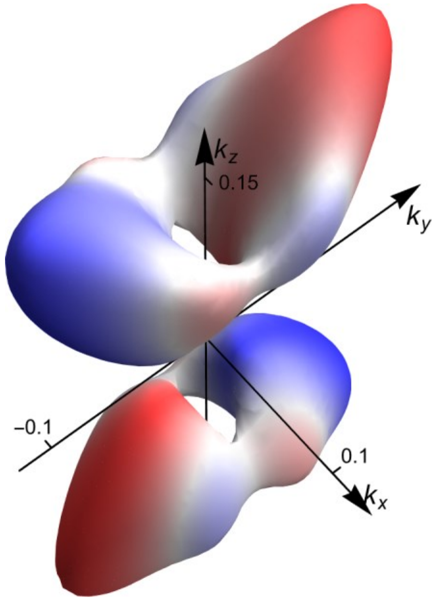
# Strain-Induced Spin Splitter Effect and Piezomagnetism in Altermagnets

Kirill Belashchenko

*University of Nebraska-Lincoln*

Maxim Khodas,[1] Sai Mu,[2] Igor Mazin[3]

[1]*Hebrew University of Jerusalem*
[2]*University of South Carolina*
[3]*George Mason University*



SPICE Workshop, October 2025

# Outline

- Spin-splitter effect in MnTe induced by strain and SOC
- Spin-splitter effect in anisotropic ferromagnets
- Piezomagnetism in altermagnets

# Motivation: spin current sources for spin torque devices

Typical source: spin Hall material
(heavy metals like Pt, W, Ta)

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{M} \times \frac{d\mathbf{M}}{dt} + \gamma \boldsymbol{\tau}$$

Manchon *et al.*, RMP **91**, 035004 (2019)

- ☐ Magnetization switching (MRAM)
- ☐ High-frequency nano-oscillators
- ☐ Spin-wave excitation and manipulation
- ☐ Driving force for magnetic textures

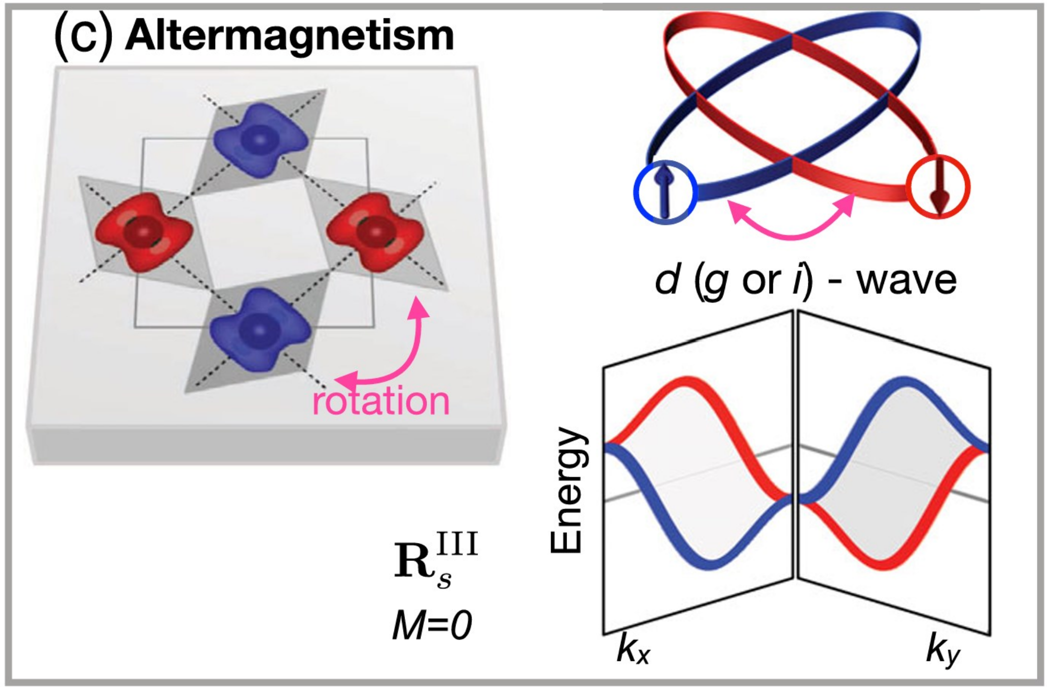
Spin Hall conductivity: antisymmetric, TR-even tensor

$$j_{s,j}^{i} = \sigma_{\text{SH}} \epsilon_{ijk} E_k$$

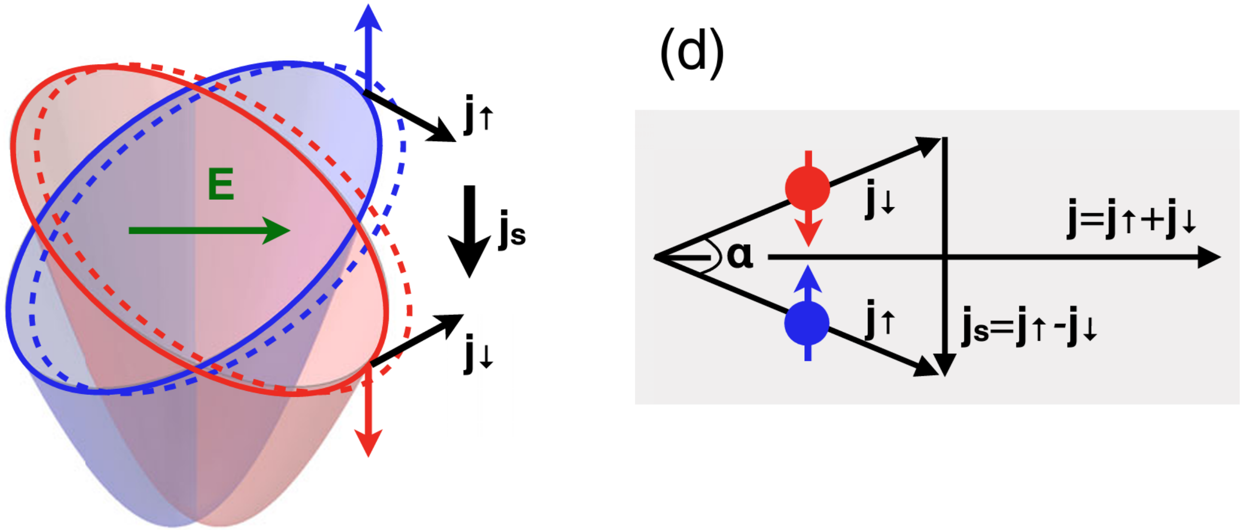
Limitations of spin-Hall sources:

- In-plane spin polarization (without symmetry breaking)
- Low efficiency, especially for z-polarized spin current

# Altermagnets and the spin-splitter effect



Šmejkal *et al.*, PRX 12, 040501 (2022)

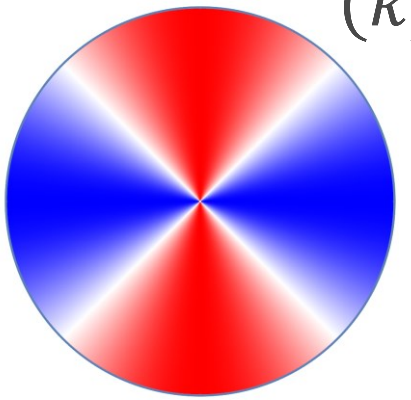


González-Hernández *et al.*, PRL 126, 127701 (2021)
Bose *et al.*, Nat. Electron. 5, 267 (2022)

- Transverse spin current without spin-orbit coupling
- Symmetric, TR-odd spin conductivity tensor: $j_i^s = \sigma_{ik}^s E_k$
- Spin-splitter angle $\theta_{SS} = j_\perp^s / j$
- Spin polarization along the Néel vector, PMA switching
- Select an AM domain via net M (e.g. piezomagnetic)

# Altermagnets: material candidates

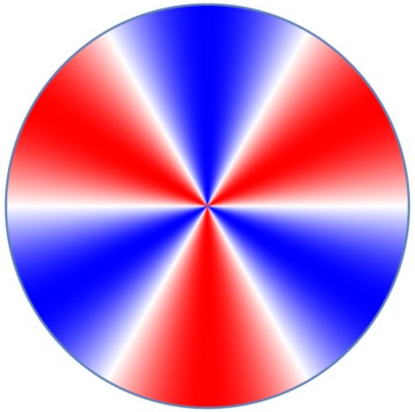
*d*-wave



RuO$_2$: Estimated $\theta_{SS} \sim 0.3$ (LDA+U)
But it is nonmagnetic

Lieb oxychalcogenides (KV$_2$Se$_2$O, etc.)
Promising but unexplored (yet)

*g*-wave



CrSb (705 K, metal)
MnTe (310 K, semiconductor)
$\theta_{SS} = 0$ by symmetry (3-fold axis)

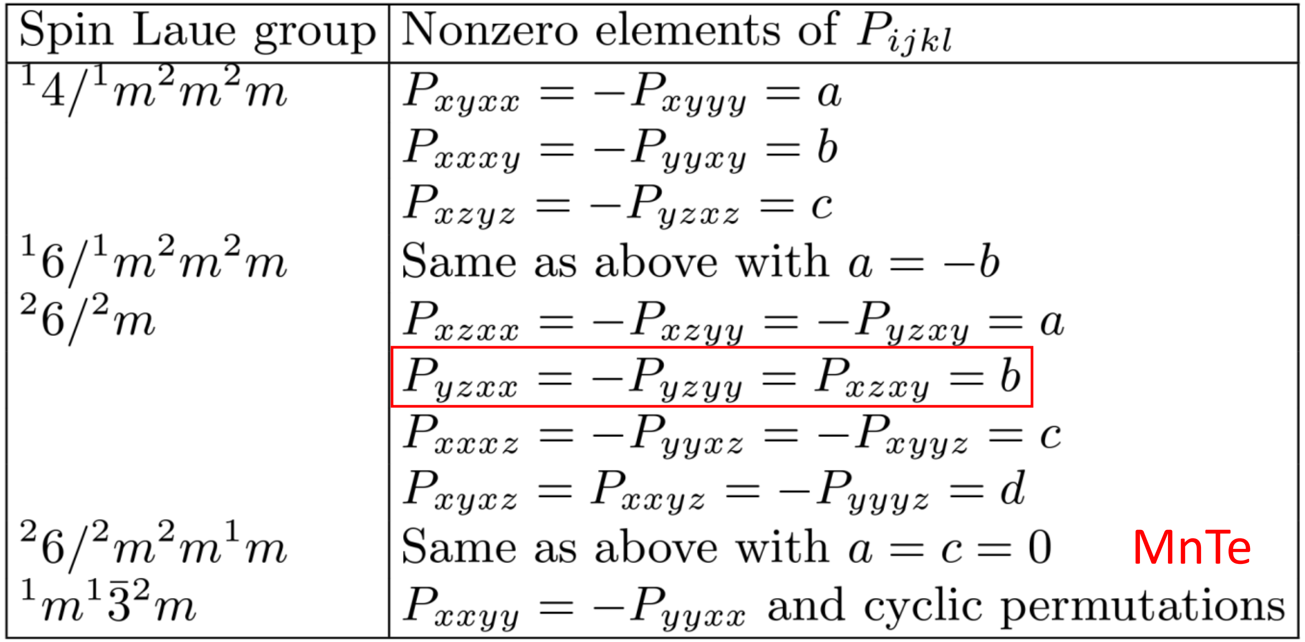
- ☐ SOC with in-plane **L** makes MnTe effectively *d*-wave, but only weakly
- ☐ Symmetry-breaking **strain** induces **strong** *d*-wave character in MnTe

# Outline

- Spin-splitter effect in MnTe induced by strain or SOC PRL 134, 086701 (2025)
- Spin-splitter effect in anisotropic ferromagnets
- Piezomagnetism in altermagnets

# Piezo-spin-galvanic tensor in g- and i-wave altermagnets

$\sigma_{ij}^{s} = P_{ijkl}\varepsilon_{kl}$, non-relativistic limit

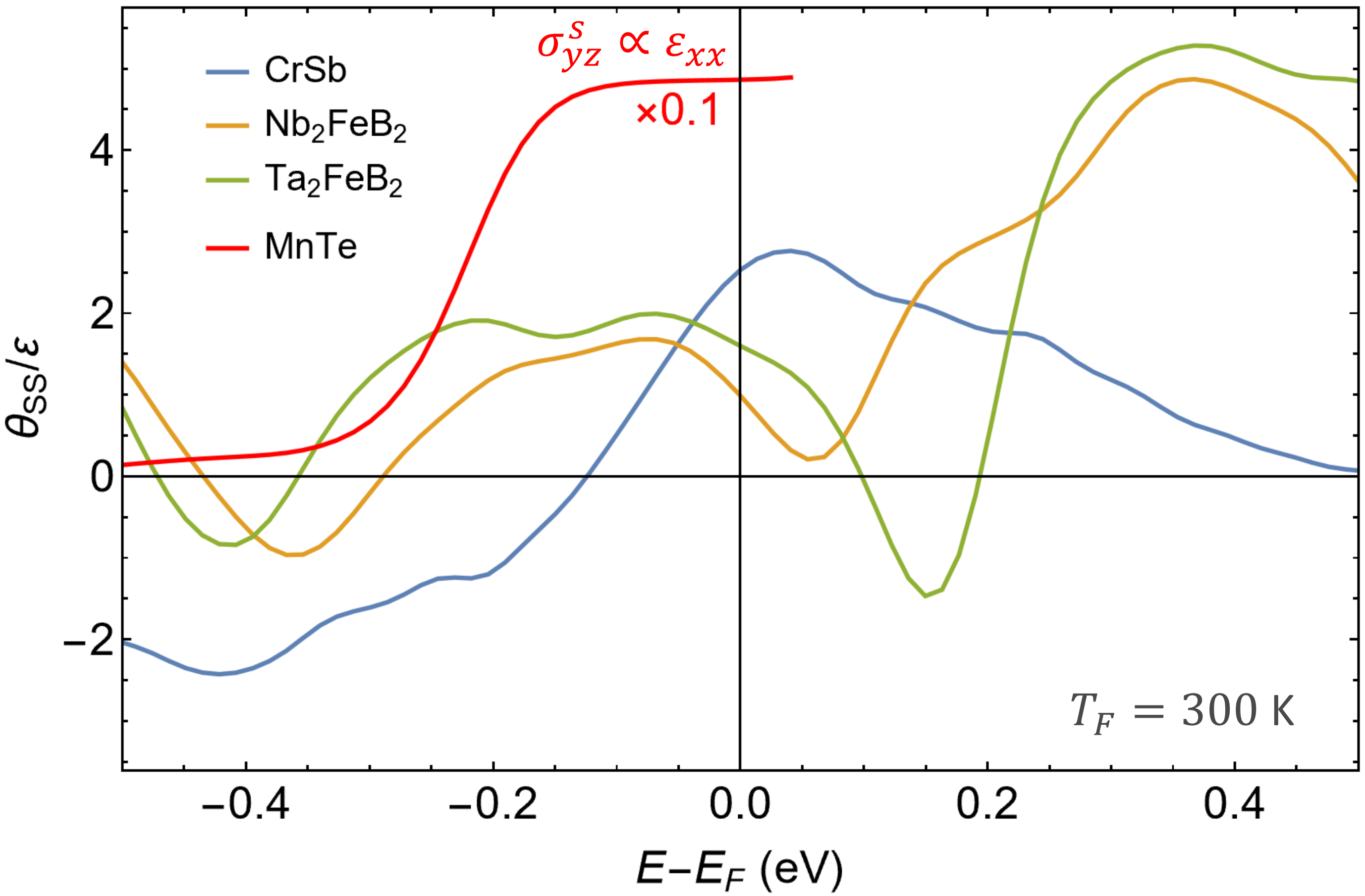


| Spin Laue group | Nonzero elements of $P_{ijkl}$ |
|---|---|
| $^{1}4/^{1}m^{2}m^{2}m$ | $P_{xyxx} = -P_{xyyy} = a$ |
| | $P_{xxxy} = -P_{yyxy} = b$ |
| | $P_{xzyz} = -P_{yzxz} = c$ |
| $^{1}6/^{1}m^{2}m^{2}m$ | Same as above with $a = -b$ |
| $^{2}6/^{2}m$ | $P_{xzxx} = -P_{xzyy} = -P_{yzxy} = a$ |
| | $\boxed{P_{yzxx} = -P_{yzyy} = P_{xzxy} = b}$ |
| | $P_{xxxz} = -P_{yyxz} = -P_{xyyz} = c$ |
| | $P_{xyxz} = P_{xxyz} = -P_{yyyz} = d$ |
| $^{2}6/^{2}m^{2}m^{1}m$ | Same as above with $a = c = 0$ MnTe |
| $^{1}m^{1}\bar{3}^{2}m$ | $P_{xxyy} = -P_{yyxx}$ and cyclic permutations |

We will consider $b = P_{yzxx}$ in MnTe

# Strain-induced spin-splitter effect in g-wave altermagnets

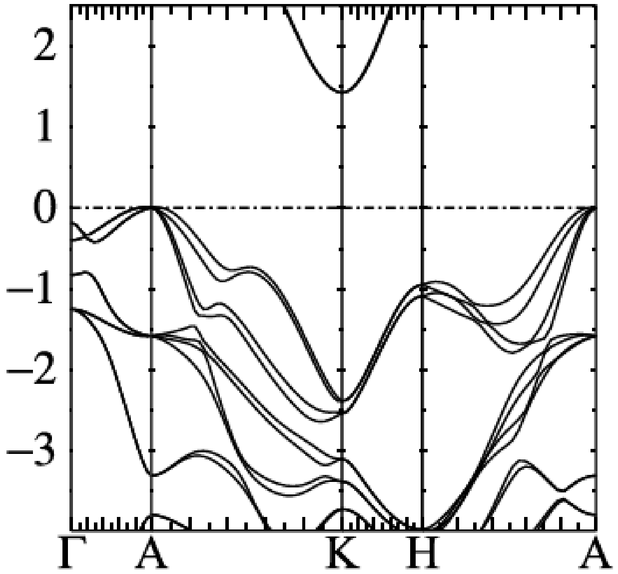
Boltzmann approximation with *ab initio* band structure (without SOC)



- Typical gauge factor $\frac{\theta_{SS}}{\varepsilon} \sim 1$ (too small for applications)
- **Large effect** in MnTe near VBM
- Where does it come from?

# MnTe: a g-wave altermagnet

- Hexagonal NiAs structure (ABAC stacking of Mn-Te-Mn-Te)
- $T_N = 310\ \mathrm{K}$, collinear order with magnetic moments along $[1\bar{1}00]$
- Semiconductor, naturally hole-doped $n \sim 10^{18}\ \mathrm{cm}^{-3}$, can also be heavily doped



QSGW with SOC
Faria *et al.*, PRB 107, L100417 (2023)

4-fold VBM at A point, slightly above $\Gamma$ (can be tuned)
Valence bands dominated by Te $p_x$, $p_y$

Inversion center only on Mn sites, not Te

g-wave: $\sigma^s_{\alpha\beta} = 0$ at zero strain

# MnTe: $k \cdot p$ model for VBM at A-point (Te $p_x$, $p_y$ states)

$$\hat{H}_h = -a_\parallel k_\parallel^2 - a_z k_z^2 + t_\Delta \left[ (k_x^2 - k_y^2)\tau_z + 2k_x k_y \tau_x \right]$$

Heavy and light hole bands

$$+ t_z k_z \sigma_z (k_x \tau_x + k_y \tau_z)$$

Altermagnetic hybridization via Mn (from spin group)

$$+ \lambda_1 (k_x \tau_z - k_y \tau_x)\sigma_y + \lambda_2 k_z \tau_z \sigma_x + \lambda_3 k_z \tau_y \sigma_x + \lambda_4 k_x \tau_y \sigma_y$$

Spin-orbit coupling (allowed with TRS present)

$$+ \Delta_0 \tau_z$$

Tiny SOC induced by magnetism (2 meV)

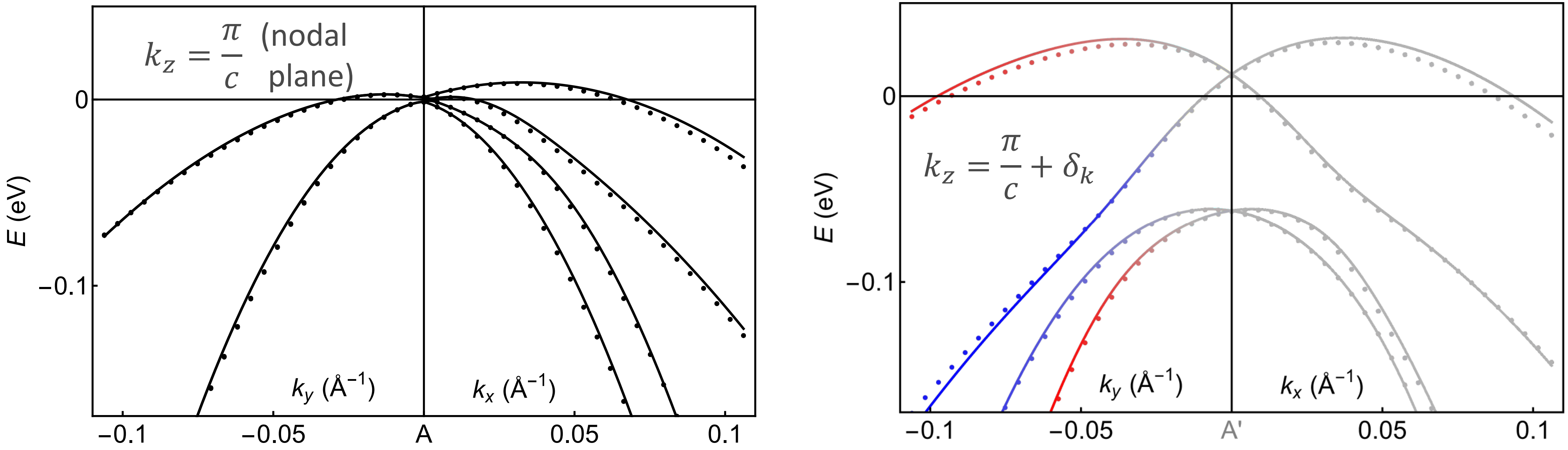
$\sigma_z$: true spin

$\sigma_x, \sigma_y$: pseudospin (spin/sublattice)

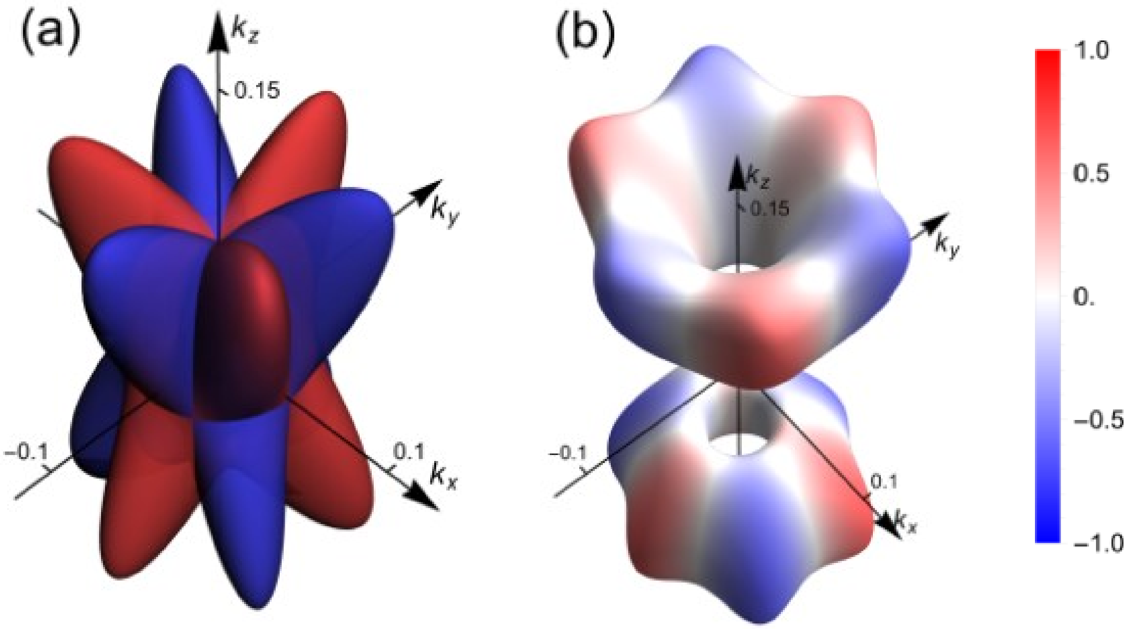
$\tau_\alpha$: orbital doublet $(p_x, p_y)$

**L** ∥ $[1\bar{1}00]$ here but the model allows arbitrary **L**

Fits to *ab initio* bands near A



# MnTe: Hole isosurfaces



No SOC

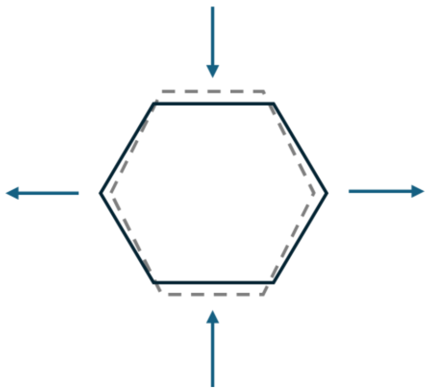
With SOC

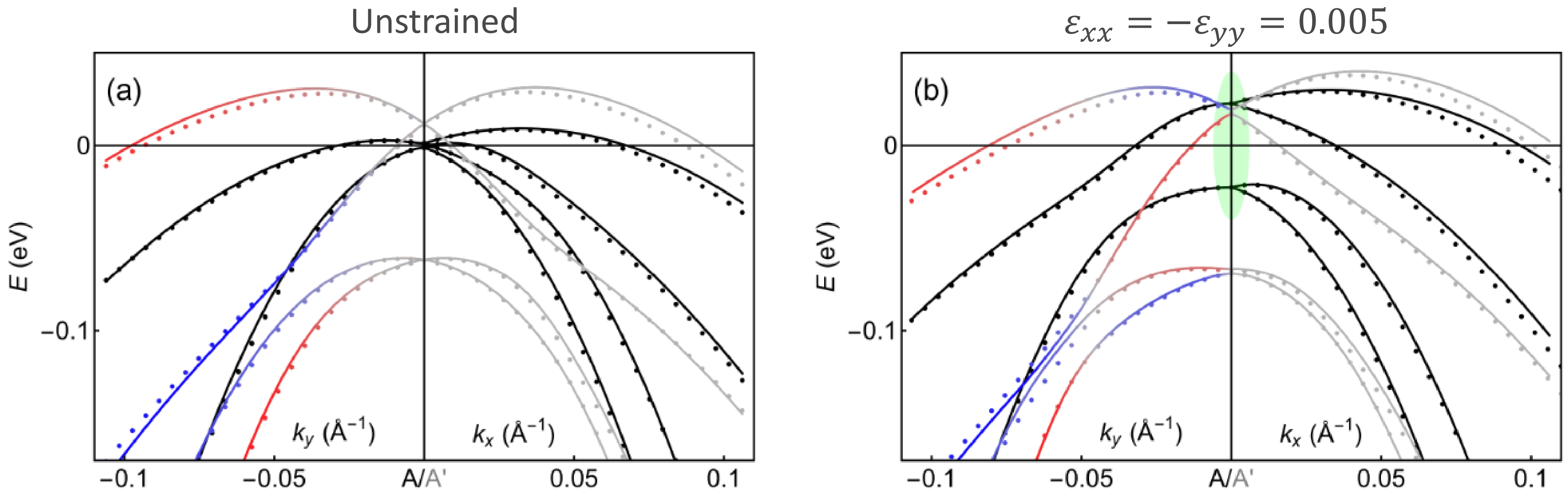
$n = 10^{19}$ cm$^{-3}$

- Strong spin mixing by SOC
- Persistent spin texture near the A point (small transverse spin component)

# MnTe: Strain-induced crystal field splitting at the VBM

$$\begin{aligned}\widehat{H}_h = & -a_\parallel k_\parallel^2 - a_z k_z^2 + t_\Delta \left[\left(k_x^2 - k_y^2\right)\tau_z + 2k_x k_y \tau_x\right] \\ & + t_z k_z \sigma_z \left(k_x \tau_x + k_y \tau_z\right) \\ & + \lambda_1 \left(k_x \tau_z - k_y \tau_x\right)\sigma_y + \lambda_2\, k_z \tau_z \sigma_x + \lambda_3\, k_z \tau_y \sigma_x + \lambda_4\, k_x \tau_y \sigma_y \\ & + (\Delta_0 + \Delta_s)\tau_z \qquad \text{Strain-induced CF splitting}\end{aligned}$$

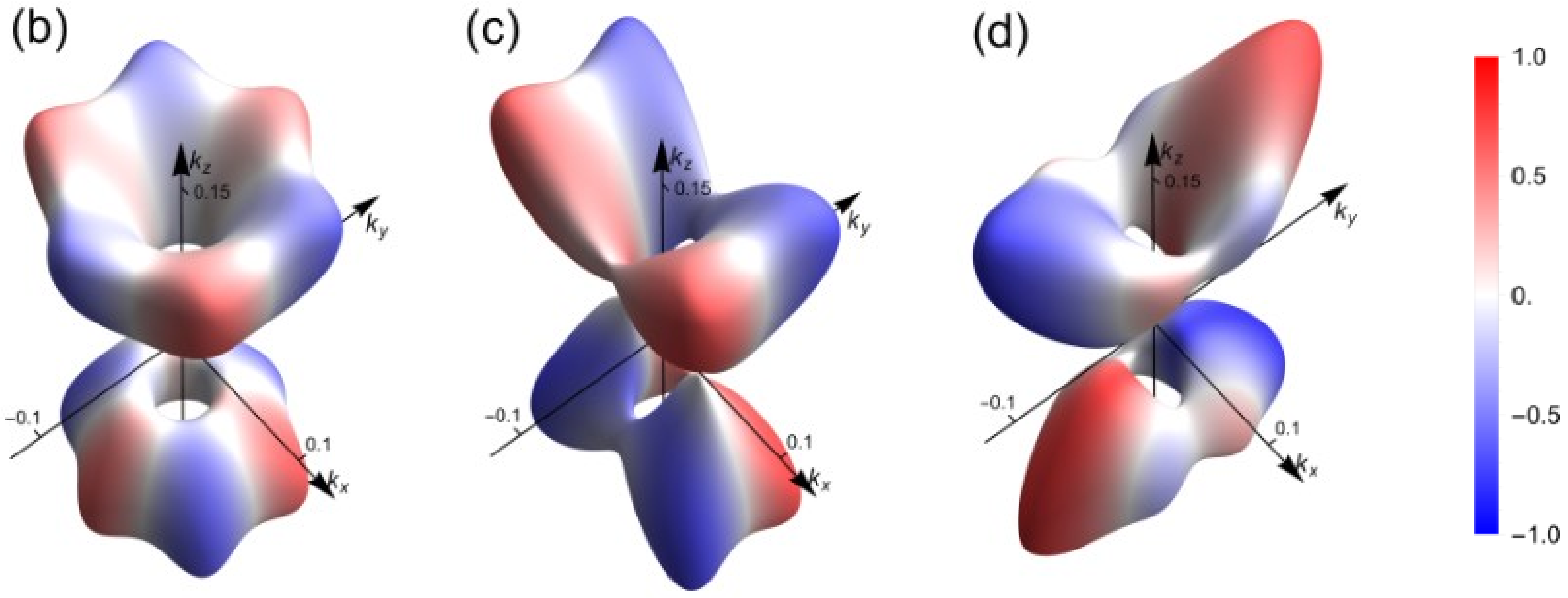



Unstrained

$\varepsilon_{xx} = -\varepsilon_{yy} = 0.005$

$\Delta_s = 21.3$ meV

# MnTe: Hole isosurfaces under strain



$\varepsilon = 0$ $\varepsilon = 0.01$ $\varepsilon = -0.01$

$n = 10^{19}$ cm$^{-3}$

- Strain turns *g*-wave into *d*-wave

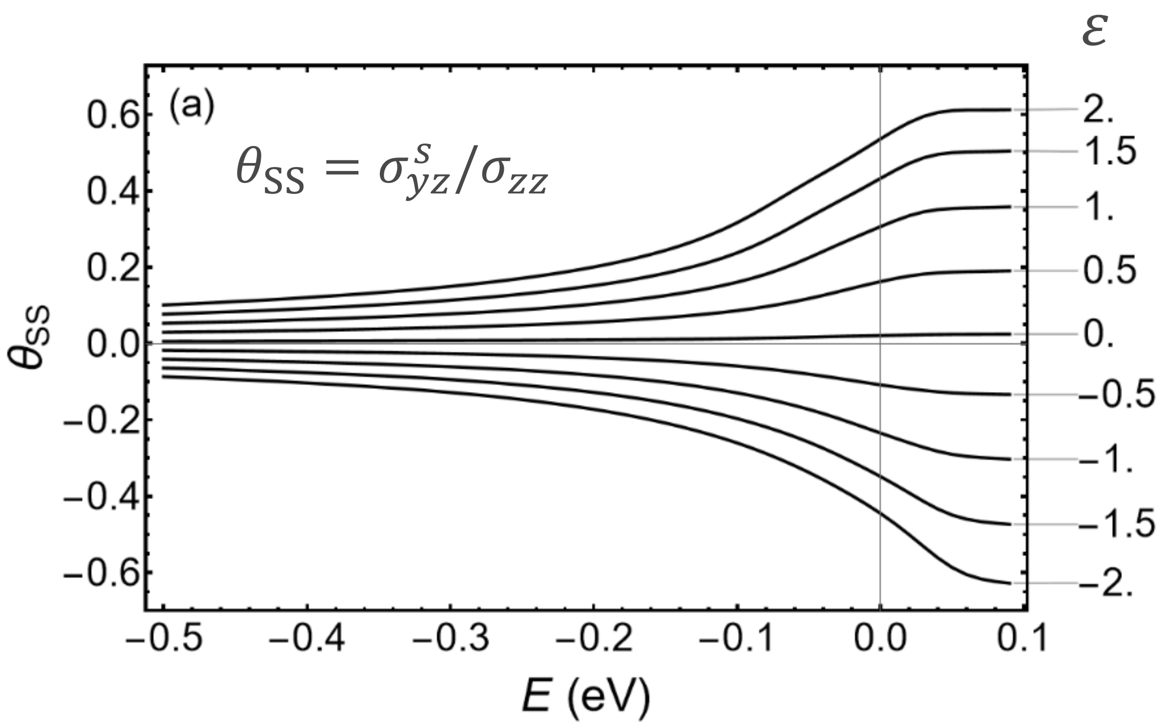
$$\varepsilon_{xx} = -\varepsilon_{yy} = \varepsilon/2$$

# MnTe: Spin-splitter angle under strain

Boltzmann approximation:

$$\sigma_{\alpha\beta} = \tau \sum_n \int v_{n\alpha} v_{n\beta} \frac{\partial f(E_n)}{\partial \mu} \frac{d^3k}{(2\pi)^3}$$

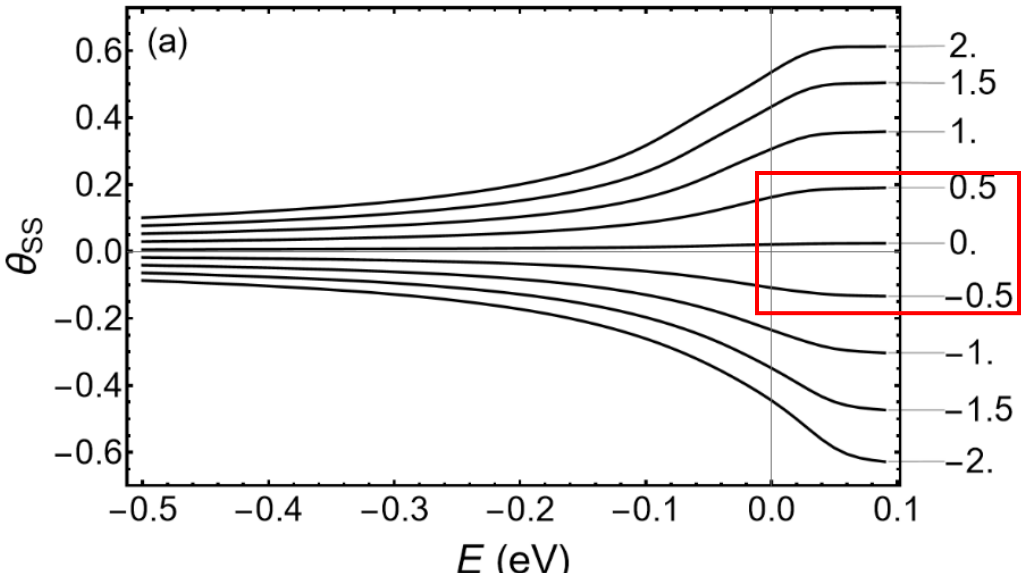
$$\sigma^s_{\alpha\beta} = \tau \sum_n \int v_{n\alpha} v_{n\beta} s_{ny} \frac{\partial f(E_n)}{\partial \mu} \frac{d^3k}{(2\pi)^3}$$

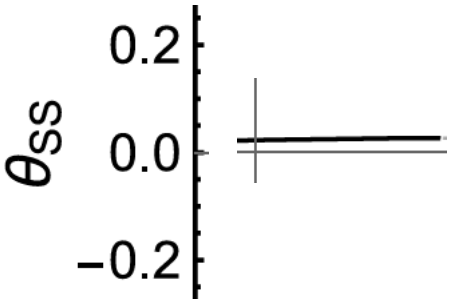


- Spin splitter gauge factor $\frac{\theta_{SS}}{\varepsilon} > 30$ near VBM (would be >100 without SOC)

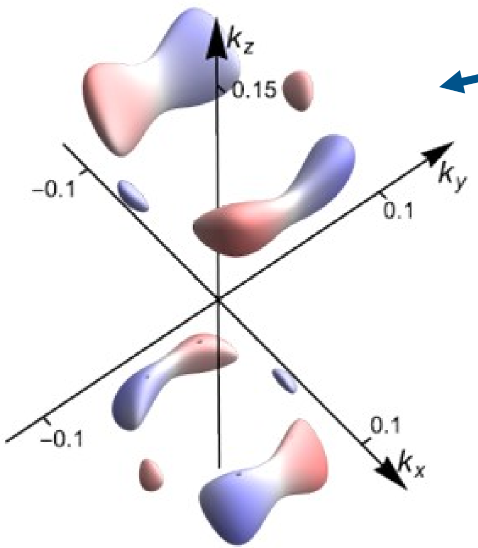
PRL 134, 086701 (2025)

# MnTe: SOC-induced spin-splitter effect (weak-ish)





- Finite $\theta_{SS} \approx 0.02$ at zero strain

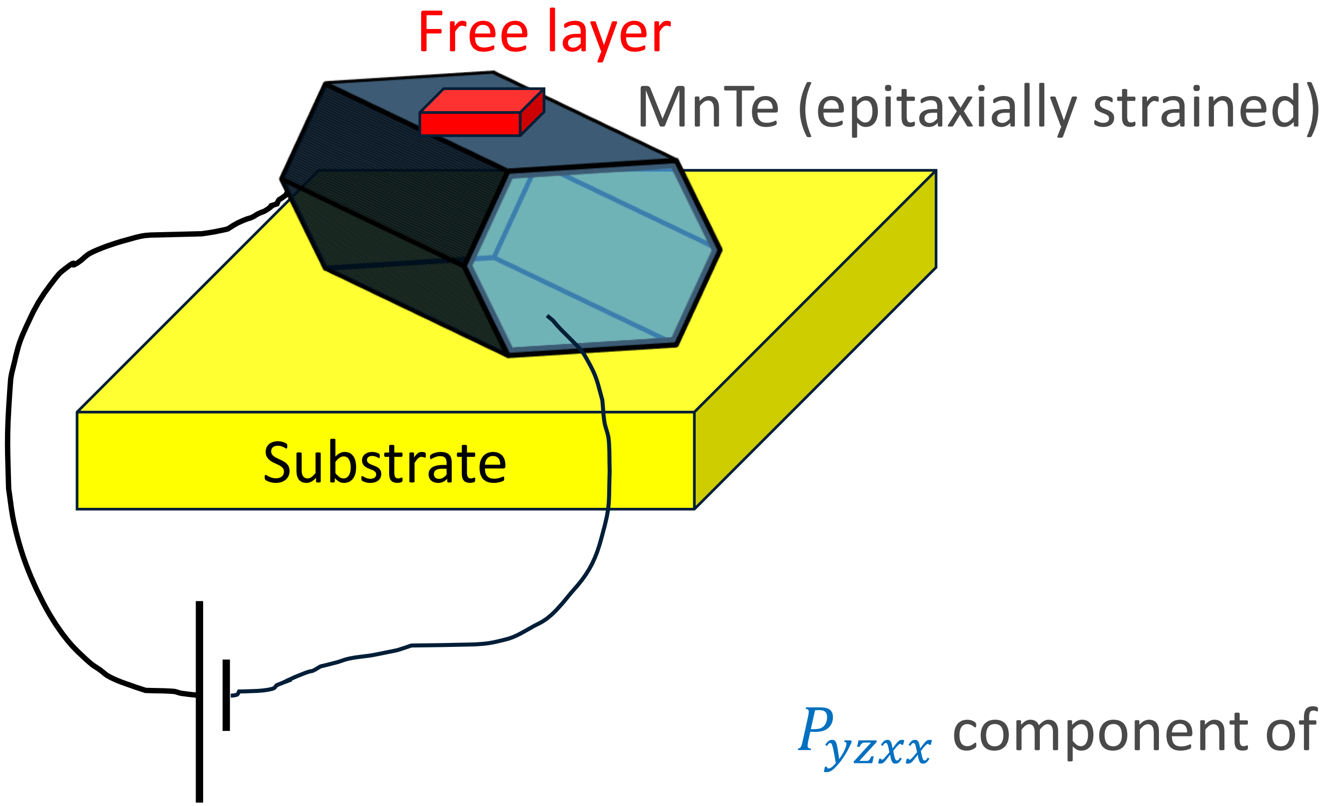


Iso-surface at 1 meV below VBM at zero strain

- Spin-orbit coupling effectively reduces the spin-momentum profile to *d*-wave in the *yz* plane

PRL 134, 086701 (2025)

# Suggested measurement geometry

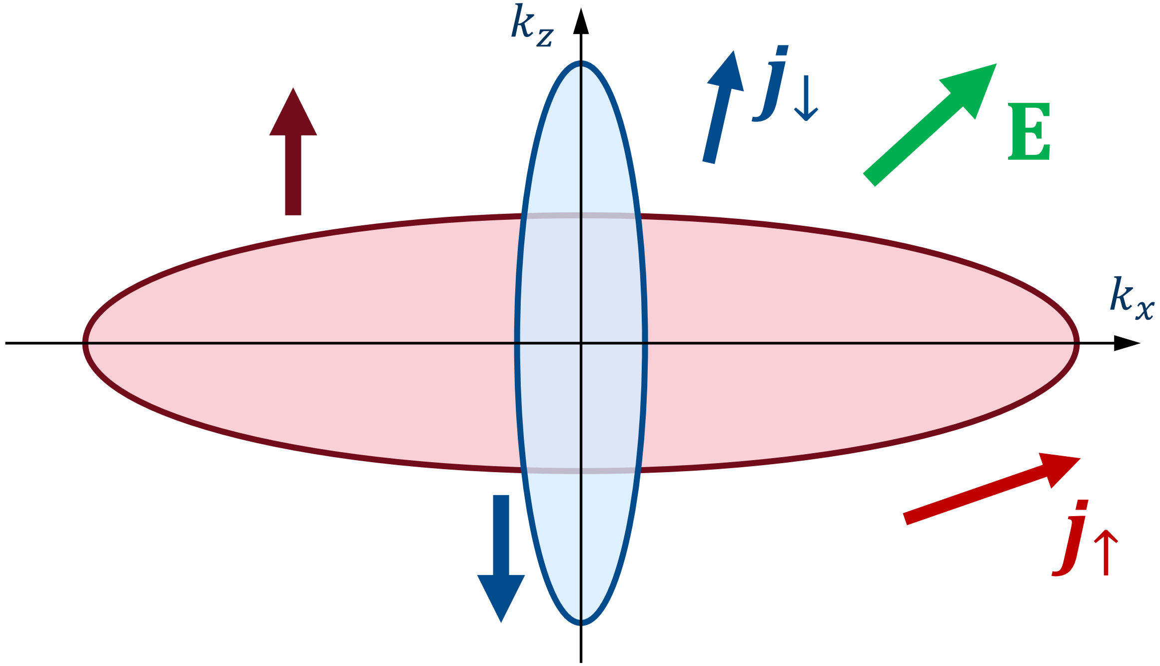


$P_{yzxx}$ component of the piezo-spin-galvanic tensor

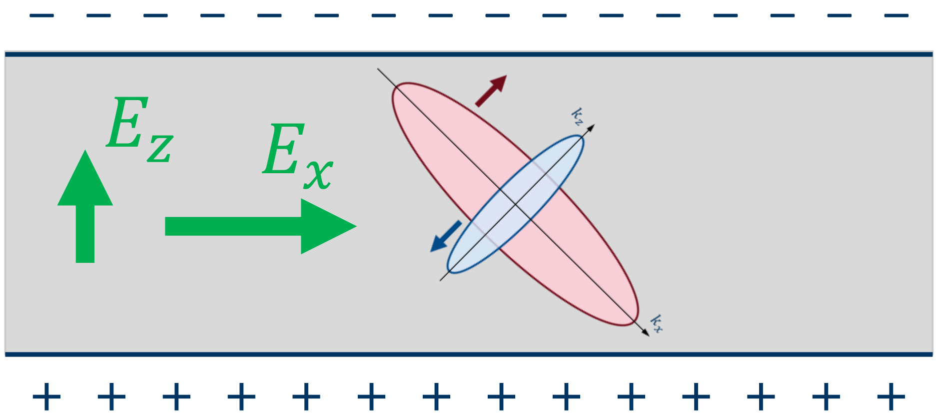
# Outline

- Spin-splitter effect in MnTe induced by strain and SOC
- Spin-splitter effect in anisotropic ferromagnets PRB 109, 054409 (2024)
- Piezomagnetism in altermagnets

# Spin splitter effect in an anisotropic ferromagnet



Film or multilayer geometry: $j_z = 0$

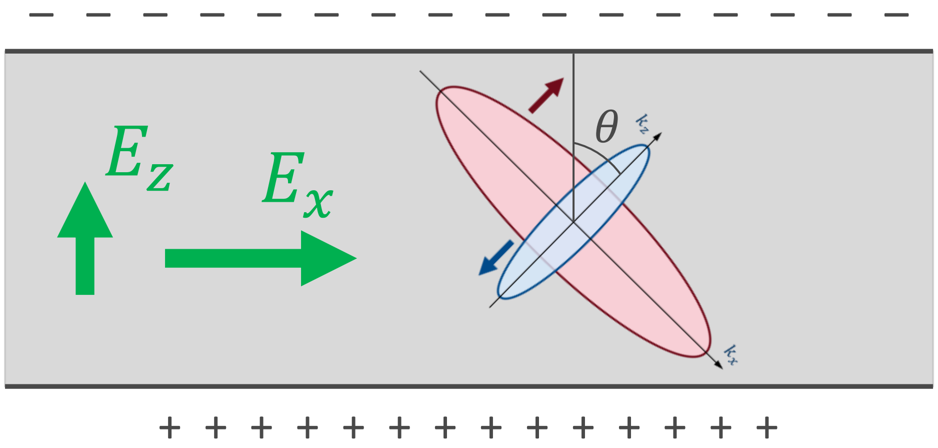


$j_x^\uparrow = \sigma_{xx}^\uparrow E_x + \sigma_{xz}^\uparrow E_z$, etc.

- Anisotropic bulk ferromagnet: transverse charge and spin currents for a generic orientation of **E**

Impose $j_z = 0$ and find $\theta_{SS} = j_{sz}/j_x$

# Spin splitter angle in a FM film with open boundary conditions



Under open boundary conditions:

$$\boxed{\theta_{\mathrm{SS}} = \frac{1}{2}(\beta_1 - \beta_2)\sin 2\theta}$$

$$|\theta_{\mathrm{SS}}| \leq 1$$

$$\beta_i = \frac{\sigma_i^{\uparrow} - \sigma_i^{\downarrow}}{\sigma_i^{\uparrow} + \sigma_i^{\downarrow}}$$

$i = 1,2,3$ (principal axis)

- Pure transverse spin current due to anisotropic transport spin polarization $\beta$
- Estimate $\beta_1 - \beta_2$ using Boltzmann calculations

# Anisotropy of transport spin polarization

$$\boxed{\theta_{SS} = \frac{1}{2}(\beta_1 - \beta_2)\sin 2\theta}$$

$$\beta_i = \frac{\sigma_i^{\uparrow} - \sigma_i^{\downarrow}}{\sigma_i^{\uparrow} + \sigma_i^{\downarrow}} \equiv P_\sigma$$

*i* = 1,2: principal directions

- Without SOC, conductivity is purely extrinsic
- Relaxation time approximation (weak to moderate disorder)

$$\sigma_{\alpha\beta}^{\lambda} = e^2 \sum_n \int v_{n\alpha}^{\lambda} v_{n\beta}^{\lambda} \tau_n^{\lambda} \frac{\partial f(E_n^{\lambda})}{\partial \mu} \frac{d^3k}{(2\pi)^3}$$

Assume $\tau$ depends only on spin: $\sigma_{\alpha\beta}^{\lambda} = e^2 \tau_\lambda K_{\alpha\beta}^{\lambda}$

$$\beta_1 - \beta_2 = \frac{(1 - P_\tau^2)(\boldsymbol{P_1} - \boldsymbol{P_2})}{(1 + P_1 P_\tau)(1 + P_2 P_\tau)}$$

$$P_i = \frac{K_i^{\uparrow} - K_i^{\uparrow}}{K_i^{\uparrow} + K_i^{\uparrow}}$$

- $\beta_1 - \beta_2$ is largely controlled by $P_1 - P_2$, which is a band structure property

# Estimated spin-splitter angles (for $P_\tau = 0$)

| Compound | $P_x$ | $P_z$ | $\lvert\theta_{SS}\rvert$ | Suggested substrate | MCIA (Å²) |
|---|---|---|---|---|---|
| MnGa | 0.07 | 0.64 | 0.29 | **$NdGaO_3$(011)/(101)** | 49 |
| MnSb | 0.91 | 0.33 | 0.29 | $MgF_2$(110)/(10-11) | 82 |
| MnAl | 0.05 | 0.55 | 0.25 | **$NdGaO_3$(011)/(101)** | 49 |
| MnAlGe | 0.42 | 0.88 | 0.23 | **a-$TiO_2$(101)/(101)** | 83 |
| MnAs | 0.75 | 0.29 | 0.23 | GaN(10-11)/(10-11) | 116 |
| $Mn_2Sb$ | 0.35 | -0.09 | 0.22 | $MgF_2$(101)/(111) | 78 |
| MnBi | 0.86 | 0.42 | 0.22 | $WTe_2$(0001)/(10-11) | 89 |
| $Be_{12}Cr$ | -0.13 | -0.54 | 0.21 | | |
| CrTe | 0.30 | 0.68 | 0.19 | $LiAlO_2$(110)/(10-11) | 90 |
| $Fe_2B$ | 0.37 | 0.06 | 0.16 | $LiGaO_2$(010)/(101) | 33 |
| $Fe_3Ge$ | 0.16 | 0.46 | 0.15 | Cu(100)/(10-11) | |
| $YFe_3$ | -0.08 | 0.21 | 0.15 | | |
| $Fe_5Si_3$ | 0.51 | 0.20 | 0.15 | GaAs(100)/(11-21) | 67 |
| $Co_2B$ | 0.34 | 0.63 | 0.14 | $LiGaO_2$(010)/(101) | 32 |
| $Mn_2Ga_5$ | 0.47 | 0.20 | 0.14 | $TiO_2$(100)/(101) | 83 |
| $Fe_3B$ | 0.47 | 0.70 | 0.12 | | |
| $Fe_8N$ | 0.03 | 0.26 | 0.12 | $BaTiO_3$(110)/(101) | 48 |
| $HfFe_2$ | -0.26 | -0.05 | 0.10 | MgO(100)/(11-21) | 72 |
| $Fe_3Sn$ | 0.72 | 0.52 | 0.10 | $LaAlO_3$(100)/(10-11) | 70 |
| CoPt | 0.23 | 0.07 | 0.08 | **$YAlO_3$(011)/(101)** | 51 |
| $Fe_2P$ | 0.28 | 0.43 | 0.07 | LiF(110)/(10-11) | 71 |
| $Fe_3N$ | -0.14 | -0.01 | 0.07 | C(0001)/(11-21) | 79 |
| $YCo_5$ | 0.51 | 0.63 | 0.06 | GaN(10-11)/(10-11) | 57 |
| $Mn_5Ge_3$ | 0.54 | 0.66 | 0.06 | $BaF_2$(100)/(11-21) | 76 |
| FePt | 0.34 | 0.24 | 0.05 | **$NdGaO_3$(011)/(101)** | 51 |
| FeNi | 0.27 | 0.36 | 0.04 | BN(0001)/(101) | 11 |
| FePd | 0.37 | 0.31 | 0.03 | **$NdGaO_3$(011)/(101)** | 51 |
| $Fe_3P$ | 0.70 | 0.77 | 0.03 | $Fe_2O_3$(0001)/(101) | 91 |
| $Fe_5B_2P$ | 0.23 | 0.17 | 0.03 | $GdScO_3$(001)/(101) | 64 |
| $MnAu_4$ | 0.45 | 0.49 | 0.02 | GaSe(0001)/(101) | 51 |
| $VAu_4$ | -0.54 | -0.58 | 0.02 | | |
| $ZrFe_2$ | -0.38 | -0.41 | 0.02 | | |

63%: $\tilde{\theta}_{SS} > 0.1$
29%: $\tilde{\theta}_{SS} > 0.2$
Comparable to the best heavy-metal sources (Pt, W)

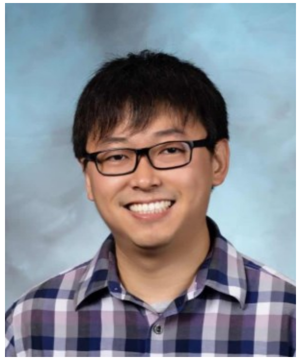
MCIA: minimal coincident area

# Outline

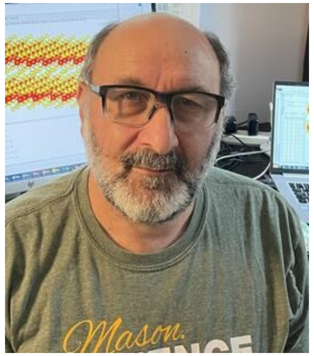
- Spin-splitter effect in MnTe induced by strain and SOC
- Spin-splitter effect in anisotropic ferromagnets
- Piezomagnetism in altermagnets

arXiv:2506.06257

Maxim Khodas



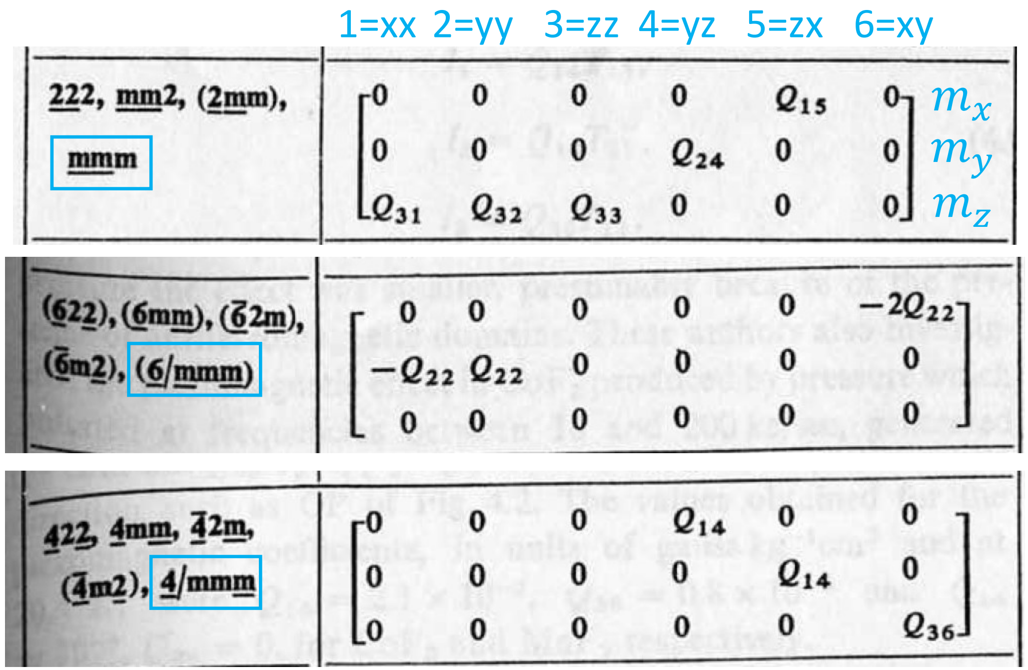
Sai Mu



Igor Mazin

# Piezomagnetism in altermagnets: Standard MPG description

- Standard analysis is based on magnetic point groups, all tabulated [R. Birss, Symmetry and Magnetism, 1964]
  Same PM tensors determine strain-induced AHC (Takahashi *et al.*, 2025)

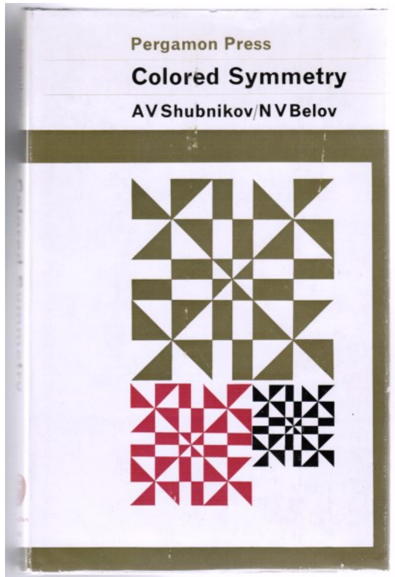


|  |  |  |
|---|---|---|
| $\underline{22}2$, $\underline{mm}2$, $(\underline{2m}m)$, $\boxed{\underline{mm}m}$ | $\begin{array}{c|cccccc} & 1=xx & 2=yy & 3=zz & 4=yz & 5=zx & 6=xy \\ \hline m_x & 0 & 0 & 0 & 0 & Q_{15} & 0 \\ m_y & 0 & 0 & 0 & Q_{24} & 0 & 0 \\ m_z & Q_{31} & Q_{32} & Q_{33} & 0 & 0 & 0 \end{array}$ | MnTe: NiAs-type, $\mathbf{L} \parallel [1\bar{1}00]$ |
| $(\underline{6}2\underline{2})$, $(\underline{6}m\underline{m})$, $(\bar{\underline{6}}2\underline{m})$, $(\bar{\underline{6}}m\underline{2})$, $\boxed{(\underline{6}/m\underline{mm})}$ | $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -2Q_{22} \\ -Q_{22} & Q_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ | CrSb: NiAs-type, $\mathbf{L} \parallel [0001]$ |
| $\underline{4}2\underline{2}$, $\underline{4}m\underline{m}$, $\bar{\underline{4}}2\underline{m}$, $(\bar{\underline{4}}m\underline{2})$, $\boxed{\underline{4}/mm\underline{m}}$ | $\begin{bmatrix} 0 & 0 & 0 & Q_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{14} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{36} \end{bmatrix}$ | (Mn,Fe,Co)$F_2$: rutile, $\mathbf{L} \parallel [001]$ |

| | |
|---|---|
| Drawbacks: | PM tensor tied to a specific orientation of $\mathbf{L}$, no relation for different orientations<br>No insight into which terms are large or small<br>Nothing about the mechanisms of PM |
| Alternative: | Start with the nontrivial spin point group, allowing $\mathbf{L}$ to take arbitrary orientation |

# Nontrivial spin group/antisymmetry group/$\Gamma_N$ irrep

- In collinear magnets, **nontrivial spin group** = **Shubnikov's black-and-white (antisymmetry) group** (Turek 2022)
- The 1D spin-splitting irrep $\Gamma_N$ of the point group (McClarty and Rau, 2024) contains the same information as the nontrivial spin point group
    - ❑ 1D irreps define halving subgroups while treating **L** as a scalar
    - ❑ Elements with character $(-1)$ in $\Gamma_N$ come with a color swap in the antisymmetry group



Pergamon Press

Colored Symmetry

AV Shubnikov/N V Belov

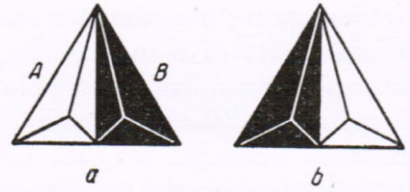


FIG. 72. Enantiomorphic figures with one antiplane of symmetry.

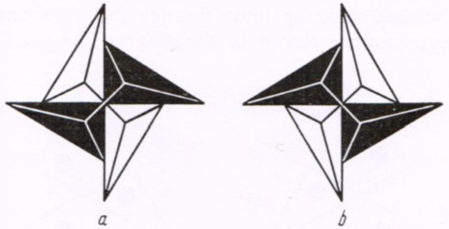
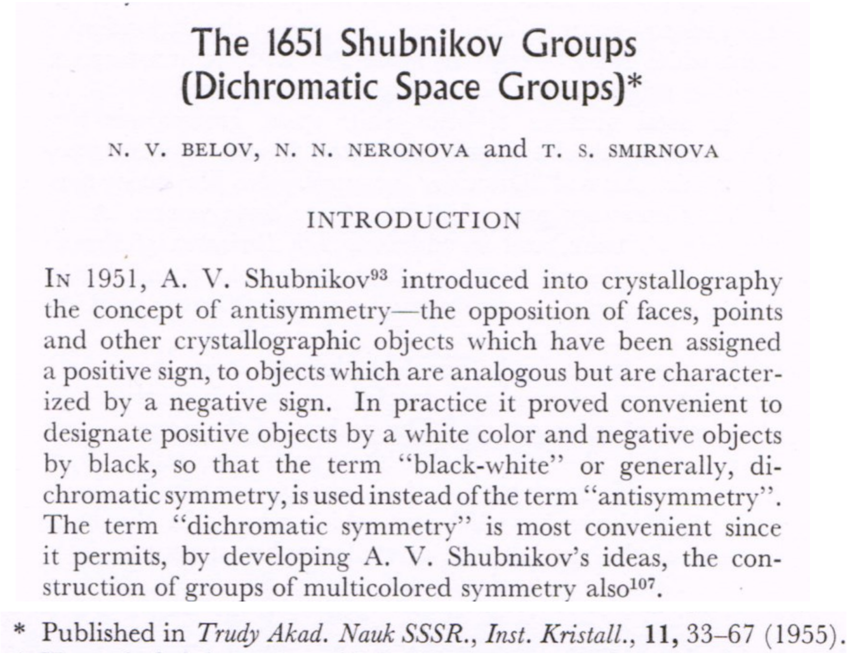


FIG. 77. Enantiomorphic figures with a four-fold antiaxis.



## The 1651 Shubnikov Groups (Dichromatic Space Groups)*

N. V. BELOV, N. N. NERONOVA and T. S. SMIRNOVA

### INTRODUCTION

IN 1951, A. V. Shubnikov[93] introduced into crystallography the concept of antisymmetry—the opposition of faces, points and other crystallographic objects which have been assigned a positive sign, to objects which are analogous but are characterized by a negative sign. In practice it proved convenient to designate positive objects by a white color and negative objects by black, so that the term "black-white" or generally, dichromatic symmetry, is used instead of the term "antisymmetry". The term "dichromatic symmetry" is most convenient since it permits, by developing A. V. Shubnikov's ideas, the construction of groups of multicolored symmetry also[107].

* Published in *Trudy Akad. Nauk SSSR., Inst. Kristall.*, **11,** 33–67 (1955).

# Piezomagnetism in altermagnets: Non-relativistic limit

- Purely longitudinal magnetization in the presence of strain: $\Delta F = \mathbf{L} \cdot \mathbf{M} f_P(\hat{\varepsilon})$
- Free energy invariants from the **nontrivial spin Laue group** (spin point group + inversion)

Spin Laue group

| Type | SLG | Prefactors in the free energy invariants |
|---|---|---|
| $d$-wave | ${}^2m_z{}^2m_y{}^1m_x$ | $\varepsilon_{yz}$ |
| | ${}^24/{}^1m$ | $\varepsilon_{xx} - \varepsilon_{yy},\ \varepsilon_{xy}$ |
| | ${}^24/{}^1m{}^2m_y{}^1m_d$ | $\varepsilon_{xy}$ Rutile (MnF$_2$, RuO$_2$); Lieb lattice (KV$_2$Se$_2$O, etc.) |
| | ${}^22/{}^2m$ | $\varepsilon_{xz},\ \varepsilon_{yz}$ |
| $g$-wave | ${}^14/{}^1m{}^2m{}^2m$ | $\varepsilon_{xy}(\varepsilon_{xx} - \varepsilon_{yy})$ |
| | ${}^1\bar{3}{}^2m_y$ | $(\varepsilon_{xx} - \varepsilon_{yy})\varepsilon_{yz} + 2\varepsilon_{xy}\varepsilon_{xz}$ |
| | ${}^26/{}^2m$ | $(\varepsilon_{xx} - \varepsilon_{yy})\varepsilon_{yz} + 2\varepsilon_{xy}\varepsilon_{xz},\ (\varepsilon_{xx} - \varepsilon_{yy})\varepsilon_{xz} - 2\varepsilon_{xy}\varepsilon_{yz}$ |
| | ${}^26/{}^2m{}^2m_y{}^1m_x$ | $(\varepsilon_{xx} - \varepsilon_{yy})\varepsilon_{yz} + 2\varepsilon_{xy}\varepsilon_{xz}$ MnTe, CrSb |
| $i$-wave | ${}^16/{}^1m{}^2m{}^2m$ | $\varepsilon_{xy}[3(\varepsilon_{xx} - \varepsilon_{yy})^2 - 4\varepsilon_{xy}^2],\ \varepsilon_{xy}(\varepsilon_{yz}^2 - \varepsilon_{xz}^2) + (\varepsilon_{xx} - \varepsilon_{yy})\varepsilon_{xz}\varepsilon_{yz}$ |
| | ${}^1m{}^1\bar{3}{}^2m$ | $(\varepsilon_{xx} - \varepsilon_{yy})(\varepsilon_{yy} - \varepsilon_{zz})(\varepsilon_{zz} - \varepsilon_{xx}),\ (\varepsilon_{xx} - \varepsilon_{yy})\varepsilon_{xy}^2 + (\varepsilon_{yy} - \varepsilon_{zz})\varepsilon_{yz}^2 + (\varepsilon_{zz} - \varepsilon_{xx})\varepsilon_{xz}^2$ |

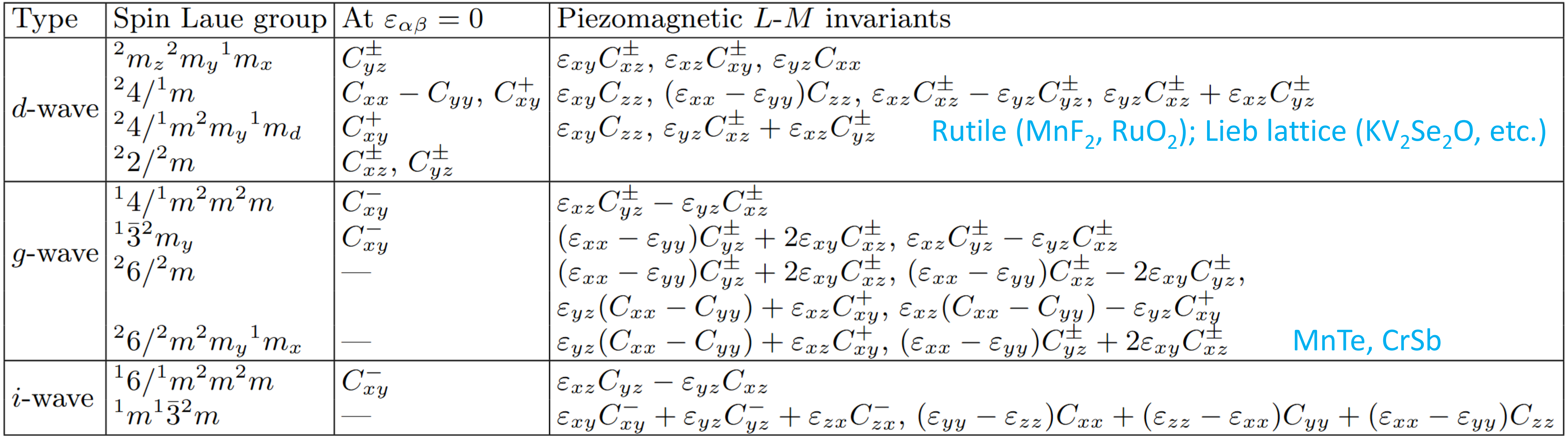
In the non-relativistic limit: *d*-wave altermagnets: $M \propto \varepsilon$ (linear PM)
*g*-wave AM: $M \propto \varepsilon^2$
*i*-wave AM: $M \propto \varepsilon^3$

- Where M is allowed, it may be suppressed due to integer band occupation in the insulating case. But spin splitting at the $\Gamma$ point is always allowed and determined by the same invariants.

# Piezomagnetism in altermagnets: Bilinear $L \cdot M$ invariants

- Invariants determined by the **nontrivial spin point group** (or equivalently PG + $\Gamma_L$)
- $\mathbf{M}$ can be both longitudinal and transverse to $\mathbf{L}$

$$C_{\alpha\beta} \equiv L_\alpha M_\beta$$
$$C^{\pm}_{\alpha\beta} \equiv C_{\alpha\beta} \pm C_{\beta\alpha}$$



| Type | Spin Laue group | At $\varepsilon_{\alpha\beta} = 0$ | Piezomagnetic $L$-$M$ invariants |
|---|---|---|---|
| $d$-wave | ${}^2m_z{}^2m_y{}^1m_x$ | $C^{\pm}_{yz}$ | $\varepsilon_{xy}C^{\pm}_{xz}$, $\varepsilon_{xz}C^{\pm}_{xy}$, $\varepsilon_{yz}C_{xx}$ |
| | ${}^24/{}^1m$ | $C_{xx} - C_{yy}$, $C^{+}_{xy}$ | $\varepsilon_{xy}C_{zz}$, $(\varepsilon_{xx} - \varepsilon_{yy})C_{zz}$, $\varepsilon_{xz}C^{\pm}_{xz} - \varepsilon_{yz}C^{\pm}_{yz}$, $\varepsilon_{yz}C^{\pm}_{xz} + \varepsilon_{xz}C^{\pm}_{yz}$ |
| | ${}^24/{}^1m{}^2m_y{}^1m_d$ | $C^{+}_{xy}$ | $\varepsilon_{xy}C_{zz}$, $\varepsilon_{yz}C^{\pm}_{xz} + \varepsilon_{xz}C^{\pm}_{yz}$ Rutile (MnF$_2$, RuO$_2$); Lieb lattice (KV$_2$Se$_2$O, etc.) |
| | ${}^22/{}^2m$ | $C^{\pm}_{xz}$, $C^{\pm}_{yz}$ | |
| $g$-wave | ${}^14/{}^1m{}^2m{}^2m$ | $C^{-}_{xy}$ | $\varepsilon_{xz}C^{\pm}_{yz} - \varepsilon_{yz}C^{\pm}_{xz}$ |
| | ${}^1\bar{3}{}^2m_y$ | $C^{-}_{xy}$ | $(\varepsilon_{xx} - \varepsilon_{yy})C^{\pm}_{yz} + 2\varepsilon_{xy}C^{\pm}_{xz}$, $\varepsilon_{xz}C^{\pm}_{yz} - \varepsilon_{yz}C^{\pm}_{xz}$ |
| | ${}^26/{}^2m$ | — | $(\varepsilon_{xx} - \varepsilon_{yy})C^{\pm}_{yz} + 2\varepsilon_{xy}C^{\pm}_{xz}$, $(\varepsilon_{xx} - \varepsilon_{yy})C^{\pm}_{xz} - 2\varepsilon_{xy}C^{\pm}_{yz}$,<br>$\varepsilon_{yz}(C_{xx} - C_{yy}) + \varepsilon_{xz}C^{+}_{xy}$, $\varepsilon_{xz}(C_{xx} - C_{yy}) - \varepsilon_{yz}C^{+}_{xy}$ |
| | ${}^26/{}^2m{}^2m_y{}^1m_x$ | — | $\varepsilon_{yz}(C_{xx} - C_{yy}) + \varepsilon_{xz}C^{+}_{xy}$, $(\varepsilon_{xx} - \varepsilon_{yy})C^{\pm}_{yz} + 2\varepsilon_{xy}C^{\pm}_{xz}$ MnTe, CrSb |
| $i$-wave | ${}^16/{}^1m{}^2m{}^2m$ | $C^{-}_{xy}$ | $\varepsilon_{xz}C_{yz} - \varepsilon_{yz}C_{xz}$ |
| | ${}^1m{}^1\bar{3}{}^2m$ | — | $\varepsilon_{xy}C^{-}_{xy} + \varepsilon_{yz}C^{-}_{yz} + \varepsilon_{zx}C^{-}_{zx}$, $(\varepsilon_{yy} - \varepsilon_{zz})C_{xx} + (\varepsilon_{zz} - \varepsilon_{xx})C_{yy} + (\varepsilon_{xx} - \varepsilon_{yy})C_{zz}$ |

- Many AM spin groups allow finite $\mathbf{M}$ at zero strain, for some $\mathbf{L}$ (already tabulated by Roig *et al.*, PRL 2025)
- All SLG allow linear PM terms inducing the components of $\mathbf{M}$ that are forbidden at zero strain
- $C^{-}_{\alpha\beta}$ terms correspond to strain-induced DMI (first-order in SOC, hence dominant – Brian Andersen's talk)
- Angular dependence (on $\mathbf{L}$) is automatically included

# Piezomagnetism in altermagnets: CrSb and MnTe siblings

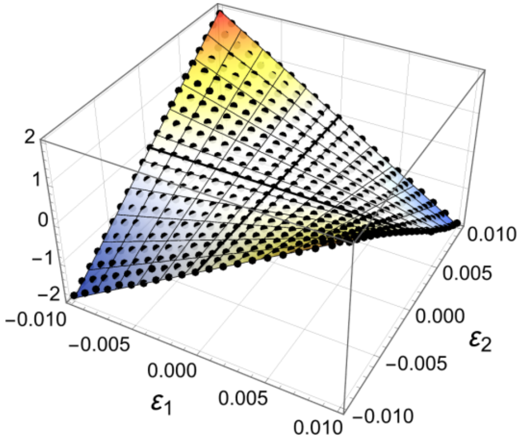
| | **CrSb** | **MnTe** |
|---|---|---|
| Orientation of **L** | $[0001]$ | $[1\bar{1}00]$ - we take $\mathbf{L} \parallel \hat{y}$ |
| Magnetic point group | $6'/m'mm'$ | $m'm'm$ |
| PM tensor from MPG | $\Lambda_{xxy} = 2\Lambda_{yxx} = -2\Lambda_{yyy}$ | $\Lambda_{xxz}, \Lambda_{yyz}, \Lambda_{zxx}, \Lambda_{zyy}, \Lambda_{zzz}$ (all distinct) |
| Spin Laue group | ${}^{2}6/{}^{2}m_z{}^{2}m_y{}^{1}m_x$ | $C_{\alpha\beta} \equiv L_\alpha M_\beta$ |
| Non-relativistic limit | No linear piezomagnetism; $M \propto \varepsilon^2$ | $C^{\pm}_{\alpha\beta} \equiv C_{\alpha\beta} \pm C_{\beta\alpha}$ |
| Linear $L \cdot M$ invariants | $\varepsilon_{yz}(C_{xx} - C_{yy}) + \varepsilon_{xz}C^{+}_{xy}, \quad (\varepsilon_{xx} - \varepsilon_{yy})C^{\pm}_{yz} + 2\,\varepsilon_{xy}C^{\pm}_{xz}$ | |
| Explicitly | $\varepsilon_{yz}(L_x M_x - L_y M_y) + \varepsilon_{xz}(L_x M_y + L_y M_x), \quad (\varepsilon_{xx} - \varepsilon_{yy})L_y M_z + 2\,\varepsilon_{xy}L_x M_z, \quad (\varepsilon_{xx} - \varepsilon_{yy})L_z M_y + 2\,\varepsilon_{xy}L_z M_x$ | |
| For given **L** axis | $[(\varepsilon_{xx} - \varepsilon_{yy})M_y + 2\,\varepsilon_{xy}M_x]L_z$ | $[-\varepsilon_{yz}M_y + \varepsilon_{xz}M_x]L_y, \quad (\varepsilon_{xx} - \varepsilon_{yy})L_y M_z$ |
| PM tensor from $L \cdot M$ | $\Lambda_{xxy} = 2\Lambda_{yxx} = -2\Lambda_{yyy} = \Lambda$ | $\Lambda_{xxz} = -\Lambda_{yyz} = \Lambda_1, \Lambda_{zxx} = -\Lambda_{zyy} = \Lambda_2$ |
| New information from spin point group compared to MPG | - Same tensor structure as from MPG<br>- DMI contributes to $\Lambda$, can be large | - Two relations among components<br>- DMI contributes to $\Lambda_2$ but not $\Lambda_1$<br>- $\Lambda_2 \gg \Lambda_1$ expected (1st order in SOC)<br>- $\Lambda_{zzz} \approx 0$ to linear order in L; obvious because $\varepsilon_{zz}$ does not lower symmetry |

# Mechanisms of piezomagnetic response

## Band filling effect

(non-relativistic, $\mathbf{M} \parallel \mathbf{L}$)

- Strain → inequivalent sublattices
- Finite M (if metallic)
- Linear PM only in *d*-wave AM.
  Example: oxyselenides like $V_2Se_2O$
  Ma *et al.,* Nat. Commun. 2021
- Quadratic PM in CrSb:



$$M = a[(\varepsilon_{xx} - \varepsilon_{yy})\varepsilon_{yz} + 2\varepsilon_{xy}\varepsilon_{xz}]$$
$$a = -39\ \mu_B/\text{f.u.}$$

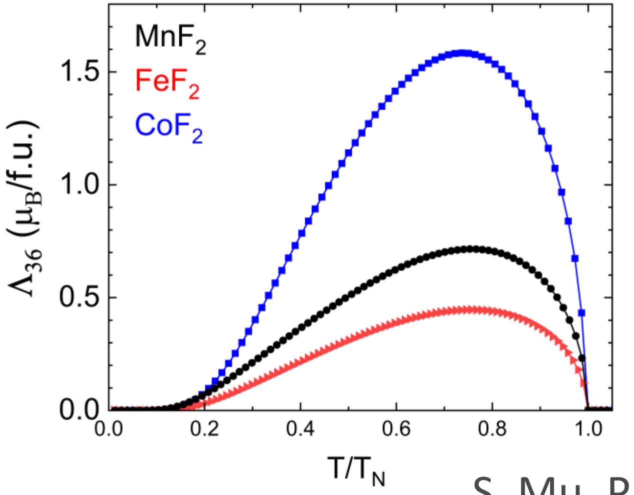
## Thermally-driven

(non-relativistic, $\mathbf{M} \parallel \mathbf{L}$)

- Strain → inequivalent sublattices
- $J_{AA} \neq J_{BB}$, hence $M_A(T) \neq M_B(T)$ regardless of metal/insulator
- Maximum $M(T)$ at finite $T$, similar to longitudinal ME effect

$$\boxed{\Lambda_{36} = \frac{\chi(T)L(T)}{\mu}\frac{\Delta}{\varepsilon}} \qquad 2\Delta = J_{AA} - J_{BB}$$

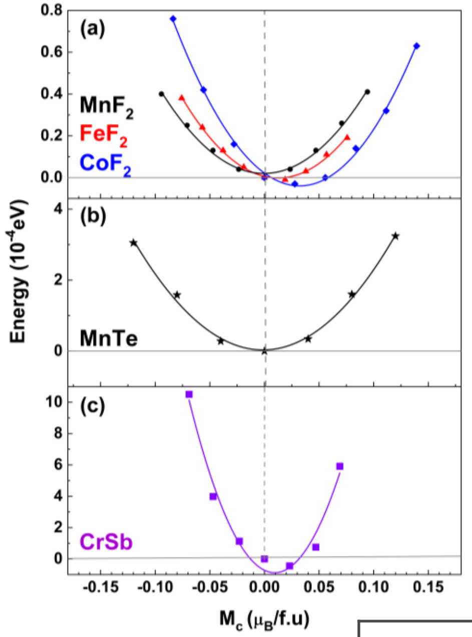
Ab initio $J_{ij}$ + mean-field



S. Mu, PhD thesis (2014)

## Strain-induced DMI

(relativistic, $\mathbf{M} \perp \mathbf{L}$)



$\mu_B$/f.u.

| | | Calc | Exp |
|---|---|---|---|
| $MnF_2$ | $\Lambda_{xyz}$ | ~ 0 | 0.1 |
| $FeF_2$ | $\Lambda_{xyz}$ | 0.4 | |
| $CoF_2$ | $\Lambda_{xyz}$ | 0.9 | 9.0 |
| MnTe | $\Lambda_{zxx}$ | ~ 0 | |
| CrSb | $\Lambda_{yxx}$ | ~0.5 | |

# Summary

- Spin splitter effect in MnTe: weak SOC-induced (0.02), giant strain induced (gauge factor >30)
- Spin splitter effect in noncubic crystalline FM film with tilted axes, $\theta_{SS} \sim 0.1\text{-}0.2$
- Piezomagnetism in altermagnets (free energy invariants from spin groups, mechanisms, ab initio)

PRL 134, 086701 (2025)
PRB 109, 054409 (2024)
M. Khodas, S. Mu, I. Mazin & KB, arXiv:2506.06257
Sai Mu, PhD thesis, UNL 2014

DOE DE-SC0024284