Superconductivity in Altermagnets

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Theory of unconventional magnetism: exploring altermagnets and beyond

October 23rd, 2025

Outline

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Introduction: Altermagnetism and Superconductivity

Results:

- Finite-momentum and field-induced superconductivity
- Perfect superconducting diode effect
- Constraints on superconducting pairing
- Altermagnetism and superconductivity in Sr_2RuO_4

Acknowledgements

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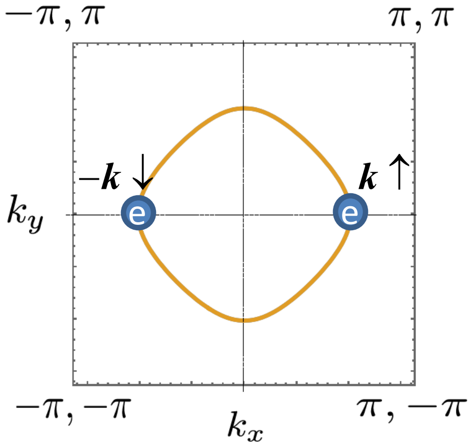
Wallenberg Initiative Materials Science for Sustainability

Superconductivity in altermagnets

Intrinsic (or in heterostructures)

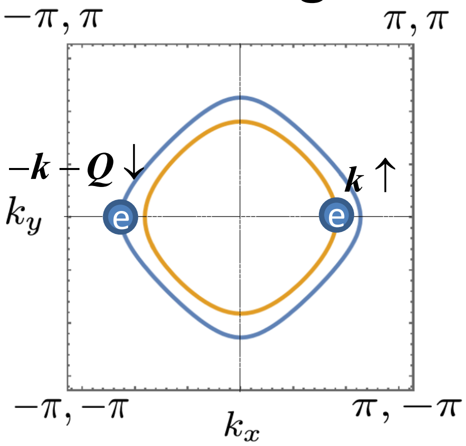
- La_2CuO_4, Sr_2RuO_4 , …

Metal



BCS pairing
(zero momentum pairing)

Ferromagnet

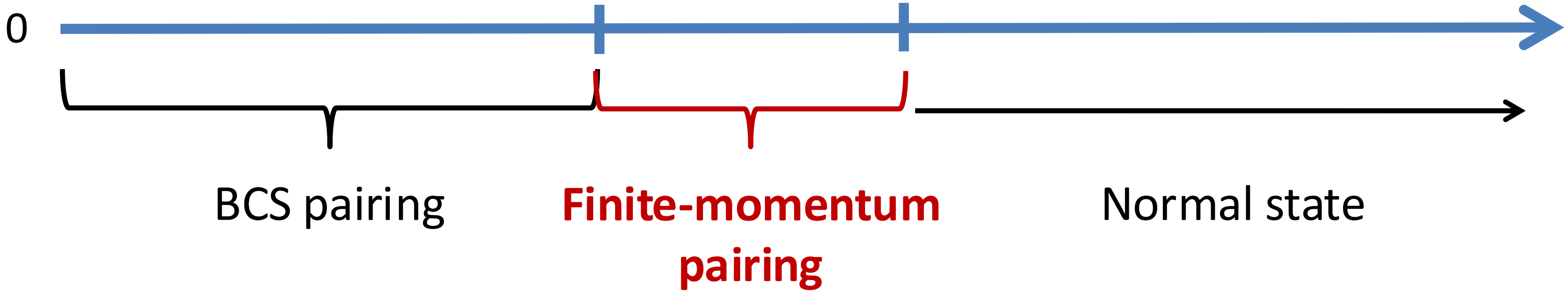


Finite momentum pairing
(finite center-of-mass Cooper pairs)

Šmejkal, Sinova, and Jungwirth PRX 12, 031042 (2022), Autieri, Cuono, Chakraborty, Gentile, ABS, PRB 112, 014412 (2025)

Finite momentum pairing

Increased spin-splitting (external magnetic field)



Fulde-Ferrel (FF): $\Delta = |\Delta_0| e^{i\boldsymbol{Q}.\boldsymbol{r}}$

PR 135, A550 (1964)

Larkin-Ovchinnikov (LO): $\Delta = \Delta_0 \cos(\boldsymbol{Q}.\boldsymbol{r})$

ZETF 47, 1136 (1964)

Superconductivity in altermagnets

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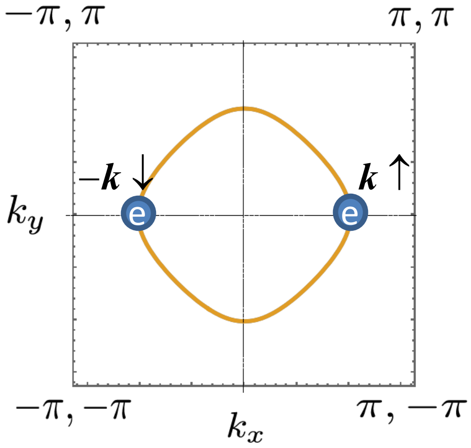
Intrinsic (or in heterostructures)

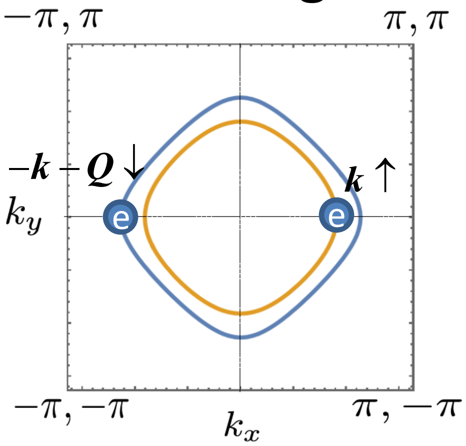
– La_2CuO_4, Sr_2RuO_4 , …

Metal



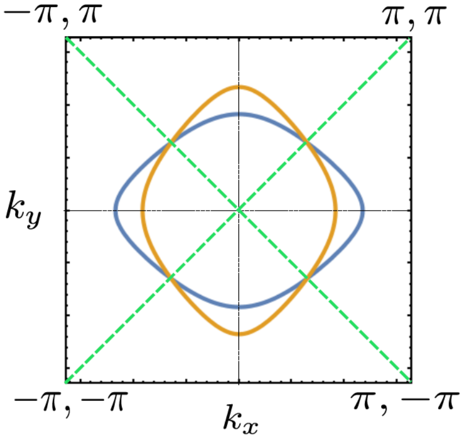
BCS pairing
(zero momentum pairing)

Ferromagnet



Finite momentum pairing
(finite center-of-mass Cooper pairs)

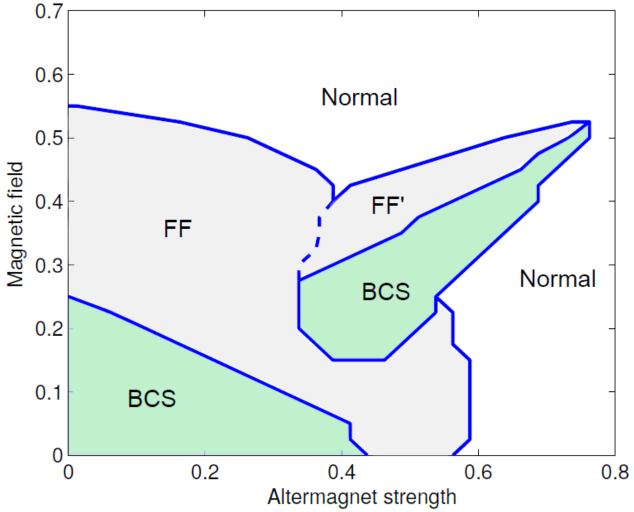
Altermagnet



Pairing?

Šmejkal, Sinova, and Jungwirth PRX 12, 031042 (2022), Autieri, Cuono, Chakraborty, Gentile, ABS, PRB 112, 014412 (2025)

Possibilities for superconducting pairing?



Modeling SC in altermagnets

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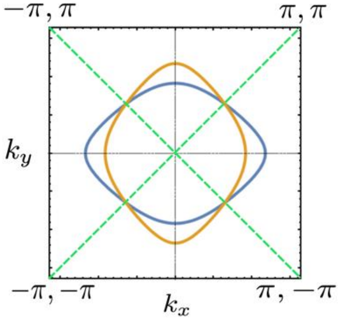
$$H = \sum_{k,\sigma} \underbrace{\left(\xi_k - \sigma \frac{t_{\mathrm{am}}}{2}(\cos(k_x) - \cos(k_y))\right.}_{\text{Altermagnet band structure}} + \underset{\uparrow}{\sigma B}) c^\dagger_{k\sigma} c_{k\sigma} + \sum_{k,k',q} V_{k,k'} c^\dagger_{k+q\uparrow} c^\dagger_{-k+q\downarrow} c_{-k'+q\downarrow} c_{k'+q\uparrow}$$

Altermagnet band structure — External magnetic field — Superconducting pairing



d-wave spin-split Fermi surfaces (FS)

Solve self-consistently for $\Delta_k^Q = \sum_{k'} V_{k,k'} \langle c^\dagger_{k'+Q/2\uparrow} c^\dagger_{-k'+Q/2\downarrow} \rangle$

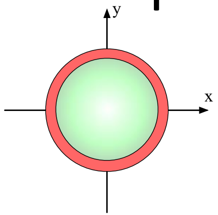
Minimize ground state energy $E(Q)$ to find optimal Q^*

Chakraborty and ABS, PRB 110, L060508 (2024)

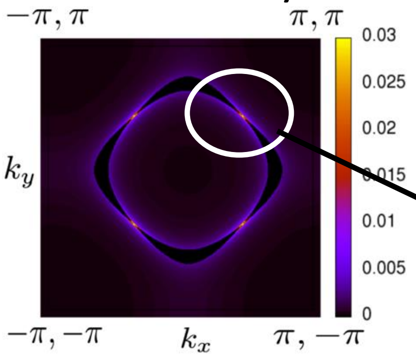
Conventional *s*-wave pairing

$$H = \sum_{k,\sigma}(\xi_k - \sigma\frac{t_{\rm am}}{2}(\cos(k_x) - \cos(k_y)) \quad)c^\dagger_{k\sigma}c_{k\sigma} + \sum_{k,k',q} V_{k,k'}c^\dagger_{k+q\uparrow}c^\dagger_{-k+q\downarrow}c_{-k'+q\downarrow}c_{k'+q\uparrow}$$

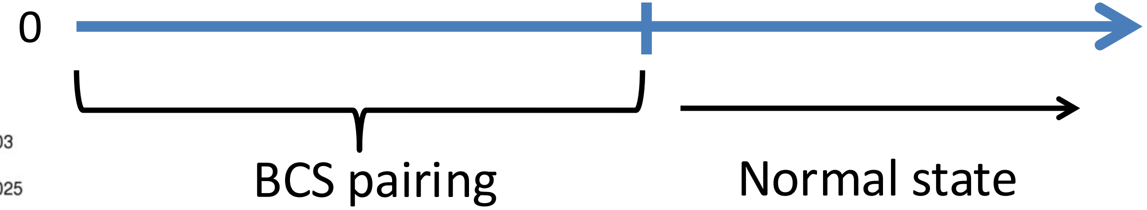
s-wave pairing (V constant, on-site attraction)



$\Delta_{\boldsymbol{k}} = \Delta_0$



Increased spin-splitting (t_{am})



Sufficient condensation of BCS pairs at AM nodes

(*s*-wave not likely in AMs)

Chakraborty and ABS, PRB 110, L060508 (2024); PRB 112, 014516 (2025)

Nodal *d*-wave pairing

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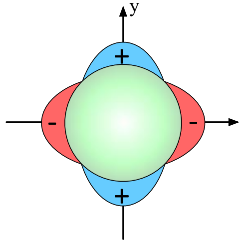
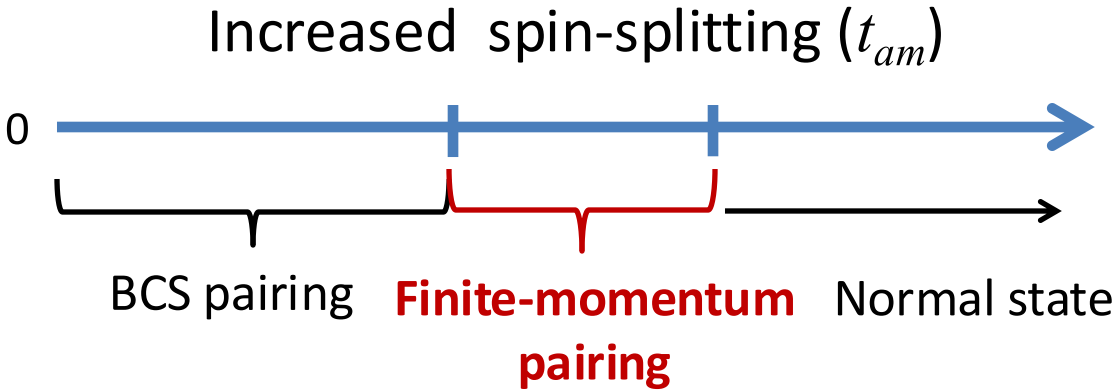
$$H = \sum_{k,\sigma} \left(\xi_k - \sigma \frac{t_{\mathrm{am}}}{2}(\cos(k_x) - \cos(k_y))\right) c^{\dagger}_{k\sigma} c_{k\sigma} + \sum_{k,k',q} V_{k,k'} c^{\dagger}_{k+q\uparrow} c^{\dagger}_{-k+q\downarrow} c_{-k'+q\downarrow} c_{k'+q\uparrow}$$

d-wave pairing (NN attraction)



$$\Delta_{\boldsymbol{k}} = \Delta_0 \left(\cos \boldsymbol{k}_x - \cos \boldsymbol{k}_y\right)$$



Chakraborty and ABS, PRB 110, L060508 (2024)

Phase diagram – Yoda ear

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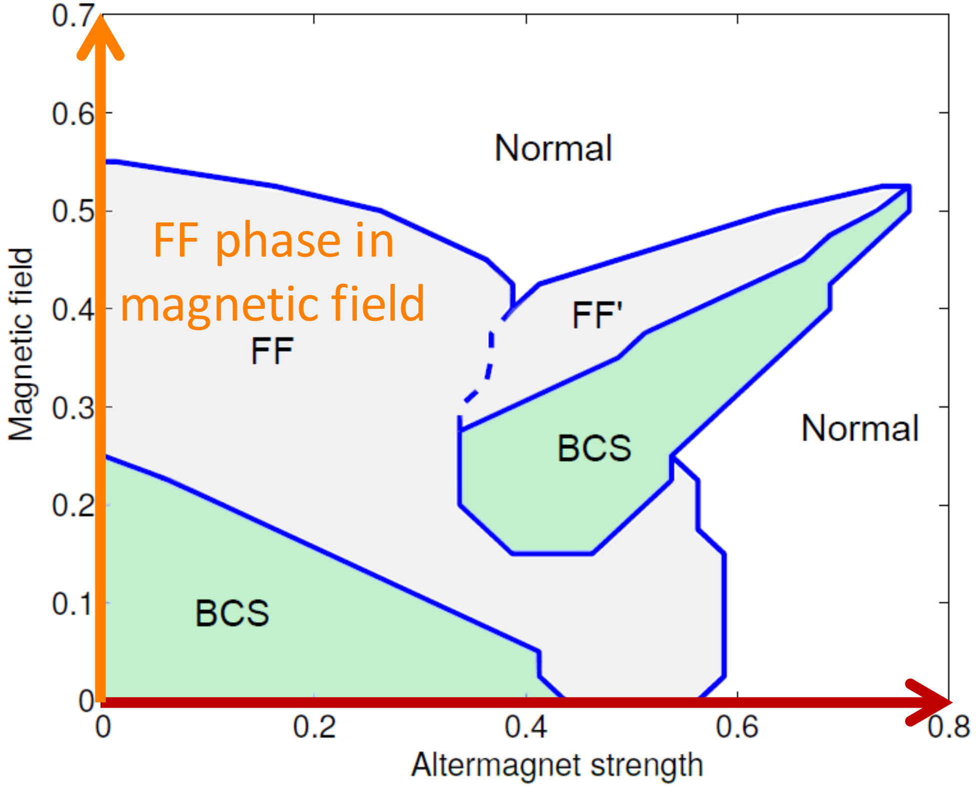
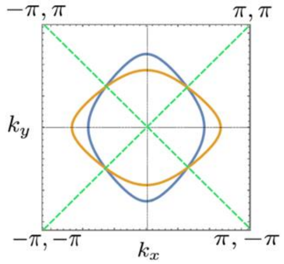
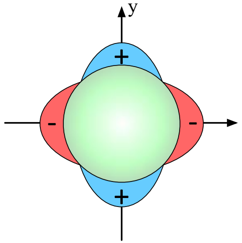


Yoda

FS AM nodes



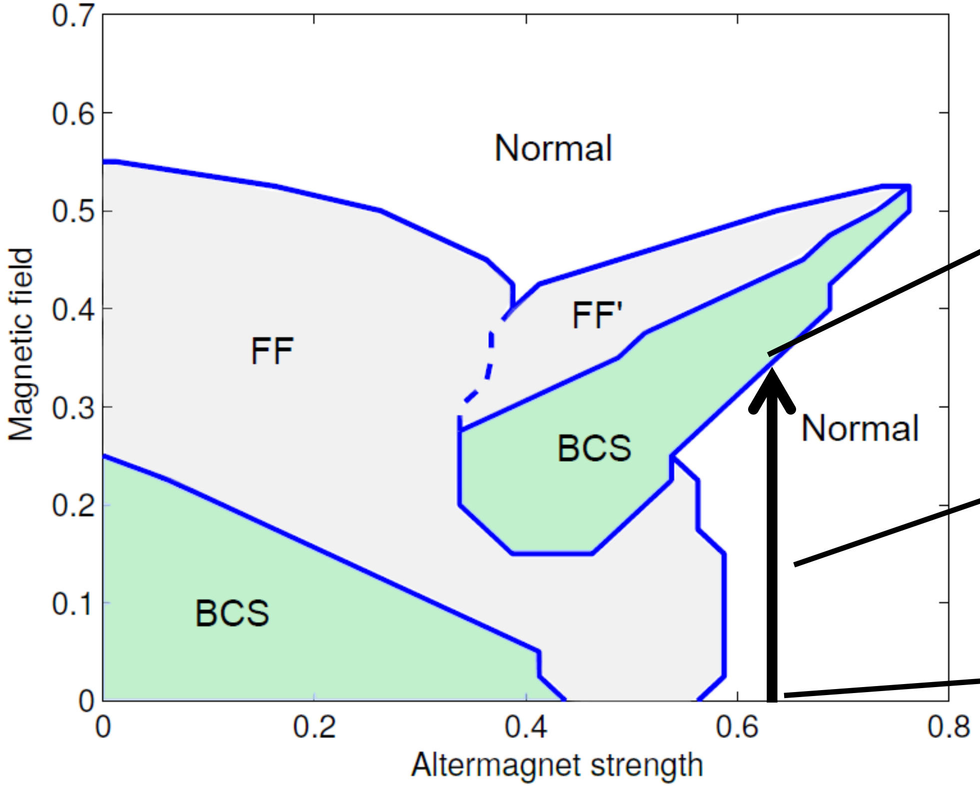
SC nodes

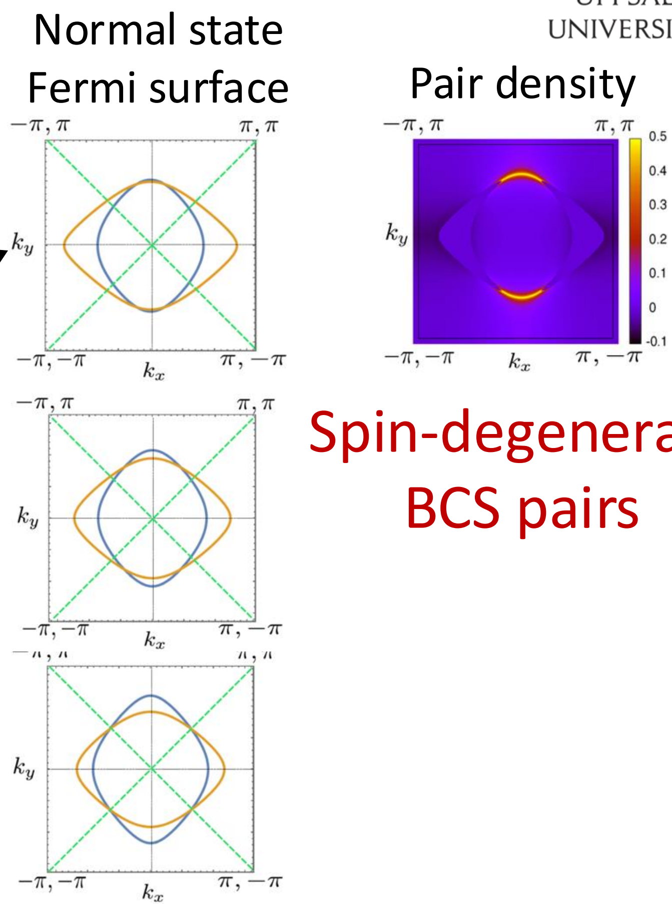


FF phase extends SC region for aligned AM and SC nodes

Chakraborty and ABS, PRB 110, L060508 (2024)

Field-induced superconductivity





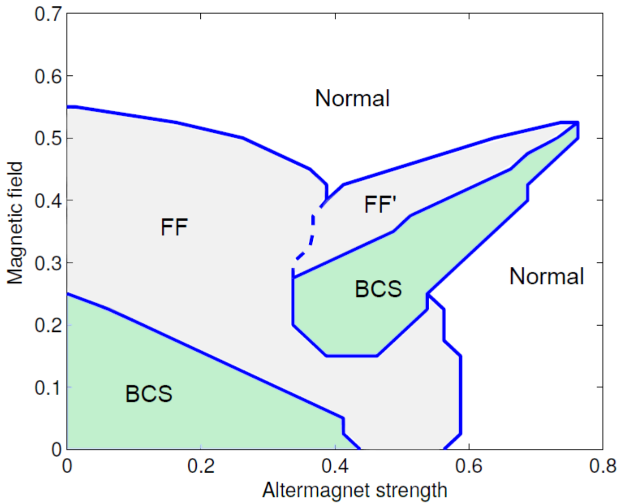
Spin-degenerate BCS pairs

Chakraborty and ABS, PRB 110, L060508 (2024)

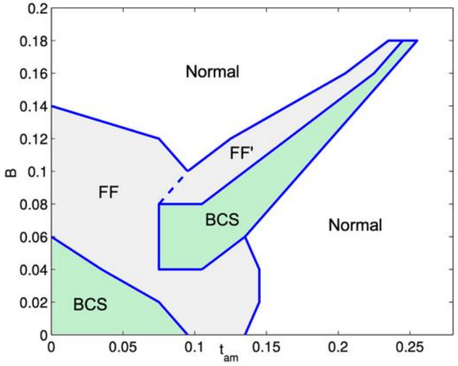
Summary

Superconductivity in altermagnets

- Finite-momentum (FF) pairing, when SC and AM nodes align
- Field-induced superconductivity at high fields



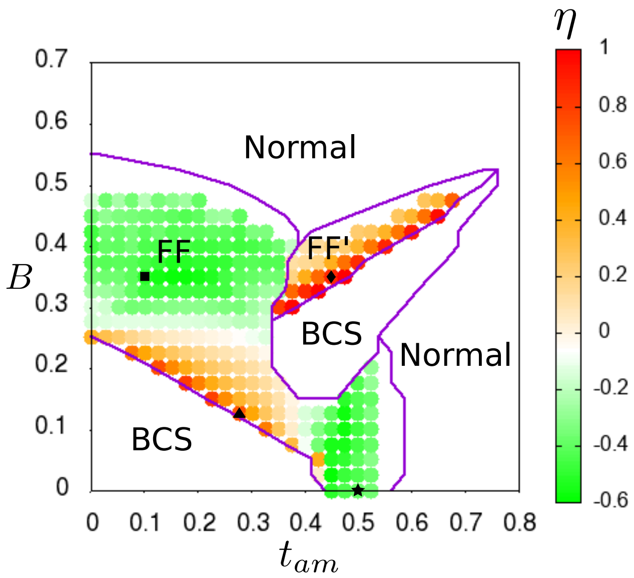
Weaker SC



t = 100 meV →
B = 0.01 ~ 16 T

Chakraborty and Black-Schaffer, PRB 110, L060508 (2024)

Perfect superconducting diode effect



Superconducting diode effect (SDE)

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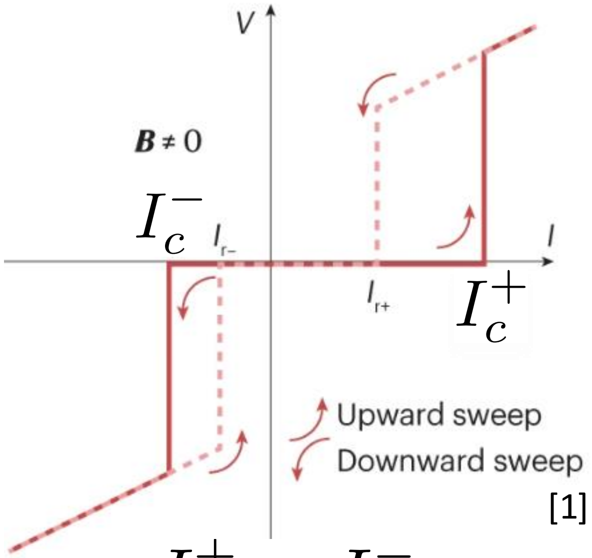
Dissipationless current in only one direction



$$I_c^+ > I_c^-$$

- Requires broken time-reversal and inversion symmetries
- FF states natural candidates [2]
- Efficiency $\eta = \frac{I_c^+ - I_c^-}{I_c^+ + I_c^-}$ ($\lesssim$ 60% [3])

[1]: Nadeem et al, Nat. Rev. Phys. 5, 558 (2023);
[2] Yuan and Fu, PNAS 119, e2119548119 (2022); [3]: Ghosh et al, Nat. Mater. 23, 612 (2024)

SDE in superconducting altermagnet

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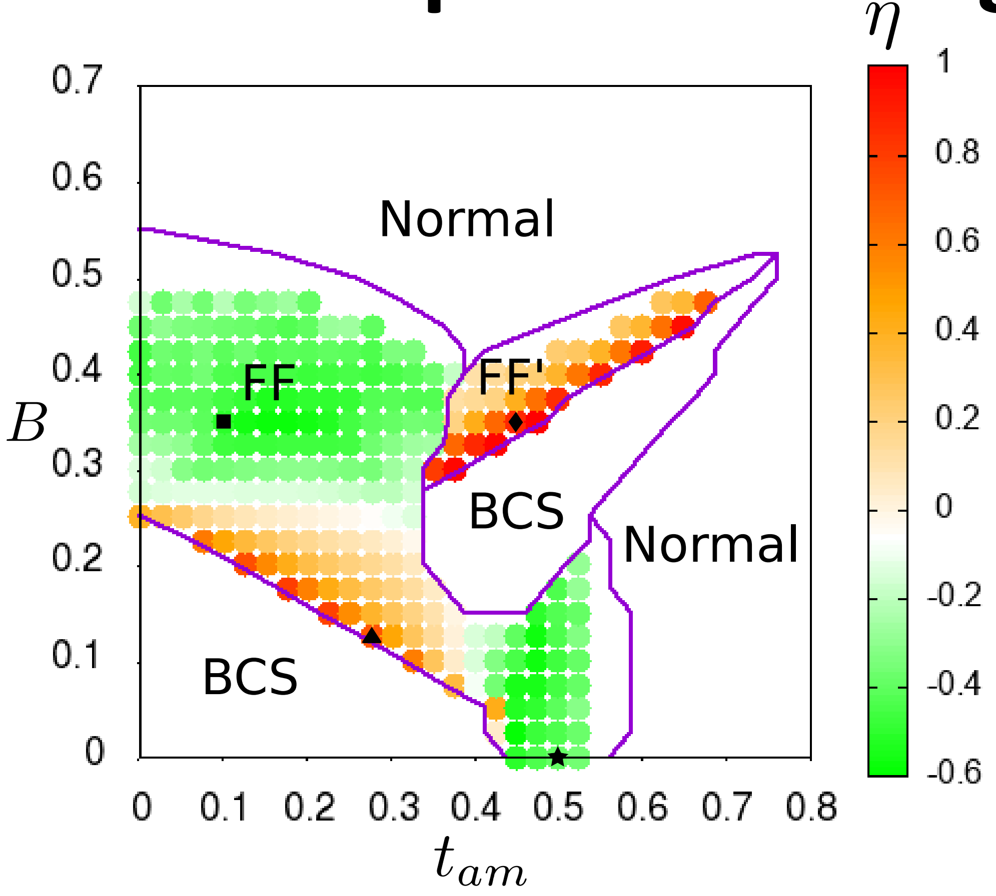

Spin-singlet *d*-wave pairing

Self-consistent solution for each Q, ground state at $Q*$

Extract current at each Q:

- $I(Q*) = 0$
- Max currents gives η

- Large η in FF regions
- **η = 100% at FF-BCS transitions**

Chakraborty and ABS, PRL 135, 026001 (2025)

SDE deep in FF phase

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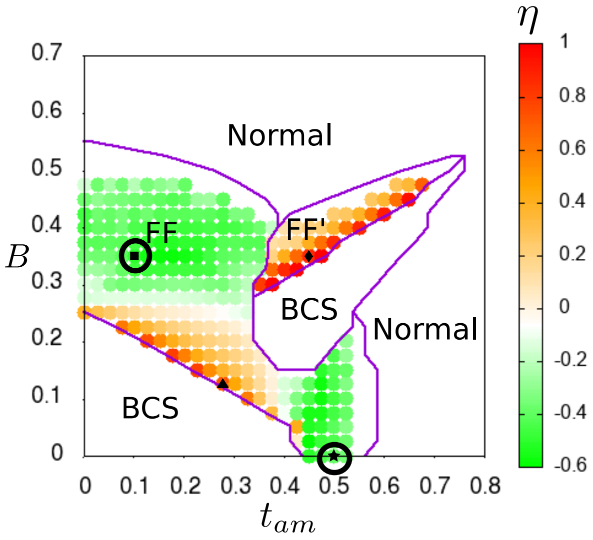
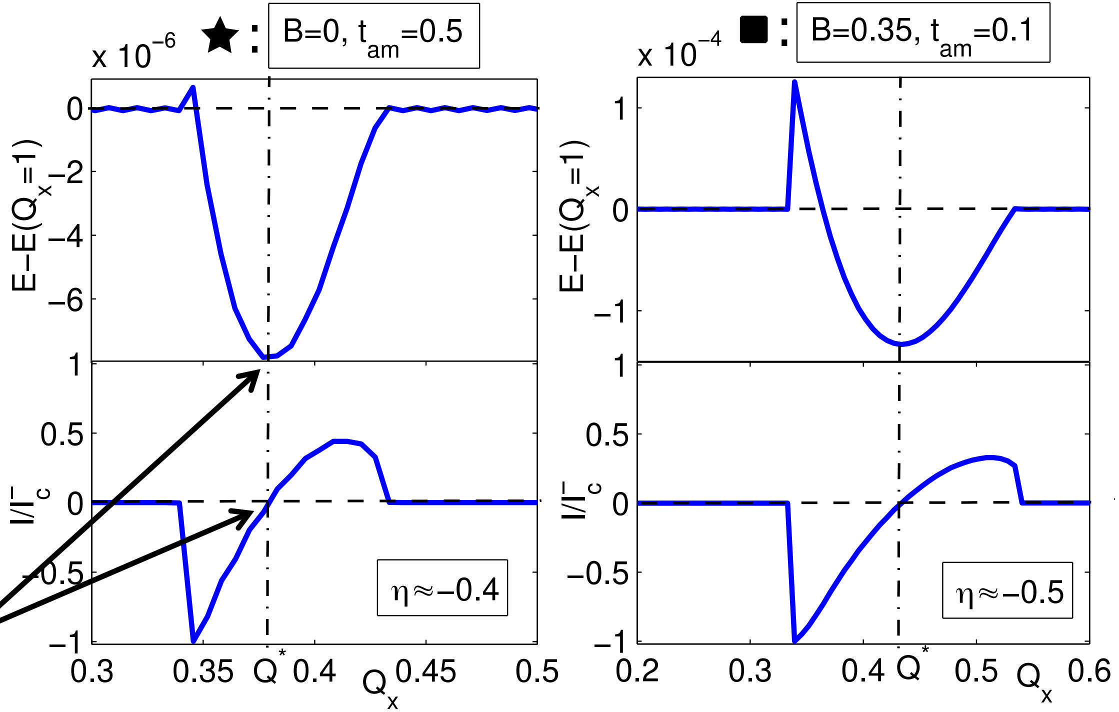


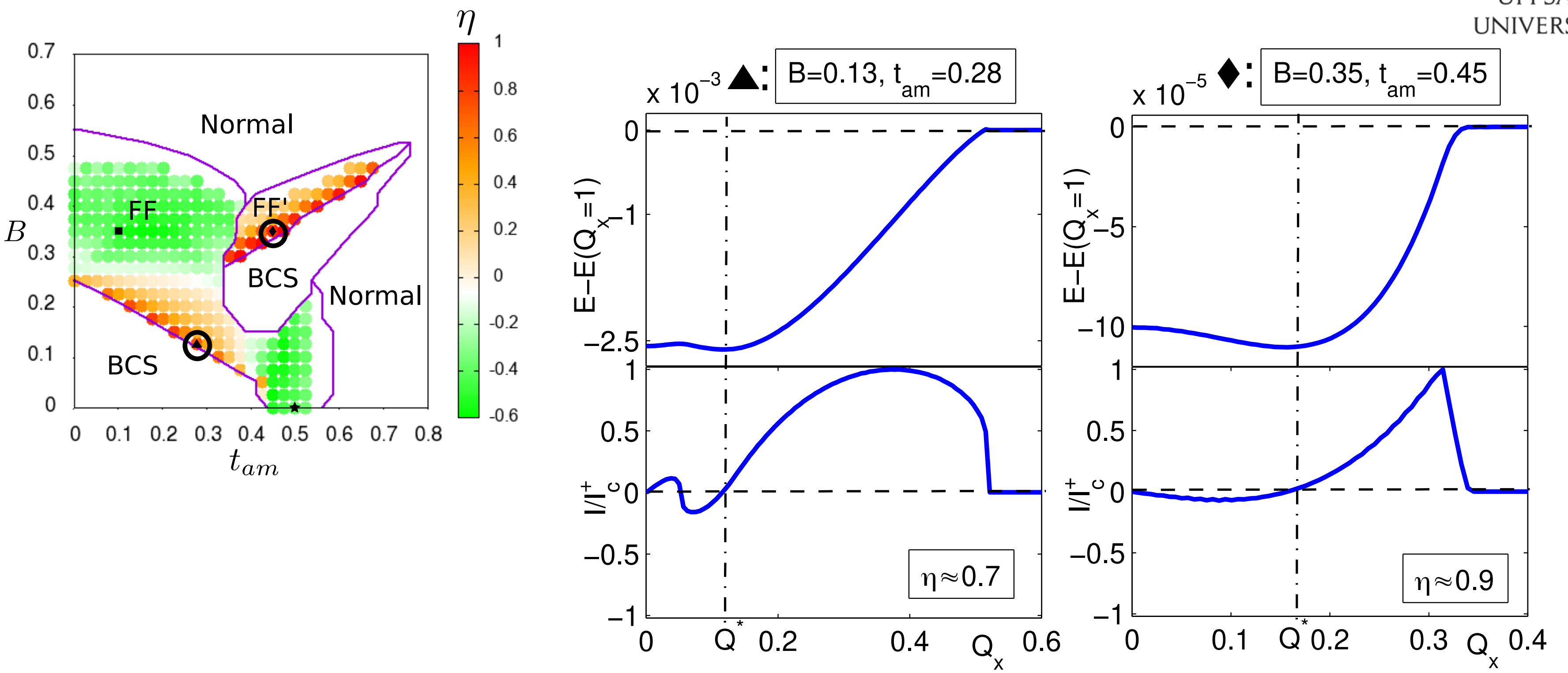


- No current in ground state Q^*
- $I = -e\frac{\partial E}{\partial Q}$

Deep in FF phase: $\eta \sim$ -50%

Chakraborty and ABS, PRL 135, 026001 (2025)

SDE at FF-BCS transitions



BCS and FF minima: large η

BCS maximum, FF minimum: η → 100%

Chakraborty and ABS, PRL 135, 026001 (2025)

Topological protection of perfect SDE

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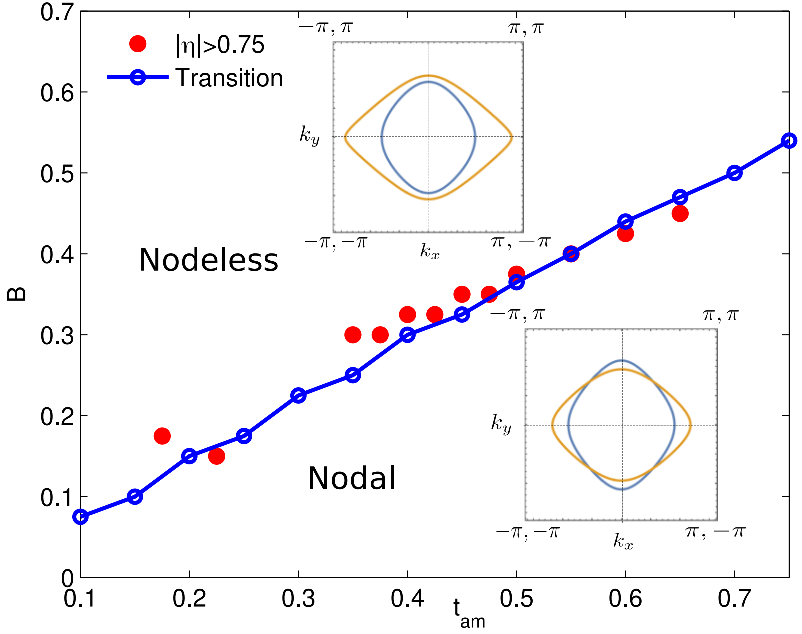

Topologically protected nodal-to-nodeless transition [1]

→ large-η regions

[1]: Fernandes et al, PRB 109, 024404 (2024)

Chakraborty and ABS, PRL 135, 026001 (2025)

Summary

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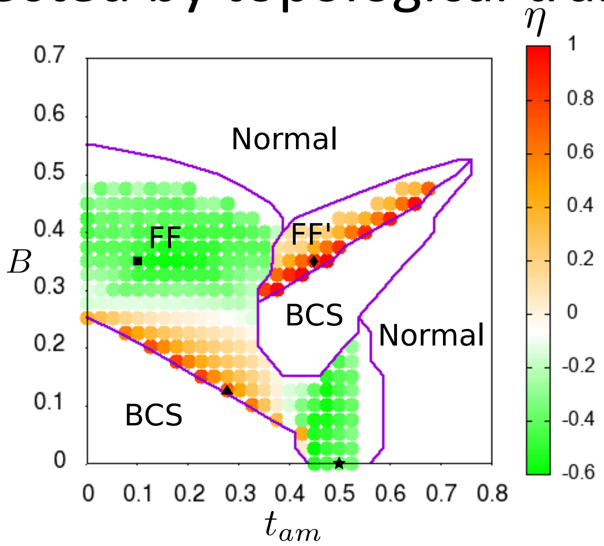
Superconducting diode effect (SDE) in altermagnets

- Large throughout FF regions
- Perfect SDE, protected by topological transition

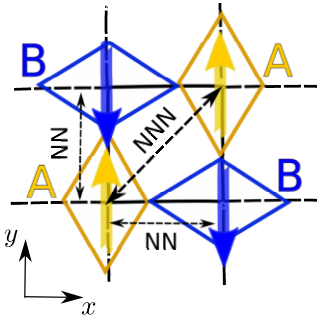


Chakraborty and Black-Schaffer, PRL 135, 026001 (2025)

Constraints on superconducting pairing

Are there constraints on superconducting symmetries in altermagnets?

Altermagnet normal state

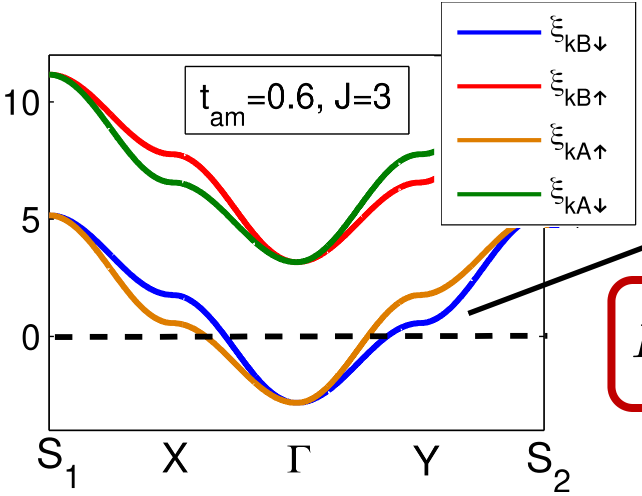


$$H_{4b} = \sum_k \Psi^\dagger \hat{H}_{4b} \Psi, \quad \Psi^\dagger = \left(c^\dagger_{kA\uparrow}, c^\dagger_{kB\uparrow}, c^\dagger_{kA\downarrow}, c^\dagger_{kB\downarrow}\right)$$

Spin → σ_0 ; Sublattice → τ_0

$$\hat{H}_{4b} = \left(-2t(\cos k_x + \cos k_y) - \mu\right)\sigma_0\tau_0 - \frac{t_{\mathrm{am}}}{2}(\cos k_x - \cos k_y)\sigma_0\tau_z - J\sigma_z\tau_z$$

Spin-degenerate dispersion — Crystal field — Exchange field



$$H_{2b} = \sum_k \Psi^\dagger_{2b} \hat{H}_{2b} \Psi_{2b}, \quad \Psi^\dagger_{2b} = \left(c^\dagger_{kA\uparrow}, c^\dagger_{kB\downarrow}\right)$$

$$\hat{H}_{2b} = \left(-2t(\cos k_x + \cos k_y) - \mu\right) s_0 - \frac{t_{\mathrm{am}}}{2}(\cos k_x - \cos k_y) s_z$$

Spin & sublattice → s_0

Chakraborty and ABS, PRB 112, 014516 (2025)

Superconducting pairing

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Spin symmetry (S)	Momentum symmetry (P)	Form factor	Real-space structure	Occurrence
Spin-singlet ($S = -1$)	s-wave ($P = 1$)	1	Onsite	✕

$$\Psi_{2b}^{\dagger} = \left(c_{kA\uparrow}^{\dagger}, c_{kB\downarrow}^{\dagger}\right)$$

No onsite spin-singlet pairs

→

No conventional *s*-wave pairing

Chakraborty and ABS, PRB 112, 014516 (2025)

Superconducting pairing

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Spin symmetry (S)	Momentum symmetry (P)	Form factor	Real-space structure	Occurrence
Spin-singlet ($S=-1$)	s-wave ($P=1$)	1	Onsite	✗
	Extended s-wave ($P=1$)	$\cos k_x + \cos k_y$	NN	✓
	$d_{x^2-y^2}$-wave ($P=1$)	$\cos k_x - \cos k_y$	NN	✓
	d_{xy}-wave ($P=1$)	$\sin k_x \sin k_y$	NNN	✗

$$\Psi^\dagger_{2b} = \left(c^\dagger_{kA\uparrow}, c^\dagger_{kB\downarrow}\right)$$

Needed for finite-momentum pairing & superconducting diode effect

Chakraborty and ABS, PRB 112, 014516 (2025)

Superconducting pairing

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Spin symmetry (S)	Momentum symmetry (P)	Form factor	Real-space structure	Occurrence
Spin-singlet ($S=-1$)	s-wave ($P=1$)	1	Onsite	✗
	Extended s-wave ($P=1$)	$\cos k_x + \cos k_y$	NN	✓
	$d_{x^2-y^2}$-wave ($P=1$)	$\cos k_x - \cos k_y$	NN	✓
	d_{xy}-wave ($P=1$)	$\sin k_x \sin k_y$	NNN	✗
Equal-spin-triplet ($S=1$)	p-wave ($P=-1$)	$\sin k_x$, $\sin k_y$	NN	✗
Mixed-spin-triplet ($S=1$)	p-wave ($P=-1$)	$\sin k_x$, $\sin k_y$	NN	✓

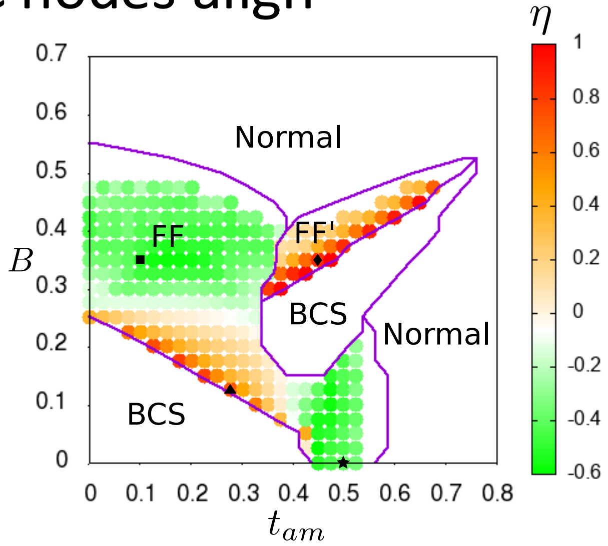
Superconductivity in altermagnets:

- **Spin-singlet *d*-wave** (or extended *s*-wave)
- **Mixed-spin-triplet *p*-wave**

Chakraborty and ABS, PRB 112, 014516 (2025)

Superconductivity in altermagnets

- Finite-momentum SC, when AM and SC nodes align
- Field-induced SC at high fields
- Large SDE throughout FF regions
- Perfect SDE protected by topology at BCS-FF transition



- No conventional *s*-wave or 'simplest' equal-spin *p*-wave SC

Chakraborty and Black-Schaffer, PRB 110, L060508 (2024); PRL 135, 026001 (2025); PRB 112, 014516 (2025)

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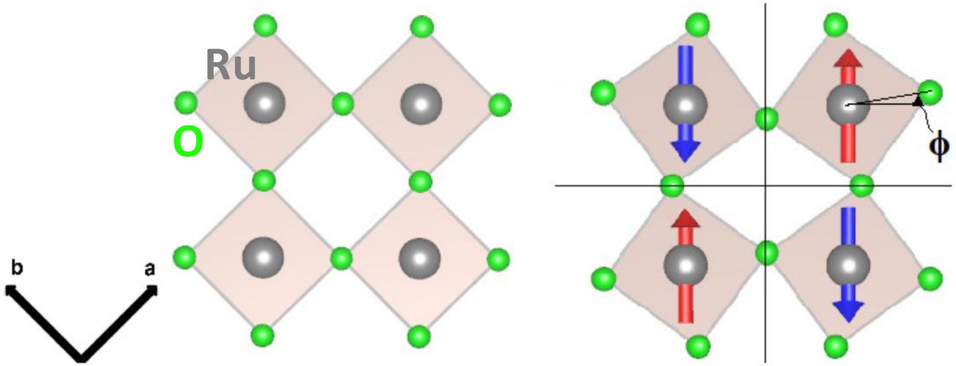
Altermagnetism in Sr_2RuO_4

Sr_2RuO_4: an intrinsic superconductor and altermagnet?

Altermagnetism in Sr_2RuO_4?



Distortion of oxygen octahedron:

- Soft phonon mode
- Surface distorted (ϕ >7°) [1]

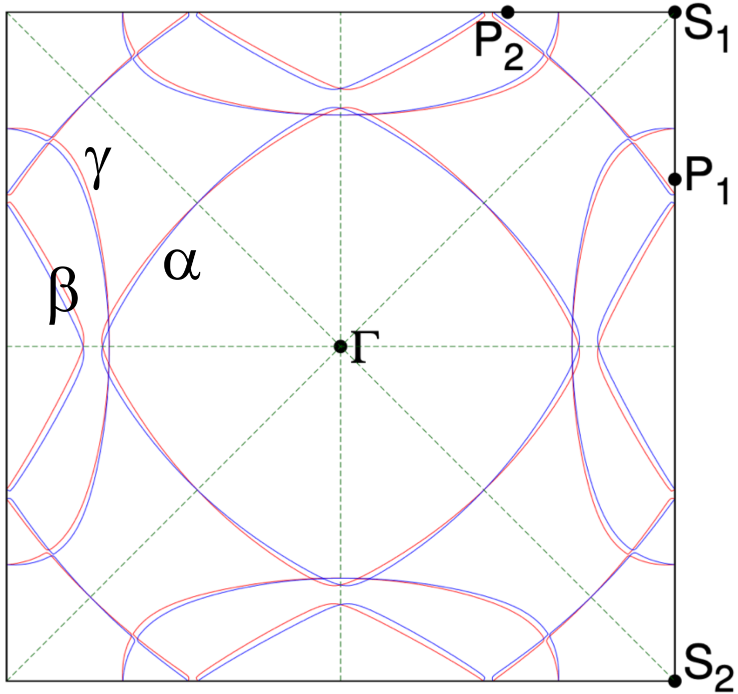
DFT+U calculations & tight-binding modeling assuming finite octahedral distortion angle ϕ

Single RuO_2 layer and bulk Sr_2RuO_4

[1]: Damascelli et al, PRL 85, 5194 (2004)
Autieri, Cuono, Chakraborty, Gentile, ABS, PRB 112, 014412 (2025)

g-wave AM in single layer Sr_2RuO_4

Spin-split Fermi surface
(U = 1.0 eV and ϕ = 8°)



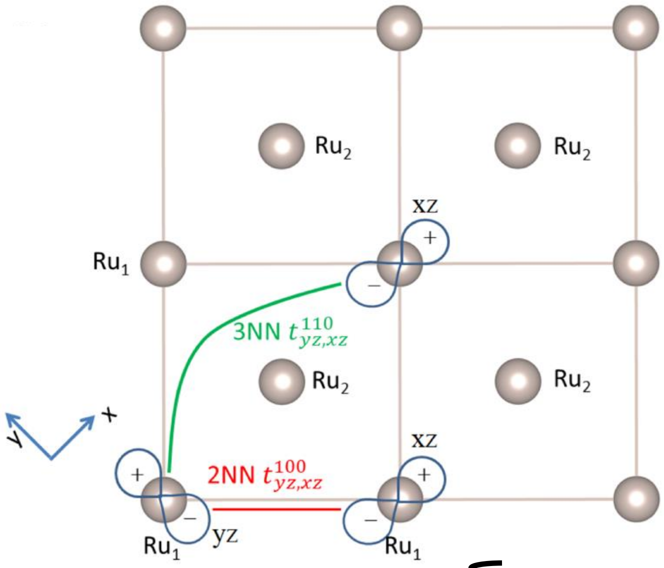
AM with *g*-wave symmetry

Spin-splitting primarily on α,β bands = Ru d(xz),d(yz) orbitals

γz orbitals

Autieri, Cuono, Chakraborty, Gentile, ABS, PRB 112, 014412 (2025)

Tight-binding modeling



$$H_{\gamma z} = H^0_{\gamma z} + H^{\mathrm{AM}}_{\gamma z}$$

$$H^{\mathrm{AM}}_{\gamma z} = \begin{pmatrix} -\frac{\Delta_{\gamma z}}{2} & 4t^{110}_{xz,yz}\sin k_x \sin k_y \\ 4t^{110}_{xz,yz}\sin k_x \sin k_y & -\frac{\Delta_{\gamma z}}{2} \end{pmatrix}$$

Interorbital NNN hybridization for finite ϕ

$\sim t_{\mathrm{am}}$

On-site spin-splitting

$\sim J$

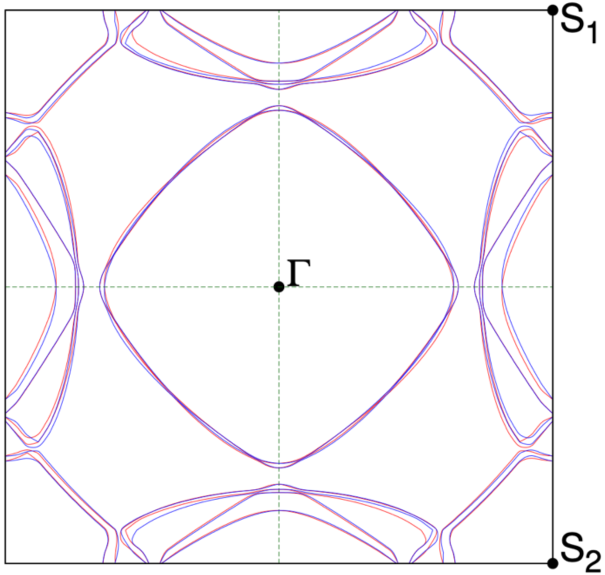
$$\rightarrow \begin{cases} \mathcal{H}^0_{\gamma z} = H^0_{\gamma z}\sigma^{spin}_0 \sigma^{site}_0 \\ \mathcal{H}^{\mathrm{AM}}_{\gamma z} = -\frac{\Delta_{\gamma z}}{2}\sigma^{spin}_z \sigma^{orbital}_0 \sigma^{site}_z \\ \qquad + 4t^{110}_{xz,yz}\sin k_x \sin k_y \sigma^{spin}_0 \sigma^{orbital}_x \sigma^{site}_z \end{cases}$$

Orbital-selective *g*-wave altermagnet

Autieri, Cuono, Chakraborty, Gentile, ABS, PRB 112, 014412 (2025)

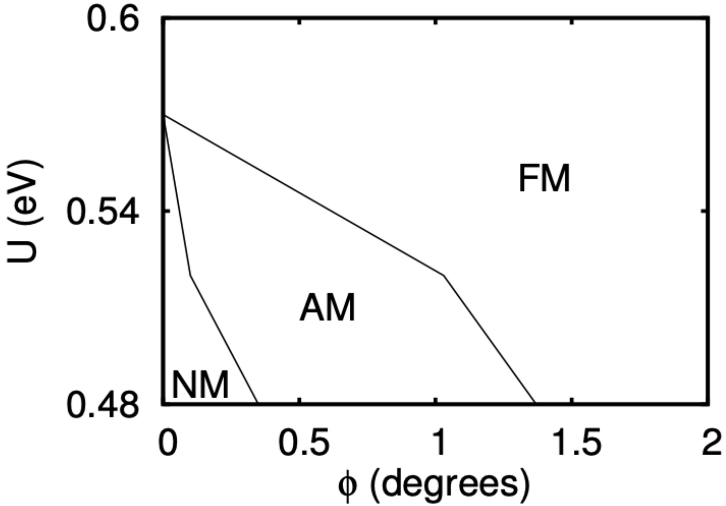
d(xy)-wave AM in bulk Sr_2RuO_4

Spin-split Fermi surface
(U = 1.0 eV and ϕ = 8°)



***d*(xy)-wave AM** due to interlayer hybridization

Phase diagram



Including SOC: Weak ferromagnetism from canted moments out of ab-plane (through DM interaction)

Autieri, Cuono, Chakraborty, Gentile, ABS, PRB 112, 014412 (2025)

SC and AM in Sr_2RuO_4

AM spin-splitting ~1 meV at ϕ = 1°, cp to SC gap ~350 μeV

SC: Likely spin-singlet

AM: Opposite spins on NN, NNNN, … sites

If spin-singlet, intraorbital, γz pairing:

→ *d*(x^2-y^2)-wave or *g*-wave SC or combinations thereof

→ *d*+i*g*: simplest possibility agreeing with exp [1]

(can also be interorbital and/or *xy*-orbital pairing)

[1]: See e.g. Maeno, Ikeda, Mattoni, Nat. Phys. 20, 1712 (2024)
Autieri, Cuono, Chakraborty, Gentile, ABS, PRB 112, 014412 (2025)