Superconducting Phases of Metallic Altermagnets

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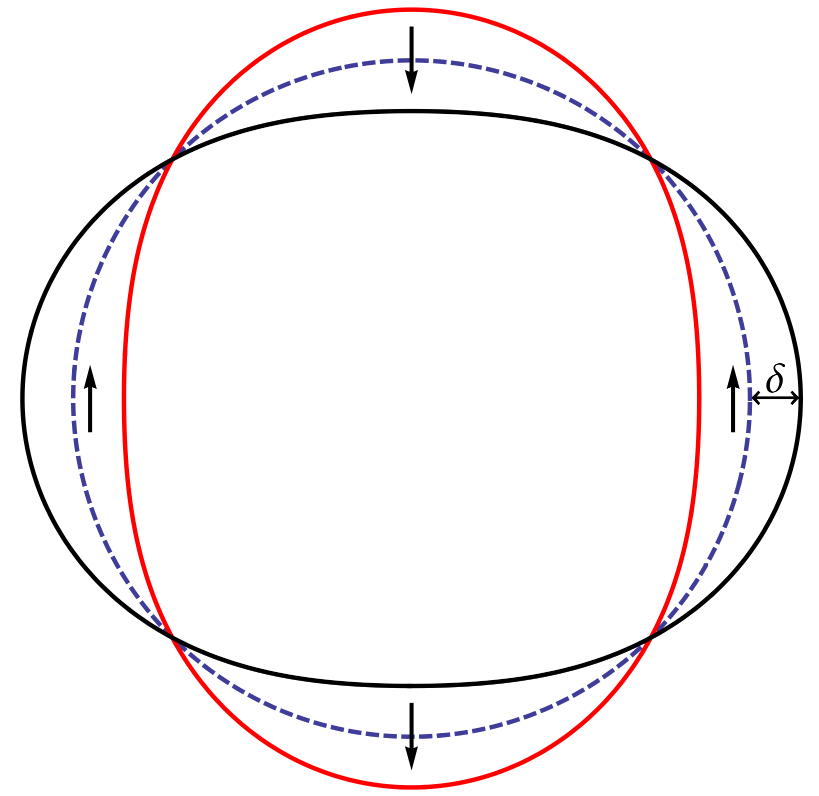
Electronic Liquid Crystal Phases

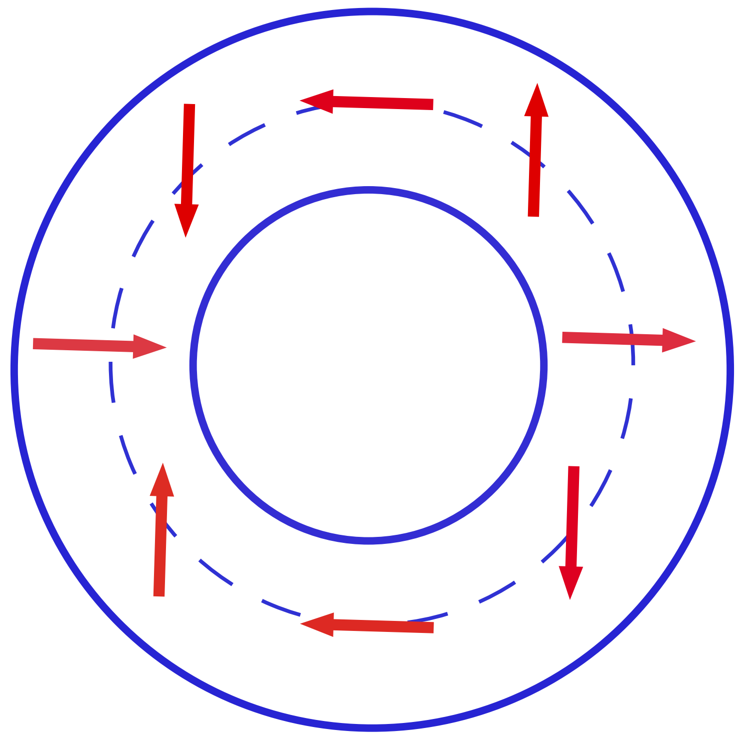
- Phases of strongly correlated electronic systems which break translation and/or rotational invariance (Kivelson, Fradkin and Emery, Nature 1998)
- Nematic phases: anisotropic phases of electronic fluids which break spontaneously rotational (point group) symmetry which are invariant under translations and spin reversal
- Simplest example of an electronic liquid crystal phase of matter
- If the coupling to the lattice these metallic states are non-Fermi liquids
- Nematic phases are seen in many different strongly correlated systems
- 2DEGs in magnetic fields
- cuprate and iron superconductors
- bilayer $Sr_3Ru_2O_7$ and several other materials (see e.g. Fradkin et al Annu. Rev. Cond. Matt. Phys. 2010, Fradkin, Kivelson and Tranquada RMP 2015, and Fernandes, Orth and Schmalian Annu. Rev. Cond. Matt. Phys. 2019)
- These phases are often found to be intertwined with SC orders
- Nematic phases are often realized as vestigial phases

Altermagnets: an example of nematic phases in the spin triplet channel

Gorkov and Sokol, PRL 1992; Kivelson *et al* RMP 2003; C. Wu, K. Sun, EF, and S.C. Zhang, PRB 2007
Šmejkal, Sinova and Jungwirth 2022, Jungwirth, Fernandes, EF, MacDonald, Sinova, Newton 2025

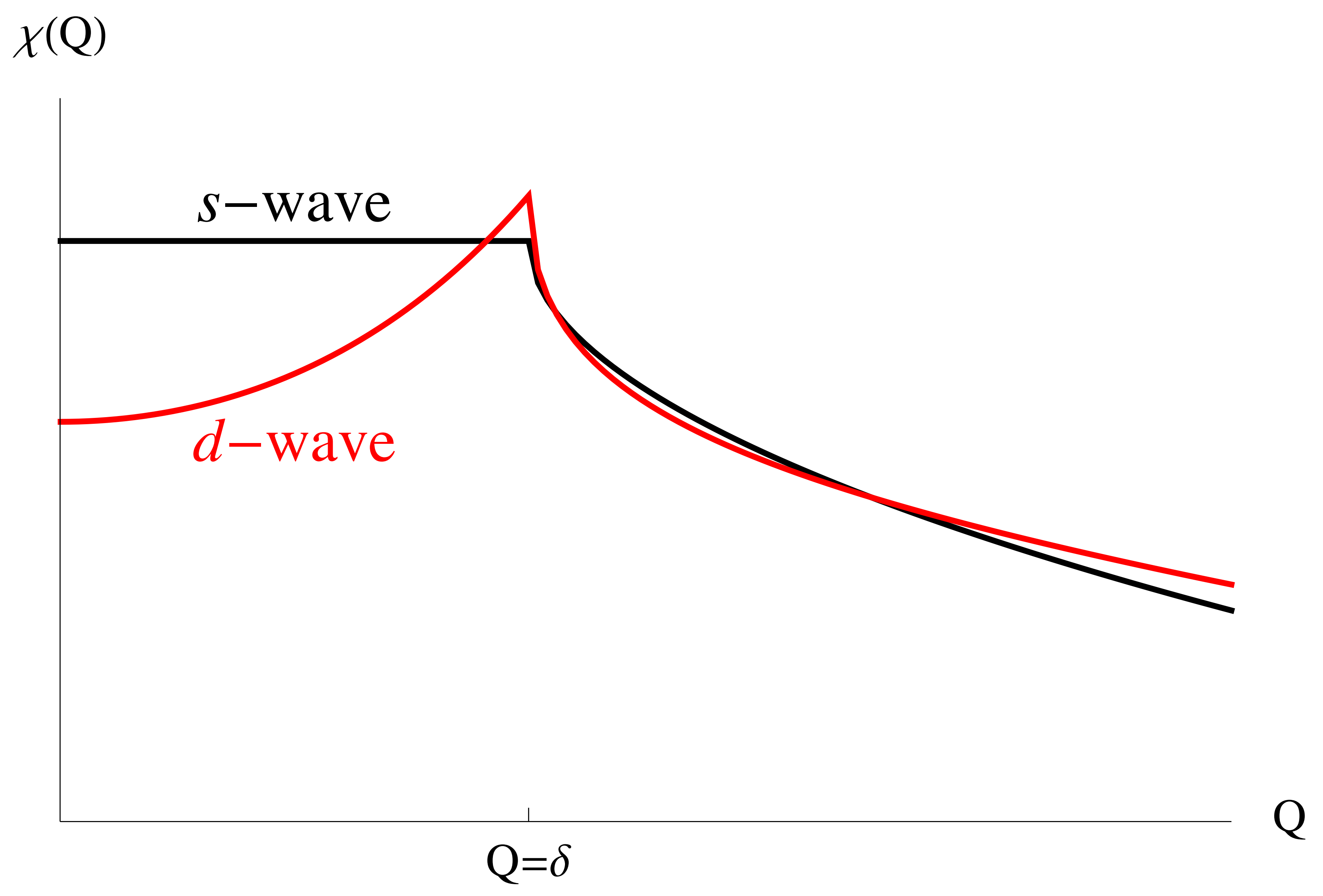


- Nematic states in the *spin triplet channel* with orbital angular momentum l
- Collinear phases known as altermagnets
- Originally known as “nematic-spin-nematic” or “α phase”
- Non-collinear phases (“β phase”)
- If the angular momentum is l=2 the α state is invariant under a $\pi/2$ rotation followed by a spin flip (altermagnet)
- In the β state the spin polarization winds around the (split) FS
- Several candidates for altermagnets (usually insulators)
- Many possible mechanisms: Pomeranchuk instability of a FL, symmetry breaking in multi-band systems, …
- In the continuum the symmetry is ~ SO(3)⊗SO(2) (broken to point group)

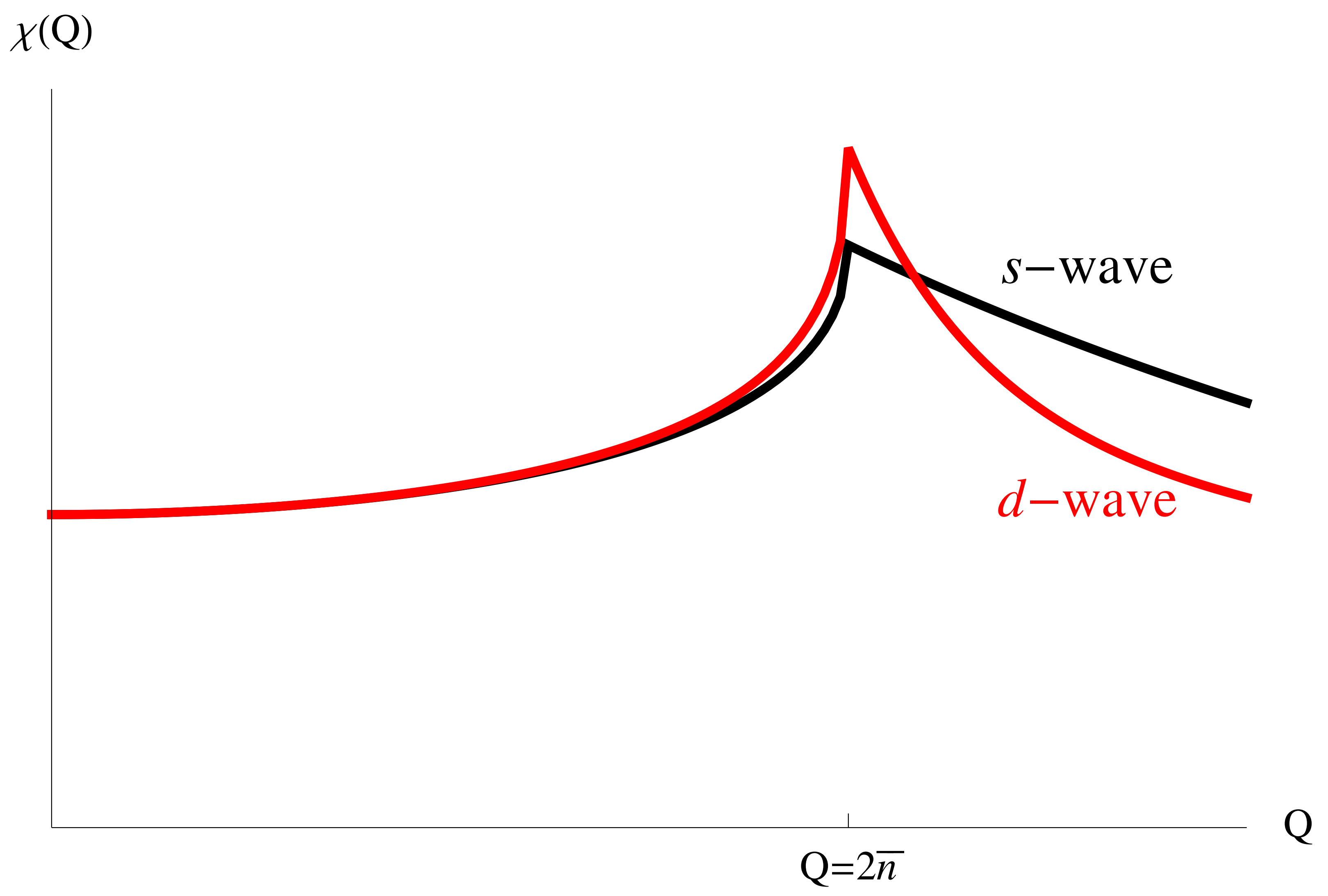


Spin-singlet superconductivity in nematic phases in the spin triplet channel
Soto-Garrido and EF, PRB 2014

- Spin-Triplet p-wave pairing susceptibility is always divergent and p-wave SC is always present
- In the altermagnet metallic phase, near its onset, singlet and triplet SC channels and non-uniform SC states can compete with p-wave depending on the strengths of the interactions



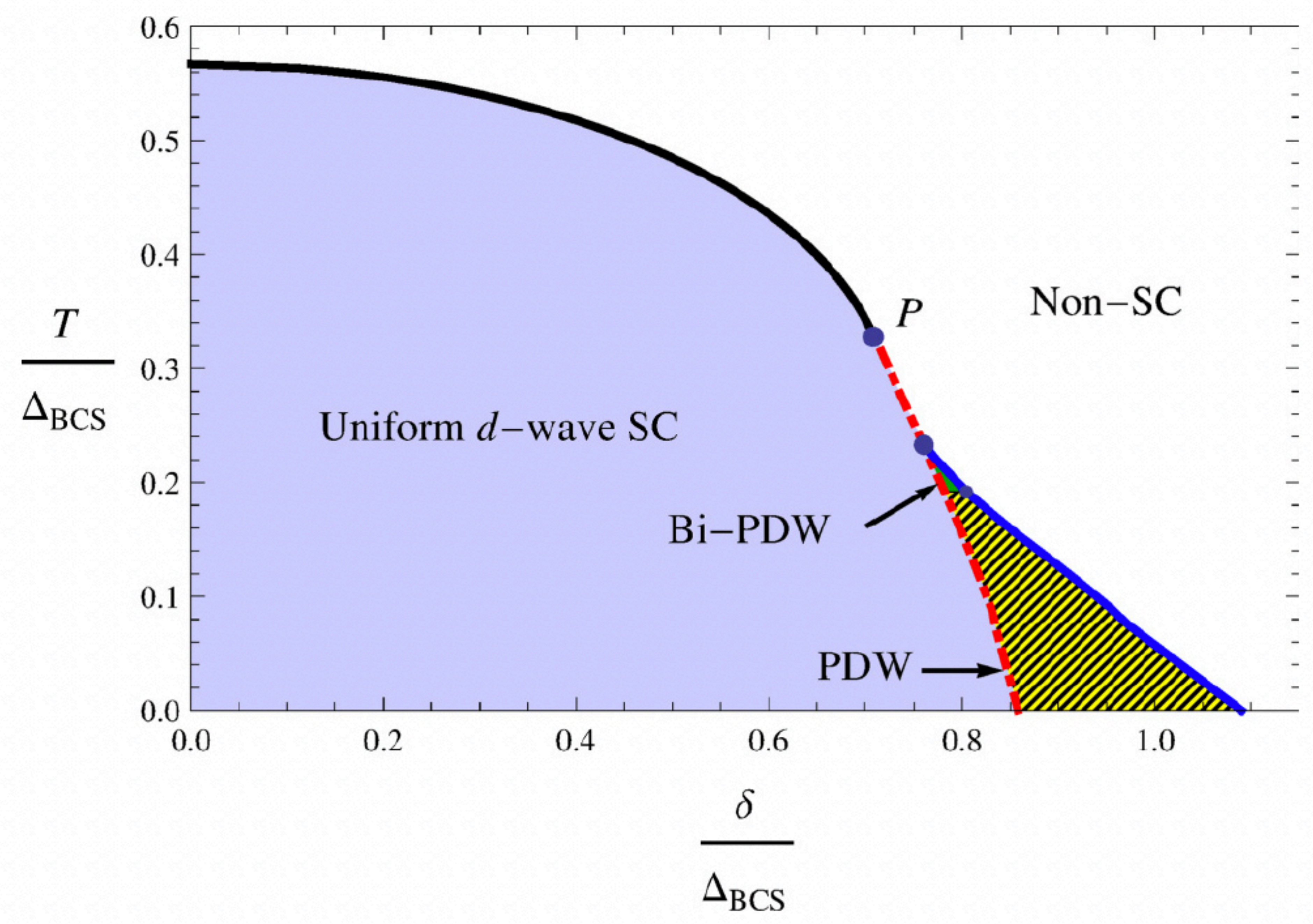
Spin-singlet pair field susceptibility in the s-wave and d-wave channels in the α (altermagnet) phase

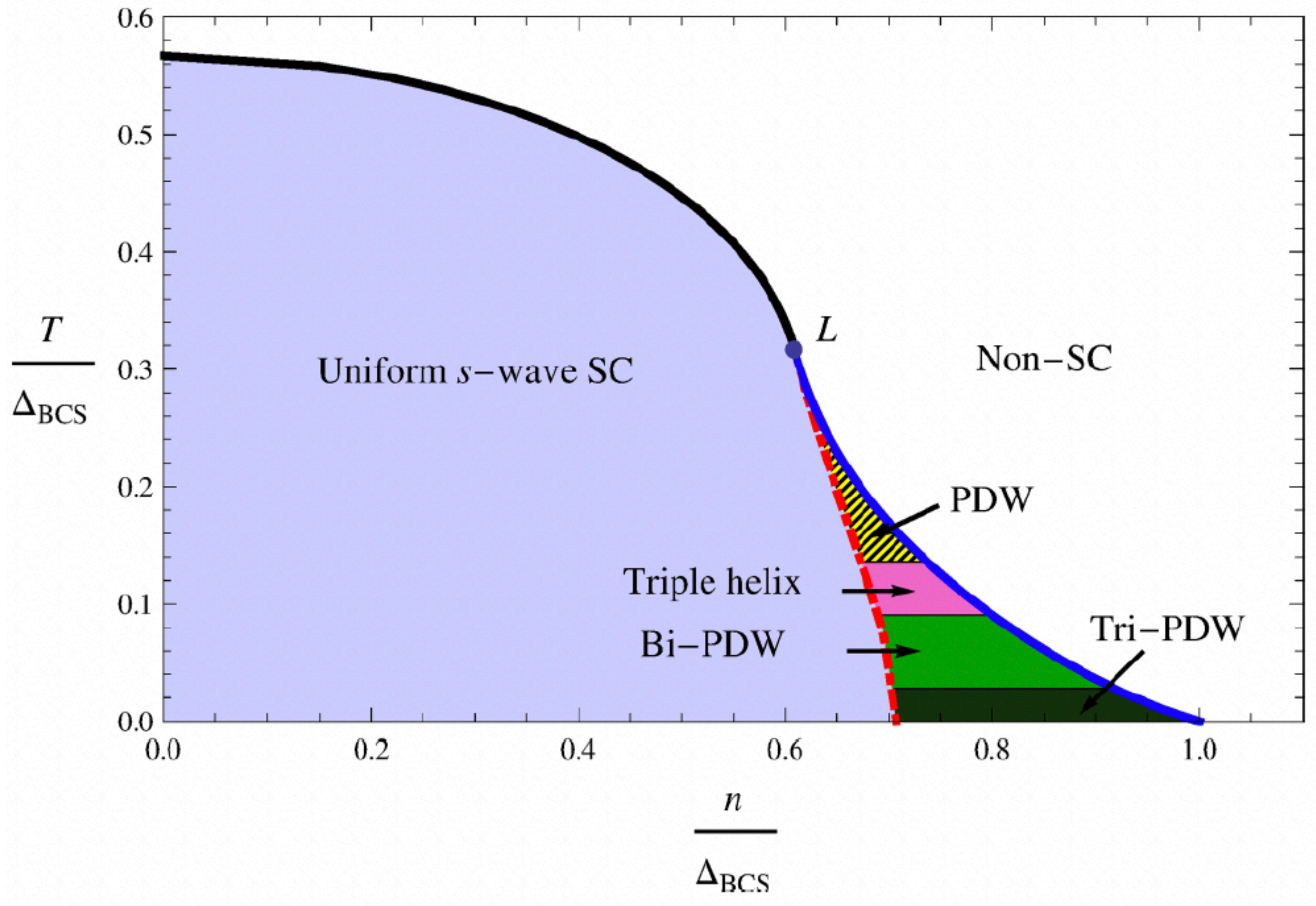


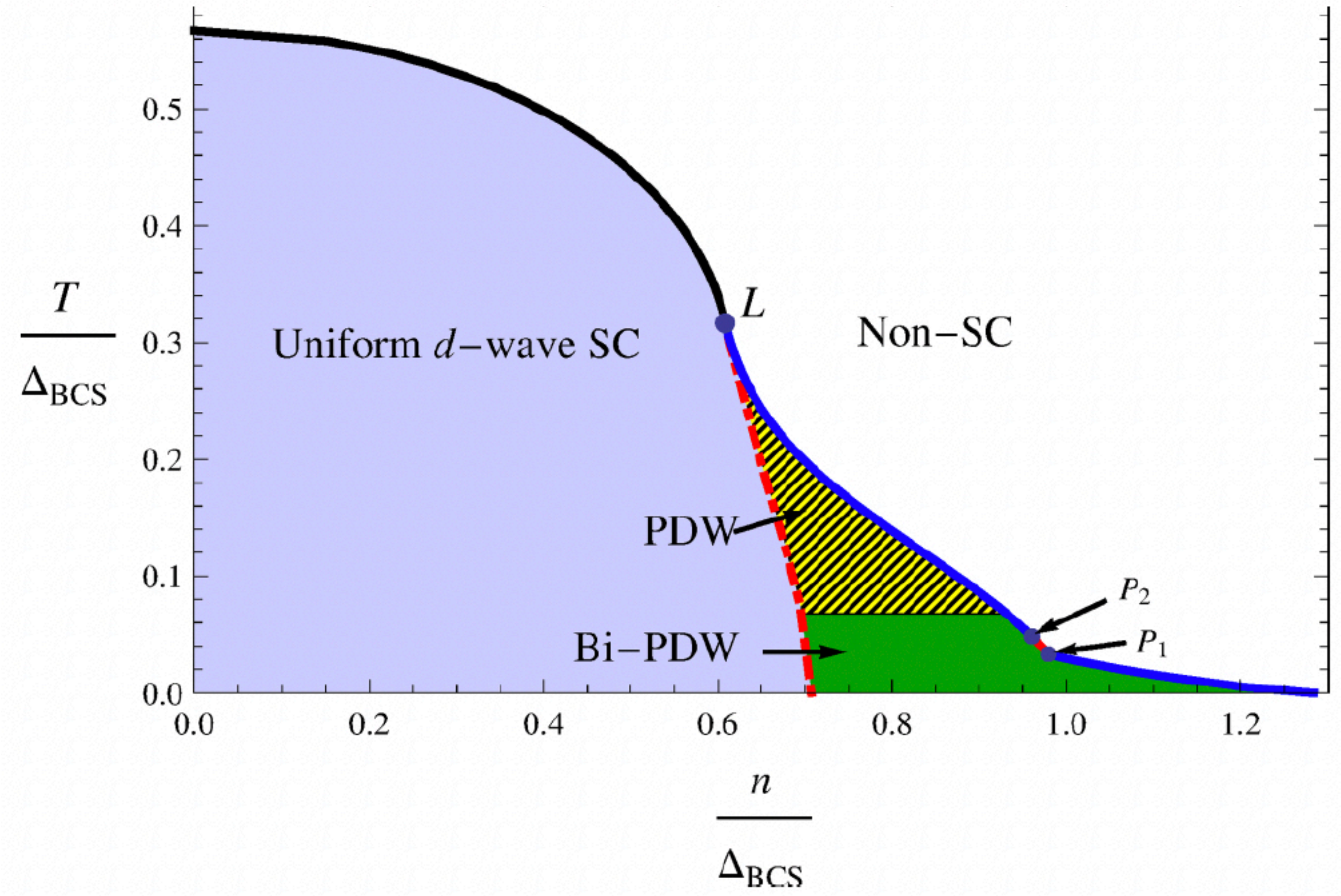
Spin-singlet pair field susceptibility in the s-wave and d-wave channels in the β phase

Spin-singlet superconductivity in nematic phases in the spin triplet channel

- Complex phase diagrams with uniform and non-uniform orders
- Uniform d-wave SC and PDW (“LO”) phases exist in the α phase
- In the β phase we can have s or d wave uniform SC states as well as PDWs and helical SC (“FF”) states



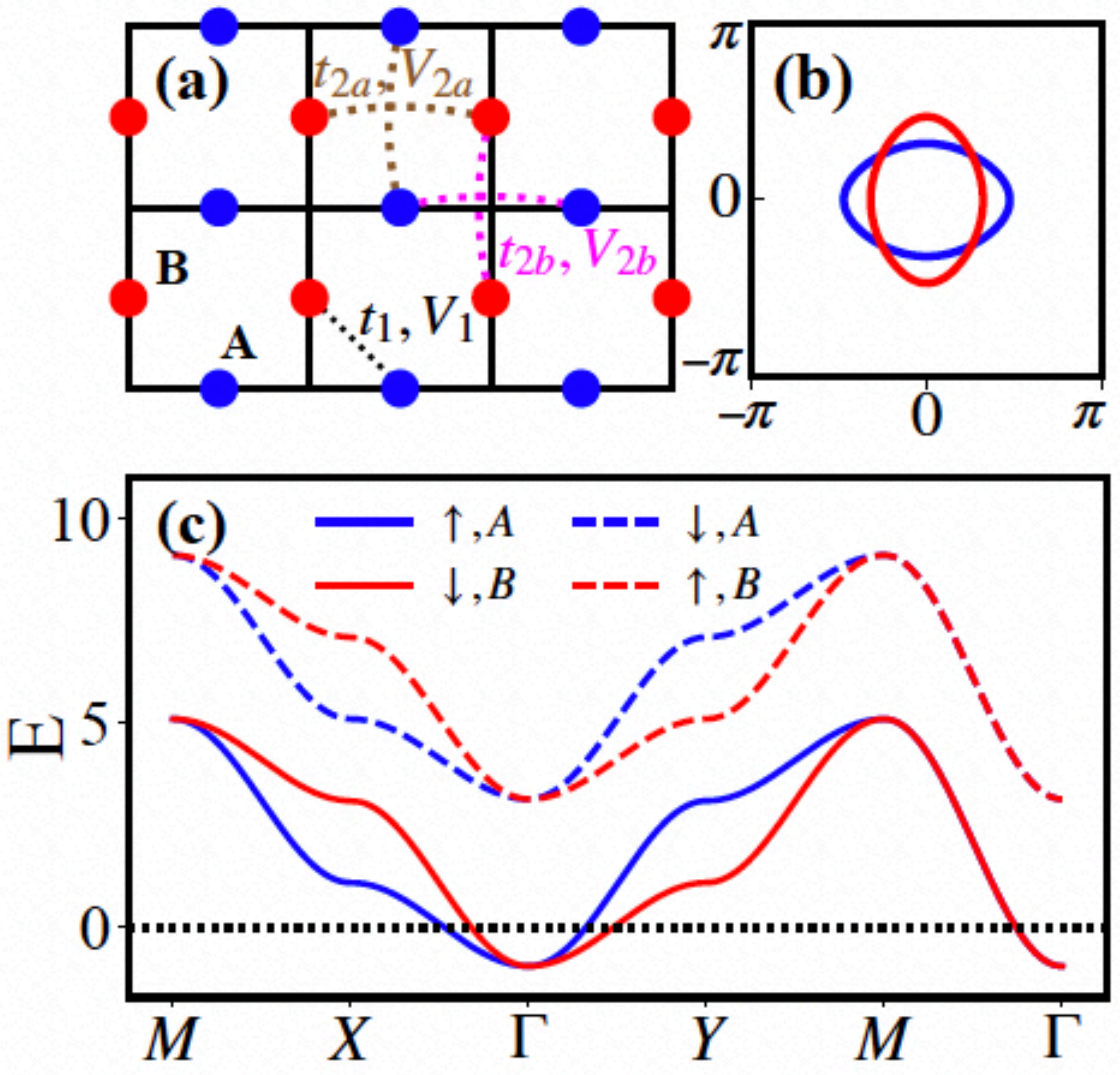




P-wave SC Phases of Altermagnets on the Lieb Lattice

Xuan Zou, Rafael Fernandes and EF, 2025

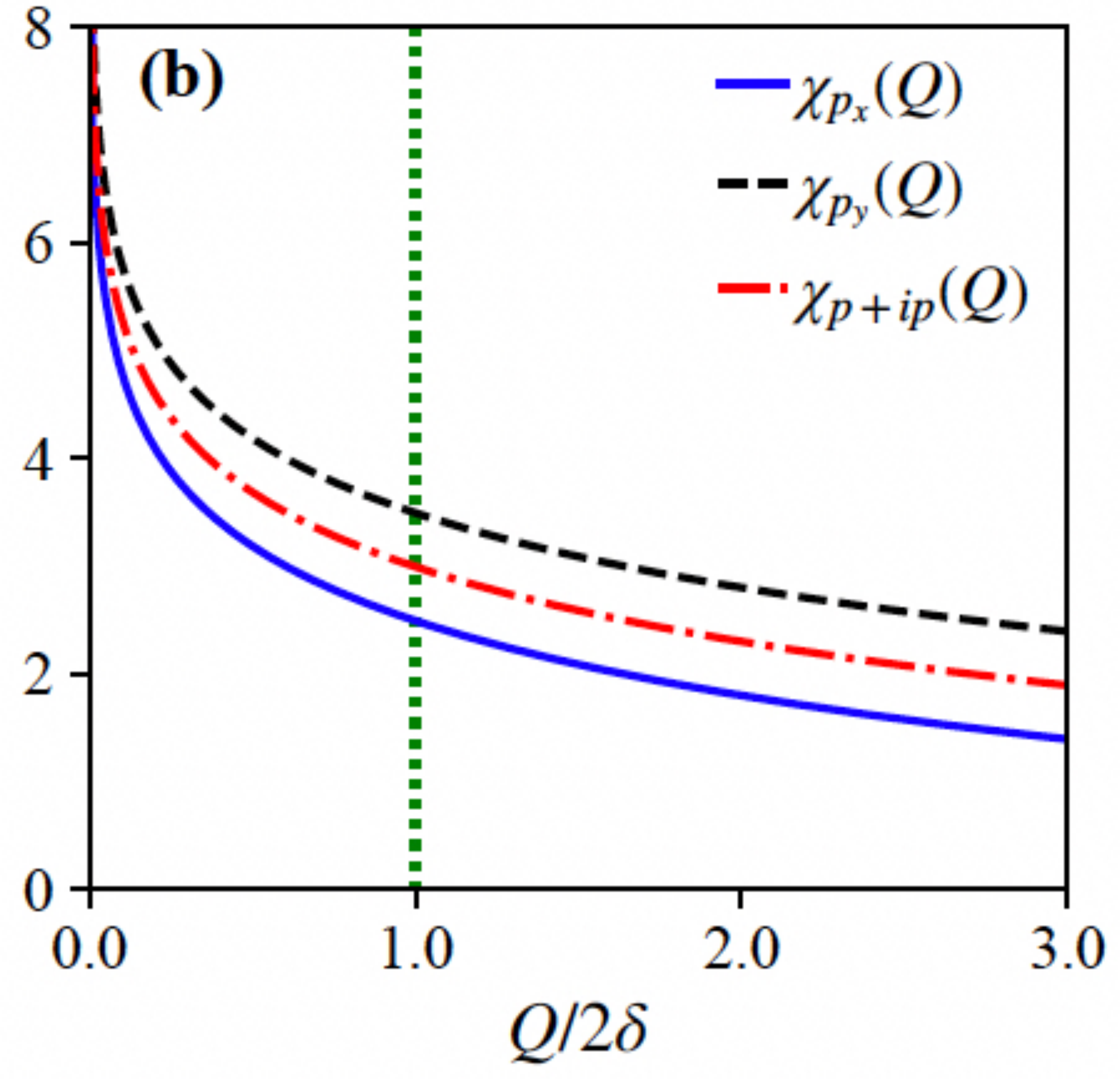
- Lattice models of altermagnets typically have several bands, e.g. the Lieb lattice
- The model has two sublattices, A and B, each with two spin polarizations, ↑ and ↓, with hopping amplitudes t_1 (between A and B) and t_{2a} and t_{2b} between the same sublattice
- Altermagnet: A and B have opposite spin polarization which splits the bands
- For the choice of chemical potential μ only the A↑ and B↓ (or viceversa) bands cross the Fermi energy
- The resulting band structure closely resembles the α phase



Antonenko, Fernandes Venderbos, 2025

Phases of P-wave SC states of Altermagnets

- Attractive interactions in the same sub lattice (and spin polarization)
- Each band, A (spin ↑) and B (spin ↓), has p-wave pairing (always nested) and is strongest in $p_x \pm i p_y$ channel
- $\Delta_s(\boldsymbol{k}) = -\Delta_s^x \sin k_x - \Delta_s^y \sin k_y$, with s=A, B
- These SC states are represented in terms of *four* complex order parameter fields Δ_A^x and Δ_A^y, and Δ_B^x and Δ_B^y, respectively.
- Parametrization: $(\Delta_s^x, \Delta_s^y) = \Delta_s \exp(i\, \theta_s)\, (\cos \alpha_s, \exp(i\, \beta_s) \sin \alpha_s)$
- We include the effects of nematic and loop order fluctuations which enable symmetry-allowed terms in the Landau theory
- We find that $\beta_s = \pm\pi/2 \Rightarrow p_x + i p_y$ order with p_x and p_y having ≠ strengths (unless α_s $\pi/4$)

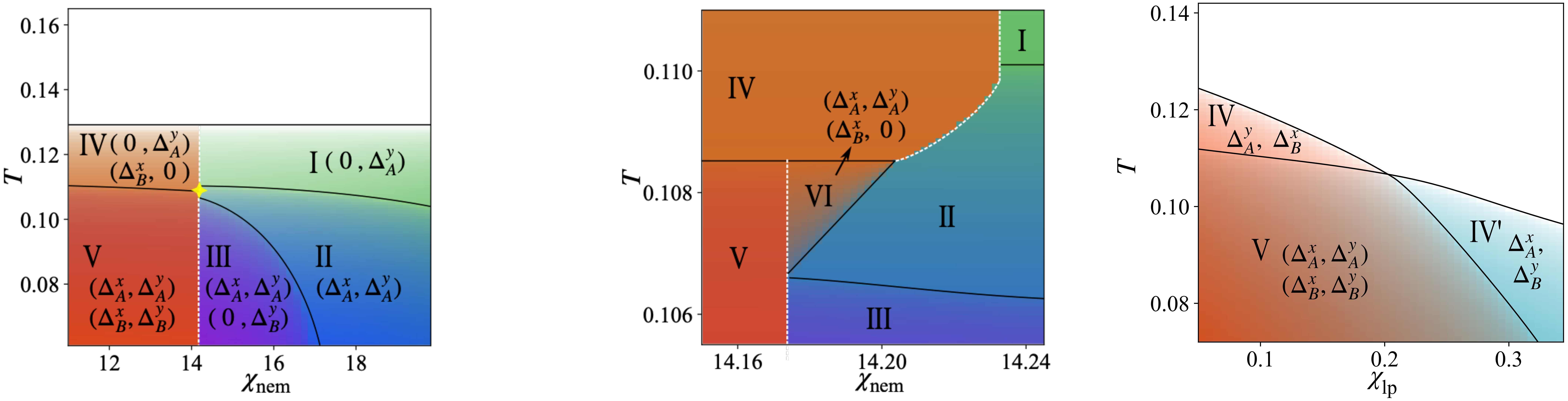


Landau Free Energy

- $r_x \sim T\text{-}T_c^x$ and $r_y \sim T\text{-}T_c^y$ are the usual Landau parameters, corrected by quantum nematic and loop current fluctuations (lower T_c)
- The 2nd, 3rd and 4th coefficients also get corrected
- Same applies to the coefficients u_x, u_y and u_{xy}
- C, D, and D' are due to quantum nematic and current loop current fluctuations (compete)
- E is due entirely to quantum loop current fluctuations
- All coefficients are obtained in a Gorkov expansion about Tc
- The 4th and and last quartic terms force the relative phases of Δ_s^x and Δ_s^y to be the same and equal to $\pm\pi/2$
- $\Rightarrow \mathbb{Z}_2$ ("Ising") global time-reversal symmetry
- If the last term is absent this symmetry becomes $\mathbb{Z}_2 \otimes \mathbb{Z}_2$
- The total global symmetry is $U(1)_A \otimes U(1)_B \otimes \mathbb{Z}_2$

$$
\begin{aligned}
F =& r_x\left(|\Delta_x^A|^2 + |\Delta_y^B|^2\right) + r_y\left(|\Delta_y^A|^2 + |\Delta_x^B|^2\right)\\
&+ u_x\left(|\Delta_x^A|^4 + |\Delta_y^B|^4\right)\\
&+ u_y\left(|\Delta_y^A|^4 + |\Delta_x^B|^4\right)\\
&+ u_{xy}\left(|\Delta_x^A|^2|\Delta_y^A|^2 + |\Delta_x^B|^2|\Delta_y^B|^2\right)\\
&+ \frac{u_{xy}}{4}\left[(\Delta_x^A\Delta_y^{A*})^2 + (\Delta_x^B\Delta_y^{B*})^2 + \text{c.c.}\right]\\
&+ C\left(|\Delta_x^A|^2|\Delta_x^B|^2 + |\Delta_y^A|^2|\Delta_y^B|^2\right)\\
&+ D|\Delta_x^A|^2|\Delta_y^B|^2 + D'\Delta_y^A|^2|\Delta_x^B|^2\\
&- E\left(\Delta_x^{A*}\Delta_x^B\Delta_y^A\Delta_y^{B*} + \text{c.c.}\right).
\end{aligned}
$$

Phase Diagrams of the SC States of Altermagnets



- Nematic fluctuations ($\chi_{\rm nem}$) and loop order fluctuations ($\chi_{\rm lp}$) lead to phases in which different components of the altermagnet SC have expectation values
- The SC states generally break time reversal and are nematic order

Outlook: Vestigial Phases, Fractionalized Vortices and Topological Order

- Multi-component SC states which break time reversal and/or point group symmetry
- Phase diagrams with unusual richness
- Multi-component SC states host fractionalized vortices
- Kosterlitz-Thouless phase transitions (in 2D)
- Vortex melting leads to phases with partially melted orders and higher charge SC condensates
- Phases with vestigial time reversal breaking and/or nematic order
- Time-reversal breaking suggest that these SC phases can be topological SC states