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Elastic Coupling in Altermagnets

Theory of Unconventional Magnetism: exploring altermagnets and beyond, Mainz, Oct. 2025

Jörg Schmalian
Karlsruhe Institute of Technology

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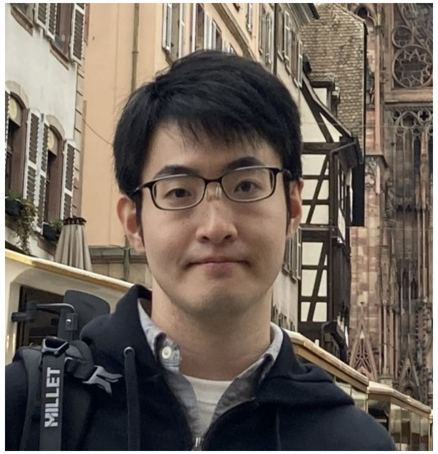

Keigo Takahashi
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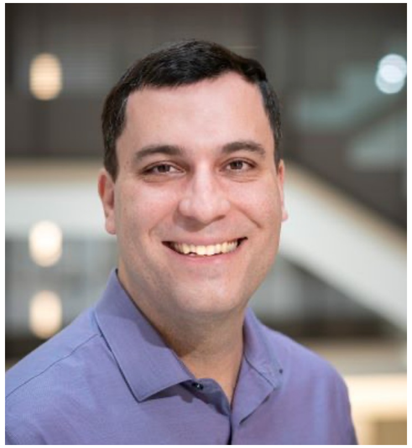

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Elasto-Q-Mat

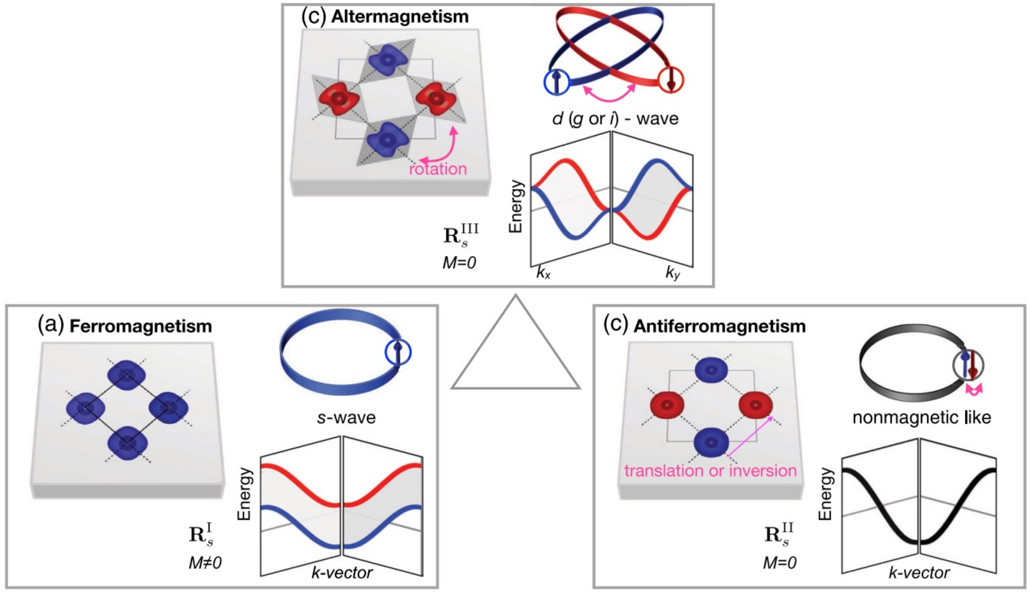
DFG Deutsche Forschungsgemeinschaft
German Research Foundation

CRC– TRR 288

- C. R. W. Steward, R. M. Fernandes, and J. S., PRB 108, 144418 (2023)
- C. R. W. Steward, G. Palle, M. Garst, J. S., and I. Jang, PRB 112, 064431 (2025)
- K. Takahashi, C. R. W. Steward, M. Ogata, R. M. Fernandes, and J. S. PRB 111, 184408 (2025)
- A. R. Chakraborty, J. S., and R. M. Fernandes, PRB 112, 035146 (2025)

altermagnets:

- crystalline translations unchanged
- broken time reversal (?)
- no net magnetization
- inversion even



L. Šmejkal, J. Sinova, and T. Jungwirth, Phys. Rev. X 12, 040501 (2022).

altermagnets:

- crystalline translations unchanged
- broken time reversal ⟵
- no net magnetization
- inversion even

Über die
Operation der Zeitumkehr in der Quantenmechanik.

Von

E. Wigner.

Vorgelegt von M. Born in der Sitzung am 25. November 1932.

Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse (1932).

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altermagnets:

- crystalline translations unchanged
- broken time reversal
- no net magnetization ⟵
- inversion even ⟵

irred. rep.	polynomial
A_{1g}	$z^2,\ x^2+y^2$
A_{2g}	ferromag. $xy\left(x^2-y^2\right)$
B_{1g}	x^2-y^2
B_{2g}	xy
E_g	ferromag. (xz, yz)
A_{1u}	z
A_{2u}	$xyz\left(x^2-y^2\right)$
B_{1u}	odd parity $z\left(x^2-y^2\right)$
B_{2u}	xyz
E_u	(x, y)

e.g. tetragonal systems

→ TRS-odd order parameter transforms under one of the irreps of the point group other that the magnetic dipole

→ 3 "pure" tetragonal am states
2 states with at least small magnetization

R. M. Fernandes, V. S. de Carvalho, T. Birol, and R. G. Pereira, PRB **109**, 024404 (2024)

order parameter

$$\phi \sim \sum_{\boldsymbol{k}ab} c_a^\dagger(\boldsymbol{k})\, f^\alpha(\boldsymbol{k})\, J_{ab}^\alpha c_b(\boldsymbol{k})$$

form factor

angular momentum operator

single band: $\phi_{B_{1g}} \sim \sum_{\boldsymbol{k}ab} \sin k_x \sin k_y c_a^\dagger(\boldsymbol{k})\, \sigma_{ab}^z c_b(\boldsymbol{k})$

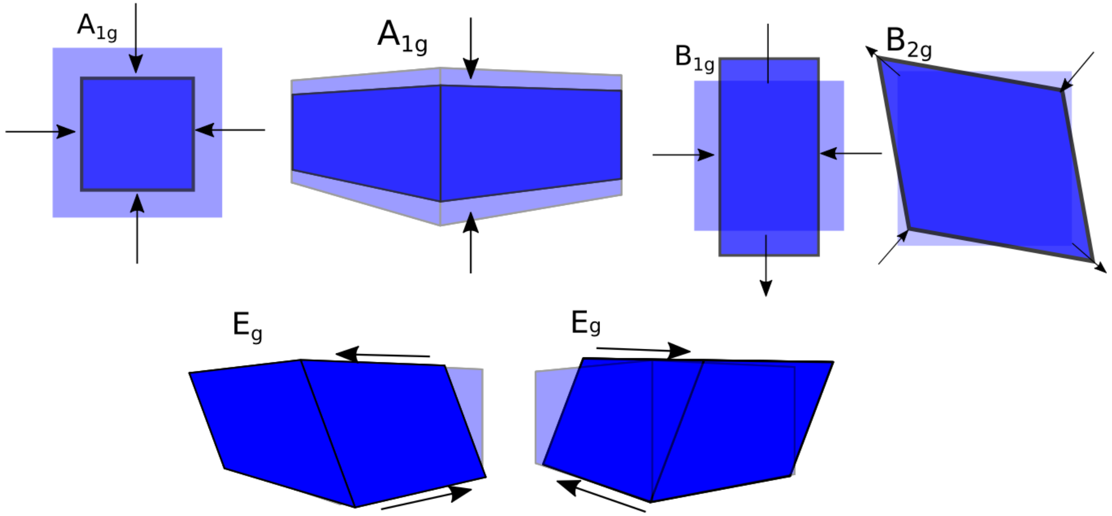
Lieb lattice: $\phi_{B_{2g}} \sim \sum_{\boldsymbol{k}} (\cos k_x - \cos k_y)\, c^\dagger(\boldsymbol{k})\, \tau^0 \sigma^z c(\boldsymbol{k}) + \sum_{\boldsymbol{k}} c^\dagger(\boldsymbol{k})\, \tau^z \sigma^z c(\boldsymbol{k})$

C. R. W. Steward, R. M. Fernandes, and J. S., PRB **108**, 144418 (2023); P. A. McClarty and J. G. Rau, PRL **132**, 176702 (2024)
R. M. Fernandes, V. S. de Carvalho, T. Birol, and R. G. Pereira, PRB **109**, 024404 (2024)

strain

$$\varepsilon_{\alpha\beta} = \frac{1}{2}\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha\right)$$

displacement (acoustic phonon)



piezomagnetism

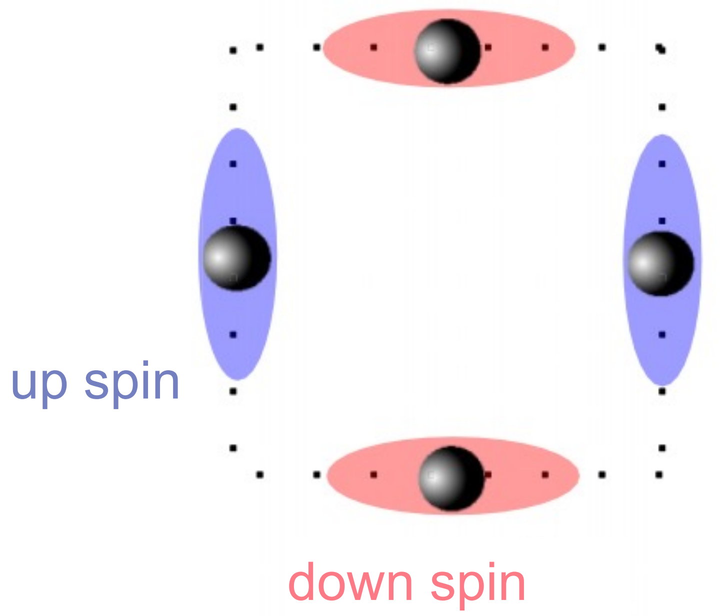
order parameter transforms under irrep. Γ^{-}

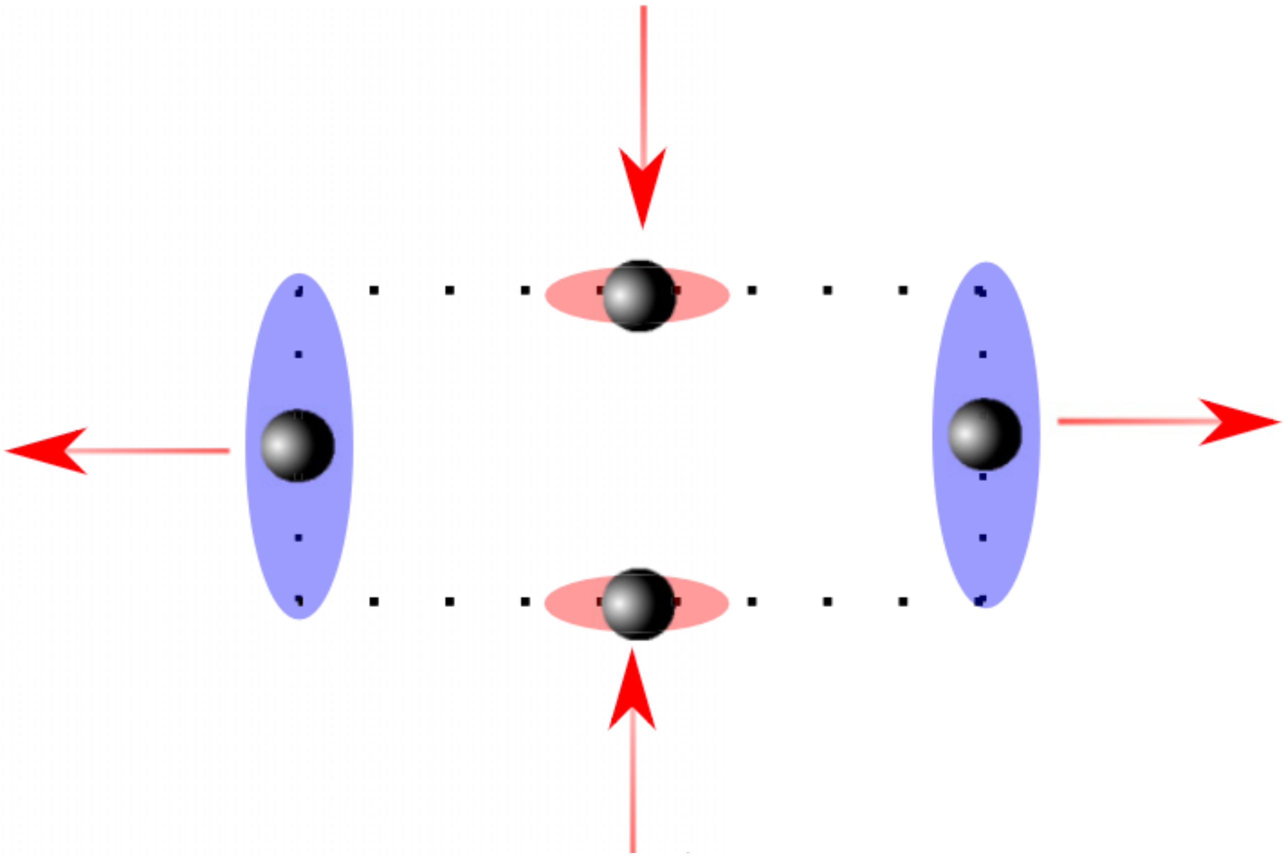
$$H_{\mathrm{piezo-mag.}} = \int d^3x \lambda^i_{\alpha\gamma\delta} B_\alpha \phi^i \varepsilon_{\gamma\delta}$$

$\Gamma^{-}_{B_\alpha} \otimes \Gamma^{-} \otimes \Gamma_{\varepsilon}$ contains trivial representation

→ strain-induced magnetization $M_\alpha = -\lambda^i_{\alpha\gamma\delta} \phi^i \varepsilon_{\gamma\delta}$

piezomagnetism



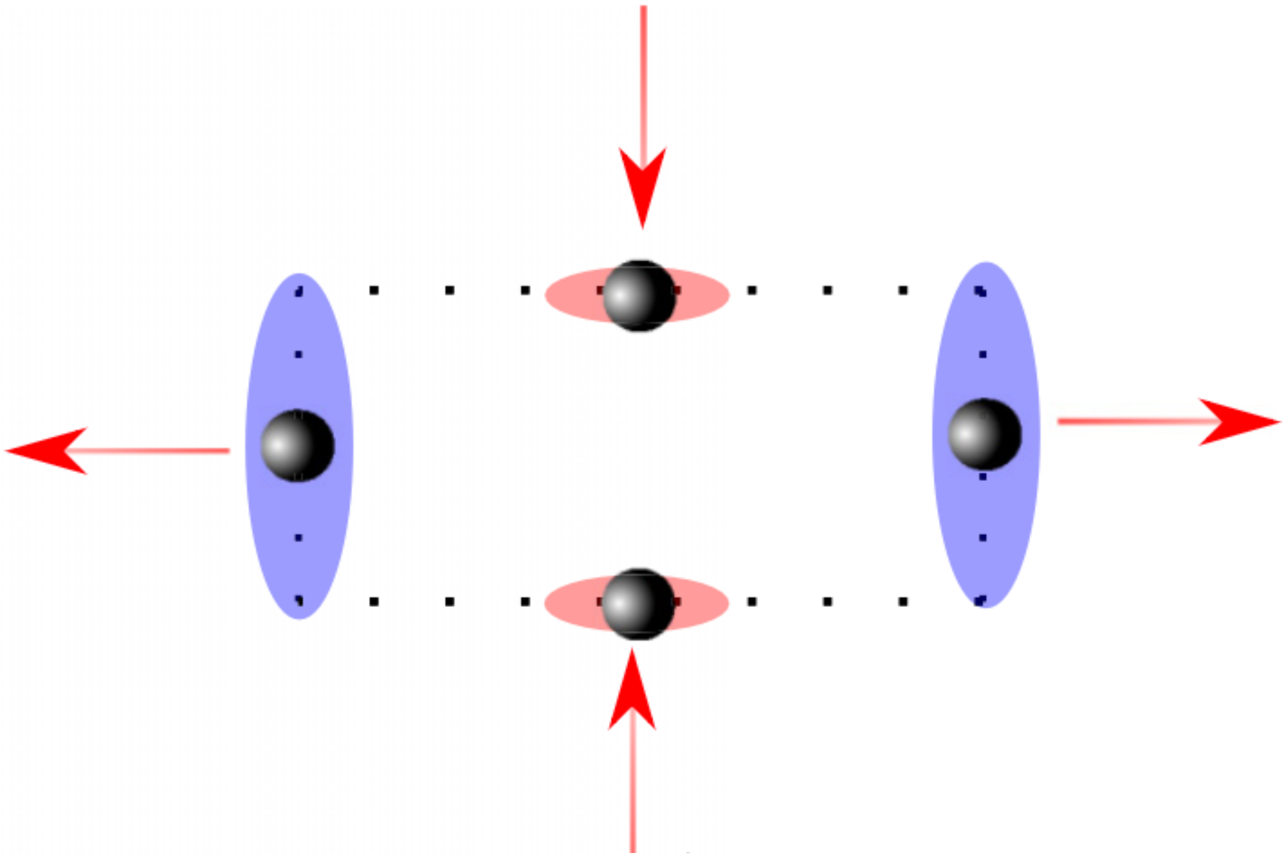


no strain

strain → net magnetization

piezomagnetism

D_{4h} $(4/mmm)$ point group	
AM irrep	piezomagnetic coupling
A_{1g}^-	$\lambda\phi\left(\epsilon_{yz}H_x - \epsilon_{xz}H_y\right)$
B_{1g}^-	$\lambda\phi\left(\epsilon_{yz}H_x + \epsilon_{xz}H_y\right)$ $\lambda'\phi\epsilon_{xy}H_z$
B_{2g}^-	$\lambda\phi\left(\epsilon_{xz}H_x - \epsilon_{yz}H_y\right)$ $\lambda'\phi\epsilon_{x^2-y^2}H_z$
FM irrep	
A_{2g}^-	$\lambda\phi\epsilon_{A_{1g}}H_z$ $\lambda'\phi\left(\epsilon_{zx}H_x + \epsilon_{yz}H_y\right)$
E_g^-	$\lambda\epsilon_{A_{1g}}\left(\phi_1 H_x + \phi_2 H_y\right)$ $\lambda'\epsilon_{x^2-y^2}\left(\phi_1 H_x - \phi_2 H_y\right)$ $\lambda''\epsilon_{xy}\left(\phi_1 H_y + \phi_2 H_x\right)$ $\lambda'''\left(\phi_1\epsilon_{xz} + \phi_2\epsilon_{yz}\right)H_z$



strain → net magnetization

C. R. W. Steward, R. M. Fernandes, and J. S., PRB **108**, 144418 (2023)
R. M. Fernandes, V. S. de Carvalho, T. Birol, and R. G. Pereira, PRB **109**, 024404 (2024)

piezomagnetism

$$H_{\text{piezo}-\text{mag.}} = \int d^3x \lambda^i_{\alpha\gamma\delta} B_\alpha \phi^i \left(\boldsymbol{x}\right) \varepsilon_{\gamma\delta}\left(\boldsymbol{x}\right)$$

$$\varepsilon_{\gamma\delta}\left(\boldsymbol{x}\right) = \varepsilon^{\text{phonon}}_{\gamma\delta}\left(\boldsymbol{x}\right) + \varepsilon^{\text{defects}}_{\gamma\delta}\left(\boldsymbol{x}\right) + \varepsilon^{\text{rand.}}_{\gamma\delta}\left(\boldsymbol{x}\right)$$

changes the altermagnetic interactions

- enhances overall coupling
- changes range / character of the interaction

dislocations

- nucleation of 1D order

randomness

- random-field Ising model with variable disorder strength

piezomagnetism

$$H_{\text{piezo}-\text{mag.}} = \int d^3x \lambda^i_{\alpha\gamma\delta} B_\alpha \phi^i(\boldsymbol{x})\, \varepsilon_{\gamma\delta}(\boldsymbol{x})$$

$$\varepsilon_{\gamma\delta}(\boldsymbol{x}) = \varepsilon^{\text{phonon}}_{\gamma\delta}(\boldsymbol{x}) + \varepsilon^{\text{defects}}_{\gamma\delta}(\boldsymbol{x}) + \varepsilon^{\text{rand.}}_{\gamma\delta}(\boldsymbol{x})$$

changes the altermagnetic interactions
- **enhances overall coupling**
- changes range / character of the interaction

dislocations
- nucleation of 1D order

randomness
- **random-field Ising model with variable disorder strength**

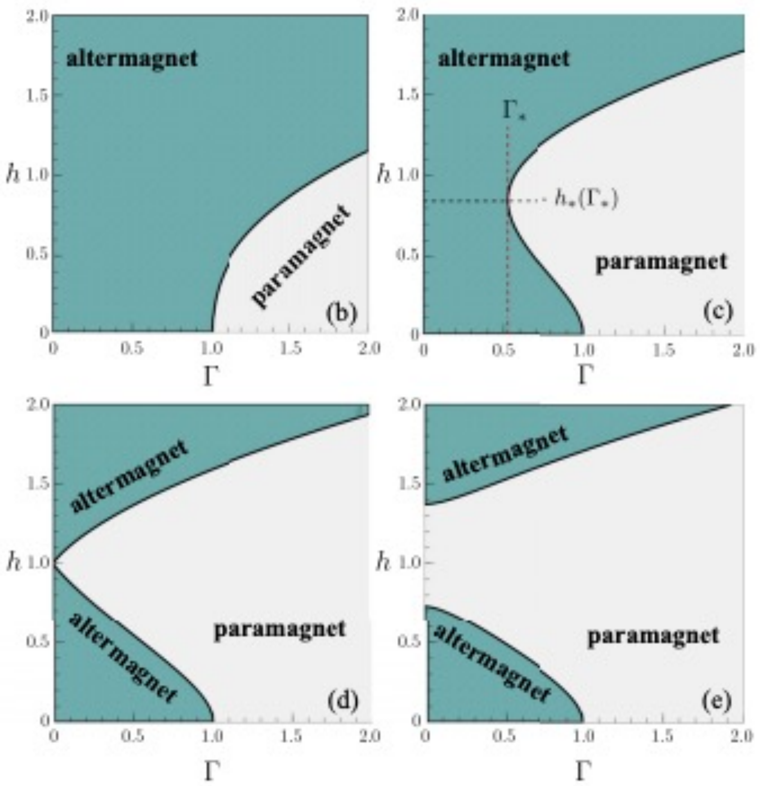
field-induced random strain

$$\mathcal{H} = -J\sum_{\langle ij\rangle}\tau_i^z\tau_j^z - \Gamma\sum_i \tau_i^x - \frac{N}{2}C_0\varepsilon_0^2 - \lambda B_z\sum_i\left(\varepsilon_0+\varepsilon_i^{\mathrm{rand}}\right)\tau_i^z$$

solution in the infinite-range limit

$$-J\sum_{\langle ij\rangle}\tau_i^z\tau_j^z \to -\frac{J}{N}\sum_{i<j}\tau_i^z\tau_j^z$$

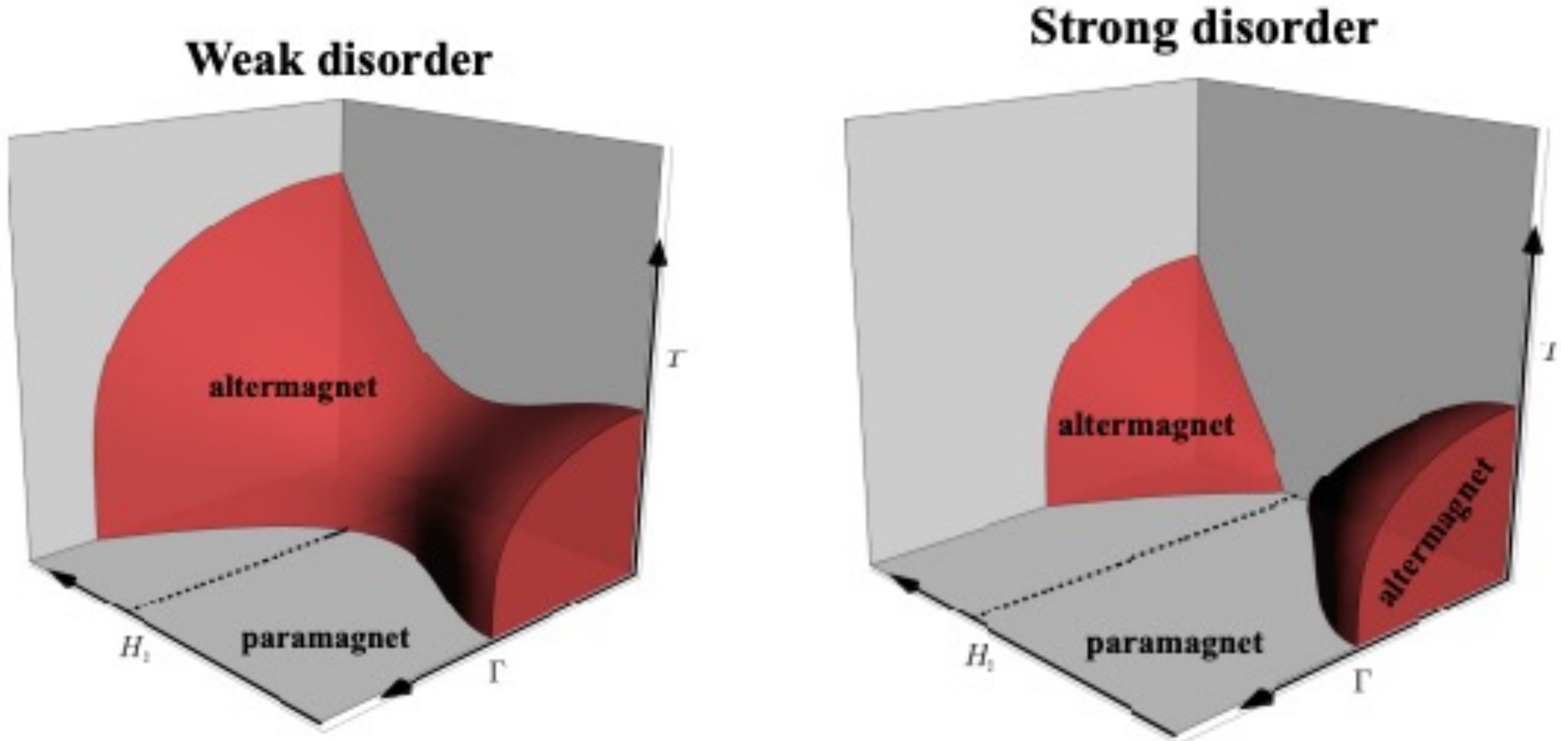
competition between strain-enhanced interactions and random-field suppression of altermagnetism



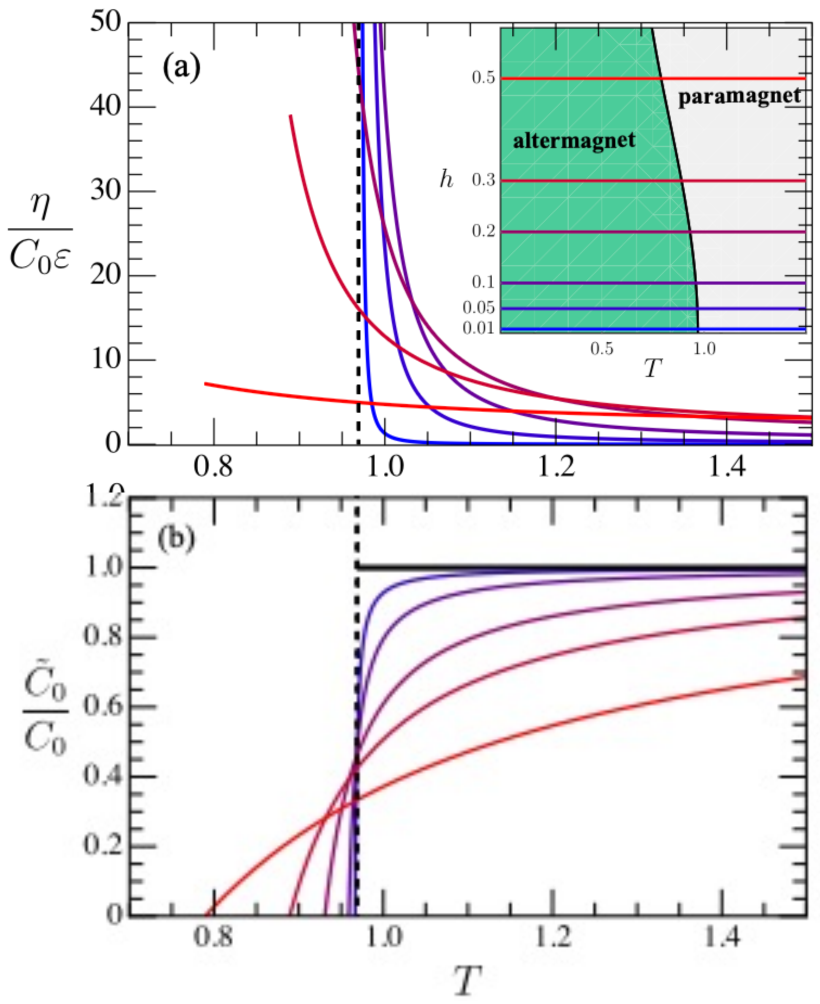
A. R. Chakraborty, J. S., and R. M. Fernandes, PRB 112, 035146 (2025)

field-induced random strain

finite
temperatures



field-induced random strain



elasto-caloric effect

$$\eta = \left.\frac{\partial T}{\partial \varepsilon}\right|_S = -\frac{T}{c}\left.\frac{\partial S}{\partial \varepsilon}\right|_T$$

field-induced lattice softening

$$\tilde{C}_0(B \neq 0, T \to T_c) \to 0$$

piezomagnetism

$$H_{\mathrm{piezo-mag.}} = \int d^3x \lambda^i_{\alpha\gamma\delta} B_\alpha \phi^i(\boldsymbol{x})\, \varepsilon_{\gamma\delta}(\boldsymbol{x})$$

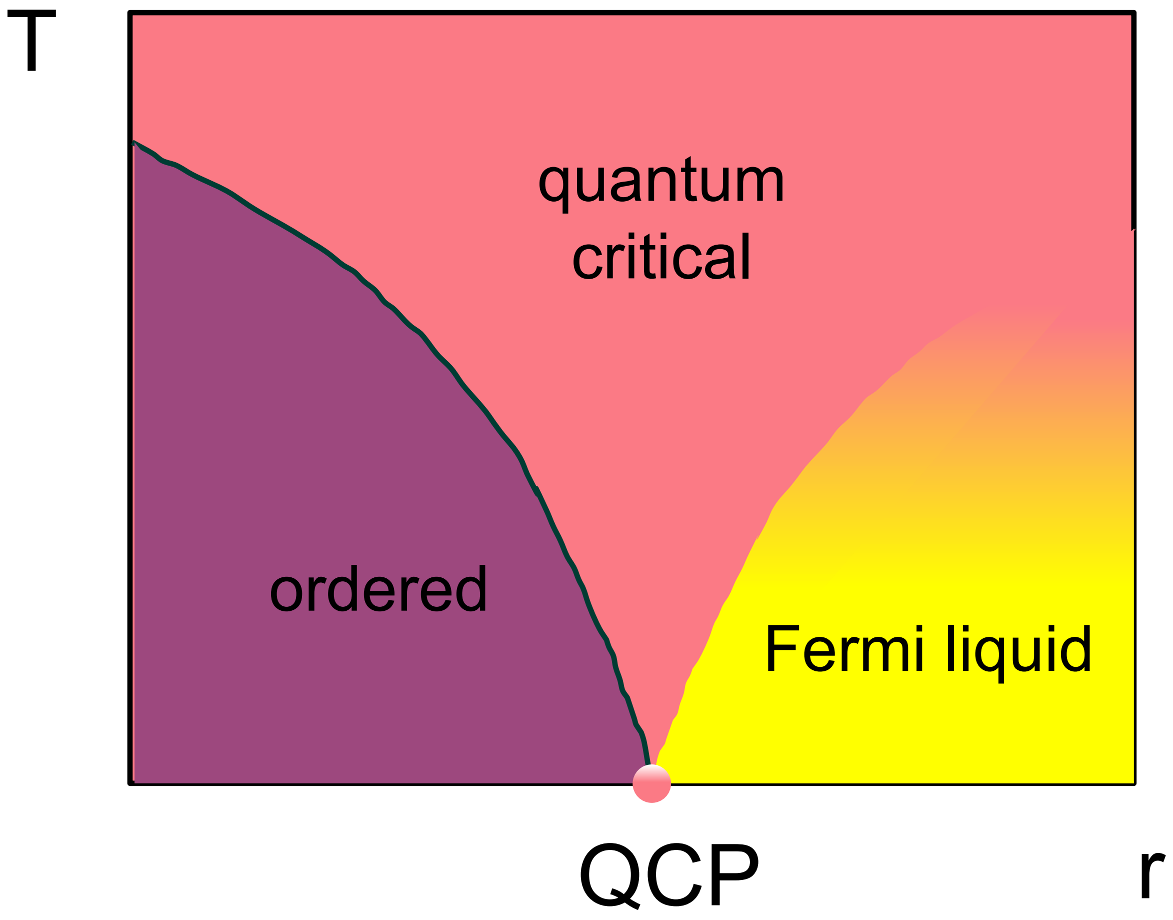
$$\varepsilon_{\gamma\delta}(\boldsymbol{x}) = \varepsilon^{\mathrm{phonon}}_{\gamma\delta}(\boldsymbol{x}) + \varepsilon^{\mathrm{defects}}_{\gamma\delta}(\boldsymbol{x}) + \varepsilon^{\mathrm{rand.}}_{\gamma\delta}(\boldsymbol{x})$$

changes the altermagnetic interactions
- enhances overall coupling
- **changes range / character of the interaction**

dislocations
- nucleation of 1D order

randomness
- random-field Ising model with variable disorder strength

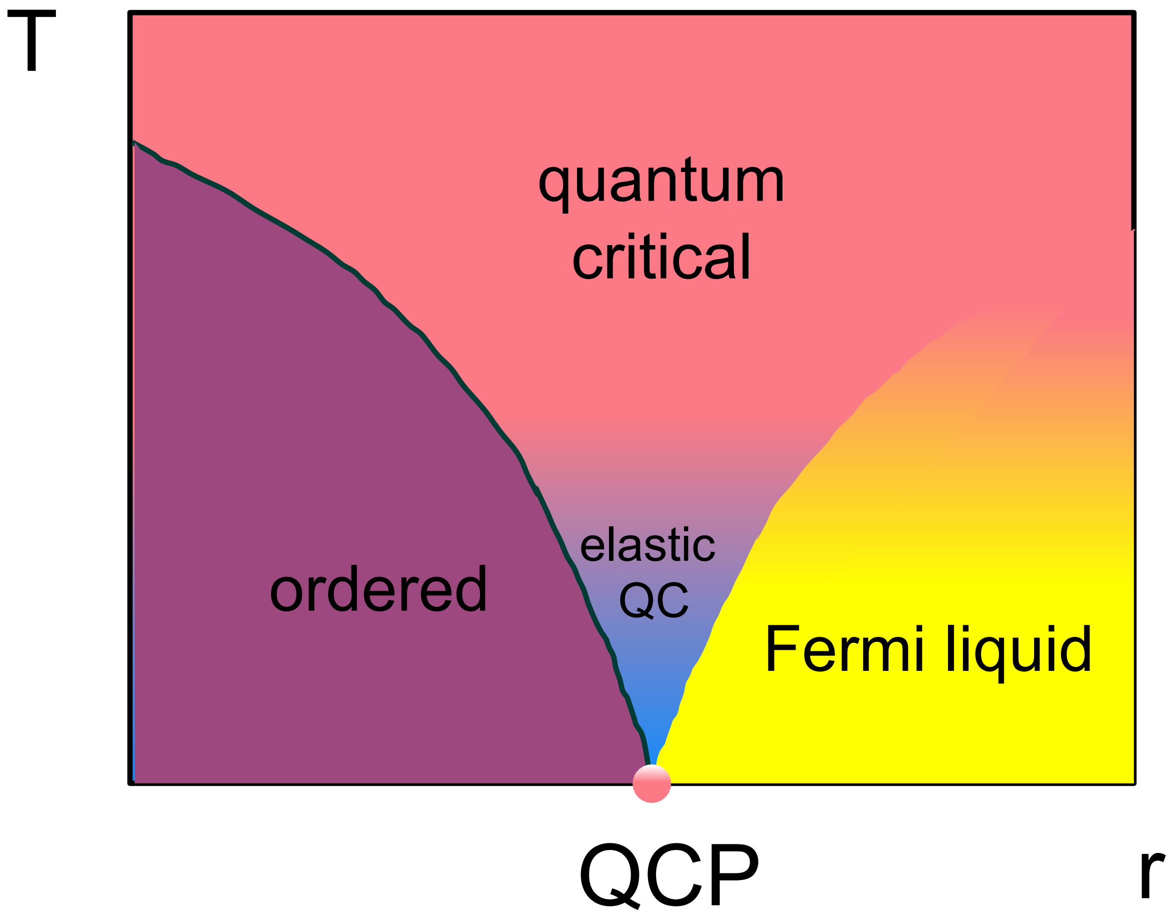
quantum criticality



d=3, itinerant QCP
(Ising nematic, altermagnetic)

$$\gamma(T) = c(T)/T \propto \log(T_0/T)$$

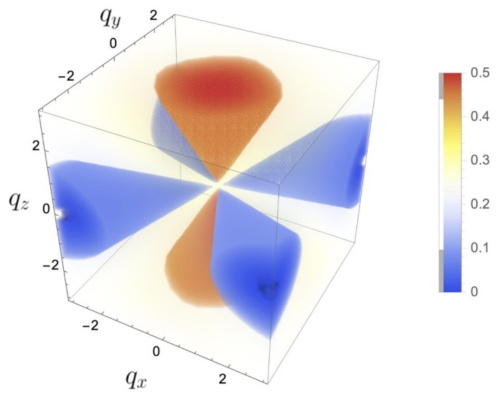
quantum criticality



d=3, itinerant QCP
(Ising nematic, altermagnetic)

$$\gamma(T) = c(T)/T \propto \log(T_0/T)$$

including elastic couplings

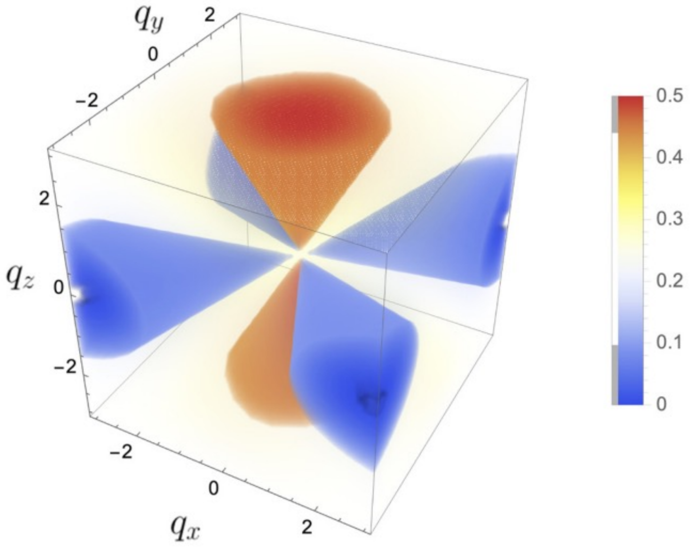


only soft along
specific directions

$$\gamma(T) = c(T)/T \propto const.$$

I. Paul and M. Garst, Phys. Rev. Lett. **118**, 227601 (2017).

directional softness of critical modes



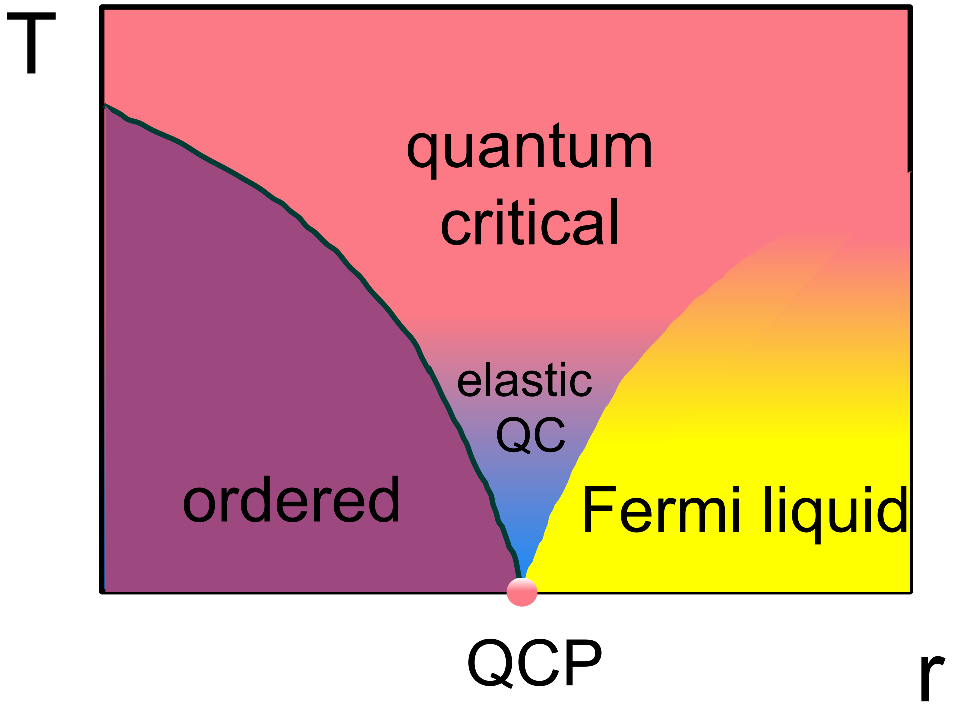
$$\chi\left(\boldsymbol{q}, \omega = 0\right) \propto \frac{1}{q^2 + c_1 \left(\frac{q_x^2 - q_y^2}{q^2}\right)^2 + c_2 \frac{q_z^2}{q^2}}$$

due to soft phonons

- reduces critical fluctuations
- well-defined fermionic quasi-particles at QCP

A. Levanyuk and A. Sobyanin, JETP Lett. **11**, 371 (1970), R. A. Cowley, PRB 13, 4877 (1976).

Is the elastic QCP a Fermi-liquid fixed point?



elasto-caloric effect

$$\eta = \frac{\partial T}{\partial \varepsilon}\Big|_S = -\frac{T}{c}\frac{\partial S}{\partial \varepsilon}\Big|_T$$

$$\frac{\eta(T)}{T} \propto \log(T_0/T)$$

not a Fermi-liquid fixed point

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Scaling arguments

$$\omega(\boldsymbol{q})^2 = \boldsymbol{q}_{\mathrm{hard}}^2 + q_{\mathrm{soft}}^4$$

$$d_{\mathrm{eff}} \to 2(d-1)+1$$

$$(d = 3 \to d_{\mathrm{eff}} = 5)$$

singular part of the free energy density

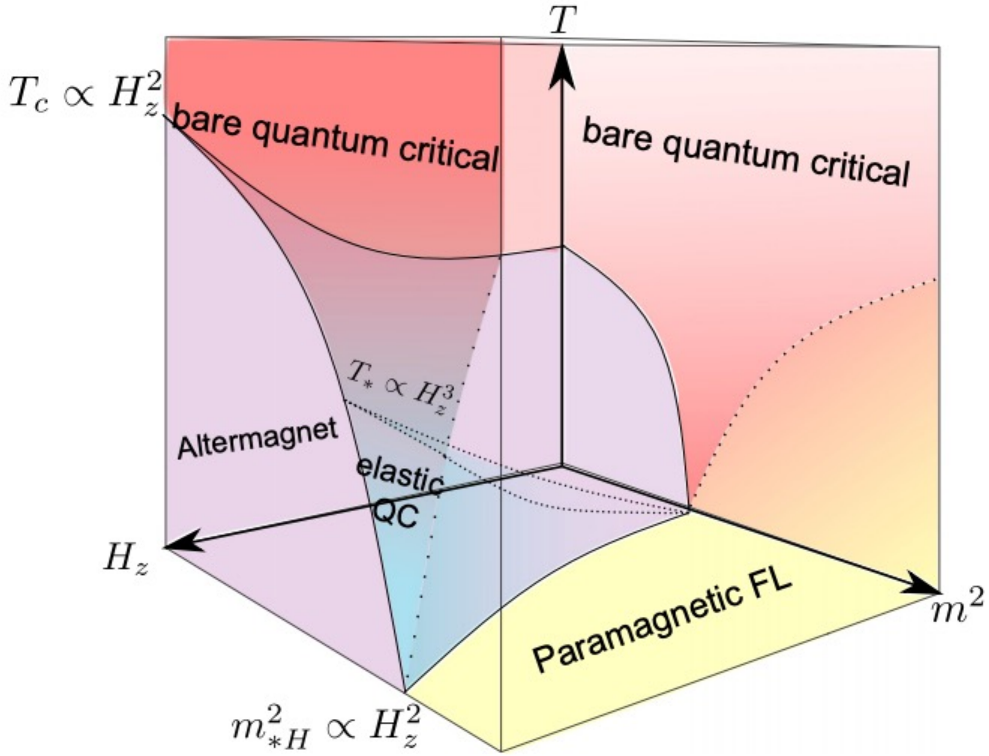
$$f_{\mathrm{sing}}(r,T) = b^{-(d_{\mathrm{eff}}+z)} f_{\mathrm{sing}}\left(b^{1/\nu} r, b^z T\right)$$

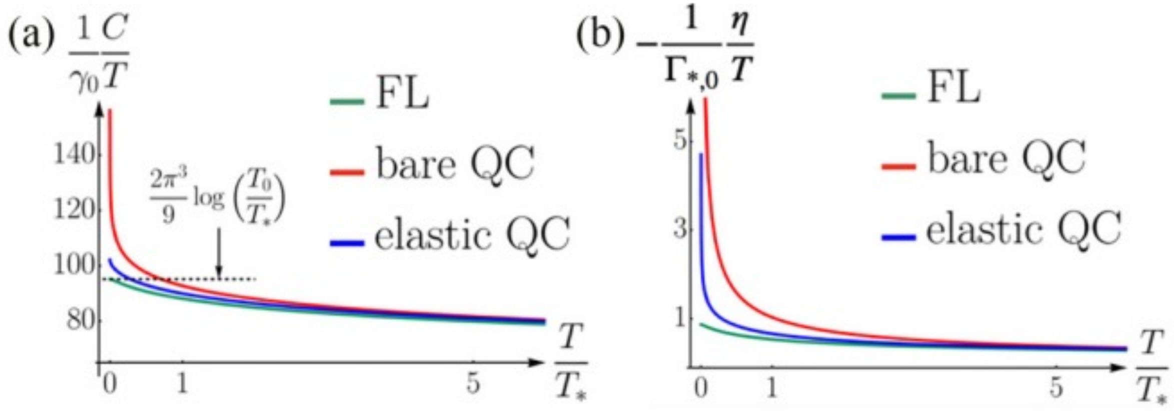
heat capacity: $\gamma_{\mathrm{sing}} = -\frac{\partial^2 f_{\mathrm{sing}}}{\partial T^2} \propto T^{\frac{d_{\mathrm{eff}}-z}{z}} \ll \gamma_{\mathrm{FL}}$

elastocaloric effect: $\frac{\eta}{T} = \frac{\frac{\eta}{T}\big|_{\mathrm{FL}} + cT^{\frac{d_{\mathrm{eff}}-\nu^{-1}-x}{x}}}{\gamma_{\mathrm{FL}} + c' T^{\frac{d_{\mathrm{eff}}-z}{z}}}$

C. R. W. Steward, G. Palle, M. Garst, J. S., and I. Jang, PRB 112, 064431 (2025)

Field-tuned elastic QCP in altermagnets





tunable crossover between “bare” QCP and elastic QCP

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anomalous Hall conductivity

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single-particle Berry-curvature contribution

$$\sigma_{\alpha\beta}^{(A)}\left(\omega\right)=-\frac{e^2}{\hbar}\sum_{\boldsymbol{k},l}\Omega_{\alpha\beta}^{(l)}\left(\boldsymbol{k}\right)\Theta\left(\omega-\xi_{\boldsymbol{k},l}\right)$$

$$\Omega_{\alpha\beta}^{(l)}\left(\boldsymbol{k}\right)=i\left[\left\langle\frac{\partial u_{\boldsymbol{k}l}}{\partial k_\alpha}\,\middle|\,\frac{\partial u_{\boldsymbol{k}l}}{\partial k_\beta}\right\rangle-\left[\alpha\leftrightarrow\beta\right]\right]$$

D_{4h} $(4/mmm)$ point group	
AM irrep	piezomagnetic coupling
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B_{1g}^-	$\lambda\phi\left(\epsilon_{yz}H_x+\epsilon_{xz}H_y\right)$ $\lambda'\phi\epsilon_{xy}H_z$
B_{2g}^-	$\lambda\phi\left(\epsilon_{xz}H_x-\epsilon_{yz}H_y\right)$ $\lambda'\phi\epsilon_{x^2-y^2}H_z$
FM irrep	
A_{2g}^-	$\lambda\phi\epsilon_{A_{1g}}H_z$ $\lambda'\phi\left(\epsilon_{zx}H_x+\epsilon_{yz}H_y\right)$
E_g^-	$\lambda\epsilon_{A_{1g}}\left(\phi_1H_x+\phi_2H_y\right)$ $\lambda'\epsilon_{x^2-y^2}\left(\phi_1H_x-\phi_2H_y\right)$ $\lambda''\epsilon_{xy}\left(\phi_1H_y+\phi_2H_x\right)$ $\lambda'''\left(\phi_1\epsilon_{xz}+\phi_2\epsilon_{yz}\right)H_z$

$\sigma_{\alpha\beta}^{(A)}\left(\omega\right)\neq 0$ only for states with finite moment

N Nagaosa, J Sinova, S Onoda, AH MacDonald, NP Ong RMP 82, 1539 (2010)

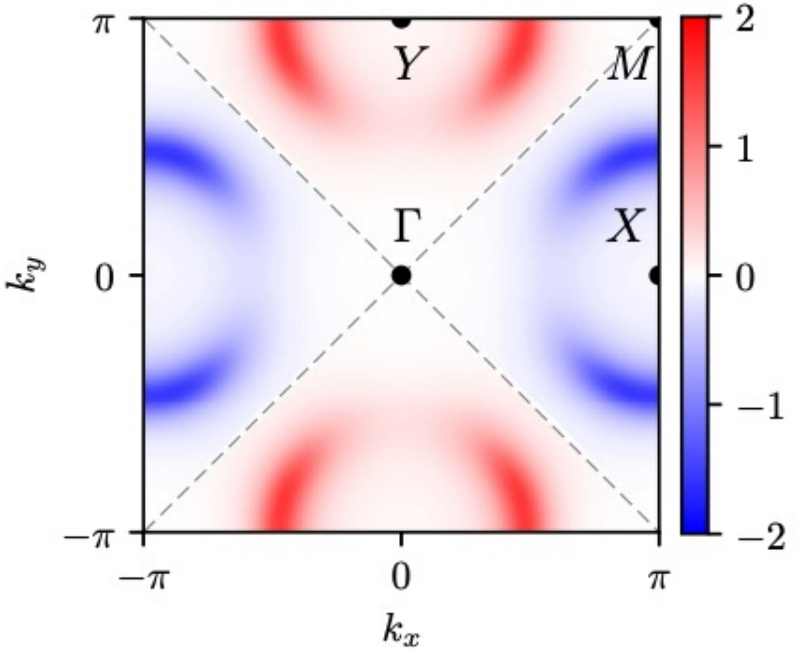
anomalous Hall conductivity

single-particle Berry-curvature contribution

$$\sigma^{(A)}_{\alpha\beta}(\omega) = -\frac{e^2}{\hbar}\sum_{\boldsymbol{k},l}\Omega^{(l)}_{\alpha\beta}(\boldsymbol{k})\,\Theta(\omega - \xi_{\boldsymbol{k},l})$$

$$\Omega^{(l)}_{\alpha\beta}(\boldsymbol{k}) = i\left[\left\langle \frac{\partial u_{\boldsymbol{k}l}}{\partial k_\alpha} \,\middle|\, \frac{\partial u_{\boldsymbol{k}l}}{\partial k_\beta} \right\rangle - [\alpha \leftrightarrow \beta]\right]$$

Berry curvature quadrupole



K. Takahashi, et al,.. PRB 111, 184408 (2025)

pure altermagnet → $\sigma^{(A)}_{\alpha\beta}(\omega) = 0$

elasto-Hall conductivity

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$$j_\alpha = \sigma^{(A)}_{\alpha\beta} E_\beta + \nu^{(A)}_{\alpha\beta\gamma\delta} E_\beta \varepsilon_{\gamma\delta}$$

vanishes for pure altermagnets

when is it nonzero?

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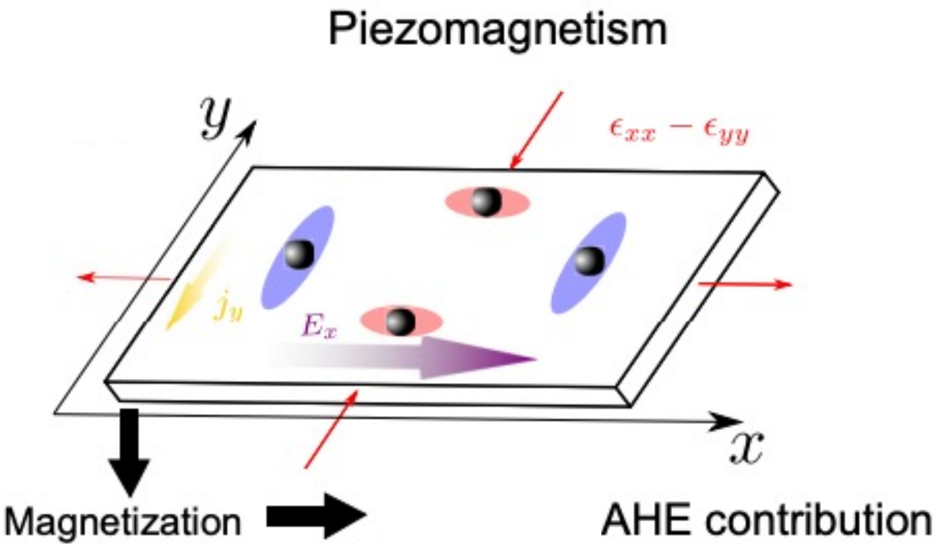
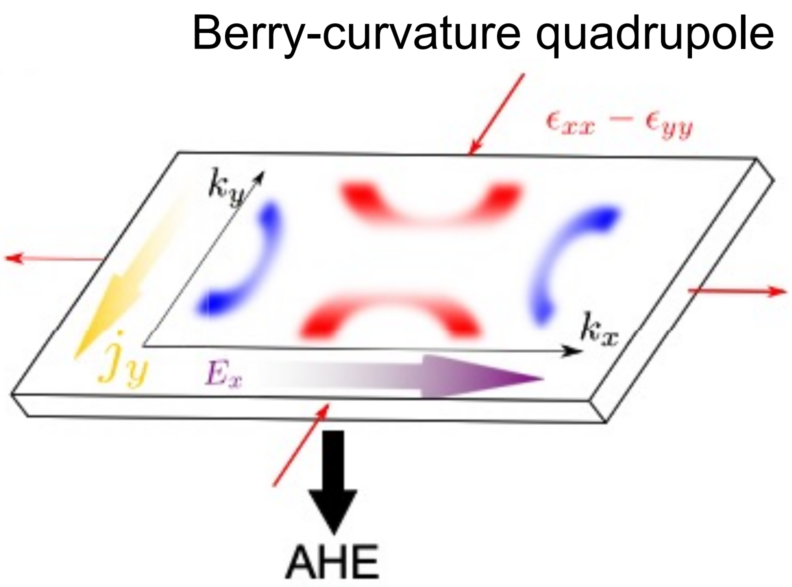
elasto-Hall conductivity





$$M_\kappa = \Lambda_{\kappa\beta\gamma}\varepsilon_{\gamma\delta}$$

$$\Lambda_{\kappa\gamma\delta} = -\lambda^i_{\kappa\gamma\delta}\phi^i$$

$$j_\alpha = \nu^{(A)}_{\alpha\beta\gamma\delta}E_\beta\varepsilon_{\gamma\delta}$$

$\nu^{(A)}_{\alpha\beta\gamma\delta}$ transforms like $\epsilon_{\alpha\beta\kappa}\Lambda_{\kappa\gamma\delta}$

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elasto-Hall conductivity

add strain
$$\mathcal{H}_{\boldsymbol{k}} = \mathcal{H}_{\boldsymbol{k},0} + \mathcal{H}_{\boldsymbol{k},\varepsilon}$$

$$\mathcal{H}_{\boldsymbol{k},0} = \sum_{l} \xi_{\boldsymbol{k}l} \left| u_{\boldsymbol{k},l} \right\rangle \left\langle u_{\boldsymbol{k},l} \right|$$ (unstrained system)

$$\mathcal{H}_{\boldsymbol{k},\varepsilon} = \varepsilon_{\alpha\beta} \sum_{ll'} \gamma_{ll'}^{\alpha\beta} \left(\boldsymbol{k}\right) \left| u_{\boldsymbol{k},l} \right\rangle \left\langle u_{\boldsymbol{k},l'} \right|$$ (strain contribution)

strain correction to the Berry curvature

$$\delta\Omega_{\alpha\beta}^{(l)}\left(\boldsymbol{k}\right) = \Gamma_{\alpha\beta\gamma\delta}^{(l)}\left(\boldsymbol{k}\right) \varepsilon_{\gamma\delta}$$

$$\Gamma_{\alpha\beta\gamma\delta}^{(l)}\left(\boldsymbol{k}\right) = -2 \left[\partial_{k_\alpha} \sum_{l' \neq l} \frac{\mathrm{Im}\left(\gamma_{ll'}^{\gamma\delta}\left(\boldsymbol{k}\right) \left\langle u_{\boldsymbol{k}l} \mid \partial_{k_\beta} u_{\boldsymbol{k};'} \right\rangle \right)}{\xi_{\boldsymbol{k},l} - \xi_{\boldsymbol{k},l'}} - \left(\alpha \leftrightarrow \beta\right) \right]$$

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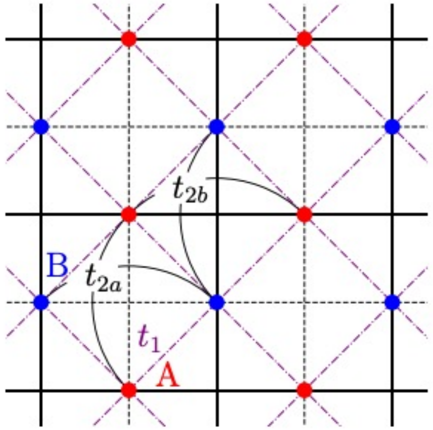
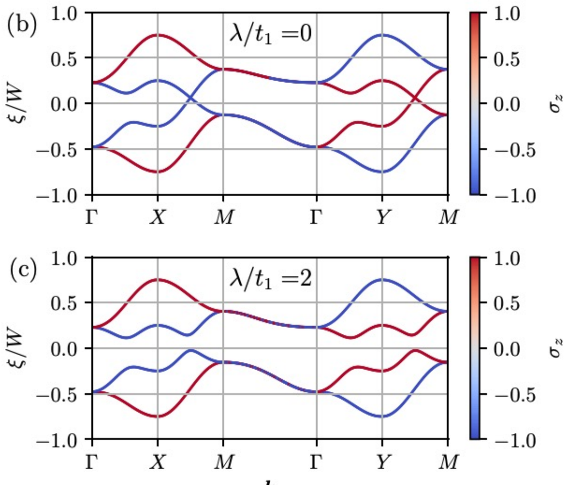
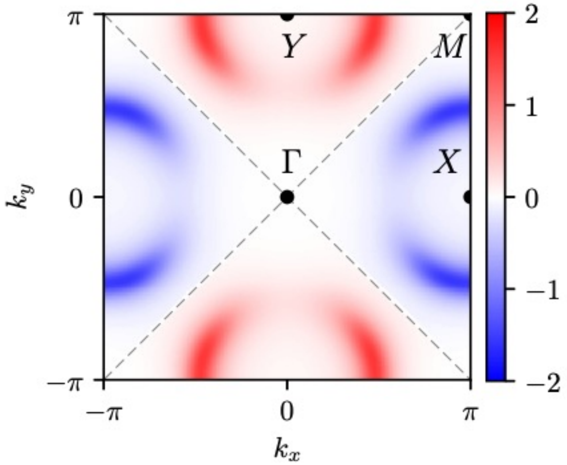
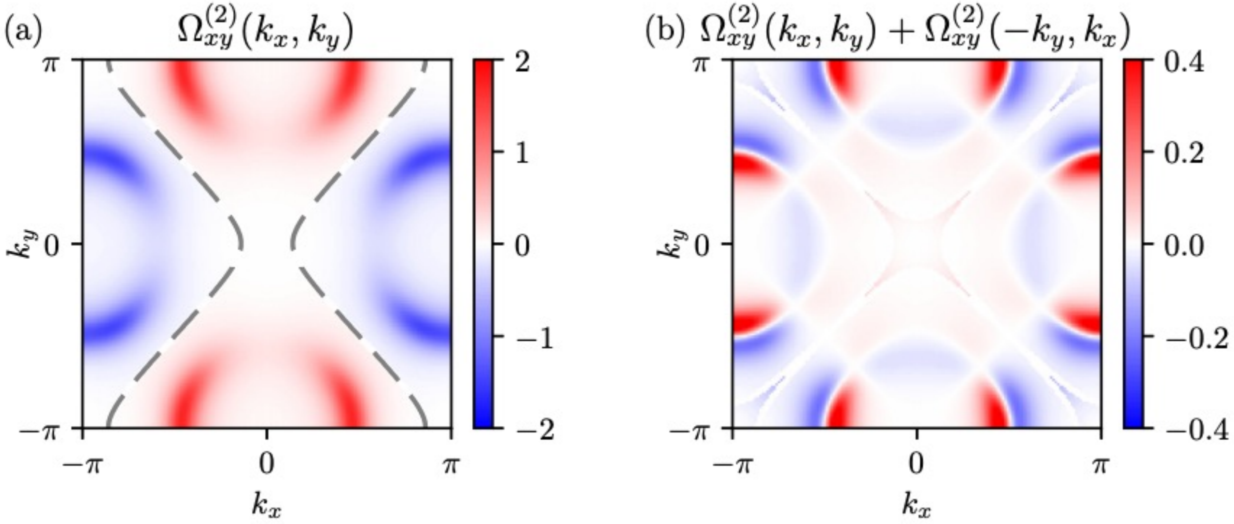
example: Lieb lattice







strain



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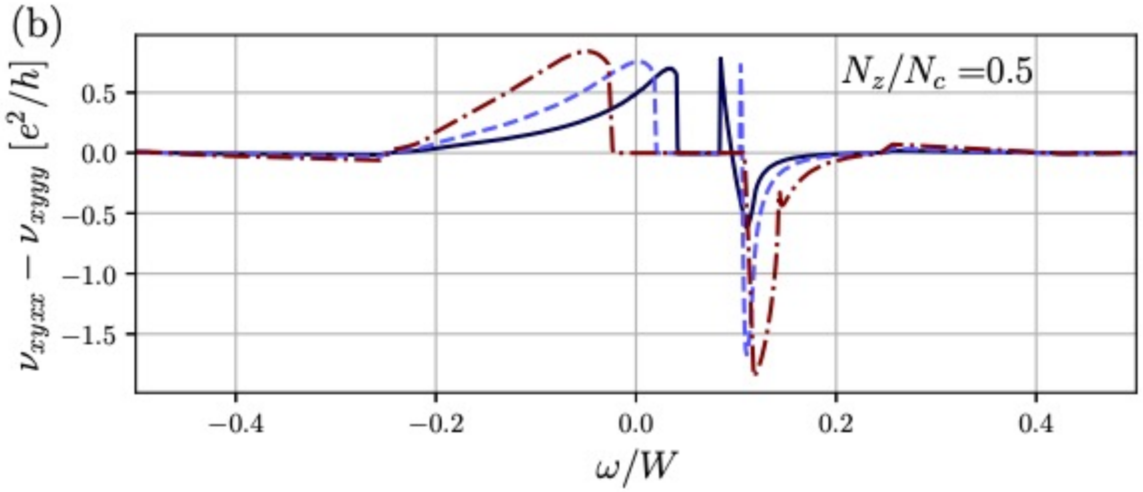
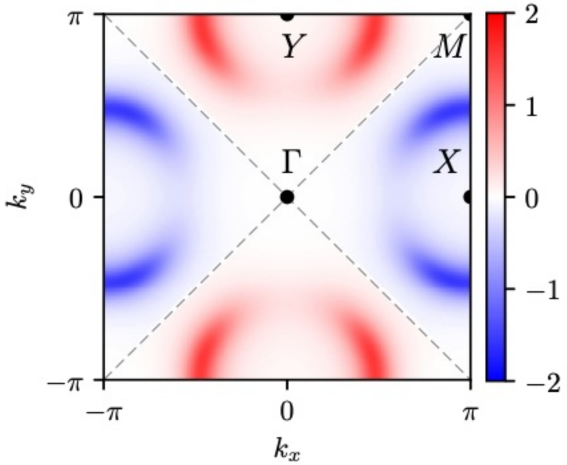
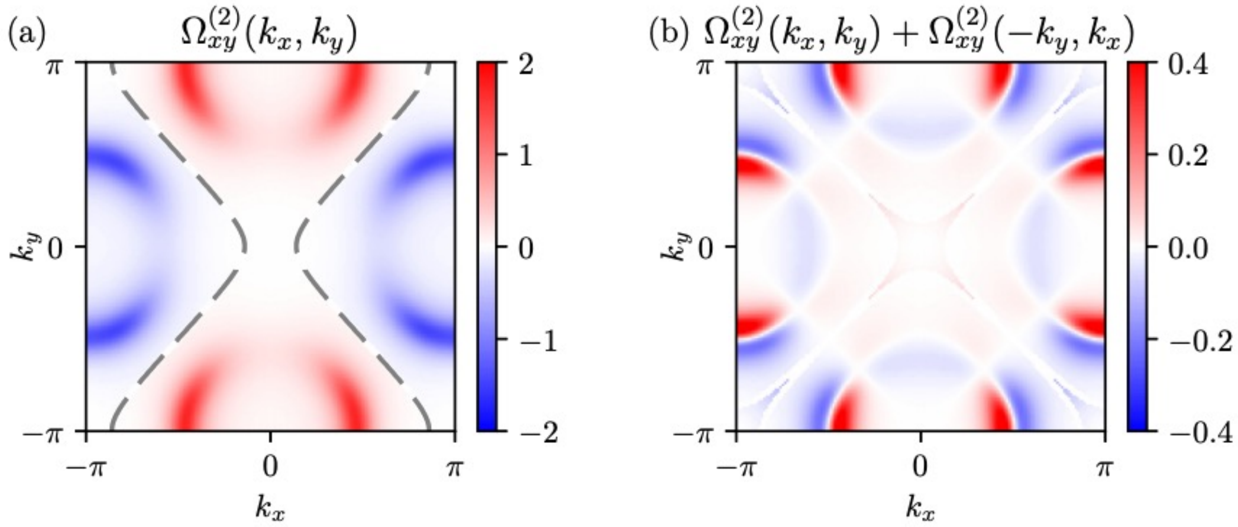


elasto-Hall conductivity



strain



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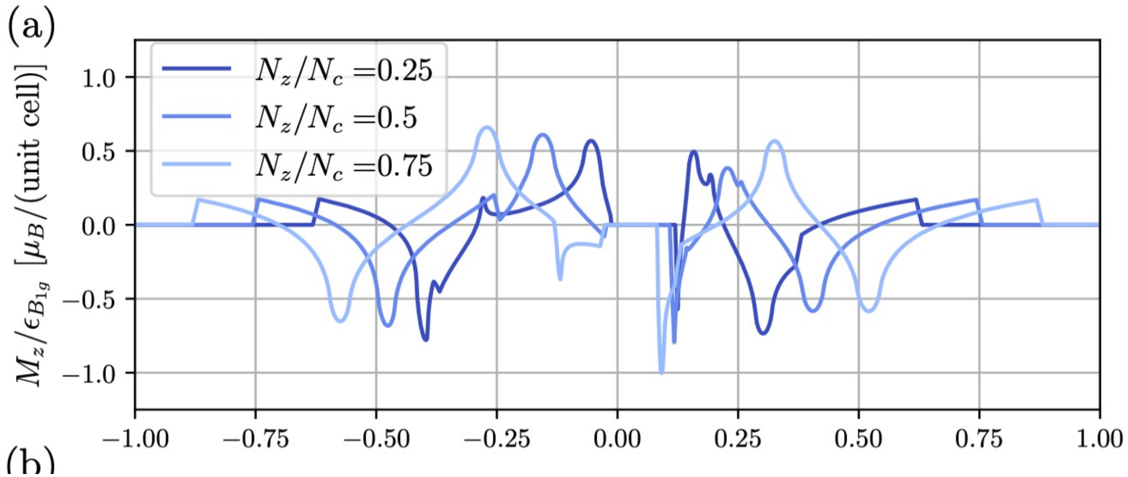
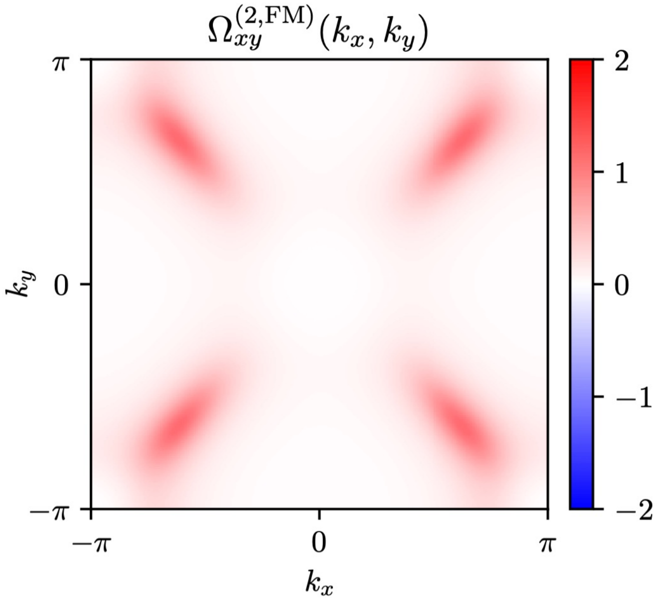
“piezomagnetic contribution”

$$M_\kappa = \Lambda_{\kappa\beta\gamma}\varepsilon_{\gamma\delta}$$





direct contribution due to strain-induced magnetization:

- no quadrupole
- not sensitive to Dirac points

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classification of the strain geometry needed to induce AHE

	AM irrep.	MPG	Elasto-Hall conductivity
orthorh., D_{2h} (mmm)	A_{1g}^-	mmm (8.1.24)	ν_{yzyz}, ν_{zxxz}, ν_{xyxy}
trigon. D_{3d} ($\bar{3}m$)	A_{1g}^-	$\bar{3}m$ (20.1.71)	$\nu_{xzxx} = \nu_{zxyy} = \nu_{zyxy}$, $\nu_{xzxz} = \nu_{yzyz}$
tetrag., D_{4h} ($4/mmm$)	A_{1g}^-	$4/mmm$ (15.1.53)	$\nu_{yzyz} = -\nu_{zxxz}$
	B_{1g}^-	$4'/mm'm$ (15.4.56)	$\nu_{yzyz} = \nu_{zxxz}$, ν_{xyxy}
	B_{2g}^-	$4'/mm'm$ (15.4.56)	$\nu_{yzxz} = -\nu_{zxyz}$, $\nu_{xyxx} = -\nu_{xyyy}$
hexag., D_{6h} ($6/mmm$)	A_{1g}^-	$6/mmm$ (27.1.100)	$\nu_{yzyz} = -\nu_{zxxz}$
	B_{1g}^-	$6'/m'mm'$ (27.5.104)	$\nu_{yzxy} = \nu_{zxxx} = -\nu_{zxyy}$
	B_{2g}^-	$6'/m'mm'$ (27.5.104)	$\nu_{zxxy} = \nu_{yzyy} = -\nu_{yzxx}$
	E_{2g}^-	$(1, 0)$ mmm (8.1.24)	ν_{yzyz}, ν_{zxxz}, ν_{xyxy}
		$(1, \frac{1}{\sqrt{3}})$ $m'm'm$ (8.4.27)	ν_{xyxx}, ν_{xyyy}, ν_{xyzz}, ν_{xzyz}, ν_{yzxz}
		$(1, b)$ $2/m$ (5.1.12)	ν_{xzxx}, ν_{xzyy}, ν_{xzzz}, ν_{xyyz}, ν_{yzyz}, ν_{xzxz}, ν_{xyxy}, ν_{yzxy}
cubic, O_h ($m\bar{3}m$)	A_{1g}^-	$m\bar{3}m$ (29.1.109)	–
	A_{2g}^-	$m\bar{3}m'$ (32.4.121)	$\nu_{yzyz} = \nu_{zxxz} = \nu_{xyxy}$
	E_g^-	$(1, 0)$ $4/mmm$ (15.1.53)	$\nu_{yzyz} = -\nu_{zxxz}$
		$(1, \frac{1}{\sqrt{3}})$ $4'/mm'm$ (15.4.56)	$\nu_{yzyz} = \nu_{zxxz}$, ν_{xyxy}
		$(1, b)$ mmm (8.1.24)	ν_{yzyz}, ν_{zxxz}, ν_{xyxy}
	T_{2g}^-	$(1, 0, 0)$ $4'/mm'm$ (15.4.56)	$\nu_{yzxz} = -\nu_{zxyz}$, $\nu_{xyxx} = -\nu_{xyyy}$
		$(1, 1, 0)$ $m'm'm$ (8.4.27)	ν_{yzxx}, ν_{yzyy}, ν_{yzzz}, ν_{xyxz}, ν_{xzxy}
		$(1, 1, 1)\bar{3}m$ (20.1.71)	$\nu_{xzxy} = \nu_{zyyy} = \nu_{yzxx}$, $\nu_{xzxz} = \nu_{yzyz}$
		$(1, b, 0)$ $2'/m'$ (5.5.16)	ν_{xyxx}, ν_{yzxx}, ν_{xyyy}, ν_{yzyy} ν_{xyzz}, ν_{yzzz}, ν_{xzyz}, ν_{xyxz}, ν_{yzxz}, ν_{xzxy}
		$(1, 1, c)$ $2/m$ (5.1.12)	ν_{xzxx}, ν_{xzyy}, ν_{xzzz}, ν_{xyyz}, ν_{yzyz}, ν_{xzxz}, ν_{xyxy}, ν_{yzxy}
		$(1, b, c)$ -1 (2.1.3)	all elements $\nu_{\alpha\beta\gamma\delta}$ are nonzero where $\alpha \neq \beta$

$$j_\alpha = \nu^{(A)}_{\alpha\beta\gamma\delta} E_\beta \varepsilon_{\gamma\delta}$$

$$\nu^{(A)}_{\alpha\beta\gamma\delta} = -\nu^{(A)}_{\beta\alpha\gamma\delta}$$

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so far: piezomagnetism

$$H_{\text{piezo}-\text{mag.}} = \int d^3x \lambda^i_{\alpha\gamma\delta} B_\alpha \phi^i(\boldsymbol{x})\, \varepsilon_{\gamma\delta}(\boldsymbol{x})$$

dynamic coupling at zero field

$$H_{\text{dyn}} = \int d^3x \mu^i_{\gamma\delta} \pi^i(\boldsymbol{x})\, \varepsilon_{\gamma\delta}(\boldsymbol{x})$$

coupling to the conjugate momentum $\left[\phi^i(\boldsymbol{x}), \pi^j(\boldsymbol{x}')\right] = i\delta^{ij}\delta(\boldsymbol{x} - \boldsymbol{x}')$

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dynamical stress-strain and Hall viscosity

$$\sigma_{\alpha\beta} = C_{\alpha\beta\gamma\delta}\varepsilon_{\gamma\delta} - \eta_{\alpha\beta\gamma\delta}\partial_\tau\varepsilon_{\gamma\delta}$$

elastic constants

viscosity

non-dissipative contribution:
(broken TRS)

$$\eta^H \equiv \eta_{xyxx}(B_z) = -\eta_{xyxx}(-B_z) = -\eta_{xxxy}(B_z) = \cdots$$

$$S_{\mathrm{Hall}} = -\eta^H \int d\tau \int d^3x \left[\varepsilon_{xx} - \varepsilon_{yy}\right] 2\partial_\tau \varepsilon_{xy}$$

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dynamic coupling at zero field

$$S_{\mathrm{dyn}} = -\mu^{i}_{\alpha\beta} \int d\tau \int d^3x \varepsilon_{\alpha\beta}\left(\boldsymbol{x},\tau\right) \partial_\tau \phi^i \left(\boldsymbol{x},\tau\right)$$

$$\Updownarrow$$

$$S_{\mathrm{Hall}} = -\eta^{H} \int d\tau \int d^3x \left[\varepsilon_{xx} - \varepsilon_{yy}\right] 2\partial_\tau \varepsilon_{xy}$$

dynamics of the order parameter induces non-dissipative stress

$$\mu^{i}_{\alpha\beta}\partial_\tau \phi^i \left(\boldsymbol{x},\tau\right) \leftrightarrow \eta^{H} \partial_\tau \varepsilon_{\alpha\beta}$$

analogous to Hall viscosity response

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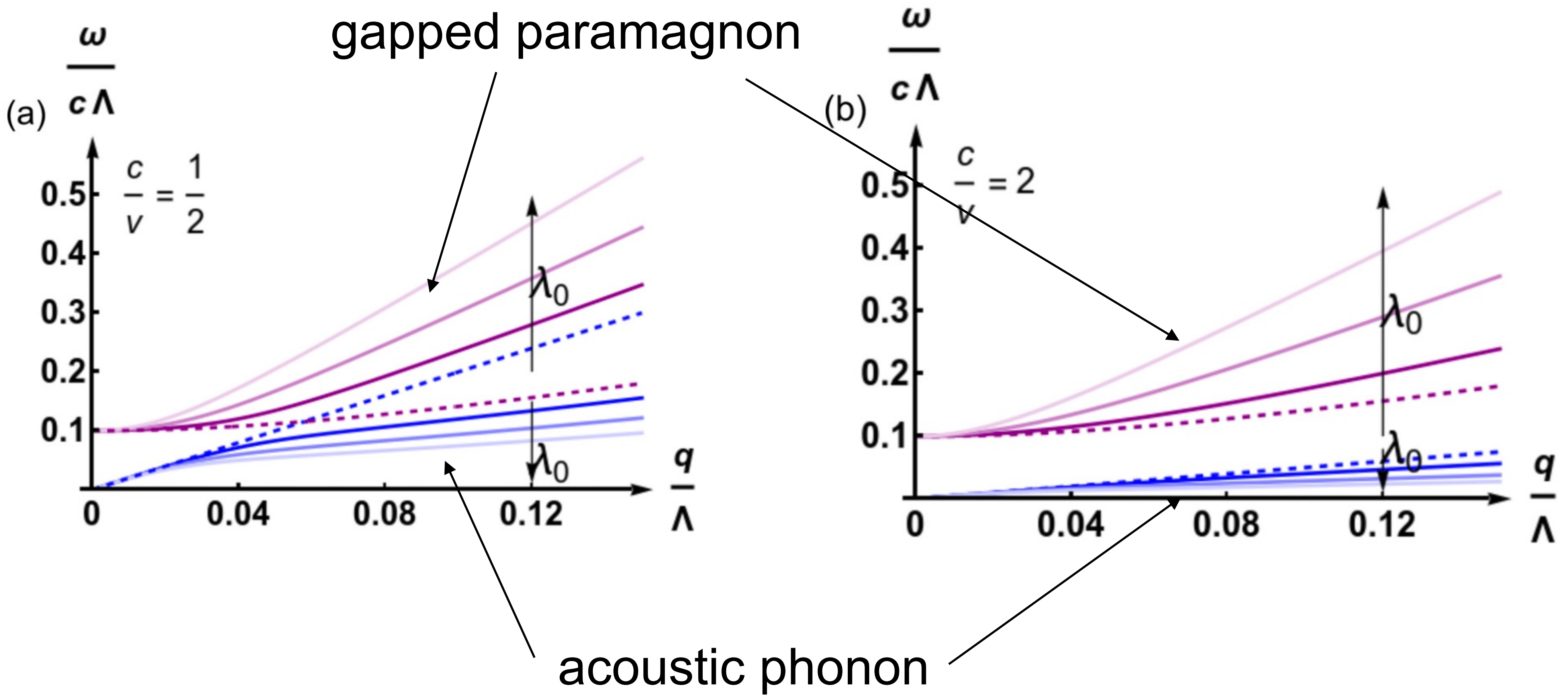
T-dependent level repulsion (no magnetic field, no AM long-range order)



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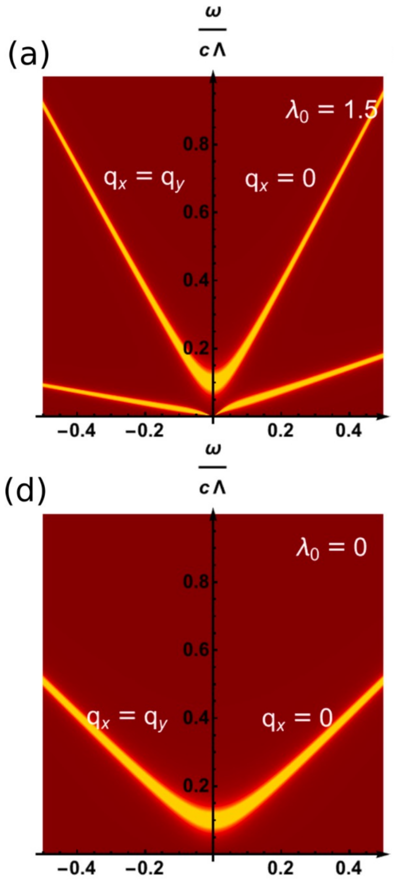
altermagnetic polarons



paramagnon

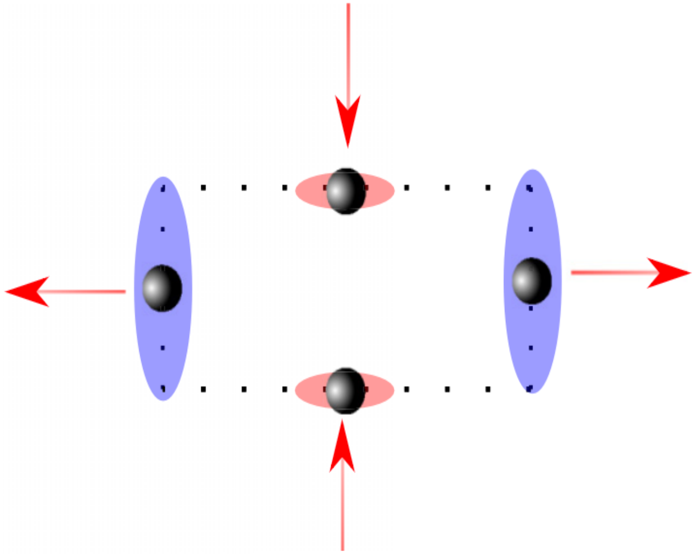
symmetry-sensitive
mixing of
phonon + paramagnon

Conclusions

coupling to strain can be exploited to detect and manipulate altermagnetism

piezo-magnetism

elasto-Hall conductivity



modified
quantum criticality

Hall viscosity

dynamical stresses

tunable randomness