Altermagnetism and anti-altermagnetism in technicolor

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- P.G. Radaelli, “Tensorial approach to altermagnetism”, *Phys. Rev. B* **110**, 214428 (2024).
- P.G. Radaelli & G. Gurung,” Color symmetry and altermagneticlike spin textures in noncollinear antiferromagnets”, *Phys. Rev. B* **112**, 014431 (2025)

Exploring altermagnets and beyond, Mainz 20 to 24 October 2025

A neutron scatterer view of magnetism (circa 2010)

	Γ	k≠Γ	multi-k	Γ+multi-k
Collinear	FM, FiM PT-AFM (ME) Non-PT AFM	SDW **ES-Multiferroics**	**C-E (Manganites)**	
Non -collinear	*PT-AFM (ME)* *Non-PT AFM* **FM spin ice**	**Helicoidal (cycloids, helices), MFs**	**Skyrmion lattice** Triangular lattice	**Fan-structures Transverse conical (Multiferroics)**

Strong magnetization

No magnetisation (PT)

Can have weak mag. or canting

"Fashionable stuff" in bold

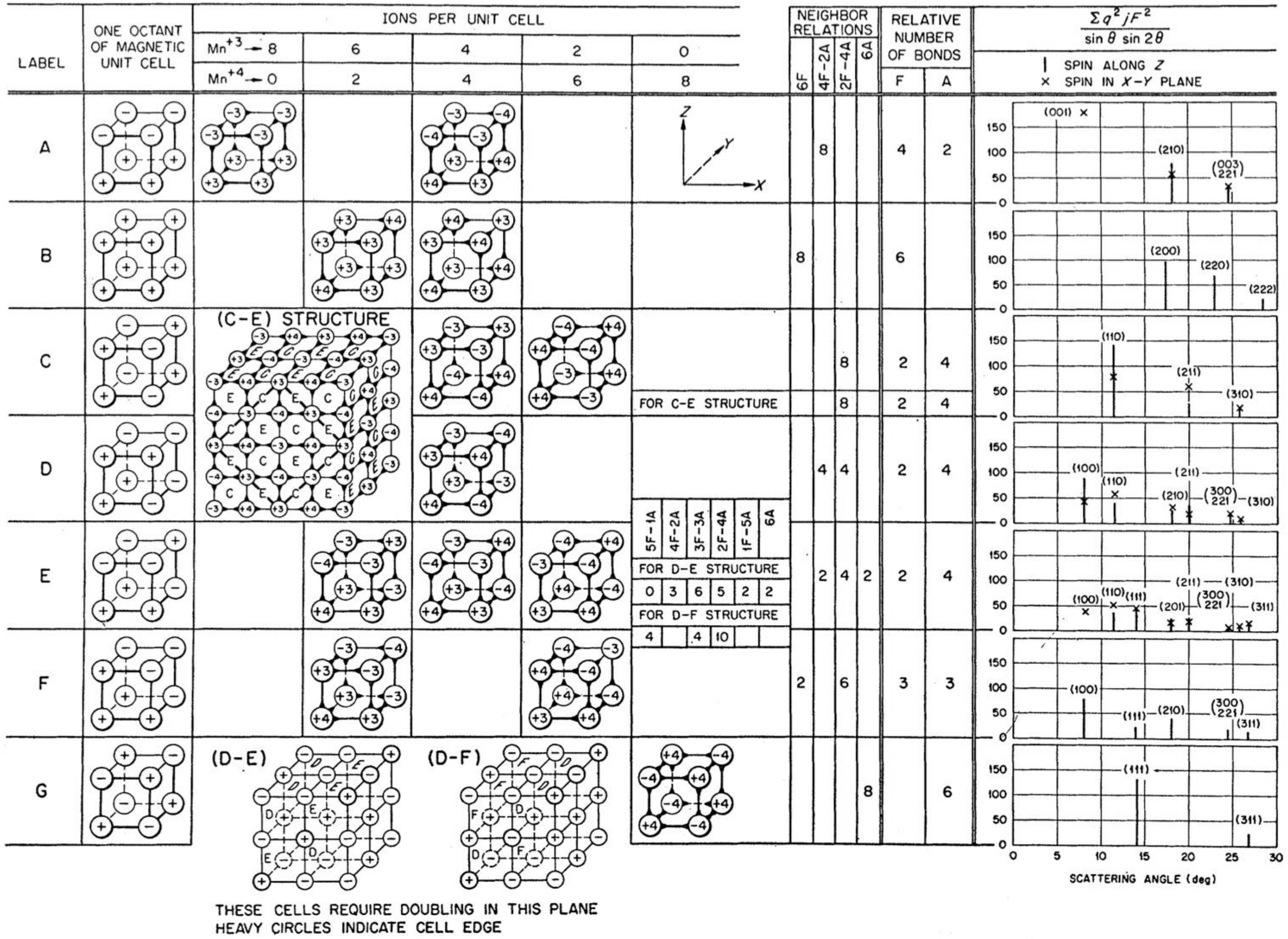
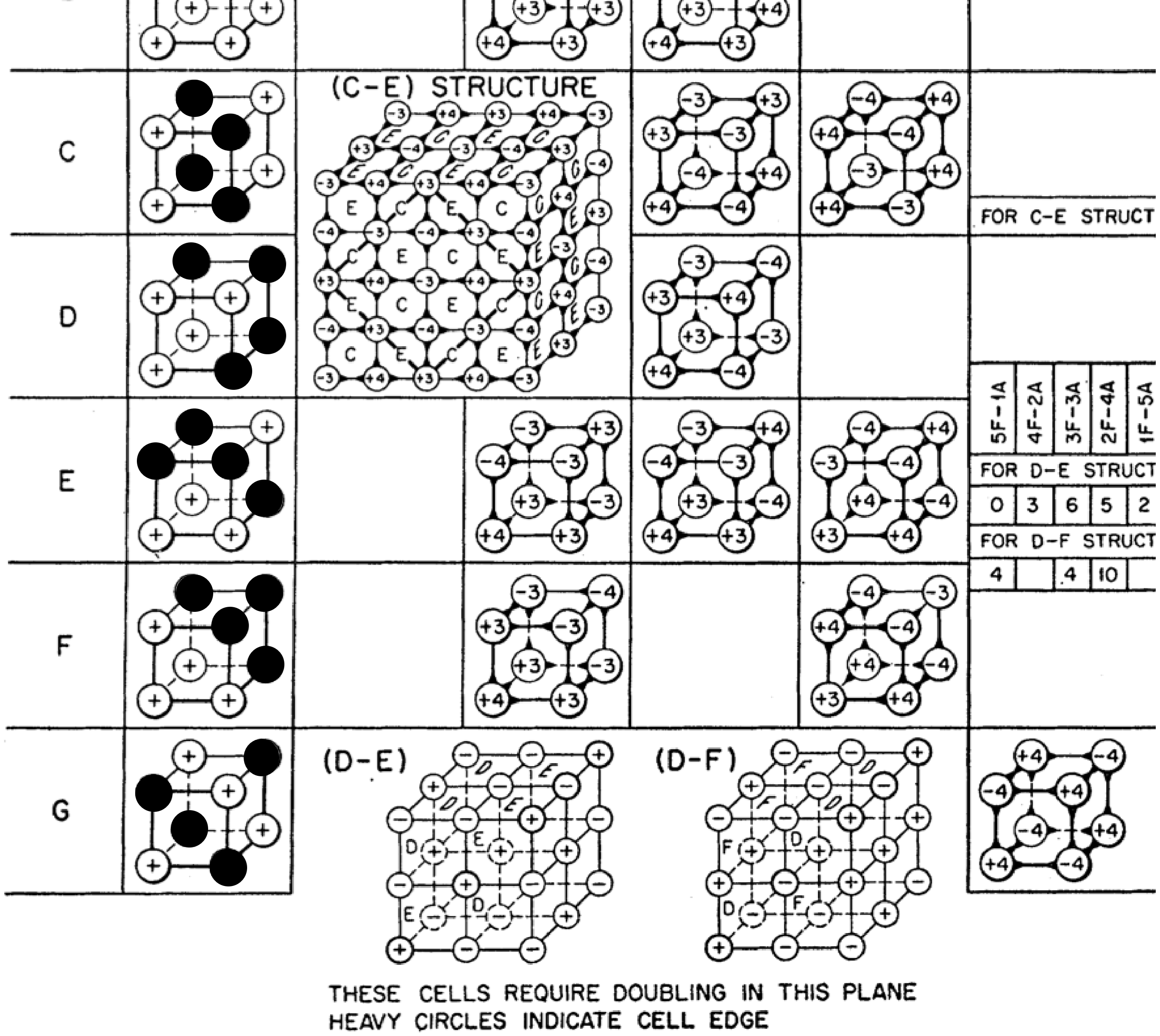


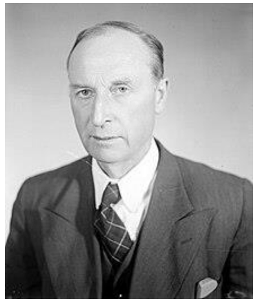
FIG. 18. Scheme of magnetic structures and related information. A, B, C, G, and $(C\text{-}E)$ definitely observed and some evidence for D and F. Ion ordering schemes represent arrangements consistent with certain coupling criteria. Arrowheads are a schematic representation of Goodenough's semicovalent exchange coupling.

Wollan, E. O. & Koehler, W. C. Neutron Diffraction Study of the Magnetic Properties of the Series of Perovskite-Type Compounds [(1 − x) La, x Ca] Mn O 3. *Phys. Rev.* **100**, 545–563 (1955).



Wollan, E. O. & Koehler, W. C. Neutron Diffraction Study of the Magnetic Properties of the Series of Perovskite-Type Compounds $[(1-x)\ La,\ x\,Ca]\,Mn\ O_3$. *Phys. Rev.* **100**, 545–563 (1955).

Scalar ordering and magnetic groups*



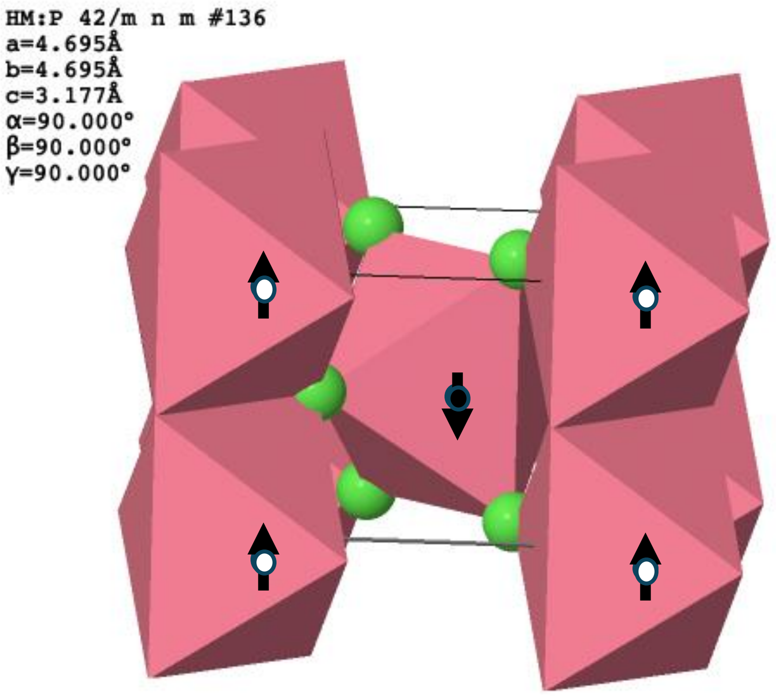
- Just like axial-vector ordering, TRO scalar ordering can ‘described’ by Shubnikov groups.
- For a given magnetic structure, axial and scalar ordering correspond to different *irreps* of the parent groups and are therefore associated with different groups.
- The *scalar* MPG generally has *higher symmetry* than the *axial* MPG.

See talk by Kirill Belashchenko on Wed.

*Heinrich Heesch, *On the Structural Theory of Plane Symmetry Groups.* Z. Krist. 71 95-102 (1929)
Alex M. *Zamorzaev, A. M. (1953). Generalization of the Fedorov groups (PhD)* *Leningrad State University*.
Tavger, B. A. & Zaitsev, V. M. Magnetic Symmetry of Crystals. *J. Exptl. Theor. Phys* **3**, 564–568 (1956).

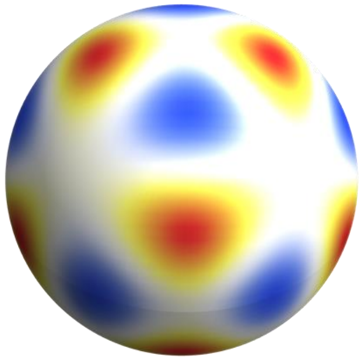
Altermagnetism – SGs, and "Scalar Shubnikov"

- MPG describes the rotational part of the *exact symmetry* (crystal + magnetic).
- MPG also describes the symmetry of the allowed momentum-space textures (incl. SOC)
- SG describes the symmetry of the momentum-space textures in the absence of SOC ("idealized").
- *Q:* What do SGs describe in *real space?*



A: the SG/SS is the MPG of a "simplified" magnetic structures in which moments have been replaced by B&W scalars

Tensorial expansions



- Texture in momentum space are naturally described in terms of Cartesian tensors. For ***k*/-*k*** symmetric textures, the tensor are of *odd rank* and totally symmetric in ***k***.

$$[\mathbf{s}_{nk}]_i = T^{(odd)}_{i,\alpha\beta\ldots} k_\alpha k_\beta \ldots$$

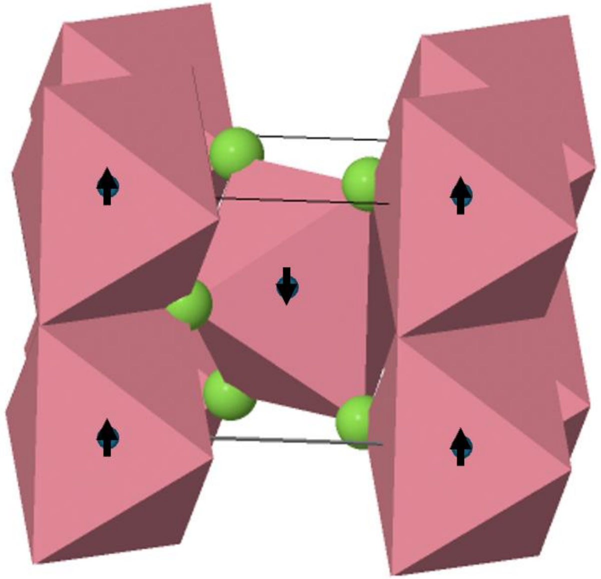
- One can define an even-rank *scalar* tensor that is totally symmetric by the 'scalar' Shubnikov or SG, and decompose the full tensor as follows:

$$T^{(odd)}_{i,\alpha\beta\ldots} = L_i \tilde{T}^{(alt)}_{\alpha\beta\ldots} + T^{(res)}_{i,\alpha\beta..} \qquad \mathbf{L} = \boldsymbol{S}_1 - \boldsymbol{S}_2$$

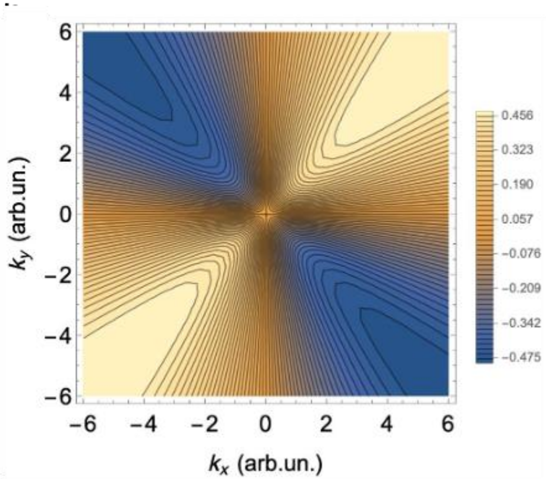
May also contain components // **L**

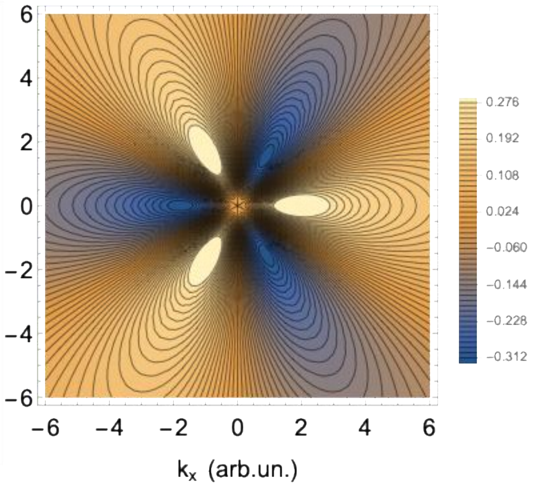
Altermagnetic analysis and macroscopic properties

CoF_2



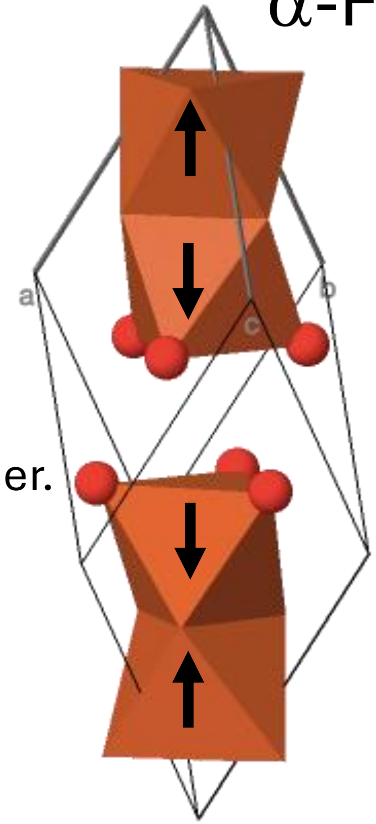
d-wave





g-wave

α-Fe_2O_3



Both are *altermagnetic*, both *piezomagnetic[1]*.

- Non-SOC piezomagnetism from asymmetric exchange (no SOC, finite T) [2] and g≠2 (SOC)
- *xy* strain makes sublattices *inequivalent*.
- No weak FM/XMCD if unstrained.

- Non-SOC piezomagnetism *forbidden* at the lowest order.
- At RT, weak FM, XMCD, *topology*....(SOC)

[1] Dzialoshinskii, I. E., J. Exptl. Theor. Phys. 33, 33, 807–808 (1957); A. S. Borovik-Romanov, Sov. Phys. JTP **11**, 1088 (1960).
[2] M. Khodas, Sai Mu, I. I. Mazin, K. D. Belashchenko, arXiv:2506.06257 (Cond. Mat.)

Generalisation to non-collinear structures

PRL **119**, 187204 (2017) PHYSICAL REVIEW LETTERS week ending 3 NOVEMBER 2017

Spin-Polarized Current in Noncollinear Antiferromagnets

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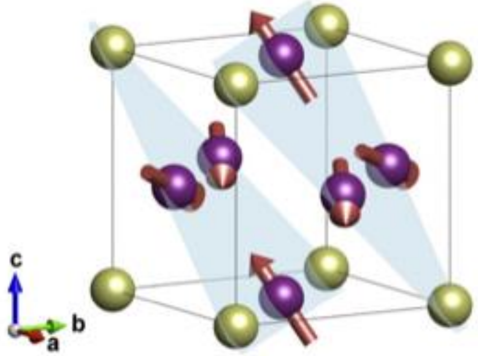
Noncollinear antiferromagnets, such as Mn_3Sn and Mn_3Ir, were recently shown to be analogous to ferromagnets in that they have a large anomalous Hall effect. Here we show that these materials are similar to ferromagnets in another aspect: the charge current in these materials is spin polarized. In addition, we show that the same mechanism that leads to the spin-polarized current also leads to a transverse spin current, which has a distinct symmetry and origin from the conventional spin Hall effect. We illustrate the existence of the spin-polarized current and the transverse spin current by performing *ab initio* microscopic calculations and by analyzing the symmetry. We discuss possible applications of these novel spin currents, such as an antiferromagnetic metallic or tunneling junction.

DOI: 10.1103/PhysRevLett.119.187204



Mn_3Ir

1. Nakatsuji, S., Kiyohara, N. & Higo, T. Large anomalous Hall effect in a non-collinear antiferromagnet at room temperature. *Nat. 2015 5277577* **527**, 212–215 (2015).
2. Kübler, J. & Felser, C. Non-collinear antiferromagnets and the anomalous Hall effect. *Europhys. Lett.* **108**, 67001 (2014).
3. Chen, H., Niu, Q. & Macdonald, A. H. Anomalous hall effect arising from noncollinear antiferromagnetism. *Phys. Rev. Lett.* **112**, 017205 (2014).

Generalisation to non-collinear structures

PRL 119, 187204 (2017) PHYSICAL REVIEW LETTERS week ending 3 NOVEMBER 2017

Spin-Polarized Current in Noncollinear Antiferromagnets

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any
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(Receiv ... ember 2017)

Noncc ... analogous to
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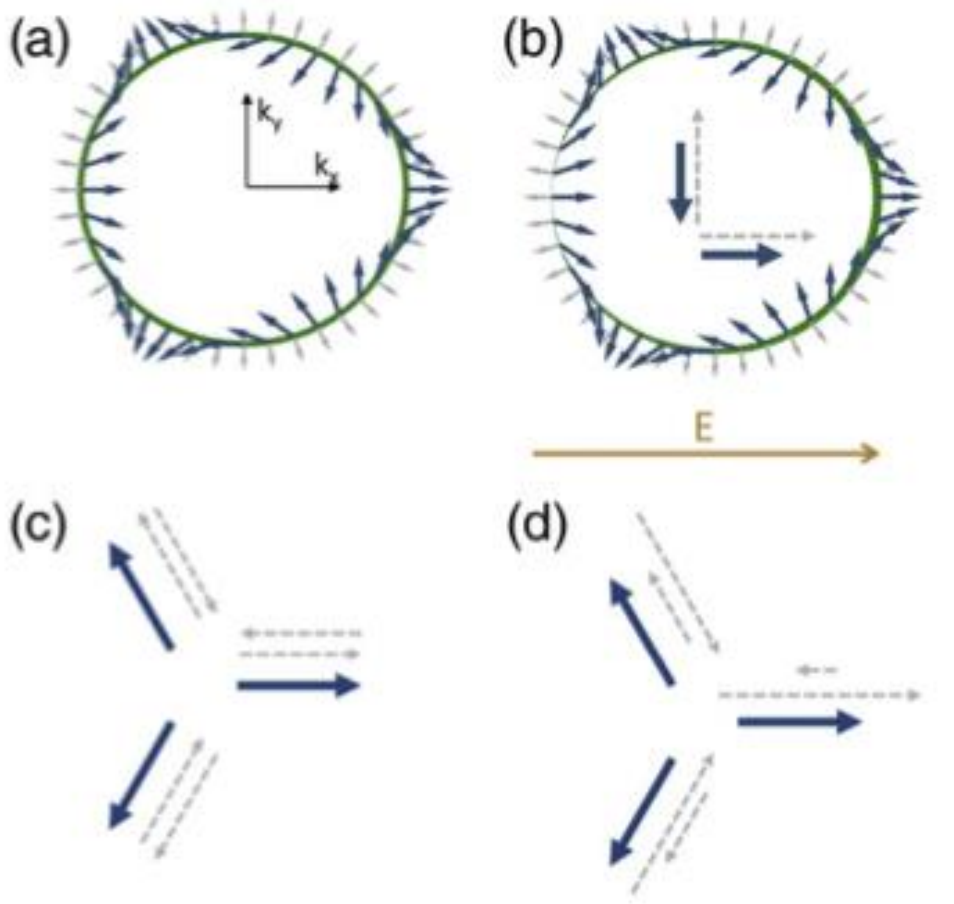
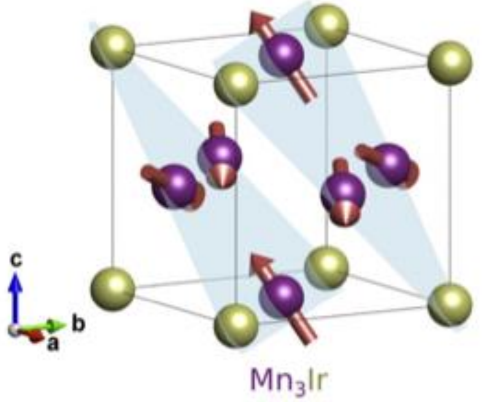




1. Nakatsuji, S., Kiyohara, ... llinear antiferromagnet at room temperature. *Nat. 2015*
2. Kübler, J. & Felser, C. No ... ll effect. *Europhys. Lett.* **108**, 67001 (2014).
3. Chen, H., Niu, Q. & Mac ... ncollinear antiferromagnetism. *Phys. Rev. Lett.* **112**, 017

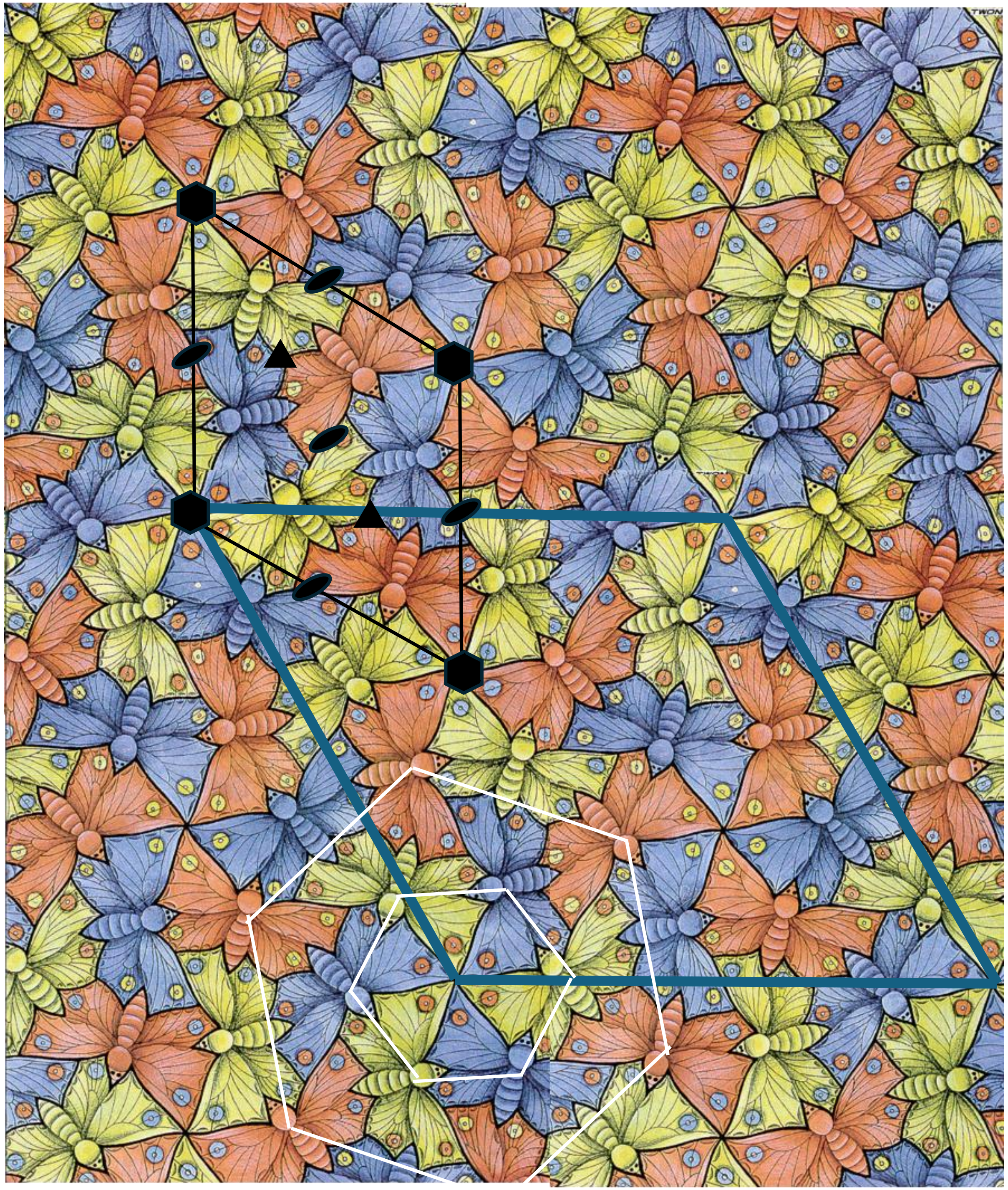
Generalisation to non-collinear structures

- Symmetry operators in **scalar Shubnikov groups** (or SG) consist of ordinary space operation acting on atomic positions, *composed with* B/W permutation.
- There is no reason why permutations should be limited to two colours only.
- Colour Groups (CSFs, CPGs) comprise operators that combine spatial transformations with permutations. They are well known due to the Escher drawings, but did not find much use in CMP so far.
- Alternatively, one can also use non-binary SGs, but IMHO CPGs are simpler and more intuitive.

$$T^{(odd)}_{i,\alpha\beta\ldots} = \sum_{l} L_i^{(l)} \tilde{T}^{(alt-like)}_{(l)\alpha\beta\ldots} + T^{(res)}_{i,\alpha\beta\ldots}$$

Sum over ‘colours’

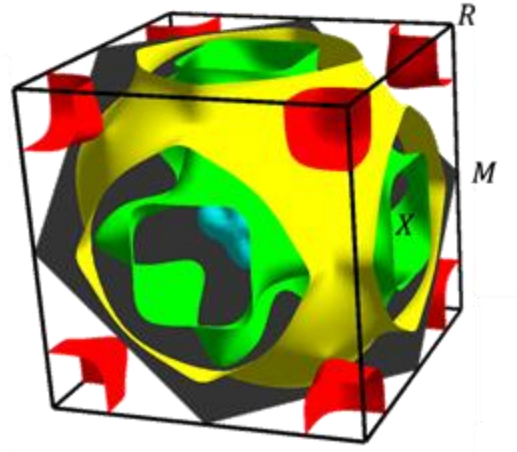
p6

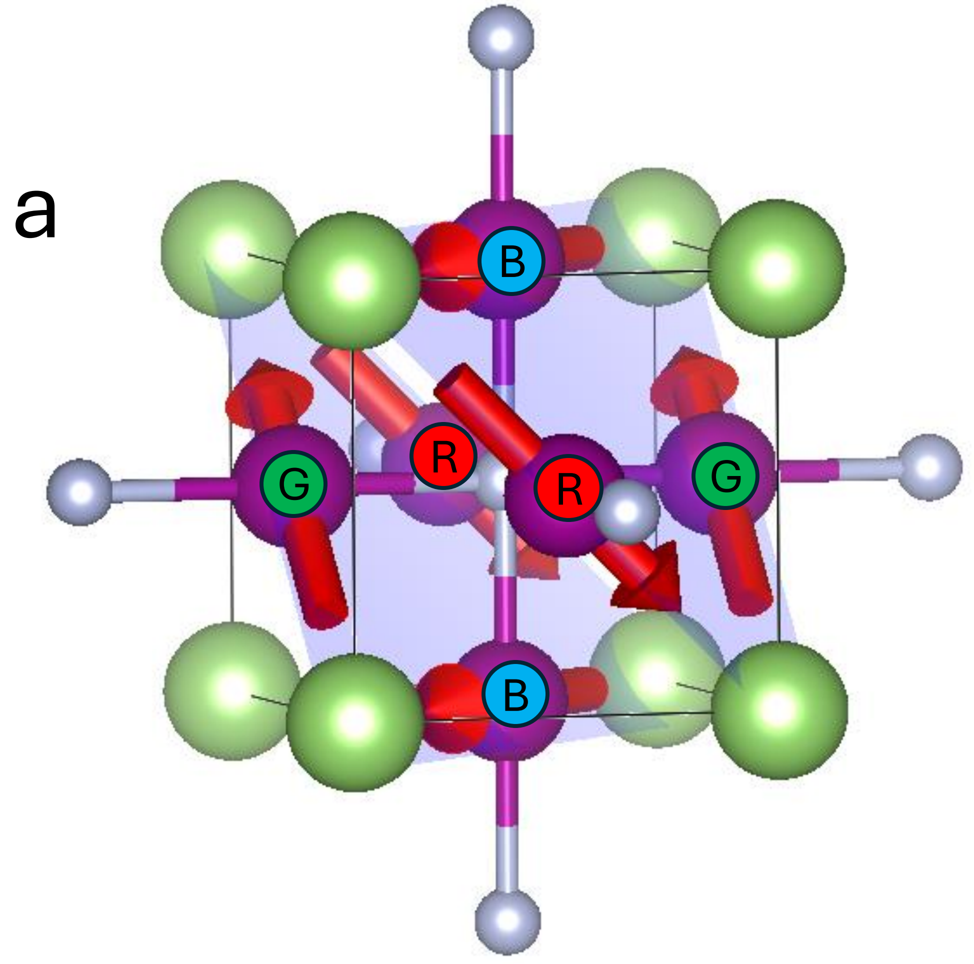


Colour symmetry – procedure

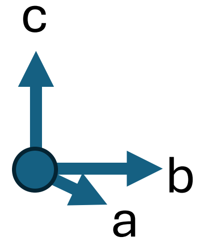
- Replace unique real-space moment directions $\boldsymbol{L}_i$ with unique colours and determine colour symmetry group of the simplified magnetic structure.
- Construct unique ‘coloured-scalar’ tensors and symmetrise by the colour symmetry. TR-related colour-anti-colour pairs can be associated with the same tensor.
- Re-associate each tensor with the corresponding $\boldsymbol{L}_i$ vector and re-assemble the full spin-texture tensor.

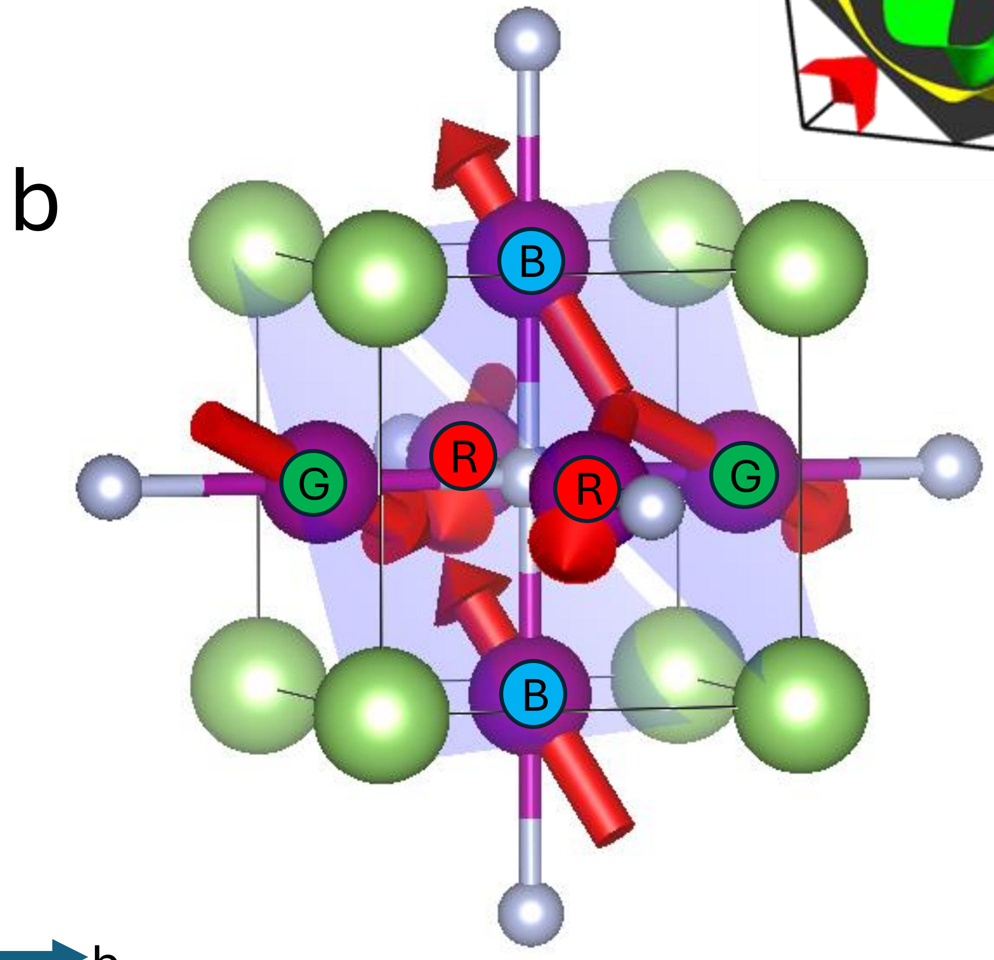
Mn_3GaN : *3-colour* permutation





Γ^{5g} ($R\bar{3}m$)

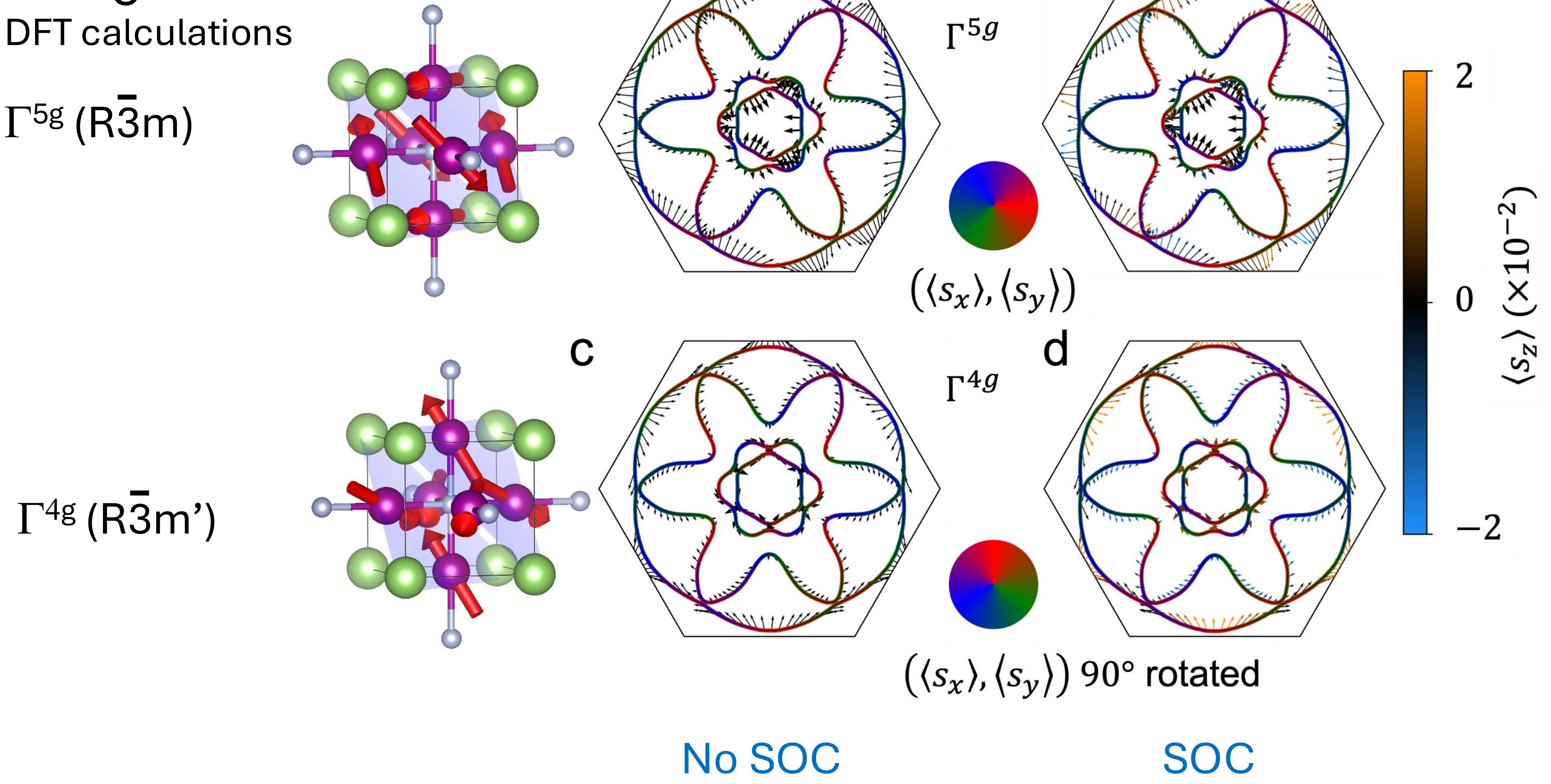




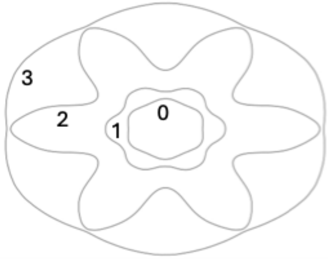
Γ^{4g} ($R\bar{3}m'$)

Mn_3GaN

DFT calculations



Band 1

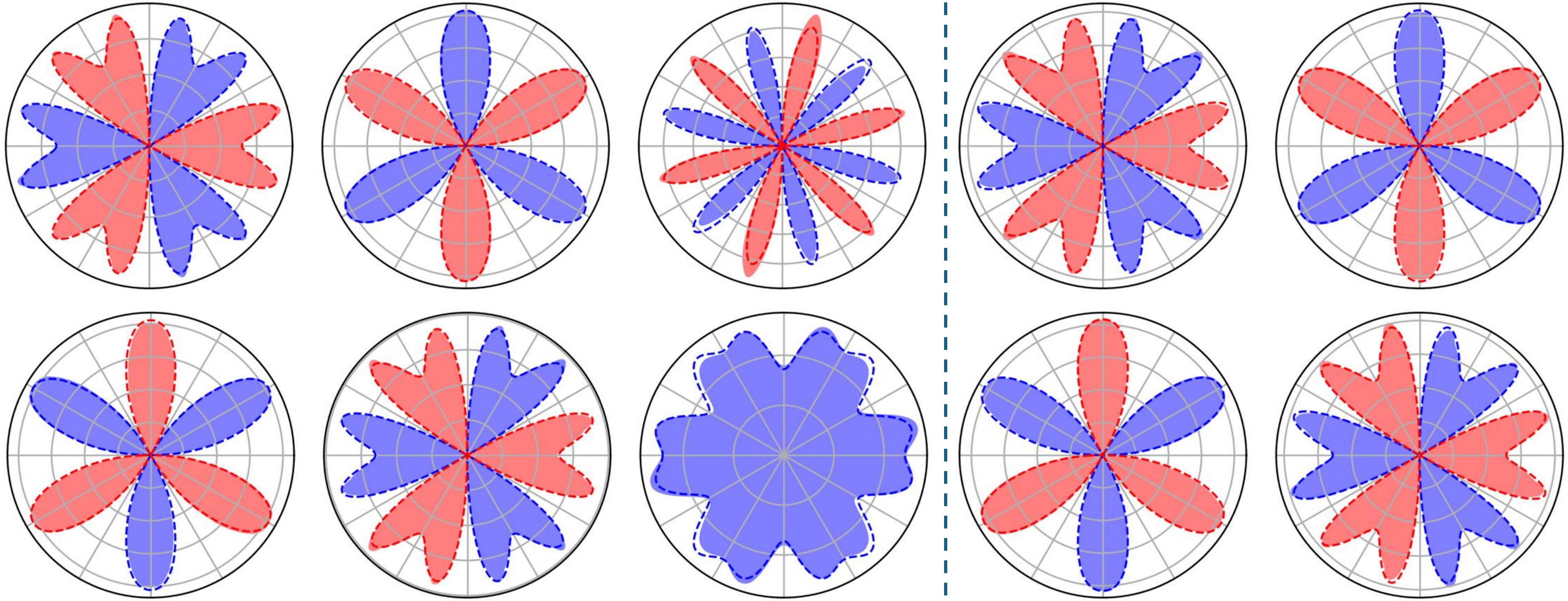


SOC | Without SOC

Radial | Tangential | Axial | Radial | Tangential

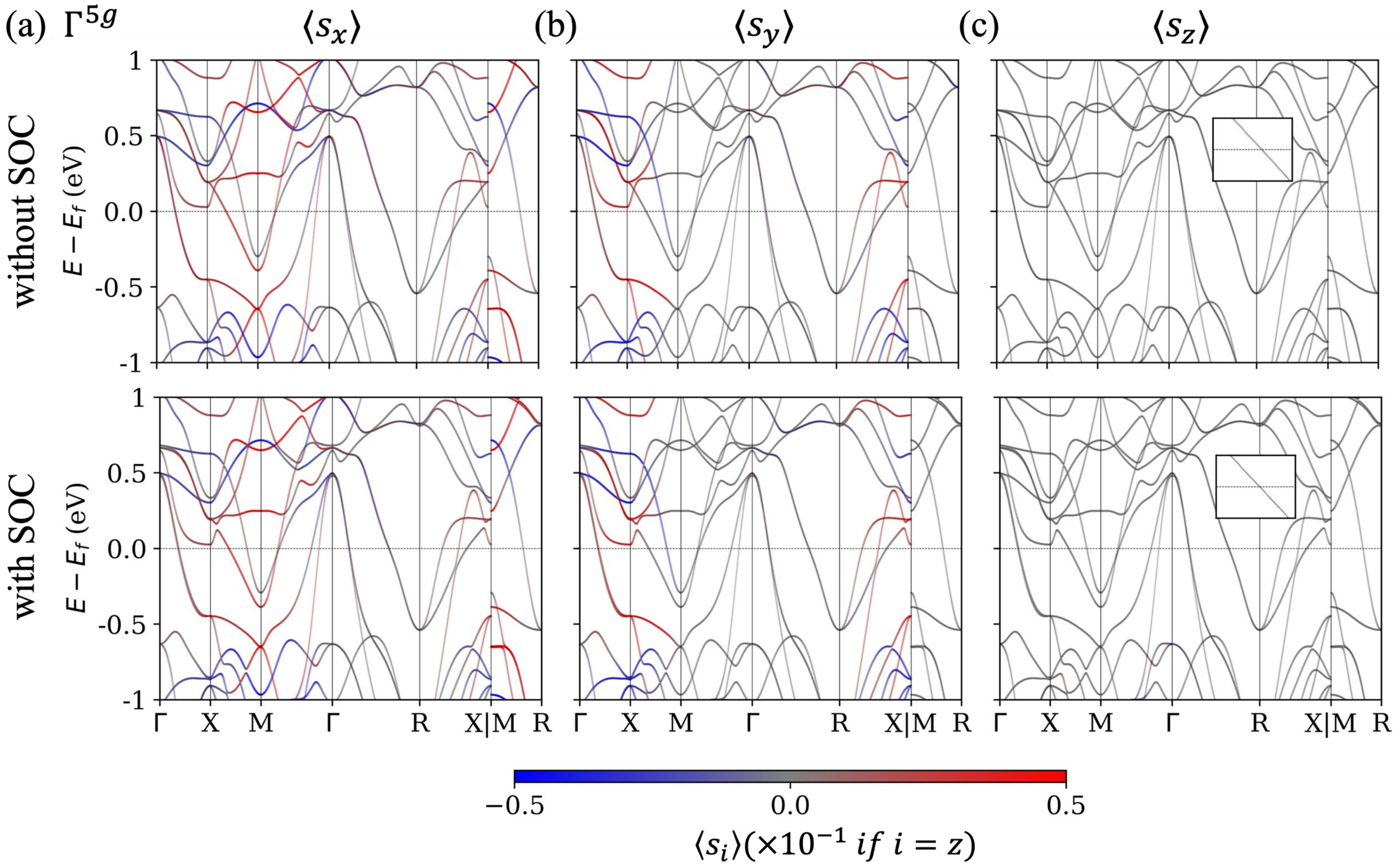
Γ_{5g} | Γ_{5g}

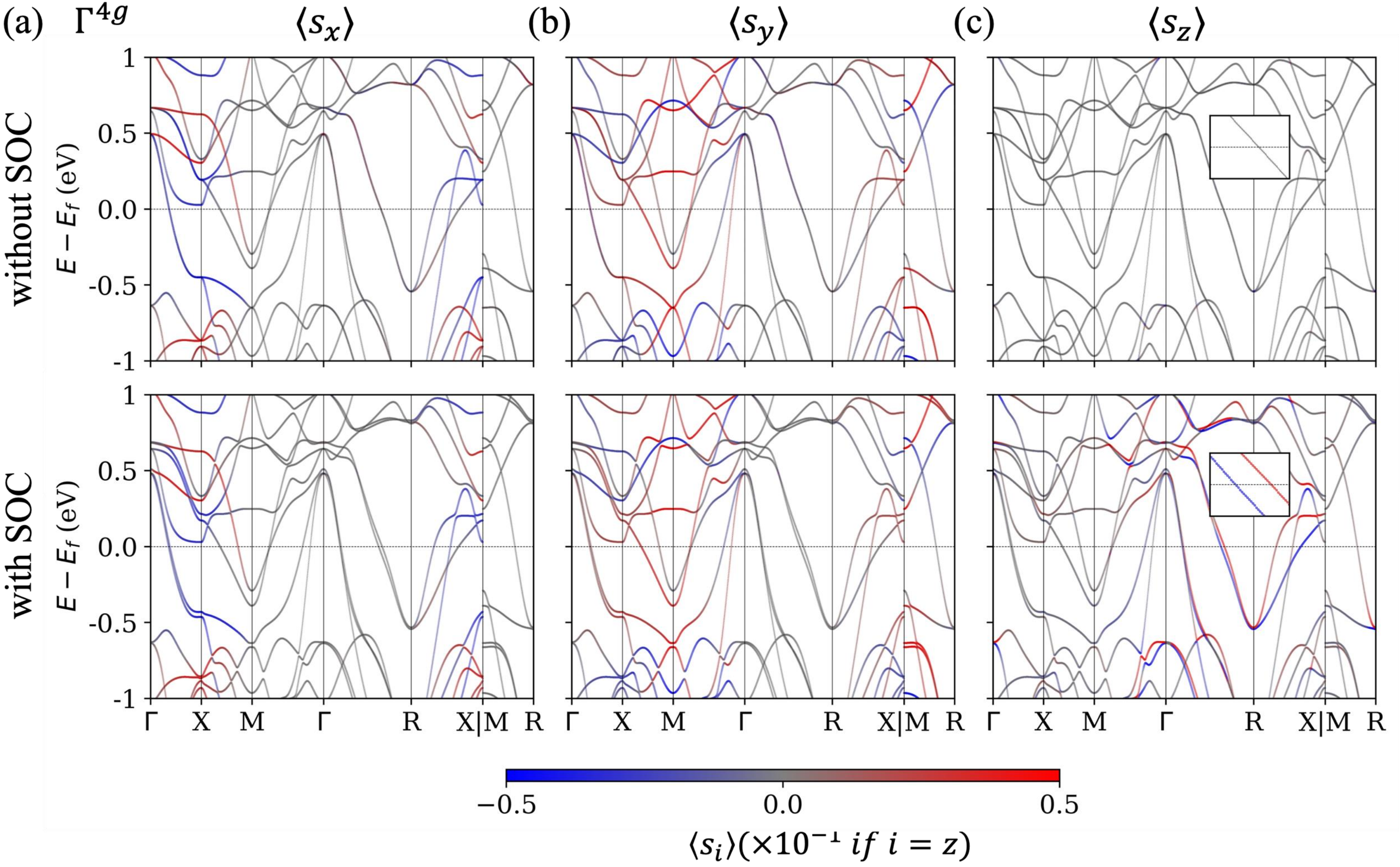
Γ_{4g} | Γ_{4g}



Dotted lines is the fit, Filled is DFT, Blue(Red) is positive(negative)

P.G. Radaelli & G. Gurung *Phys. Rev. B* **112**, 014431 (2025)


(a) Γ^{5g}
$\langle s_x \rangle$
(b)
$\langle s_y \rangle$
(c)
$\langle s_z \rangle$
without SOC
with SOC
$E - E_f$ (eV)
1
0.5
0.0
-0.5
-1
Γ X M Γ R X|M R
−0.5
0.0
0.5
$\langle s_i \rangle (\times 10^{-1}\ if\ i = z)$



(a) Γ^{4g} $\langle s_x \rangle$
(b) $\langle s_y \rangle$
(c) $\langle s_z \rangle$
without SOC
with SOC
$E - E_f$ (eV)
Γ X M Γ R X|M R
$\langle s_i \rangle (\times 10^{-1}\ if\ i = z)$

Generalisation to anti-altermagnets

Altermagnet

$$[\mathbf{s}_{nk}]_i = T^{(odd)}_{i,\alpha\beta\ldots} k_\alpha k_\beta \ldots$$

$$\mathbf{L} = \boldsymbol{S}_1 - \boldsymbol{S}_2$$

$$T^{(odd)}_{i,\alpha\beta\ldots} = L_i \tilde{T}^{(alt)}_{\alpha\beta\ldots} + T^{(res)}_{i,\alpha\beta\ldots}$$

$$[\mathbf{s}_{nk}]_i = T^{(even)}_{i,\alpha\beta\gamma\ldots} k_\alpha k_\beta k_\gamma \ldots$$

Anti-altermagnet

$$T^{(even)}_{i,\alpha\beta\gamma\ldots} = \Xi_i \tilde{T}^{(antialt)}_{\alpha\beta\gamma\ldots} + T^{(res)}_{i,\alpha\beta\gamma\ldots}$$

$$\boldsymbol{\Xi} = \boldsymbol{S}_1 \times \boldsymbol{S}_2$$

	Altermagnets	**Anti-altermagnets**
By definition	Collinear	Co-planar
$\boldsymbol{k}$ /-$\boldsymbol{k}$ sym.	Symmetric	Anti-symmetric
Forbidden symmetry	T	P
Vector/scalar Tensor rank	Odd/Even	Even/Odd
Staggered field	$\mathbf{L} = \boldsymbol{S}_1 - \boldsymbol{S}_2$	$\boldsymbol{\Xi} = \boldsymbol{S}_1 \times \boldsymbol{S}_2$
Staggered f. sym	T-odd, P-even	T-even, P-odd

Non-collinear generalisation

Sum over 'colours'

Altermagnet-like

$$T^{(odd)}_{i,\alpha\beta\ldots} = \sum_l L_i^{(l)} \tilde{T}^{(alt-like)}_{(l)\alpha\beta\ldots} + T^{(res)}_{i,\alpha\beta\ldots}$$

Anti-altermagnet-like

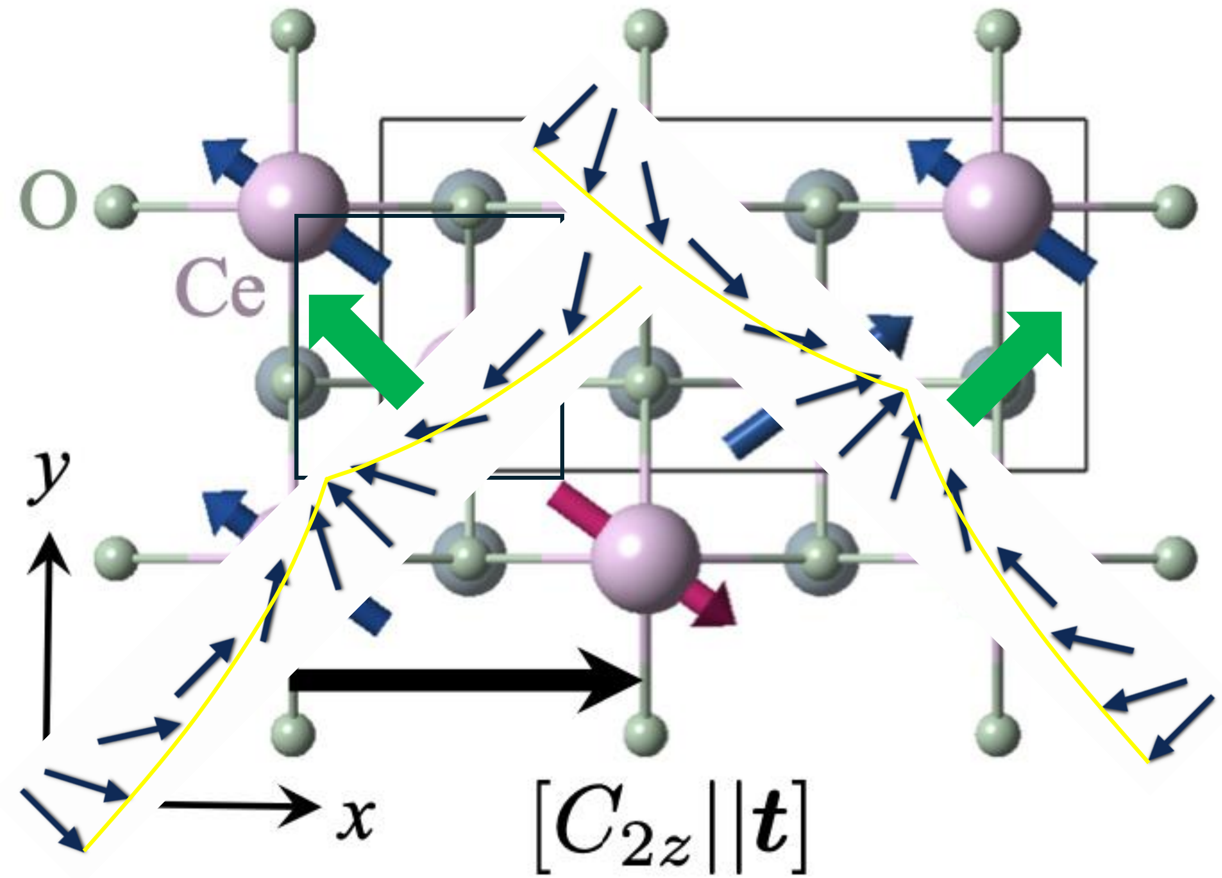
$$T^{(even)}_{i,\alpha\beta\gamma\ldots} = \sum_l \Xi_i^{(l)} \tilde{T}^{(antialt-like)}_{(l)\alpha\beta\gamma\ldots} + T^{(res)}_{i,\alpha\beta\gamma\ldots}$$

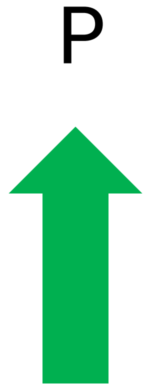
Sum over 'colours'

CeNiAsO P4/nmm (129) MPG m2m.1'

Textures allowed by MPG (Rashba)

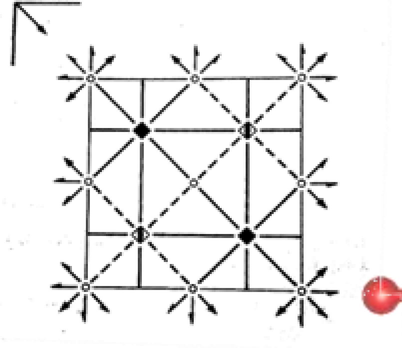
$s_x = k_z (a\, k_x^2 + b\, k_y^2 + c\, k_z^2)$
$s_y = 0$
$s_z = k_x (a' k_x^2 + b' k_y^2 + c' k_z^2)$

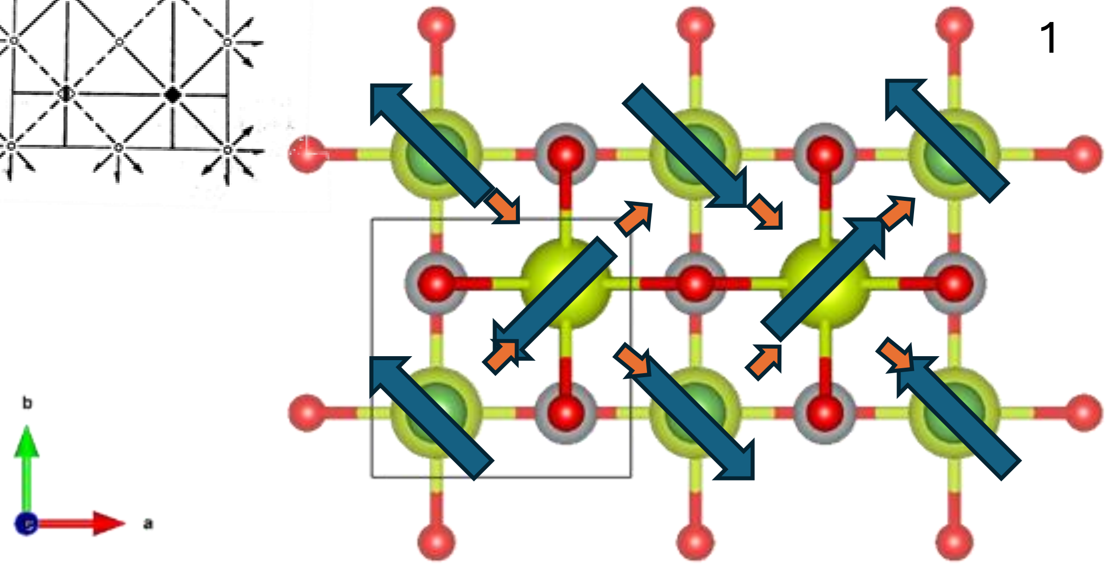


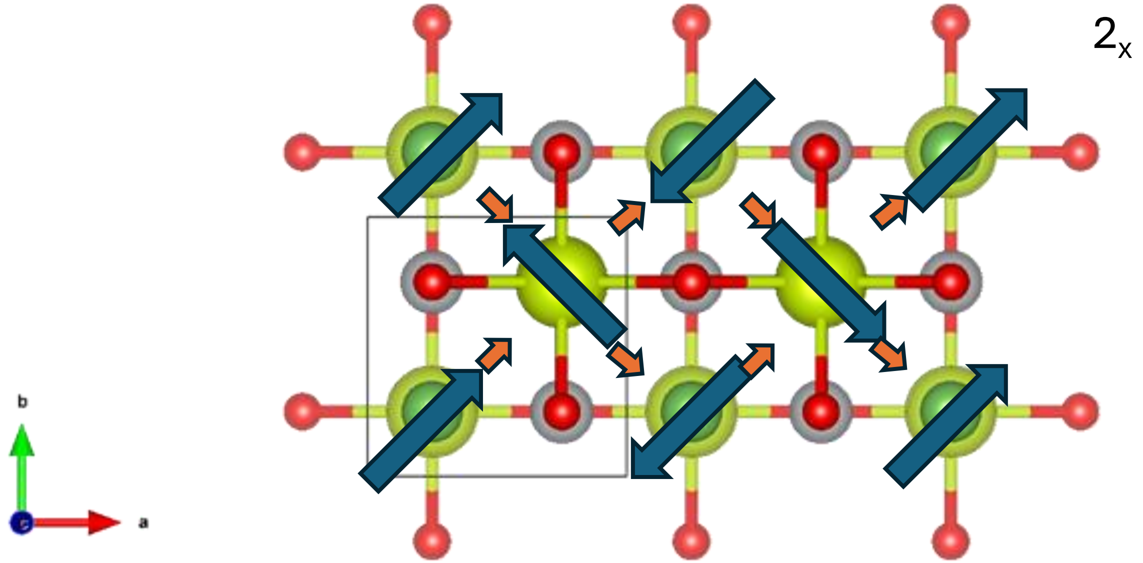


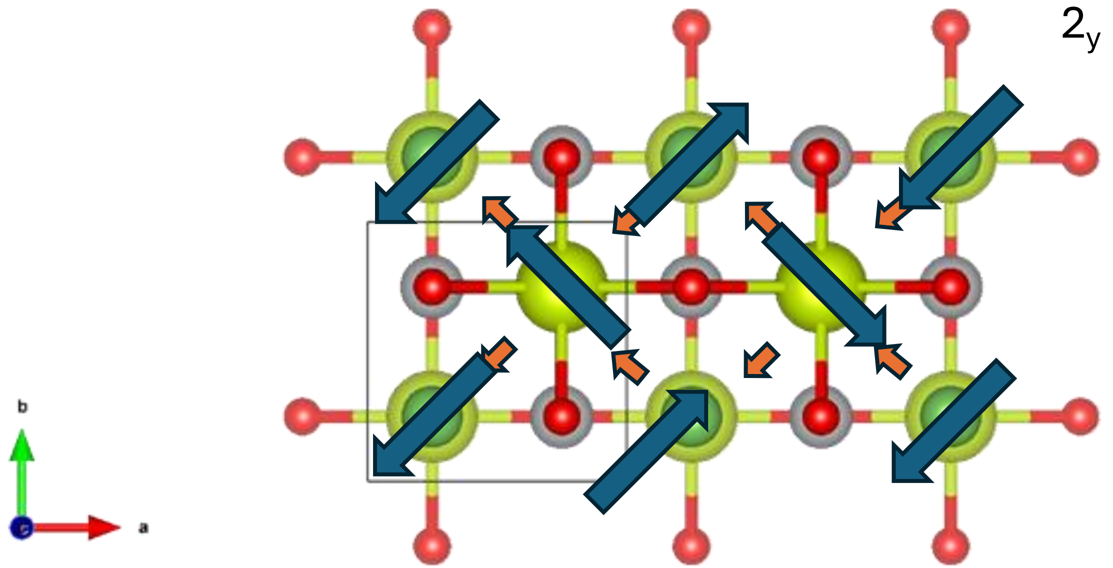
A. B. Hellenes, T. Jungwirth, R. Jaeschke-Ubiergo, *et al.* (2023), arXiv:2309.01607.
Chakraborty, A. *et al.* *Nat. Commun. 2025 161* **16**, 1–8 (2025).
Brekke, B., Sukhachov, P., Giil, H. G., Brataas, A. & Linder, *Phys. Rev. Lett.* **133**, 236703 (2024).

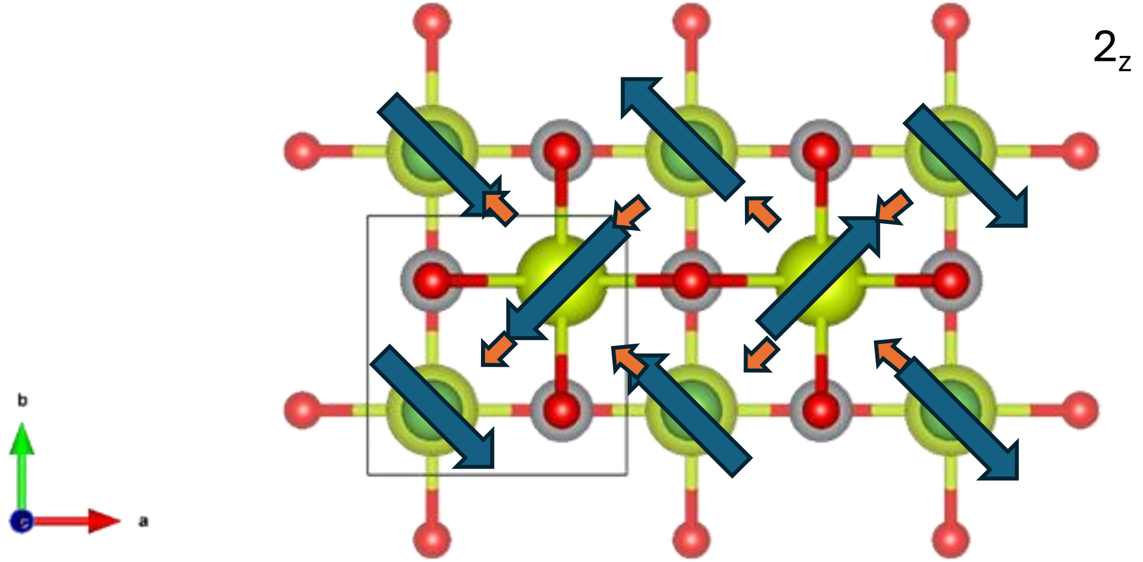
Simplified description: bond-centered ordering



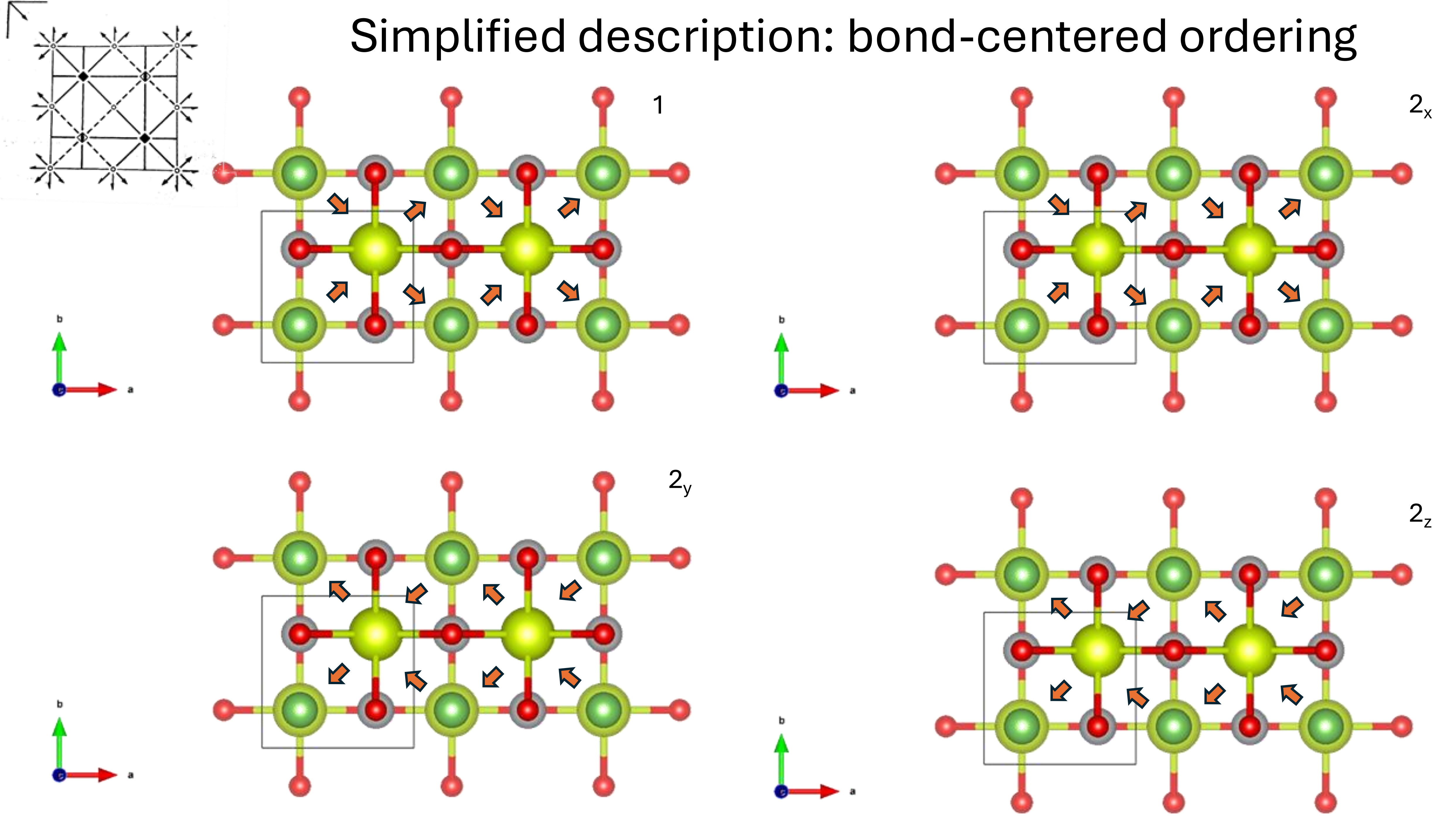








Simplified description: bond-centered ordering



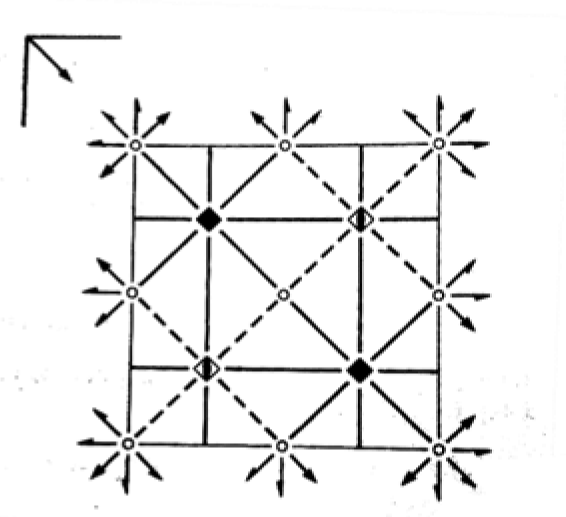
$$P\frac{2}{\breve{m}}\frac{\breve{2}}{m}\frac{\breve{2}_1}{n} \rightarrow \frac{2}{\breve{m}}\frac{\breve{2}}{m}\frac{\breve{2}}{m}$$

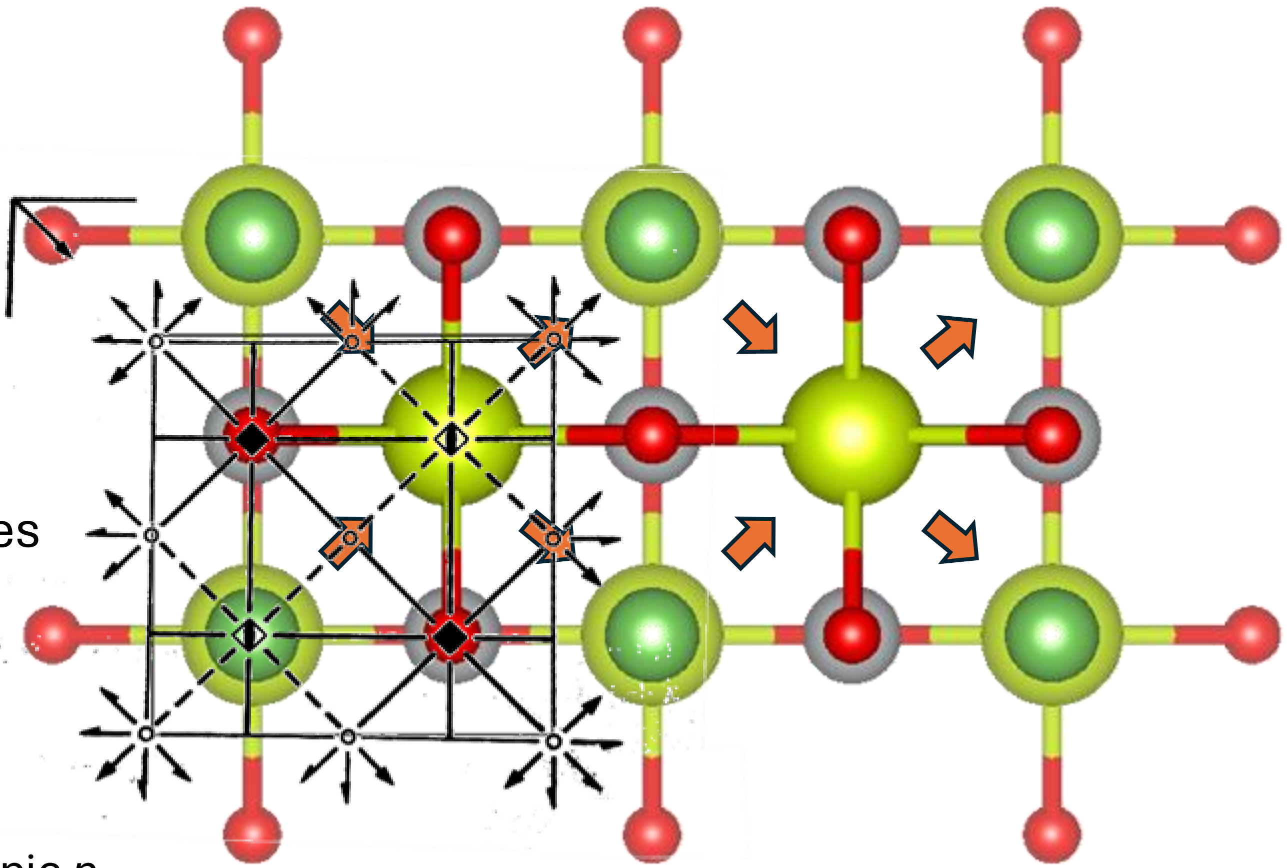
Shubnikov groups!

Table of tensor components

c_{ij}	j		
i	1	2	3
1	0	0	c_{13}
2	0	0	c_{23}
3	0	0	c_{33}
4	0	c_{23}	0
5	c_{13}	0	0
6	0	0	0

Number of independent coefficients: 3





Anti-altermagnetic Textures

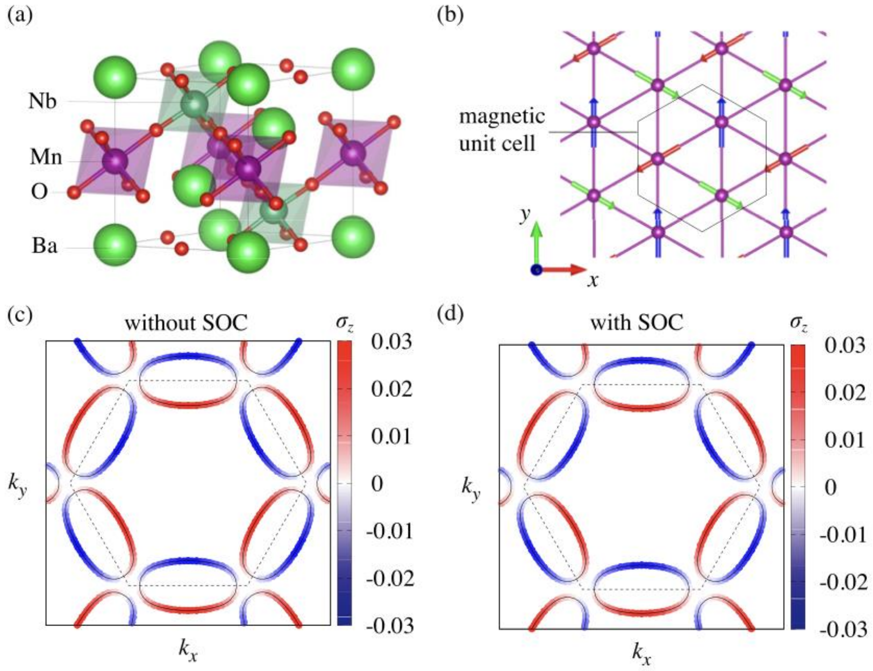
~~$s_x=k_z(a\,k_x^2+b\,k_y^2+c\,k_z^2)$~~

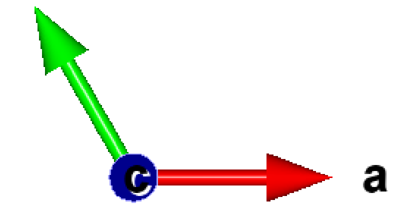
$s_y=0$

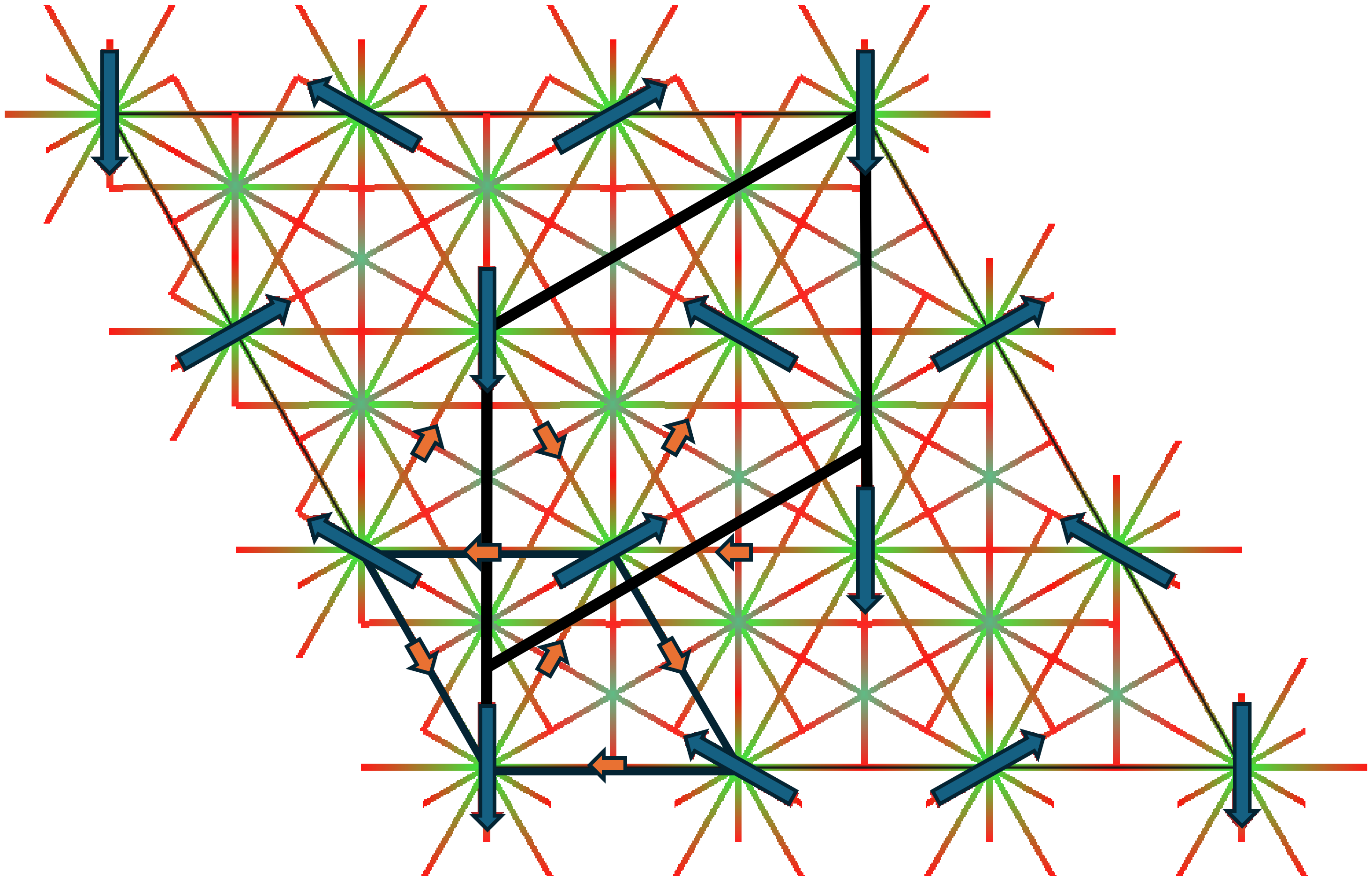
$s_z=k_x(a'k_x^2+b'k_y^2+c'k_z^2)$

Anisotropic *p*-wave

$Ba_3Nb_2NiO_9$ $P\bar{3}m$ (164) MPG: P32.1’







S. Hayami, Y. Yanagi, and H. Kusunose, Physical Review B 101, 220403 (2020).

$Ba_3Nb_2NiO_9$ $P\bar{3}m$ (164)

$$P\breve{\bar{3}}\frac{2}{\breve{m}} \rightarrow \breve{\bar{3}}\frac{2}{\breve{m}}$$

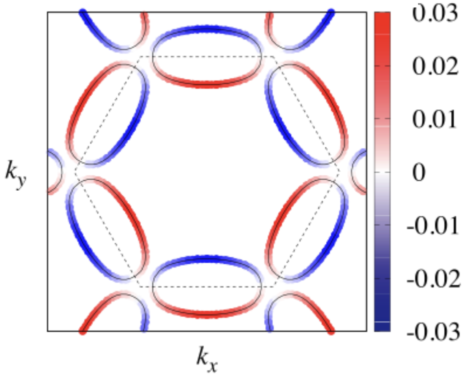
Table of tensor components

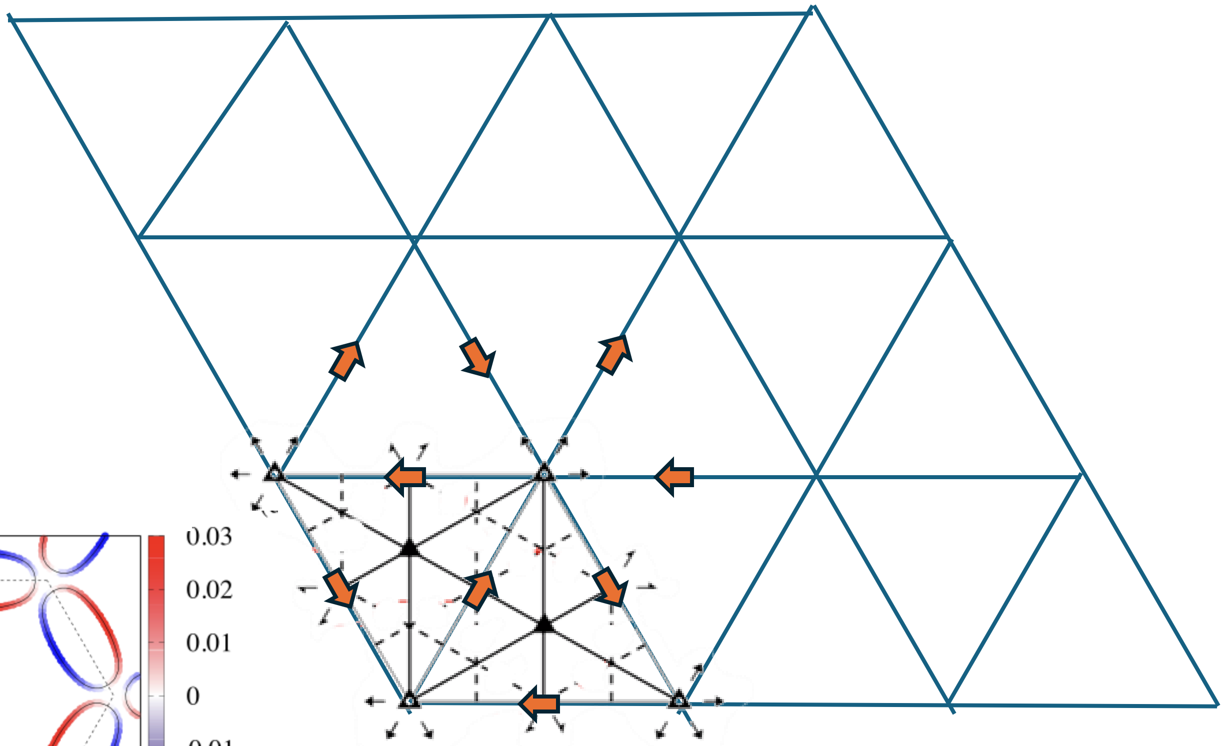
c_{ij}	j: 1	2	3
i: 1	c_{11}	0	0
2	$-c_{11}$	0	0
3	0	0	0
4	0	0	0
5	0	0	0
6	0	$-c_{11}$	0

Number of independent coefficients: 1

$s_z = k_x^3 - 3k_y^2 k_x$

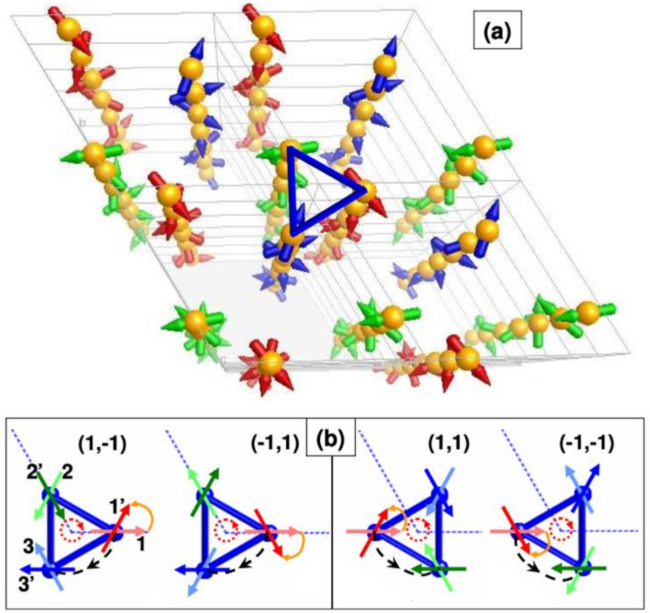
f-wave

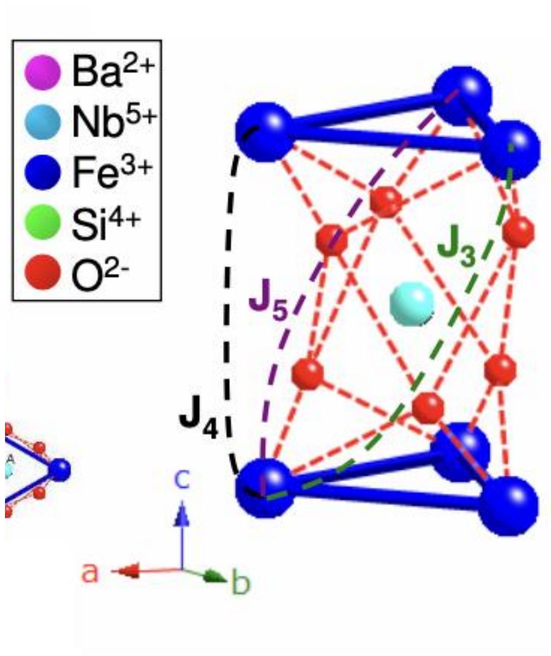




Langasite $Ba_3NbFe_3Si_2O_{14}$

SG: P321 MPG 32.1’





$$P3\breve{2} \rightarrow 3\breve{2}$$

Table of tensor components

c_{ij}	j			
i		1	2	3
	1	0	c_{12}	c_{13}
	2	0	$-c_{12}$	c_{13}
	3	0	0	c_{33}
	4	0	c_{13}	0
	5	c_{13}	0	0
	6	c_{12}	0	0

$$s_z = [2c_{13}(k_x^2 + k_y^2) + c_{33} k_z^2] k_z - c_{12} k_y (-3 k_x^2 + k_y^2)$$

Anisotropic p-wave + *f*-wave

Marty, K. *et al. Phys. Rev. Lett.* **101**, 247201 (2008).

Questions?