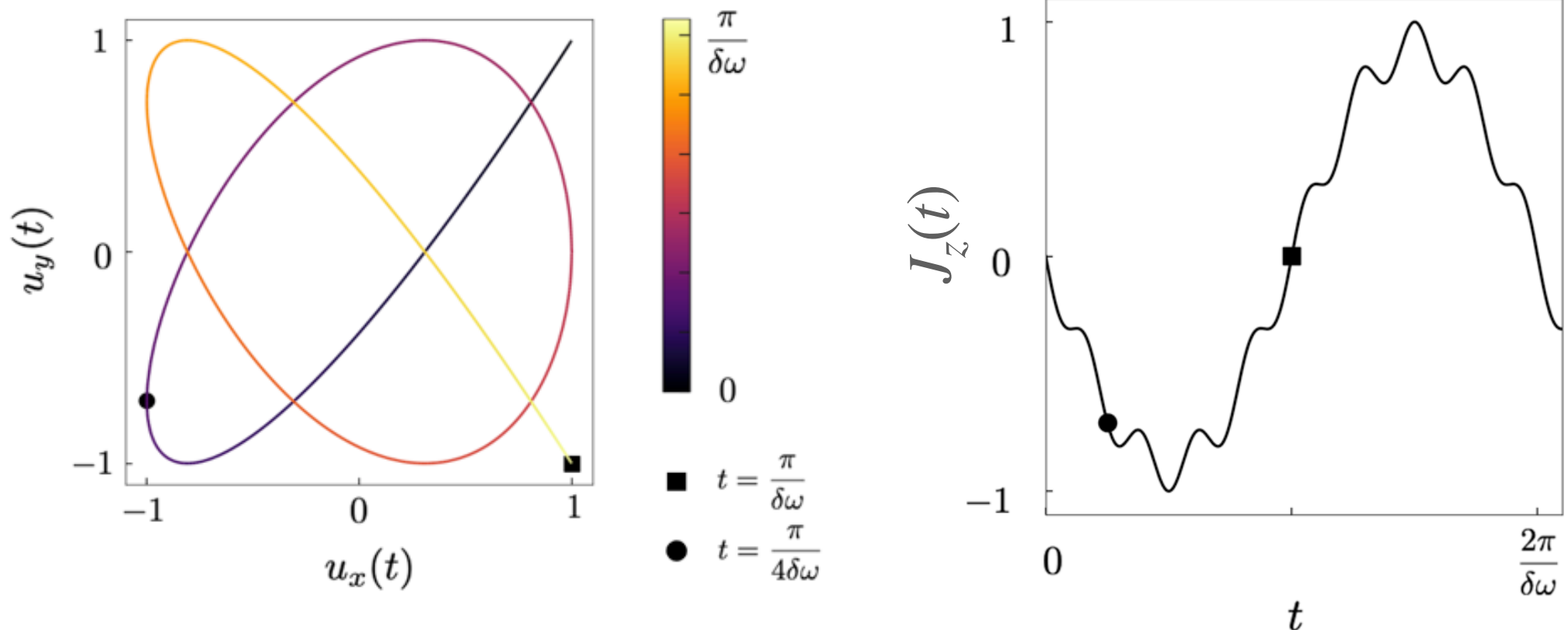
- Relative phase $\delta\omega \cdot t$ modulates the instantaneous ellipticity of the displacement field

No instantaneous ellipticity if the modes are degenerate, i.e., $\delta\omega = 0$

A classical analogy

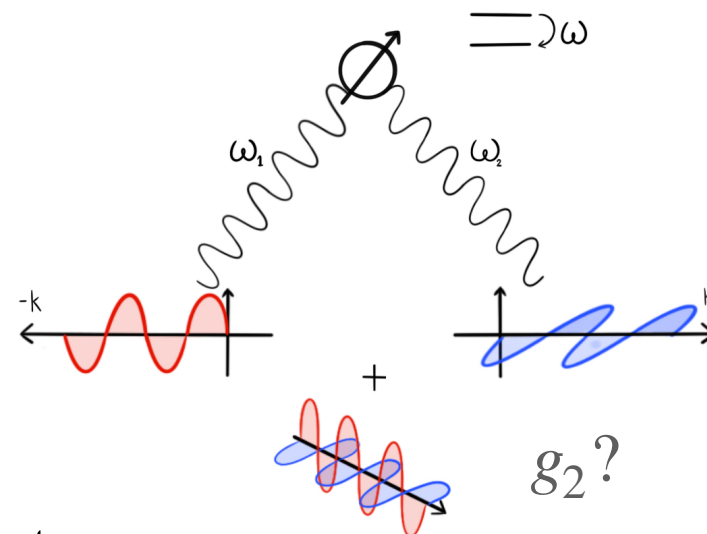
Instantaneous classical angular momentum
(vanishing time average)

$$J_z(t) = -\frac{\delta}{2} \sin((2\omega_0 + \delta)t) - \frac{2\omega_0 + \delta}{2} \sin(\delta t)$$



Summary (Part II)

- The vacuum of an inversion and time-reversal symmetric crystal can exhibit finite phonon angular momentum fluctuations.
- Key requirement : lack of mode degeneracy
- Non-commutativity of angular momentum operator & classical analogy
- A probe that couples bilinearly to phonon operators may generate/detect this phenomenon (optical phonons + Raman?)
- To do : temperature effects, geometric phases, distinguishability between “virtual” and “real” angular momentum noise, etc ..



Thank you for the attention!