CORRELATED ELECTRONIC PHENOMENA IN ALTERMAGNETS

Rafael M. Fernandes

University of Illinois Urbana-Champaign

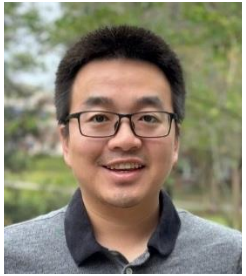
SPICE Workshop

Theory of Unconventional Magnetism: exploring altermagnets and beyond

Oct 21st, 2025

Collaborators

Yiming Wu
(Stanford)



Yuxuan Wang
(Florida)

Wu, Wang & RMF
Phys. Rev. Lett. ***135***, *156001 (2025)*

A. Chakraborty
(Illinois)

F. Yang
(Minnesota)

T. Birol
(Minnesota)

Chakraborty, Yang, Birol & RMF
arXiv:2509.26596 (2005)

Outline

1. Superconductivity mediated by altermagnetic fluctuations.
2. Interaction-driven orbital altermagnetism on the kagome lattice.

Outline

1. Superconductivity mediated by altermagnetic fluctuations.

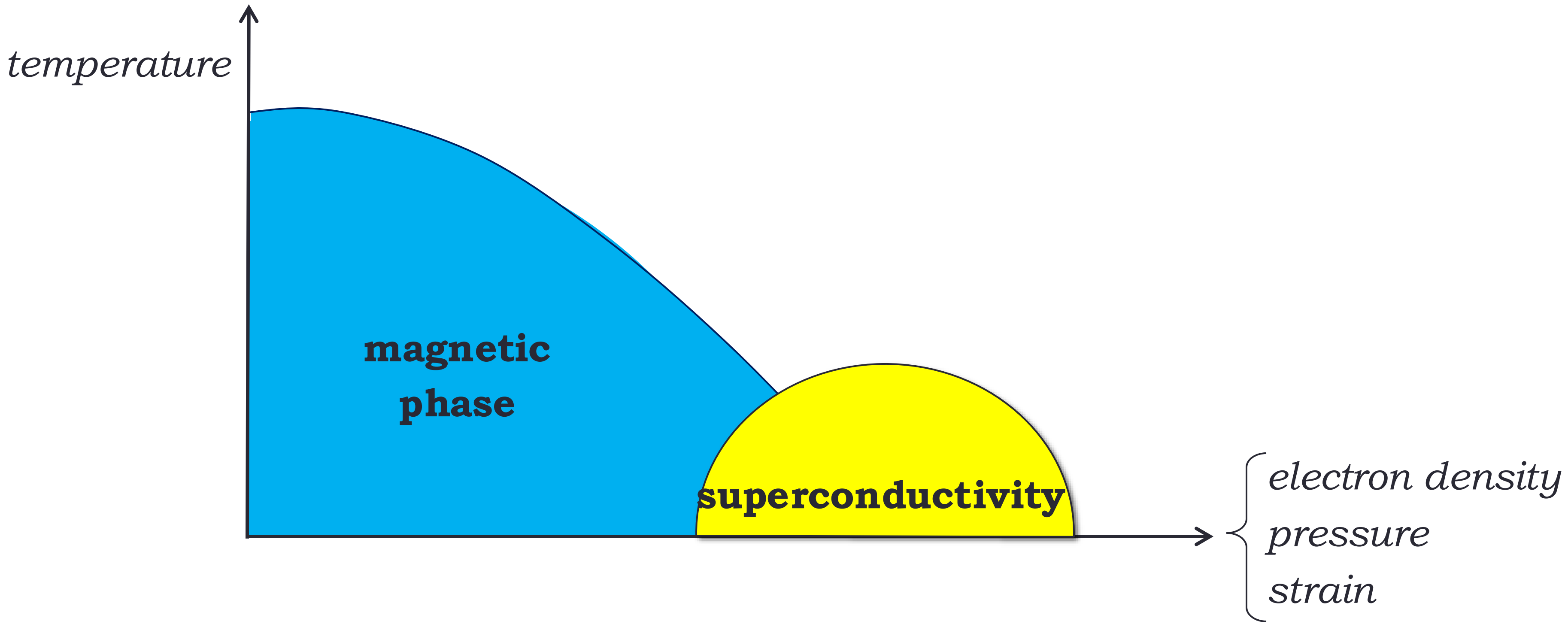
2. Interaction-driven orbital altermagnetism on the kagome lattice.

Wu, Wang & RMF

Phys. Rev. Lett. ***135***, *156001 (2025)*

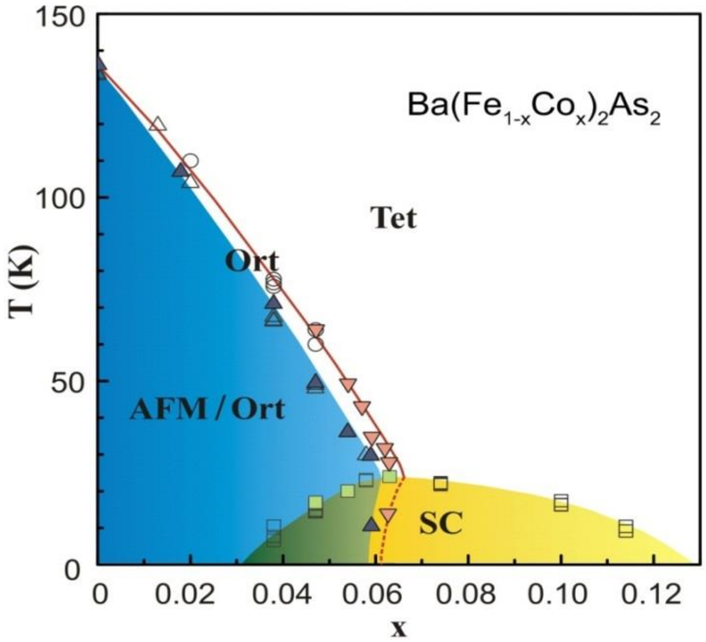
Superconductivity mediated by magnetic fluctuations

- A common theme: a magnetically ordered state is suppressed, and its fluctuations give rise to unconventional superconductivity.

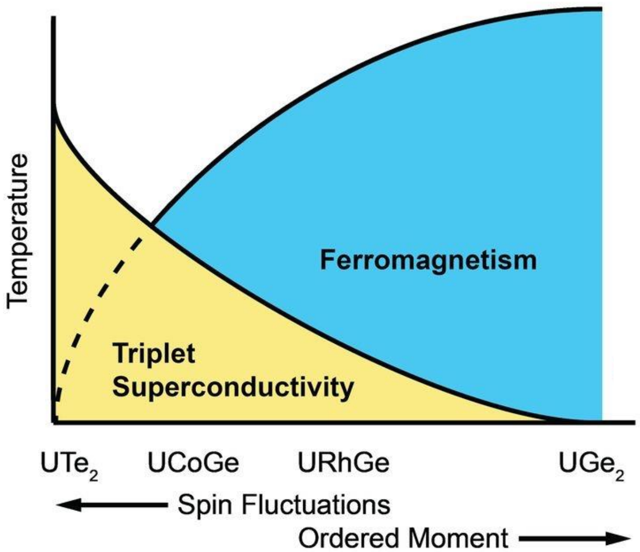


Superconductivity mediated by magnetic fluctuations

- A common theme: a magnetically ordered state is suppressed, and its fluctuations give rise to unconventional superconductivity.



Nandi et al, PRL (2010)

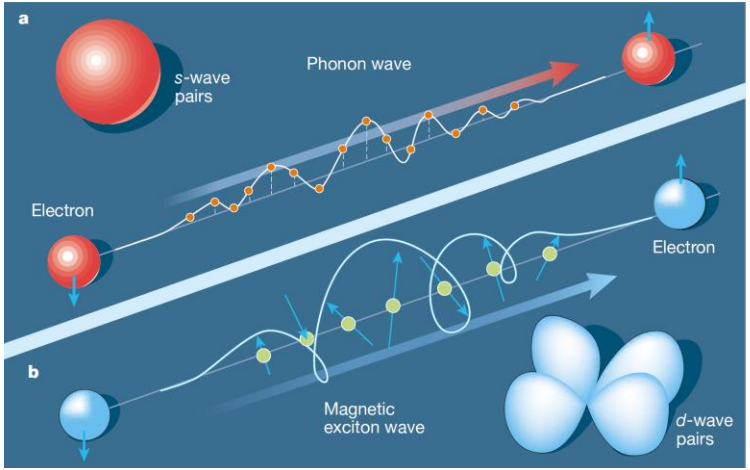


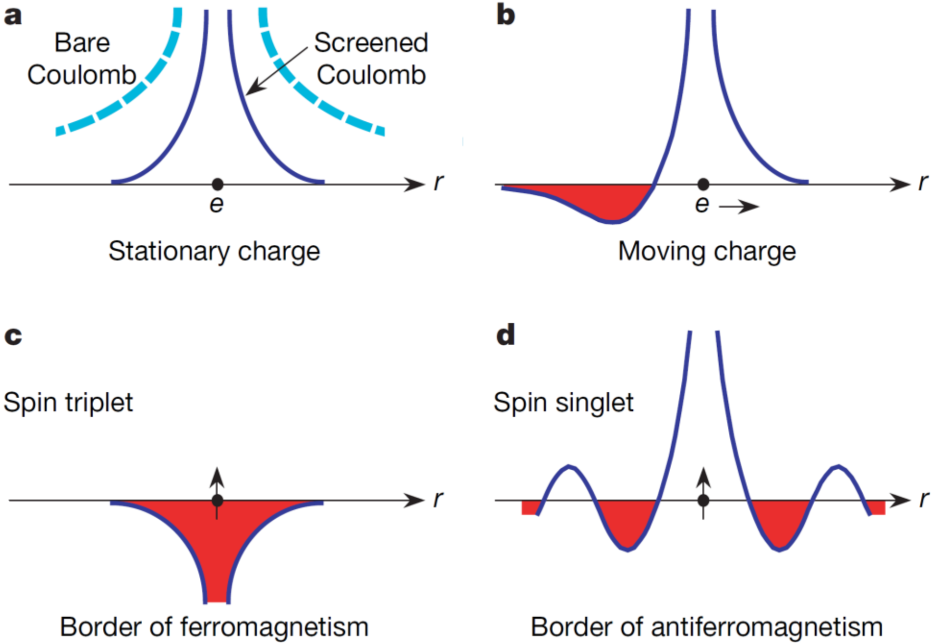
Ran et al, Science (2019)

Superconductivity mediated by magnetic fluctuations

- Microscopic models for unconventional superconductivity mediated by magnetic fluctuations.

figure from: Monthoux, Pines & Lonzarich, Nature (2007)

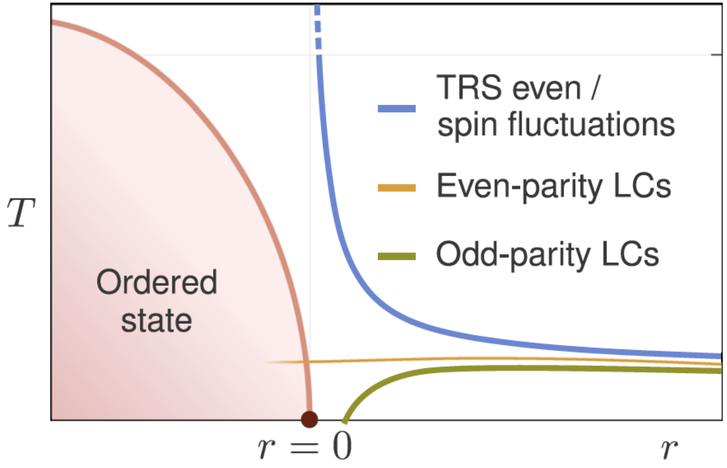




Scalapino, Pines, Varma, Woefle, Rice, Chubukov, Schmalian, Millis, Hirschfeld, Mazin, Sachdev, P Lee, Senthil, Kivelson, DH Lee, Berg, Raghu, RMF, and many others…

Superconductivity mediated by magnetic fluctuations

- **Antiferromagnetic** fluctuations (peaked at finite wave-vectors) generally favor **singlet** pairing.
- **Ferromagnetic** fluctuations generally favor **triplet** pairing.



Palle, Ojajärvi, RMF & Schmalian, Science Adv. (2024)

- **Altermagnetic** phases break time-reversal and rotational symmetries but preserve translations. **Triplet** pairing is expected from fluctuations.

Superconductivity mediated by magnetic fluctuations

- But altermagnets have a sublattice degree of freedom. Room for new physics?

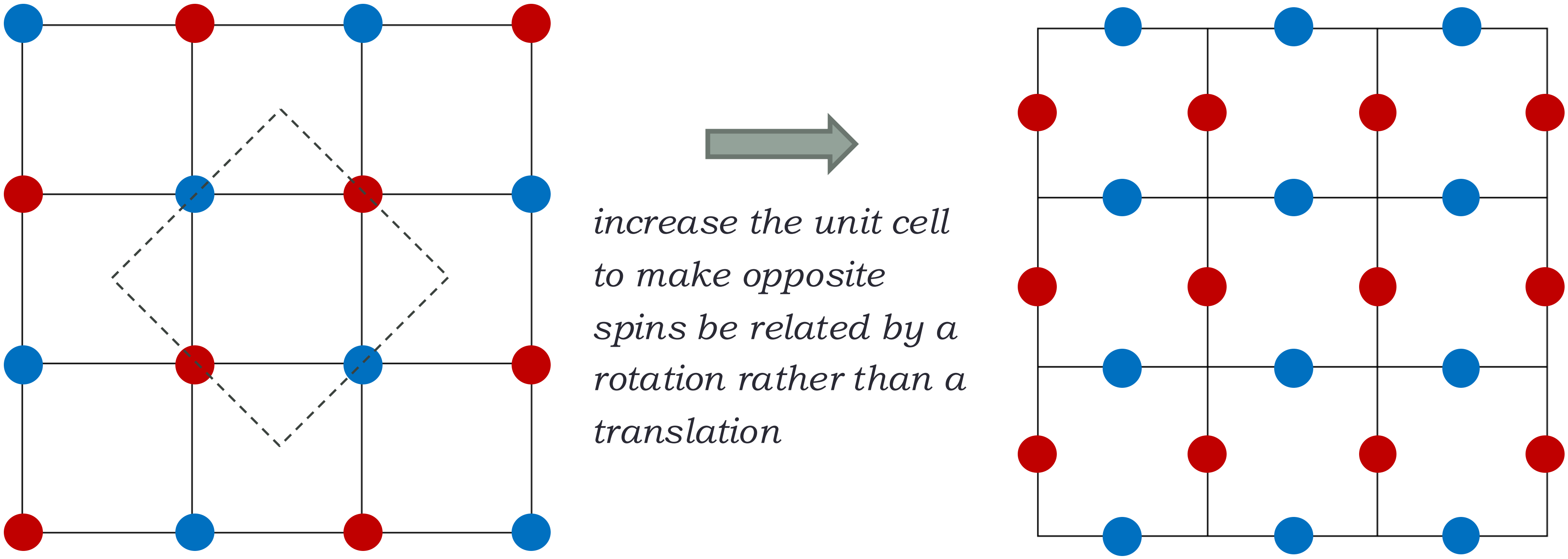
Notes on altermagnetism and superconductivity

Igor I. Mazin

Department of Physics & Astronomy, George Mason University, Fairfax, VA 22030, USA and Quantum Science and Engineering Center, George Mason University, Fairfax, VA 22030, USA.

From the formal point of view, the issue is that $\mathbf{q} \approx \mathbf{0}$ the spin susceptibility may have importand internal structure, and has to be written as $\chi(\mathbf{q}, \mathbf{r}_1, \mathbf{r}_2)$, where $\mathbf{r}$ is defined inside the first unit cell, or as a matrix in reciprocal vectors, $\chi(\mathbf{q}+\mathbf{G}, \mathbf{q}+\mathbf{G}')$. The corresponding vertex will be determined by the variation of the (non-magnetic) one-electron Green function with respect to a fluctuation generating opposite magnetic moments on the two sublattices. A more detailed theory than that usually used for ferro- or antiferromagnetic spin fluctuations needs to be developed, and it is not unreasonable to assume that both triplet and singlet pairing can be iduced by AM spin fluctuations, depending on the details.

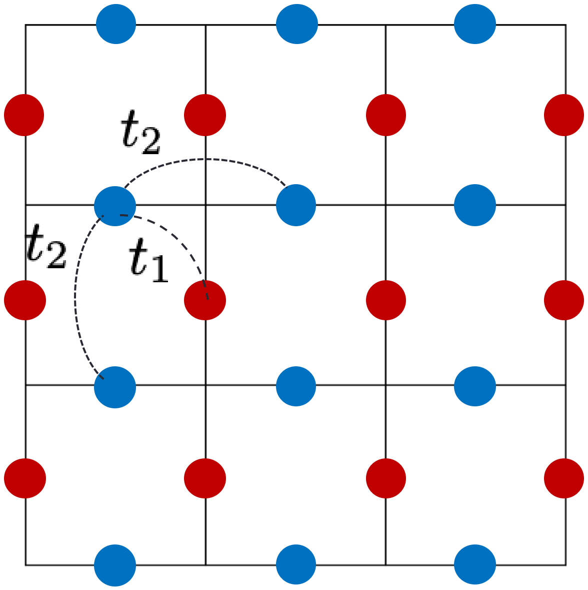
Toy model for altermagnetism: the Lieb lattice



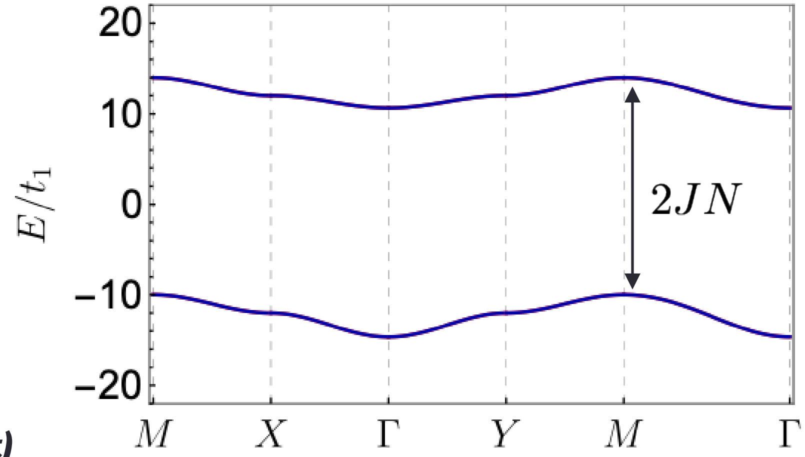
increase the unit cell to make opposite spins be related by a rotation rather than a translation

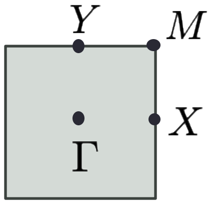
Toy model for altermagnetism: the Lieb lattice

- Sublattice: $\boldsymbol{\tau}$
- Spin: $\boldsymbol{\sigma}$
- Staggered magnetization: $\boldsymbol{N}$



$$\mathcal{H}_0(\boldsymbol{k}) = -4t_1 \cos\frac{k_x}{2}\cos\frac{k_y}{2}\tau_x - 2t_2(\cos k_x + \cos k_y)\tau_0 + J\tau_z \boldsymbol{N}\cdot\boldsymbol{\sigma}$$



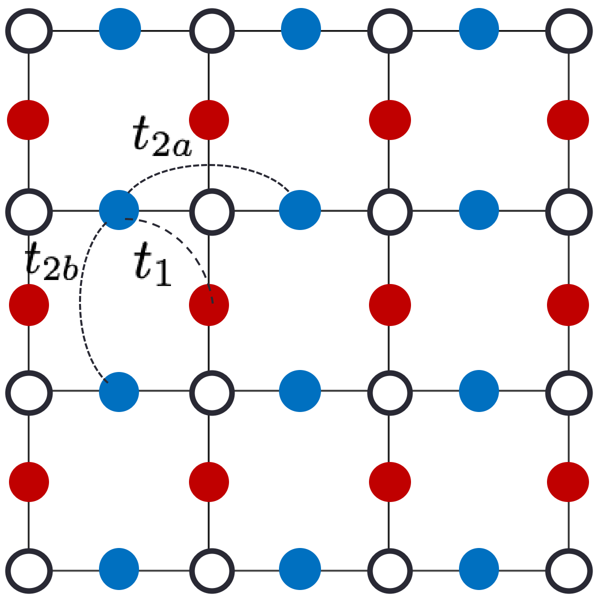


two spin-degenerate bands: no spin-splitting (antiferromagnet)

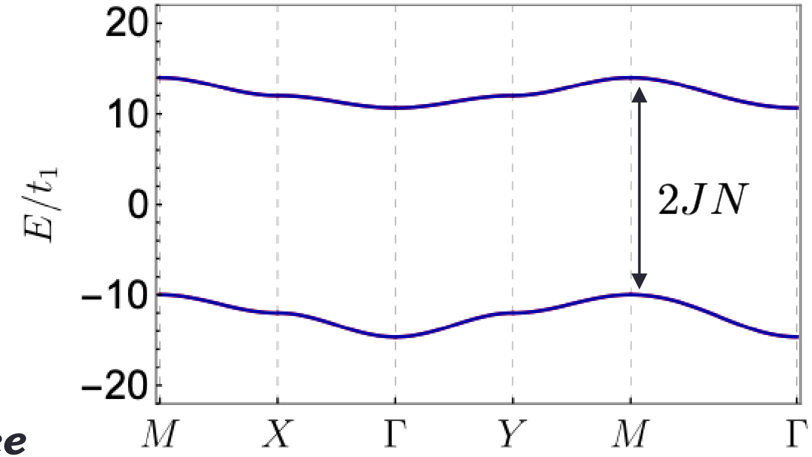
Toy model for altermagnetism: the Lieb lattice

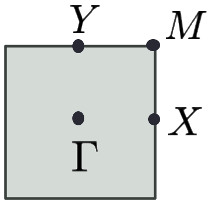
- Sublattice: $\boldsymbol{\tau}$
- Spin: $\boldsymbol{\sigma}$
- Staggered magnetization: $\boldsymbol{N}$

$$\mathcal{H}_0(\boldsymbol{k}) = -4t_1 \cos\frac{k_x}{2}\cos\frac{k_y}{2}\tau_x - 2t_2(\cos k_x + \cos k_y)\tau_0 + J\tau_z \boldsymbol{N}\cdot\boldsymbol{\sigma}$$



Include a non-magnetic atom to enforce the symmetries: Lieb lattice





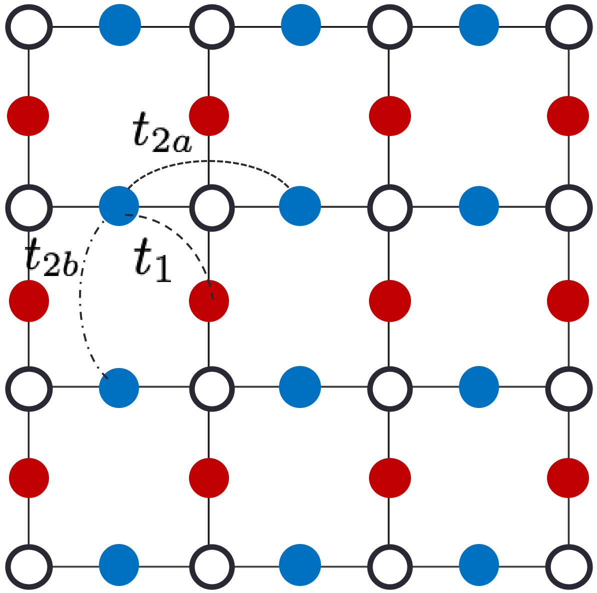
Toy model for altermagnetism: the Lieb lattice

Antonenko, RMF, Venderbos, PRL (2025)

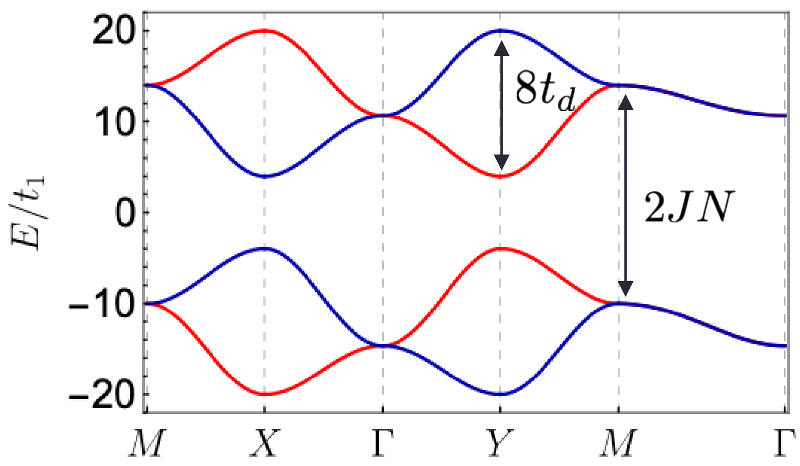
for a related model: Brekke et al, PRB (2023)

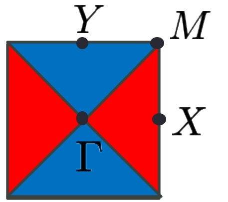
- Sublattice: $\boldsymbol{\tau}$
- Spin: $\boldsymbol{\sigma}$
- Staggered magnetization: $\boldsymbol{N}$

$$t_d \equiv t_{2a} - t_{2b}$$



$$\mathcal{H}_0(\boldsymbol{k}) = -4t_1 \cos\frac{k_x}{2}\cos\frac{k_y}{2}\tau_x - 2t_2(\cos k_x + \cos k_y)\tau_0 - 2t_d(\cos k_x - \cos k_y)\tau_z + J\tau_z \boldsymbol{N}\cdot\boldsymbol{\sigma}$$



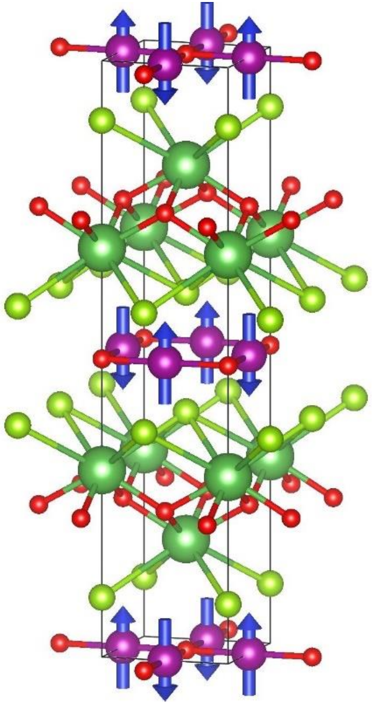


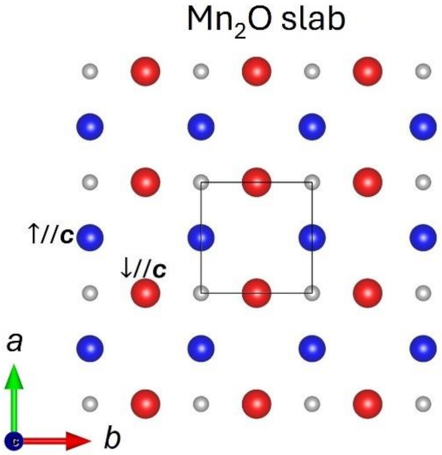
$d_{x^2-y^2}$ **-wave altermagnet**

magnitude of the spin-splitting is set by the hopping (lattice)

Toy model for altermagnetism: the Lieb lattice

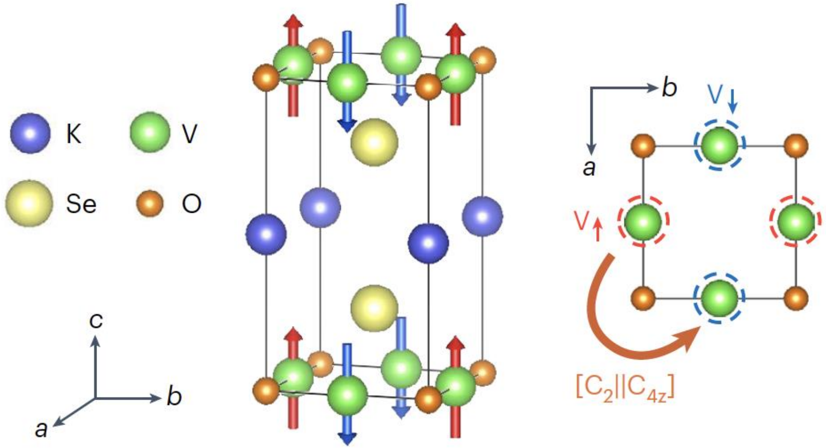
- Mott insulating $La_2O_3Mn_2Se_2$





Wei, …, RMF, …, Ji, PRM (2025)

- Metallic AV_2Se_2O

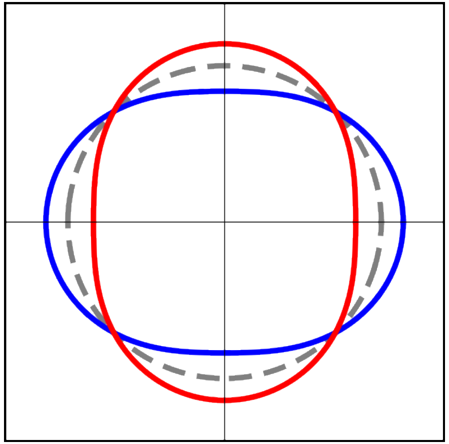


Jiang et al, Nature Phys (2025)

Zhang et al, Nature Phys (2025)

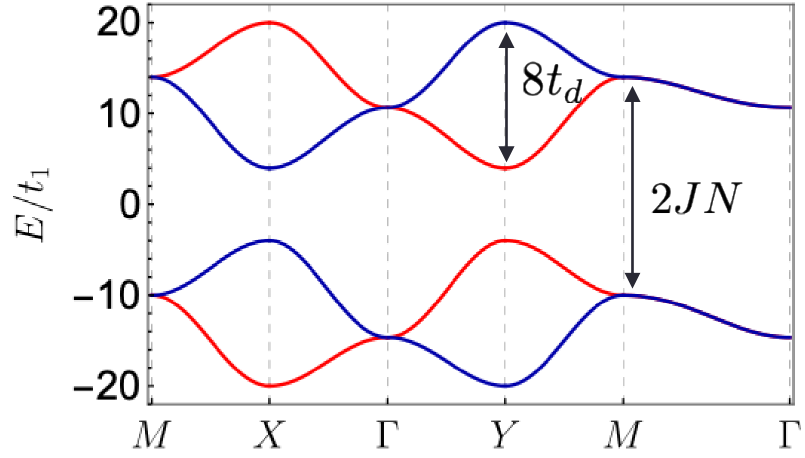
Same symmetry, different microscopic mechanisms

Pomeranchuk instability



$$E_\sigma(\mathbf{k}) = E^0(\mathbf{k}) + \phi(k_x^2 - k_y^2)\sigma$$

Altermagnetism

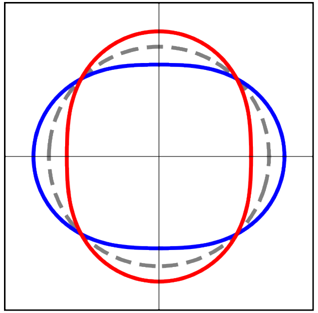


$$E_\sigma(\mathbf{k}) = E^0(\mathbf{k}) + \frac{JNt_d}{4t_1}(k_x^2 - k_y^2)\sigma$$

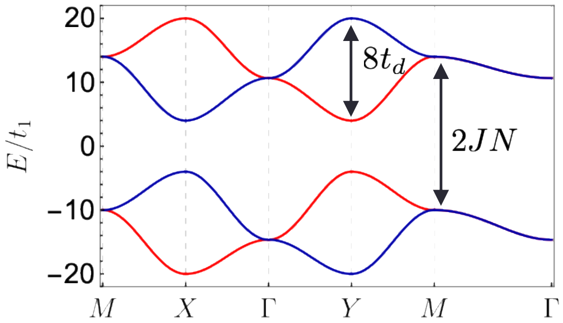
Superconductivity mediated by AM fluctuations

- In the long-wavelength low-energy regime, AM fluctuations should be identical to fluctuations associated with an *l = 2* spin-triplet Pomeranchuk transition, resulting in a **p-wave triplet pairing state**.

- But altermagnetism arises from the combination of two different energy scales. For energy scales larger than t_d, it resembles an antiferromagnetic phase, which favors **singlet**. Can AM fluctuations promote singlet pairing?



$$E_\sigma(\mathbf{k}) = E^0(\mathbf{k}) + \phi(k_x^2 - k_y^2)\sigma$$

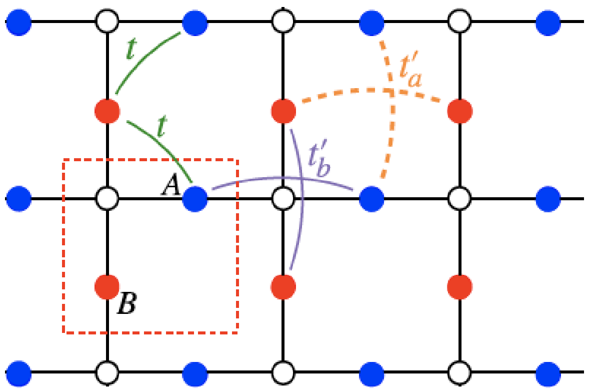


$$E_\sigma(\mathbf{k}) = E^0(\mathbf{k}) + \frac{JNt_d}{4t_1}(k_x^2 - k_y^2)\sigma$$

Lieb lattice model: AM-mediated pairing

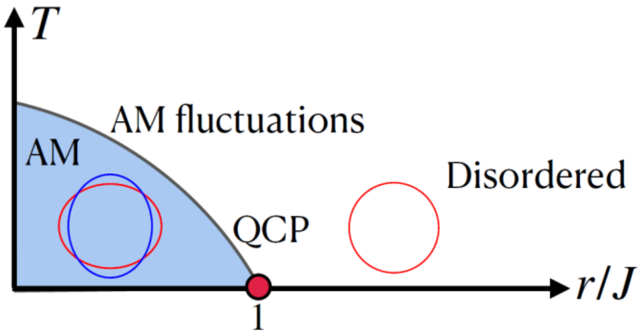
- A low-energy model for electron-electron interactions on the Lieb lattice mediated by AM fluctuations.

free electrons



AM fluctuations that are enhanced upon approaching a putative QCP

$$\chi_{\boldsymbol{q}}^{-1} = r - J\cos\frac{q_x}{2}\cos\frac{q_y}{2}$$



$$S_0 = -\int_k \psi_k^\dagger \hat{G}_0^{-1}(k)\psi_k + \int_q N_q D^{-1}(q) N_{-q}$$

$$S_{\rm int} = \frac{g}{\sqrt{2}}\int_{k,q} N_q \psi_k^\dagger(\tau_3\sigma_3)\psi_{k+q}.$$

Lieb lattice model: AM-mediated pairing

- Sublattice degrees of freedom impact the symmetries of the gap function.

$$\psi_{\boldsymbol{k}} = (A_{\uparrow,\boldsymbol{k}}, A_{\downarrow,\boldsymbol{k}}, B_{\uparrow,\boldsymbol{k}}, B_{\downarrow,\boldsymbol{k}})^T \qquad \psi^T_{-\boldsymbol{k}}(\tau_\mu \sigma_\nu)\psi_{\boldsymbol{k}}$$

- The Hamiltonian as written in the sublattice base is not periodic under a shift by a primitive reciprocal lattice vector:

$$\begin{cases} \boldsymbol{G}_1 = (2\pi, 0) \\ \boldsymbol{G}_2 = (0, 2\pi) \end{cases}$$

$$\mathcal{H}(\boldsymbol{k} + \boldsymbol{G}_{1,2}) = \tau_3 \mathcal{H}(\boldsymbol{k}) \tau_3$$

- This has implications for the symmetry of the gap function: if pairing involves electrons from **different sublattices**, the gap function is **odd** under this shift. If it involves electrons from the **same sublattice**, it is **even**.

$$\hat{\Delta}(\boldsymbol{k} + \boldsymbol{G}_{1,2}) = \pm \hat{\Delta}(\boldsymbol{k})$$

Lieb lattice model: AM-mediated pairing

- A low-energy model for electron-electron interactions on the Lieb lattice mediated by AM fluctuations.
- Use Fierz identities to derive the gap equation: gap function is a 4x4 matrix in the sublattice and spin degrees of freedom.

> sublattice: $\boldsymbol{\tau}$
> spin: $\boldsymbol{\sigma}$

$$\psi_{\boldsymbol{k}} = (A_{\uparrow,\boldsymbol{k}}, A_{\downarrow,\boldsymbol{k}}, B_{\uparrow,\boldsymbol{k}}, B_{\downarrow,\boldsymbol{k}})^T$$

$$S_C = -\sum_{i=1}^{6}\sum_{\boldsymbol{k},\boldsymbol{p}}\left(\psi_{\boldsymbol{k}}^{\dagger}\Lambda^i\psi_{-\boldsymbol{k}}^{*}\right)\chi_{\boldsymbol{k},\boldsymbol{p}}\left(\psi_{-\boldsymbol{p}}^{\mathrm{T}}\Lambda^i\psi_{\boldsymbol{p}}\right)$$

see also Brekke et al, PRB (2023), where pairing mediated by the double exchange of magnons was considered

Lieb lattice model: AM-mediated pairing

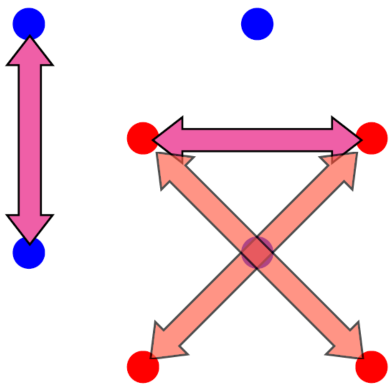
- sublattice: $\boldsymbol{\tau}$
- spin: $\boldsymbol{\sigma}$

- AM fluctuations provide attractive pairing interaction in 4 SC channels.

$$\begin{cases} \hat{\Delta}_{(1)}(\boldsymbol{k}) = \Delta_{(1)}\sigma_{\uparrow}[\tau_0 v(\boldsymbol{k}) + \tau_3 v'(\boldsymbol{k})] \\ \hat{\Delta}_{(2)}(\boldsymbol{k}) = \Delta_{(2)}\sigma_{\uparrow}[\tau_0 v(\tilde{\boldsymbol{k}}) + \tau_3 v'(\tilde{\boldsymbol{k}})] \\ \hat{\Delta}_{(3)}(\boldsymbol{k}) = \Delta_{(3)}\sigma_{\downarrow}[\tau_0 v(\boldsymbol{k}) - \tau_3 v'(\boldsymbol{k})] \\ \hat{\Delta}_{(4)}(\boldsymbol{k}) = \Delta_{(4)}\sigma_{\downarrow}[\tau_0 v(\tilde{\boldsymbol{k}}) - \tau_3 v'(\tilde{\boldsymbol{k}})] \end{cases}$$

- *pairs are made out of electrons from the same sublattice.*
- *fourfold degenerate triplet odd-parity state with S=1*

p-wave



$\sin k_x \pm \sin k_y$

$\sin(k_x \pm k_y)$

 : triplet $(S_z = \pm 1)$

Lieb lattice model: AM-mediated pairing

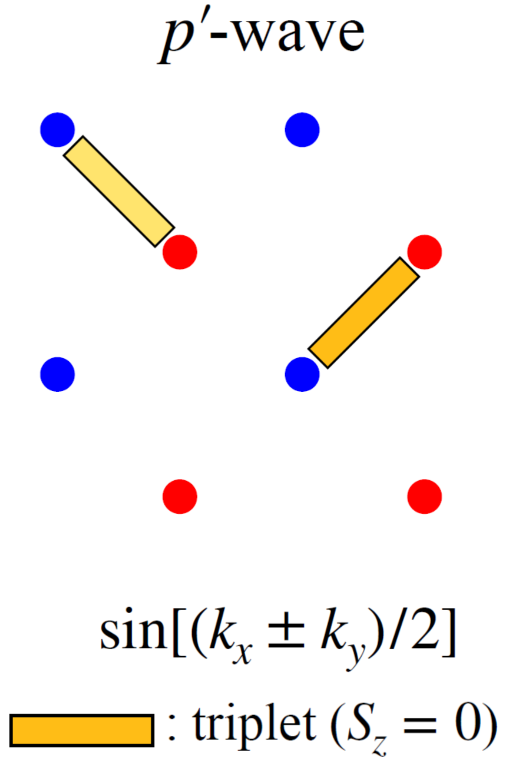
- sublattice: $\boldsymbol{\tau}$
- spin: $\boldsymbol{\sigma}$

- AM fluctuations provide attractive pairing interaction in 4 SC channels.

$$\begin{cases} \hat{\Delta}_1(\boldsymbol{k}) = \Delta_1 u(\boldsymbol{k})\tau_1\sigma_1 \\ \hat{\Delta}_2(\boldsymbol{k}) = \Delta_2 u(\tilde{\boldsymbol{k}})\tau_1\sigma_1 \end{cases}$$

- *pairs are made out of electrons from different sublattices:* ***intra-unit-cell pairing***
- *twofold degenerate triplet odd-parity state with S=1, S_z=0*
- *the gap function changes sign upon a shift by a reciprocal lattice vector*

$$\hat{\Delta}(\boldsymbol{k} + \boldsymbol{G}_{1,2}) = -\hat{\Delta}(\boldsymbol{k})$$



Lieb lattice model: AM-mediated pairing

- sublattice: $\boldsymbol{\tau}$
- spin: $\boldsymbol{\sigma}$

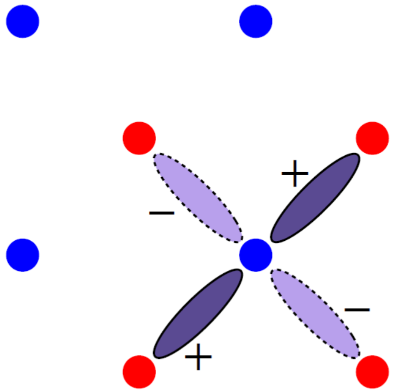
- AM fluctuations provide attractive pairing interaction in 4 SC channels.

$$\hat{\Delta}_d(\boldsymbol{k}) = \Delta \sin\frac{k_x}{2}\,\sin\frac{k_y}{2}\,\tau_1\sigma_2$$

- *pairs are made out of electrons from different sublattices:* ***intra-unit-cell pairing***
- ***singlet d-wave*** *state with symmetry-enforced nodes along the main axes*

$$\hat{\Delta}(\boldsymbol{k}+\boldsymbol{G}_{1,2}) = -\hat{\Delta}(\boldsymbol{k})$$

d'-wave



$$\sin\frac{k_x}{2}\sin\frac{k_y}{2}$$


: singlet

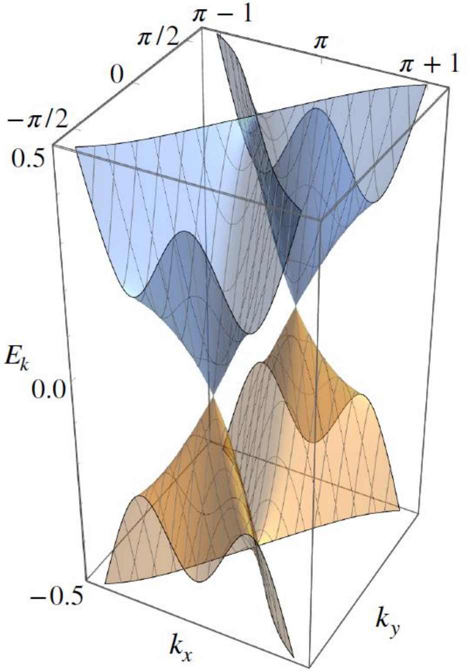
Lieb lattice model: AM-mediated pairing

- sublattice: $\boldsymbol{\tau}$
- spin: $\boldsymbol{\sigma}$

- AM fluctuations provide attractive pairing interaction in 4 SC channels.

$$\hat{\Delta}_d(\boldsymbol{k}) = \Delta \sin\frac{k_x}{2}\, \sin\frac{k_y}{2}\, \tau_1 \sigma_2$$

- *pairs are made out of electrons from different sublattices:* ***intra-unit-cell pairing***
- ***singlet d-wave*** *state with symmetry-enforced nodes along the main axes*
- *additional nodes can appear along the Brillouin zone boundaries due to the projection of the inter-sublattice pairing onto the Fermi surface.*

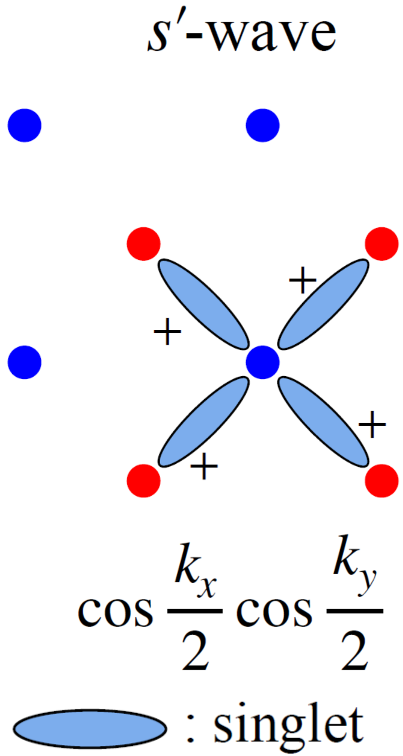


Lieb lattice model: AM-mediated pairing

- sublattice: $\boldsymbol{\tau}$
- spin: $\boldsymbol{\sigma}$

- AM fluctuations provide attractive pairing interaction in 4 SC channels.

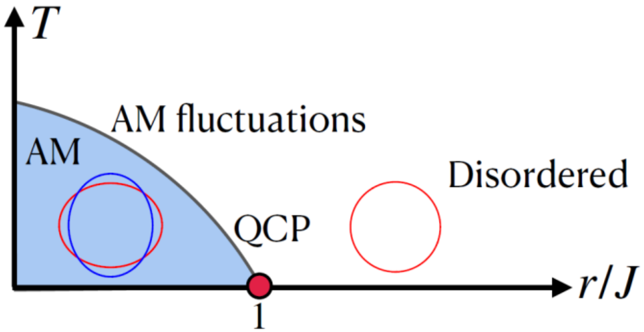
$$\hat{\Delta}_s(\boldsymbol{k}) = \Delta \cos\frac{k_x}{2}\,\cos\frac{k_y}{2}\,\tau_1\sigma_2$$



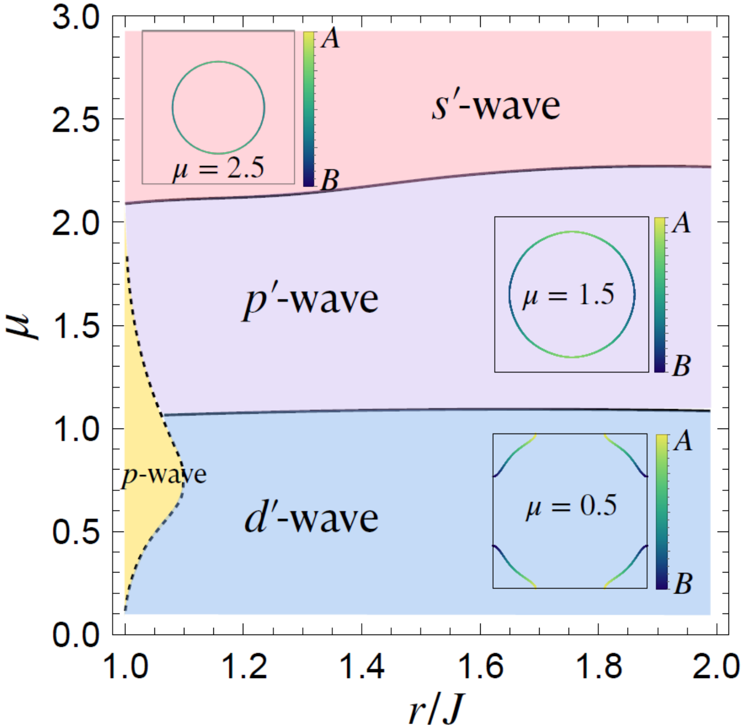
- *pairs are made out of electrons from different sublattices:* ***intra-unit-cell pairing***
- ***singlet s-wave*** *state with nodes along the Brillouin zone boundaries.*
- *linear nodes can become quadratic due to projection onto the Fermi surface.*

Intra-unit-cell pairing mediated by AM fluctuations

- Phase diagram: shorter-range fluctuations favor intra-unit-cell pairing, whereas longer-range fluctuations give the anticipated p-wave state.

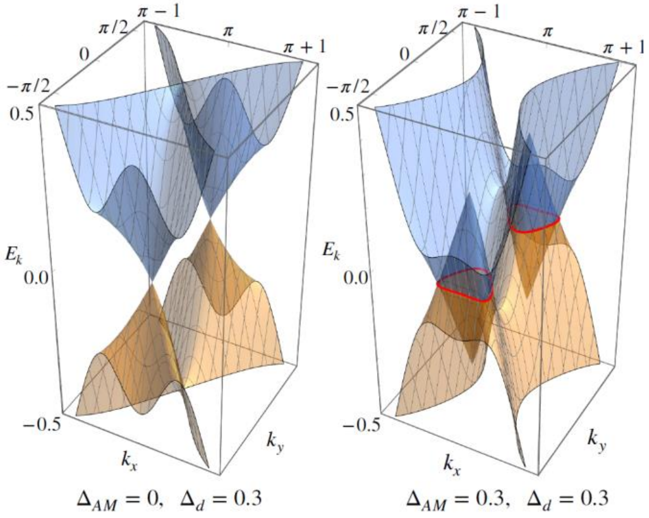


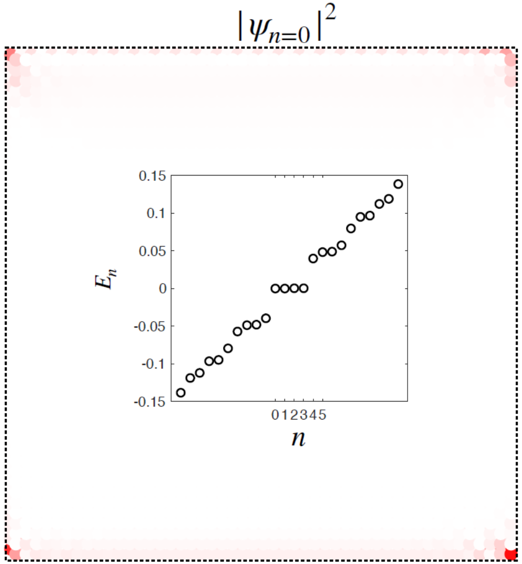
> *Symmetry of the intra-unit-cell pairing state depends on properties of the Fermi surface.*



Coexistence between SC and AM order

- Coexistence with altermagnetic order endows the superconducting state with non-trivial topological features.





d'-wave* + *AM: *induces an s-wave component, giving rise to an s+id' state.*

p-wave* + *AM: *higher-order topology with corner Majorana zero modes*

Outline

1. Superconductivity mediated by altermagnetic fluctuations.

2. Interaction-driven orbital altermagnetism on the kagome lattice.

Chakraborty, Yang, Birol & RMF
arXiv:2509.26596 (2005)

Altermagnetism in the kagome lattice

- Altermagnetic order (collinear compensated magnet invariant under rotation + time-reversal) is naturally realized in non-Bravais lattices with an even number of sublattices (Wyckoff positions with even multiplicity).

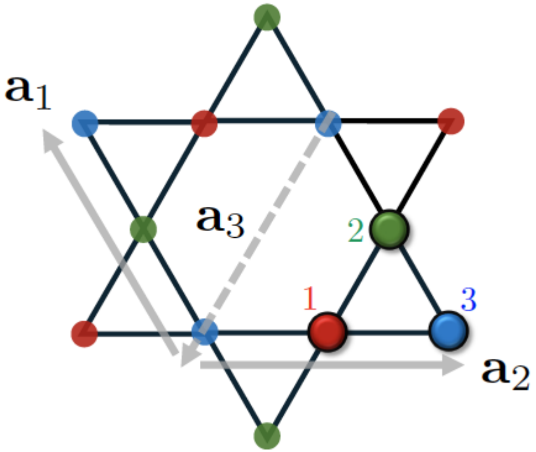
TABLE XII. Space group Wyckoff positions supporting altermagnetism, and the irreps $\Gamma_{\mathbf{N}}$ under which **N** transforms.

PG	SG	WP	$\Gamma_{\mathbf{N}}$
2	3	$\{2e\}$	$\{B\}$
	4	$\{2a\}$	$\{B\}$
	5	$\{4c, 2b, 2a\}$	$\{B\}$
m	6	$\{2c\}$	$\{A''\}$
	7	$\{2a\}$	$\{A''\}$
	8	$\{4b\}$	$\{A''\}$
	9	$\{4a\}$	$\{A''\}$
2/m	10	$\{4o\}$	$\{B_g\}$
	11	$\{4f, 2d, 2c, 2b, 2a\}$	$\{B_g\}$
	12	$\{8j, 4h, 4g, 4f, 4e, 2d, 2b\}$	$\{B_g\}$
	13	$\{4g, 2d, 2c, 2b, 2a\}$	$\{B_g\}$
	14	$\{4e, 2d, 2c, 2b, 2a\}$	$\{B_g\}$
	15	$\{8f, 4e, 4d, 4c, 4b, 4a\}$	$\{B_g\}$

Smolyanyuk et al, SciPost (2024)
Schiff et al, PRR (2025)

Altermagnetism in the kagome lattice

- Can altermagnetic-like states be realized in lattices with an **odd** number of sublattices, like the kagome lattice?

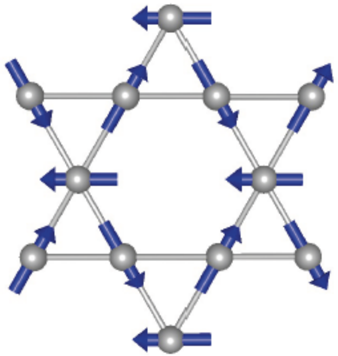


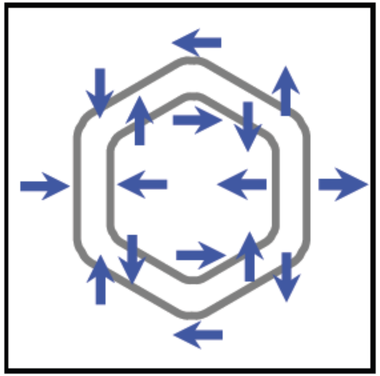
collinear moments on the 3 sublattices will necessarily yield an uncompensated magnet

Altermagnetism in the kagome lattice

- Can altermagnetic-like states be realized in lattices with an **odd** number of sublattices, like the kagome lattice?
  - Option 1: non-collinear moments (but now band structure has a spin texture).

Mn_3Sn

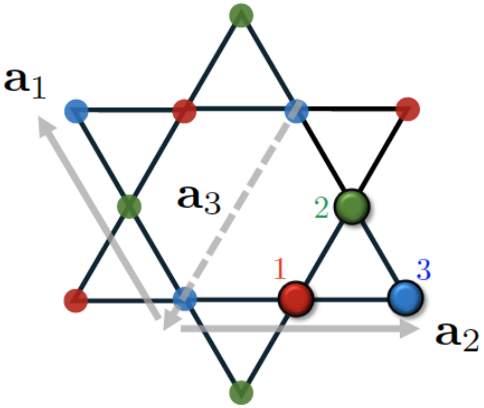


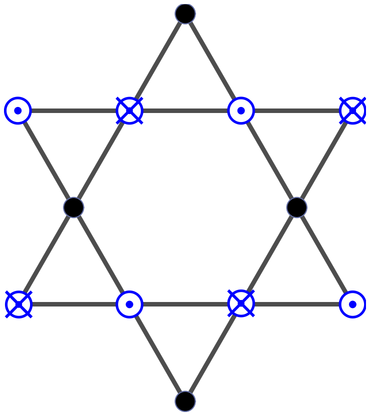


Nakatsuji et al, Nature (2015)

Altermagnetism in the kagome lattice

- Can altermagnetic-like states be realized in lattices with an **odd** number of sublattices, like the kagome lattice?
  - Option 1: non-collinear moments (but now band structure has a spin texture).
  - Option 2: collinear but **non-uniform** moments.

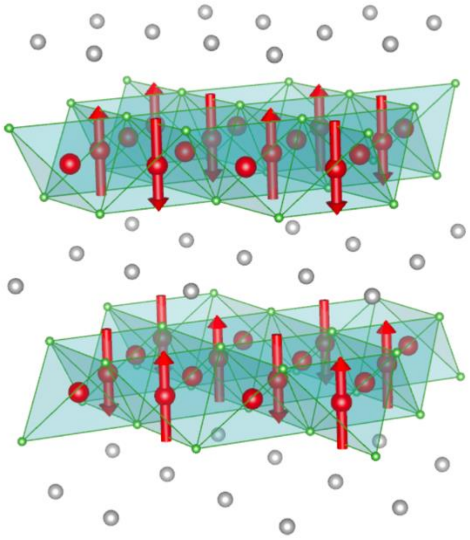




- ***moments vanish on a third of the sites***
- ***symmetries of a d-wave altermagnet***

Altermagnetism in the kagome lattice

- Non-uniform collinear magnetic states have been found in correlated electronic systems.
  - Hole-doped iron-based superconductor: “interference” between two symmetry-related spin-density wave order parameters.



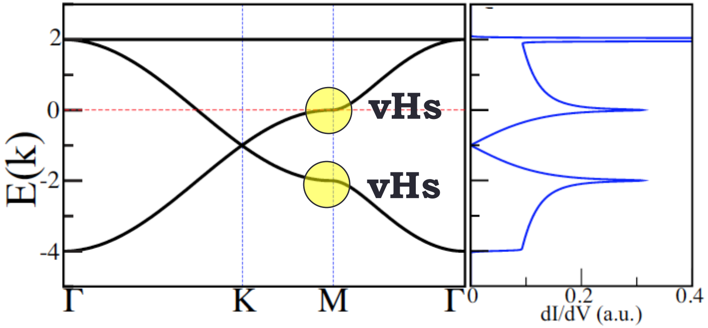
Allred et al, Nature Phys (2016)

- ***loop-current states naturally give rise to non-uniform collinear orbital moments.***
- ***interacting kagome lattices can be unstable towards loop-current states.***

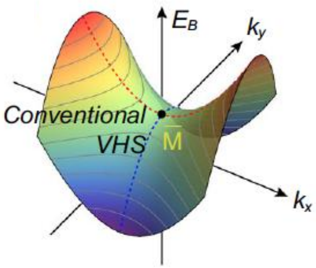
see also the case of Mn_5Si_3,
Reichlova et al, Nature Comm (2024)

Interacting kagome metals: loop-current order

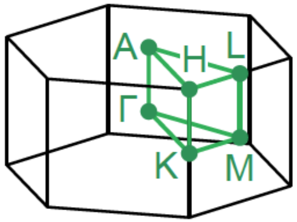
- Generic band dispersion of the kagome lattice has saddle-points at *M*: enhanced density of states due to **van Hove singularities** (vHs).



Feng et al, PRB (2021)



Hu et al, Nat Comm (2022)

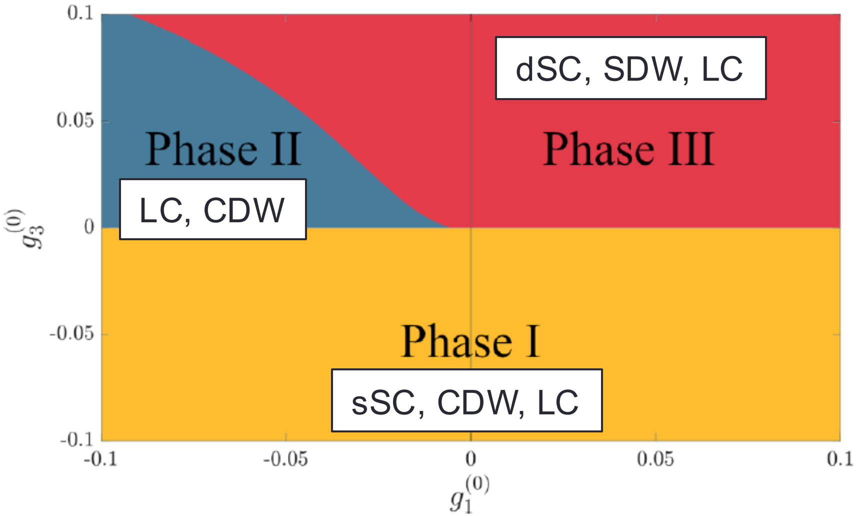


Ratcliff et al, PRM (2021)

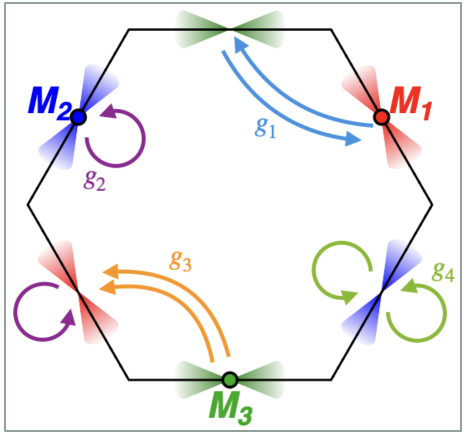
- When the chemical potential is close to the vHs, even weak interactions can drive the system across an instability.

Interacting kagome metals: loop-current order

- Low-energy interacting models involving the vHs find both charge density waves (CDW) and loop-current (LC) instabilities.



for a recente review:
RMF, Birol, Ye & Vanderbilt, arxiv (2025)



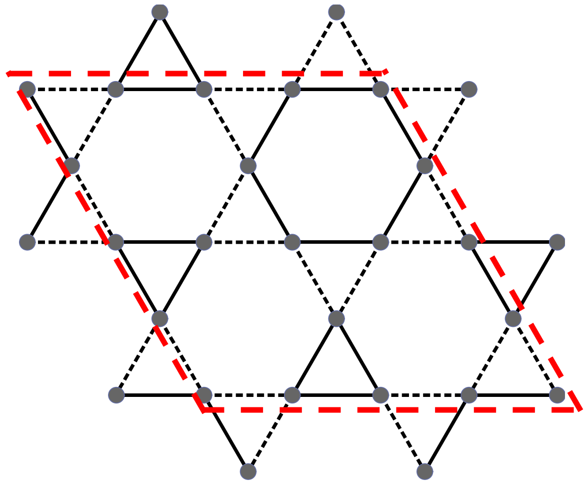
Park, Ye, Balents, PRB (2021)
Feng, Zhang, Jiang, Hu, PRB (2021)
Lin & Nandkishore, PRB (2021)
Denner, Thomale, Neupert, PRL (2021)

Interacting kagome metals: loop-current order

- Low-energy interacting models involving the vHs find both charge density waves (CDW) and loop-current (LC) instabilities.
- All instabilities have wave-vector (½, ½): unit cell is quadrupled.

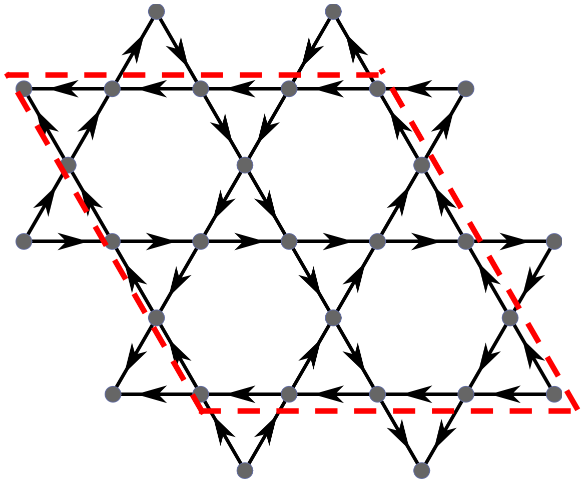
bond charge order

➢ translation symmetry breaking



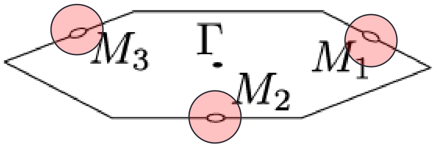
loop current order

➢ translation + time-reversal symmetry breaking



Interacting kagome metals: loop-current order

- Landau free-energy analysis: CDW and LC orders are intertwined.



bond charge order

$$\mathbf{W} = (W_1, W_2, W_3)$$

loop current order

$$\mathbf{\Phi} = (\Phi_1,\ \Phi_2,\ \Phi_3)$$

$$\mathcal{F}_{\text{cdw}}(\mathbf{W}) = \frac{a_w}{2}(\mathbf{W}\cdot\mathbf{W}) + \frac{\gamma_w}{3}W_1W_2W_3 + \frac{u_w}{4}(\mathbf{W}\cdot\mathbf{W})^2 + \frac{\lambda_w}{4}(W_1^2W_2^2 + W_2^2W_3^2 + W_3^2W_1^2)$$

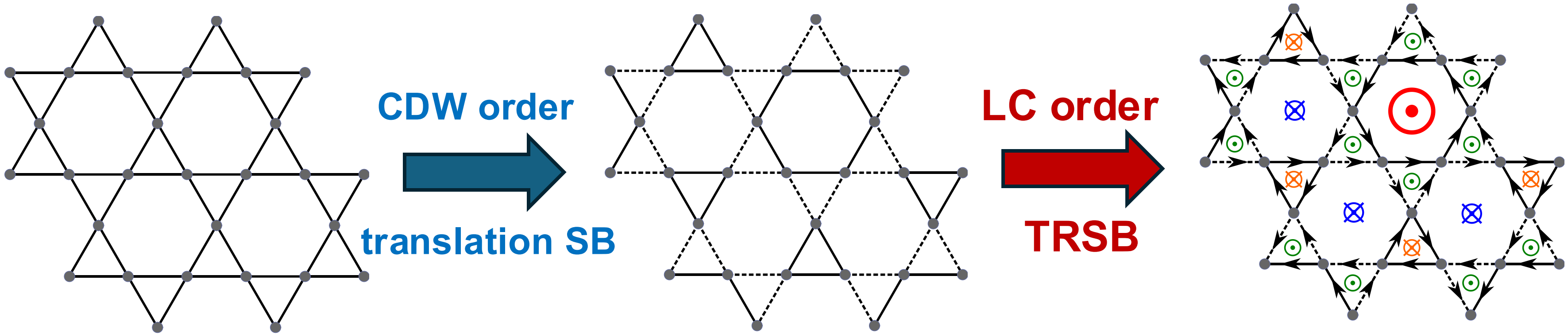
$$\mathcal{F}_{\text{lc}}(\mathbf{\Phi}) = \frac{a_\Phi}{2}(\mathbf{\Phi}\cdot\mathbf{\Phi}) + \frac{u_\Phi}{4}(\mathbf{\Phi}\cdot\mathbf{\Phi})^2 + \frac{\lambda_\Phi}{4}(\Phi_1^2\Phi_2^2 + \Phi_2^2\Phi_3^2 + \Phi_3^2\Phi_1^2)$$

$$\mathcal{F}_{\text{mixed}}(\mathbf{W},\mathbf{\Phi}) = \frac{\gamma}{3}(\Phi_1\Phi_2W_3 + \Phi_2\Phi_3W_1 + \Phi_3\Phi_1W_2) + \frac{\lambda_1}{4}(W_1W_2\Phi_1\Phi_2 + W_2W_3\Phi_2\Phi_3 + W_3W_1\Phi_3\Phi_1)$$
$$+ \frac{\lambda_2}{4}(W_1^2\Phi_1^2 + W_2^2\Phi_2^2 + W_3^2\Phi_3^2) + \frac{\lambda_3}{4}(\mathbf{W}\cdot\mathbf{W})(\mathbf{\Phi}\cdot\mathbf{\Phi})$$

Christensen, Birol, Andersen & RMF, PRB (2022)

Interacting kagome metals: loop-current order

- Wide parameter regime in which loop-current order emerges inside the bond CDW phase.

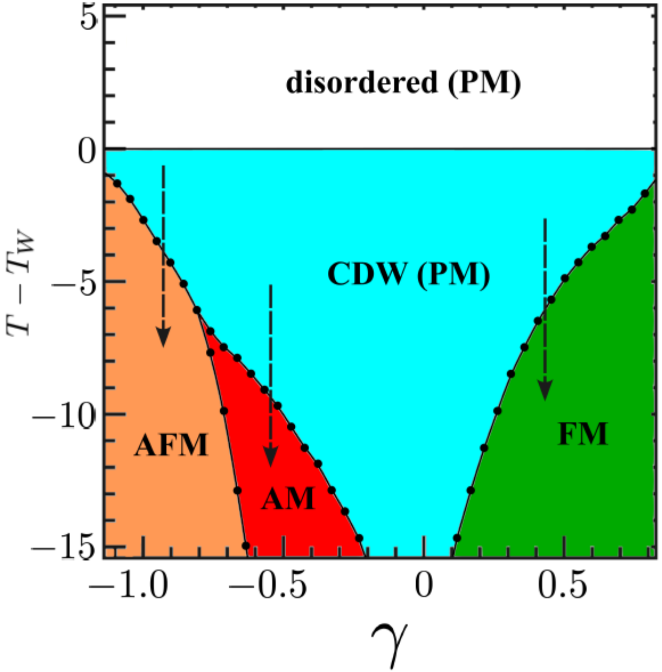


- In this case, the loop-current order becomes a **Q=0** orbital magnetically ordered state. Ferromagnetic, antiferromagnetic, or altermagnetic?

Šmejkal, Sinova & Jungwirth, PRX (2022)

Non-uniform ferro-, antiferro-, and alter-magnetism

- Landau phase diagram + symmetry analysis.

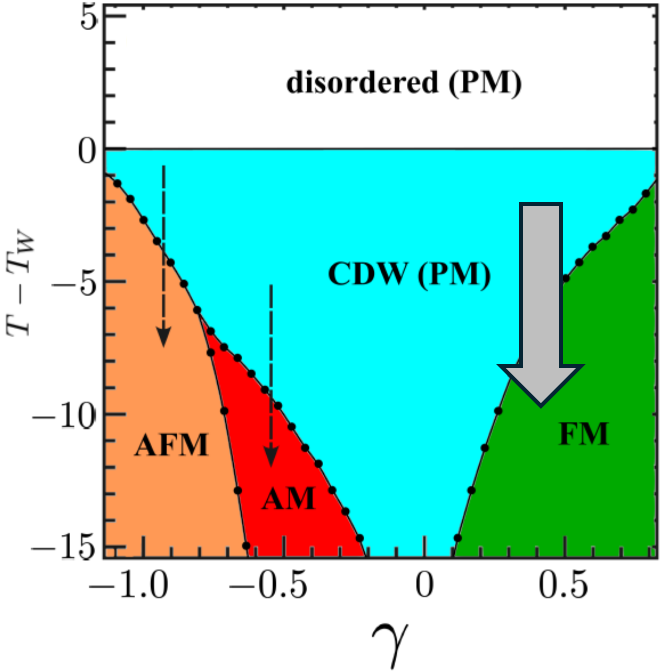


similar analysis to:
Christensen, Birol, Andersen & RMF, PRB (2021)

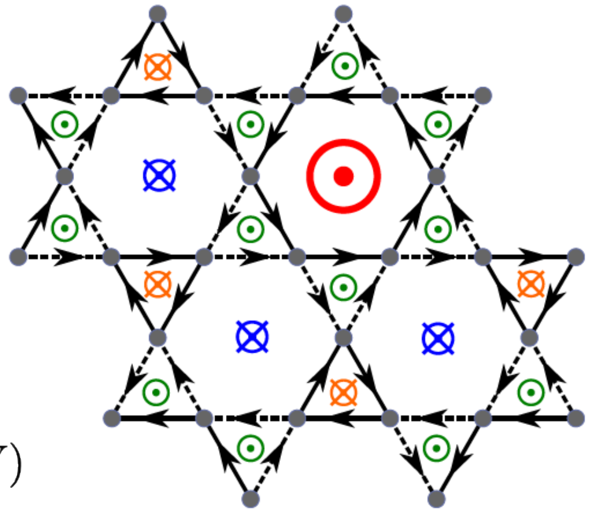
$$\mathcal{F}_{\text{mixed}}(\mathbf{W}, \mathbf{\Phi}) = \frac{\gamma}{3}(\Phi_1\Phi_2 W_3 + \Phi_2\Phi_3 W_1 + \Phi_3\Phi_1 W_2)$$

Non-uniform ferro-, antiferro-, and alter-magnetism

- Landau phase diagram + symmetry analysis.



$$\mathbf{W} = (W, W, W)$$
$$\mathbf{\Phi} = (\Phi, \Phi, \Phi)$$

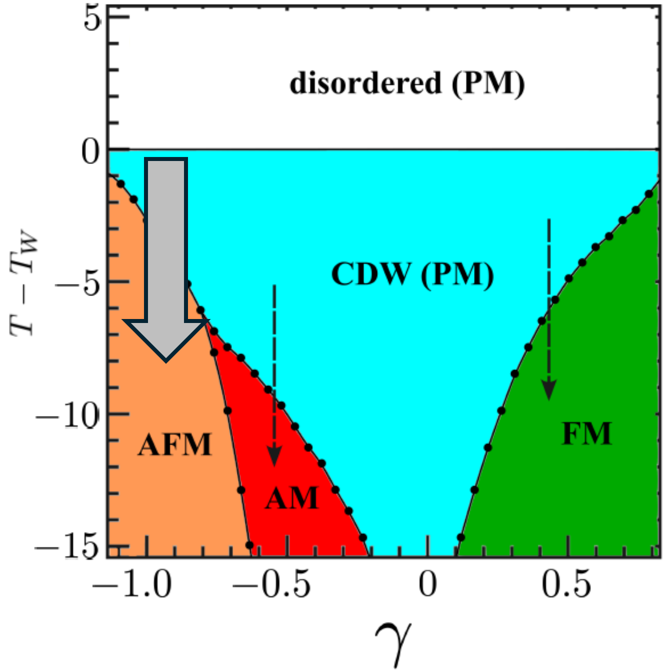


ferromagnet

> ***non-uniform moments do not cancel out because of the underlying CDW bond order***

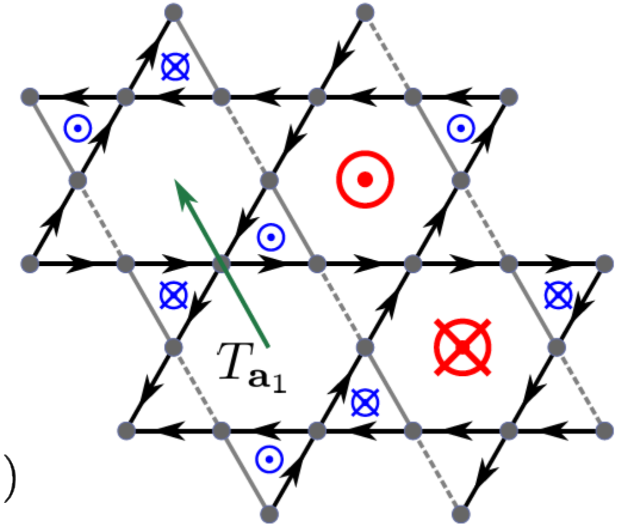
Non-uniform ferro-, antiferro-, and alter-magnetism

- Landau phase diagram + symmetry analysis.



$\mathbf{W} = (0, W, 0)$

$\mathbf{\Phi} = (\Phi, 0, \Phi)$

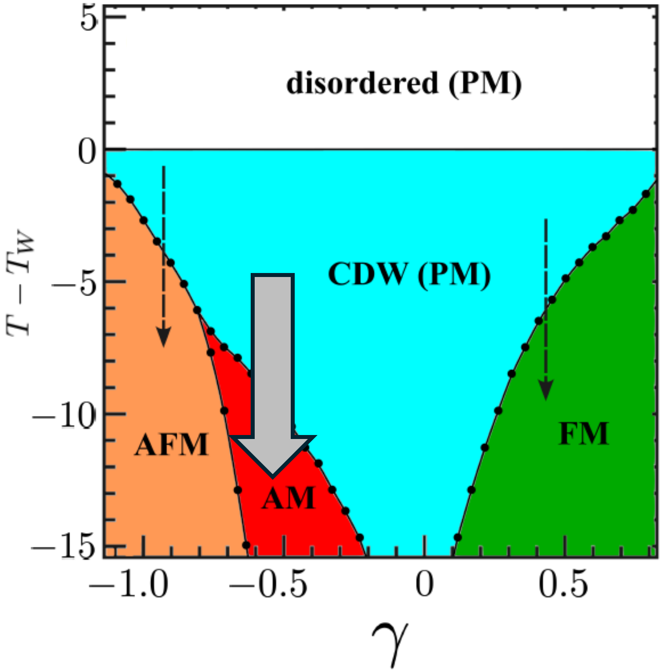


antiferromagnet

- ➢ ***compensated non-uniform moments related by a translation***
- ➢ ***three-fold rotational symmetry breaking***

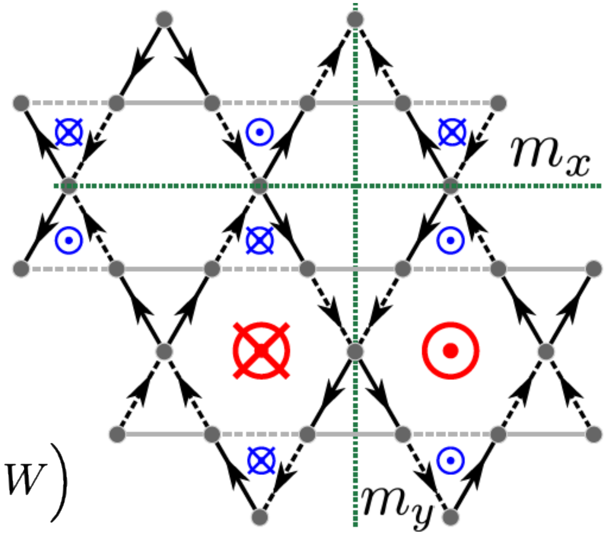
Non-uniform ferro-, antiferro-, and alter-magnetism

- Landau phase diagram + symmetry analysis.



$$\mathbf{W} = \left(W, \tilde{W}, W\right)$$

$$\mathbf{\Phi} = (\Phi, 0, -\Phi)$$



d-wave altermagnet

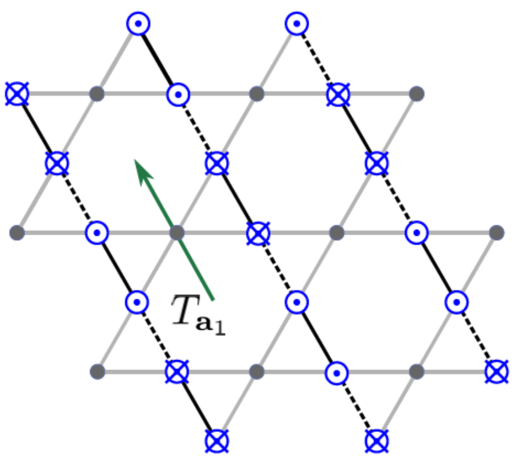
- ***compensated non-uniform moments related by a mirror***
- ***three-fold rotational symmetry breaking***

Non-uniform ferro-, antiferro-, and alter-magnetism

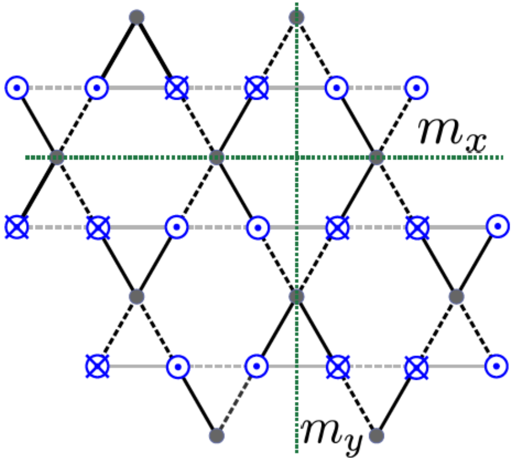
phase	OP configuration	magnetic subgroup	TR-odd multipoles	TR-even multipoles	properties
0	(W, W, W)	$P6/mmm.1'$	-	-	3Q CDW
1	(W, W, W) (Φ, Φ, Φ)	$P6/mm'm'$	$\mathcal{M} = W\Phi$	-	ferromagnetic
2	$(W, 0, 0)$ $(0, \Phi, \Phi)$	$Cmmm.1'_a$	-	$\mathcal{Q} = \Phi^2\langle -1, \sqrt{3}\rangle$	antiferromagnetic nematic
3	(W, W', W) $(\Phi, 0, -\Phi)$	$Cmmm.1$	$\mathcal{N} = W\Phi\langle 1, 0\rangle$ $\Upsilon = -W\Phi^3$	$\mathcal{Q} = \Phi^2\langle 1, 0\rangle$	d-wave altermagnetic i-wave altermagnetic nematic
4	(W, W', W) (Φ, Φ', Φ)	$Cm'm'm$	$\mathcal{N} = (W\Phi - W'\Phi')\langle 0, 1\rangle$ $\mathcal{M} = 2W\Phi + W'\Phi'$	$\mathcal{Q} = (\Phi^2 - \Phi'^2)\langle 1, 0\rangle$	d-wave altermagnetic ferromagnetic nematic
5	$(W, 0, 0)$ $(0, \Phi, \Phi')$	$P2/m.1'_a$	-	$\mathcal{Q} = \langle \Phi'^2 - 2\Phi^2, \sqrt{3}\Phi'^2\rangle$ $\mathcal{G} = \Phi^2\Phi'^2(\Phi^2 - \Phi'^2)$	antiferromagnetic nematic ferroaxial
6	$(W, 0, 0)$ $(\Phi, 0, 0)$	$Pm'm'm$	$\mathcal{M} = W\Phi$ $\mathcal{N} = W\Phi\langle \sqrt{3}, 1\rangle$	$\mathcal{Q} = \Phi^2\langle 1, -\sqrt{3}\rangle$	ferromagnetic d-wave altermagnetic nematic
7	(W, W', W'') (Φ, Φ', Φ'')	$P2/m.1$	all	all	all

Non-uniform ferro-, antiferro-, and alter-magnetism

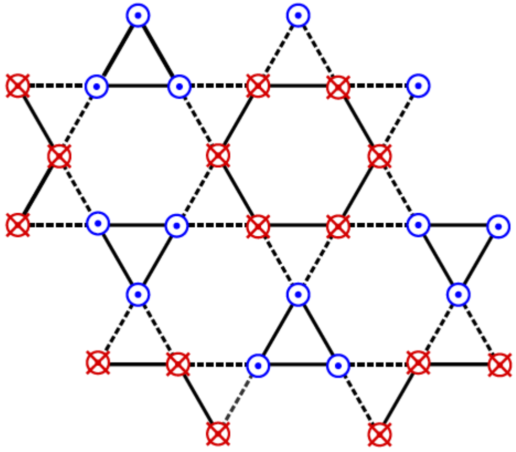
- These are orbital magnetic moments that live on the centers of the plaquettes.
- When spin-orbit coupling (SOC) is turned on, spins on the kagome sites become ordered too.



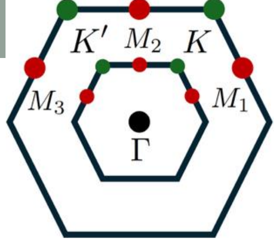
antiferromagnet



d-wave altermagnet



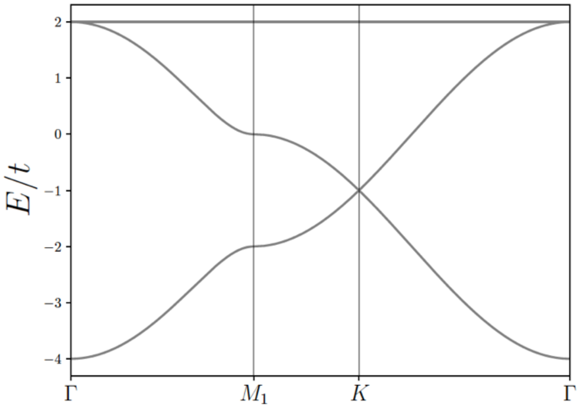
ferromagnet



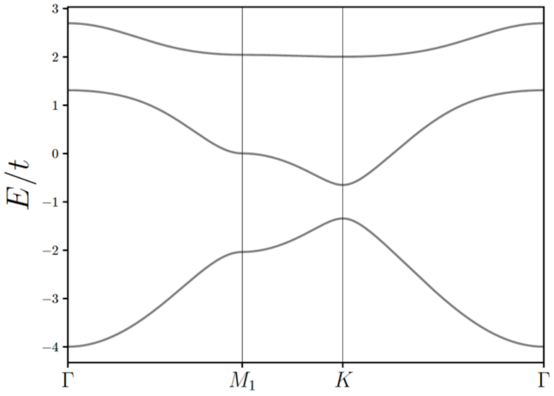
Band structure spin-splitting

- Microscopic tight-binding model: with SOC, band structure shows the characteristic spin-splitting pattern of each phase.

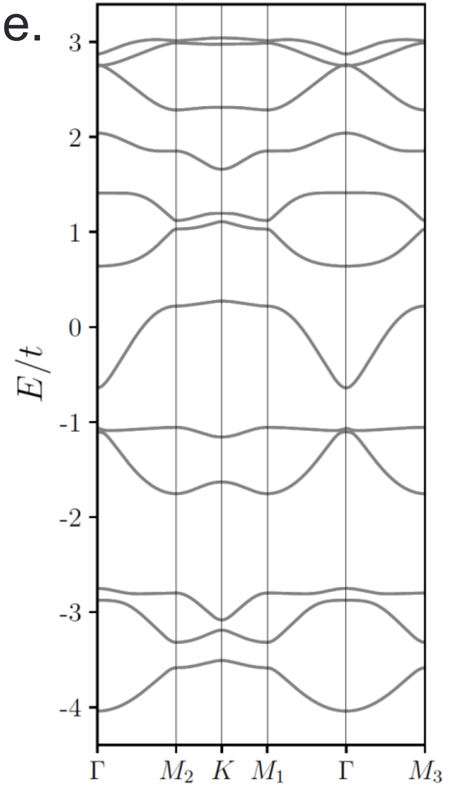
$$\mathcal{H}_0 = \sum_{i,\mathbf{r},\alpha} \left[(-t + \mathrm{i}\lambda_{\mathrm{soc}} \sigma_z^{\alpha\alpha}) d^\dagger_{j\mathbf{r},\alpha} (d_{l\mathbf{r},\alpha} + d_{l\mathbf{r}+\mathbf{a}_i,\alpha}) + \mathrm{H.c.} \right]$$

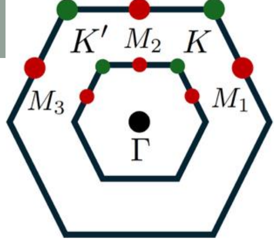


+ SOC



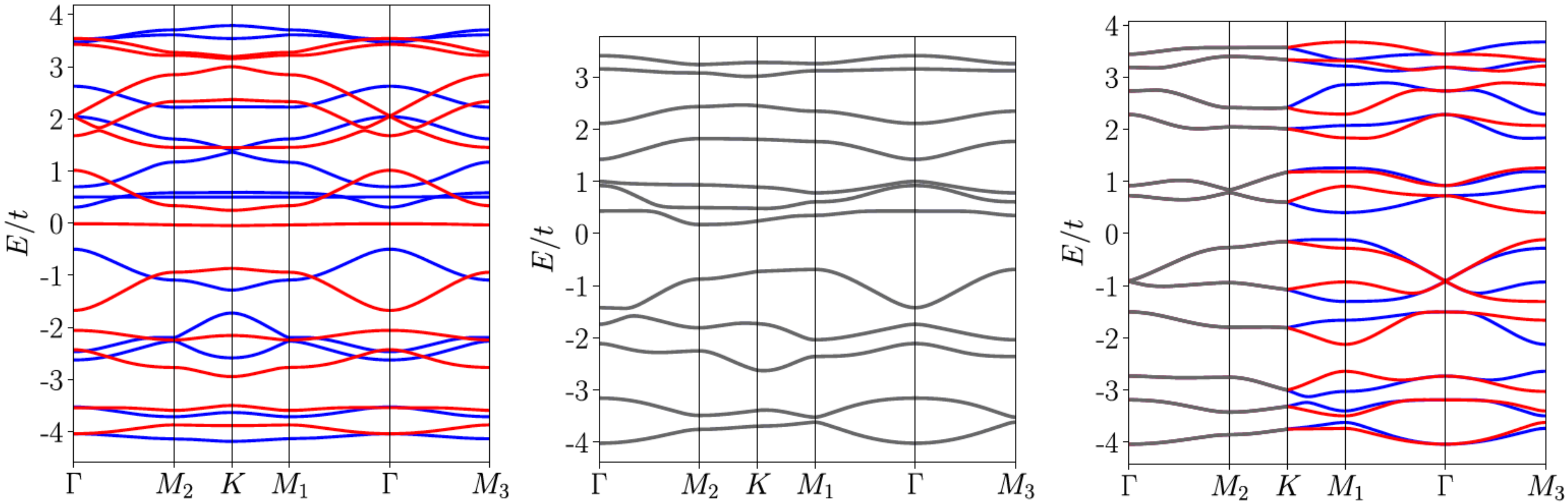
+ CDW





Band structure spin-splitting

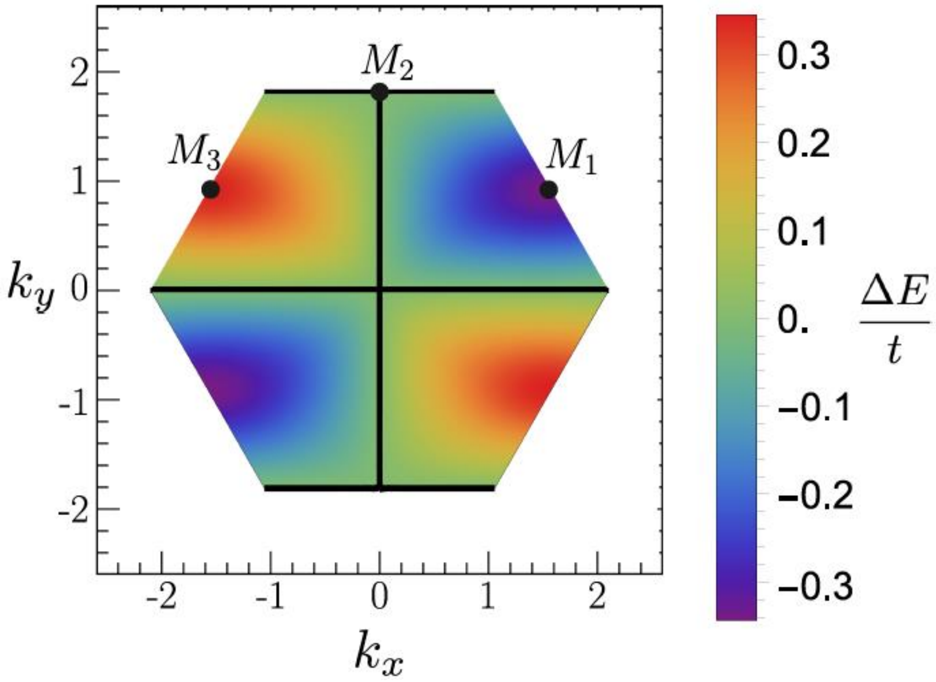
- Microscopic tight-binding model: with SOC, band structure shows the characteristic spin-splitting pattern of each phase.



s-wave ferromagnet **antiferromagnet** **d-wave altermagnet**

Band structure spin-splitting

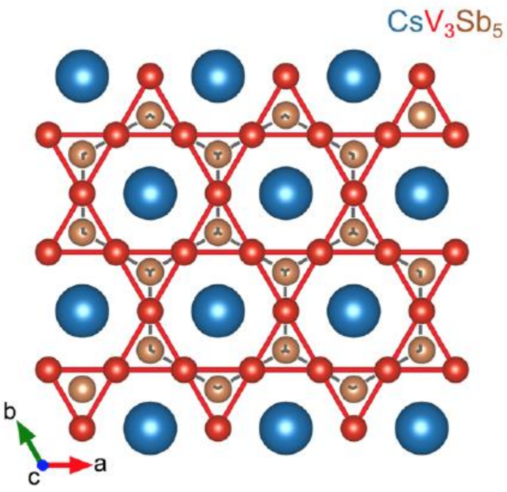
- The *d*-wave altermagnetic spin-splitting breaks three-fold rotational symmetry due to a non-linear coupling to an *i*-wave altermagnetic component.



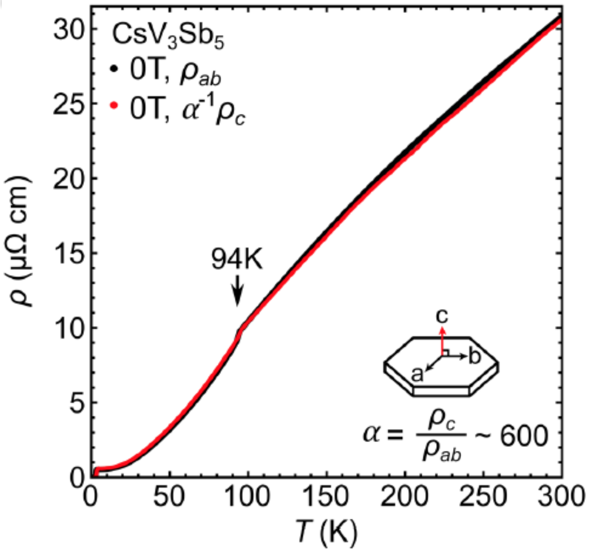
Possible material realization

- AV_3Sb_5 are kagome superconductors that display CDW order where time-reversal symmetry-breaking has been reported (μSR, STM).

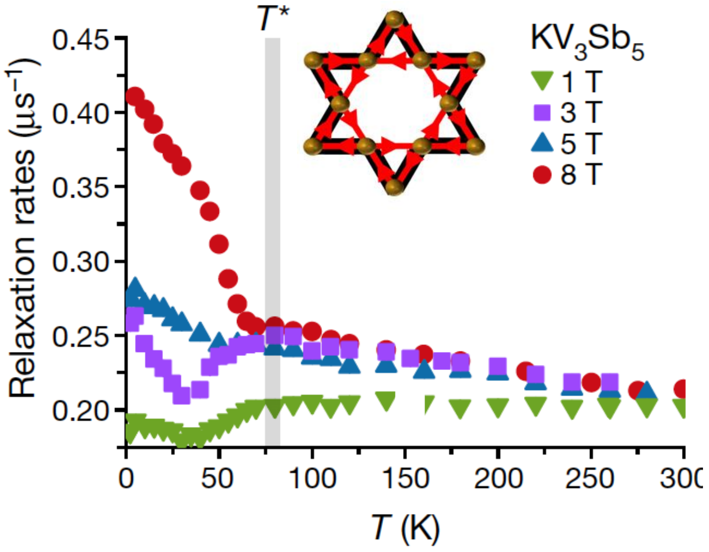
(*A* = Cs, K, Rb)



Ortiz et al, PRL (2020)



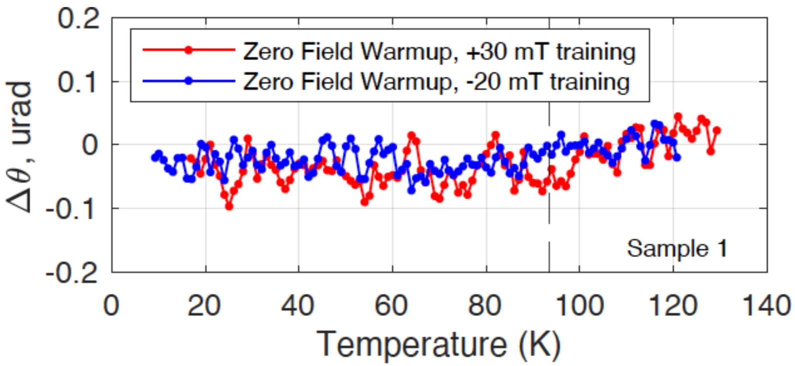
Ortiz et al, PRM (2019)



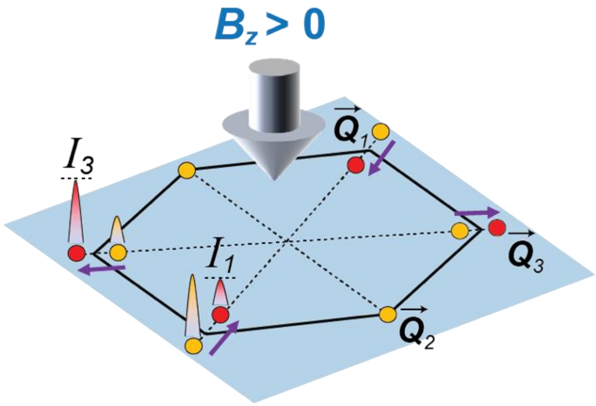
Mielke et al, Nature (2022)

Possible material realization

- Absence of net magnetization (Kerr effect, although there are conflicting data) and reports of piezomagnetism (STM) and C_{3z} symmetry breaking (transport) are consistent with a d-wave orbital altermagnetic state.



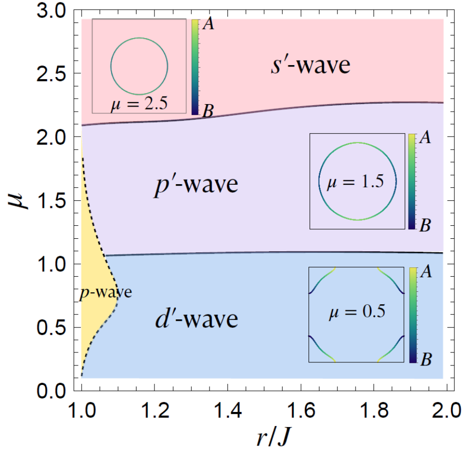
Saykin et al, PRL (2023)
see also: Xu et al, Nat Phys (2022)

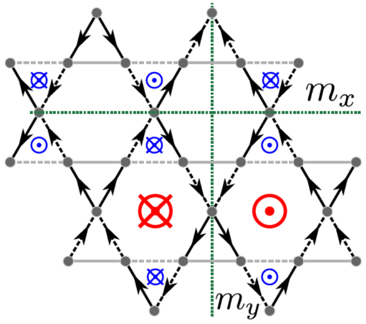


Xing, …, RMF, Madhavan, Nature (2024)
see also: Jiang et al, Nat Materials (2021)

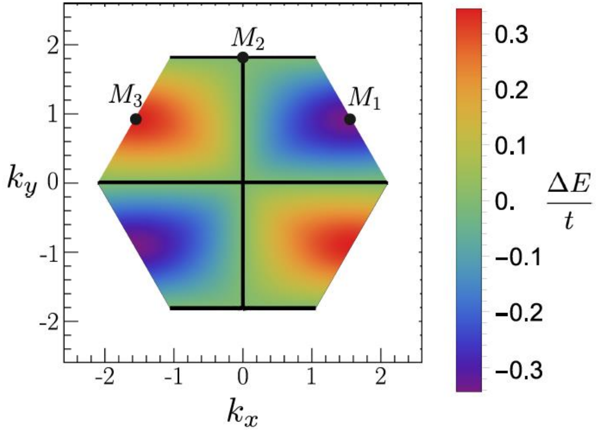
spin-resolved ARPES measurements could directly verify it.

Summary





d-wave altermagnet



- Altermagnetic fluctuations can mediate intra-unit-cell singlet superconductivity, which reveals the importance of the sublattice degrees of freedom.
- Collinear altermagnetic-like states can be realized in non-Bravais lattices with an odd number of sublattices provided that the magnetic moments are non-uniform.
- Correlations can promote an orbital d-wave altermagnetic phase on the kagome lattice by intertwining bond-charge and loop-current orders, with possible realization in AV_3Sb_5.

Wu, Wang & RMF, *Phys. Rev. Lett.* ***135***, *156001 (2025)*

Chakraborty, Yang, Birol & RMF, *arXiv:2509.26596 (2005)*