Persistent spin currents

in Superconducting Altermagnets

Marcel Franz, October 2025
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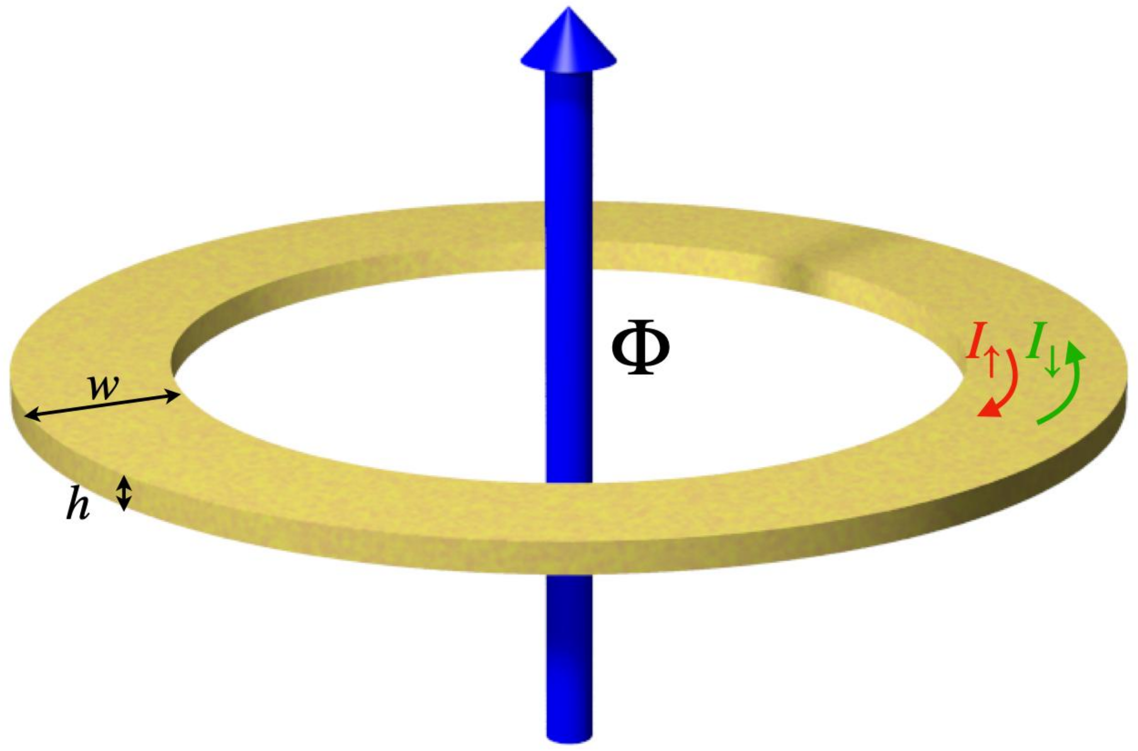

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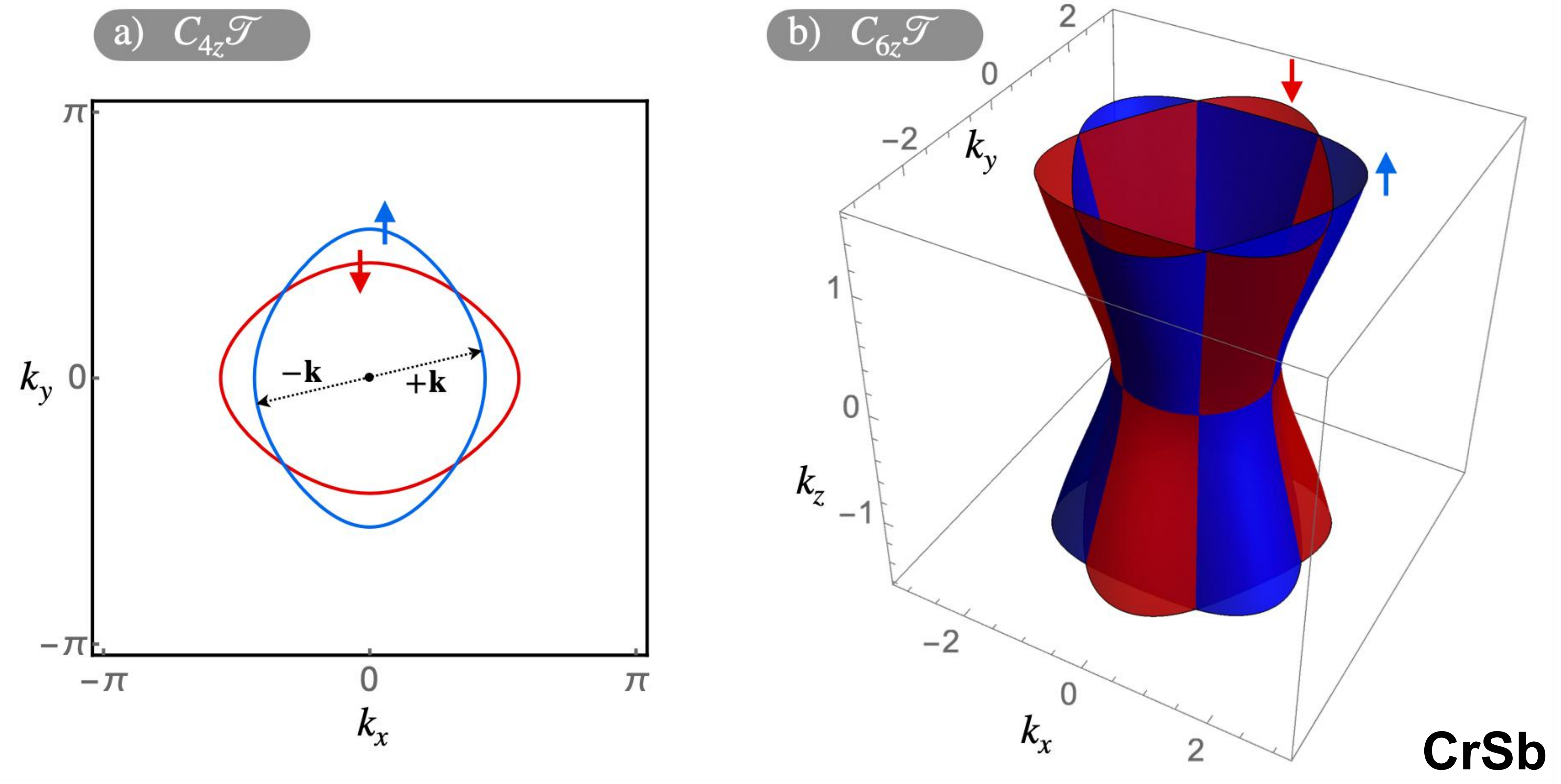
arXiv:2509.03774, arXiv:2507.22139

I.— Motivation: Altermagnetic metals may form unconventional SUPERCONDUCTORS

- Altermagnets are characterized by spin-split Fermi surfaces but vanishing total magnetization.
- An interesting question arises: *"When cooled to a low temperature what are the natural superconducting instabilities of metallic altermagnets?"*
- Answer: One expects unconventional spin-triplet, odd-parity superconducting orders!



Natural zero-momentum Cooper pairs: "equal-spin triplet" channel

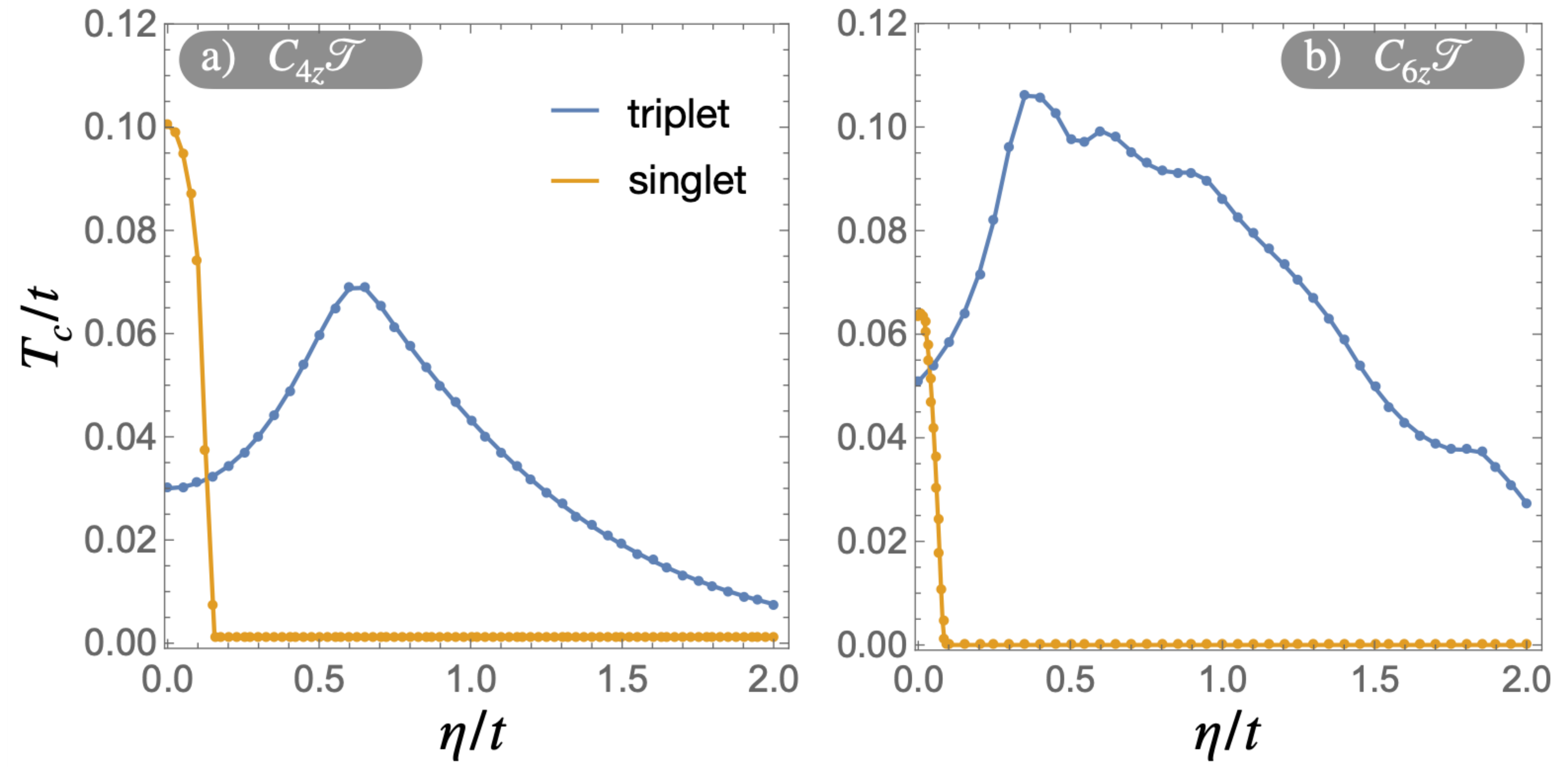
$|\uparrow\uparrow\rangle(p_x \pm ip_y)$ and $|\downarrow\downarrow\rangle(p_x \pm ip_y)$

I.—Motivation: Altermagnetic metals may form unconventional SUPERCONDUCTORS

- This intuition is indeed confirmed by a model calculation for both $C_{4z}\mathcal{T}$ and $C_{6z}\mathcal{T}$ altermagnets:

$$h_0(\boldsymbol{k}) = -2t(\cos k_x + \cos k_y) - 2\eta\sigma_z(\cos k_x - \cos k_y)$$

+ weak attraction V_1 on n.n. sites

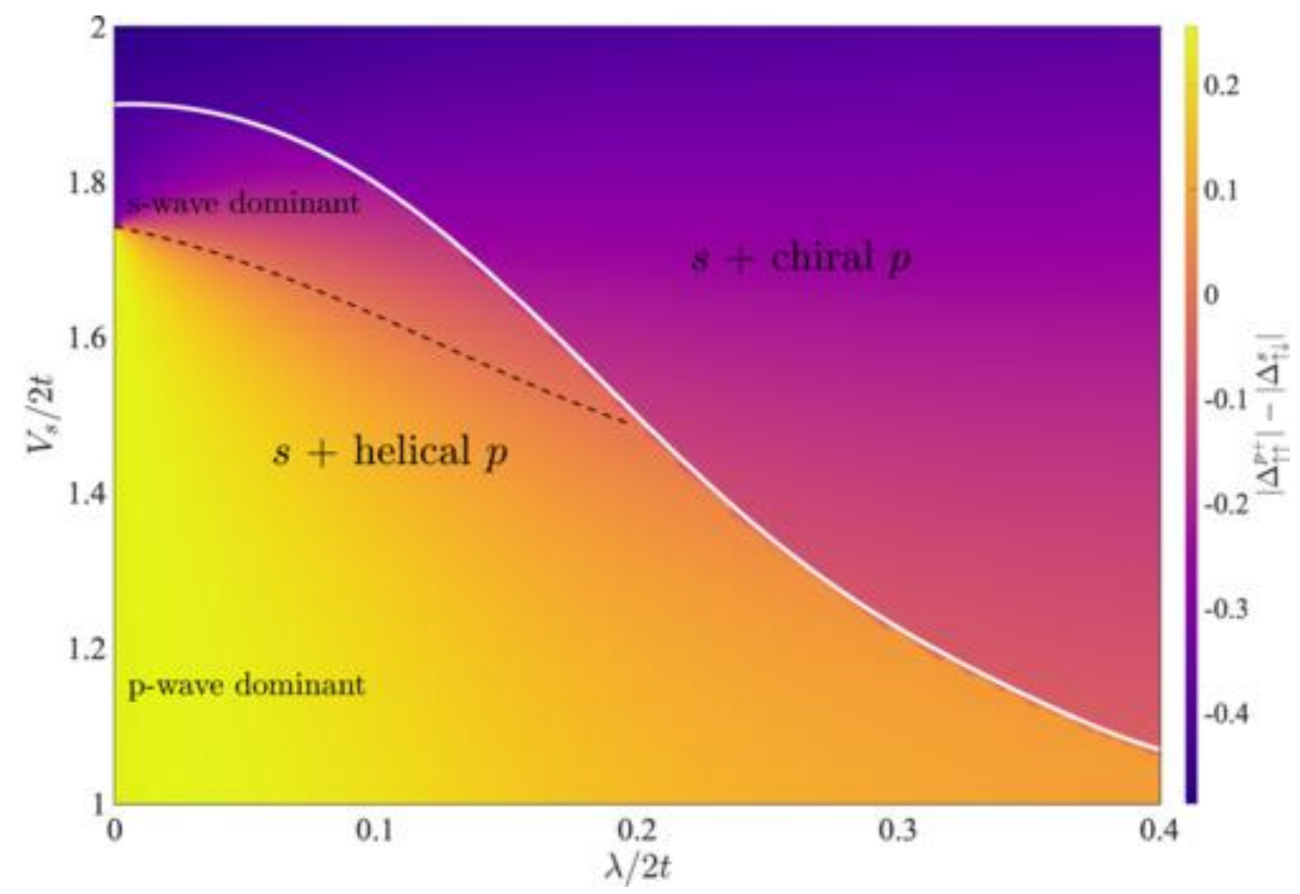


This leads to gap equations for singlet and triplet order parameters:

$$\Delta_0 = \frac{V_1}{2N}\sum_{\boldsymbol{k}} \frac{\Delta_0 C_{\boldsymbol{k}}^2}{\epsilon_{\boldsymbol{k}}} \frac{\sinh\beta\epsilon_{\boldsymbol{k}}}{\cosh\beta\xi_{\boldsymbol{k}-} + \cosh\beta\epsilon_{\boldsymbol{k}}},$$ **singlet** ***s*-wave**

$$\Delta_\sigma = \frac{V_1}{2N}\sum_{\boldsymbol{k}} \frac{\Delta_\sigma |S_{\boldsymbol{k}\sigma}|^2}{E_{\boldsymbol{k}\sigma}} \tanh\frac{1}{2}\beta E_{\boldsymbol{k}\sigma},$$ **triplet** $p_x \pm ip_y$

$$C_{\boldsymbol{k}} = \cos k_x + \cos k_y, \quad S_{\boldsymbol{k}\sigma} = \sin k_x \pm i\sin k_y,$$



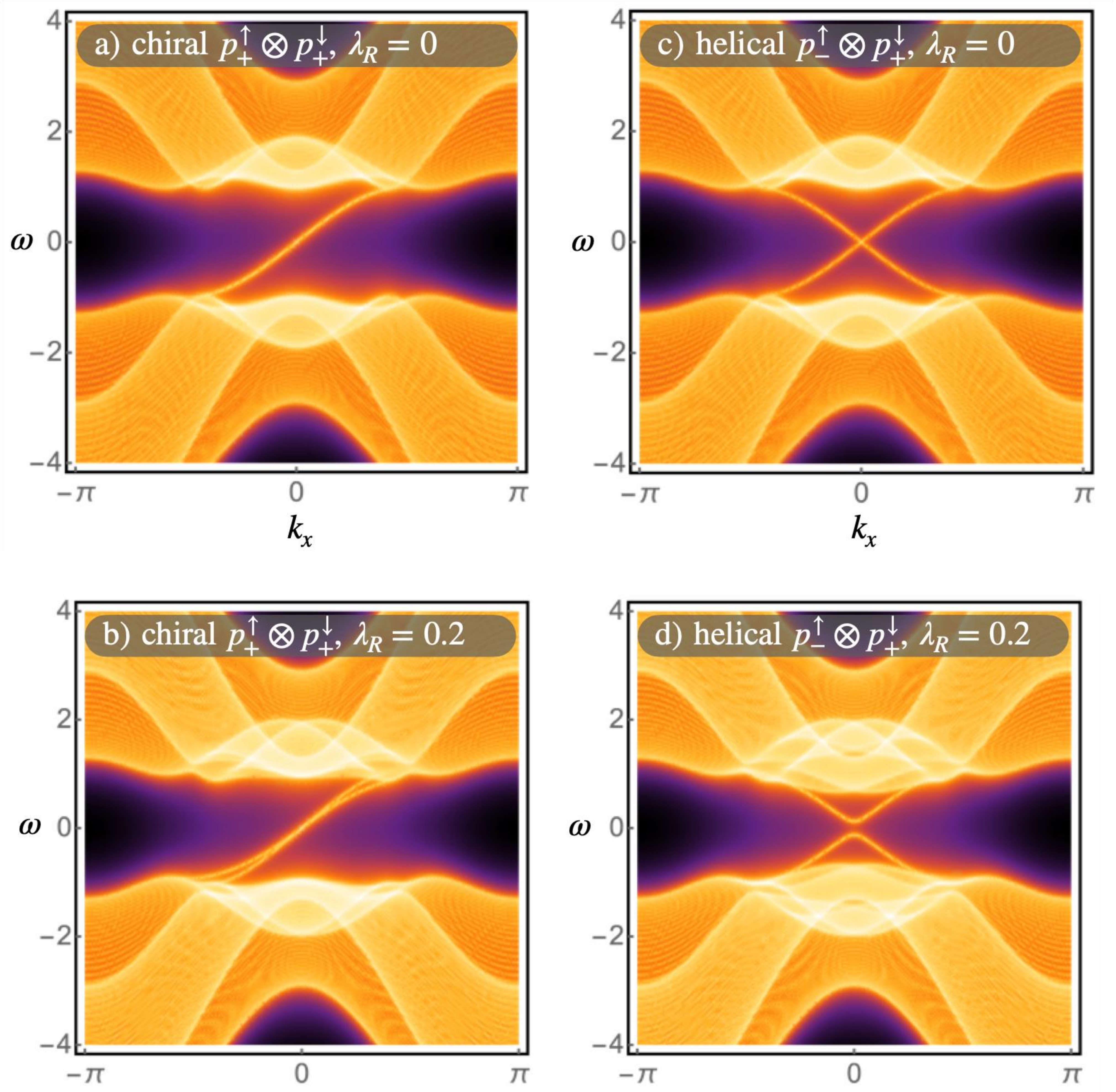
D. Zhu, Z.-Y. Zhuang, Z. Wu, and Z. Yan, PRB 108, 184505 (2023).

I.—Sidenote: Non-trivial Topology

Chiral *p*-wave superconductors are TOPOLOGICAL, specifically there are four distinct possibilities:

Ground state	Abbreviation	Chern number	Property
$\lvert\uparrow\uparrow(p_x+ip_y)\rangle \otimes \lvert\downarrow\downarrow(p_x+ip_y)\rangle$	$p_+^\uparrow \otimes p_+^\downarrow$	$+2$	chiral
$\lvert\uparrow\uparrow(p_x-ip_y)\rangle \otimes \lvert\downarrow\downarrow(p_x+ip_y)\rangle$	$p_-^\uparrow \otimes p_+^\downarrow$	0	helical
$\lvert\uparrow\uparrow(p_x+ip_y)\rangle \otimes \lvert\downarrow\downarrow(p_x-ip_y)\rangle$	$p_+^\uparrow \otimes p_-^\downarrow$	0	helical
$\lvert\uparrow\uparrow(p_x-ip_y)\rangle \otimes \lvert\downarrow\downarrow(p_x-ip_y)\rangle$	$p_-^\uparrow \otimes p_-^\downarrow$	-2	chiral

- The two chiral phases are stable, protected by a non-zero Chern number $C = \pm 2$.
- The two helical phases are fragile: helical edge modes are gapped out by perturbations that break the $U(1)^\uparrow \times U(1)^\downarrow$ spin symmetry, e.g. Rashba spin-orbit coupling or in-plane applied magnetic field.



II. — Persistent spin currents and pure spin supercurrents

We argued that a natural SC ground state of the metallic altermagnet is equal spin chiral *p*-wave

$$|\uparrow\uparrow\rangle(p_x \pm ip_y) \text{ and } |\downarrow\downarrow\rangle(p_x \pm ip_y)$$

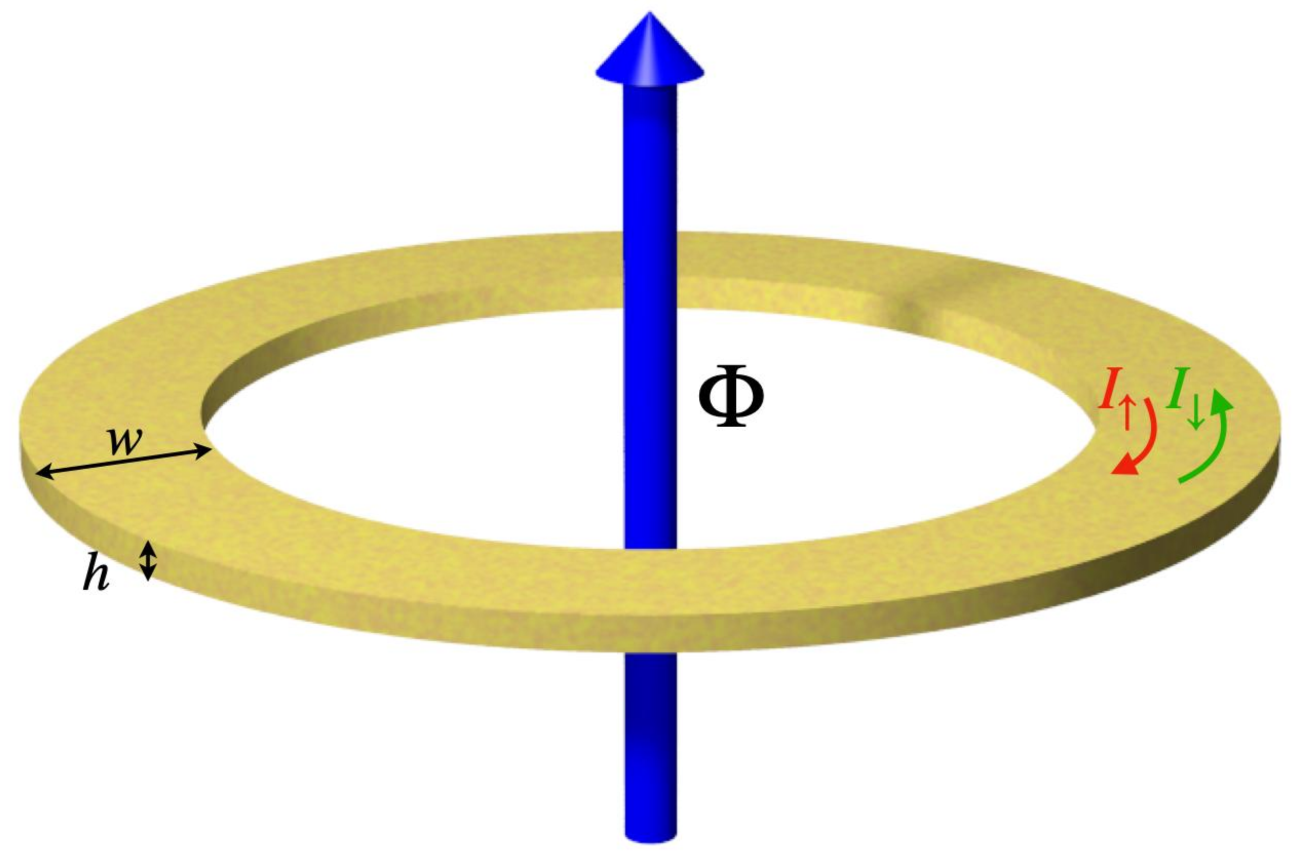
In the strict non-relativistic limit the spin-up and spin-down condensates are decoupled:

$$H = \begin{pmatrix} h^{\uparrow}_{\mathrm{BdG}} & 0 \\ 0 & h^{\downarrow}_{\mathrm{BdG}} \end{pmatrix}$$

One thus expects an interesting possibility of spin-polarized persistent currents in these materials!

In the following we briefly discuss the following novel effects:

A. Pure spin supercurrent in a ring
B. Effect of spin-orbit coupling and magnetic disorder on persistent spin currents
C. "Spin-current dynamo effect" — generation of spin-polarized supercurrent in a *d*-wave altermagnet



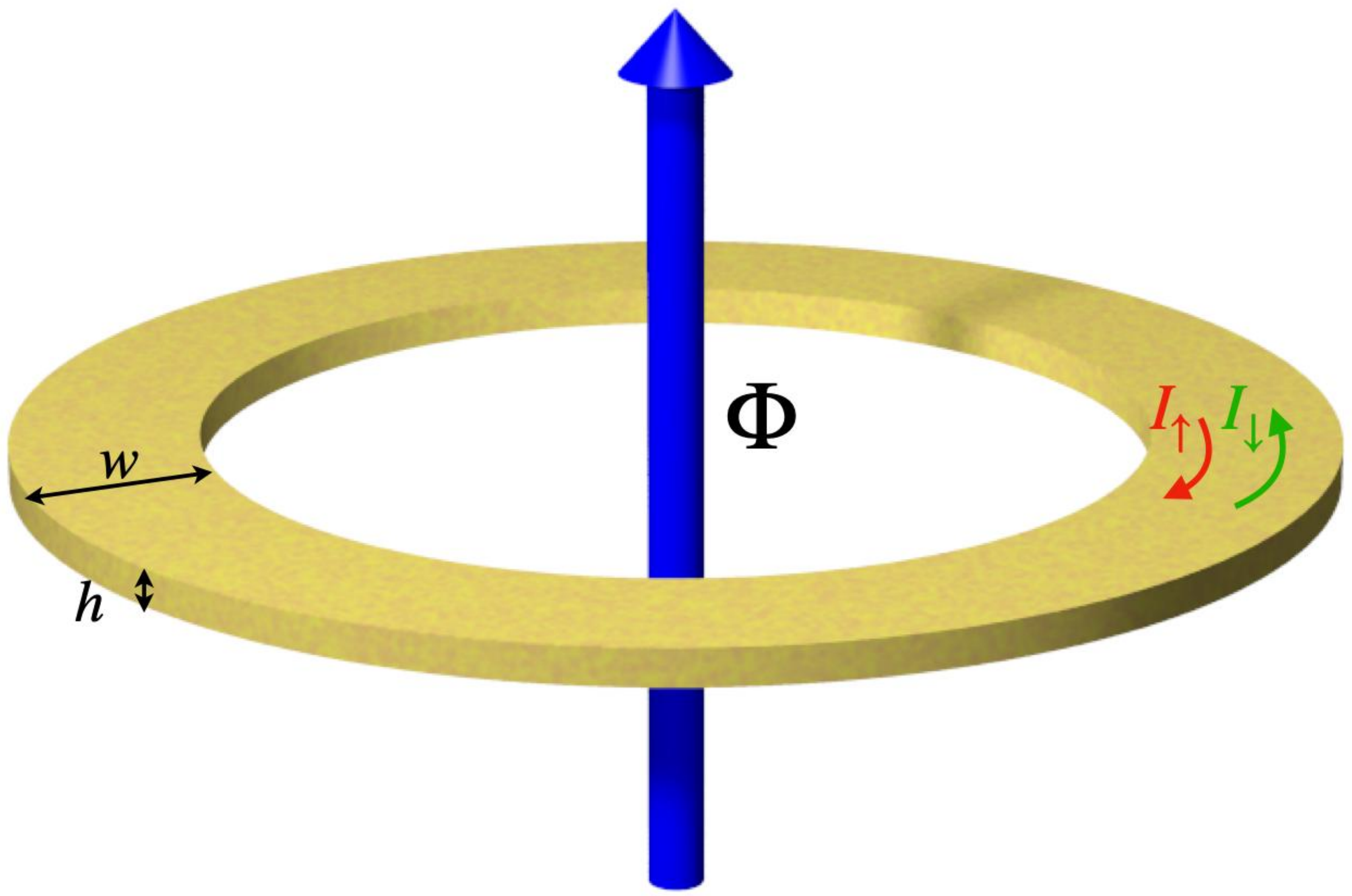
II.A — Pure persistent spin current in a ring

Consider **Ginzburg-Landau free energy** density:

$$f[\psi_\uparrow, \psi_\downarrow] = \sum_\sigma f_\sigma[\psi_\sigma] \boxed{-\frac{1}{2}D(\psi_\uparrow^*\psi_\downarrow + \text{c.c.})} + \frac{B^2}{8\pi},$$

$$f_\sigma[\psi] = \alpha|\psi|^2 + \frac{1}{2}\beta|\psi|^4 + \gamma_\sigma|(i\nabla + \frac{2e}{\hbar c}\boldsymbol{A})\psi|^2.$$

Here $D \propto \lambda_R^2$ is the coupling between the condensates proportional to SOC.



Write $\psi_\sigma = \psi_0 e^{i\varphi_\sigma}$ and define the **current density** per spin as

$$\boldsymbol{j}_\sigma = 4e\frac{\gamma\psi_0^2}{\hbar}(\nabla\varphi_\sigma - \frac{2e}{\hbar c}\boldsymbol{A}).$$

The total current, per spin, then is

$$I_\sigma = S\hat{\theta}\cdot\boldsymbol{j}_\sigma = I_0(n_\sigma - \Phi/\Phi_0), \qquad I_0 = 4eS\frac{\gamma\psi_0^2}{\hbar}\frac{2\pi}{\ell}$$

with $\Phi_0 = \frac{hc}{2e}$ the flux quantum and $n_\sigma = 0, \pm1, \pm2, \ldots$ the vorticity.

When $J = 0$ the total free energy associated with the supercurrent can be expressed as:

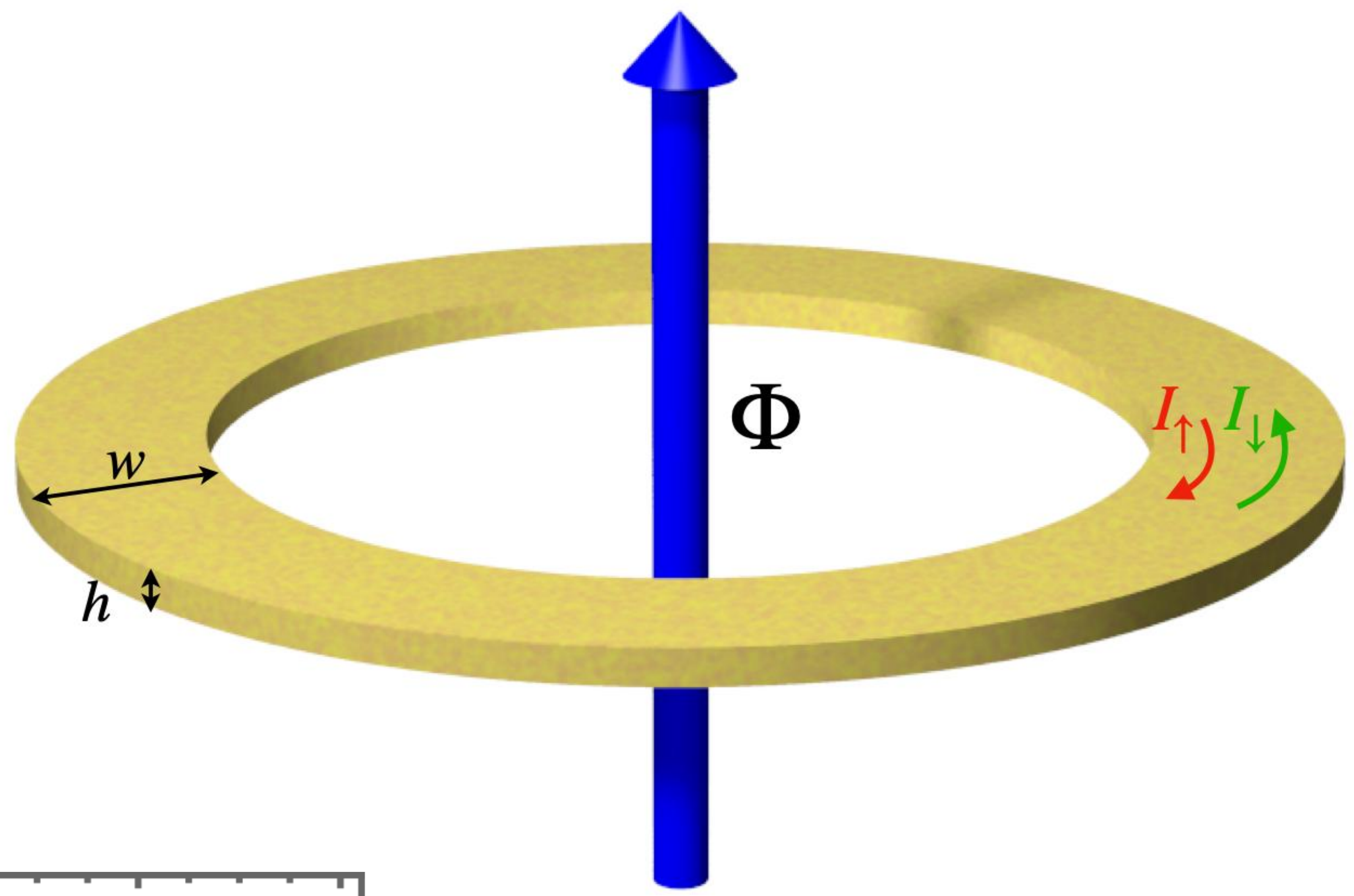
$$F = \frac{1}{2}L_K(I_\uparrow^2 + I_\downarrow^2) + \frac{1}{2}L_G(I_\uparrow + I_\downarrow)^2.$$

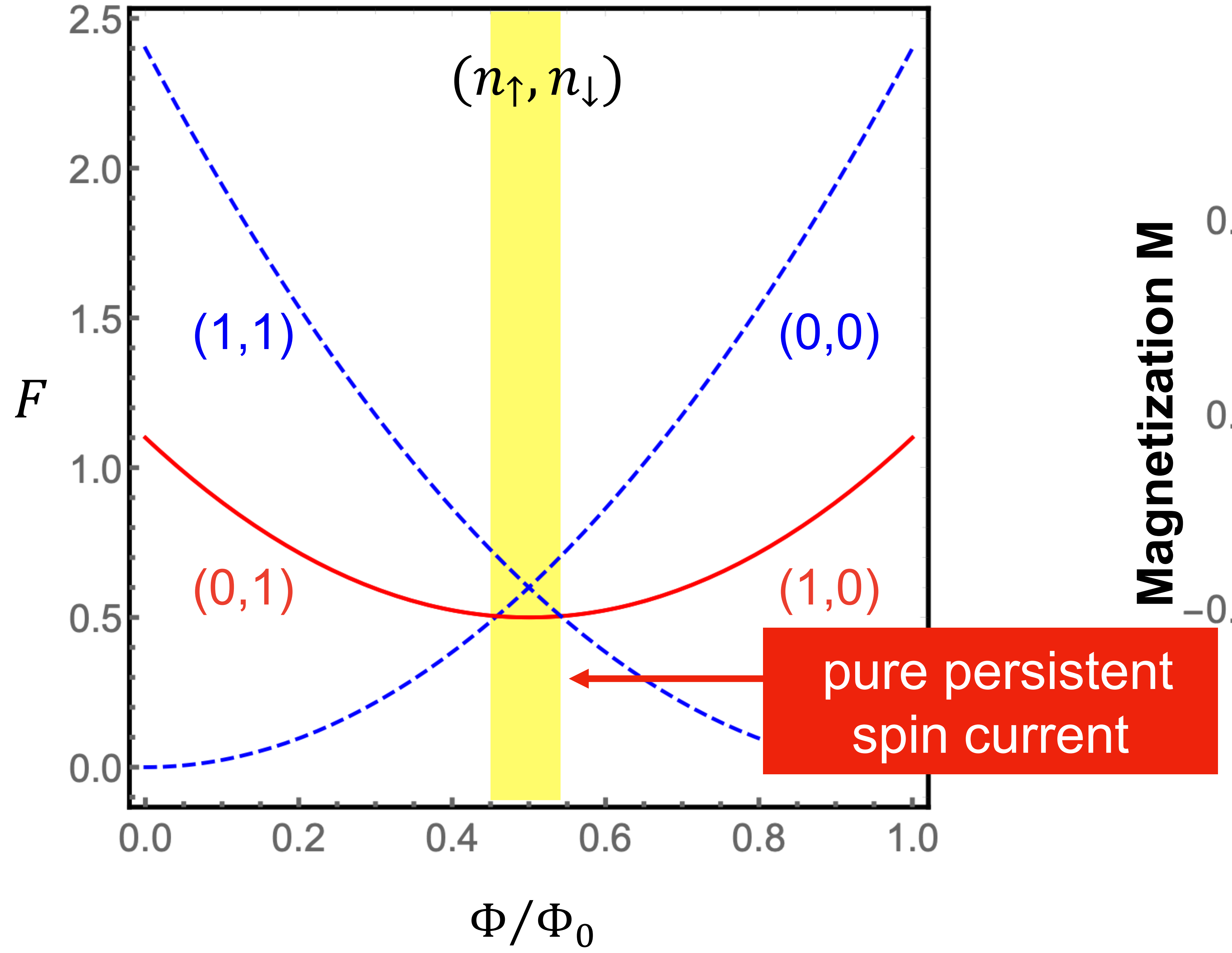
superflow energy **magnetic energy**

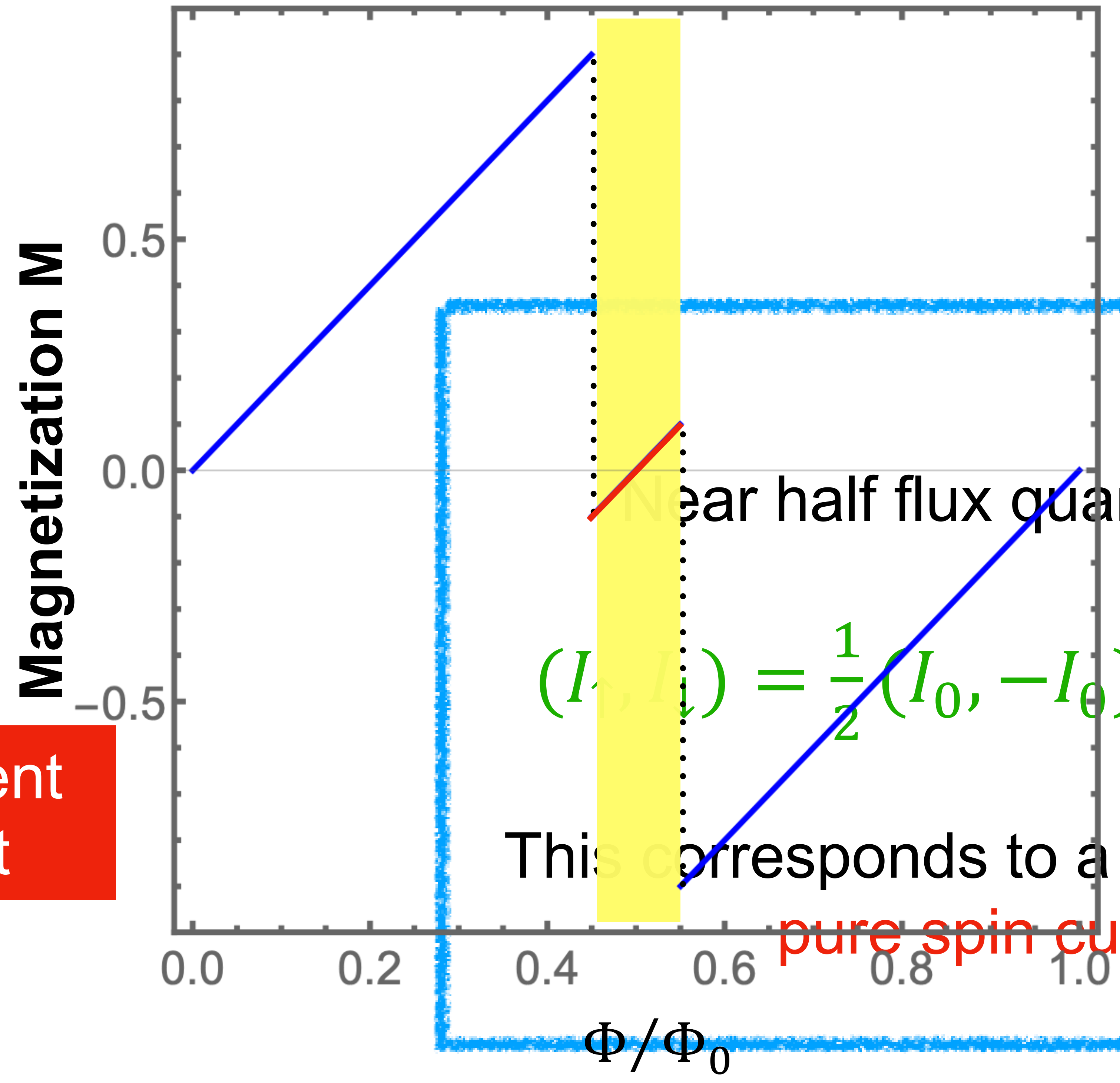
II.A — Pure persistent spin current in a ring

$$F = \frac{1}{2} L_K (I_\uparrow^2 + I_\downarrow^2) + \frac{1}{2} L_G (I_\uparrow + I_\downarrow)^2 .$$

$$I_\sigma = I_0(n_\sigma - \Phi/\Phi_0), n_\sigma = 0, \pm 1, \pm 2, \ldots$$







Near half flux quantum F is minimized when

$(I_\uparrow, I_\downarrow) = \frac{1}{2}(I_0, -I_0)$ or $(I_\uparrow, I_\downarrow) = \frac{1}{2}(-I_0, I_0)$

This corresponds to a "charge counterflow" regime —> pure spin current flows in the ring.

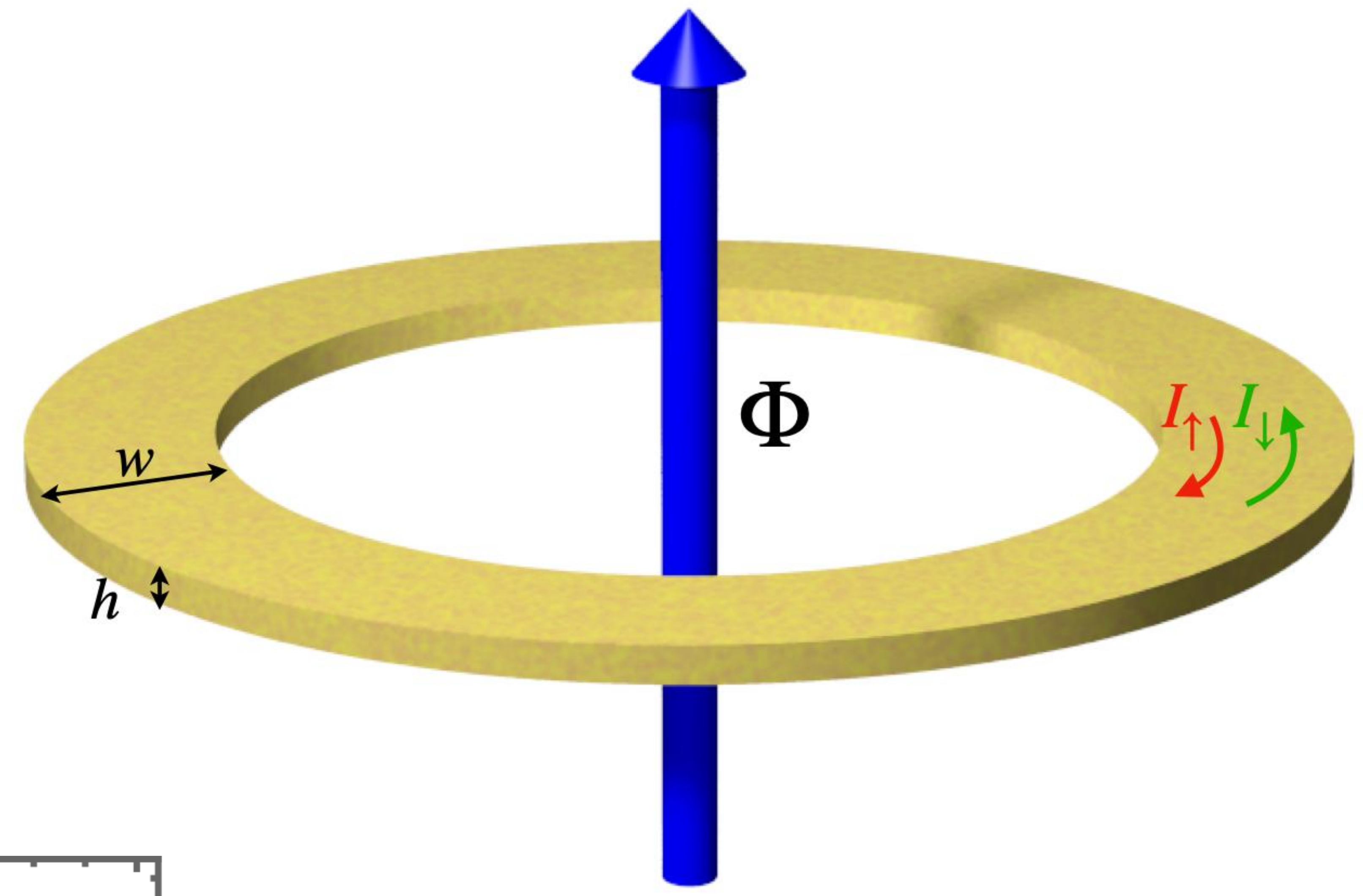
Can be detected via **magnetization measurement** — no spin-current probe is necessary!

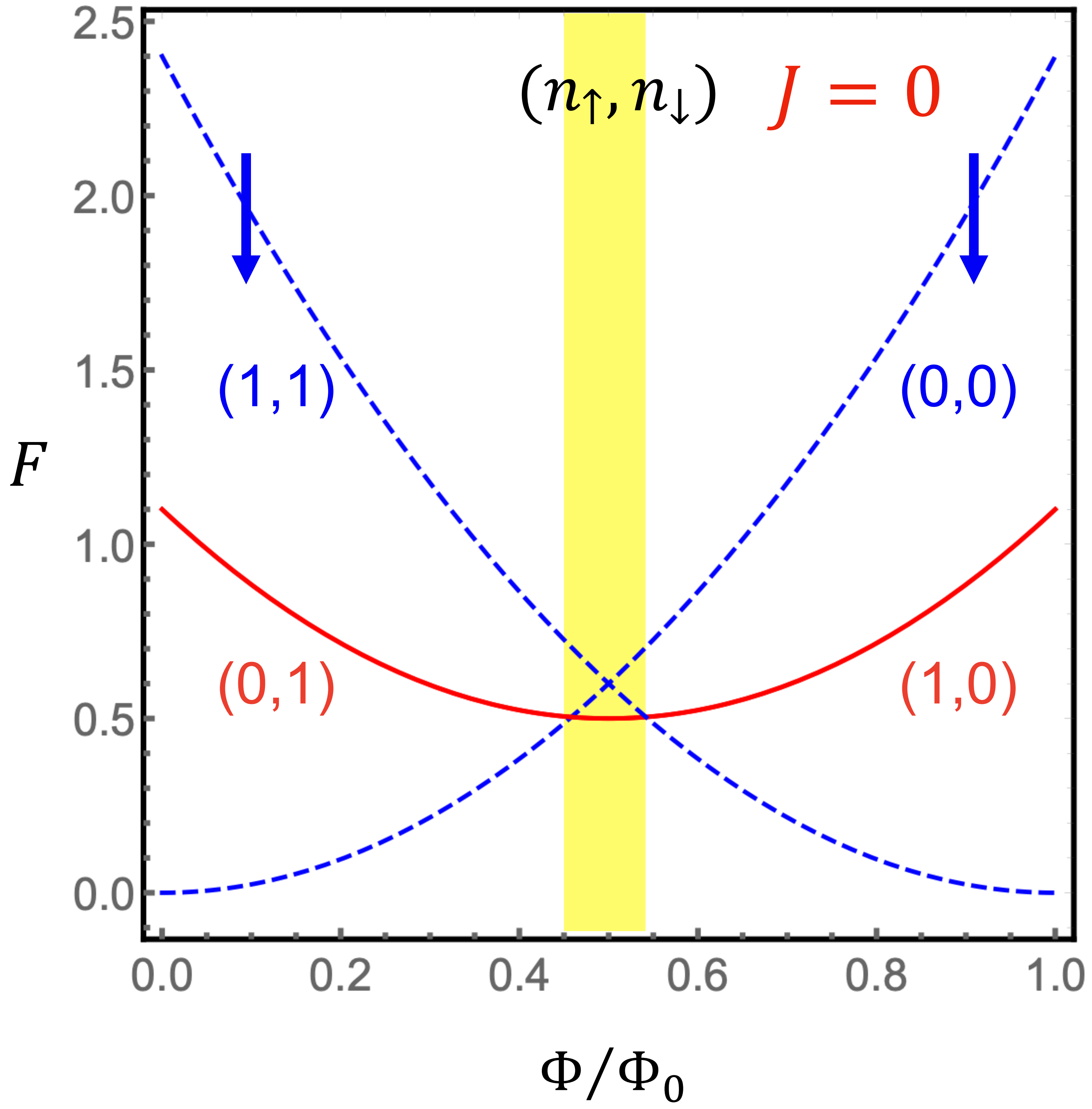
II.A — Pure persistent spin current in a ring

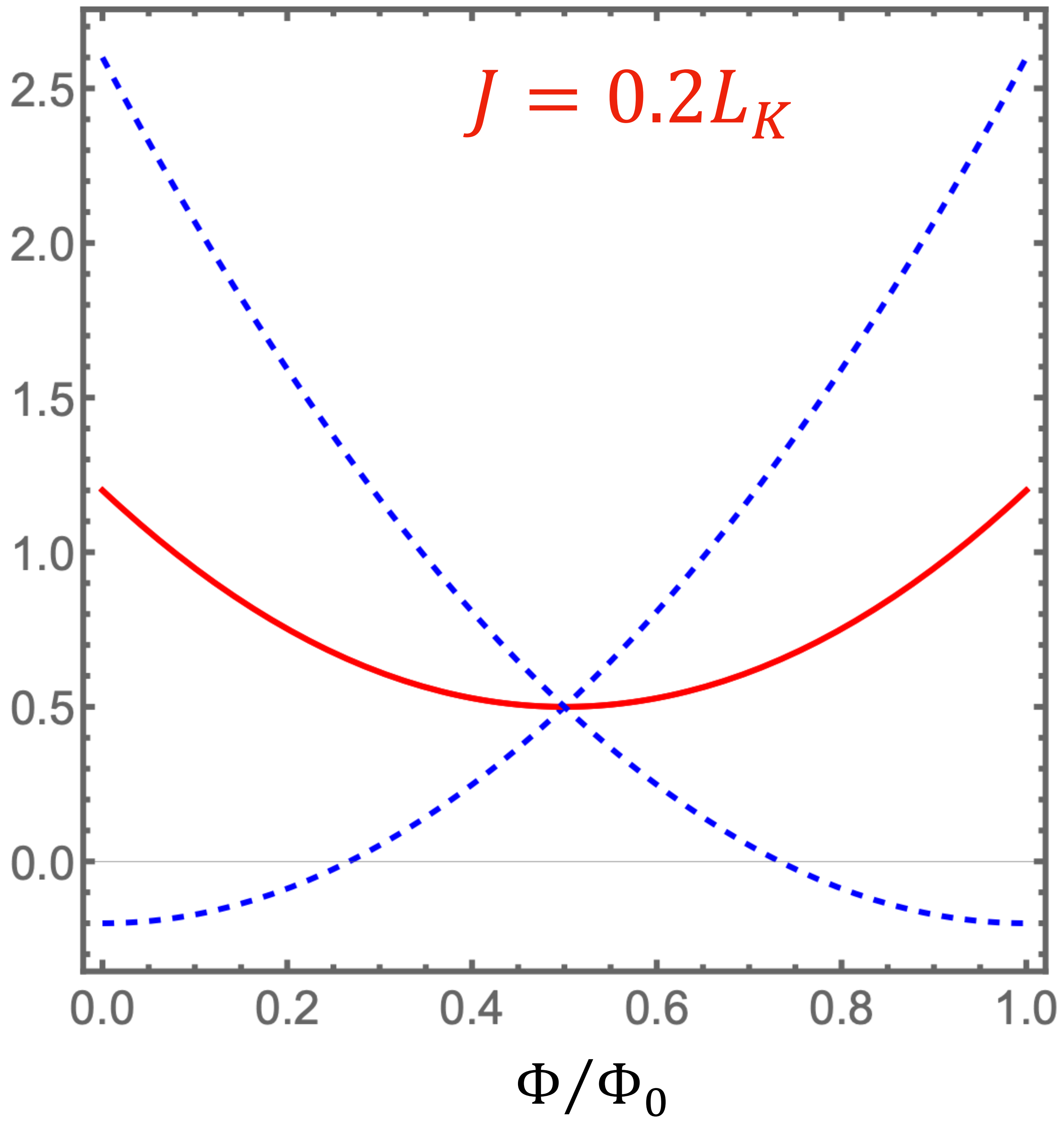
Q: Will spin current survive in the presence of non-zero *D* ?

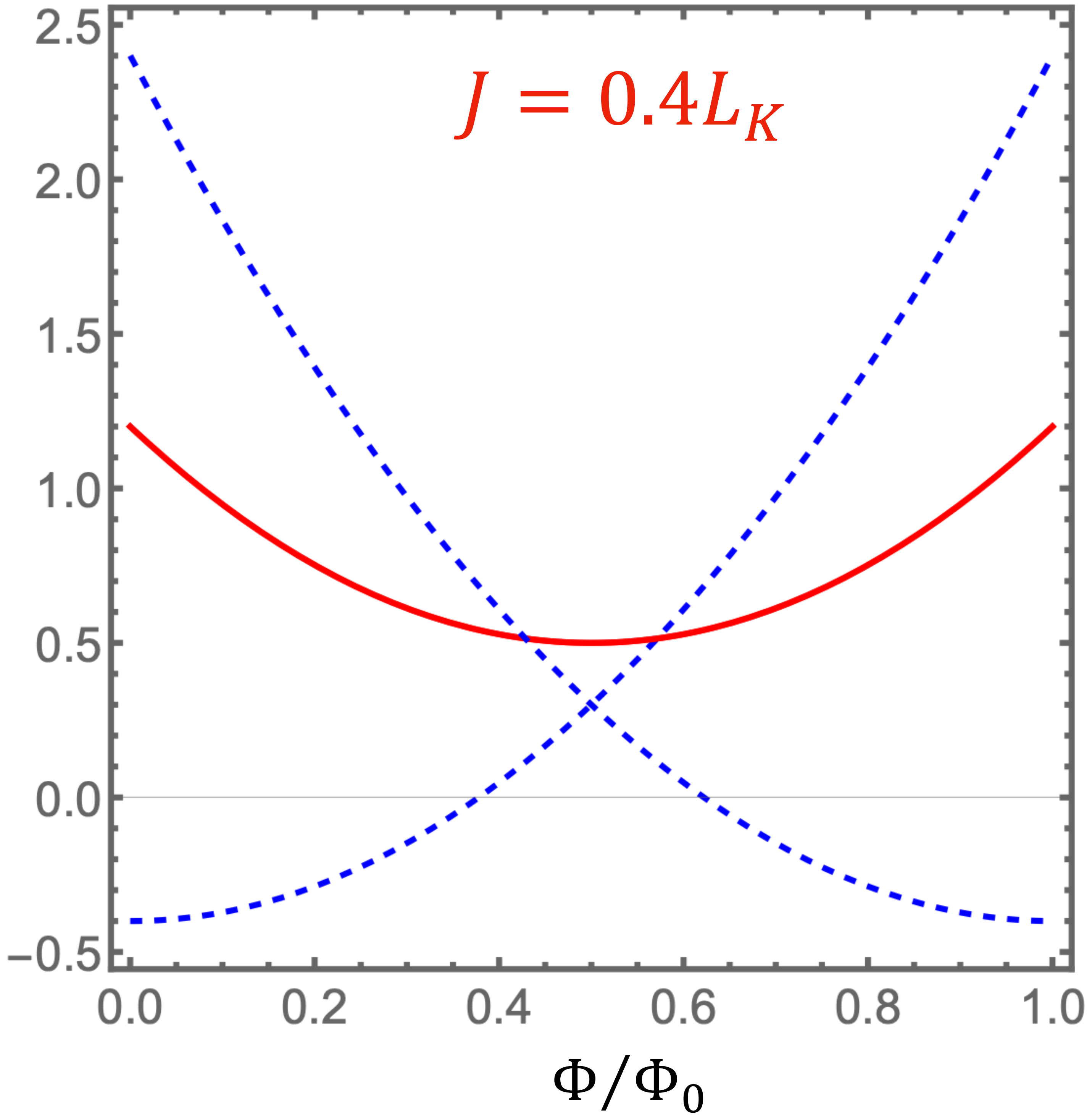
$$f[\psi_\uparrow,\psi_\downarrow]=\sum_\sigma f_\sigma[\psi_\sigma]-\frac{1}{2}D(\psi_\uparrow^*\psi_\downarrow+\text{c.c.})+\frac{B^2}{8\pi},$$

$\sim -J\cos(\varphi_\uparrow-\varphi_\downarrow), J = D\psi_0^2$





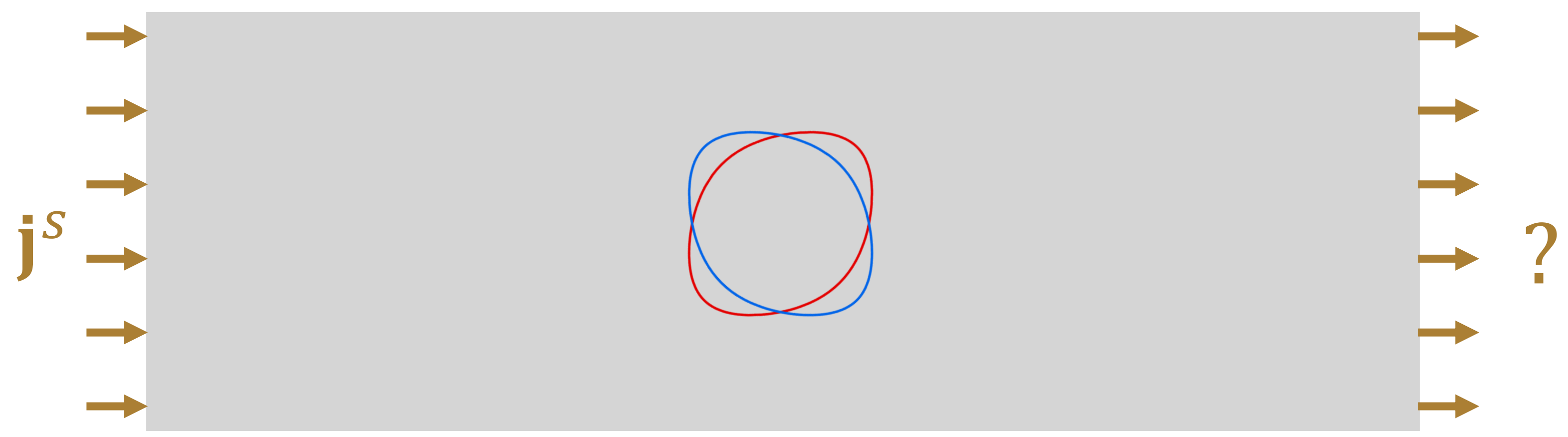




II.B — Spin current in a long wire

Q: Will spin current survive in the presence of non-zero J ?

$$f[\psi_\uparrow,\psi_\downarrow] = \sum_\sigma f_\sigma[\psi_\sigma] - \frac{1}{2}D(\psi_\uparrow^*\psi_\downarrow + \text{c.c.}) + \frac{B^2}{8\pi},$$



To analyze this situation we first write the Josephson free energy

$$f[\varphi_\uparrow,\varphi_\downarrow] = f_0 + \sum_\sigma \gamma\psi_0^2(\nabla\varphi_\sigma - \frac{2e}{\hbar c}\boldsymbol{A})^2 - J\cos(\varphi_\uparrow - \varphi_\downarrow)$$

and recast it in terms of $\quad \Omega = \frac{1}{2}(\varphi_\uparrow + \varphi_\downarrow), \quad \varphi = \varphi_\uparrow - \varphi_\downarrow$

$$f[\Omega,\varphi] = \gamma\psi_0^2\left[2(\nabla\Omega - \frac{2e}{\hbar c}\boldsymbol{A})^2 + \frac{1}{2}(\nabla\varphi)^2\right] - J\cos\varphi.$$

Spin current density is given by

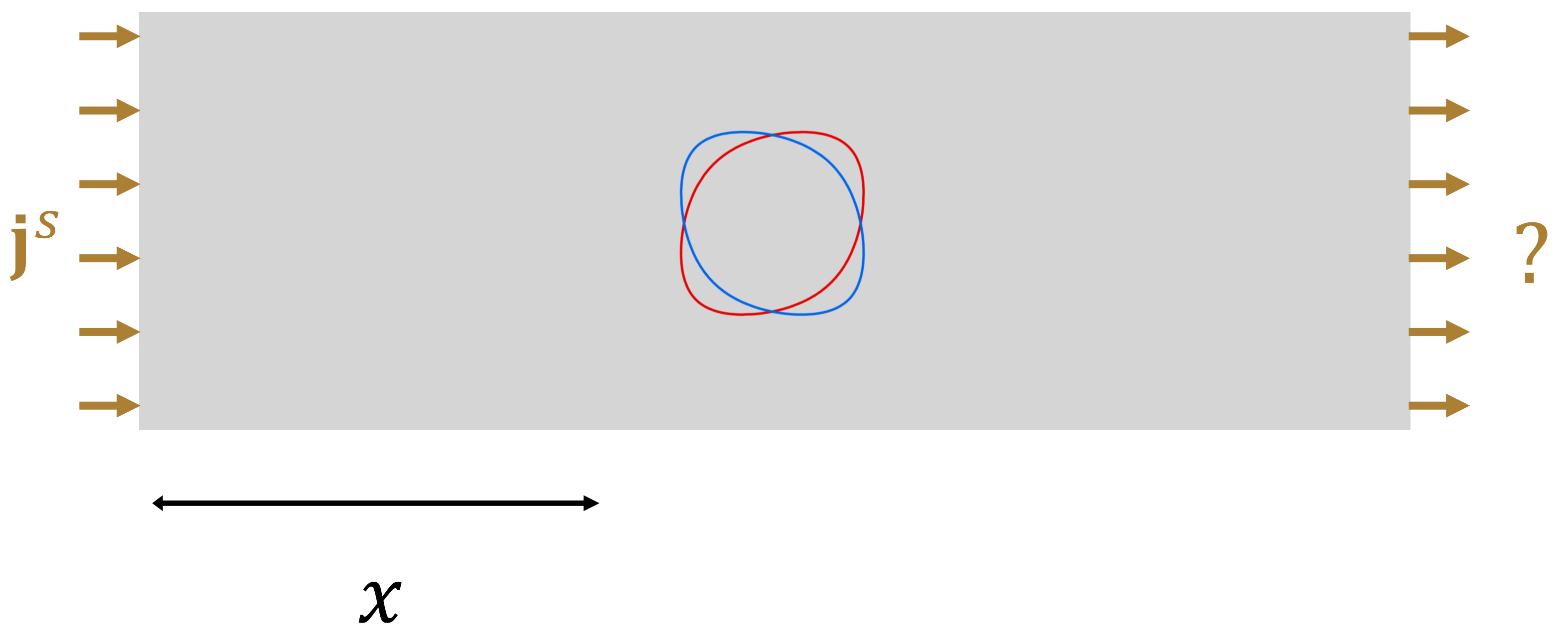
$$\boldsymbol{j}^s = \boldsymbol{j}_\uparrow - \boldsymbol{j}_\downarrow = 4e\frac{\gamma\psi_0^2}{\hbar}\nabla\varphi.$$

A: To understand the behavior of spin current we must analyze the equation of motion for the phase φ.

II.B — Spin current in a long wire

Q: Will spin polarization survive in the presence of non-zero *J* ?

$$f[\Omega,\varphi] = \gamma\psi_0^2\left[2(\nabla\Omega - \frac{2e}{\hbar c}\boldsymbol{A})^2 + \frac{1}{2}(\nabla\varphi)^2\right] - J\cos\varphi.$$



The equation of motion follows from $\delta F/\delta\varphi = 0$ and reads

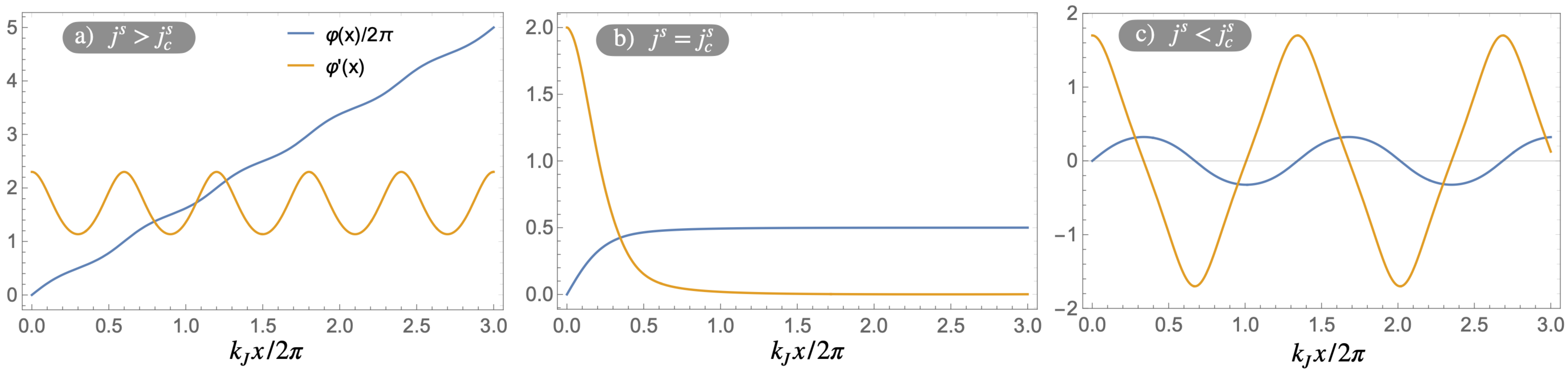
$$\frac{d^2\varphi}{dx^2} + k_J^2\sin\varphi = 0,$$

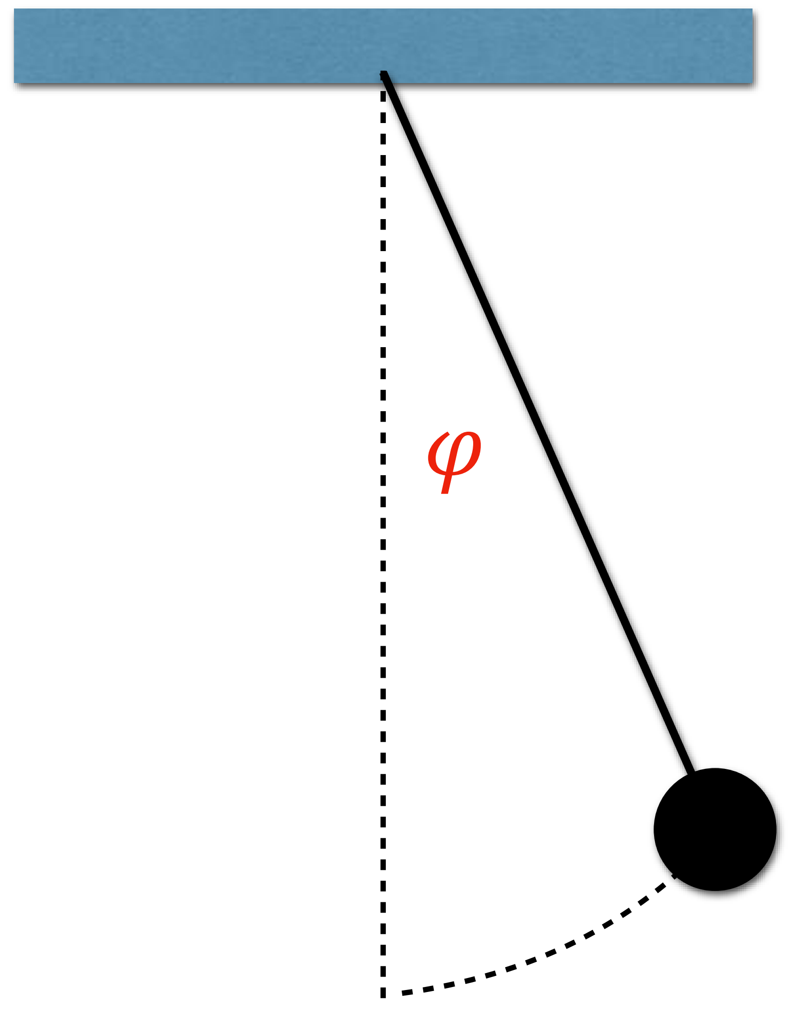
$$k_J = \sqrt{J/\gamma\psi_0^2}$$

This is analogous to the eq. of motion for a mechanical pendulum

$$\frac{d^2\varphi}{dt^2} + \omega^2\sin\varphi = 0,$$

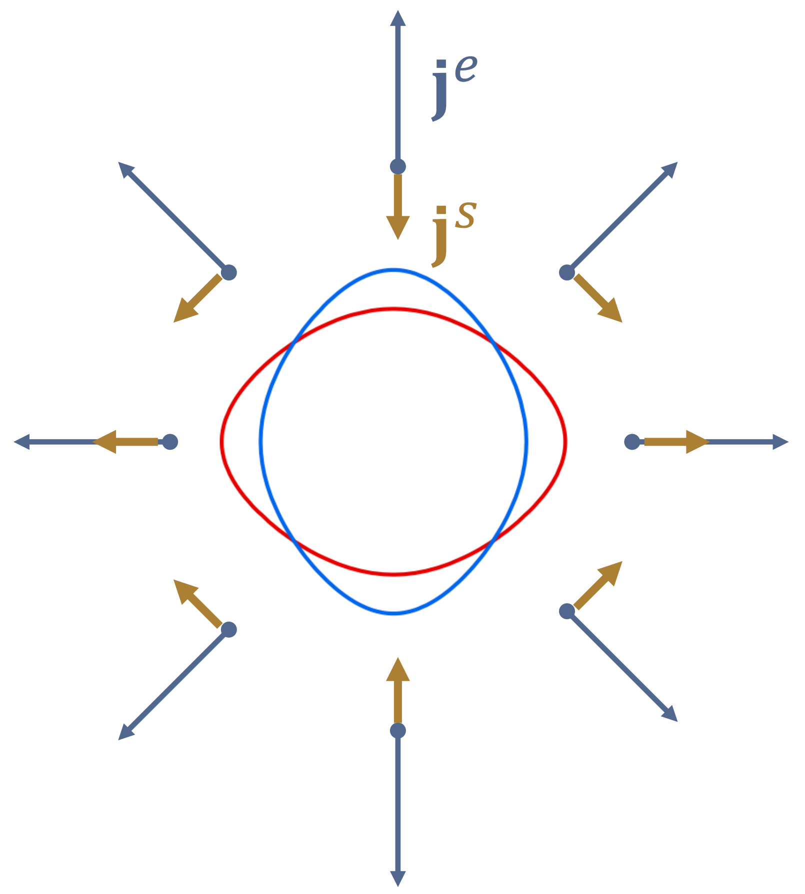
$$\omega = \sqrt{g/l}$$

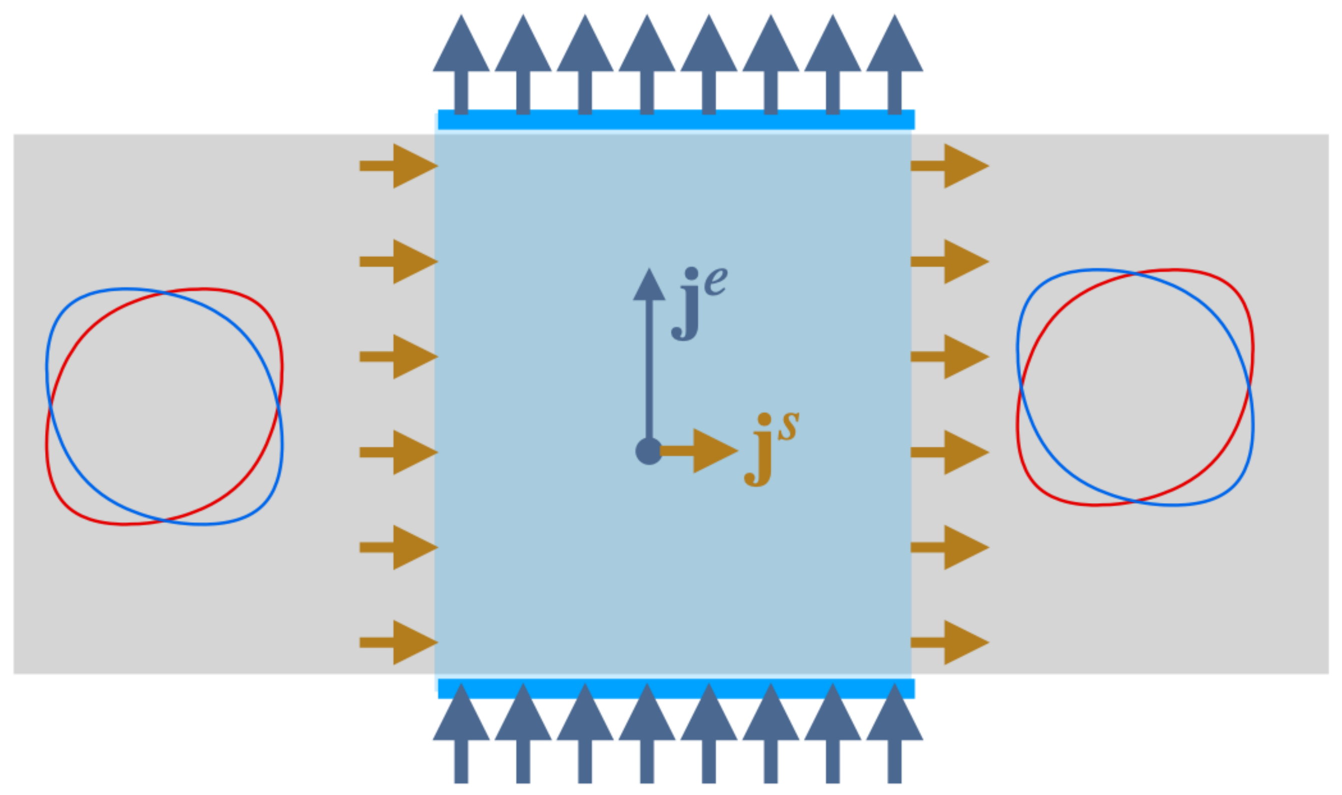


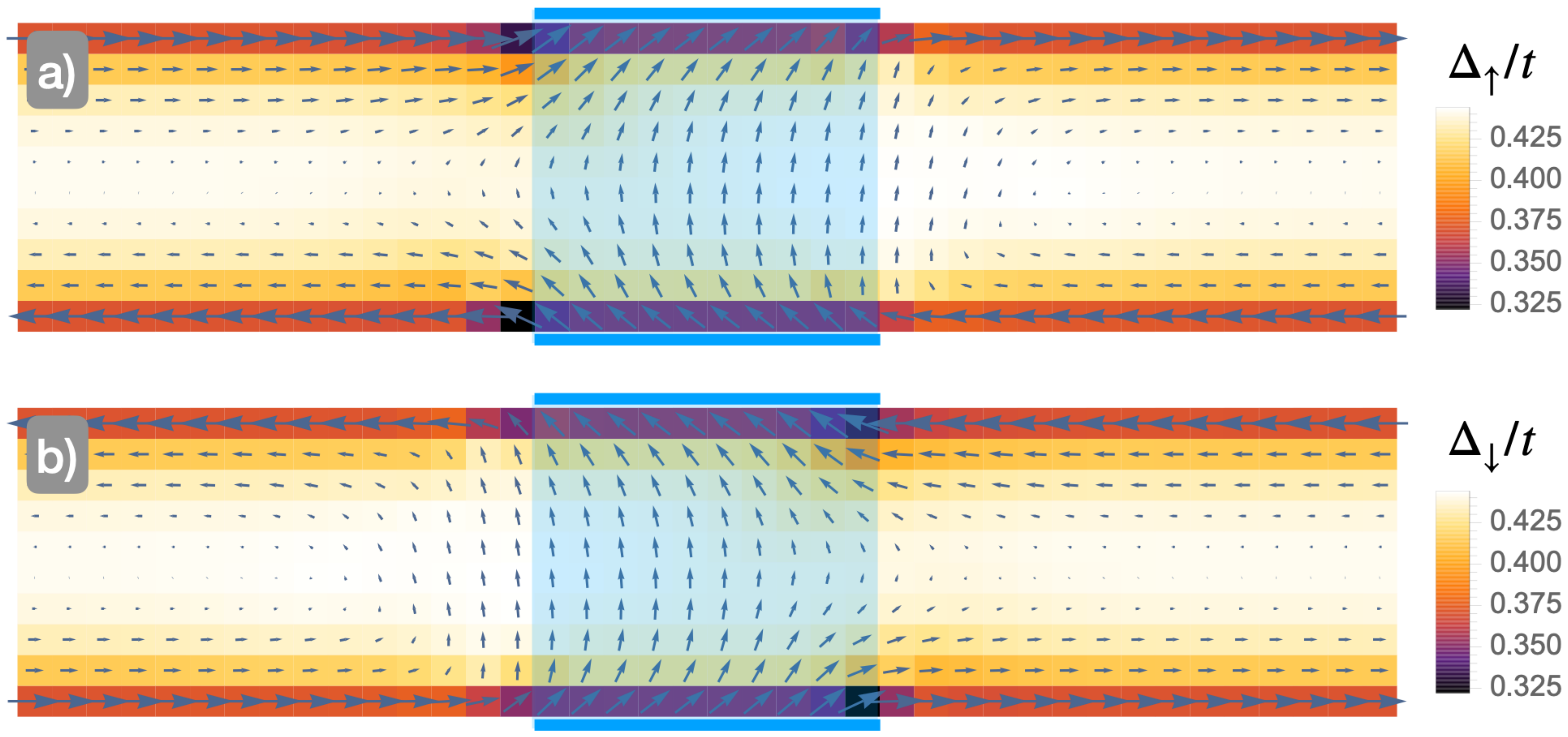


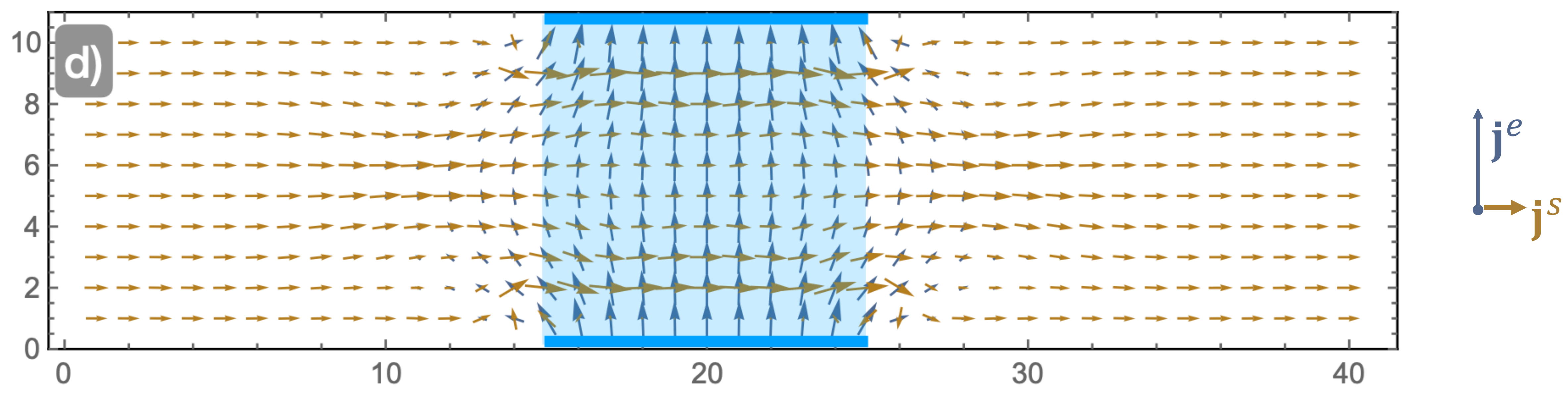
II.C — Spin-current dynamo effect

- A d-wave SC altermagnet can be used to generate spin current from applied charge current
- We call this “spin-current dynamo effect”
- We illustrate this effect below based on self-consistent solutions of the Bogoliubov-de Gennes theory for equal spin triplet chiral p-wave SC

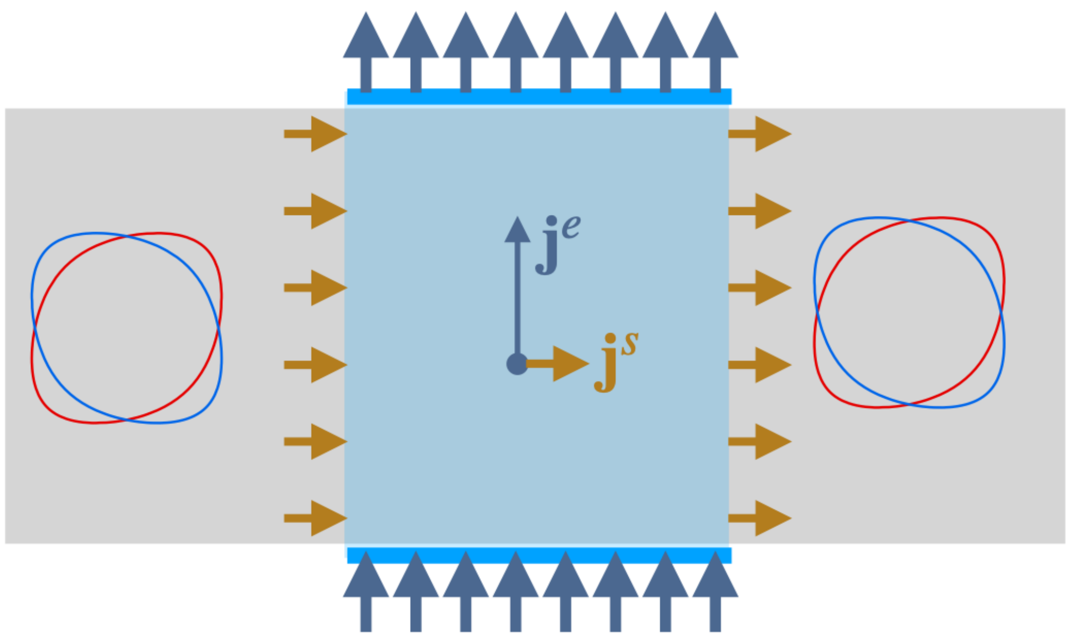




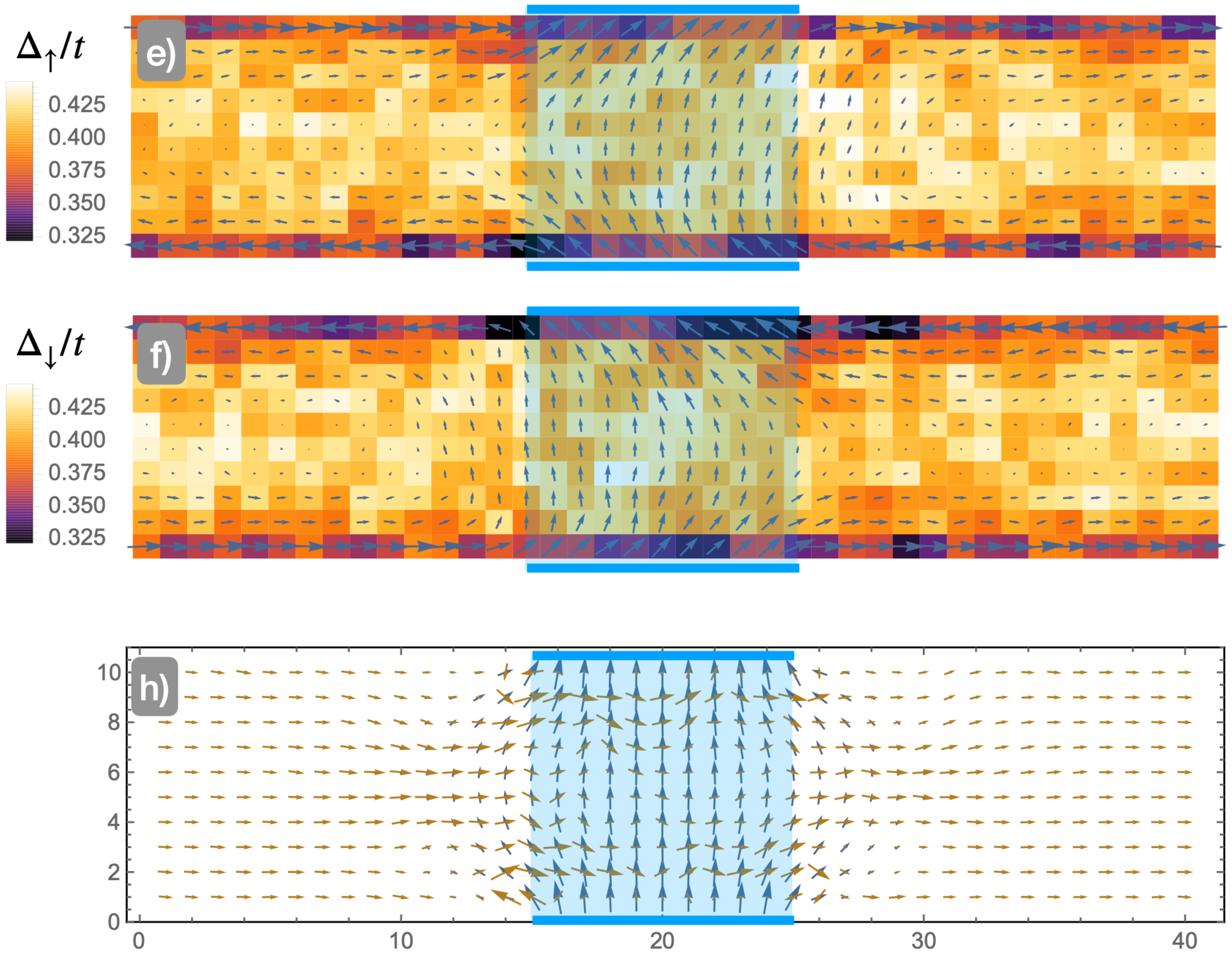




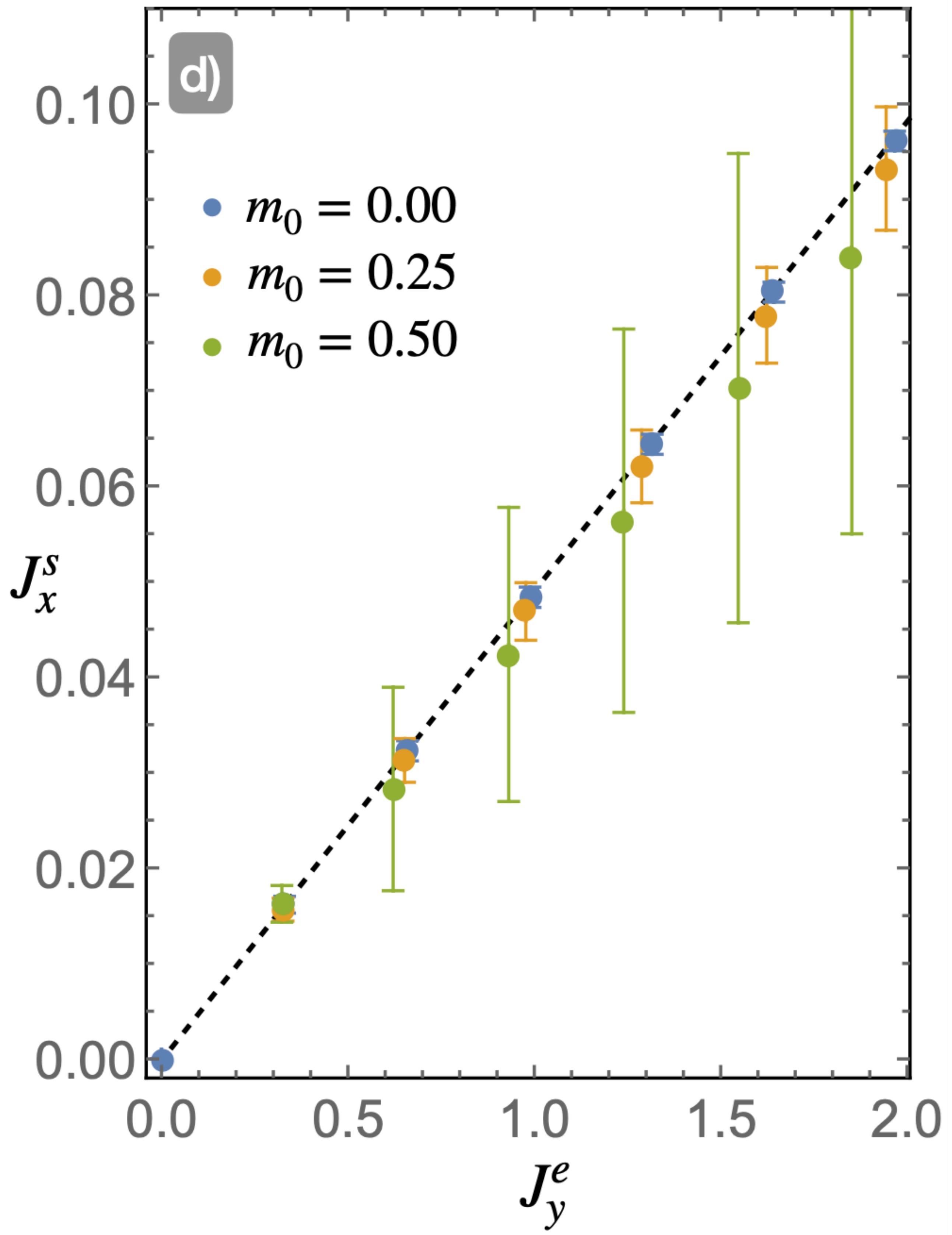
II.C — Spin-current dynamo effect



- **Q:** Is the dynamo effect robust against magnetic disorder?



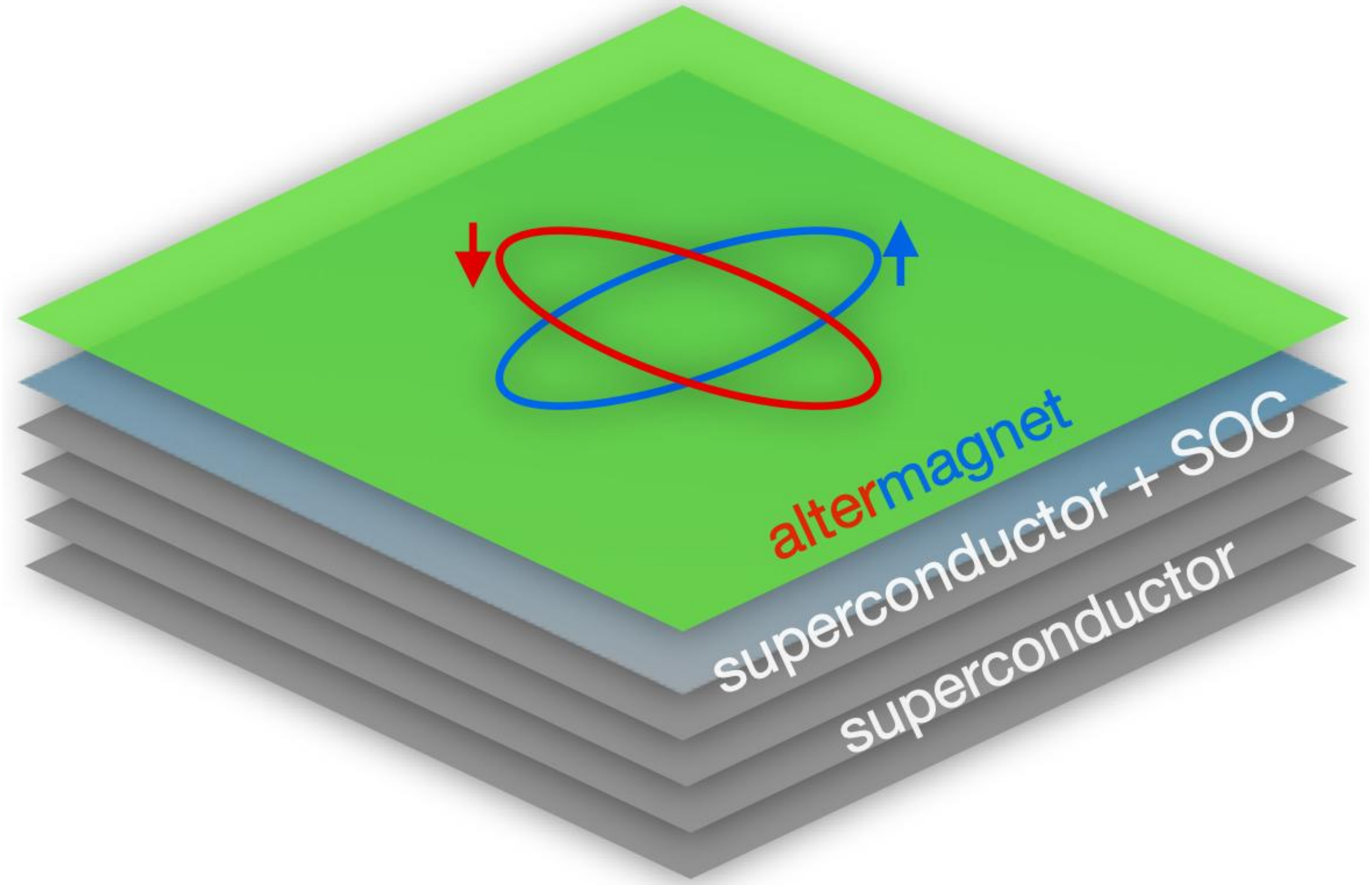
- **A:** Yes, the dynamo effect is robust against magnetic disorder.



III. — Proximity-induced SC order in altermagnets

Q: Can we induce triplet SC order in altermagnets by proximity effect?

$$f[\psi_s, \psi_\uparrow, \psi_\downarrow] = f_s[\psi_s] + \sum_\sigma f_\sigma[\psi_\sigma] - \frac{1}{2} D(\psi_\uparrow^* \psi_s + \psi_\downarrow^* \psi_s + \mathrm{c.c.}) + \cdots$$



$$\psi_s \sim |\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)s\rangle$$

These triplet/singlet terms are symmetry-allowed when ψ_σ represent

$\psi_\uparrow \sim |\uparrow\uparrow(p_x - ip_y)\rangle$ and $\psi_\downarrow \sim |\downarrow\downarrow(p_x + ip_y)\rangle$

Both have ZERO total angular momentum J_z and hence transform under rotations just like ψ_s.

In addition, in-plane inversion symmetry $(x, y) \to (-x, -y)$ must be broken at the interface —> require Rashba SOC to generate *p*-wave SC by proximity effect.

Bottom line:

Equal-spin triplet chiral *p*-wave SC can be induced in an altermagnet by proximity effect using an ordinary spin-singlet superconductor IF there is Rashba SOC present at the interface.

This conclusion follows from the symmetry analysis and is supported by a microscopic model calculation.

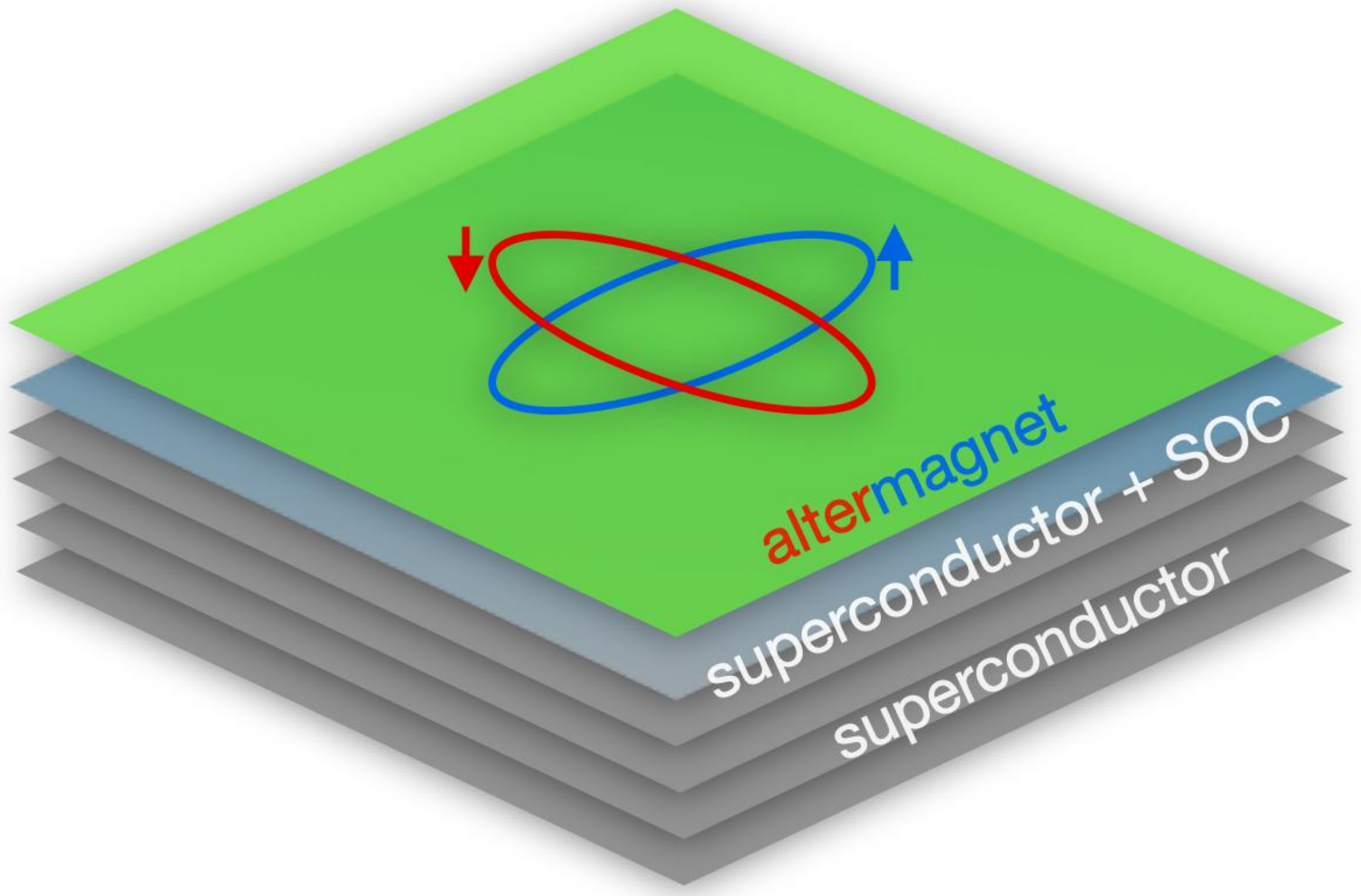
III. — Proximity-induced SC order in altermagnets

Microscopic model:

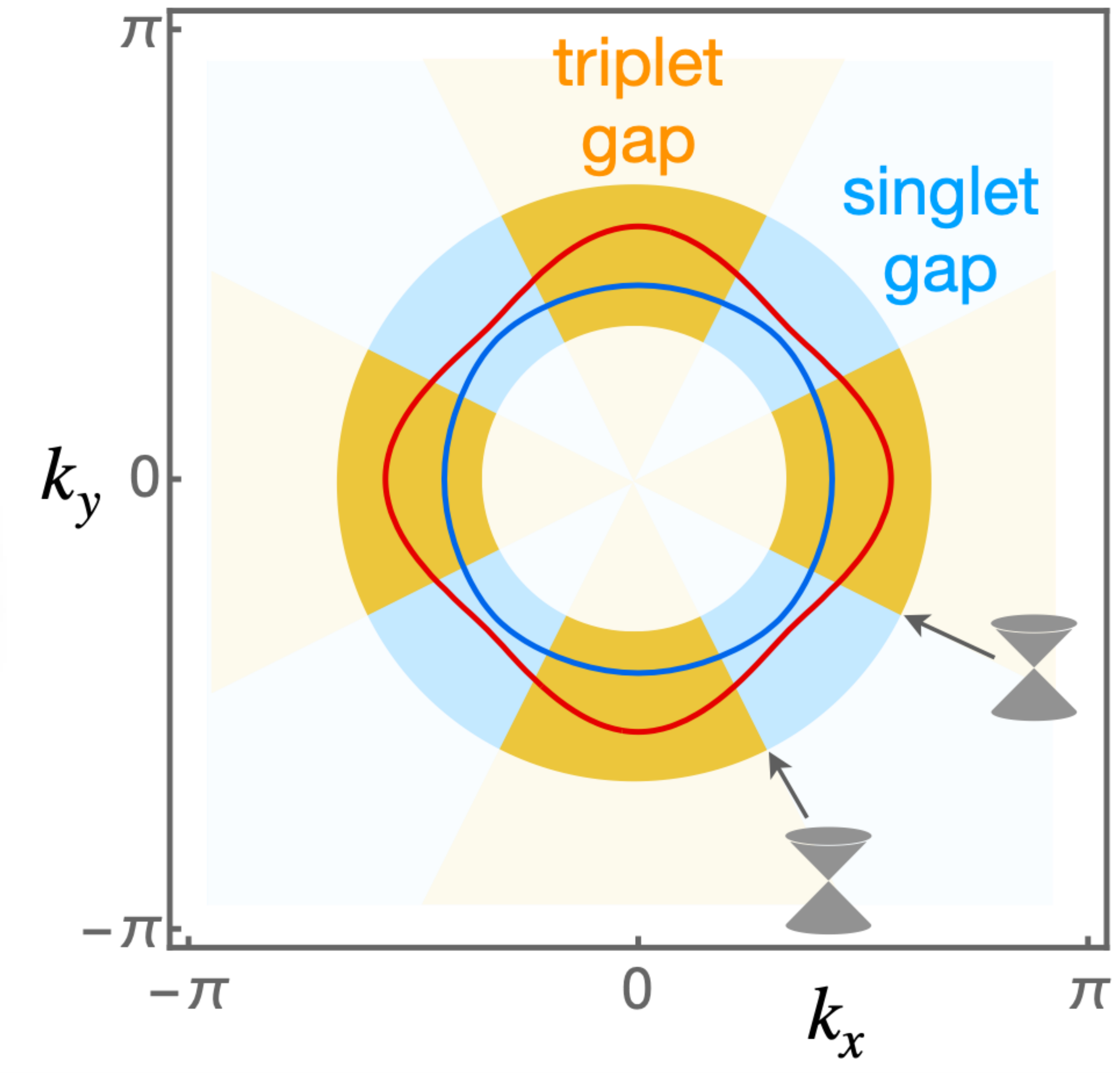
$$\mathcal{H}_{\mathrm{ALM}} = \sum_{\mathbf{k}} [\xi'_{\mathbf{k}} + 2\sigma^z \eta_0 (\cos k_x - \cos k_y)]_{\sigma\sigma'} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma'}$$

$$\mathcal{H}_{\mathrm{SC}} = \sum_{\mathbf{k}} \left[\xi_{\mathbf{k}} + 2\lambda_R (\sigma^x \sin k_y - \sigma^y \sin k_x)\right]_{\sigma\sigma'} d^\dagger_{\mathbf{k}\sigma} d_{\mathbf{k}\sigma'} + \Delta_0 \sum_{\mathbf{k}} \left(d^\dagger_{\mathbf{k}\uparrow} d^\dagger_{-\mathbf{k}\downarrow} + \mathrm{h.c.}\right)$$

$$\mathcal{H}_g = g \sum_{\mathbf{k}} \left(c^\dagger_{\mathbf{k}\sigma} d_{\mathbf{k}\sigma} + \mathrm{h.c.}\right)$$



- An interesting **nodal** superconductor emerges, comprising a mix of singlet and triplet gaps
- This structure will be visible in spectroscopy and thermodynamics
- The spin-current dynamo effect remains present



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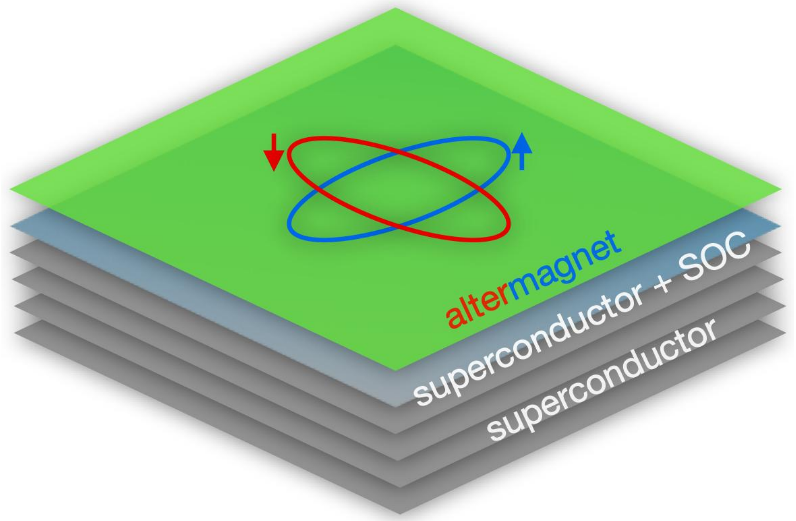
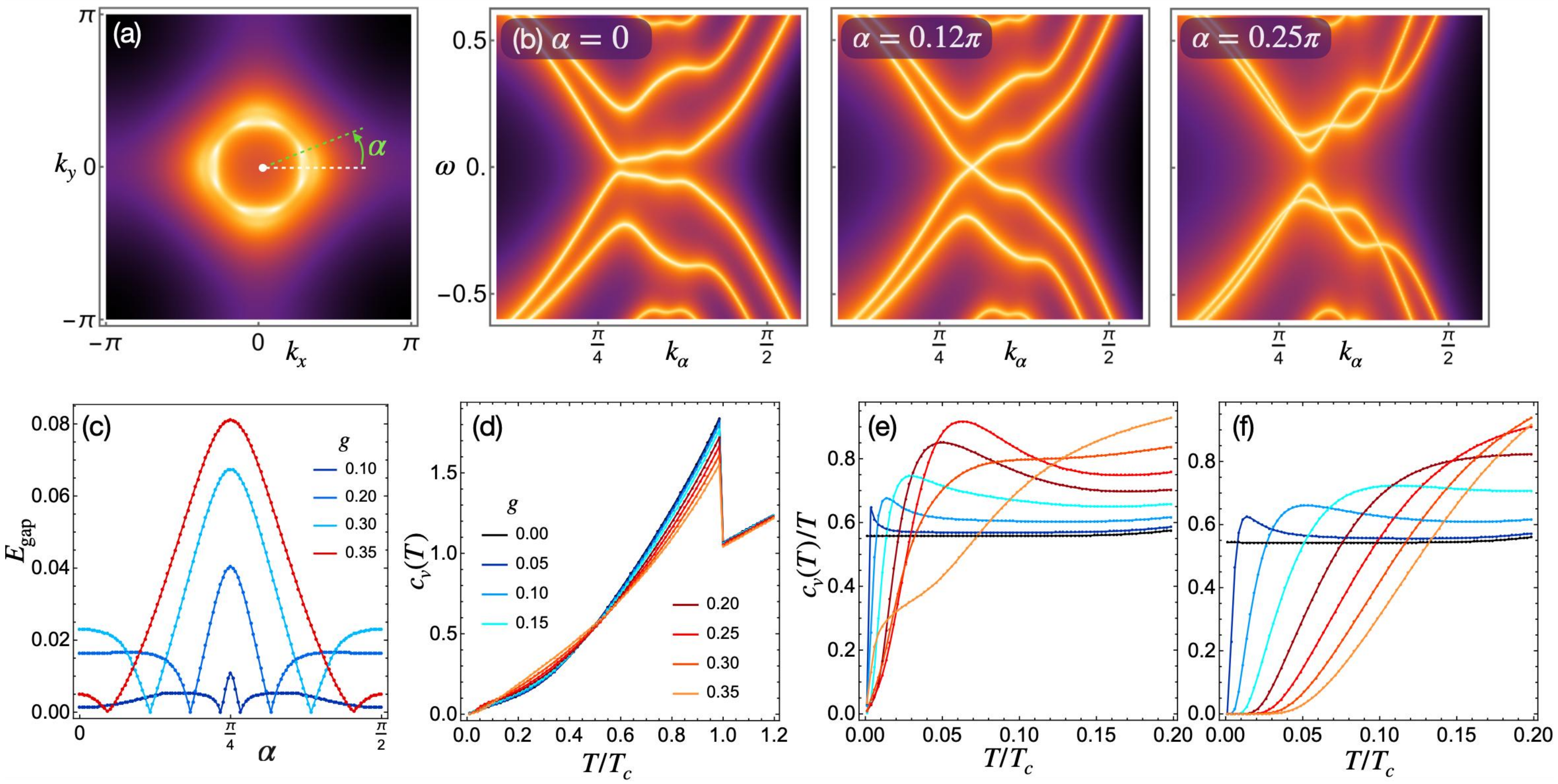


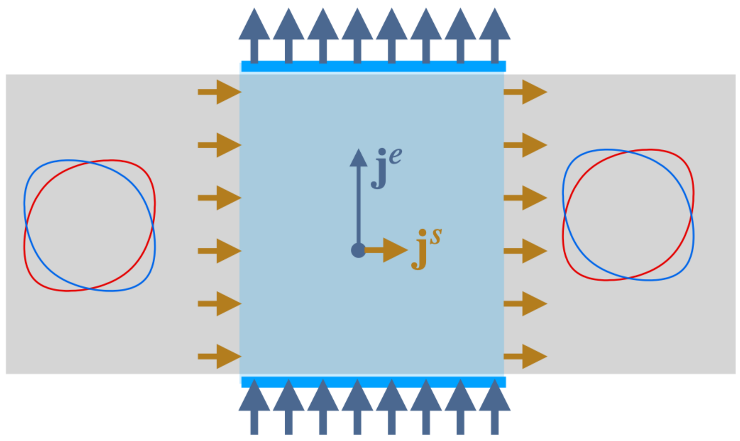
Spectroscopy and thermodynamics:

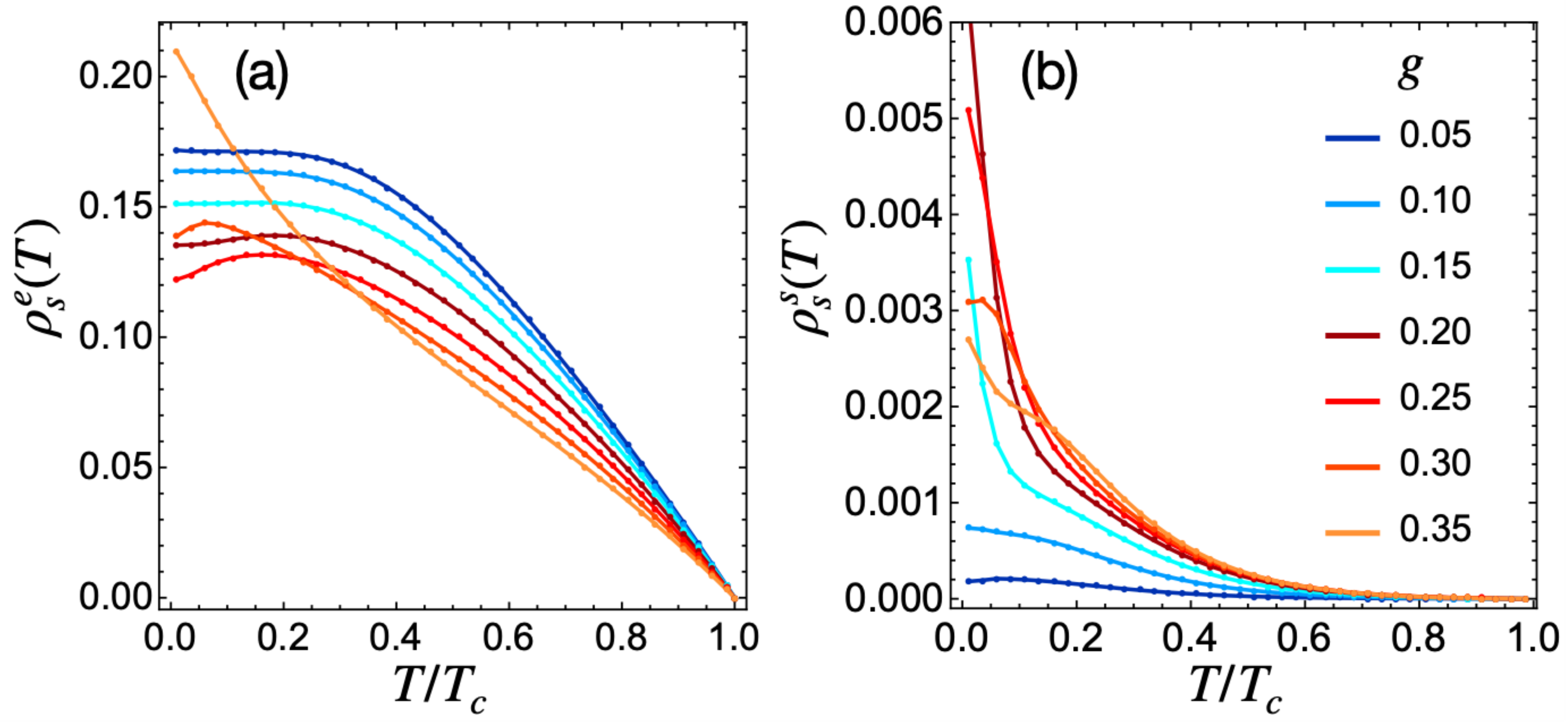


Material candidates:

SC	ALM	mismatch	symmetry
Al	$Rb_{1-\delta}V_2Te_2O$	0.04%	tetragonal
NbS	$FeSb_2$	0.61%, 3.21%	orthorhombic
Nb_4Se_8	$FeBr_3$	2.23%	hexagonal
$CaKFe_4As_4$	KV_2Se_2O	2.87%	tetragonal
Pb	OsO_2	7.64%	tetragonal

Charge & spin transport:

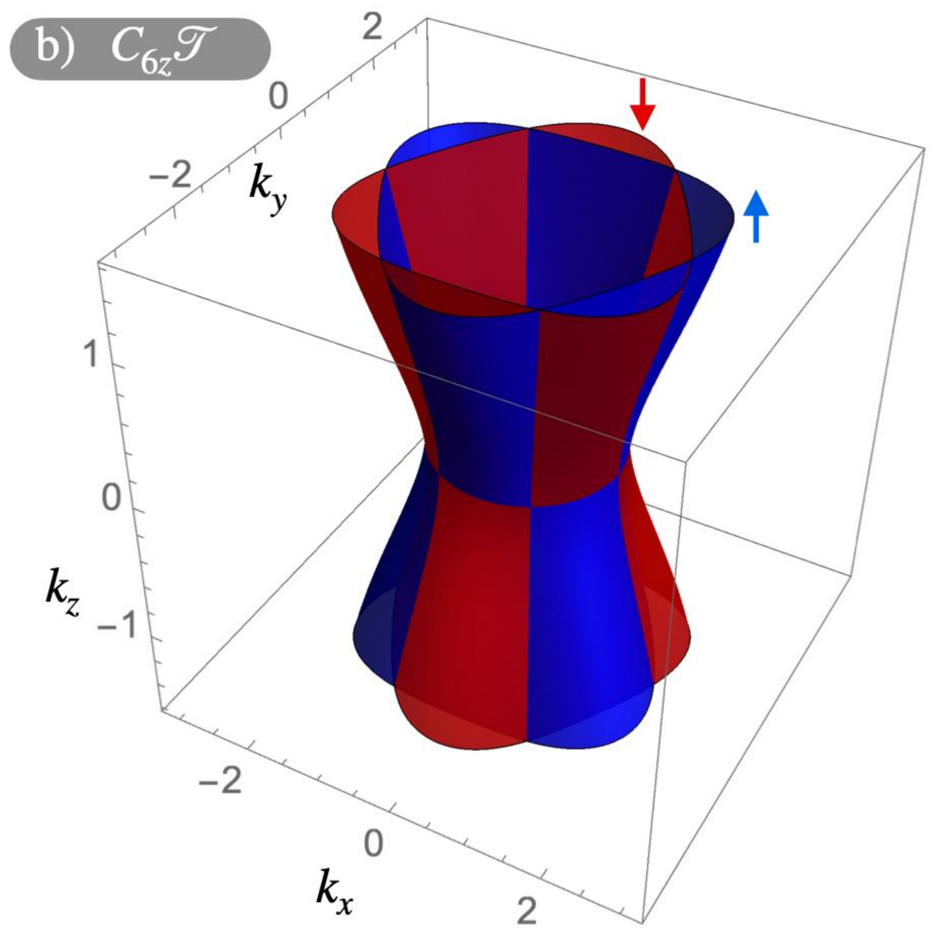


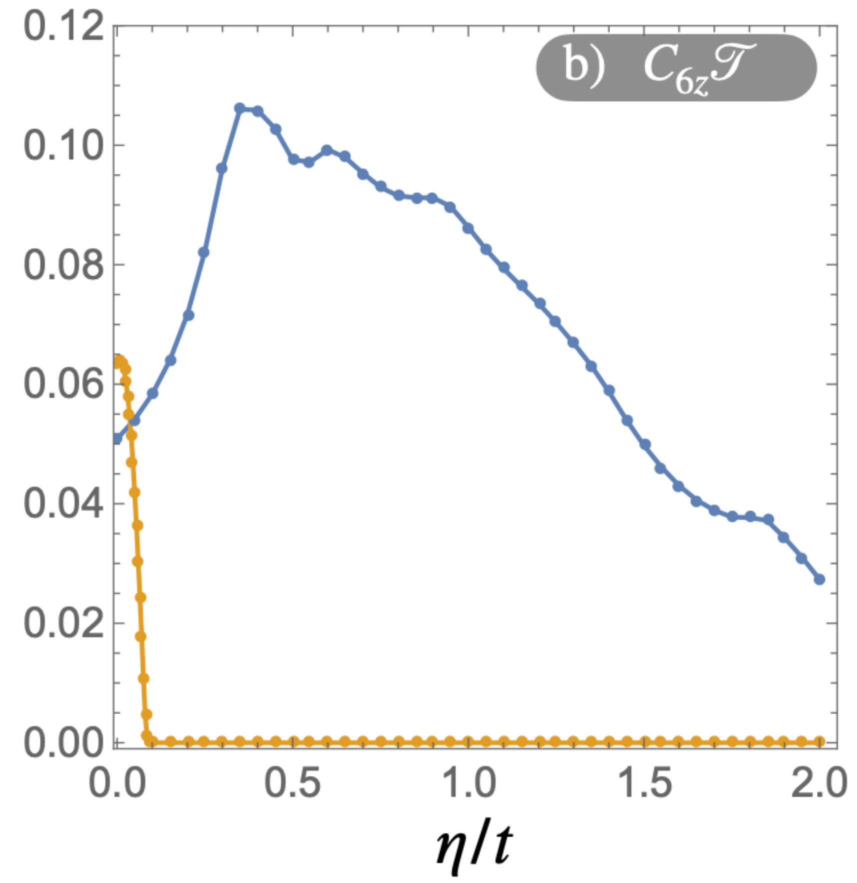


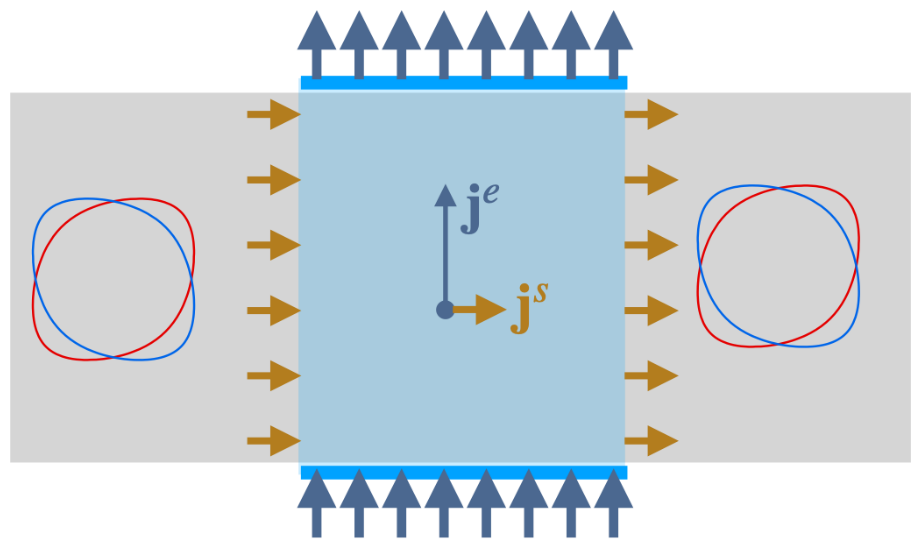
IV. — Conclusions

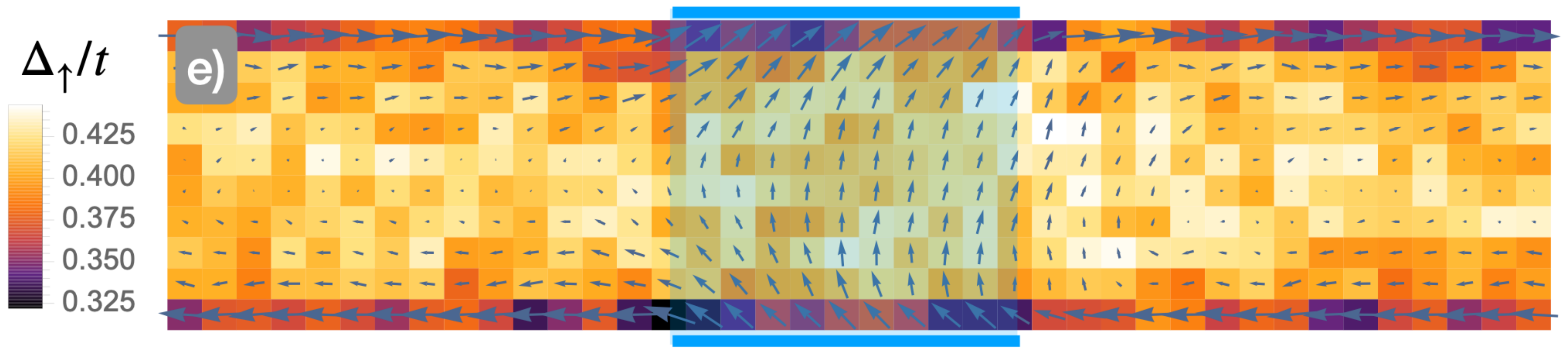
1. Metallic altermagnets are likely to produce unconventional equal-spin triplet superconductors with chiral *p*-wave order parameter
2. In such superconductors persistent spin currents can be observed in various geometries:
   A. Persistent spin current in a ring with half-integer flux
   B. Robust to SOC
   C. Spin-current dynamo effect robust to strong magnetic disorder
3. Some of these effects also occur in a proximitized altermagnet

arXiv:2509.03774, arXiv:2507.22139









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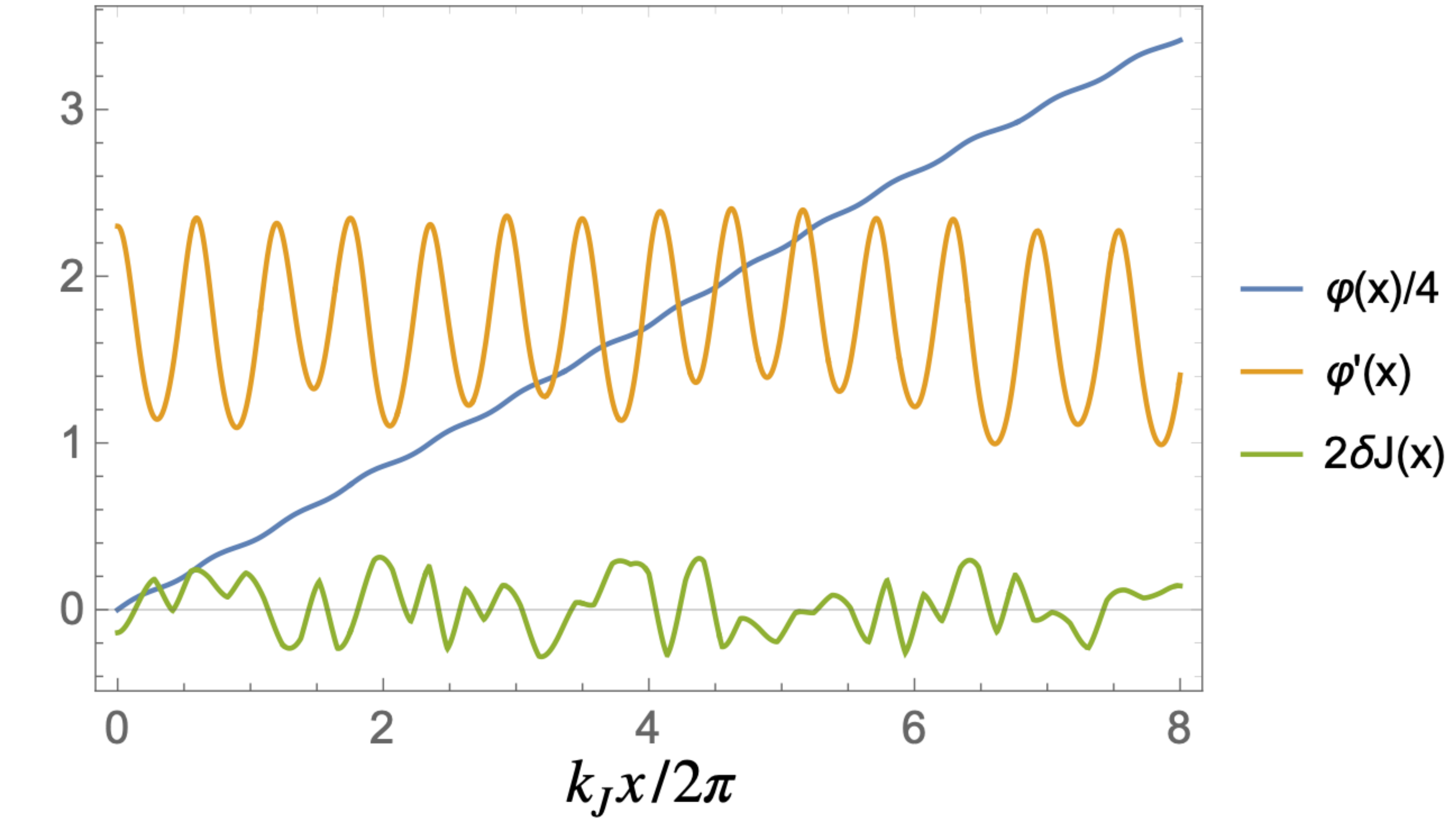
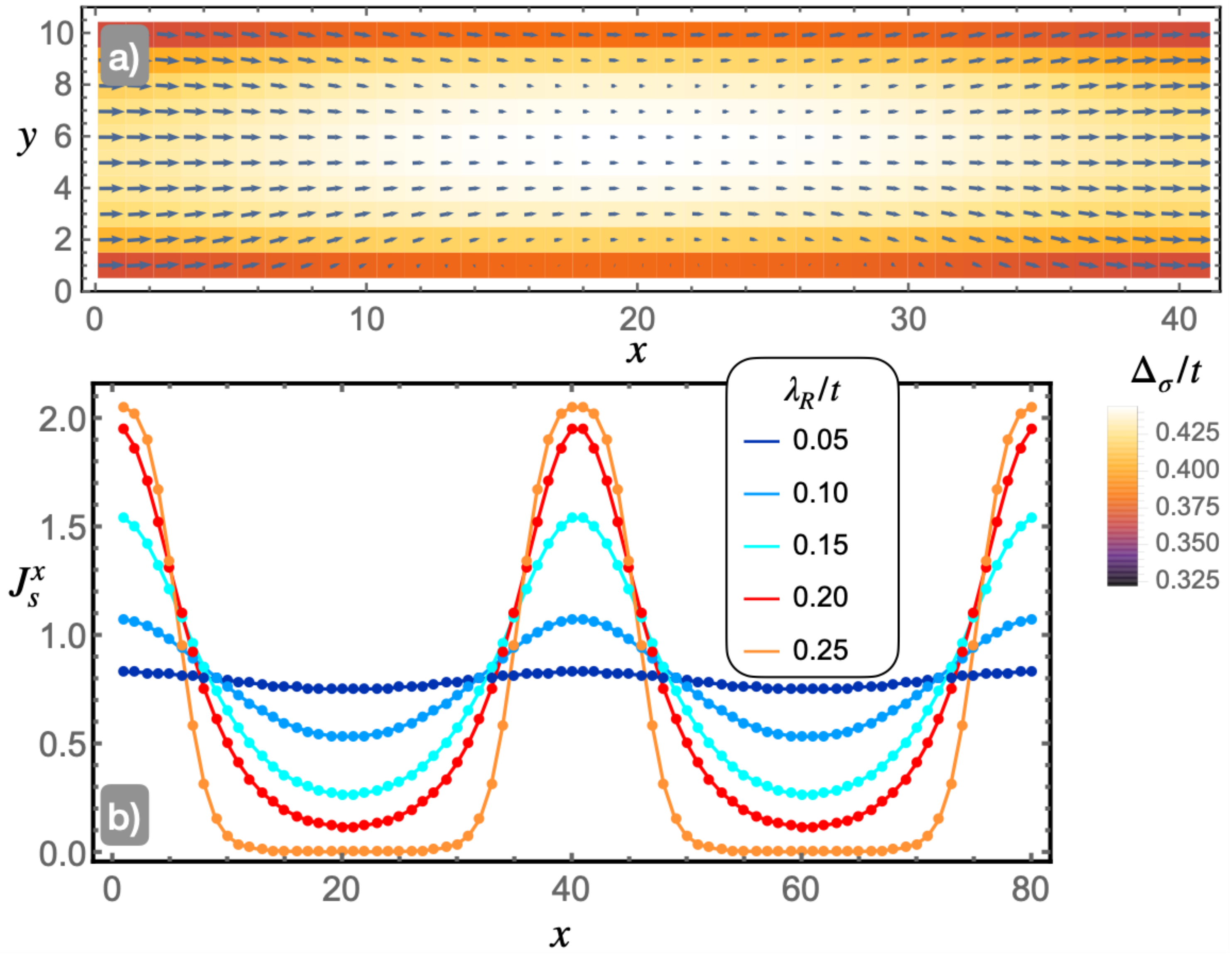
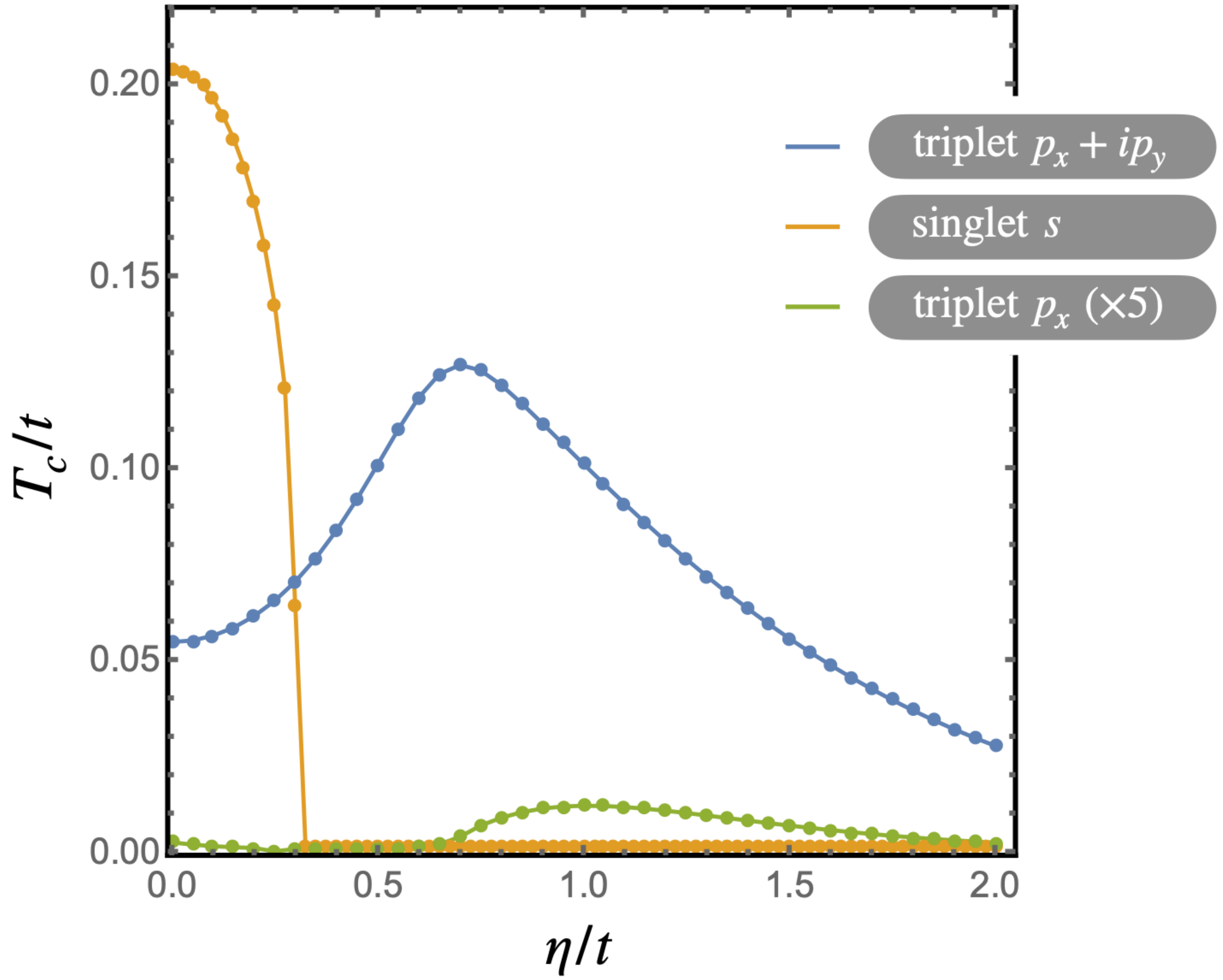






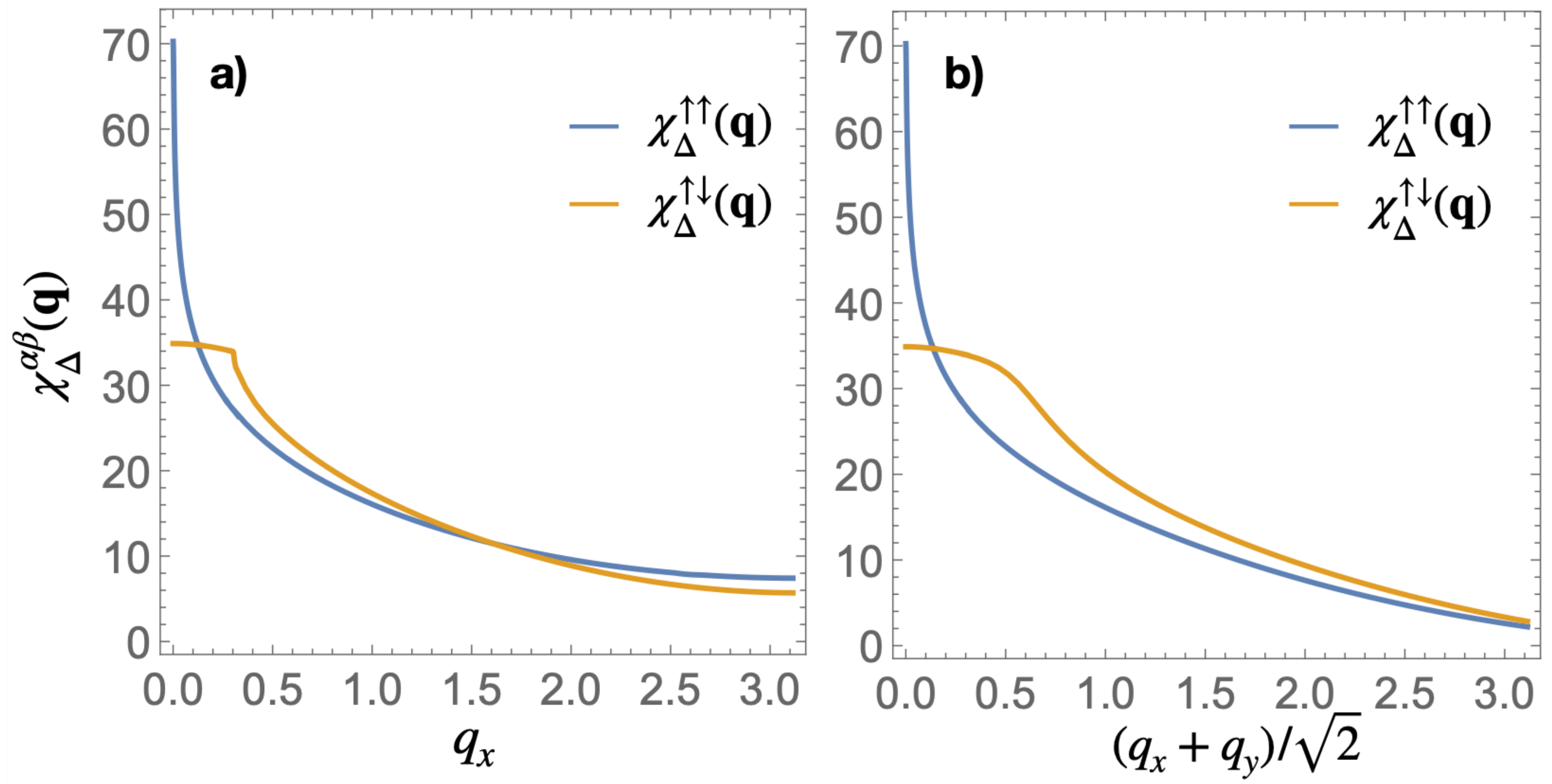
$$\Delta_0 = \frac{V_1}{2N} \sum_{\boldsymbol{k}} \frac{\Delta_0 C_{\boldsymbol{k}}^2}{\epsilon_{\boldsymbol{k}}} \frac{\sinh \beta \epsilon_{\boldsymbol{k}}}{\cosh \beta \xi_{\boldsymbol{k}-} + \cosh \beta \epsilon_{\boldsymbol{k}}},$$

$$\Delta_\sigma = \frac{V_1}{2N} \sum_{\boldsymbol{k}} \frac{\Delta_\sigma |S_{\boldsymbol{k}\sigma}|^2}{E_{\boldsymbol{k}\sigma}} \tanh \frac{1}{2} \beta E_{\boldsymbol{k}\sigma},$$

$$C_{\boldsymbol{k}} = \cos k_x + \cos k_y, \quad S_{\boldsymbol{k}\sigma} = \sin k_x \pm i \sin k_y.$$

Finite momentum pairing

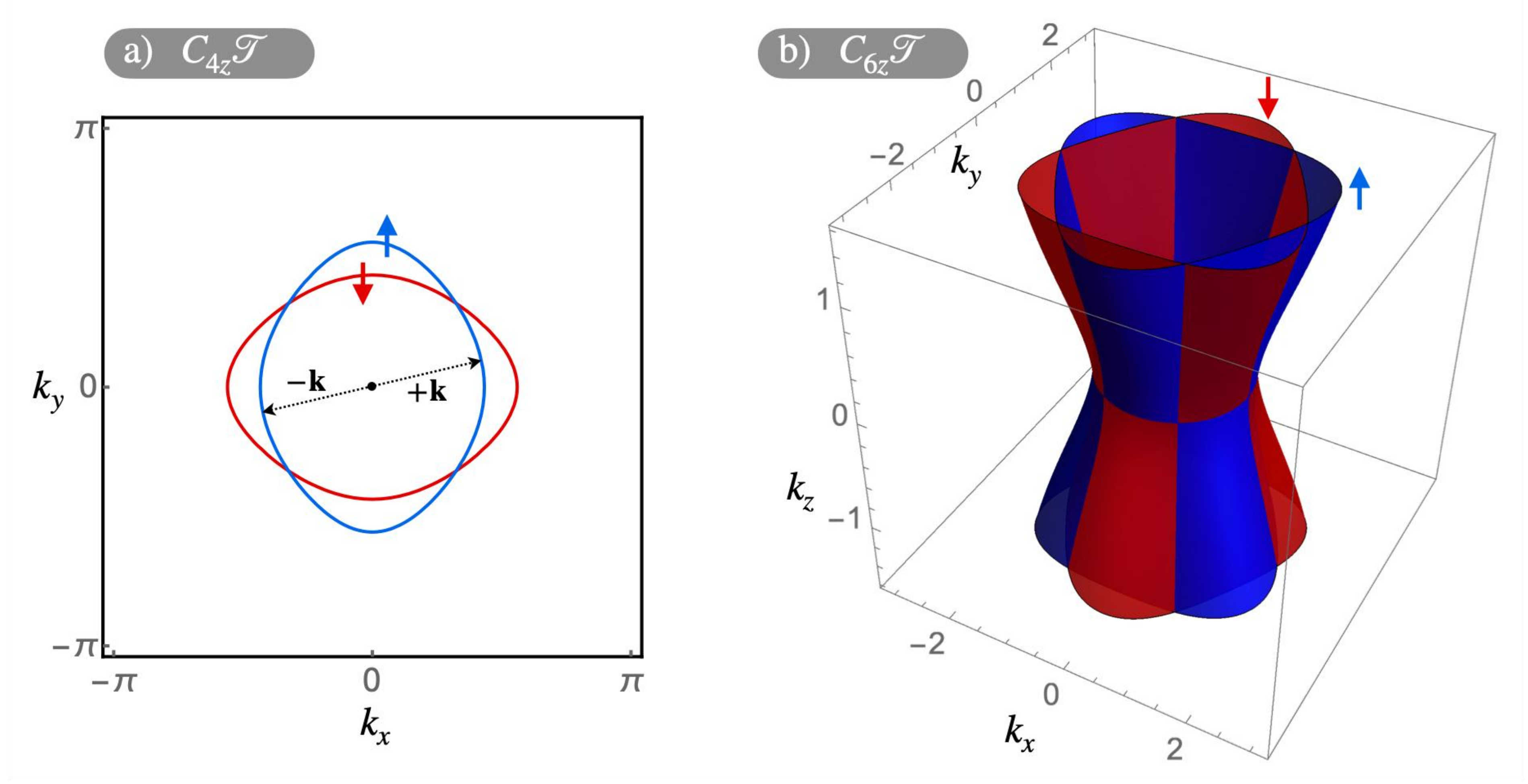
Pair susceptibilities:



$$\chi_\Delta^{\alpha\beta}(\omega,\boldsymbol{q}) = \int_{-\infty}^{\infty} dt e^{-i\omega t}\langle\hat{\Delta}_{\boldsymbol{q}}^{\alpha\beta}(t)^\dagger\hat{\Delta}_{\boldsymbol{q}}^{\alpha\beta}(0)\rangle$$

$$\hat{\Delta}_{\boldsymbol{q}}^{\alpha\beta} = \sum_{\boldsymbol{k}} g_{\boldsymbol{k}} c^\dagger_{\boldsymbol{k}+\boldsymbol{q}\alpha} c^\dagger_{-\boldsymbol{k}\beta}$$

$$\chi_\Delta^{\alpha\beta}(\boldsymbol{q}) = \sum_{\boldsymbol{k}} |g_{\boldsymbol{k}} + g_{\boldsymbol{k}-\boldsymbol{q}}|^2 \frac{1-n_F(\xi_{\boldsymbol{k}\alpha}) - n_F(\xi_{\boldsymbol{q}-\boldsymbol{k}\beta})}{\xi_{\boldsymbol{k}\alpha}+\xi_{\boldsymbol{q}-\boldsymbol{k}\beta}}.$$



Nested FS case: