Topological transition from nodal to nodeless Zeeman splitting in altermagnets

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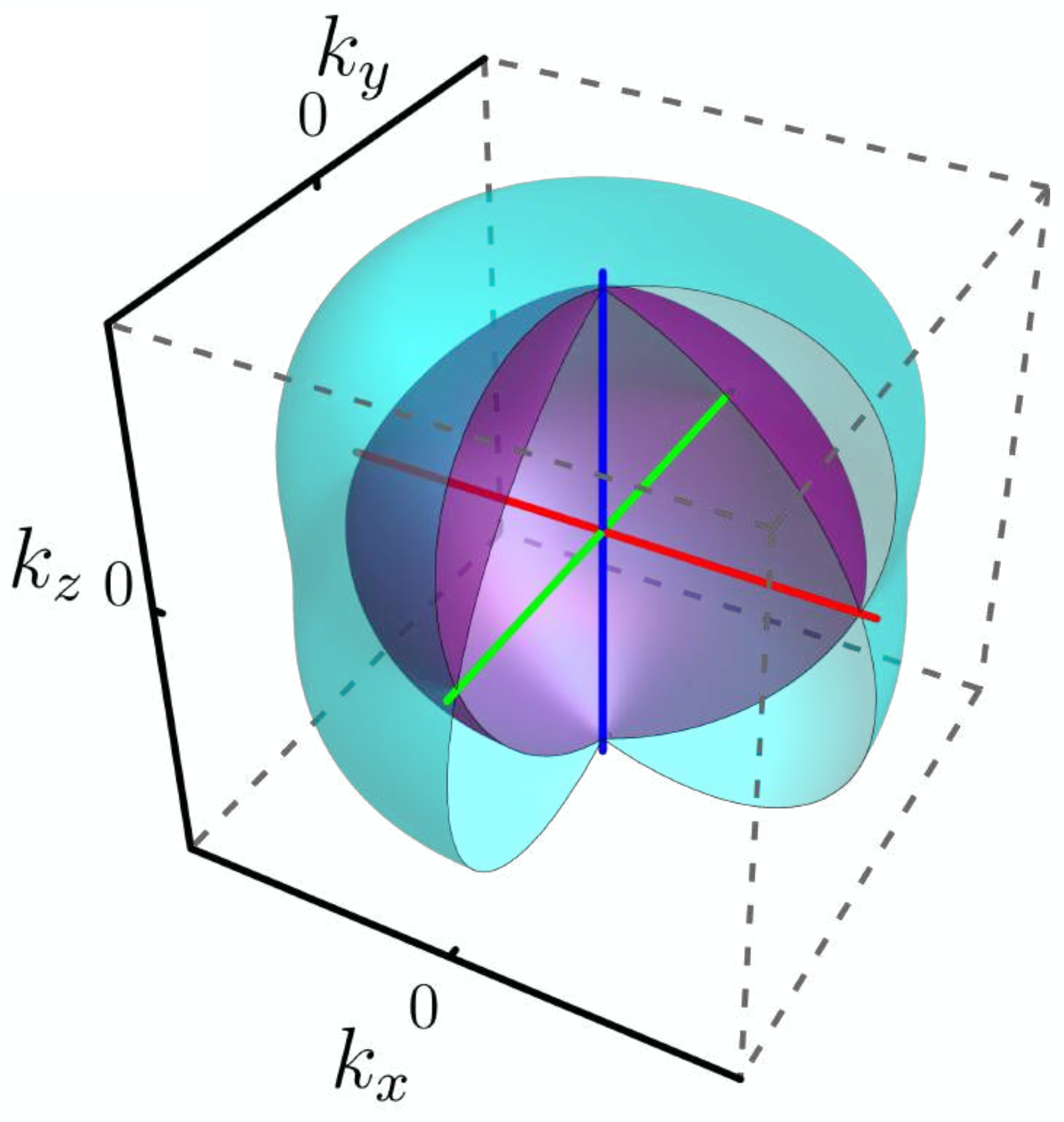


SPICE Workshop Theory of Unconventional Magnetism: exploring altermagnets and beyond
Mainz, October 22, 2025

Different types of collinear magnetic order

Ferromagnetic (FM)

time reversal ✗

translation ✓

$$\tilde{\varepsilon}_{\mathbf{k}\uparrow} \neq \tilde{\varepsilon}_{\mathbf{k}\downarrow}$$

Antiferromagnetic (AFM)

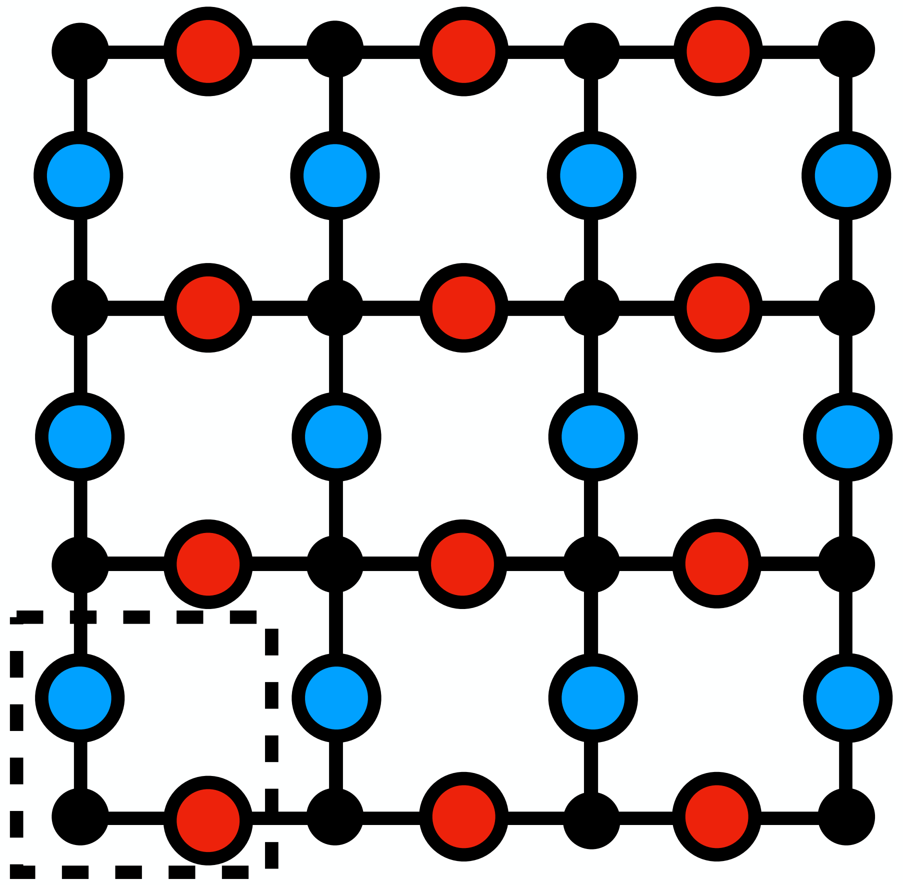
time reversal ✗

translation ✗

time reversal x translation ✓

$$\tilde{\varepsilon}_{\mathbf{k}\uparrow} = \tilde{\varepsilon}_{\mathbf{k}\downarrow}$$

Altermagnetic (AM)



time reversal ✗

translation ✓

rotation (R) ✗

time reversal x rotation ✓

$$\varepsilon_{\uparrow}(\mathbf{k}) = \varepsilon_{\downarrow}(\mathcal{R} \cdot \mathbf{k})$$

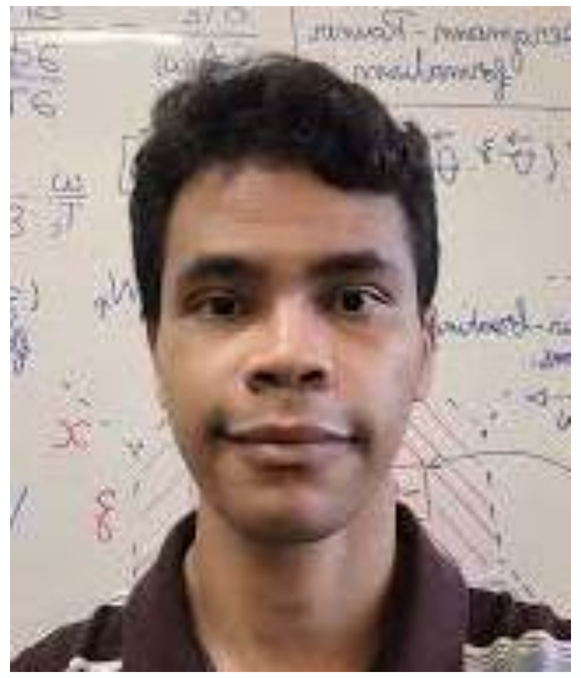
[Šmejkal, Sinova, Jungwirth, PRX 2022]

Outline

- Effect of the crystalline environment (i.e. spin-orbit coupling) in 3D?
- Response to an external magnetic field; does the altermagnetic phase survive?
  - topological transition for field directions that respect mirror symmetries.

[PRB **109**, 024404 (2024)]

Rafael Fernandes



Vanuildo Carvalho

Turan Birol

Low-energy model

Consider metallic altermagnets, low energies near Γ point.

$$H = \sum_{\mathbf{k},s} \varepsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k},s} c_{\mathbf{k},s} + H_{\text{int}} \qquad \varepsilon_{\mathbf{k}} = \frac{\mathbf{k}^2}{2m}$$

$$H_{\text{int}} = -\lambda \sum_{i} \sum_{\mathbf{k},s,s'} \Phi^i c^{\dagger}_{\mathbf{k},s} [\mathbf{d}_i(\mathbf{k}) \cdot \boldsymbol{\sigma}_{ss'}] c_{\mathbf{k},s'}$$

component index

$$i = 1, \ldots, \text{dimension of irrep}$$

Assume inversion symmetry, no ferromagnetic moment.

Order parameter Φ^i transforms as time-reversal-odd, even-parity irrep of centrosymmetric point group.

Point groups and AM order parameters

$$H_{\text{int}} = -\lambda \sum_i \sum_{\mathbf{k},s,s'} \Phi^i c^\dagger_{\mathbf{k},s} [\mathbf{d}_i(\mathbf{k}) \cdot \boldsymbol{\sigma}_{ss'}] c_{\mathbf{k},s'}$$

time reversal ❌ inversion ✓

$$\mathbf{d}(-\mathbf{k}) = \mathbf{d}(\mathbf{k})$$

"Pure" altermagnet: no s-wave component.

[Fernandes, Carvalho, Birol, RP, PRB 2024]

Point group	AM irrep	$\mathbf{d}_i(\mathbf{k}) \equiv (d_{i,x},\, d_{i,y},\, d_{i,z})$
D_{2h}	A_{1g}^-	$(k_yk_z,\, \eta_1 k_xk_z,\, \eta_2 k_xk_y)$
D_{4h}	A_{1g}^-	$\left(k_yk_z, -k_xk_z,\, \eta\, k_xk_y\left(k_x^2-k_y^2\right)\right)$
	B_{1g}^-	$(k_yk_z,\, k_xk_z,\, \eta\, k_xk_y)$
	B_{2g}^-	$\left(-k_xk_z,\, k_yk_z,\, \eta\left(k_x^2-k_y^2\right)\right)$
D_{6h}	A_{1g}^-	$\left(k_yk_z, -k_xk_z,\, \eta\, k_xk_y\left(k_x^2-3k_y^2\right)\left(3k_x^2-k_y^2\right)\right)$
	B_{1g}^-	$\left(k_x^2-k_y^2, -2k_xk_y,\, \eta\, k_xk_z\left(k_x^2-3k_y^2\right)\right)$
	B_{2g}^-	$\left(2k_xk_y,\, k_x^2-k_y^2,\, \eta\, k_yk_z\left(3k_x^2-k_y^2\right)\right)$
	E_{2g}^-	$\begin{cases}(k_yk_z,\, k_xk_z,\, 2\eta\, k_xk_y) & ,\, i=1\\ \left(k_xk_z, -k_yk_z, \eta\left(k_x^2-k_y^2\right)\right) & ,\, i=2\end{cases}$
O_h	A_{1g}^-	$\left(k_yk_z\left(k_y^2-k_z^2\right),\, k_xk_z\left(k_z^2-k_x^2\right),\, k_xk_y\left(k_x^2-k_y^2\right)\right)$
	A_{2g}^-	$(k_yk_z,\, k_xk_z,\, k_xk_y)$
	E_g^-	$\begin{cases}\sqrt{3}\,(k_yk_z,\, -k_xk_z,\, 0) & ,\, i=1\\ (-k_yk_z,\, -k_xk_z,\, 2k_xk_y) & ,\, i=2\end{cases}$
	T_{2g}^-	$\begin{cases}\left(k_xk_z, -k_yk_z,\, \eta\left(k_x^2-k_y^2\right)\right) & ,\, i=1\\ \left(\eta\left(k_y^2-k_z^2\right),\, k_xk_y, -k_xk_z\right) & ,\, i=2\\ \left(-k_xk_y,\, \eta\left(k_z^2-k_x^2\right),\, k_yk_z\right) & ,\, i=3\end{cases}$

Point groups and AM order parameters

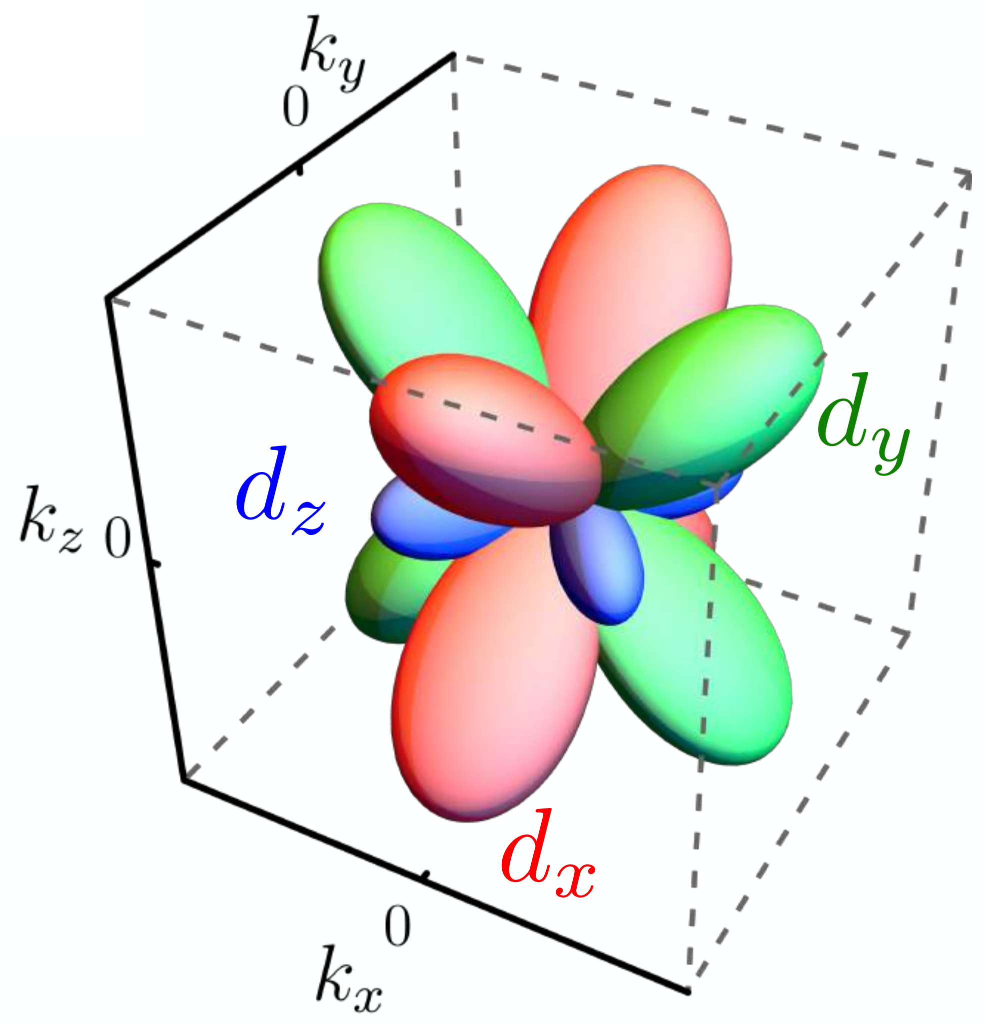
$$H_{\rm int} = -\lambda \sum_i \sum_{\mathbf{k},s,s'} \Phi^i c^\dagger_{\mathbf{k},s} [\mathbf{d}_i(\mathbf{k}) \cdot \boldsymbol{\sigma}_{ss'}] c_{\mathbf{k},s'}$$

time reversal ✗ inversion ✓

$$\mathbf{d}(-\mathbf{k}) = \mathbf{d}(\mathbf{k})$$

Example:

D_{4h} B^-_{1g} : $(k_y k_z, k_x k_z, \eta k_x k_y)$



$\eta < 1$

SOC in 3D

↓

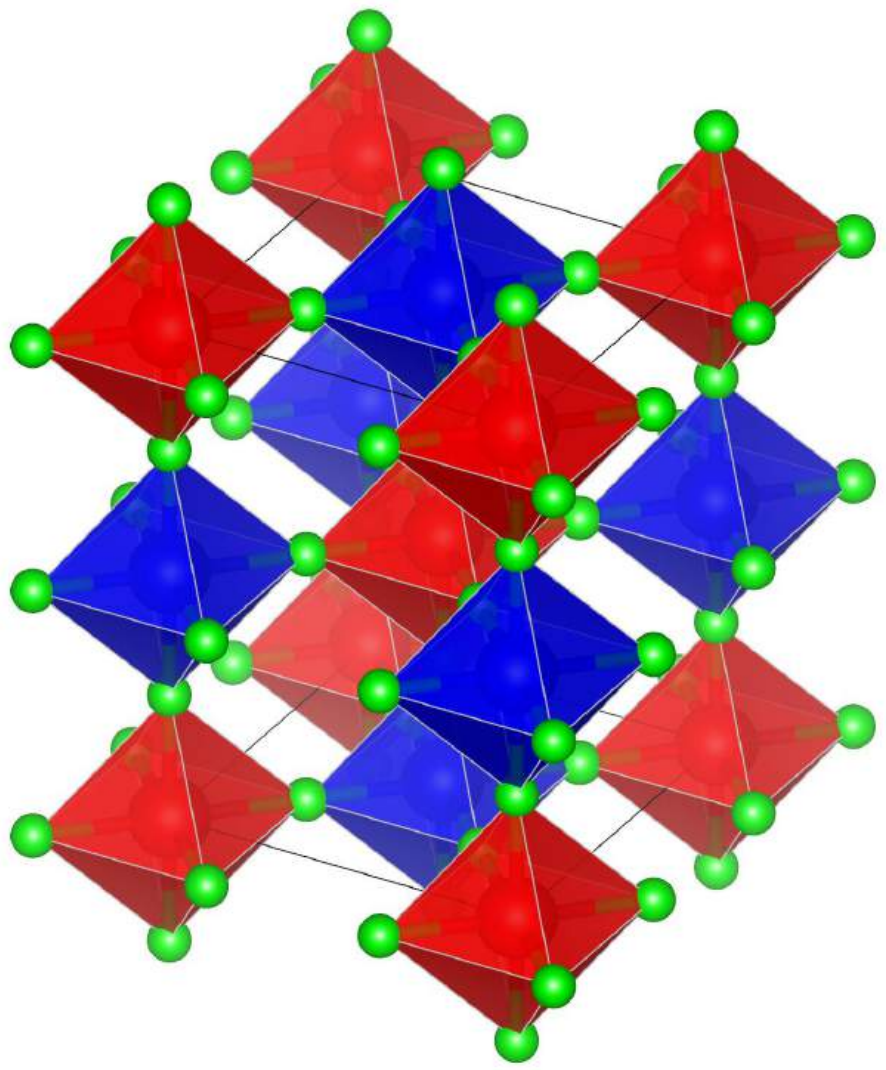
noncollinear

[Fernandes, Carvalho, Birol, RP, PRB 2024]

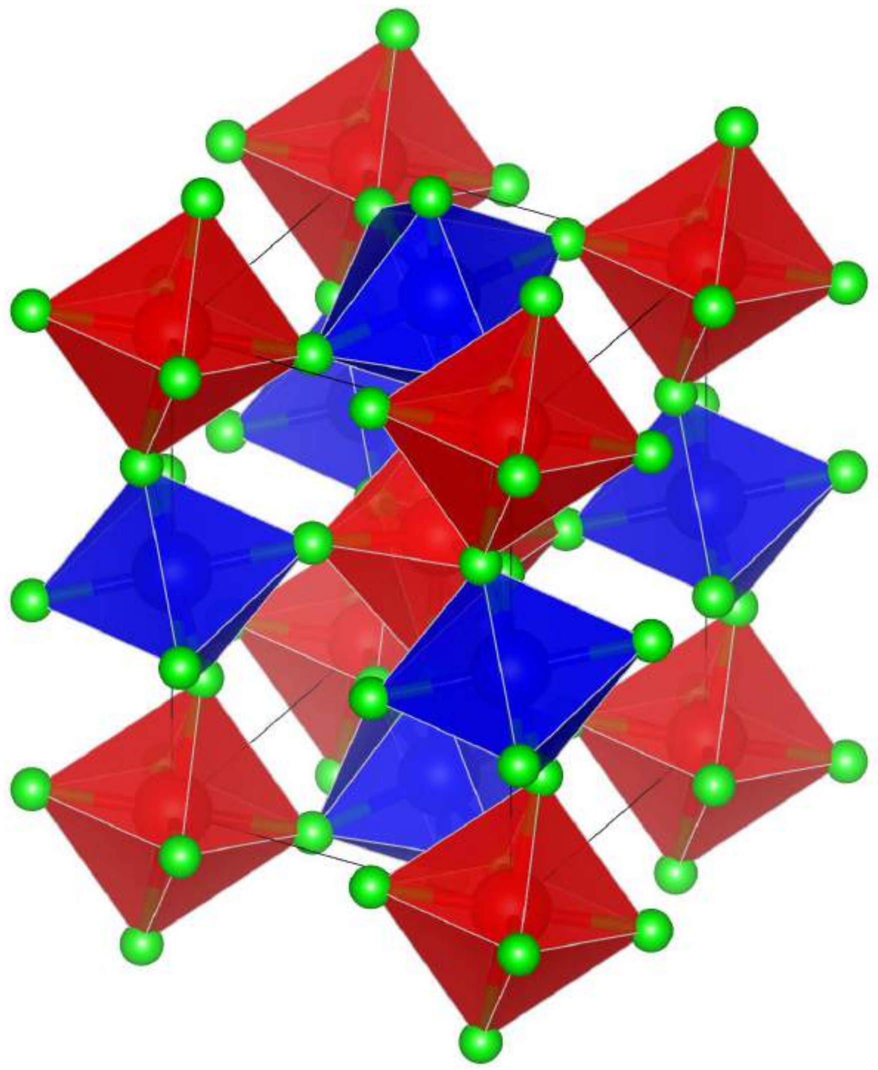
Example: perovskites (ABO_3)

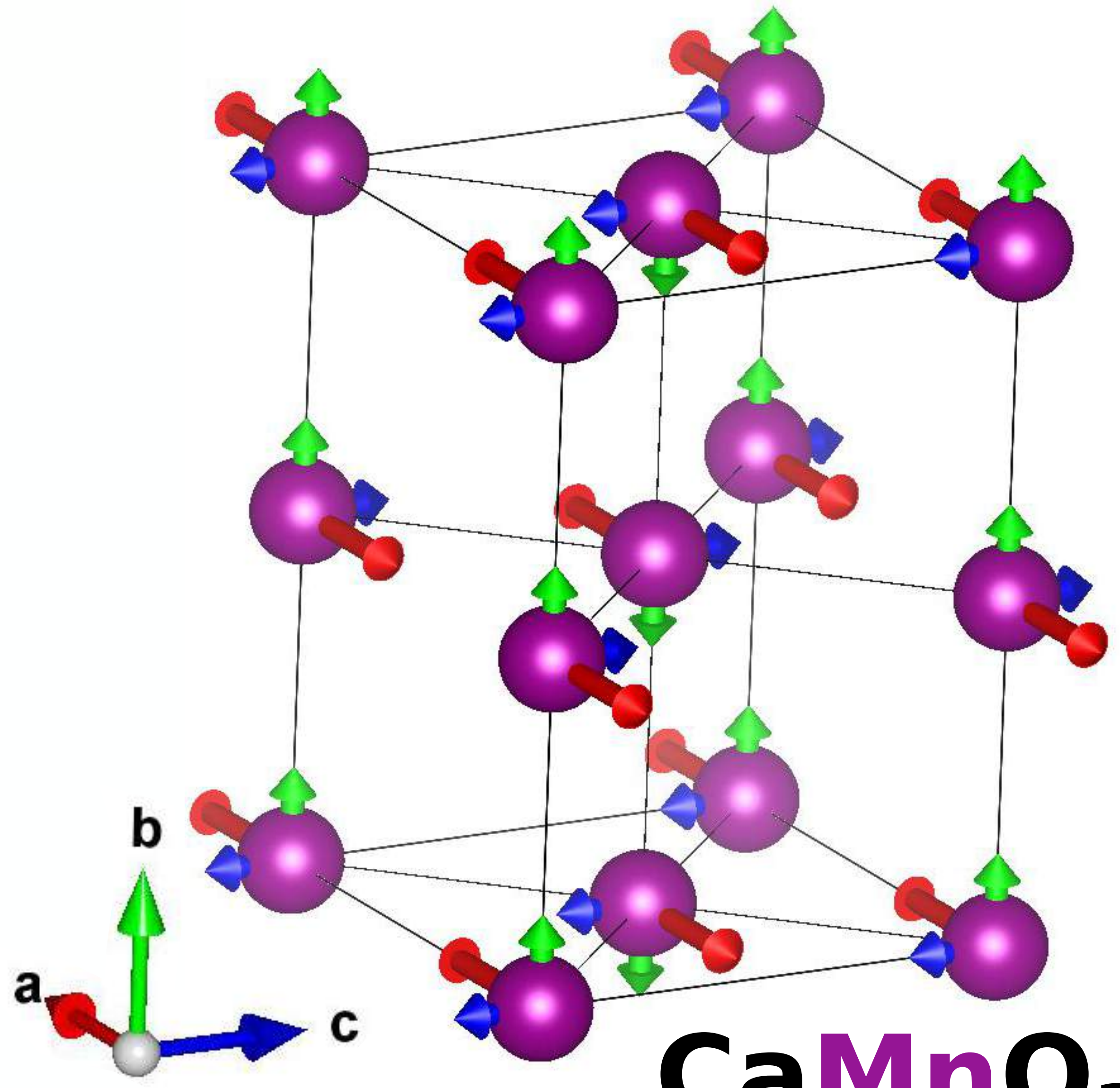
Oxygen octahedra rotate to accommodate the A atom, leading to a larger unit cell with orthorhombic symmetry (*Pnma*). AF becomes AM!

First-principle calculations for orthorhombic crystals show noncollinear magnetic moments, e.g. $G_aC_bA_c$.



octahedral rotations





$CaMnO_3$

Dispersion relation and nodal lines

1D representations: Ising-like order parameter Φ (odd under time reversal).

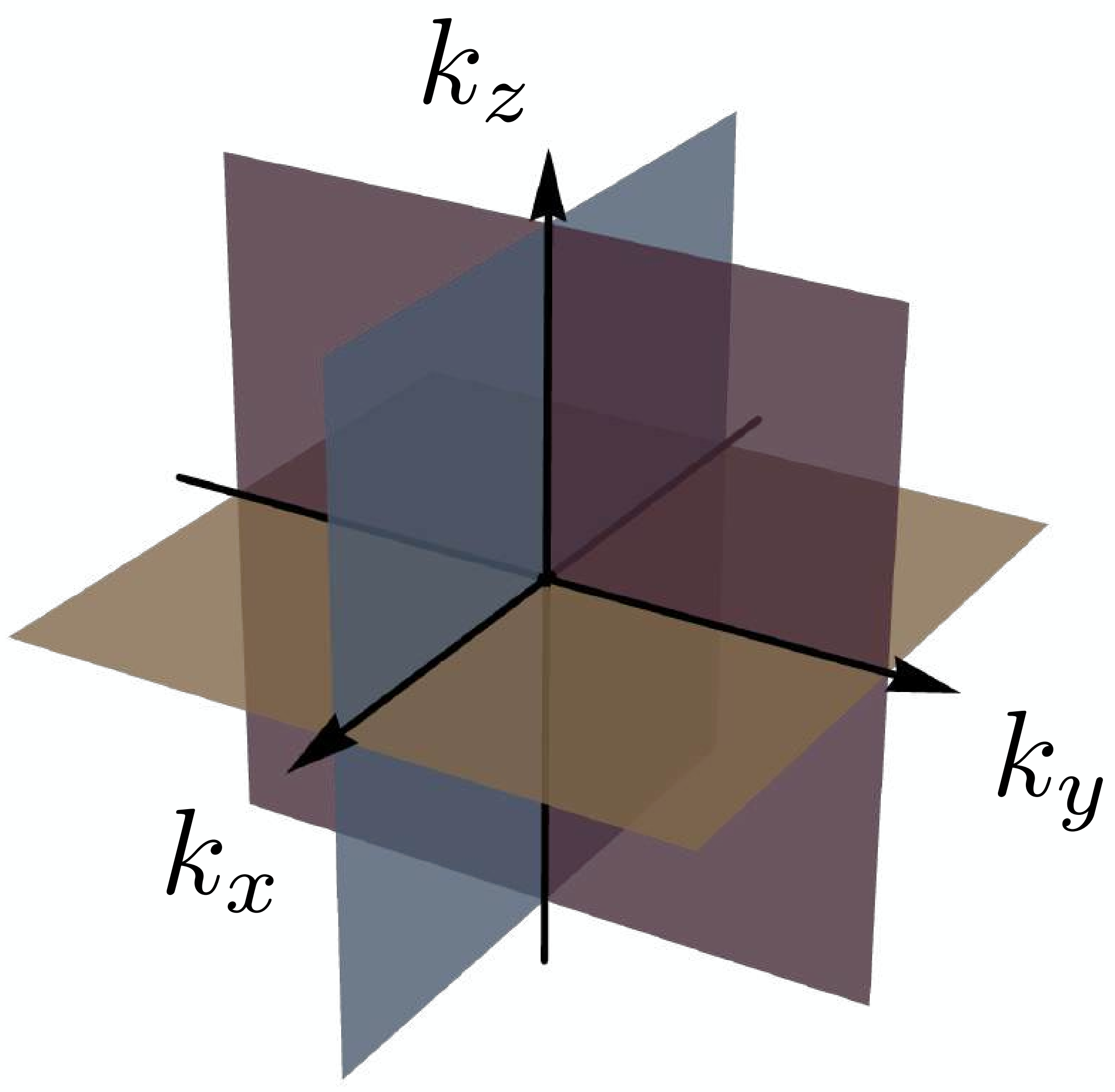
$$H = H_0 + H_{\text{int}} = \sum_{\mathbf{k}} c_{\mathbf{k}}^{\dagger} \mathcal{H}(\mathbf{k}) c_{\mathbf{k}}$$

$$\mathcal{H}(\mathbf{k}) = \varepsilon_{\mathbf{k}} \mathbb{I} - \lambda \Phi \, \mathbf{d}(\mathbf{k}) \cdot \boldsymbol{\sigma}$$

Dispersion: $\boxed{E_{\pm}(\mathbf{k}) = \varepsilon_{\mathbf{k}} \pm \lambda \Phi \left| \mathbf{d}(\mathbf{k}) \right|}$

Spin degeneracy: $E_{+}(\mathbf{k}) = E_{-}(\mathbf{k}) \Leftrightarrow \mathbf{d}(\mathbf{k}) = 0$

Example: B_{1g}^{-} representation of D_{4h}

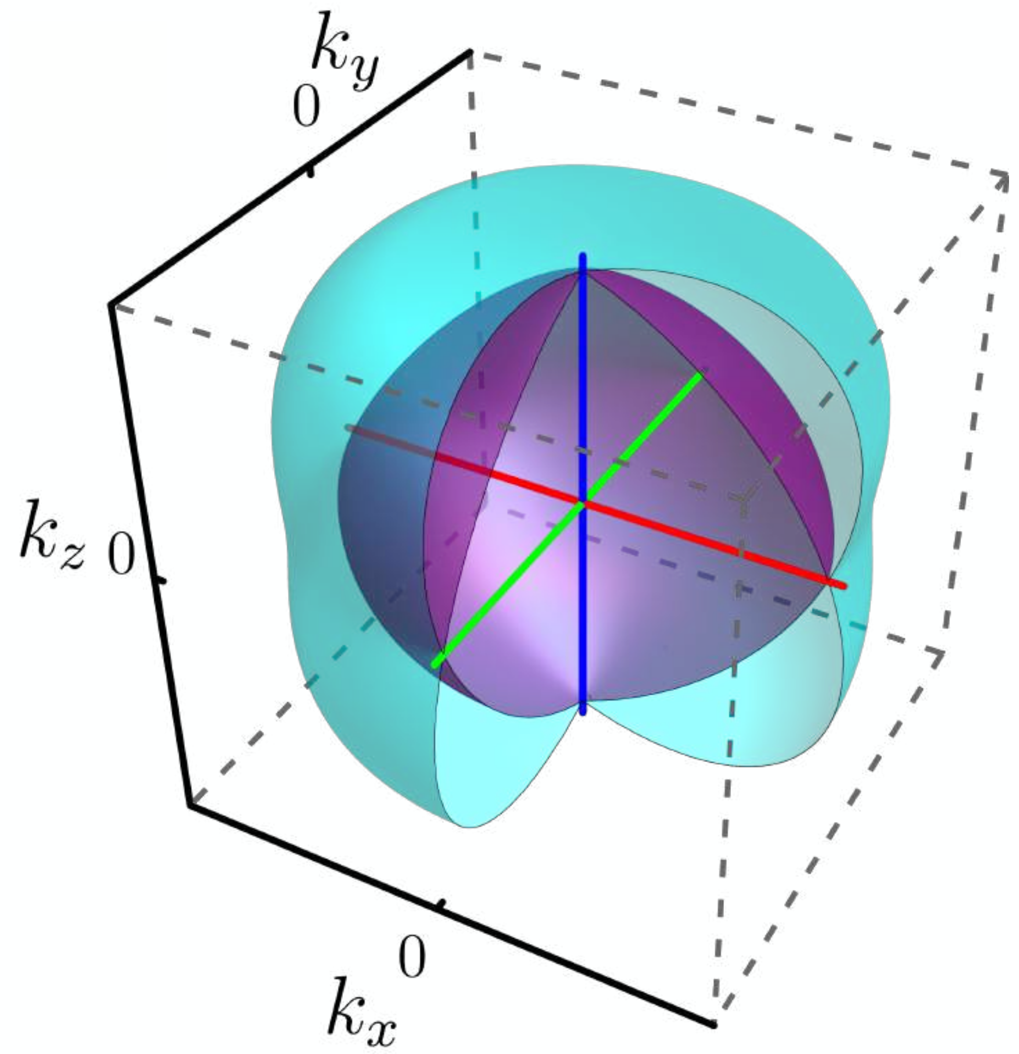


$$\mathbf{d}(\mathbf{k}) = (k_y k_z, k_x k_z, \eta k_x k_y) = 0$$

nodal lines

Fermi surfaces and spin texture

Fermi surfaces: $E_{\pm}(\mathbf{k}) = \frac{\mathbf{k}^2}{2m} \pm \lambda\Phi|\mathbf{d}(\mathbf{k})| = E_F$

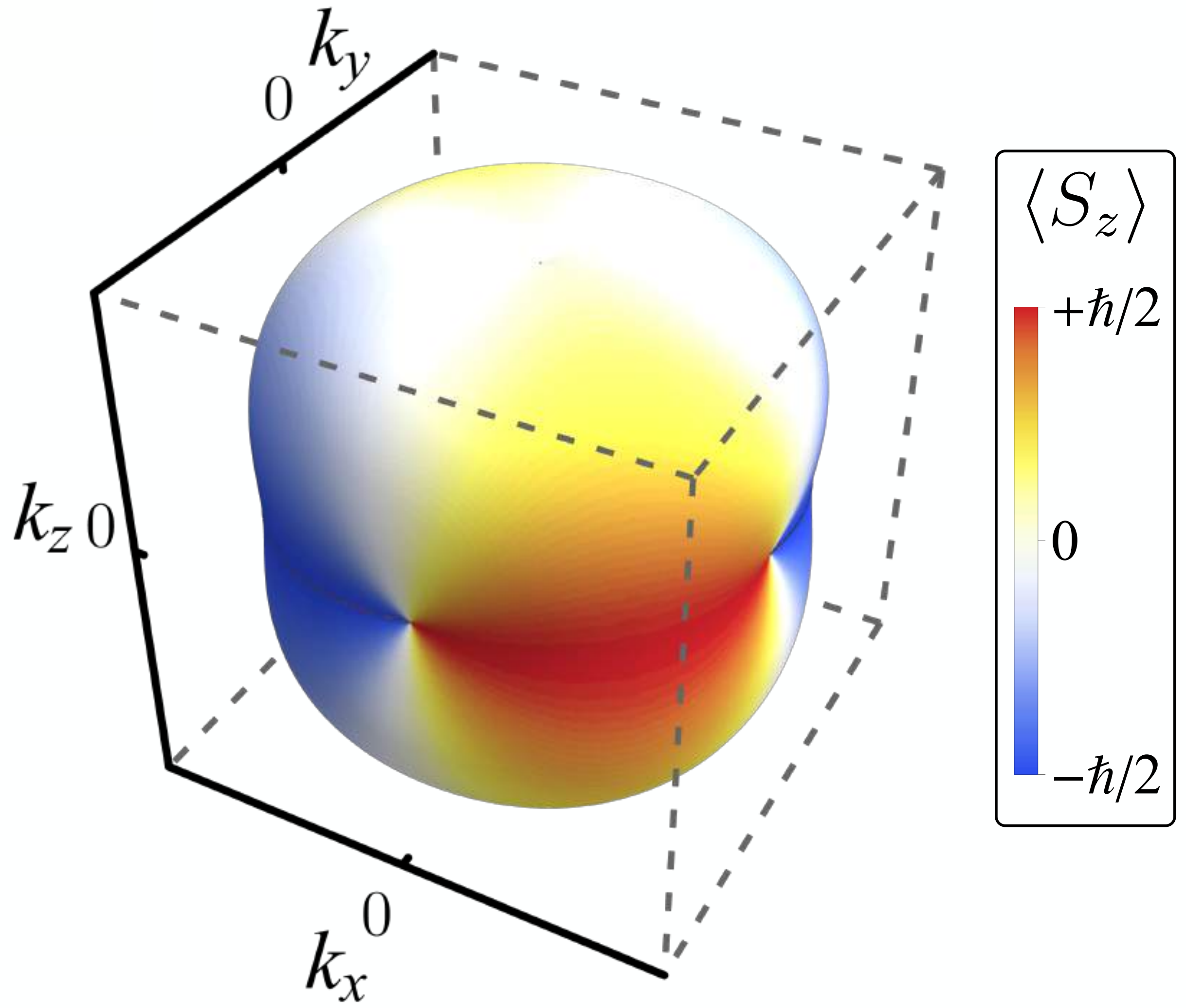


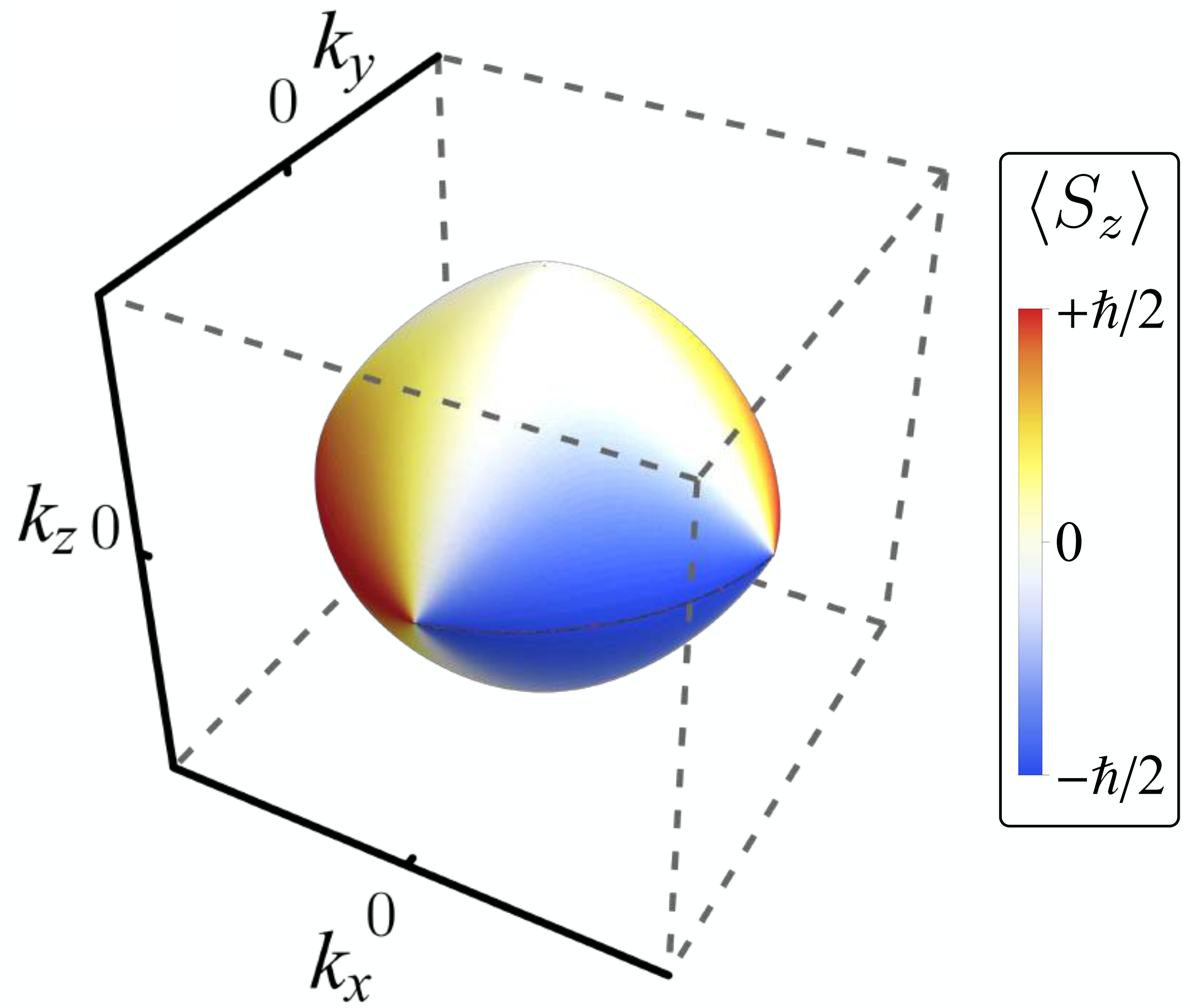
Inner and outer Fermi surfaces touch at pinch points where the sphere $k = k_F$ intercepts the nodal lines.

$$\mathbf{d}(\mathbf{k}) = (k_y k_z, k_x k_z, \eta k_x k_y)$$

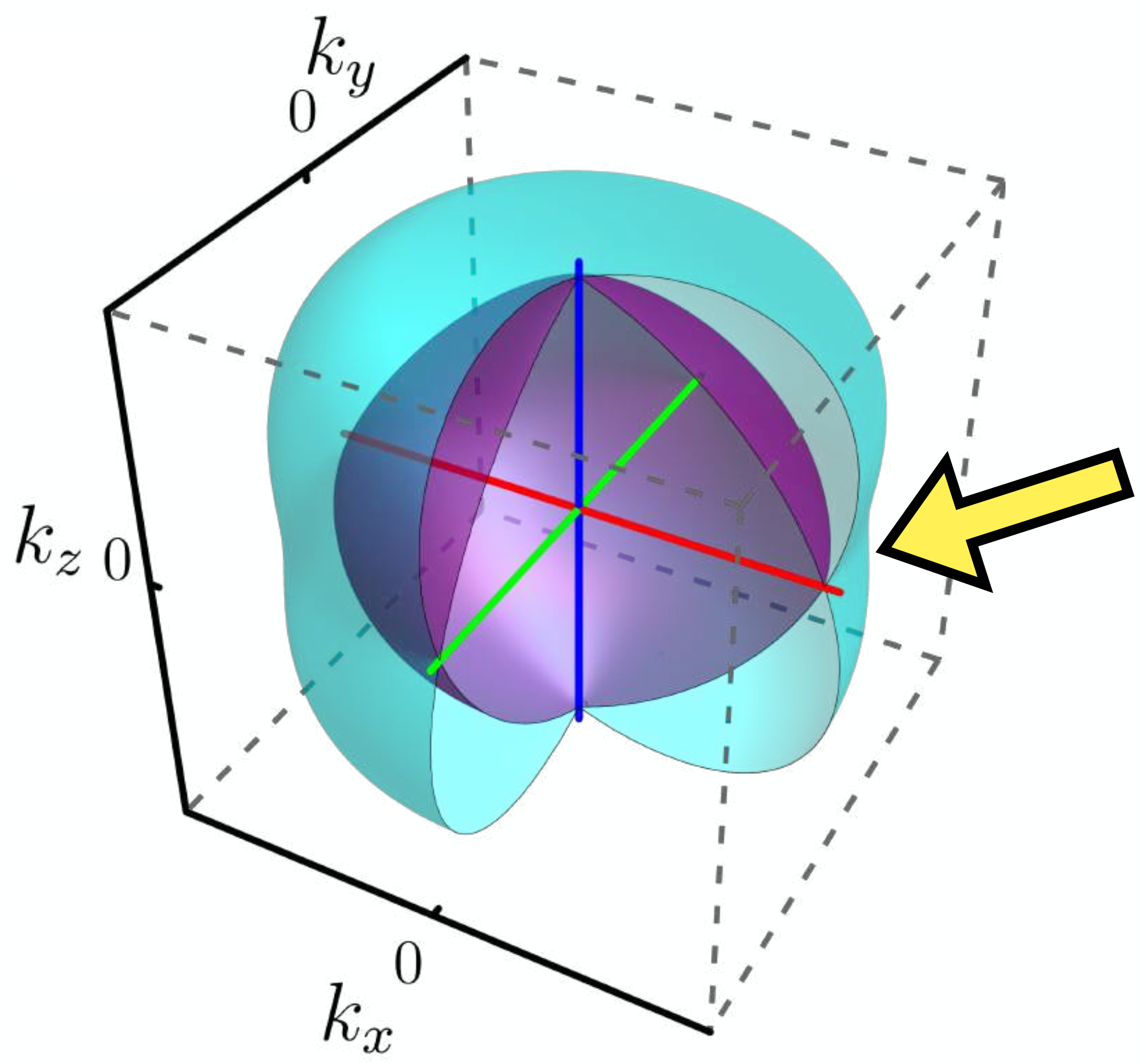
$$k_z = 0 \Rightarrow \mathbf{d} = \eta k_x k_y \hat{\mathbf{z}}$$

Spin polarization rotates along a path in the plane perpendicular to the nodal line.





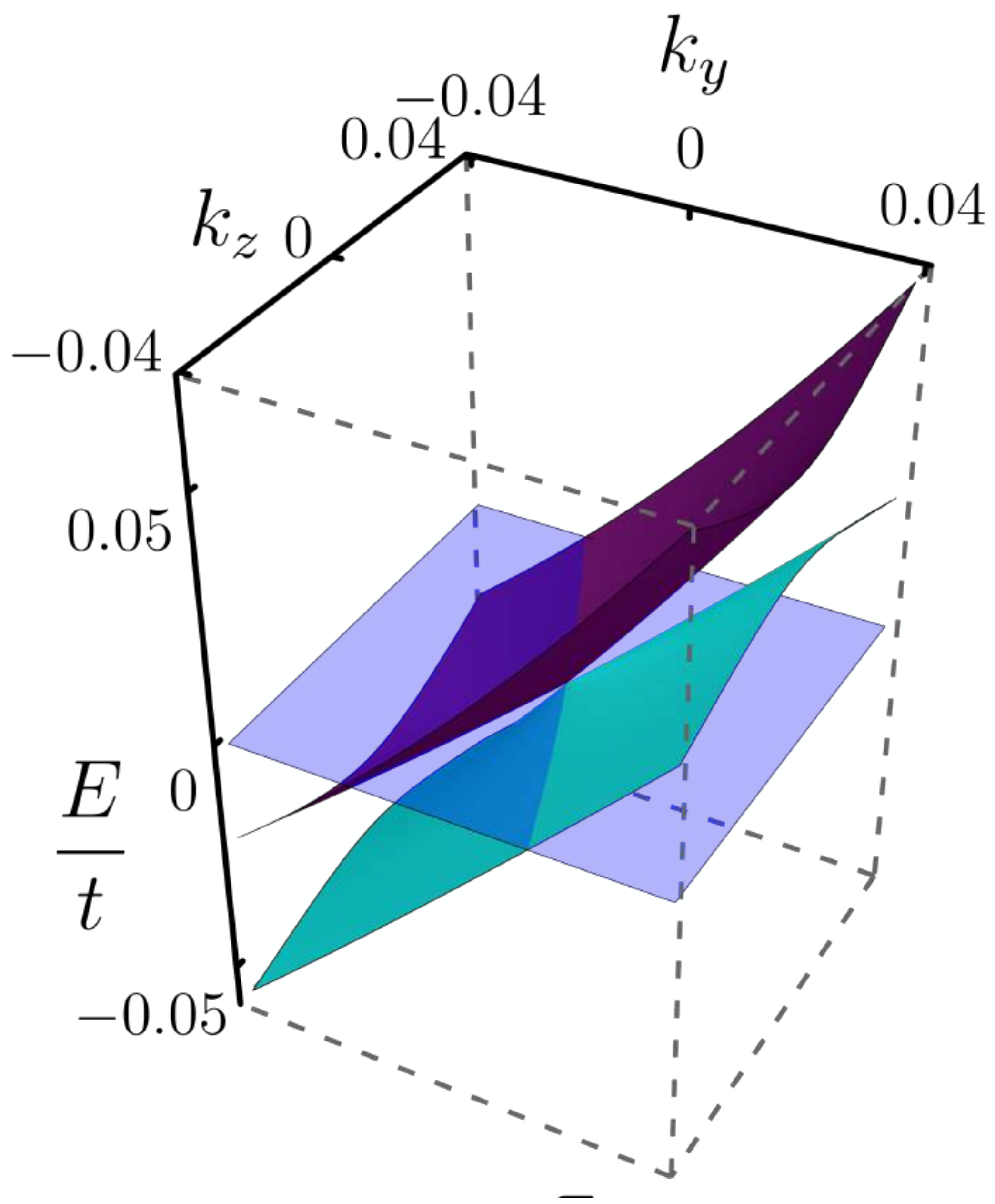
Vicinity of the pinch points



Expand Hamiltonian around a given pinch point:

$$\mathbf{k} = k_F \hat{\mathbf{x}} + \mathbf{q} \qquad\qquad |\mathbf{q}| \ll k_F$$

$$\boxed{H_{\mathbf{q}} = v_F q_x \sigma^0 + \lambda \Phi k_F \left(q_z \sigma^y + \eta q_y \sigma^z \right)}$$



Nontrivial Berry phase around nodal line.

Cf. effective Hamiltonian for type-II Weyl semimetals.

[Soluyanov, Gresch, Wang, Wu, Troyer, Dai, Bernevig, Nature 2015]

Robustness of nodal structure

Characterize phase by momentum-dependent spin splitting:

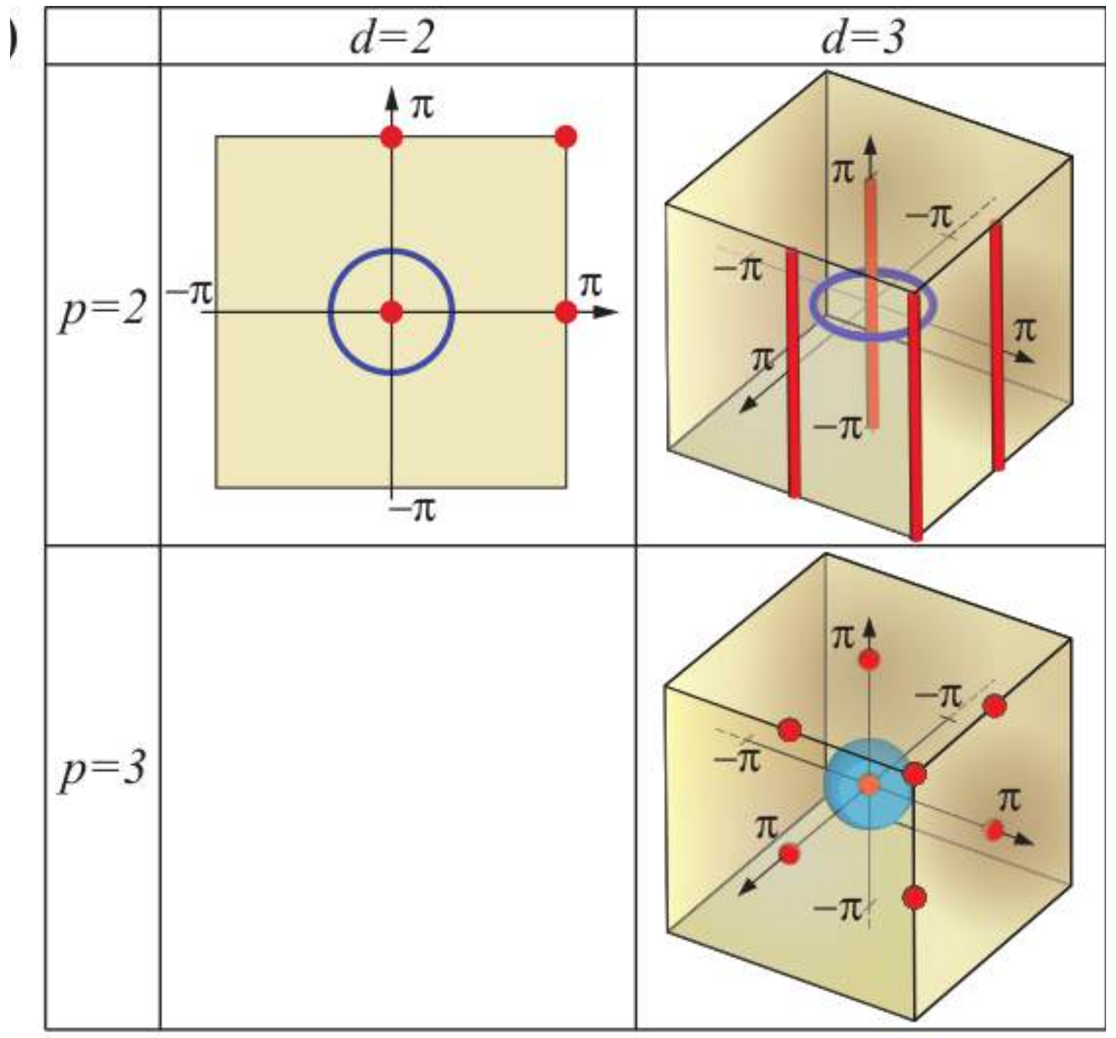
$$\Delta E(\mathbf{k}) \equiv E_+(\mathbf{k}) - E_-(\mathbf{k}) \propto |\mathbf{d}(\mathbf{k})|$$

$\mathbf{d}(\mathbf{k}) = 0 \;\Rightarrow\;$ 3 nonlinear equations for 3 components of **k**

Robust against perturbations? Equivalent to asking about stability of the nodal manifold in

$$\mathcal{H}_{\text{int}}(\mathbf{k}) = -\lambda\Phi\,\mathbf{d}(\mathbf{k})\cdot\boldsymbol{\sigma}$$

Related to classification of gapless topological phases.



[Review: Chiu, Teo, Schnyder, Ryu, RMP 2016]

Symmetry-protected nodal lines

Considering only non-spatial symmetries (charge conjugation, time reversal and chiral symmetry), nodal lines are topologically trivial.

class\\δ	T	C	S
A	0	0	0
AIII	0	0	1
AI	+	0	0
BDI	+	+	1
D	0	+	0
DIII	−	+	1
AII	−	0	0
CII	−	−	1
C	0	−	0
CI	+	−	1

FS1	p=8	p=1	p=2	p=3	p=4	p=5	p=6	p=7
FS2	p=2	p=3	p=4	p=5	p=6	p=7	p=8	p=1
TI/TSC	d=1	d=2	d=3	d=4	d=5	d=6	d=7	d=8
A	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AI	0	0^a	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2^{a,b}$	$\mathbb{Z}_2^b$	$\mathbb{Z}$
BDI	$\mathbb{Z}$	0	0^a	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2^{a,b}$	$\mathbb{Z}_2^b$
D	$\mathbb{Z}_2^b$	$\mathbb{Z}$	0	0^a	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2^{a,b}$
DIII	$\mathbb{Z}_2^{a,b}$	$\mathbb{Z}_2^b$	$\mathbb{Z}$	0	0^a	0	$2\mathbb{Z}$	0
AII	0	$\mathbb{Z}_2^{a,b}$	$\mathbb{Z}_2^b$	$\mathbb{Z}$	0	0^a	0	$2\mathbb{Z}$
CII	$2\mathbb{Z}$	0	$\mathbb{Z}_2^{a,b}$	$\mathbb{Z}_2^b$	$\mathbb{Z}$	0	0^a	0
C	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2^{a,b}$	$\mathbb{Z}_2^b$	$\mathbb{Z}$	0	0^a
CI	0^a	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2^{a,b}$	$\mathbb{Z}_2^b$	$\mathbb{Z}$	0

$$C = i\sigma^y \mathcal{K}$$

$$C^{-1}\mathcal{H}_{\text{int}}(-\mathbf{k})C = -\mathcal{H}_{\text{int}}(\mathbf{k})$$

However…

[Chiu & Schnyder, PRB 2014; Knoll & Timm, PRB 2022]

Symmetry-protected nodal lines

Nodal lines belong to mirror planes. Stable against perturbations that respect mirror symmetry.

Reflection	top. insul. and top. SC	d=1	d=2	d=3	d=4	d=5	d=6	d=7	d=8
	FS within mirror plane at high-sym. point	p=8	p=1	p=2	p=3	p=4	p=5	p=6	p=7
	FS within mirror plane off high-sym. point	p=2	p=3	p=4	p=5	p=6	p=7	p=8	p=1
R	A	$M\mathbb{Z}$	0	$M\mathbb{Z}$	0	$M\mathbb{Z}$	0	$M\mathbb{Z}$	0
R_+	AIII	0	$M\mathbb{Z}$	0	$M\mathbb{Z}$	0	$M\mathbb{Z}$	0	$M\mathbb{Z}$
R_-	AIII	$M\mathbb{Z}\oplus\mathbb{Z}$	0	$M\mathbb{Z}\oplus\mathbb{Z}$	0	$M\mathbb{Z}\oplus\mathbb{Z}$	0	$M\mathbb{Z}\oplus\mathbb{Z}$	0
R_+,R_{++}	AI	$M\mathbb{Z}$	0	0	0	$2M\mathbb{Z}$	0	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}_2^{\text{a,b}}$
	BDI	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$	0	0	0	$2M\mathbb{Z}$	0	$M\mathbb{Z}_2^{\text{a,b}}$
	D	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$	0	0	0	$2M\mathbb{Z}$	0
	DIII	0	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$	0	0	0	$2M\mathbb{Z}$
	AII	$2M\mathbb{Z}$	0	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$	0	0	0
	CII	0	$2M\mathbb{Z}$	0	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$	0	0
	C	0	0	$2M\mathbb{Z}$	0	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$	0
	CI	0	0	0	$2M\mathbb{Z}$	0	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$
R_-,R_{--}	AI	0	0	$2M\mathbb{Z}$	0	$T\mathbb{Z}_2^{\text{a,b,c}}$	$\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$	0
	BDI	0	0	0	$2M\mathbb{Z}$	0	$T\mathbb{Z}_2^{\text{a,b,c}}$	$\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$
	D	$M\mathbb{Z}$	0	0	0	$2M\mathbb{Z}$	0	$T\mathbb{Z}_2^{\text{a,b,c}}$	$\mathbb{Z}_2^{\text{a,b}}$
	DIII	$\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$	0	0	0	$2M\mathbb{Z}$	0	$T\mathbb{Z}_2^{\text{a,b,c}}$
	AII	$T\mathbb{Z}_2^{\text{a,b,c}}$	$\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$	0	0	0	$2M\mathbb{Z}$	0
	CII	0	$T\mathbb{Z}_2^{\text{a,b,c}}$	$\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$	0	0	0	$2M\mathbb{Z}$
	C	$2M\mathbb{Z}$	0	$T\mathbb{Z}_2^{\text{a,b,c}}$	$\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$	0	0	0
	CI	0	$2M\mathbb{Z}$	0	$T\mathbb{Z}_2^{\text{a,b,c}}$	$\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$	0	0
R_{-+}	BDI, CII	$2\mathbb{Z}$	0	$2M\mathbb{Z}$	0	$2\mathbb{Z}$	0	$2M\mathbb{Z}$	0
R_{+-}	DIII, CI	$2M\mathbb{Z}$	0	$2\mathbb{Z}$	0	$2M\mathbb{Z}$	0	$2\mathbb{Z}$	0
R_{+-}	BDI	$M\mathbb{Z}\oplus\mathbb{Z}$	0	0	0	$2M\mathbb{Z}\oplus 2\mathbb{Z}$	0	$M\mathbb{Z}_2\oplus\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}_2\oplus\mathbb{Z}_2^{\text{a,b}}$
R_{-+}	DIII	$M\mathbb{Z}_2\oplus\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}_2\oplus\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}\oplus\mathbb{Z}$	0	0	0	$2M\mathbb{Z}\oplus 2\mathbb{Z}$	0
R_{+-}	CII	$2M\mathbb{Z}\oplus 2\mathbb{Z}$	0	$M\mathbb{Z}_2\oplus\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}_2\oplus\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}\oplus\mathbb{Z}$	0	0	0
R_{-+}	CI	0	0	$2M\mathbb{Z}\oplus 2\mathbb{Z}$	0	$M\mathbb{Z}_2\oplus\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}_2\oplus\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}\oplus\mathbb{Z}$	0

a

$$R_{\hat{\mathbf{n}}} = \boldsymbol{\sigma}\cdot\hat{\mathbf{n}}$$

$$R_{\hat{\mathbf{n}}}\mathcal{H}_{\text{int}}(\mathcal{R}_{\hat{\mathbf{n}}}\mathbf{k})R_{\hat{\mathbf{n}}} = \mathcal{H}_{\text{int}}(\mathbf{k})$$

[Chiu & Schnyder, PRB 2014]

Symmetry-protected nodal lines

Nodal lines belong to mirror planes. Stable against perturbations that respect mirror symmetry.

FS within mirror plane off high-sym. point

Refl			d=2	d=3	d=4	d=5	d=6	d=7	d=8
			p=1	p=2	p=3	p=4	p=5	p=6	p=7
			p=3	p=4	p=5	p=6	p=7	p=8	p=1
R	A	$M\mathbb{Z}$	0	$M\mathbb{Z}$	0	$M\mathbb{Z}$	0	$M\mathbb{Z}$	0
R_+	AIII	0	$M\mathbb{Z}$	0	$M\mathbb{Z}$	0	$M\mathbb{Z}$	0	$M\mathbb{Z}$
R_-	AIII	$M\mathbb{Z}\oplus\mathbb{Z}$	0	$M\mathbb{Z}\oplus\mathbb{Z}$	0	$M\mathbb{Z}\oplus\mathbb{Z}$	0	$M\mathbb{Z}\oplus\mathbb{Z}$	0
R_+,R_{++}	AI	$M\mathbb{Z}$	0	0	0	$2M\mathbb{Z}$	0	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}_2^{\text{a,b}}$
	BDI	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$	0	0	0	$2M\mathbb{Z}$	0	$M\mathbb{Z}_2^{\text{a,b}}$
	D	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$	0	0	0	$2M\mathbb{Z}$	0
	DIII	0	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$	0	0	0	$2M\mathbb{Z}$
	AII	$2M\mathbb{Z}$	0	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$	0	0	0
	CII	0	$2M\mathbb{Z}$	0	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$	0	0
	C	0	0	$2M\mathbb{Z}$	0	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$	0
	CI	0	0	0	$2M\mathbb{Z}$	0	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$
R_-,R_{--}	AI	0	0	$2M\mathbb{Z}$	0	$T\mathbb{Z}_2^{\text{a,b,c}}$	$\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$	0
	BDI	0	0	0	$2M\mathbb{Z}$	0	$T\mathbb{Z}_2^{\text{a,b,c}}$	$\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$
	D	$M\mathbb{Z}$	0	0	0	$2M\mathbb{Z}$	0	$T\mathbb{Z}_2^{\text{a,b,c}}$	$\mathbb{Z}_2^{\text{a,b}}$
	DIII	$\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$	0	0	0	$2M\mathbb{Z}$	0	$T\mathbb{Z}_2^{\text{a,b,c}}$
	AII	$T\mathbb{Z}_2^{\text{a,b,c}}$	$\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$	0	0	0	$2M\mathbb{Z}$	0
	CII		$T\mathbb{Z}_2^{\text{a,b,c}}$	$\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$	0	0	0	$2M\mathbb{Z}$
	C		0	$T\mathbb{Z}_2^{\text{a,b,c}}$	$\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$	0	0	0
	CI	$2M\mathbb{Z}$	$2M\mathbb{Z}$	0	$T\mathbb{Z}_2^{\text{a,b,c}}$	$\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}$	0	0
R_{-+}	BDI, CII	$2\mathbb{Z}$	0	$2M\mathbb{Z}$	0	$2\mathbb{Z}$	0	$2M\mathbb{Z}$	0
R_{+-}	DIII, CI	$2M\mathbb{Z}$	0	$2\mathbb{Z}$	0	$2M\mathbb{Z}$	0	$2\mathbb{Z}$	0
R_{+-}	BDI	$M\mathbb{Z}\oplus\mathbb{Z}$	0	0	0	$2M\mathbb{Z}\oplus 2\mathbb{Z}$	0	$M\mathbb{Z}_2\oplus\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}_2\oplus\mathbb{Z}_2^{\text{a,b}}$
R_{-+}	DIII	$M\mathbb{Z}_2\oplus\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}_2\oplus\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}\oplus\mathbb{Z}$	0	0	0	$2M\mathbb{Z}\oplus 2\mathbb{Z}$	0
R_{+-}	CII	$2M\mathbb{Z}\oplus 2\mathbb{Z}$	0	$M\mathbb{Z}_2\oplus\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}_2\oplus\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}\oplus\mathbb{Z}$	0	0	0
R_{-+}	CI	0	0	$2M\mathbb{Z}\oplus 2\mathbb{Z}$	0	$M\mathbb{Z}_2\oplus\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}_2\oplus\mathbb{Z}_2^{\text{a,b}}$	$M\mathbb{Z}\oplus\mathbb{Z}$	0

$$R_{\hat{\mathbf{n}}} = \boldsymbol{\sigma}\cdot\hat{\mathbf{n}}$$

$$R_{\hat{\mathbf{n}}}\mathcal{H}_{\text{int}}(\mathcal{R}_{\hat{\mathbf{n}}}\mathbf{k})R_{\hat{\mathbf{n}}} = \mathcal{H}_{\text{int}}(\mathbf{k})$$

$$R_{\hat{\mathbf{n}}}C = -CR_{\hat{\mathbf{n}}}$$

Topological invariant from mirror eigenvalues.

Topological transition induced by magnetic field

Add Zeeman term as a perturbation: $\boxed{\mathcal{H}_Z = \mathbf{h} \cdot \boldsymbol{\sigma}}$

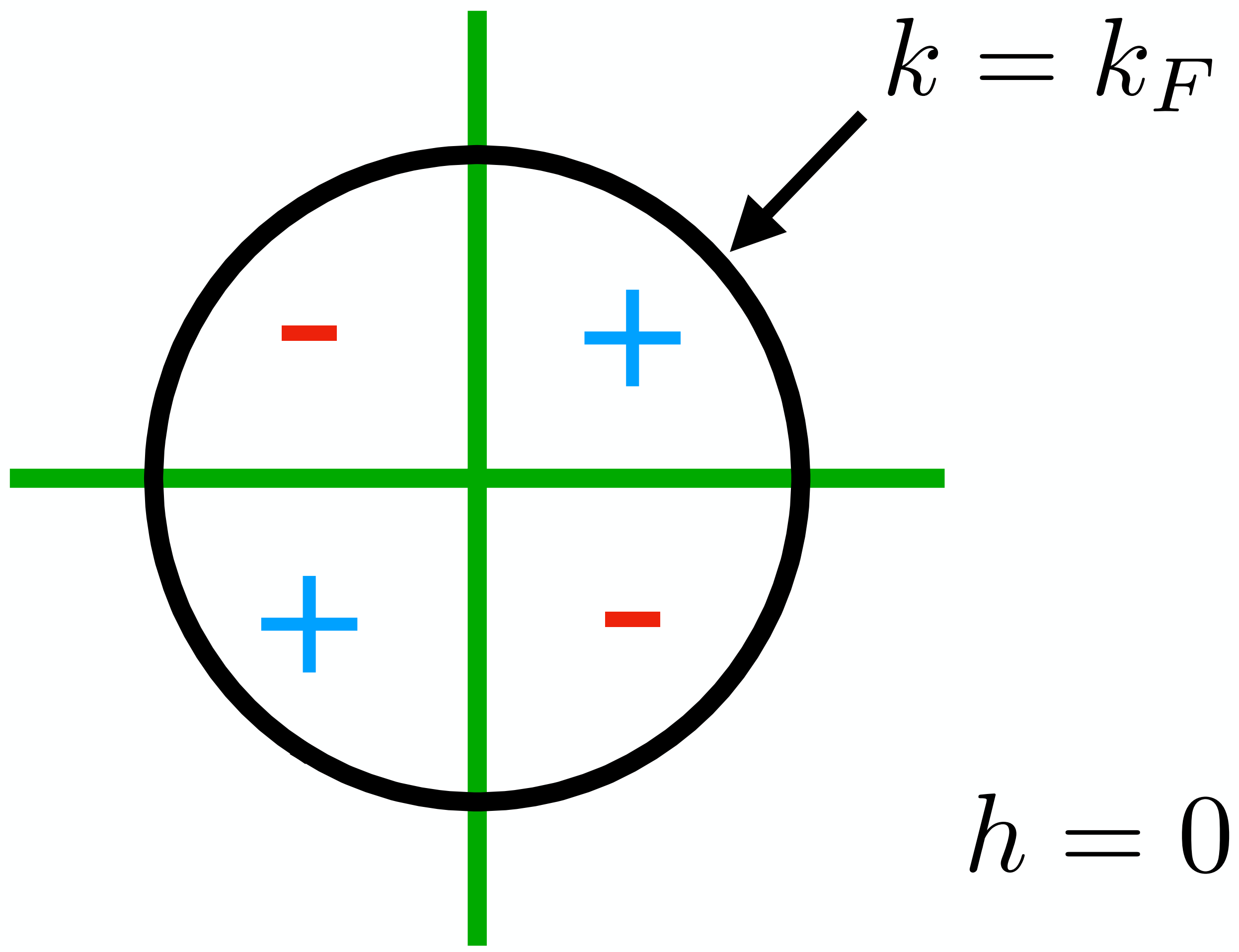
Equivalent to shifting d vector: $\tilde{\mathbf{d}}(\mathbf{k}) = \mathbf{d}(\mathbf{k}) - \frac{\mathbf{h}}{\lambda \Phi}$

Special case: **h** perpendicular to mirror plane that contains the nodal line.

$$\mathbf{d}(\mathbf{k}) = (k_y k_z, k_x k_z, \eta k_x k_y)$$

plane $k_z = 0$ field $\mathbf{h} = h\hat{\mathbf{z}}$

nodal lines: $\tilde{\mathbf{d}}(\mathbf{k}) = 0 \Rightarrow \boxed{k_x k_y = \frac{h}{\lambda \Phi \eta}}$



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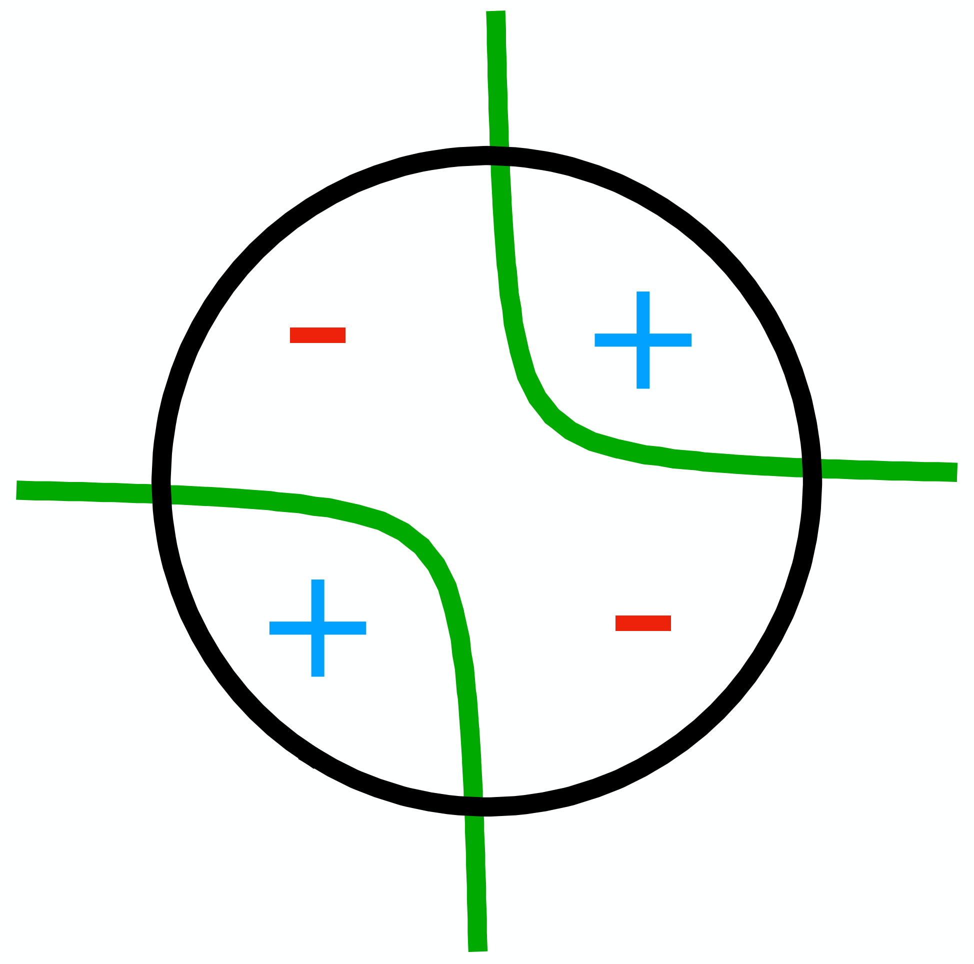

$h > 0$

Topological transition induced by magnetic field

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Equivalent to shifting d vector: $\tilde{\mathbf{d}}(\mathbf{k}) = \mathbf{d}(\mathbf{k}) - \frac{\mathbf{h}}{\lambda\Phi}$

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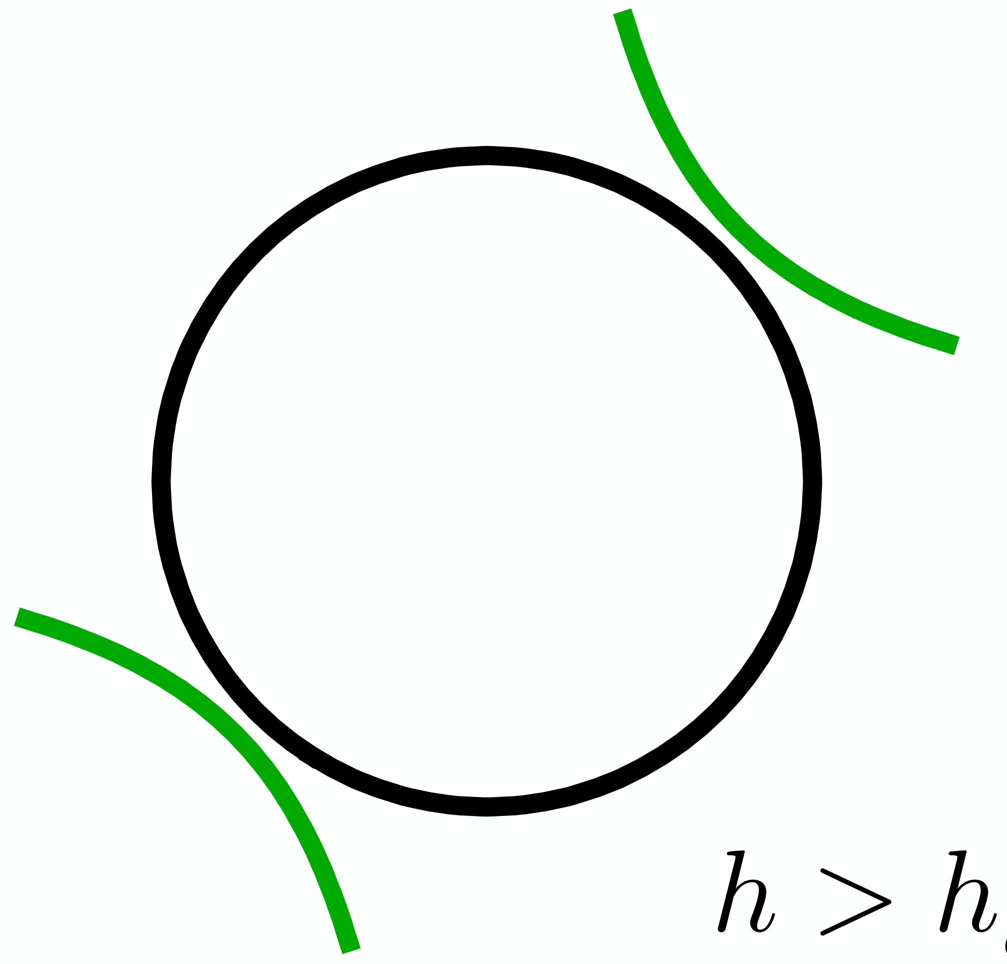
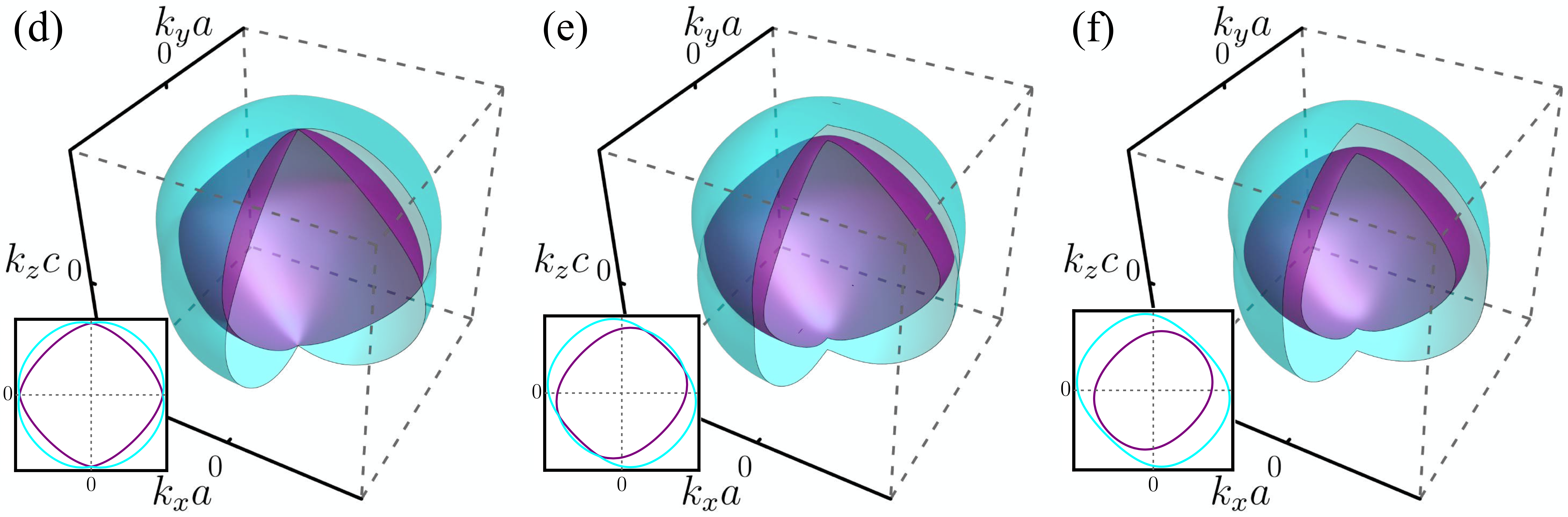


$$h > h_c^* = \frac{\lambda\Phi k_F^2}{2}$$

Topological transition induced by magnetic field

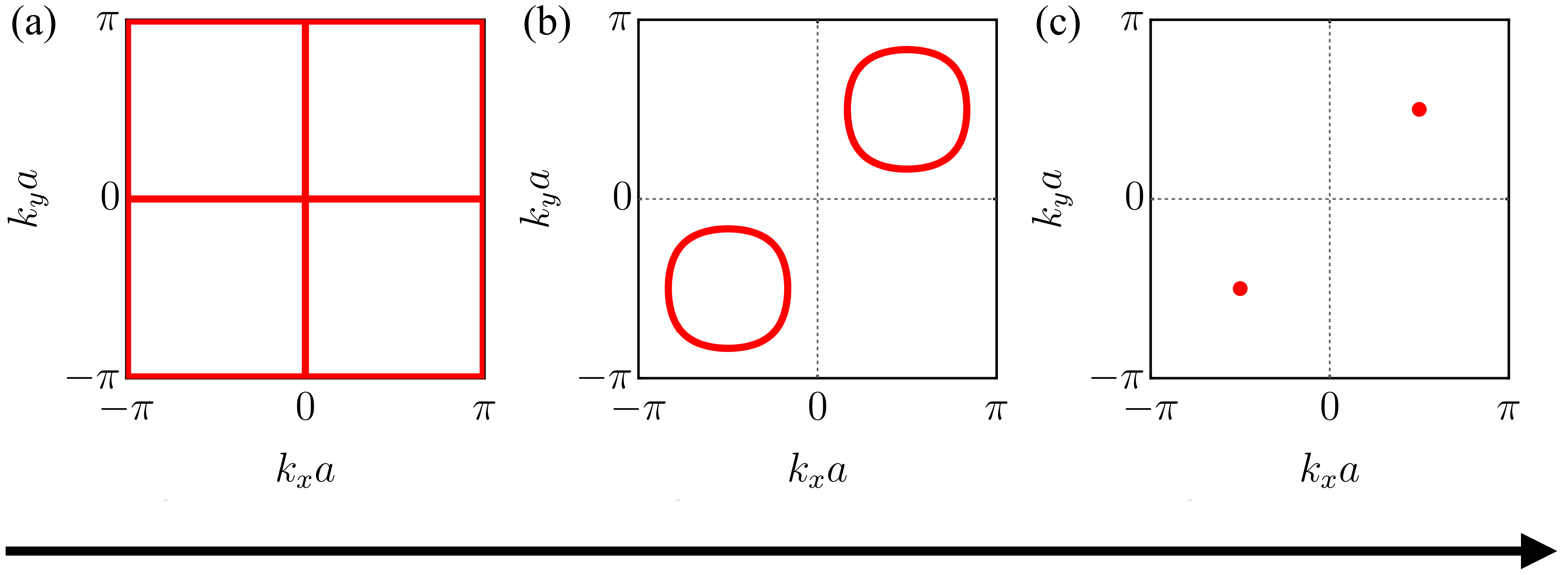
Pinch points move with increasing magnetic field until they meet. Above critical field, Fermi surfaces are fully split (FM-like).



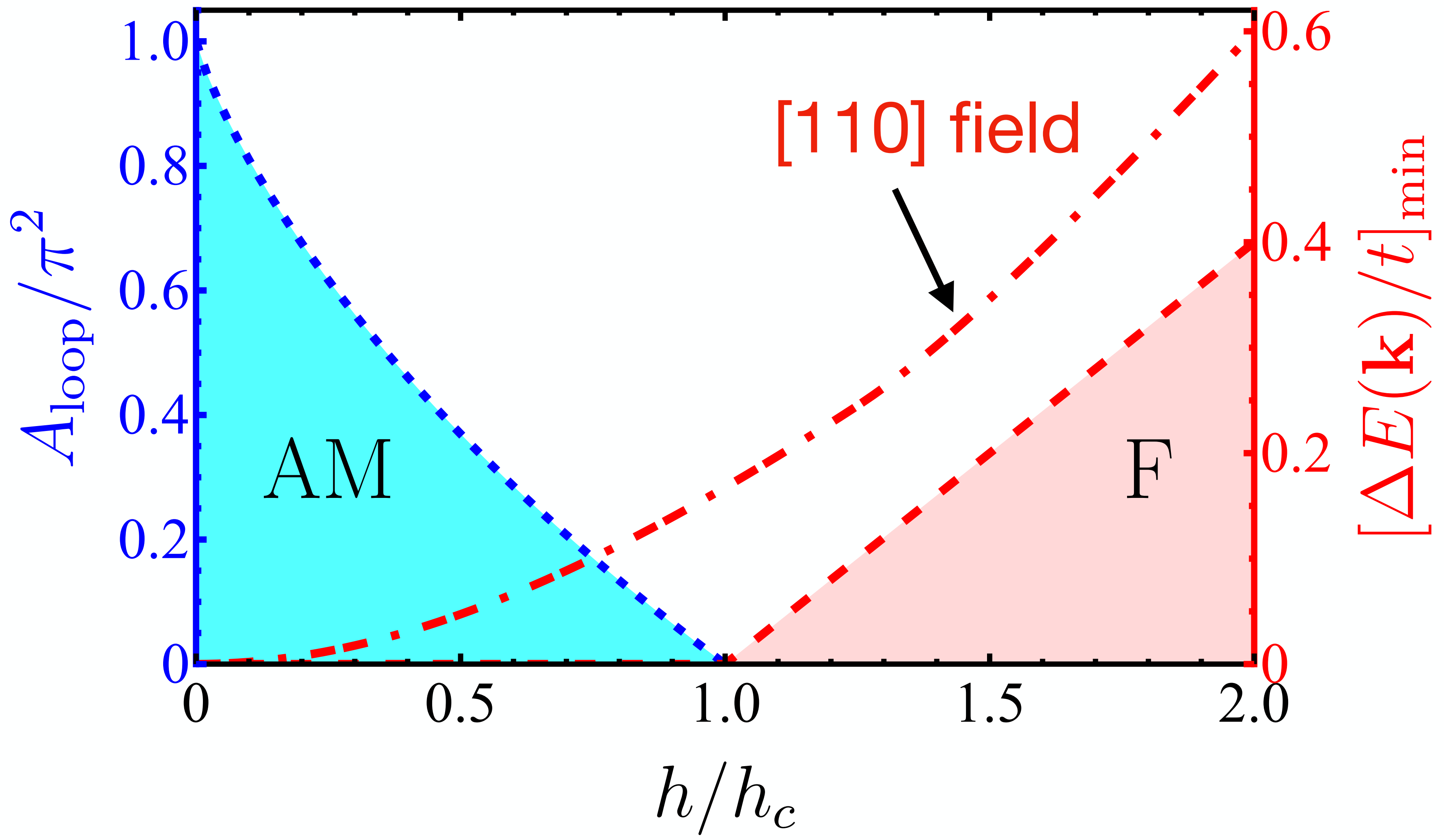
Nodal to nodeless transition

Lattice: nodal lines form closed loops that collapse at a critical magnetic field.

$$h_c = \lambda \Phi / \eta$$



increasing h



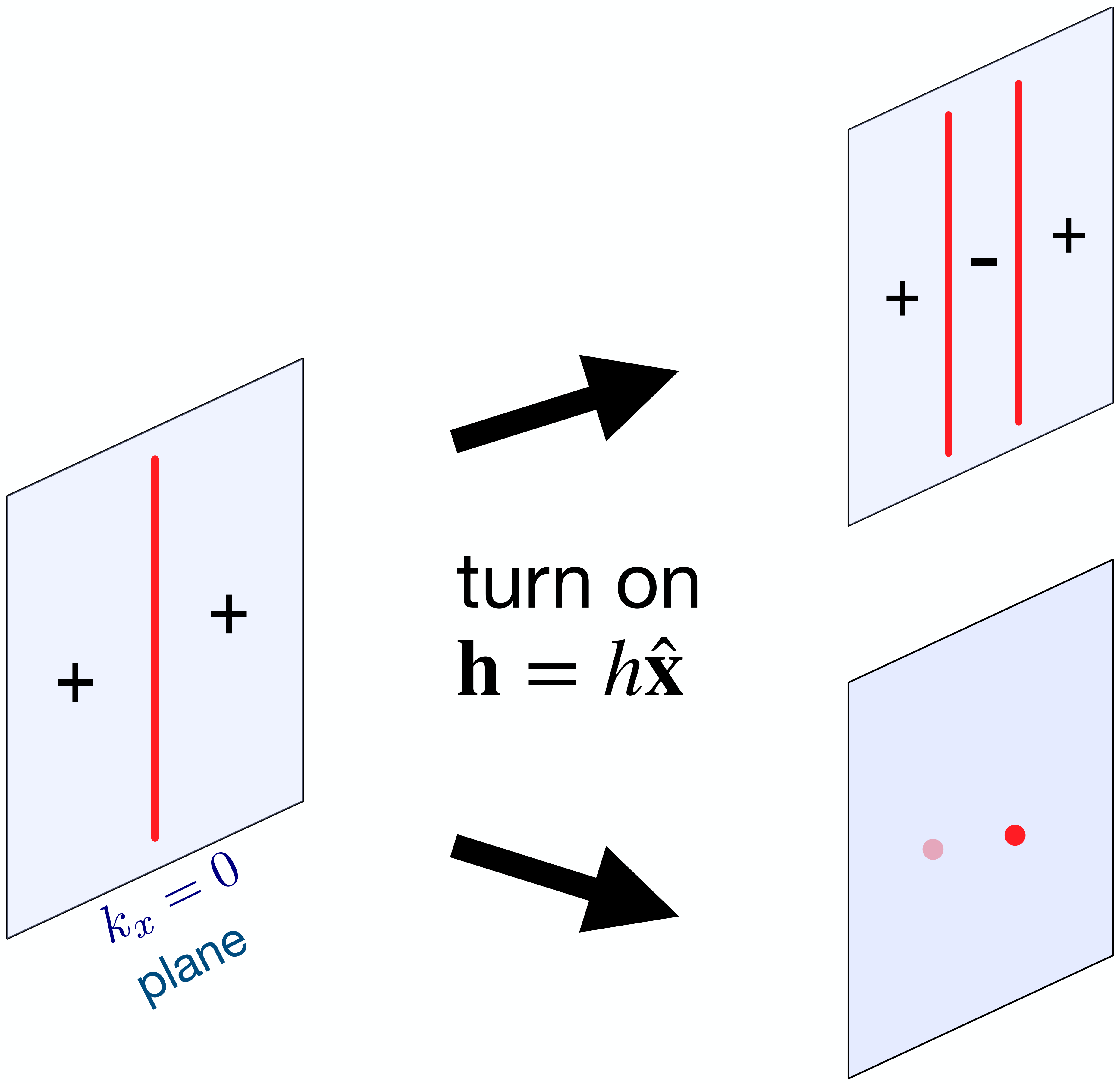
Note: transition can also be driven by strain. ε_{xy}

Another example

D_{6h}, B_{1g}^{-} : $$H(\mathbf{k}) = \varepsilon_{\mathbf{k}}\sigma^0 - \lambda\Phi(k_x^2 - k_y^2)\sigma^x + 2\lambda\Phi k_x k_y \sigma^y - \lambda\Phi\eta k_x k_z (k_x^2 - 3k_y^2)\sigma^z$$

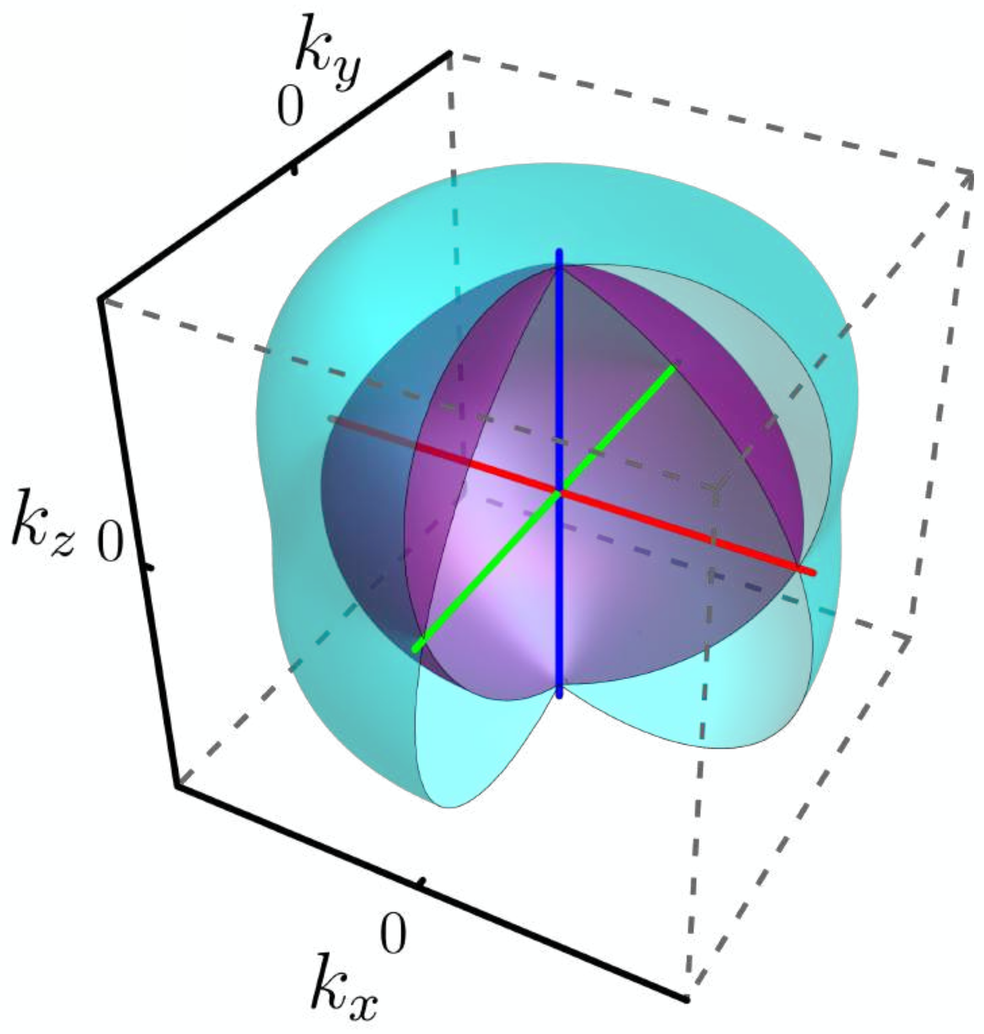
nodal line: $k_x = k_y = 0$

(2π Berry phase)



Conclusions

- Altermagnets + SOC in 3D: bands are degenerate along high-symmetry directions in momentum space (nodal lines).
- Nodal lines of altermagnets can be protected by mirror symmetries. Magnetic fields that preserve the mirror symmetry drive a topological transition from nodal to nodeless Zeeman splitting.



Thank you!