Faculty of Science

Hubbard models for non-relativistic altermagnetic spin-splitting

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Outline of presentation

Minimal models for altermagnetism

- Construction of minimal models
- Altermagnetic susceptibility and stabilisation of altermagnetic order

- Landau free energy: effects of spin-orbit coupling.
- Anomalous Hall effect and weak ferromagnetism

Local signatures of altermagnetism

- Local markers of altermagnetism

Superconductivity and altermagnetism

- What pairing states are preferred in altermagnetic metals?

Conclusions & Outlook

Collaborators


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Christian Rasmussen

M. Roig *et al*, PRL 135, 046701 (2025)
M. Roig *et al*, PRB 100, 144412 (2024)
J. Gondolf *et al*, PRB 111, 174436 (2025)
C. L. H. Rasmussen *et al*, ArXiv:2509.03247

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Minimal models for altermagnetism

Motivation:

- Ab-initio calculations: material specific
- Study the mechanisms and physical properties

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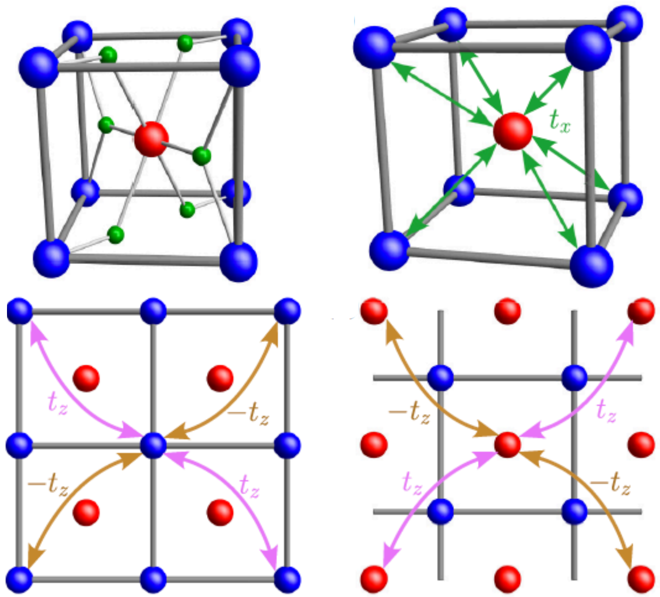
Minimal one-orbital model: sublattice τ_i, spin σ_i

$$H = \underbrace{\varepsilon_{0,\mathbf{k}}}_{\text{Dispersion}} + \underbrace{t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z}_{\text{inter/intra-sublattice hoppings}} + \tau_y \underbrace{\vec{\lambda}_{\mathbf{k}}}_{\text{SOC}} \cdot \vec{\sigma} + \tau_z \underbrace{\vec{J}}_{\text{AM OP}} \cdot \vec{\sigma}$$

τ_0, τ_x : invariant under point group symmetries

τ_y, τ_z : odd under some point group operations



$$E_{\alpha=\pm} = \varepsilon_{0,\mathbf{k}} + \alpha\left(t_{x,\mathbf{k}}^2 + (t_{z,\mathbf{k}} + \vec{J}\cdot\vec{\sigma})^2\right)^{1/2}$$

- Momentum-dependence of spin-splitting determined by point group and Wyckoff position of magnetic ions.

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Minimal one-orbital model: sublattice τ_i , spin σ_i

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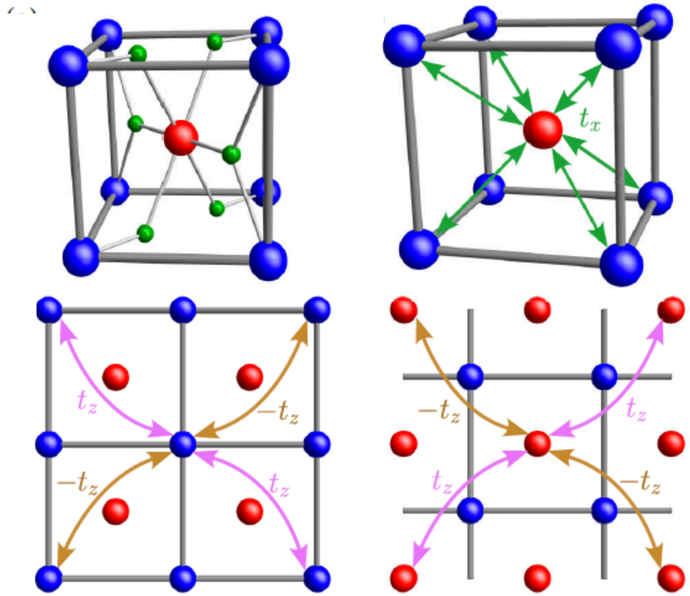


τ_0 , τ_x : invariant under point group symmetries

τ_y , τ_z : odd under some point group operations

- ➢ Can be applied to monoclinic, orthorhombic, tetragonal, hexagonal, cubic and rhombohedral materials
- ➢ Describes d-wave, g-wave and i-wave AM

M. Roig *et al*, PRB 100, 144412 (2024)

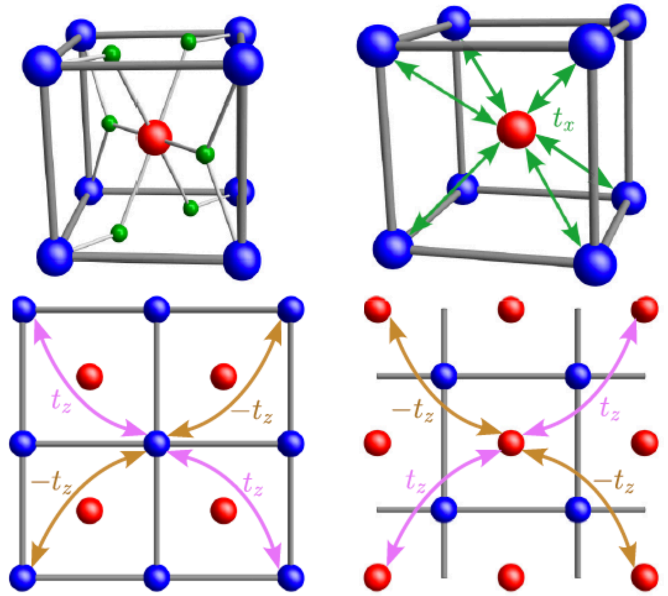
Minimal models for altermagnetism

The rutile case (“fruit fly”)

$$H = \varepsilon_{0,\mathbf{k}} + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z + \tau_y \vec{\lambda}_{\mathbf{k}} \cdot \vec{\sigma} + \tau_z \vec{J} \cdot \vec{\sigma}$$

$$t_{x,\mathbf{k}} = t_0 \cos\frac{k_x}{2}\cos\frac{k_y}{2}\cos\frac{k_z}{2}$$

$$t_{z,\mathbf{k}} = t_{z0} \sin k_x \sin k_y$$ → d-wave symmetry



Minimal models for altermagnetism

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$$H = \varepsilon_{0,\mathbf{k}} + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z + \tau_y \vec{\lambda}_{\mathbf{k}} \cdot \vec{\sigma} + \tau_z \vec{J} \cdot \vec{\sigma}$$

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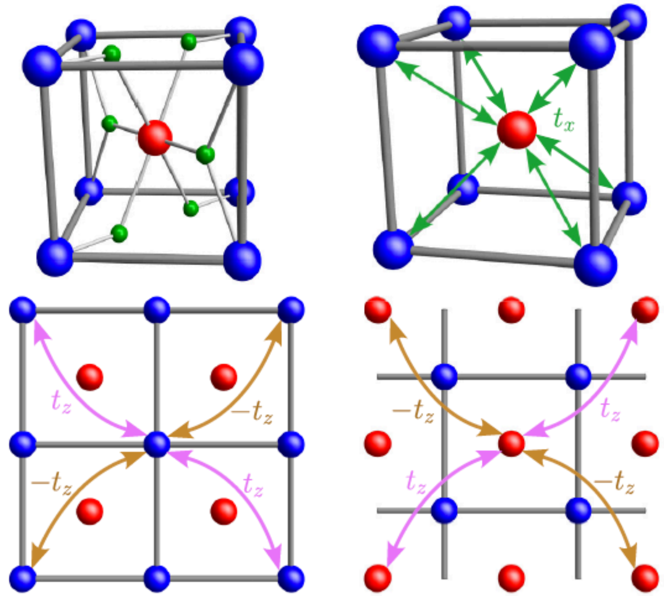
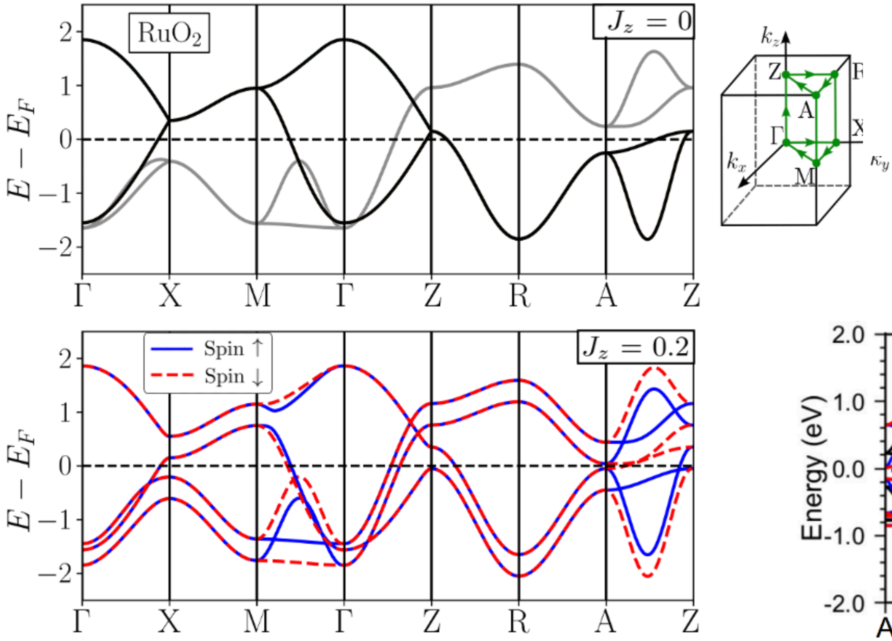
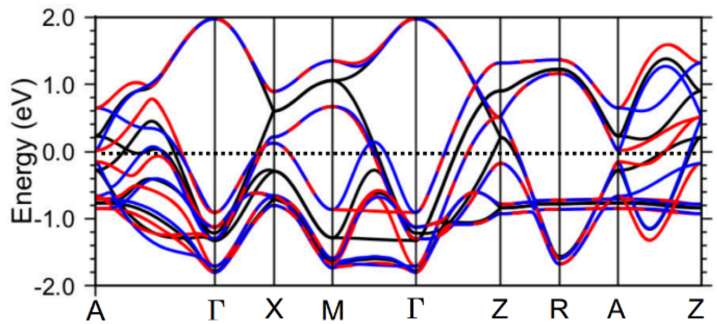


Minimal tight-binding model



Wyckoff positions +
lowest order hoppings



[Ahn *et al.*, PRB (2019)]

Minimal models for altermagnetism

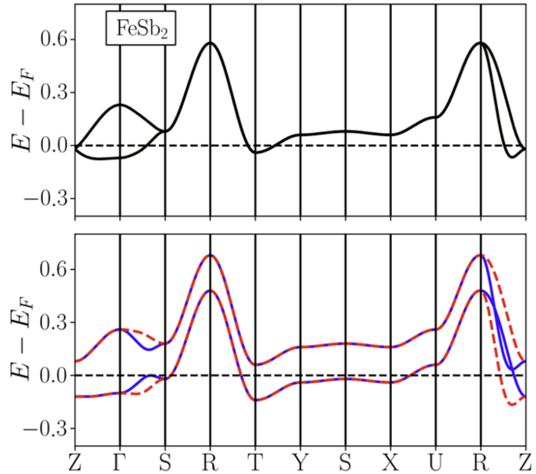
Other material candidates

$$H = \varepsilon_{0,\mathbf{k}} + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z + \tau_y \vec{\lambda}_{\mathbf{k}} \cdot \vec{\sigma} + \tau_z \vec{J} \cdot \vec{\sigma},$$

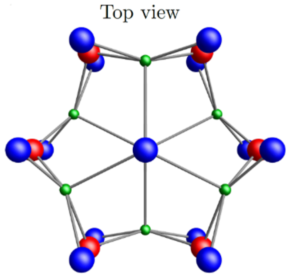
- Orthorhombic systems: κ-Cl and $FeSb_2$

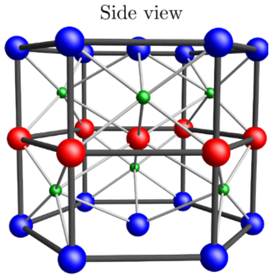
$\boxed{FeSb_2}$: [Mazin *et al.*, PNAS (2021)], [Lukoyanov *et al.*, EPJ B (2006)]

$$t_{x,\mathbf{k}} = t_0 \cos\frac{k_x}{2}\cos\frac{k_y}{2}\cos\frac{k_z}{2}$$
$$t_{z,\mathbf{k}} = t_{z0}\sin k_x \sin k_y$$



- Hexagonal systems: $\boxed{\text{CrSb or MnTe}}$





$$t_{x,\mathbf{k}} = t_4 \cos\frac{k_z}{2}$$
$$t_{z,\mathbf{k}} = t_3 \sin k_y \sin k_z (\cos\sqrt{3}k_x - \cos k_y)$$

$\Rightarrow$ g-wave symmetry

- Lieb lattice materials & 2D layer groups.

Minimal models for altermagnetism

General symmetry analysis

M. Roig *et al*, PRB 100, 144412 (2024)

Focus on centrosymmetric SG with magnetic ions on inversion sym. Wyckoff positions with multiplicity 2.

TABLE II. Space groups and Wyckoff positions that allow altermagnetism: tetragonal, rhombehedral, hexagonal, and cubic groups.

SG (P)	Wyckoff (S)	Γ_N	Spin splitting ($f_{\Gamma_N}(\mathbf{k})$)
83 (C_{4h})	2e,2f (C_{2h})	B_g	$\alpha k_x k_y + \beta(k_x^2 - k_y^2)$
84 (C_{4h})	2a-2d (C_{2h})	B_g	$\alpha k_x k_y + \beta(k_x^2 - k_y^2)$
87 (C_{4h})	4c (C_{2h})	B_g	$\alpha k_x k_y + \beta(k_x^2 - k_y^2)$
123 (D_{4h})	2e,2f (D_{2h})	B_{1g}	$k_x^2 - k_y^2$
124 (D_{4h})	2b,2d (C_{4h})	A_{2g}	$k_x k_y (k_x^2 - k_y^2)$
127 (D_{4h})	2a,2b (C_{4h})	A_{2g}	$k_x k_y (k_x^2 - k_y^2)$
127 (D_{4h})	2c,2d (D_{2h})	B_{2g}	$k_x k_y$
128 (D_{4h})	2a,2b (C_{4h})	A_{2g}	$k_x k_y (k_x^2 - k_y^2)$
131 (D_{4h})	2a-2d (D_{2h})	B_{1g}	$k_x^2 - k_y^2$
132 (D_{4h})	2a,2c (D_{2h})	B_{2g}	$k_x k_y$
136 (D_{4h})	2a,2b (D_{2h})	B_{2g}	$k_x k_y$
139 (D_{4h})	4c (D_{2h})	B_{1g}	$k_x^2 - k_y^2$
140 (D_{4h})	4c (C_{4h})	A_{2g}	$k_x k_y (k_x^2 - k_y^2)$
140 (D_{4h})	4d (D_{2h})	B_{2g}	$k_x k_y$
163 (D_{3d})	2b (S_6)	A_{2g}	$k_y k_z (k_y^2 - 3k_x^2)$
165 (D_{3d})	2b (S_6)	A_{2g}	$k_x k_z (k_x^2 - 3k_y^2)$
167 (D_{3d})	6b (S_6)	A_{2g}	$k_x k_z (k_x^2 - 3k_y^2)$
176 (C_{6h})	2b (S_6)	B_g	$\alpha k_y k_z (k_y^2 - 3k_x^2)$ $+\beta k_x k_z (k_x^2 - 3k_y^2)$
192 (D_{6h})	2b (C_{6h})	A_{2g}	$k_x k_y (k_x^2 - 3k_y^2)(k_y^2 - 3k_x^2)$
193 (D_{6h})	2b (D_{3d})	B_{2g}	$k_x k_z (k_x^2 - 3k_y^2)$
194 (D_{6h})	2a (D_{3d})	B_{1g}	$k_y k_z (3k_x^2 - k_y^2)$
223 (O_h)	2a (D_{3d})	A_{2g}	$k_x^4 (k_y^2 - k_z^2)$ $+k_y^4 (k_z^2 - k_x^2)$ $+k_z^4 (k_x^2 - k_y^2)$

Minimal models for altermagnetism

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Focus on centrosymmetric SG with magnetic ions on inversion sym. Wyckoff positions with multiplicity 2.

TABLE IV. Tight-binding coefficients for space groups with two atoms per unit cell at the inversion center, but without nodal plane. The same abbreviation applies as in the previous table.

SG	τ_x	τ_z	$\tau_y\sigma_x$	$\tau_y\sigma_y$	$\tau_y\sigma_z$
13(2a-2d)	$c_{z/2}, s_x s_{z/2}$	$s_y(s_x, s_z)$	$c_{z/2}, s_x s_{z/2}$	$s_y(s_{z/2}, s_x c_{z/2})$	$c_{z/2}, s_x s_{z/2}$
49(2a-2d)	$c_{z/2}$	$s_x s_y$	$s_x s_{z/2}$	$s_y s_{z/2}$	$c_{z/2}$
83(2e,2f)	$c_{x/2}c_{y/2}$, $s_{x/2}s_{y/2}(c_x - c_y)$	$(c_x - c_y), s_x s_y$	$\lambda_1 s_{x/2}c_{y/2}s_z$ $+\lambda_2 c_{x/2}s_{y/2}s_z$	$-\lambda_1 c_{x/2}s_{y/2}s_z$ $+\lambda_2 s_{x/2}c_{y/2}s_z$	$s_{x/2}s_{y/2}$, $c_{x/2}c_{y/2}(c_x - c_y)$
84(2a,2b)	$c_{z/2}$	$(c_x - c_y), s_x s_y$	$(\lambda_1 s_x + \lambda_2 s_y)s_{z/2}$	$(-\lambda_1 s_y + \lambda_2 s_x)s_{z/2}$	$c_{z/2}(c_x - c_y)$
84(2c,2d)	$c_{x/2}c_{y/2}c_{z/2}$, $s_{x/2}s_{y/2}c_{z/2}$	$(c_x - c_y), s_x s_y$	$\begin{pmatrix}+\lambda_1 s_{x/2}c_{y/2}\\ +\lambda_2 c_{x/2}s_{y/2}\end{pmatrix}s_{z/2}$	$\begin{pmatrix}-\lambda_1 c_{x/2}s_{y/2}\\ +\lambda_2 s_{x/2}c_{y/2}\end{pmatrix}s_{z/2}$	$c_{x/2}c_{y/2}c_{z/2}(c_x - c_y)$, $s_{x/2}s_{y/2}c_{z/2}(c_x - c_y)$
123(2e,2f)	$c_{x/2}c_{y/2}$	$(c_x - c_y)$	$\lambda c_{x/2}s_{y/2}s_z$	$\lambda s_{x/2}c_{y/2}s_z$	$s_{x/2}s_{y/2}$
124(2b,2d)	$c_{z/2}$	$s_x s_y(c_x - c_y)$	$\lambda s_x s_{z/2}$	$\lambda s_y s_{z/2}$	$c_{z/2}$
131(2a,2b)	$c_{z/2}$	$(c_x - c_y)$	$\lambda s_y s_{z/2}$	$\lambda s_x s_{z/2}$	$s_x s_y c_{z/2}$
131(2c,2d)	$c_{x/2}c_{y/2}c_{z/2}$	$(c_x - c_y)$	$\lambda c_{x/2}s_{y/2}s_{z/2}$	$\lambda s_{x/2}c_{y/2}s_{z/2}$	$s_{x/2}s_{y/2}c_{z/2}$
132(2a,2c)	$c_{z/2}$	$s_x s_y$	$\lambda s_x s_{z/2}$	$-\lambda s_y s_{z/2}$	$c_{z/2}(c_x - c_y)$
163(2b)	$c_{z/2}$, $f_x(3f_y^2 - f_x^2)s_{z/2}$	$f_y(f_y^2 - 3f_x^2)s_z$, $f_x f_y(f_x^2 - 3f_y^2)(3f_x^2 - f_y^2)$	$\lambda_1 f_x s_{z/2}$ $+\lambda_2(f_x^2 - f_y^2)c_{z/2}$	$\lambda_1 f_y s_{z/2}$ $-2\lambda_2 f_x f_y c_{z/2}$	$c_{z/2}$, $f_x(3f_y^2 - f_x^2)s_{z/2}$
165(2b)	$c_{z/2}$, $f_y(3f_x^2 - f_y^2)s_{z/2}$	$f_x(f_x^2 - 3f_y^2)s_z$, $f_x f_y(f_x^2 - 3f_y^2)(3f_x^2 - f_y^2)$	$\lambda_1 f_x s_{z/2}$ $+2\lambda_2 f_x f_y c_{z/2}$	$\lambda_1 f_y s_{z/2}$ $+\lambda_2(f_x^2 - f_y^2)c_{z/2}$	$c_{z/2}$, $f_y(3f_x^2 - f_y^2)s_{z/2}$
192(2b)	$c_{z/2}$	$f_x f_y(f_x^2 - 3f_y^2)(3f_x^2 - f_y^2)$	$\lambda s_{z/2}f_x$	$\lambda s_{z/2}f_y$	$c_{z/2}$
223(2a)	$c_{x/2}c_{y/2}c_{z/2}$	$(c_x - c_y)(c_y - c_z)(c_z - c_x)$	$\lambda c_{x/2}s_{y/2}s_{z/2}$	$\lambda s_{x/2}c_{y/2}s_{z/2}$	$\lambda s_{x/2}s_{y/2}c_{z/2}$

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Conclusions & Outlook

Altermagnetic susceptibility

Stabilization of Altermagnetism

- Minimal model in the normal state

$$H'(\mathbf{k}) = \varepsilon_{0,\mathbf{k}} + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z$$

- Intra-orbital Hubbard interaction: $H_{\text{int}} = U\sum_{i,\mu} n_{i,\mu\uparrow}n_{i,\mu\downarrow}$

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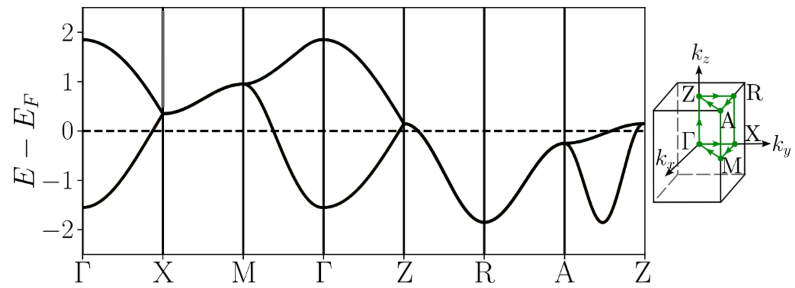
- RPA susceptibility: $[\chi_{\text{RPA}}(\mathbf{q}, iq_n)]^{\mu_1,\mu_2}_{\mu_3,\mu_4} = \left[\chi_0(\mathbf{q}, iq_n)\left(1 - U\chi_0(\mathbf{q}, iq_n)\right)^{-1}\right]^{\mu_1,\mu_2}_{\mu_3,\mu_4}$

Altermagnetic susceptibility

Stabilization of Altermagnetism



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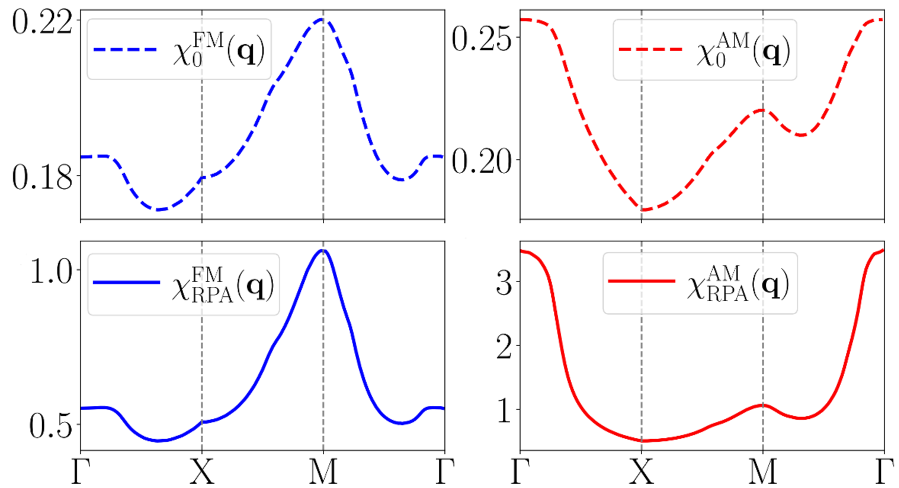
- Ferromagnetic vs altermagnetic channels

$$\begin{cases} \chi^{\text{FM}}_{\text{RPA}}(\mathbf{q},\omega) = \sum_{\mu,\nu} [\chi_{\text{RPA}}(\mathbf{q}, iq_n \to \omega + i\eta)]^{\mu,\mu}_{\nu,\nu} \\ \chi^{\text{AM}}_{\text{RPA}}(\mathbf{q},\omega) = \sum_{\mu,\nu} (-1)^{\mu}(-1)^{\nu} [\chi_{\text{RPA}}(\mathbf{q}, iq_n \to \omega + i\eta)]^{\mu,\mu}_{\nu,\nu} \end{cases}$$



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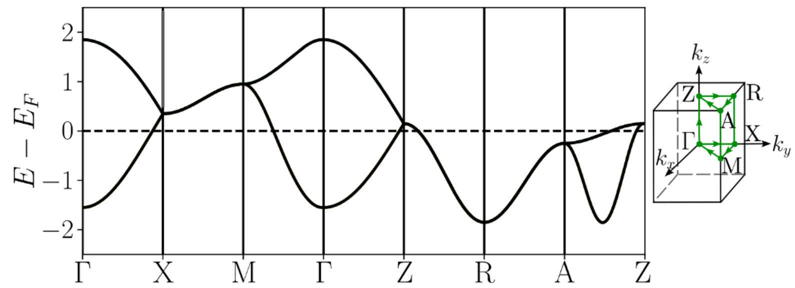

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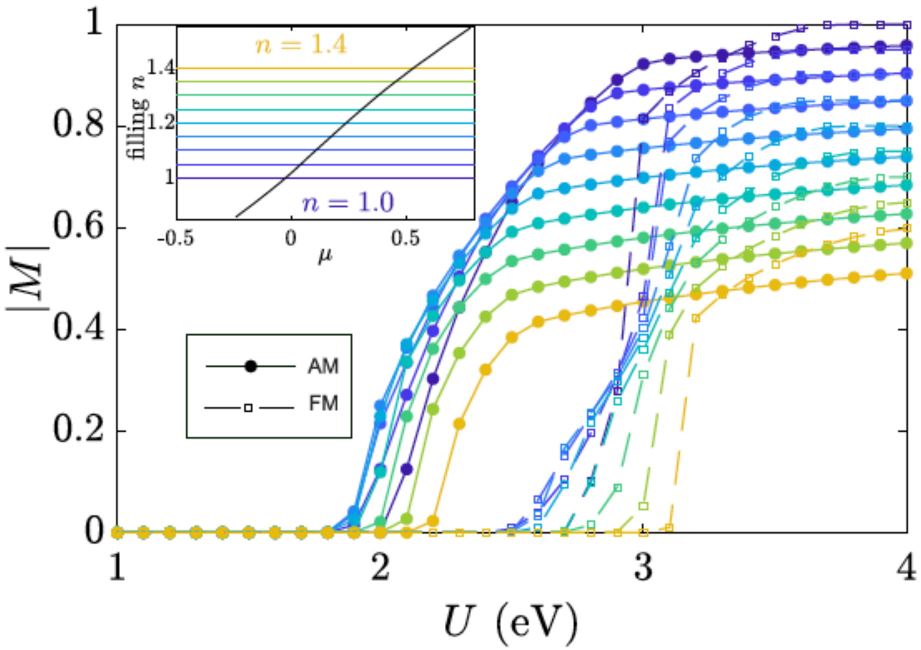

Altermagnetic susceptibility

Understanding the susceptibilities

- What stabilizes a leading AM vs FM instability?
- Susceptibilities in the two channels $\begin{cases} \chi^{\mathrm{FM}}(\mathbf{q},\omega) = \sum_{\mu,\nu}[\chi(\mathbf{q}, iq_n \to \omega + i\eta)]^{\mu,\mu}_{\nu,\nu} \\ \chi^{\mathrm{AM}}(\mathbf{q},\omega) = \sum_{\mu,\nu}(-1)^{\mu}(-1)^{\nu}[\chi(\mathbf{q}, iq_n \to \omega + i\eta)]^{\mu,\mu}_{\nu,\nu} \end{cases}$

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- Evaluation in the band basis:

$$\boxed{\chi^{\rm AM}(0) - \chi^{\rm FM}(0) = -\frac{1}{N}\sum_{\mathbf{k}} \sin^2\theta_{\mathbf{k}} \left\{ \frac{f(E^-_{\mathbf{k}}) - f(E^+_{\mathbf{k}})}{E^-_{\mathbf{k}} - E^+_{\mathbf{k}}} - \left[\left.\frac{df(\varepsilon)}{d\varepsilon}\right|_{\varepsilon=E^+_{\mathbf{k}}} + \left.\frac{df(\varepsilon)}{d\varepsilon}\right|_{\varepsilon=E^-_{\mathbf{k}}} \right] \right\}}$$

Eigenenergies: $E^{+/-}_{\mathbf{k}}$

Eigenvector:

$$u_{\mathbf{k}} = \begin{pmatrix} \sin(\theta_{\mathbf{k}}/2) \\ \cos(\theta_{\mathbf{k}}/2) \end{pmatrix}$$

1. Competition between intra- and inter-band terms
2. $\sin^2\theta_{\mathbf{k}} > 0$, $\sin\theta_{\mathbf{k}} = \dfrac{t_{x,\mathbf{k}}}{\sqrt{t^2_{z,\mathbf{k}} + t^2_{x,\mathbf{k}}}}$
3. Band degeneracies $E^+_{\mathbf{k}} - E^-_{\mathbf{k}} \to 0$ help AM

M. Roig *et al*, PRB 100, 144412 (2024)

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Conclusions & Outlook

Landau free energy & effects of SOC

- General form of free energy

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  2. Large anomalous Hall effect and weak ferromagnetism
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Large AHE from microscopic models ⟷ FM constrained by quasi-symmetries

PRB 110, 144412 (2024)

[Liu *et al.*, PRL (2024)], [Guo *et al.*, Nat. Phys. (2022)]

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P	Γ_N	$f_{\Gamma_N}(\mathbf{k})$	Lowest order invariant
C_{2h}	B_g	$\alpha k_xk_z + \beta k_yk_z$	$\alpha_1 N_xM_z,\ \alpha_2 N_yM_z$ $\alpha_3 N_zM_y,\ \alpha_4 N_zM_x$
D_{2h}	B_{1g}	k_xk_y	$\alpha_1 M_xN_y + \alpha_2 M_yN_x$
D_{2h}	B_{2g}	k_xk_z	$\alpha_1 M_yN_z + \alpha_2 M_zN_y$
D_{2h}	B_{3g}	k_yk_z	$\alpha_1 M_zN_x + \alpha_2 M_xN_z$
C_{4h}	B_g	$\alpha(k_x^2 - k_y^2)$ $+\ \beta k_xk_y$	$M_xN_y + M_yN_x,$ $M_xN_x - M_yN_y$
D_{4h}	A_{2g}	$k_xk_y(k_x^2 - k_y^2)$	$M_xN_y - M_yN_x$
D_{4h}	B_{1g}	$k_x^2 - k_y^2$	$M_xN_x - M_yN_y$
D_{4h}	B_{2g}	k_xk_y	$M_xN_y + M_yN_x$
D_{3d}	A_{2g}	$k_xk_z(k_x^2 - 3k_y^2)$	$M_xN_y - M_yN_x$
C_{6h}	B_g	$\alpha k_yk_z(k_y^2 - 3k_x^2)$ $+\beta k_xk_z(k_x^2 - 3k_y^2)$	$\alpha M_zN_y(3N_x^2 - N_y^2)$ $+\beta M_zN_x(3N_y^2 - N_x^2)$
D_{6h}	A_{2g}	$k_xk_y(k_x^2 - 3k_y^2)$ $\times(k_y^2 - 3k_x^2)$	$M_xN_y - M_yN_x$
D_{6h}	B_{1g}	$k_yk_z(3k_x^2 - k_y^2)$	$M_zN_y(3N_x^2 - N_y^2)$
D_{6h}	B_{2g}	$k_xk_z(k_x^2 - 3k_y^2)$	$M_zN_x(N_x^2 - 3N_y^2)$
O_h	A_{2g}	$k_x^4(k_y^2 - k_z^2)$ $+k_y^4(k_z^2 - k_x^2)$ $+k_z^4(k_x^2 - k_y^2)$	$M_xN_x(N_y^2 - N_z^2)$ $+M_yN_y(N_z^2 - N_x^2)$ $+M_zN_z(N_x^2 - N_y^2)$

Landau free energy & effects of SOC

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C_{2h}	B_g	$\alpha k_x k_z + \beta k_y k_z$	$\alpha_1 N_x M_z,\ \alpha_2 N_y M_z$ $\alpha_3 N_z M_y,\ \alpha_4 N_z M_x$
D_{2h}	B_{1g}	$k_x k_y$	$\alpha_1 M_x N_y + \alpha_2 M_y N_x$
D_{2h}	B_{2g}	$k_x k_z$	$\alpha_1 M_y N_z + \alpha_2 M_z N_y$
D_{2h}	B_{3g}	$k_y k_z$	$\alpha_1 M_z N_x + \alpha_2 M_x N_z$
C_{4h}	B_g	$\alpha(k_x^2 - k_y^2)$ $+\ \beta k_x k_y$	$M_x N_y + M_y N_x$, $M_x N_x - M_y N_y$
D_{4h}	A_{2g}	$k_x k_y (k_x^2 - k_y^2)$	$M_x N_y - M_y N_x$
D_{4h}	B_{1g}	$k_x^2 - k_y^2$	$M_x N_x - M_y N_y$
D_{4h}	B_{2g}	$k_x k_y$	$M_x N_y + M_y N_x$
D_{3d}	A_{2g}	$k_x k_z (k_x^2 - 3k_y^2)$	$M_x N_y - M_y N_x$
C_{6h}	B_g	$\alpha k_y k_z (k_y^2 - 3k_x^2)$ $+\beta k_x k_z (k_x^2 - 3k_y^2)$	$\alpha M_z N_y (3N_x^2 - N_y^2)$ $+\beta M_z N_x (3N_y^2 - N_x^2)$
D_{6h}	A_{2g}	$k_x k_y (k_x^2 - 3k_y^2)$ $\times (k_y^2 - 3k_x^2)$	$M_x N_y - M_y N_x$
D_{6h}	B_{1g}	$k_y k_z (3k_x^2 - k_y^2)$	$M_z N_y (3N_x^2 - N_y^2)$
D_{6h}	B_{2g}	$k_x k_z (k_x^2 - 3k_y^2)$	$M_z N_x (N_x^2 - 3N_y^2)$
O_h	A_{2g}	$k_x^4 (k_y^2 - k_z^2)$ $+k_y^4 (k_z^2 - k_x^2)$ $+k_z^4 (k_x^2 - k_y^2)$	$M_x N_x (N_y^2 - N_z^2)$ $+M_y N_y (N_z^2 - N_x^2)$ $+M_z N_z (N_x^2 - N_y^2)$

No bilinear coupling

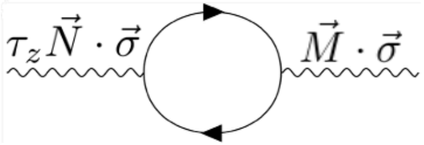
Landau free energy & effects of SOC

- Minimal model for altermagnetism

$$H_0(\mathbf{k}) = \varepsilon_{0,\mathbf{k}} + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z + \tau_y\vec{\lambda}_{\mathbf{k}}\cdot\vec{\sigma}$$

$$H' = \tau_z\vec{N}\cdot\vec{\sigma} + \vec{M}\cdot\vec{\sigma}$$

- Quadratic free energy



$$F^{(2)}_{NM} = \frac{1}{2\beta}\sum_{i\omega_n}\mathrm{Tr}[G_0\tau_z\vec{N}\cdot\vec{\sigma}G_0\vec{M}\cdot\vec{\sigma}]$$

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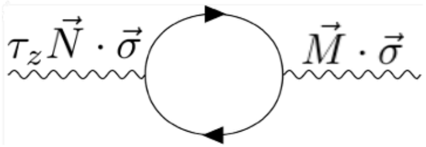

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Only antisymmetric combinations are generated to linear order in SOC

$$L(\mathbf{k}) = \left.\frac{df(\varepsilon)}{d\varepsilon}\right|_{\varepsilon=E^+_{\mathbf{k}}} + \left.\frac{df(\varepsilon)}{d\varepsilon}\right|_{\varepsilon=E^-_{\mathbf{k}}} - \frac{2[f(E^-_{\mathbf{k}}) - f(E^+_{\mathbf{k}})]}{E^-_{\mathbf{k}} - E^+_{\mathbf{k}}}$$

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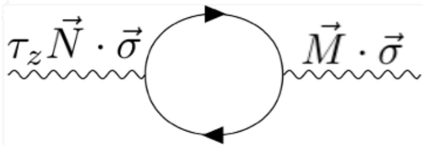

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C_{2h}	B_g	$\alpha k_x k_z + \beta k_y k_z$	$\alpha_1 N_x M_z,\ \alpha_2 N_y M_z$ $\alpha_3 N_z M_y,\ \alpha_4 N_z M_x$	✓
D_{2h}	B_{1g}	$k_x k_y$	$\alpha_1 M_x N_y + \alpha_2 M_y N_x$	✓
D_{2h}	B_{2g}	$k_x k_z$	$\alpha_1 M_y N_z + \alpha_2 M_z N_y$	✓
D_{2h}	B_{3g}	$k_y k_z$	$\alpha_1 M_z N_x + \alpha_2 M_x N_z$	✓
C_{4h}	B_g	$\alpha(k_x^2 - k_y^2)$ $+ \beta k_x k_y$	$M_x N_y + M_y N_x,$ $M_x N_x - M_y N_y$	✗
D_{4h}	A_{2g}	$k_x k_y (k_x^2 - k_y^2)$	$M_x N_y - M_y N_x$	✓
D_{4h}	B_{1g}	$k_x^2 - k_y^2$	$M_x N_x - M_y N_y$	✗
D_{4h}	B_{2g}	$k_x k_y$	$M_x N_y + M_y N_x$	✗
D_{3d}	A_{2g}	$k_x k_z (k_x^2 - 3k_y^2)$	$M_x N_y - M_y N_x$	✓
C_{6h}	B_g	$\alpha k_y k_z (k_y^2 - 3k_x^2)$ $+\beta k_x k_z (k_x^2 - 3k_y^2)$	$\alpha M_z N_y (3N_x^2 - N_y^2)$ $+\beta M_z N_x (3N_y^2 - N_x^2)$	✗
D_{6h}	A_{2g}	$k_x k_y (k_x^2 - 3k_y^2)$ $\times (k_y^2 - 3k_x^2)$	$M_x N_y - M_y N_x$	✓
D_{6h}	B_{1g}	$k_y k_z (3k_x^2 - k_y^2)$	$M_z N_y (3N_x^2 - N_y^2)$	✗
D_{6h}	B_{2g}	$k_x k_z (k_x^2 - 3k_y^2)$	$M_z N_x (N_x^2 - 3N_y^2)$	✗
O_h	A_{2g}	$k_x^4 (k_y^2 - k_z^2)$ $+k_y^4 (k_z^2 - k_x^2)$ $+k_z^4 (k_x^2 - k_y^2)$	$M_x N_x (N_y^2 - N_z^2)$ $+M_y N_y (N_z^2 - N_x^2)$ $+M_z N_z (N_x^2 - N_y^2)$	✗

Landau free energy & effects of SOC

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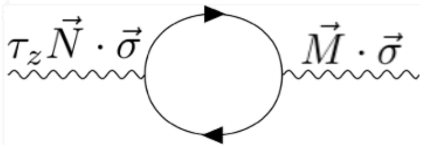

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D_{2h}	B_{2g}	k_xk_z	$\alpha_1M_yN_z + \alpha_2M_zN_y$	✓
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C_{4h}	B_g	$\alpha(k_x^2-k_y^2)$ $+\ \beta k_xk_y$	$M_xN_y + M_yN_x,$ $M_xN_x - M_yN_y$	✗
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D_{4h}	B_{1g}	$k_x^2-k_y^2$	$M_xN_x - M_yN_y$	✗
D_{4h}	B_{2g}	k_xk_y	$M_xN_y + M_yN_x$	✗
D_{3d}	A_{2g}	$k_xk_z(k_x^2-3k_y^2)$	$M_xN_y - M_yN_x$	✓
C_{6h}	B_g	$\alpha k_yk_z(k_y^2-3k_x^2)$ $+\beta k_xk_z(k_x^2-3k_y^2)$	$\alpha M_zN_y(3N_x^2-N_y^2)$ $+\beta M_zN_x(3N_y^2-N_x^2)$	✗
D_{6h}	A_{2g}	$k_xk_y(k_x^2-3k_y^2)$ $\times(k_y^2-3k_x^2)$	$M_xN_y - M_yN_x$	✓
D_{6h}	B_{1g}	$k_yk_z(3k_x^2-k_y^2)$	$M_zN_y(3N_x^2-N_y^2)$	✗
D_{6h}	B_{2g}	$k_xk_z(k_x^2-3k_y^2)$	$M_zN_x(N_x^2-3N_y^2)$	✗
O_h	A_{2g}	$k_x^4(k_y^2-k_z^2)$ $+k_y^4(k_z^2-k_x^2)$ $+k_z^4(k_x^2-k_y^2)$	$M_xN_x(N_y^2-N_z^2)$ $+M_yN_y(N_z^2-N_x^2)$ $+M_zN_z(N_x^2-N_y^2)$	✗

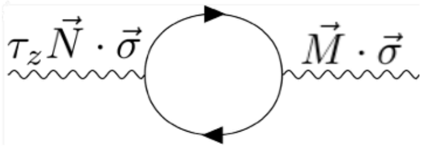
Landau free energy & effects of SOC

- Minimal model for altermagnetism

$$H_0(\mathbf{k}) = \varepsilon_{0,\mathbf{k}} + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z + \tau_y\vec{\lambda}_{\mathbf{k}}\cdot\vec{\sigma}$$

$$H' = \tau_z\vec{N}\cdot\vec{\sigma} + \vec{M}\cdot\vec{\sigma}$$

- Quadratic free energy



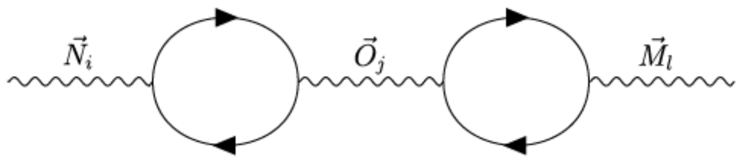
$$F^{(2)}_{NM} = \frac{1}{2\beta}\sum_{i\omega_n}\mathrm{Tr}[G_0\tau_z\vec{N}\cdot\vec{\sigma}G_0\vec{M}\cdot\vec{\sigma}]$$

$$F^{(2)}_{NM} = 2\sum_{\mathbf{k}}\frac{t_{x,\mathbf{k}}}{\tilde{E}^2_{\mathbf{k}}}L(\mathbf{k})\boxed{\vec{\lambda}_{\mathbf{k}}\cdot(\vec{M}\times\vec{N})}$$

Only antisymmetric combinations are generated to linear order in SOC

P	Γ_N	$f_{\Gamma_N}(\mathbf{k})$	Lowest order invariant	M_i linear in SOC
C_{2h}	B_g	$\alpha k_xk_z + \beta k_yk_z$	$\alpha_1N_xM_z,\ \alpha_2N_yM_z$ $\alpha_3N_zM_y,\ \alpha_4N_zM_x$	✓
D_{2h}	B_{1g}	k_xk_y	$\alpha_1M_xN_y + \alpha_2M_yN_x$	✓
D_{2h}	B_{2g}	k_xk_z	$\alpha_1M_yN_z + \alpha_2M_zN_y$	✓
D_{2h}	B_{3g}	k_yk_z	$\alpha_1M_zN_x + \alpha_2M_xN_z$	✓
C_{4h}	B_g	$\alpha(k_x^2-k_y^2)$ $+\ \beta k_xk_y$	$M_xN_y + M_yN_x$, $M_xN_x - M_yN_y$	✗
D_{4h}	A_{2g}	$k_xk_y(k_x^2-k_y^2)$	$M_xN_y - M_yN_x$	✓
D_{4h}	B_{1g}	$k_x^2-k_y^2$	$M_xN_x - M_yN_y$	✗
D_{4h}	B_{2g}	k_xk_y	$M_xN_y + M_yN_x$	✗
D_{3d}	A_{2g}	$k_xk_z(k_x^2-3k_y^2)$	$M_xN_y - M_yN_x$	✓
C_{6h}	B_g	$\alpha k_yk_z(k_y^2-3k_x^2)$ $+\beta k_xk_z(k_x^2-3k_y^2)$	$\alpha M_zN_y(3N_x^2-N_y^2)$ $+\beta M_zN_x(3N_y^2-N_x^2)$	✗
D_{6h}	A_{2g}	$k_xk_y(k_x^2-3k_y^2)$ $\times(k_y^2-3k_x^2)$	$M_xN_y - M_yN_x$	✓
D_{6h}	B_{1g}	$k_yk_z(3k_x^2-k_y^2)$	$M_zN_y(3N_x^2-N_y^2)$	✗
D_{6h}	B_{2g}	$k_xk_z(k_x^2-3k_y^2)$	$M_zN_x(N_x^2-3N_y^2)$	✗
O_h	A_{2g}	$k_x^4(k_y^2-k_z^2)$ $+k_y^4(k_z^2-k_x^2)$ $+k_z^4(k_x^2-k_y^2)$	$M_xN_x(N_y^2-N_z^2)$ $+M_yN_y(N_z^2-N_x^2)$ $+M_zN_z(N_x^2-N_y^2)$	✗

Role of secondary order parameters:



M. Roig *et al*, PRL 135, 046701 (2025)

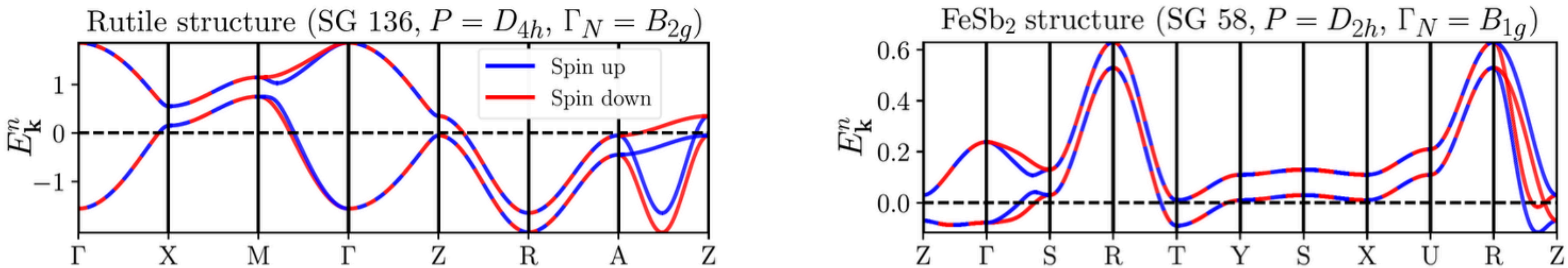
Anomalous Hall effect and weak ferromagnetism

The case of the rutile lattice and $FeSb_2$

- Microscopic model for altermagnetism

$$H = \varepsilon_{0,\mathbf{k}} + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z + \tau_y \vec{\lambda}_{\mathbf{k}} \cdot \vec{\sigma} + \tau_z \vec{N} \cdot \vec{\sigma}$$

P	Γ_N	$f_{\Gamma_N}(\mathbf{k})$	Lowest order invariant	M_i linear in SOC
D_{4h}	B_{2g}	$k_x k_y$	$M_x N_y + M_y N_x$	✗
D_{2h}	B_{1g}	$k_x k_y$	$\alpha_1 M_x N_y + \alpha_2 M_y N_x$	✓



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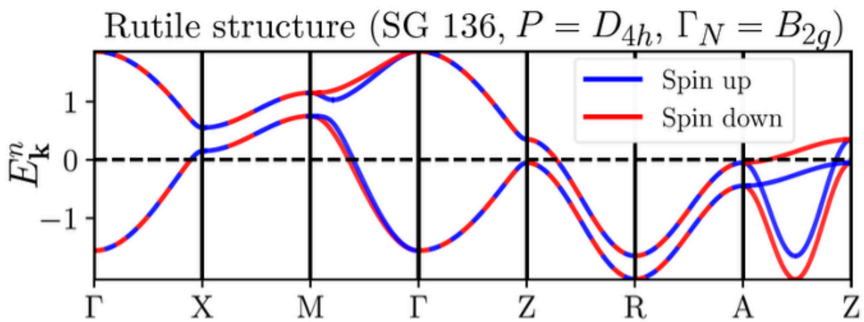
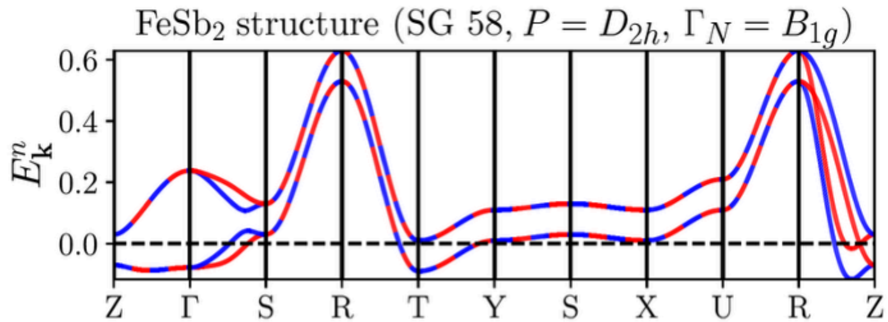
P	Γ_N	$f_{\Gamma_N}(\mathbf{k})$	Lowest order invariant	M_i linear in SOC
D_{4h}	B_{2g}	$k_x k_y$	$M_x N_y + M_y N_x$	✗
D_{2h}	B_{1g}	$k_x k_y$	$\alpha_1 M_x N_y + \alpha_2 M_y N_x$	✓





$$\vec{M} = \mu_B \sum_{a,\mathbf{k}} \langle u^a_{\mathbf{k}} | \vec{S} | u^a_{\mathbf{k}} \rangle f(E^a_{\mathbf{k}})$$

Anomalous Hall effect and weak ferromagnetism

The case of the rutile lattice and $FeSb_2$

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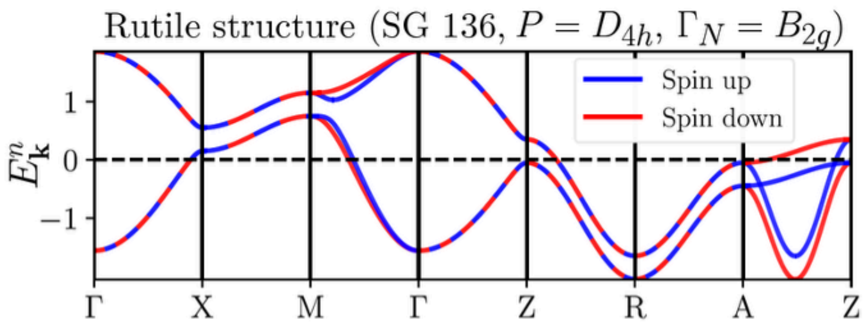
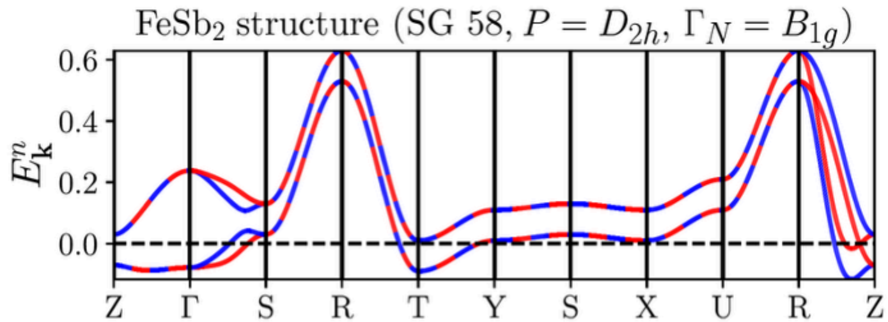
$$H = \varepsilon_{0,\mathbf{k}} + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z + \tau_y \vec{\lambda}_{\mathbf{k}} \cdot \vec{\sigma} + \tau_z \vec{N} \cdot \vec{\sigma}$$

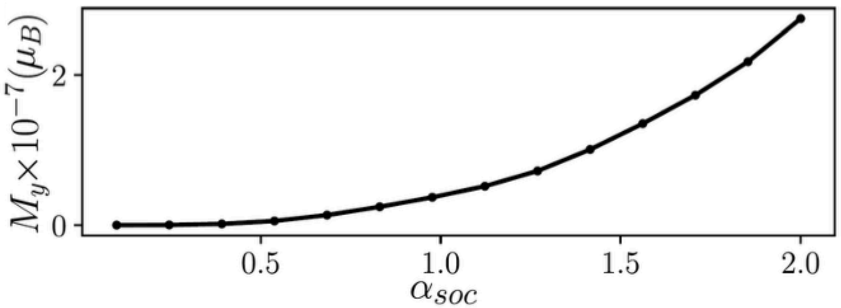
P	Γ_N	$f_{\Gamma_N}(\mathbf{k})$	Lowest order invariant	M_i linear in SOC
D_{4h}	B_{2g}	$k_x k_y$	$M_x N_y + M_y N_x$	✗
D_{2h}	B_{1g}	$k_x k_y$	$\alpha_1 M_x N_y + \alpha_2 M_y N_x$	✓

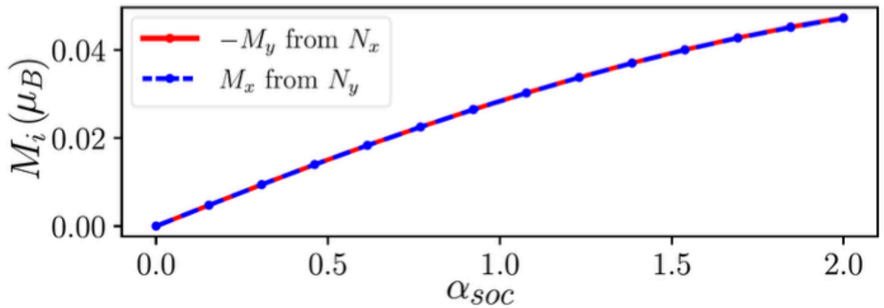




$$\vec{M} = \mu_B \sum_{a,\mathbf{k}} \langle u^a_{\mathbf{k}} | \vec{S} | u^a_{\mathbf{k}} \rangle f(E^a_{\mathbf{k}})$$

- SOC-induced magnetization





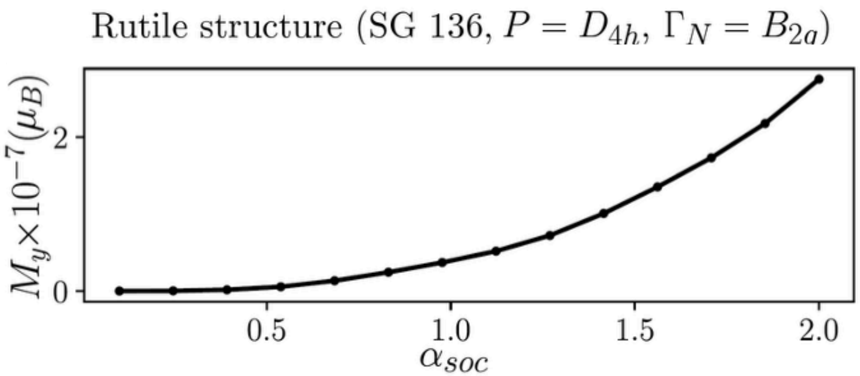
Anomalous Hall effect and weak ferromagnetism

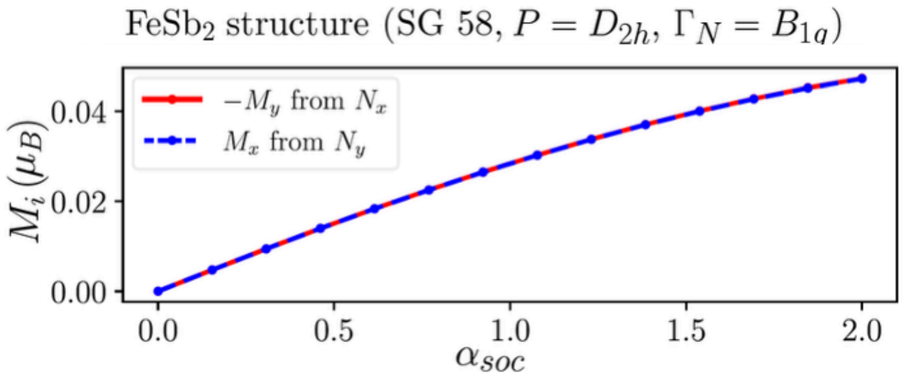
The case of the rutile lattice and $FeSb_2$

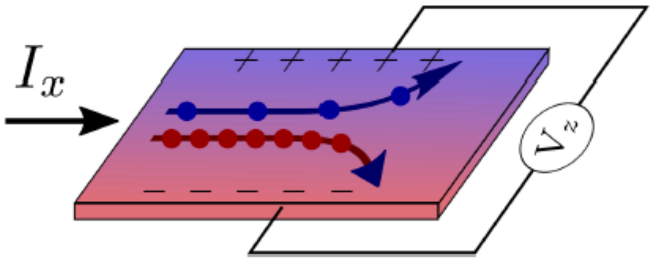
- Microscopic model for altermagnetism

$$H = \varepsilon_{0,\mathbf{k}} + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z + \tau_y \vec{\lambda}_{\mathbf{k}} \cdot \vec{\sigma} + \tau_z \vec{N} \cdot \vec{\sigma}$$

P	Γ_N	$f_{\Gamma_N}(\mathbf{k})$	Lowest order invariant	M_i linear in SOC
D_{4h}	B_{2g}	$k_x k_y$	$M_x N_y + M_y N_x$	✗
D_{2h}	B_{1g}	$k_x k_y$	$\alpha_1 M_x N_y + \alpha_2 M_y N_x$	✓







$$I_x \propto \sigma_{xz} V_z$$

$$\sigma_{ij} = -\frac{e^2}{\hbar} \int_{\mathrm{BZ}} \frac{d\mathbf{k}}{(2\pi)^3} \sum_n f_n(\mathbf{k}) \Omega_{n,ij}$$

Anomalous Hall effect and weak ferromagnetism

The case of the rutile lattice and $FeSb_2$

- Microscopic model for altermagnetism

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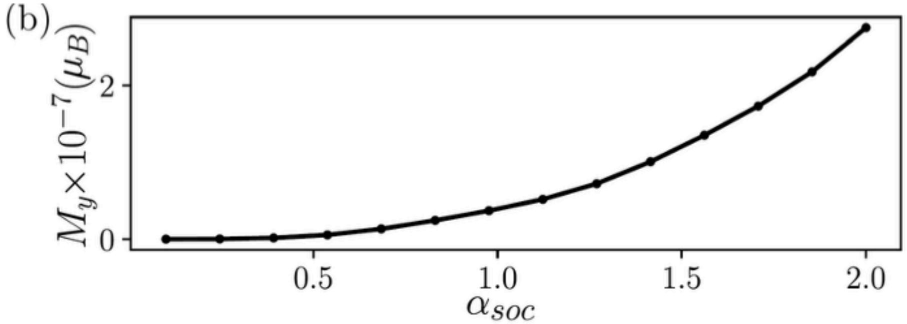
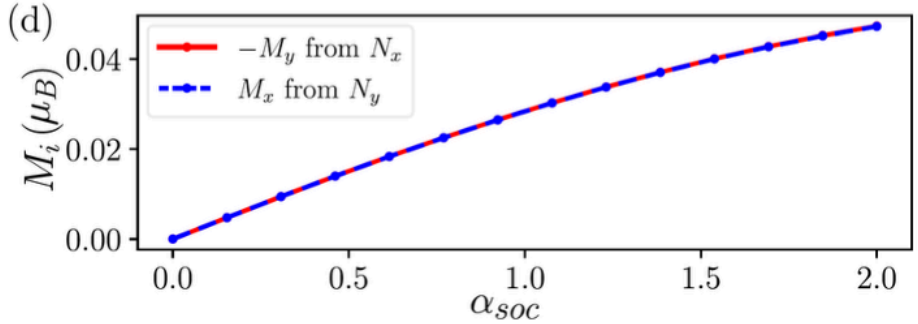
P	Γ_N	$f_{\Gamma_N}(\mathbf{k})$	Lowest order invariant	M_i linear in SOC
D_{4h}	B_{2g}	$k_x k_y$	$M_x N_y + M_y N_x$	✗
D_{2h}	B_{1g}	$k_x k_y$	$\alpha_1 M_x N_y + \alpha_2 M_y N_x$	✓

Rutile structure (SG 136, $P = D_{4h}$, $\Gamma_N = B_{2g}$)



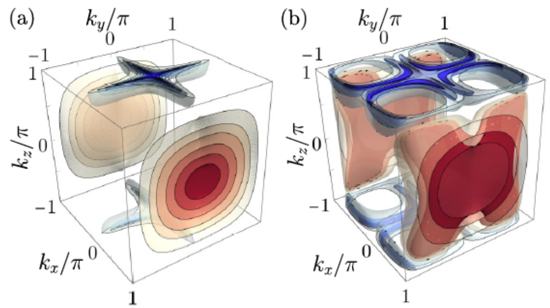
FeSb₂ structure (SG 58, $P = D_{2h}$, $\Gamma_N = B_{1g}$)



Quantum geometric tensor [Graf and Piéchon, PRB (2021)]

$$T_{n,ij} = \mathrm{Tr}\{(\partial_i P_n)(1 - P_n)(\partial_j P_n)\}, \quad \text{with} \quad P_n = |\psi_n\rangle\langle\psi_n| \quad \Rightarrow \quad \Omega_{n,ij} = -2\,\mathrm{Im}\{T_{n,ij}\}$$

Ω_{xz}



M. Roig *et al*, PRL 135, 046701 (2025)
M. Roig *et al*, PRB 100, 144412 (2024)

Anomalous Hall effect and weak ferromagnetism

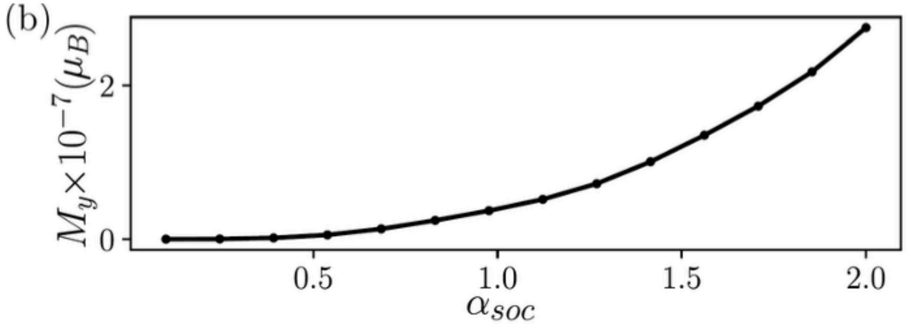
The case of the rutile lattice and $FeSb_2$

- Microscopic model for altermagnetism

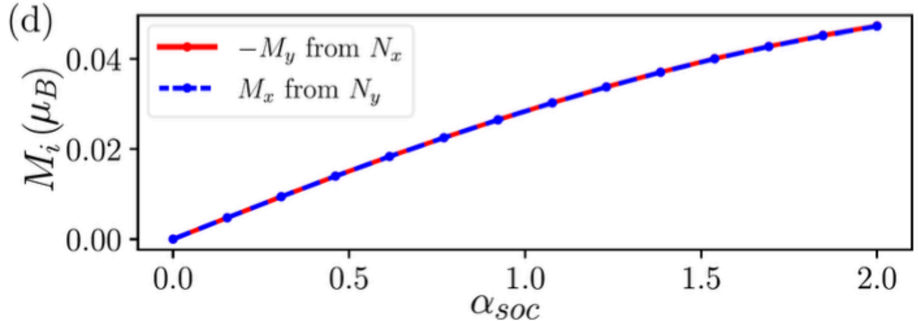
$$H = \varepsilon_{0,\mathbf{k}} + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z + \tau_y \vec{\lambda}_{\mathbf{k}} \cdot \vec{\sigma} + \tau_z \vec{N} \cdot \vec{\sigma}$$

P	Γ_N	$f_{\Gamma_N}(\mathbf{k})$	Lowest order invariant	M_i linear in SOC
D_{4h}	B_{2g}	$k_x k_y$	$M_x N_y + M_y N_x$	✗
D_{2h}	B_{1g}	$k_x k_y$	$\alpha_1 M_x N_y + \alpha_2 M_y N_x$	✓

Rutile structure (SG 136, $P = D_{4h}$, $\Gamma_N = B_{2g}$)



$FeSb_2$ structure (SG 58, $P = D_{2h}$, $\Gamma_N = B_{1g}$)



$$\Omega^{(N_l)}_{\alpha,\beta,ij} = \frac{1}{2E^3_{\alpha,\beta}} \sum_{m,n=i,j} \varepsilon_{mn} \Big[(N_l + \beta |t_{z,\mathbf{k}}|) \partial_m \lambda_{l,\mathbf{k}} \partial_n t_{x,\mathbf{k}} + \mathrm{sgn}(t_{z,\mathbf{k}}) \beta t_{x,\mathbf{k}} \partial_m t_{z,\mathbf{k}} \partial_n \lambda_{l,\mathbf{k}} + \mathrm{sgn}(t_{z,\mathbf{k}}) \beta \lambda_{l,\mathbf{k}} \partial_m t_{x,\mathbf{k}} \partial_n t_{z,\mathbf{k}} \Big],$$

$$\sigma_{ij} = -\frac{e^2}{\hbar} \int_{\mathrm{BZ}} \frac{d\mathbf{k}}{(2\pi)^3} \sum_{\alpha,\beta} f_{\alpha,\beta}(\mathbf{k}) \Omega_{\alpha,\beta,ij},$$

→ Linear in SOC

Anomalous Hall effect and weak ferromagnetism

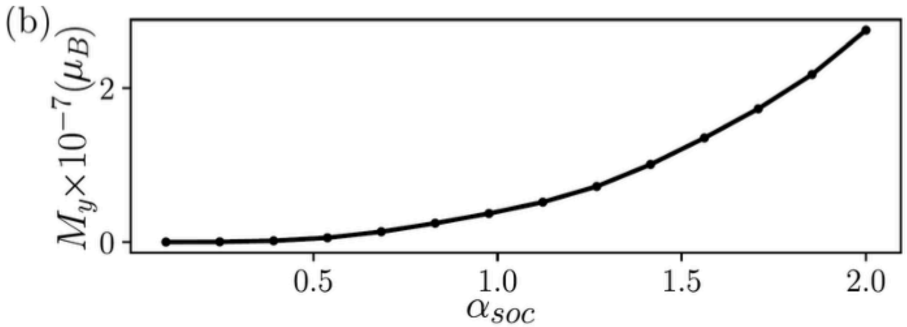
The case of the rutile lattice and $FeSb_2$

- Microscopic model for altermagnetism

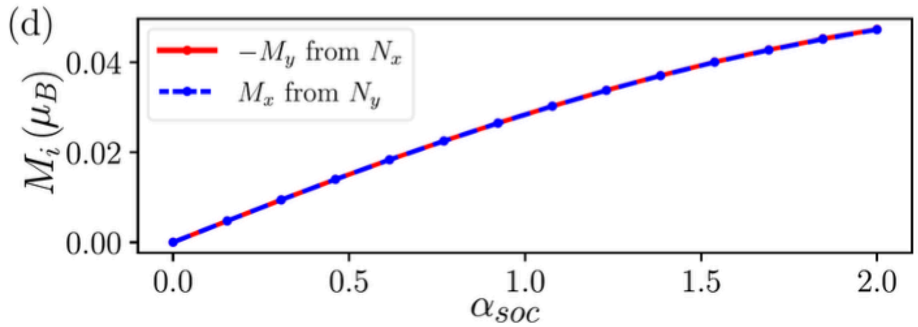
$$H = \varepsilon_{0,\mathbf{k}} + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z + \tau_y \vec{\lambda}_{\mathbf{k}} \cdot \vec{\sigma} + \tau_z \vec{N} \cdot \vec{\sigma}$$

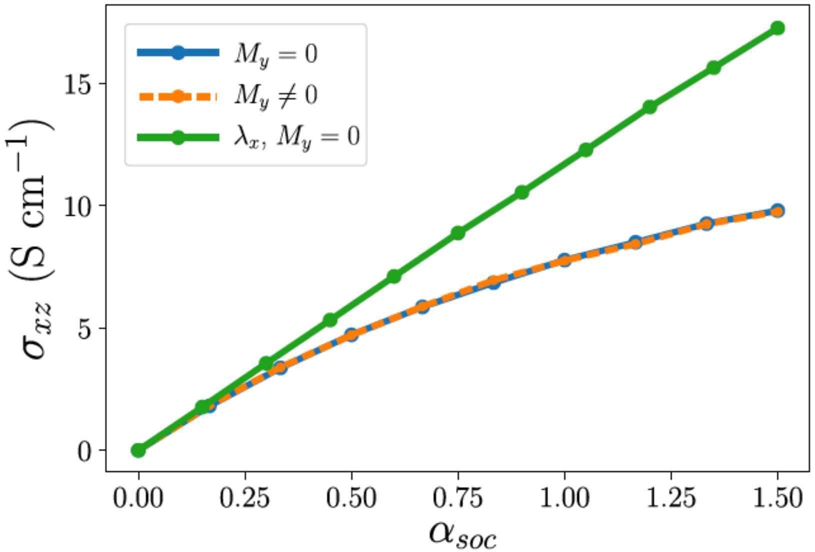
P	Γ_N	$f_{\Gamma_N}(\mathbf{k})$	Lowest order invariant	M_i linear in SOC
D_{4h}	B_{2g}	$k_x k_y$	$M_x N_y + M_y N_x$	✗
D_{2h}	B_{1g}	$k_x k_y$	$\alpha_1 M_x N_y + \alpha_2 M_y N_x$	✓

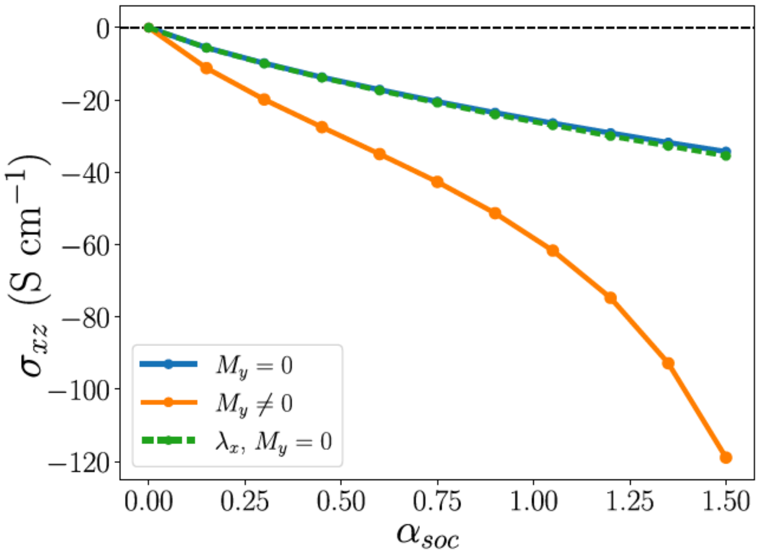
Rutile structure (SG 136, $P = D_{4h}$, $\Gamma_N = B_{2g}$)



FeSb₂ structure (SG 58, $P = D_{2h}$, $\Gamma_N = B_{1g}$)







Anomalous Hall effect and weak ferromagnetism

Comparison with DFT results

- **Comparison to FM moment identified in DFT:**

P	Γ_N	$f_{\Gamma_N}(\mathbf{k})$	Lowest order invariant	M_i linear in SOC
C_{2h}	B_g	$\alpha k_x k_z + \beta k_y k_z$	$\alpha_1 N_x M_z,\ \alpha_2 N_y M_z$ $\alpha_3 N_z M_y,\ \alpha_4 N_z M_x$	✓
D_{2h}	B_{1g}	$k_x k_y$	$\alpha_1 M_x N_y + \alpha_2 M_y N_x$	✓
D_{2h}	B_{2g}	$k_x k_z$	$\alpha_1 M_y N_z + \alpha_2 M_z N_y$	✓
D_{2h}	B_{3g}	$k_y k_z$	$\alpha_1 M_z N_x + \alpha_2 M_x N_z$	✓
C_{4h}	B_g	$\alpha(k_x^2 - k_y^2)$ $+\ \beta k_x k_y$	$M_x N_y + M_y N_x,$ $M_x N_x - M_y N_y$	✗
D_{4h}	A_{2g}	$k_x k_y (k_x^2 - k_y^2)$	$M_x N_y - M_y N_x$	✓
D_{4h}	B_{1g}	$k_x^2 - k_y^2$	$M_x N_x - M_y N_y$	✗
D_{4h}	B_{2g}	$k_x k_y$	$M_x N_y + M_y N_x$	✗
D_{3d}	A_{2g}	$k_x k_z (k_x^2 - 3k_y^2)$	$M_x N_y - M_y N_x$	✓
C_{6h}	B_g	$\alpha k_y k_z (k_y^2 - 3k_x^2)$ $+\beta k_x k_z (k_x^2 - 3k_y^2)$	$\alpha M_z N_y (3N_x^2 - N_y^2)$ $+\beta M_z N_x (3N_y^2 - N_x^2)$	✗
D_{6h}	A_{2g}	$k_x k_y (k_x^2 - 3k_y^2)$ $\times (k_y^2 - 3k_x^2)$	$M_x N_y - M_y N_x$	✓
D_{6h}	B_{1g}	$k_y k_z (3k_x^2 - k_y^2)$	$M_z N_y (3N_x^2 - N_y^2)$	✗
D_{6h}	B_{2g}	$k_x k_z (k_x^2 - 3k_y^2)$	$M_z N_x (N_x^2 - 3N_y^2)$	✗
O_h	A_{2g}	$k_x^4 (k_y^2 - k_z^2)$ $+k_y^4 (k_z^2 - k_x^2)$ $+k_z^4 (k_x^2 - k_y^2)$	$M_x N_x (N_y^2 - N_z^2)$ $+M_y N_y (N_z^2 - N_x^2)$ $+M_z N_z (N_x^2 - N_y^2)$	✗

Anomalous Hall effect and weak ferromagnetism

Comparison with DFT results

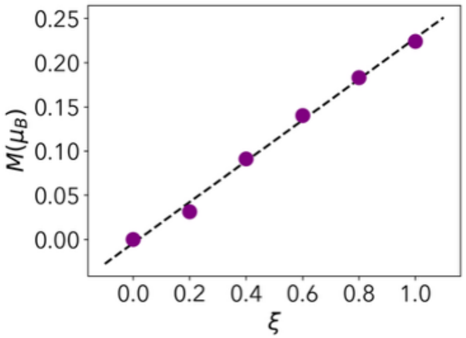
- Comparison to FM moment identified in DFT:
  - Sizable: $\{$ RuF_4 [Milivojević *et al.*, 2D Mater. (2024)]

P	Γ_N	$f_{\Gamma_N}(\mathbf{k})$	Lowest order invariant	M_i linear in SOC
C_{2h}	B_g	$\alpha k_x k_z + \beta k_y k_z$	$\alpha_1 N_x M_z$, $\alpha_2 N_y M_z$ $\alpha_3 N_z M_y$, $\alpha_4 N_z M_x$	✓
D_{2h}	B_{1g}	$k_x k_y$	$\alpha_1 M_x N_y + \alpha_2 M_y N_x$	✓
D_{2h}	B_{2g}	$k_x k_z$	$\alpha_1 M_y N_z + \alpha_2 M_z N_y$	✓
D_{2h}	B_{3g}	$k_y k_z$	$\alpha_1 M_z N_x + \alpha_2 M_x N_z$	✓
C_{4h}	B_g	$\alpha(k_x^2 - k_y^2)$ $+ \beta k_x k_y$	$M_x N_y + M_y N_x$, $M_x N_x - M_y N_y$	✗
D_{4h}	A_{2g}	$k_x k_y (k_x^2 - k_y^2)$	$M_x N_y - M_y N_x$	✓
D_{4h}	B_{1g}	$k_x^2 - k_y^2$	$M_x N_x - M_y N_y$	✗
D_{4h}	B_{2g}	$k_x k_y$	$M_x N_y + M_y N_x$	✗
D_{3d}	A_{2g}	$k_x k_z (k_x^2 - 3k_y^2)$	$M_x N_y - M_y N_x$	✓
C_{6h}	B_g	$\alpha k_y k_z (k_y^2 - 3k_x^2)$ $+\beta k_x k_z (k_x^2 - 3k_y^2)$	$\alpha M_z N_y (3N_x^2 - N_y^2)$ $+\beta M_z N_x (3N_y^2 - N_x^2)$	✗
D_{6h}	A_{2g}	$k_x k_y (k_x^2 - 3k_y^2)$ $\times (k_y^2 - 3k_x^2)$	$M_x N_y - M_y N_x$	✓
D_{6h}	B_{1g}	$k_y k_z (3k_x^2 - k_y^2)$	$M_z N_y (3N_x^2 - N_y^2)$	✗
D_{6h}	B_{2g}	$k_x k_z (k_x^2 - 3k_y^2)$	$M_z N_x (N_x^2 - 3N_y^2)$	✗
O_h	A_{2g}	$k_x^4 (k_y^2 - k_z^2)$ $+k_y^4 (k_z^2 - k_x^2)$ $+k_z^4 (k_x^2 - k_y^2)$	$M_x N_x (N_y^2 - N_z^2)$ $+M_y N_y (N_z^2 - N_x^2)$ $+M_z N_z (N_x^2 - N_y^2)$	✗

Anomalous Hall effect and weak ferromagnetism

Comparison with DFT results

- Comparison to FM moment identified in DFT:
  - Sizable: { RuF_4 [Milivojević *et al.*, 2D Mater. (2024)]



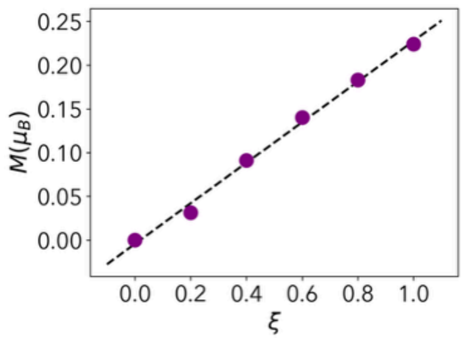
[Milivojević *et al.*, 2D Mater. (2024)]

P	Γ_N	$f_{\Gamma_N}(\mathbf{k})$	Lowest order invariant	M_i linear in SOC
C_{2h}	B_g	$\alpha k_x k_z + \beta k_y k_z$	$\alpha_1 N_x M_z,\ \alpha_2 N_y M_z$ $\alpha_3 N_z M_y,\ \alpha_4 N_z M_x$	✓
D_{2h}	B_{1g}	$k_x k_y$	$\alpha_1 M_x N_y + \alpha_2 M_y N_x$	✓
D_{2h}	B_{2g}	$k_x k_z$	$\alpha_1 M_y N_z + \alpha_2 M_z N_y$	✓
D_{2h}	B_{3g}	$k_y k_z$	$\alpha_1 M_z N_x + \alpha_2 M_x N_z$	✓
C_{4h}	B_g	$\alpha(k_x^2 - k_y^2)$ $+ \beta k_x k_y$	$M_x N_y + M_y N_x,$ $M_x N_x - M_y N_y$	✘
D_{4h}	A_{2g}	$k_x k_y (k_x^2 - k_y^2)$	$M_x N_y - M_y N_x$	✓
D_{4h}	B_{1g}	$k_x^2 - k_y^2$	$M_x N_x - M_y N_y$	✘
D_{4h}	B_{2g}	$k_x k_y$	$M_x N_y + M_y N_x$	✘
D_{3d}	A_{2g}	$k_x k_z (k_x^2 - 3k_y^2)$	$M_x N_y - M_y N_x$	✓
C_{6h}	B_g	$\alpha k_y k_z (k_y^2 - 3k_x^2)$ $+\beta k_x k_z (k_x^2 - 3k_y^2)$	$\alpha M_z N_y (3N_x^2 - N_y^2)$ $+\beta M_z N_x (3N_y^2 - N_x^2)$	✘
D_{6h}	A_{2g}	$k_x k_y (k_x^2 - 3k_y^2)$ $\times (k_y^2 - 3k_x^2)$	$M_x N_y - M_y N_x$	✓
D_{6h}	B_{1g}	$k_y k_z (3k_x^2 - k_y^2)$	$M_z N_y (3N_x^2 - N_y^2)$	✘
D_{6h}	B_{2g}	$k_x k_z (k_x^2 - 3k_y^2)$	$M_z N_x (N_x^2 - 3N_y^2)$	✘
O_h	A_{2g}	$k_x^4 (k_y^2 - k_z^2)$ $+k_y^4 (k_z^2 - k_x^2)$ $+k_z^4 (k_x^2 - k_y^2)$	$M_x N_x (N_y^2 - N_z^2)$ $+M_y N_y (N_z^2 - N_x^2)$ $+M_z N_z (N_x^2 - N_y^2)$	✘

Anomalous Hall effect and weak ferromagnetism

Comparison with DFT results

- **Comparison to FM moment identified in DFT:**
  - Sizable: RuF_4 [Milivojević *et al.*, 2D Mater. (2024)] ; $FeSb_2$ [Mazin *et al.*, PNAS (2021)]



[Milivojević *et al.*, 2D Mater. (2024)]

P	Γ_N	$f_{\Gamma_N}(\mathbf{k})$	Lowest order invariant	M_i linear in SOC
C_{2h}	B_g	$\alpha k_x k_z + \beta k_y k_z$	$\alpha_1 N_x M_z,\ \alpha_2 N_y M_z$ $\alpha_3 N_z M_y,\ \alpha_4 N_z M_x$	✓
D_{2h}	B_{1g}	$k_x k_y$	$\alpha_1 M_x N_y + \alpha_2 M_y N_x$	✓
D_{2h}	B_{2g}	$k_x k_z$	$\alpha_1 M_y N_z + \alpha_2 M_z N_y$	✓
D_{2h}	B_{3g}	$k_y k_z$	$\alpha_1 M_z N_x + \alpha_2 M_x N_z$	✓
C_{4h}	B_g	$\alpha(k_x^2 - k_y^2)$ $+ \beta k_x k_y$	$M_x N_y + M_y N_x,$ $M_x N_x - M_y N_y$	✗
D_{4h}	A_{2g}	$k_x k_y (k_x^2 - k_y^2)$	$M_x N_y - M_y N_x$	✓
D_{4h}	B_{1g}	$k_x^2 - k_y^2$	$M_x N_x - M_y N_y$	✗
D_{4h}	B_{2g}	$k_x k_y$	$M_x N_y + M_y N_x$	✗
D_{3d}	A_{2g}	$k_x k_z (k_x^2 - 3k_y^2)$	$M_x N_y - M_y N_x$	✓
C_{6h}	B_g	$\alpha k_y k_z (k_y^2 - 3k_x^2)$ $+\beta k_x k_z (k_x^2 - 3k_y^2)$	$\alpha M_z N_y (3N_x^2 - N_y^2)$ $+\beta M_z N_x (3N_y^2 - N_x^2)$	✗
D_{6h}	A_{2g}	$k_x k_y (k_x^2 - 3k_y^2)$ $\times (k_y^2 - 3k_x^2)$	$M_x N_y - M_y N_x$	✓
D_{6h}	B_{1g}	$k_y k_z (3k_x^2 - k_y^2)$	$M_z N_y (3N_x^2 - N_y^2)$	✗
D_{6h}	B_{2g}	$k_x k_z (k_x^2 - 3k_y^2)$	$M_z N_x (N_x^2 - 3N_y^2)$	✗
O_h	A_{2g}	$k_x^4 (k_y^2 - k_z^2)$ $+k_y^4 (k_z^2 - k_x^2)$ $+k_z^4 (k_x^2 - k_y^2)$	$M_x N_x (N_y^2 - N_z^2)$ $+M_y N_y (N_z^2 - N_x^2)$ $+M_z N_z (N_x^2 - N_y^2)$	✗

Anomalous Hall effect and weak ferromagnetism

Comparison with DFT results

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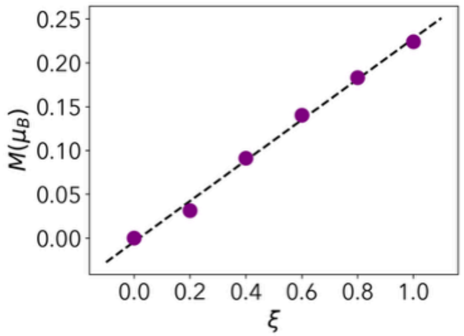
  - Sizable:
    - RuF_4 [Milivojević *et al.*, 2D Mater. (2024)]
    - $FeSb_2$ [Mazin *et al.*, PNAS (2021)]



[Milivojević *et al.*, 2D Mater. (2024)]

  - Vanishing:
    - RuO_2 [Šmejkal *et al.*, Sci. Adv. (2020)]

P	Γ_N	$f_{\Gamma_N}(\mathbf{k})$	Lowest order invariant	M_i linear in SOC
C_{2h}	B_g	$\alpha k_x k_z + \beta k_y k_z$	$\alpha_1 N_x M_z,\ \alpha_2 N_y M_z$, $\alpha_3 N_z M_y,\ \alpha_4 N_z M_x$	✓
D_{2h}	B_{1g}	$k_x k_y$	$\alpha_1 M_x N_y + \alpha_2 M_y N_x$	✓
D_{2h}	B_{2g}	$k_x k_z$	$\alpha_1 M_y N_z + \alpha_2 M_z N_y$	✓
D_{2h}	B_{3g}	$k_y k_z$	$\alpha_1 M_z N_x + \alpha_2 M_x N_z$	✓
C_{4h}	B_g	$\alpha(k_x^2 - k_y^2)$ $+ \beta k_x k_y$	$M_x N_y + M_y N_x$, $M_x N_x - M_y N_y$	✗
D_{4h}	A_{2g}	$k_x k_y (k_x^2 - k_y^2)$	$M_x N_y - M_y N_x$	✓
D_{4h}	B_{1g}	$k_x^2 - k_y^2$	$M_x N_x - M_y N_y$	✗
D_{4h}	B_{2g}	$k_x k_y$	$M_x N_y + M_y N_x$	✗
D_{3d}	A_{2g}	$k_x k_z (k_x^2 - 3k_y^2)$	$M_x N_y - M_y N_x$	✓
C_{6h}	B_g	$\alpha k_y k_z (k_y^2 - 3k_x^2)$ $+\beta k_x k_z (k_x^2 - 3k_y^2)$	$\alpha M_z N_y (3N_x^2 - N_y^2)$ $+\beta M_z N_x (3N_y^2 - N_x^2)$	✗
D_{6h}	A_{2g}	$k_x k_y (k_x^2 - 3k_y^2)$ $\times (k_y^2 - 3k_x^2)$	$M_x N_y - M_y N_x$	✓
D_{6h}	B_{1g}	$k_y k_z (3k_x^2 - k_y^2)$	$M_z N_y (3N_x^2 - N_y^2)$	✗
D_{6h}	B_{2g}	$k_x k_z (k_x^2 - 3k_y^2)$	$M_z N_x (N_x^2 - 3N_y^2)$	✗
O_h	A_{2g}	$k_x^4 (k_y^2 - k_z^2)$ $+k_y^4 (k_z^2 - k_x^2)$ $+k_z^4 (k_x^2 - k_y^2)$	$M_x N_x (N_y^2 - N_z^2)$ $+M_y N_y (N_z^2 - N_x^2)$ $+M_z N_z (N_x^2 - N_y^2)$	✗

Anomalous Hall effect and weak ferromagnetism

Comparison with DFT results

- Comparison to FM moment identified in DFT:
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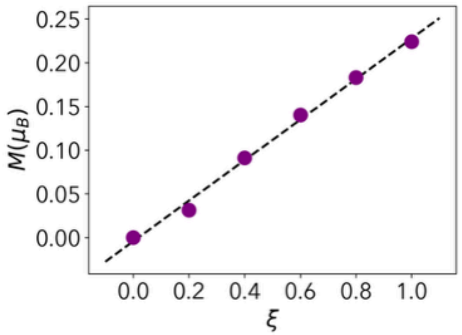

[Milivojević *et al.*, 2D Mater. (2024)]

  - Vanishing : RuO_2 [Šmejkal *et al.*, Sci. Adv. (2020)], MnTe, CrSb [Mazin, arXiv (2024)], [Autieri *et al.*, PRB (2025)]

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C_{2h}	B_g	$\alpha k_x k_z + \beta k_y k_z$	$\alpha_1 N_x M_z$, $\alpha_2 N_y M_z$, $\alpha_3 N_z M_y$, $\alpha_4 N_z M_x$	✓
D_{2h}	B_{1g}	$k_x k_y$	$\alpha_1 M_x N_y + \alpha_2 M_y N_x$	✓
D_{2h}	B_{2g}	$k_x k_z$	$\alpha_1 M_y N_z + \alpha_2 M_z N_y$	✓
D_{2h}	B_{3g}	$k_y k_z$	$\alpha_1 M_z N_x + \alpha_2 M_x N_z$	✓
C_{4h}	B_g	$\alpha(k_x^2 - k_y^2) + \beta k_x k_y$	$M_x N_y + M_y N_x$, $M_x N_x - M_y N_y$	✗
D_{4h}	A_{2g}	$k_x k_y (k_x^2 - k_y^2)$	$M_x N_y - M_y N_x$	✓
D_{4h}	B_{1g}	$k_x^2 - k_y^2$	$M_x N_x - M_y N_y$	✗
D_{4h}	B_{2g}	$k_x k_y$	$M_x N_y + M_y N_x$	✗
D_{3d}	A_{2g}	$k_x k_z (k_x^2 - 3k_y^2)$	$M_x N_y - M_y N_x$	✓
C_{6h}	B_g	$\alpha k_y k_z (k_y^2 - 3k_x^2) + \beta k_x k_z (k_x^2 - 3k_y^2)$	$\alpha M_z N_y (3N_x^2 - N_y^2) + \beta M_z N_x (3N_y^2 - N_x^2)$	✗
D_{6h}	A_{2g}	$k_x k_y (k_x^2 - 3k_y^2) \times (k_y^2 - 3k_x^2)$	$M_x N_y - M_y N_x$	✓
D_{6h}	B_{1g}	$k_y k_z (3k_x^2 - k_y^2)$	$M_z N_y (3N_x^2 - N_y^2)$	✗
D_{6h}	B_{2g}	$k_x k_z (k_x^2 - 3k_y^2)$	$M_z N_x (N_x^2 - 3N_y^2)$	✗
O_h	A_{2g}	$k_x^4 (k_y^2 - k_z^2) + k_y^4 (k_z^2 - k_x^2) + k_z^4 (k_x^2 - k_y^2)$	$M_x N_x (N_y^2 - N_z^2) + M_y N_y (N_z^2 - N_x^2) + M_z N_z (N_x^2 - N_y^2)$	✗

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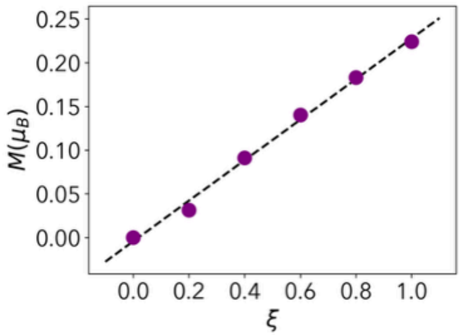

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C_{2h}	B_g	$\alpha k_x k_z + \beta k_y k_z$	$\alpha_1 N_x M_z,\ \alpha_2 N_y M_z$, $\alpha_3 N_z M_y,\ \alpha_4 N_z M_x$	✓
D_{2h}	B_{1g}	$k_x k_y$	$\alpha_1 M_x N_y + \alpha_2 M_y N_x$	✓
D_{2h}	B_{2g}	$k_x k_z$	$\alpha_1 M_y N_z + \alpha_2 M_z N_y$	✓
D_{2h}	B_{3g}	$k_y k_z$	$\alpha_1 M_z N_x + \alpha_2 M_x N_z$	✓
C_{4h}	B_g	$\alpha(k_x^2 - k_y^2)$ $+ \beta k_x k_y$	$M_x N_y + M_y N_x$, $M_x N_x - M_y N_y$	✘
D_{4h}	A_{2g}	$k_x k_y (k_x^2 - k_y^2)$	$M_x N_y - M_y N_x$	✓
D_{4h}	B_{1g}	$k_x^2 - k_y^2$	$M_x N_x - M_y N_y$	✘
D_{4h}	B_{2g}	$k_x k_y$	$M_x N_y + M_y N_x$	✘
D_{3d}	A_{2g}	$k_x k_z (k_x^2 - 3k_y^2)$	$M_x N_y - M_y N_x$	✓
C_{6h}	B_g	$\alpha k_y k_z (k_y^2 - 3k_x^2)$ $+\beta k_x k_z (k_x^2 - 3k_y^2)$	$\alpha M_z N_y (3N_x^2 - N_y^2)$ $+\beta M_z N_x (3N_y^2 - N_x^2)$	✘
D_{6h}	A_{2g}	$k_x k_y (k_x^2 - 3k_y^2)$ $\times (k_y^2 - 3k_x^2)$	$M_x N_y - M_y N_x$	✓
D_{6h}	B_{1g}	$k_y k_z (3k_x^2 - k_y^2)$	$M_z N_y (3N_x^2 - N_y^2)$	✘
D_{6h}	B_{2g}	$k_x k_z (k_x^2 - 3k_y^2)$	$M_z N_x (N_x^2 - 3N_y^2)$	✘
O_h	A_{2g}	$k_x^4 (k_y^2 - k_z^2)$ $+k_y^4 (k_z^2 - k_x^2)$ $+k_z^4 (k_x^2 - k_y^2)$	$M_x N_x (N_y^2 - N_z^2)$ $+M_y N_y (N_z^2 - N_x^2)$ $+M_z N_z (N_x^2 - N_y^2)$	✘

- General table and agrees with DFT if the magnetic order is in the spin channel

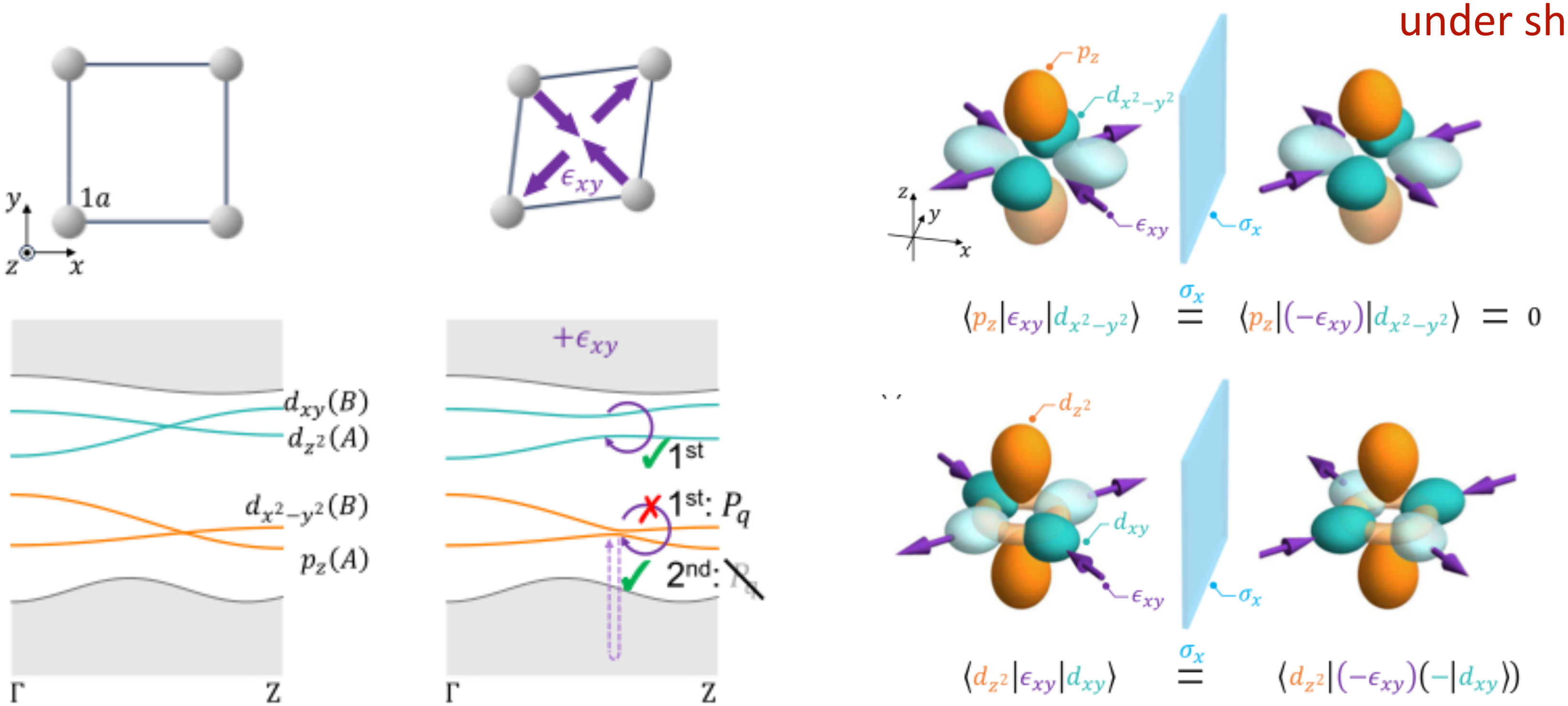
General validity: Quasi-symmetry

PHYSICAL REVIEW LETTERS **133**, 026402 (2024)

Group Theory on Quasisymmetry and Protected Near Degeneracy

Jiayu Li,[1] Ao Zhang,[1] Yuntian Liu,[1] and Qihang Liu[1,2,3,*]
[1]*Department of Physics and Shenzhen Institute for Quantum Science and Engineering (SIQSE), Southern University of Science and Technology, Shenzhen 518055, China*

Tetragonal system under shear strain



See also: [Guo *et al.*, Nat. Phys. (2022)]

Quasi-symmetry enabled by SOC

- Is the result from the microscopic models general?

- Hamiltonian without SOC

$$H = \varepsilon_{0,\mathbf{k}} + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z + \tau_z \vec{N} \cdot \vec{\sigma}$$

- Normal state: full spin-rotational invariance
- Ordered state: spin space group

Quasi-symmetry enabled by SOC

- Is the result from the microscopic models general?

- Hamiltonian without SOC

$$H = \varepsilon_{0,\mathbf{k}} + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z + \tau_z \vec{N}\cdot\vec{\sigma}$$

- Including all SOC components

$$H = \varepsilon_{0,\mathbf{k}} + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z + \tau_z \vec{N}\cdot\vec{\sigma} + \tau_y\lambda_{x,\mathbf{k}}\sigma_x + \tau_y\lambda_{y,\mathbf{k}}\sigma_y + \tau_y\lambda_{z,\mathbf{k}}\sigma_z$$

- Normal state: full spin-rotational invariance
- Ordered state: spin space group

- Normal state: no pure spin-rotational invariance
- Ordered state: magnetic space group

Quasi-symmetry enabled by SOC

- Is the result from the microscopic models general?
- Hamiltonian without SOC

$$H = \varepsilon_{0,\mathbf{k}} + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z + \tau_z\vec{N}\cdot\vec{\sigma}$$

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$$H = \varepsilon_{0,\mathbf{k}} + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z + \tau_z\vec{N}\cdot\vec{\sigma} + \tau_y\lambda_{x,\mathbf{k}}\sigma_x + \tau_y\lambda_{y,\mathbf{k}}\sigma_y + \tau_y\lambda_{z,\mathbf{k}}\sigma_z$$

- Single SOC component (linear SOC)

$$H = \varepsilon_{0,\mathbf{k}} + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z + \tau_z\vec{N}\cdot\vec{\sigma} + \tau_y\lambda_{x,\mathbf{k}}\sigma_x + \cancel{\tau_y\lambda_{y,\mathbf{k}}\sigma_y} + \cancel{\tau_y\lambda_{z,\mathbf{k}}\sigma_z}$$

- Normal state: full spin-rotational invariance
- Ordered state: spin space group

- Normal state: no pure spin-rotational invariance
- Ordered state: magnetic space group

- Normal state: spin rotational invariance along one axis (uncoupled spin from spatial rotations)

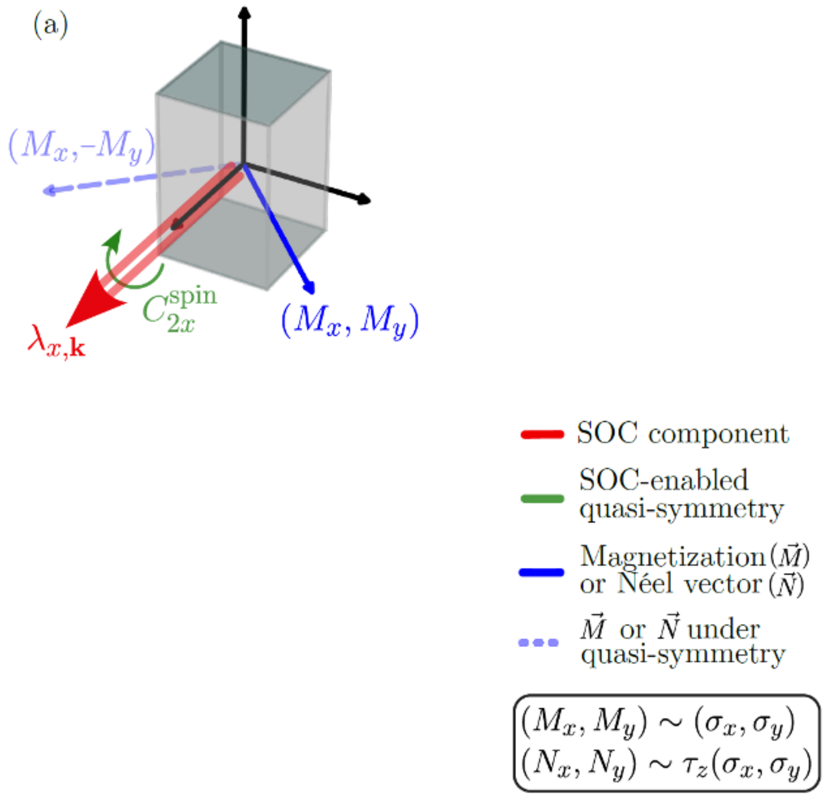
$\boxed{\text{SOC-enabled quasi-symmetries}}$

Quasi-symmetry enabled by SOC

Quasi-symmetry protection of negligible FM

- Including one SOC component

$$H = \varepsilon_{0,\mathbf{k}} + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z + \tau_z \vec{N}\cdot\vec{\sigma}$$
$$+\tau_y\lambda_{x,\mathbf{k}}\sigma_x + \cancel{\tau_y\lambda_{y,\mathbf{k}}}\sigma_y + \cancel{\tau_y\lambda_{z,\mathbf{k}}}\sigma_z$$



Quasi-symmetry enabled by SOC

Quasi-symmetry protection of negligible FM

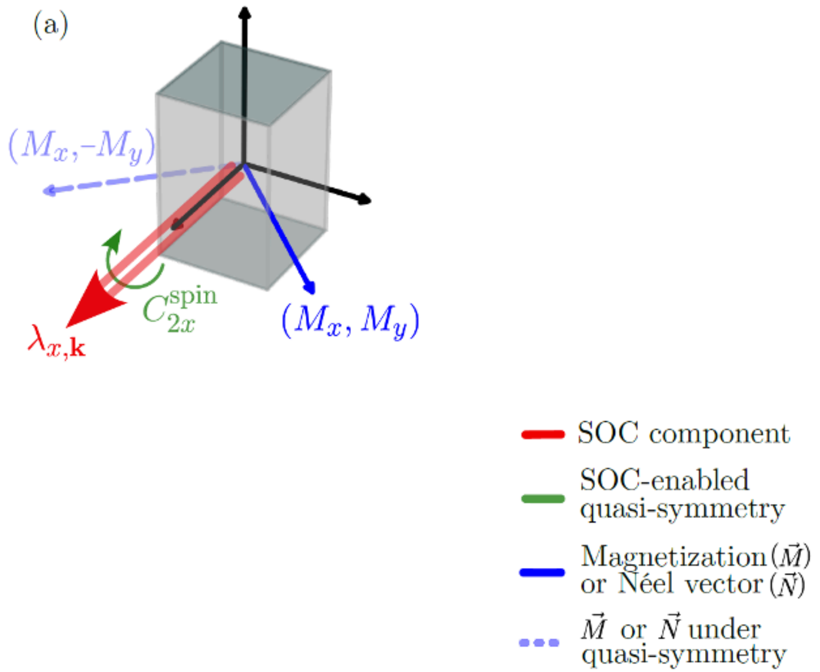
- Including one SOC component

$$H = \varepsilon_{0,\mathbf{k}} + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z + \tau_z \vec{N}\cdot\vec{\sigma}$$

$$+\tau_y\lambda_{x,\mathbf{k}}\sigma_x + \cancel{\tau_y\lambda_{y,\mathbf{k}}}\sigma_y + \cancel{\tau_y\lambda_{z,\mathbf{k}}}\sigma_z$$

- Consider the Landau coefficient

$$M_x N_y \qquad \left.\begin{array}{l} M_x \text{ is even} \\ N_y \text{ is odd} \end{array}\right\} \begin{array}{l} \lambda_{x,\mathbf{k}}\text{-linear coupling} \\ \text{not allowed} \end{array}$$



$$\boxed{\begin{array}{l}(M_x, M_y) \sim (\sigma_x, \sigma_y) \\ (N_x, N_y) \sim \tau_z(\sigma_x, \sigma_y)\end{array}}$$

Quasi-symmetry enabled by SOC

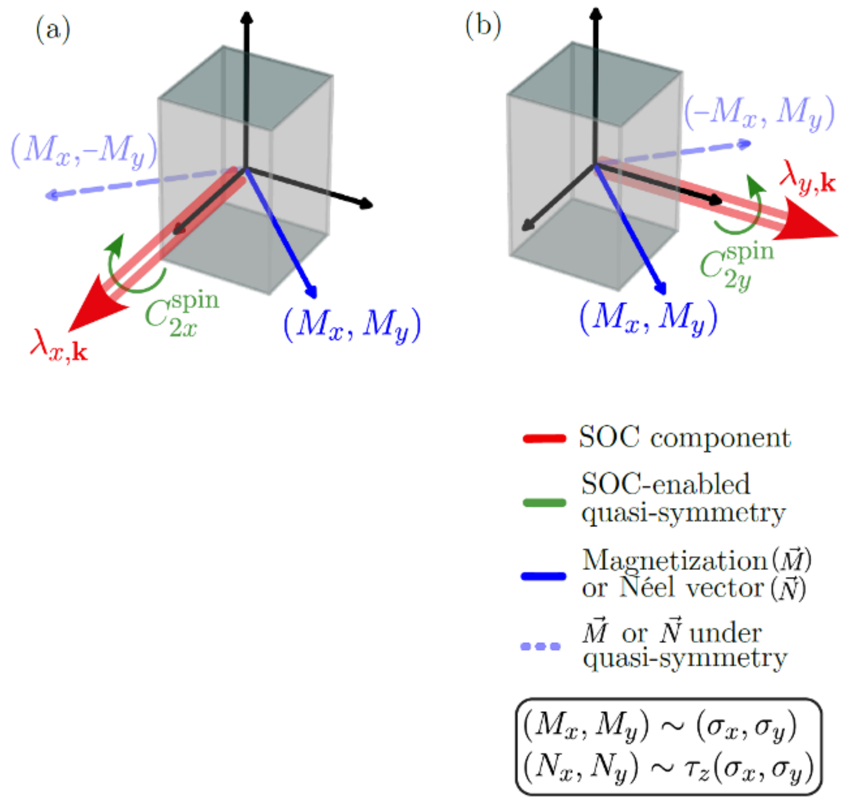
Quasi-symmetry protection of negligible FM

- Including one SOC component

$$H = \varepsilon_{0,\mathbf{k}} + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z + \tau_z \vec{N}\cdot\vec{\sigma}$$
$$+\cancel{\tau_y\lambda_{x,\mathbf{k}}\sigma_x} + \tau_y\lambda_{y,\mathbf{k}}\sigma_y + \cancel{\tau_y\lambda_{z,\mathbf{k}}\sigma_z}$$

- Consider the Landau coefficient

$$M_x N_y \qquad \left.\begin{matrix} M_x \text{ is even} \\ N_y \text{ is odd}\end{matrix}\right\} \lambda_{x,\mathbf{k}}\text{-linear coupling not allowed}$$



Quasi-symmetry enabled by SOC

Quasi-symmetry protection of negligible FM

- Including one SOC component

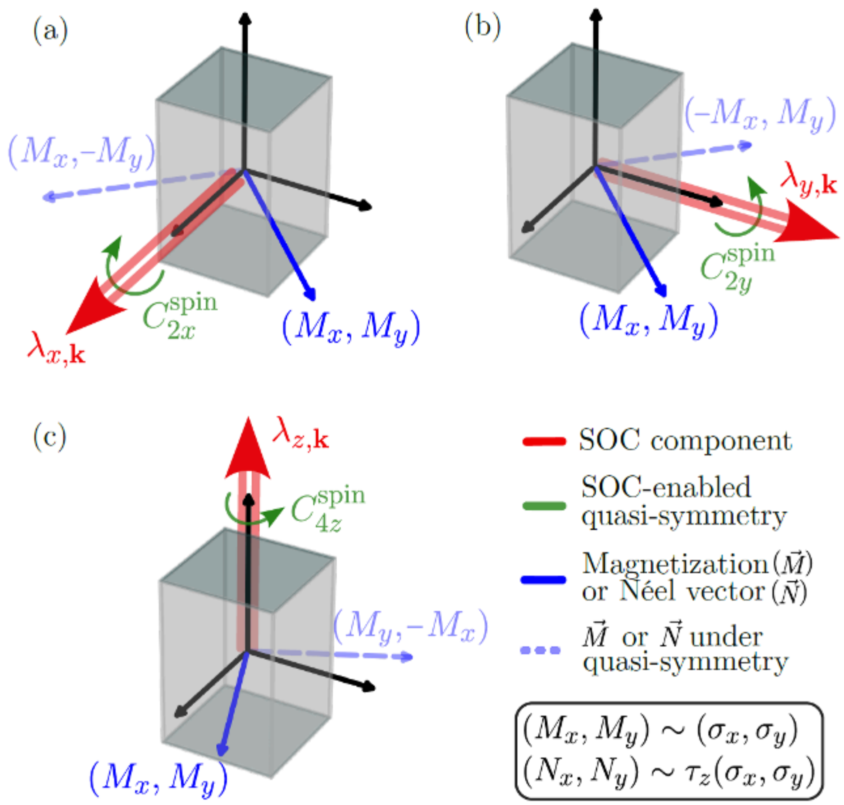
$$H = \varepsilon_{0,\mathbf{k}} + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z + \tau_z \vec{N}\cdot\vec{\sigma}$$

$$+\cancel{\tau_y\lambda_{x,\mathbf{k}}\sigma_x} + \cancel{\tau_y\lambda_{y,\mathbf{k}}\sigma_y} + \tau_y\lambda_{z,\mathbf{k}}\sigma_z$$

- Consider the Landau coefficient

$M_x N_y$ — M_x is even, N_y is odd $\Big\}$ $\lambda_{x,\mathbf{k}}$-linear coupling not allowed

$M_x N_y - M_y N_x \Big\}$ Even under C_{4z}^{spin}, $\lambda_{z,\mathbf{k}}$-linear coupling



Quasi-symmetry enabled by SOC

Quasi-symmetry protection of negligible FM

- Including one SOC component

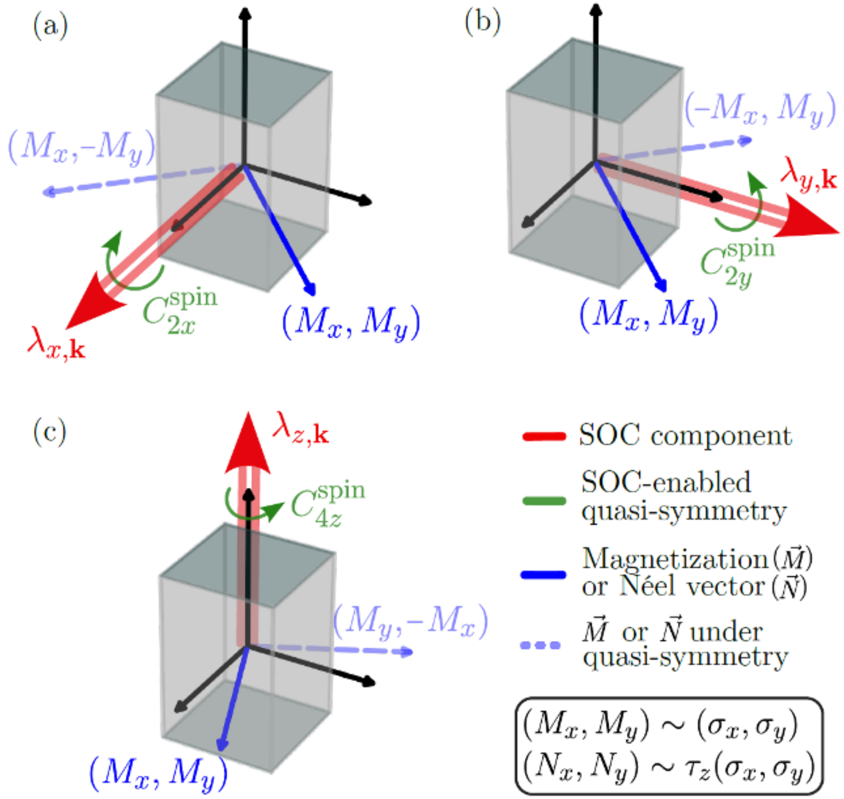
$$H = \varepsilon_{0,\mathbf{k}} + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z + \tau_z \vec{N}\cdot\vec{\sigma}$$

$$+\cancel{\tau_y\lambda_{x,\mathbf{k}}\sigma_x} + \cancel{\tau_y\lambda_{y,\mathbf{k}}\sigma_y} + \tau_y\lambda_{z,\mathbf{k}}\sigma_z$$

- Consider the Landau coefficient

$M_x N_y$ — $\left.\begin{array}{l} M_x \text{ is even} \\ N_y \text{ is odd}\end{array}\right\}$ $\lambda_{x,\mathbf{k}}$-linear coupling not allowed

$M_x N_y - M_y N_x \Big\}$ Even under C_{4z}^{spin}, $\lambda_{z,\mathbf{k}}$-linear coupling



➢ Explains the size of FM moment and demonstrates that the microscopic model leads to a general result!

Outline of presentation

Minimal models for altermagnetism

- Construction of minimal models
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- Landau free energy: effects of spin-orbit coupling.
- Anomalous Hall effect and weak ferromagnetism

Local signatures of altermagnetism

- Local markers of altermagnetism

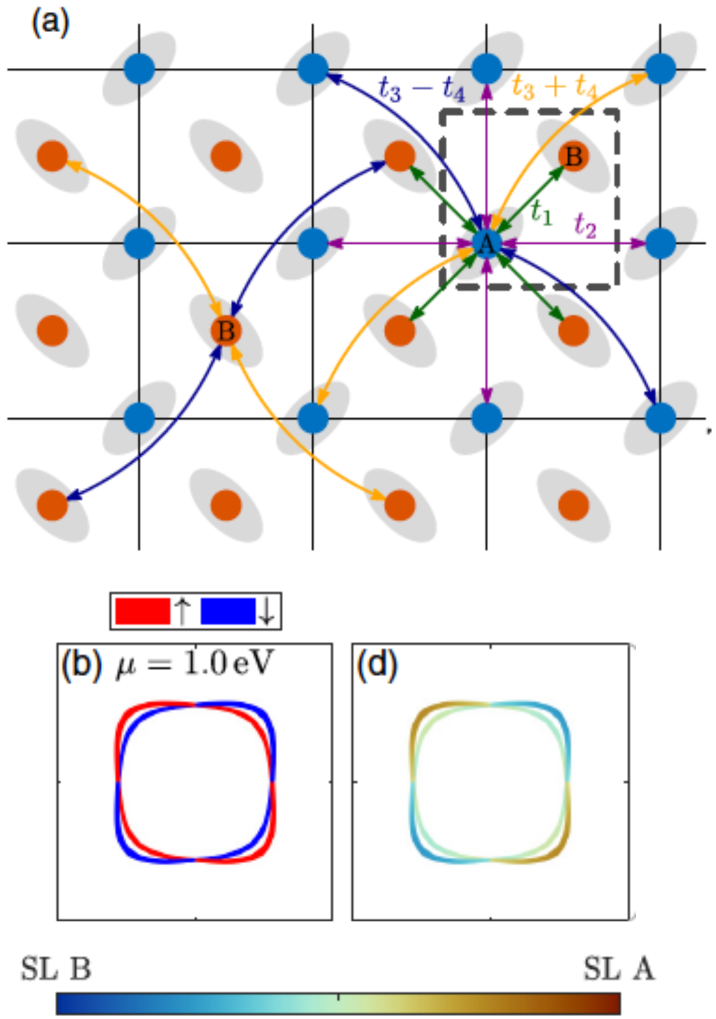
Superconductivity and altermagnetism

- What pairing states are preferred in altermagnetic metals?

Conclusions & Outlook

Local signatures of altermagnetism

$$\mathcal{H}_{\rm MM} = \varepsilon_{0,\mathbf{k}}\tau_0 + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z + \tau_y\vec{\lambda}_{\mathbf{k}}\cdot\vec{\sigma} + \tau_z\vec{N}\cdot\vec{\sigma}$$

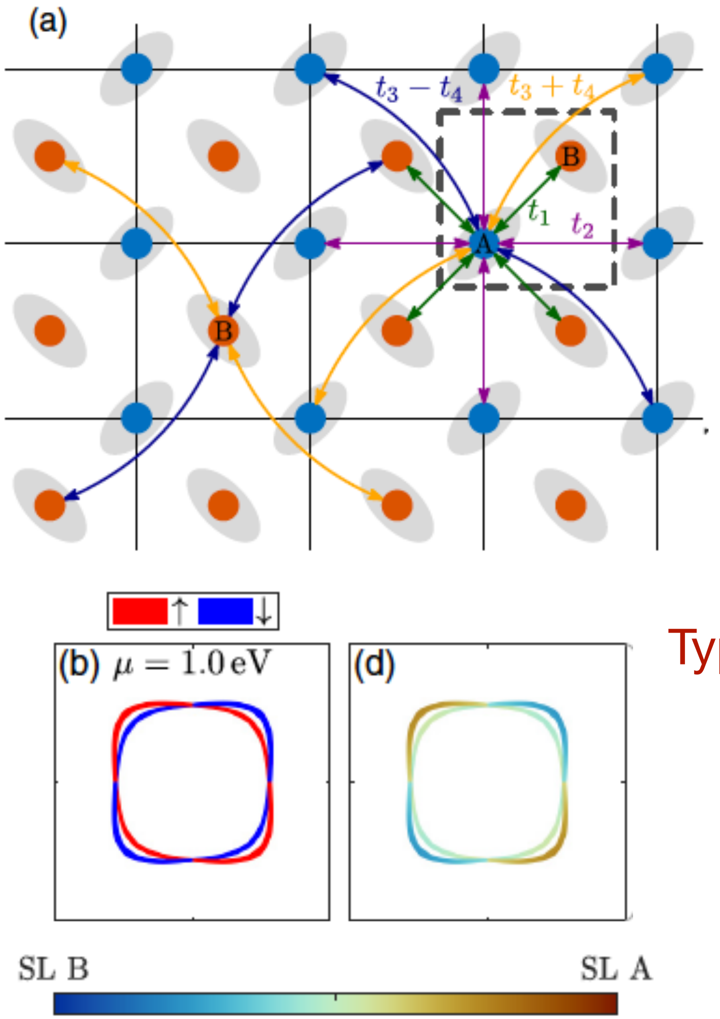


2D layer groups	Wyckoff positions	τ_x	τ_z
L7 ($p112/a$)	$2a$-$2b$	$c_{x/2}$	0
L15 ($p2_1/m11$)	$2a$-$2b$	$c_{x/2}$	$s_x s_y$
L16 ($p2/b11$)	$2a$-$2b$	$c_{y/2}$	$s_x s_y$
L17 ($p2_1/b11$)	$2a$-$2b$	$c_{x/2}c_{y/2}$	$s_x s_y$
L38 ($pmaa$)	$2a$-$2b$	$c_{x/2}$	0
L40 ($pmam$)	$2a$-$2b$	$c_{x/2}$	$s_x s_y$
L41 ($pmma$)	$2a$-$2b$	$c_{x/2}$	0
L42 ($pman$)	$2a$-$2b$	$c_{x/2}c_{y/2}$	0
L44 ($pbam$)	$2a$-$2b$	$c_{x/2}c_{y/2}$	$s_x s_y$
L51 ($p4/m$)	$2c$	$c_{x/2}c_{y/2}$	$t_{z1}(c_x - c_y) + t_{z2}s_x s_y$
L61 ($p4/mmm$)	$2c$	$c_{x/2}c_{y/2}$	$c_x - c_y$
L63 ($p4/mbm$)	$2a$	$c_{x/2}c_{y/2}$	$s_x s_y(c_x - c_y)$
L63 ($p4/mbm$)	$2b$	$c_{x/2}c_{y/2}$	$s_x s_y$

J. Gondolf *et al*, PRB 111, 174436 (2025)

Local signatures of altermagnetism

$$\mathcal{H}_{\mathrm{MM}} = \varepsilon_{0,\mathbf{k}}\tau_0 + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z + \tau_y \vec{\lambda}_{\mathbf{k}} \cdot \vec{\sigma} + \tau_z \vec{N} \cdot \vec{\sigma}$$



2D layer groups	Wyckoff positions	τ_x	τ_z
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L38 ($pmaa$)	$2a$-$2b$	$c_{x/2}$	0
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L42 ($pman$)	$2a$-$2b$	$c_{x/2}c_{y/2}$	0
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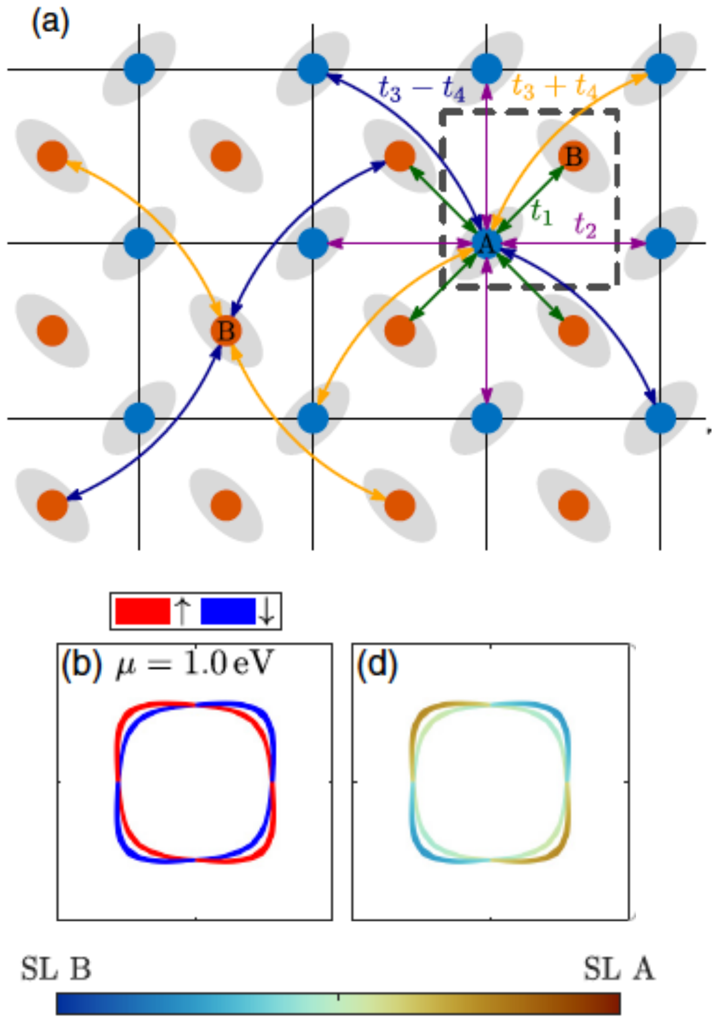
L63 ($p4/mbm$)	$2b$	$c_{x/2}c_{y/2}$	$s_x s_y$

Type IV

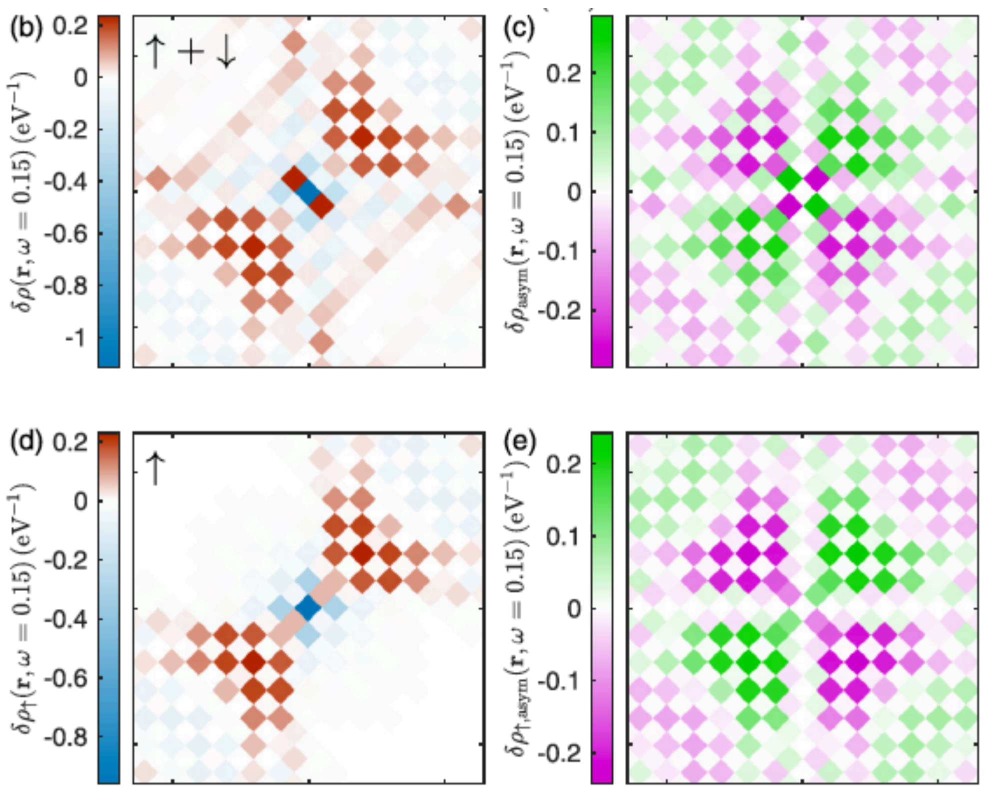
L. Bai *et al*, PRL 135, 036702 (2025) …

Local signatures of altermagnetism

$$\mathcal{H}_{\mathrm{MM}} = \varepsilon_{0,\mathbf{k}}\tau_0 + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z + \tau_y\vec{\lambda}_{\mathbf{k}}\cdot\vec{\sigma} + \tau_z\vec{N}\cdot\vec{\sigma}$$

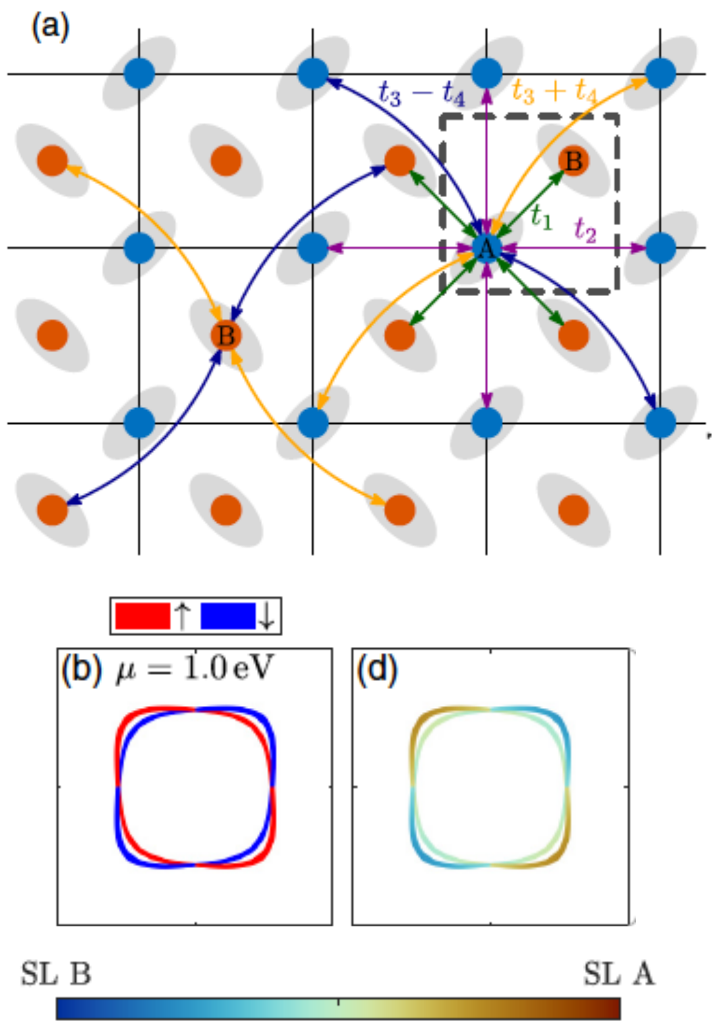


Local image of spin-splitting symmetry

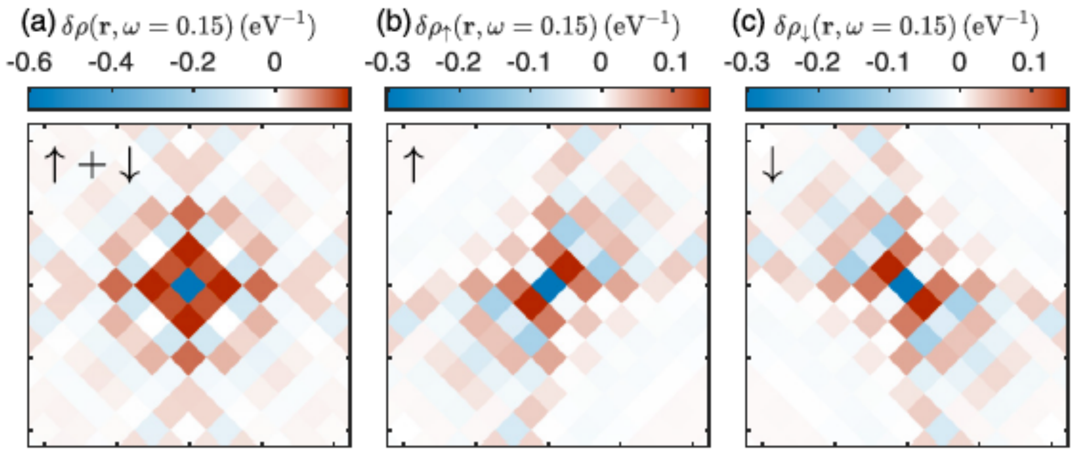


$$\delta\rho_{\sigma,\mathrm{asym}}(\mathbf{r},\omega) = \delta\rho_\sigma(\mathbf{r},\omega) - \delta\rho_\sigma(\mathcal{S}\mathbf{r},\omega).$$

Local signatures of altermagnetism



$$\mathcal{H}_{1\mathrm{SL}} = \varepsilon_{0,\mathbf{k}}\sigma_0 + N t_{z,\mathbf{k}}\sigma_z$$



Physics not correctly captured by one-band models

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Conclusions & Outlook

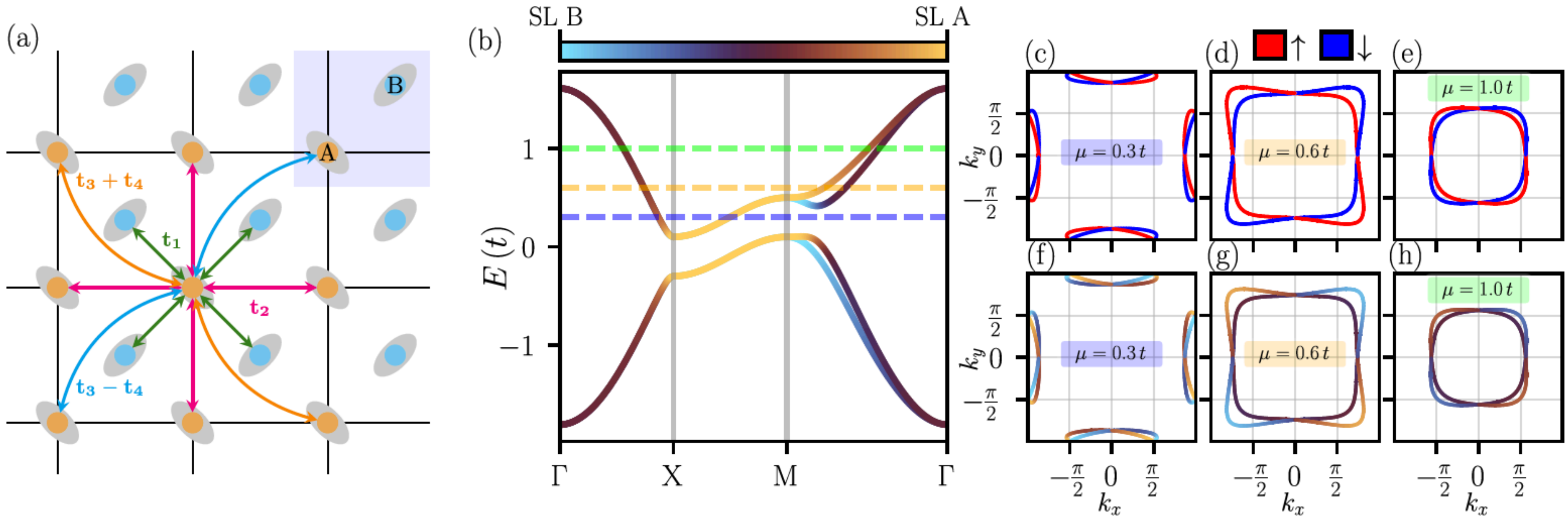
Superconductivity and altermagnetism

$$\mathcal{H}_{\mathrm{MM}} = \varepsilon_{0,\mathbf{k}}\tau_0 + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z + \tau_y\vec{\lambda}_{\mathbf{k}}\cdot\vec{\sigma} + \tau_z\vec{N}\cdot\vec{\sigma}$$

$$H_{\mathrm{int}}^{\mathrm{OS}} = -\frac{V}{N}\sum_{s,\mathbf{k},\mathbf{k}'} c^{\dagger}_{\uparrow,s,\mathbf{k}} c^{\dagger}_{\downarrow,s,-\mathbf{k}} c_{\downarrow,s,-\mathbf{k}'} c_{\uparrow,s,\mathbf{k}'}$$

Mats Barkman

Christian Rasmussen



C. L. H. Rasmussen *et al*, ArXiv:2509.03247

Superconductivity and altermagnetism

$$\mathcal{H}_{\mathrm{MM}} = \varepsilon_{0,\mathbf{k}}\tau_0 + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z + \tau_y\vec{\lambda}_{\mathbf{k}}\cdot\vec{\sigma} + \tau_z\vec{N}\cdot\vec{\sigma}$$

$$H_{\mathrm{int}}^{\mathrm{OS}} = -\frac{V}{N}\sum_{s,\mathbf{k},\mathbf{k}'} c^{\dagger}_{\uparrow,s,\mathbf{k}} c^{\dagger}_{\downarrow,s,-\mathbf{k}} c_{\downarrow,s,-\mathbf{k}'} c_{\uparrow,s,\mathbf{k}'}$$

$$\begin{pmatrix} d_{\sigma,\alpha,\mathbf{k}} \\ d_{\sigma,\beta,\mathbf{k}} \end{pmatrix} = \begin{pmatrix} l_{\sigma,\mathbf{k}} & m_{\sigma,\mathbf{k}} \\ -m^*_{\sigma,\mathbf{k}} & l_{\sigma,\mathbf{k}} \end{pmatrix} \begin{pmatrix} c_{\sigma,A,\mathbf{k}} \\ c_{\sigma,B,\mathbf{k}} \end{pmatrix}$$

Rewrite interaction via sublattice-to-band transformation.

$$l_{\sigma,\mathbf{k}} = \cos\frac{\theta_{\sigma,\mathbf{k}}}{2},$$

$$m_{\sigma,\mathbf{k}} = \frac{t_x}{|t_x|}\sin\frac{\theta_{\sigma,\mathbf{k}}}{2}$$

$$\cos\frac{\theta_{\sigma,\mathbf{k}}}{2} = \frac{1}{\sqrt{2}}\sqrt{1 + \frac{t_{z,\mathbf{k}} + \sigma N}{\sqrt{|t_{x,\mathbf{k}}|^2 + (t_{z,\mathbf{k}} + \sigma N)^2}}}$$

$$\sin\frac{\theta_{\sigma,\mathbf{k}}}{2} = \frac{-1}{\sqrt{2}}\sqrt{1 - \frac{t_{z,\mathbf{k}} + \sigma N}{\sqrt{|t_{x,\mathbf{k}}|^2 + (t_{z,\mathbf{k}} + \sigma N)^2}}}$$

Superconductivity and altermagnetism

$$\mathcal{H}_{\mathrm{MM}} = \varepsilon_{0,\mathbf{k}}\tau_0 + t_{x,\mathbf{k}}\tau_x + t_{z,\mathbf{k}}\tau_z + \tau_y\vec{\lambda}_{\mathbf{k}}\cdot\vec{\sigma} + \tau_z\vec{N}\cdot\vec{\sigma}$$

$$H_{\mathrm{int}}^{\mathrm{OS}} = -\frac{V}{N}\sum_{s,\mathbf{k},\mathbf{k}'} c^{\dagger}_{\uparrow,s,\mathbf{k}} c^{\dagger}_{\downarrow,s,-\mathbf{k}} c_{\downarrow,s,-\mathbf{k}'} c_{\uparrow,s,\mathbf{k}'}$$

$$\begin{pmatrix} d_{\sigma,\alpha,\mathbf{k}} \\ d_{\sigma,\beta,\mathbf{k}} \end{pmatrix} = \begin{pmatrix} l_{\sigma,\mathbf{k}} & m_{\sigma,\mathbf{k}} \\ -m^{*}_{\sigma,\mathbf{k}} & l_{\sigma,\mathbf{k}} \end{pmatrix} \begin{pmatrix} c_{\sigma,A,\mathbf{k}} \\ c_{\sigma,B,\mathbf{k}} \end{pmatrix}$$

$$\begin{aligned} H_{\mathrm{int}}^{\mathrm{OS}} = -\frac{V}{N}\sum_{\mathbf{k},\mathbf{k}'} \Big[& \left(l_{\uparrow,\mathbf{k}} l_{\uparrow,\mathbf{k}'} l_{\downarrow,-\mathbf{k}} l_{\downarrow,-\mathbf{k}'} + m_{\uparrow,\mathbf{k}} m^{*}_{\uparrow,\mathbf{k}'} m_{\downarrow,-\mathbf{k}} m^{*}_{\downarrow,-\mathbf{k}'}\right) d^{\dagger}_{\uparrow,\alpha,\mathbf{k}} d_{\uparrow,\alpha,\mathbf{k}'} d^{\dagger}_{\downarrow,\alpha,-\mathbf{k}} d_{\downarrow,\alpha,-\mathbf{k}'} \\ & + \left(m^{*}_{\uparrow,\mathbf{k}} m_{\uparrow,\mathbf{k}'} m^{*}_{\downarrow,-\mathbf{k}} m_{\downarrow,-\mathbf{k}'} + l_{\uparrow,\mathbf{k}} l_{\uparrow,\mathbf{k}'} l_{\downarrow,-\mathbf{k}} l_{\downarrow,-\mathbf{k}'}\right) d^{\dagger}_{\uparrow,\beta,\mathbf{k}} d_{\uparrow,\beta,\mathbf{k}'} d^{\dagger}_{\downarrow,\beta,-\mathbf{k}} d_{\downarrow,\beta,-\mathbf{k}'} \\ & + \left(l_{\uparrow,\mathbf{k}} m_{\uparrow,\mathbf{k}'} l_{\downarrow,-\mathbf{k}} m_{\downarrow,-\mathbf{k}'} + m_{\uparrow,\mathbf{k}} l_{\uparrow,\mathbf{k}'} m_{\downarrow,-\mathbf{k}} l_{\downarrow,-\mathbf{k}'}\right) d^{\dagger}_{\uparrow,\alpha,\mathbf{k}} d_{\uparrow,\beta,\mathbf{k}'} d^{\dagger}_{\downarrow,\alpha,-\mathbf{k}} d_{\downarrow,\beta,-\mathbf{k}'} \\ & + \left(m^{*}_{\uparrow,\mathbf{k}} l_{\uparrow,\mathbf{k}'} m^{*}_{\downarrow,-\mathbf{k}} l_{\downarrow,-\mathbf{k}'} + l_{\uparrow,\mathbf{k}} m^{*}_{\uparrow,\mathbf{k}'} l_{\downarrow,-\mathbf{k}} m^{*}_{\downarrow,-\mathbf{k}'}\right) d^{\dagger}_{\uparrow,\beta,\mathbf{k}} d_{\uparrow,\alpha,\mathbf{k}'} d^{\dagger}_{\downarrow,\beta,-\mathbf{k}} d_{\downarrow,\alpha,-\mathbf{k}'} \Big] + \mathrm{H.c.} \end{aligned}$$

Inherent momentum-dependent gaps
from sublattice-to-band transformation.

Superconductivity and altermagnetism

$$\mathcal{H}^{\text{OS}}_{\text{BdG}} = \begin{pmatrix} \hat{E}_{\mathbf{k}} & \hat{\Delta}_{\mathbf{k}} \\ \hat{\Delta}^*_{\mathbf{k}} & -\hat{E}_{-\mathbf{k}} \end{pmatrix},$$

$$\hat{E}_{\mathbf{k}} = \begin{pmatrix} \hat{E}^{\alpha}_{\mathbf{k}} & 0 \\ 0 & \hat{E}^{\beta}_{\mathbf{k}} \end{pmatrix}, \qquad \hat{\Delta}_{\mathbf{k}} = \begin{pmatrix} \hat{\Delta}^{\alpha}_{\mathbf{k}} & 0 \\ 0 & \hat{\Delta}^{\beta}_{\mathbf{k}} \end{pmatrix},$$

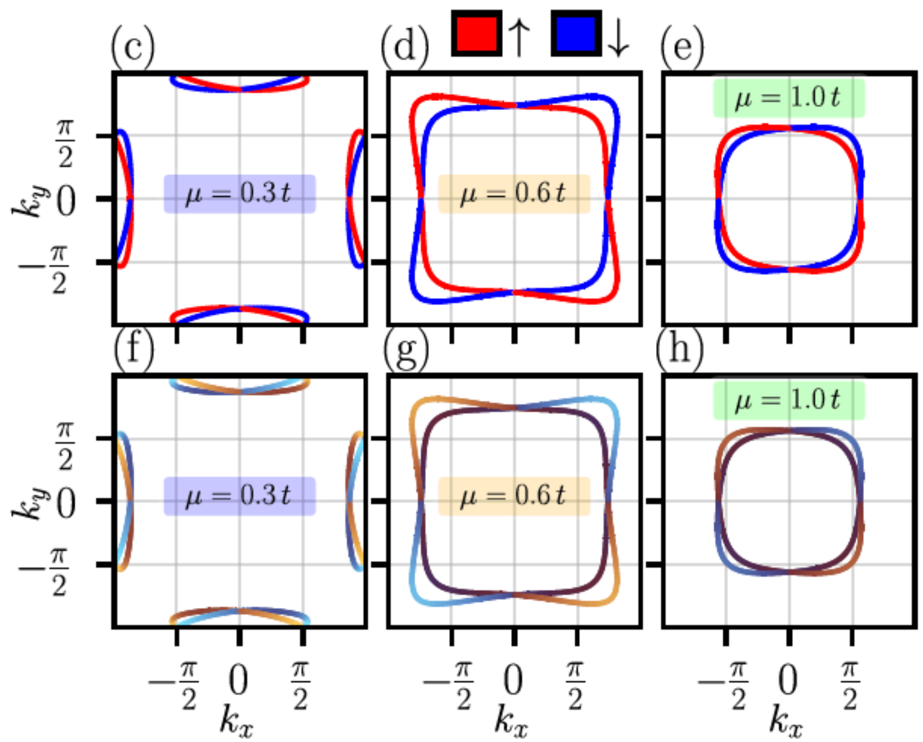
$$\hat{E}^{\lambda}_{\mathbf{k}} = \begin{pmatrix} E^{\lambda}_{\uparrow,\mathbf{k}} & 0 \\ 0 & E^{\lambda}_{\downarrow,\mathbf{k}} \end{pmatrix}, \qquad \hat{\Delta}^{\lambda}_{\mathbf{k}} = \begin{pmatrix} 0 & \Delta^{\lambda}_{\uparrow\downarrow,\mathbf{k}} \\ \Delta^{\lambda}_{\downarrow\uparrow,\mathbf{k}} & 0 \end{pmatrix}$$

$$\begin{aligned}\Delta^{\alpha}_{\uparrow\downarrow,\mathbf{k}} = -\frac{V}{N}\Big[& l_{\uparrow,\mathbf{k}} l_{\downarrow,-\mathbf{k}} \sum_{\mathbf{k}'} l_{\uparrow,\mathbf{k}'} l_{\downarrow,-\mathbf{k}'} \langle d_{\downarrow,\alpha,-\mathbf{k}'} d_{\uparrow,\alpha,\mathbf{k}'} \rangle \\ & + m_{\uparrow,\mathbf{k}} m_{\downarrow,-\mathbf{k}} \sum_{\mathbf{k}'} m^*_{\uparrow,\mathbf{k}'} m^*_{\downarrow,-\mathbf{k}'} \langle d_{\downarrow,\alpha,-\mathbf{k}'} d_{\uparrow,\alpha,\mathbf{k}'} \rangle \\ & + l_{\uparrow,\mathbf{k}} l_{\downarrow,-\mathbf{k}} \sum_{\mathbf{k}'} m_{\uparrow,\mathbf{k}'} m_{\downarrow,-\mathbf{k}'} \langle d_{\downarrow,\beta,-\mathbf{k}'} d_{\uparrow,\beta,\mathbf{k}'} \rangle \\ & + m_{\uparrow,\mathbf{k}} m_{\downarrow,-\mathbf{k}} \sum_{\mathbf{k}'} l_{\uparrow,\mathbf{k}'} l_{\downarrow,-\mathbf{k}'} \langle d_{\downarrow,\beta,-\mathbf{k}'} d_{\uparrow,\beta,\mathbf{k}'} \rangle \Big]\end{aligned}$$

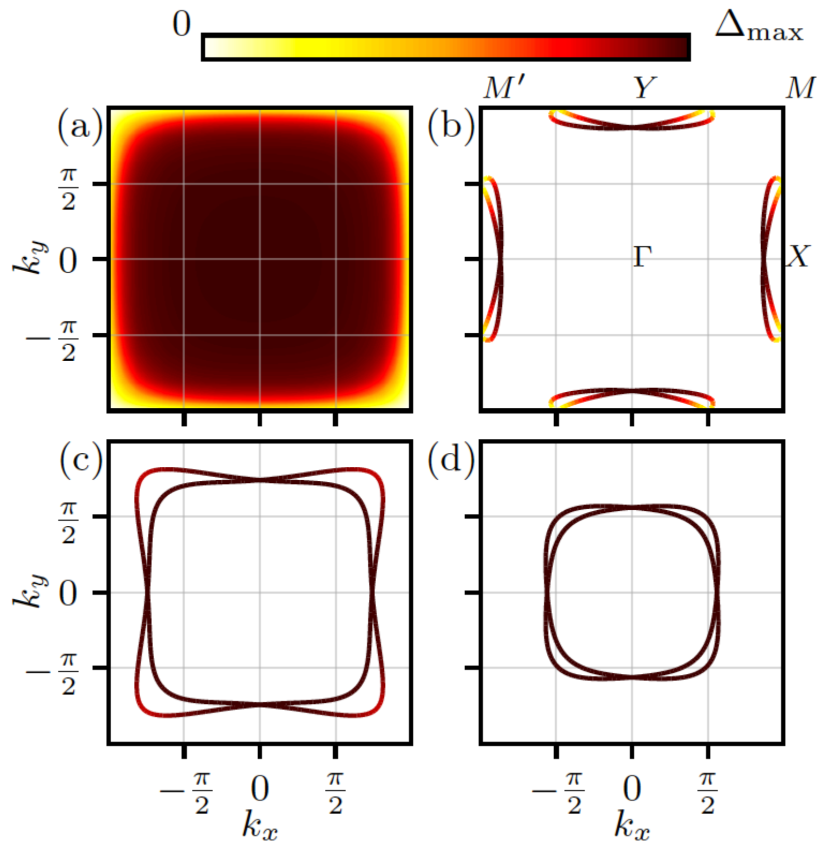
Inherent momentum-dependent gaps
from sublattice-to-band transformation.

Superconductivity and altermagnetism

Fermi surface and sublattice weights

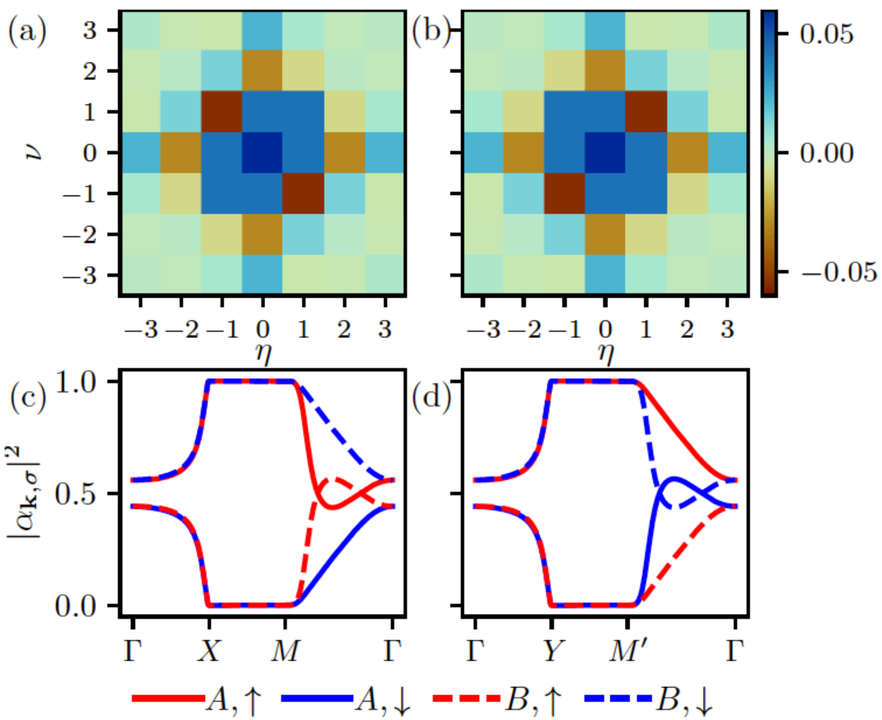


Matrix elements and superconducting gaps

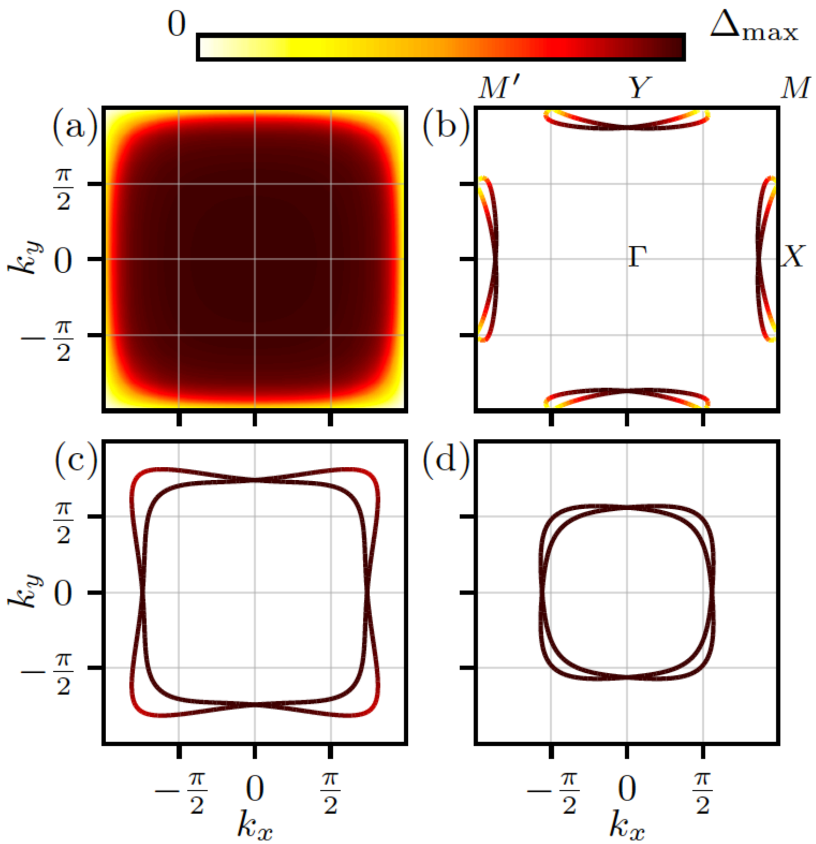


Superconductivity and altermagnetism

$$l_{\uparrow,\mathbf{k}} l_{\downarrow,-\mathbf{k}} = \sum_{\nu,\eta} c^{l}_{\nu,\eta} e^{-ik_x\nu} e^{-ik_y\eta}$$



Blocking effect!



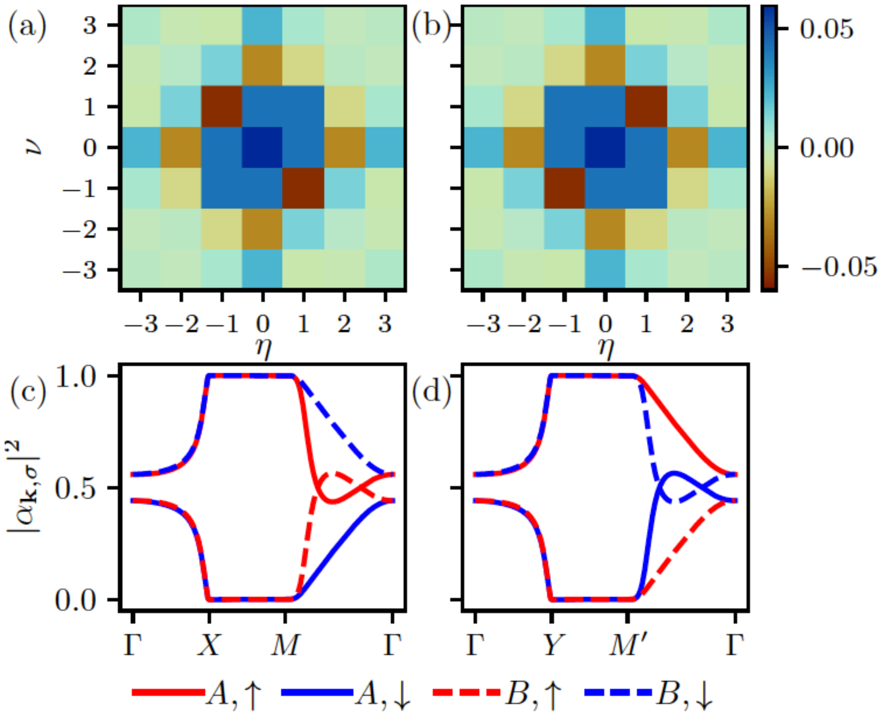
Sublattice and spin-resolved weights on the band.

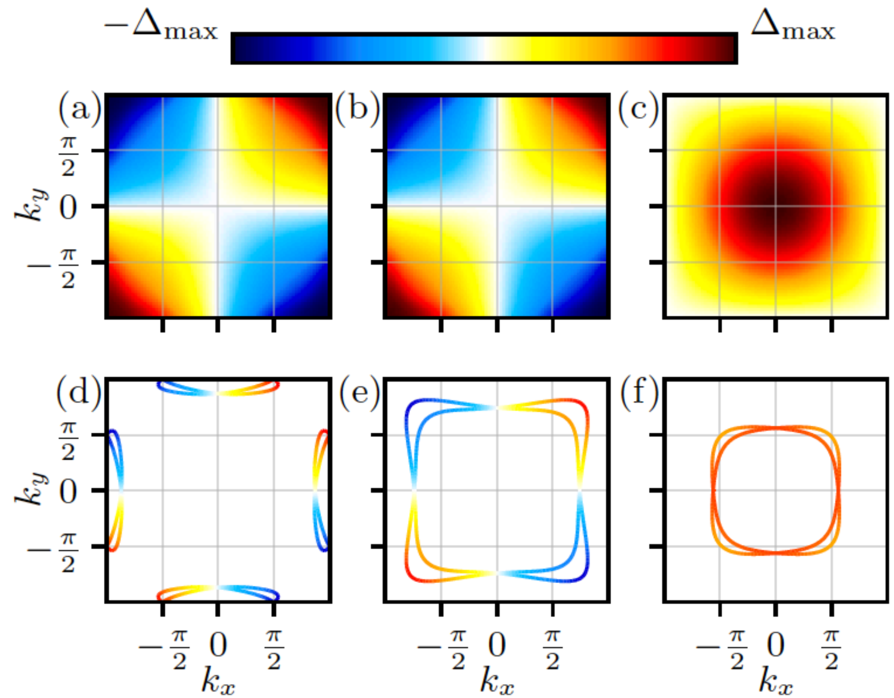
C. L. H. Rasmussen *et al,* ArXiv:2509.03247

Superconductivity and altermagnetism

$$l_{\uparrow,\mathbf{k}} l_{\downarrow,-\mathbf{k}} = \sum_{\nu,\eta} c^{l}_{\nu,\eta} e^{-ik_x\nu} e^{-ik_y\eta}$$

$$H^{\mathrm{NN}}_{\mathrm{int}} = -\frac{1}{N} \sum_{s,\mathbf{k},\mathbf{k}'} V^{\mathrm{NN}}_{\mathbf{k},\mathbf{k}',s,\bar{s}} c^{\dagger}_{\uparrow,s,\mathbf{k}} c^{\dagger}_{\downarrow,\bar{s},-\mathbf{k}} c_{\downarrow,\bar{s},-\mathbf{k}'} c_{\uparrow,s,\mathbf{k}'}$$





Sublattice and spin-resolved weights on the band.

Superconductivity and altermagnetism

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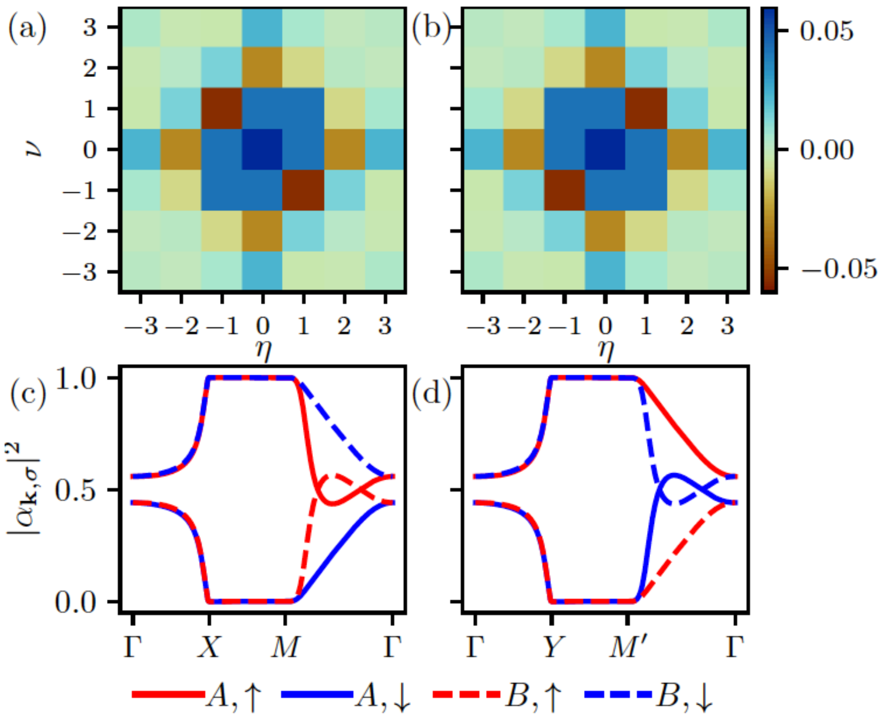

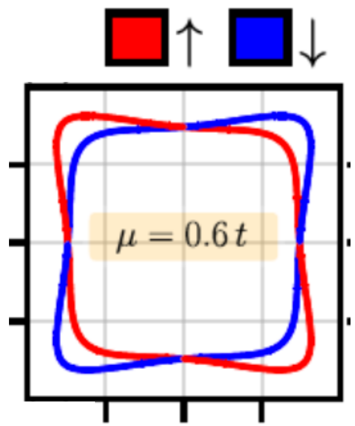

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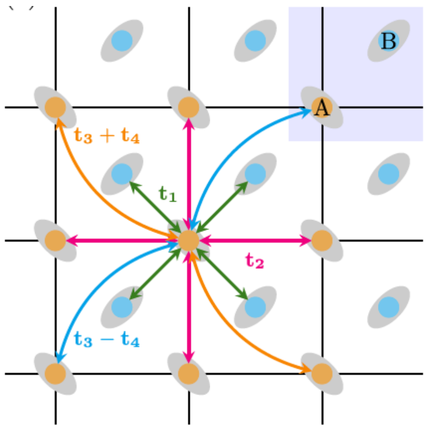
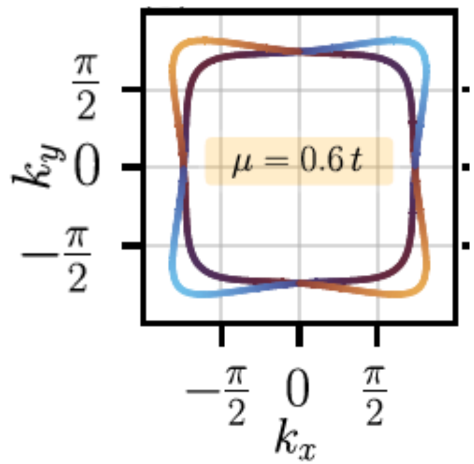
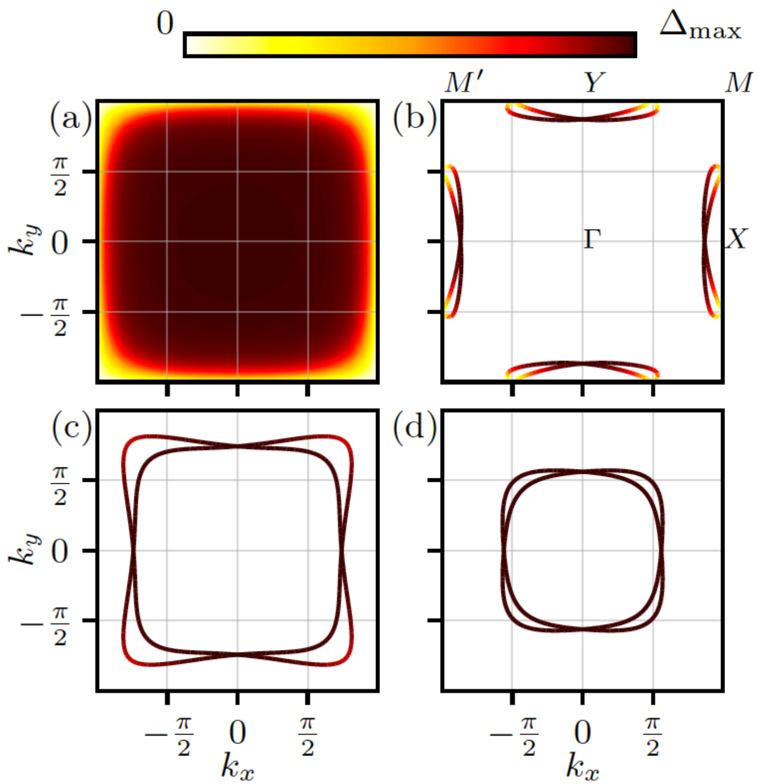
Superconductivity and altermagnetism



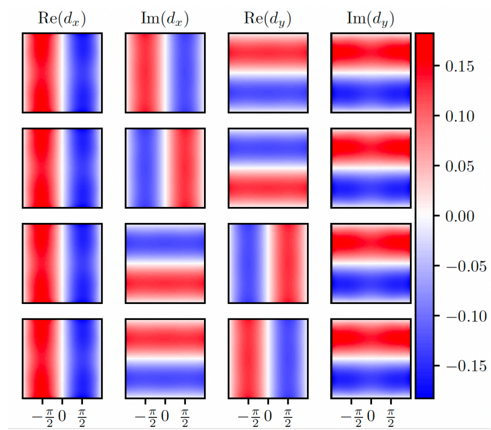


Equal-spin triplet superconductivity arises from NNN bonds!



Degenerate triplet states

$$(p_x \pm ip_y)|\uparrow\uparrow\rangle + (p_x \pm ip_y)|\downarrow\downarrow\rangle$$



C. L. H. Rasmussen *et al*, ArXiv:2509.03247

Conclusions

1. We have constructed versatile classes of minimal models for altermagnetic materials.
2. We have analysed the interplay between magnetisation and Néel order and derived the allowed lowest-order invariants.
3. There exists a general underlying quasi-symmetry enabled by SOC that constrains the size of the induced FM moment for different altermagnetic symmetries.
4. Conclusions from realistic microscopic models: only antisymmetric combinations are generated linear in SOC.
5. There exists clear local signatures of altermagnetism relevant for future STM measurements + general physics of 2D layer groups. (POSTER)

6. There exists a rich interplay between altermagnetism and unconventional superconductivity. (POSTER)