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A CRITERION FOR THE EQUIVALENCE OF FORMAL SINGULARITIES

By KONRAD MÖHRING and DUCO VAN STRATEN

Abstract. We prove a generalization of the finite determinacy theorem for isolated singularities. The maximal ideal occurring in the finite determinacy theorem is replaced by any ideal annihilating the first cotangent cohomology of a formal singularity over a Noetherian ring. An analogous result holds for finitely generated modules. As an application we give a criterion for the algebraizability of formal singularities and modules.

0. Introduction. In this paper we give a criterion for certain algebras over a noetherian ring $S$ to be isomorphic, Theorem 1.1. Informally speaking, the criterion is the following stability assertion. Let the first cotangent cohomology $T^1(R/S)$ of $R = S[[x_1, \ldots, x_n]]/I$ be annihilated by some power of an ideal $a$. Then any $S[[x_1, \ldots, x_n]]/J$, such that generators of $J$ and relations among the generators are congruent to generators and relations of $I$ modulo a sufficiently high power of $a$, is right equivalent to $R$. If $R$ is an isolated singularity over the field $k$, $T^1(R/k)$ is always annihilated by some power of the maximal ideal $(x_1, \ldots, x_n)$, because the support of $T^1(R/k)$ is contained in the singular locus; so this generalizes known results on isolated singularities.

For a list of references on the subject, we refer to the introduction of [CS93]. Our proof is similar to Hironaka’s proof of a criterion for the equivalence of isolated singularities sketched in [Hir69].

Just as for isolated singularities in Artin’s paper [Art69, Th. 3.8], we deduce from our criterion the algebraizability of a certain class of singularities, Theorem 1.3. This class includes the isolated singularities, generalizing Artin’s result. Theorem 1.5 is the analogue of our main theorem for finitely generated modules over a field.

We will use the notation $P = S[[x_1, \ldots, x_n]]$ throughout. We recall that the first cotangent cohomology $T^1(R/S)$ of an $S$-algebra $R = P/I$ is the cokernel of the natural map $\text{Der}_S(P, P) \rightarrow \text{Hom}_P(I, R)$.

1. Results. Our main theorem is this:

THEOREM 1.1. (Equivalence of singularities) Let $S$ be a noetherian commutative ring with 1, $P = S[[x_1, \ldots, x_n]]$ and $a \subset P$ an ideal such that $1 - x$ is invertible.
for all \( x \in \mathfrak{a} \) and \( P \) is \( \mathfrak{a} \)-complete, i.e., \((P, \mathfrak{a})\) is a complete Zariski ring. Let \( I \subset P \) be a proper ideal and write \( R := P/I \). Assume that \( \mathfrak{a}^a T^1(R/S) = 0 \) for some \( a \in \mathbb{N} \). For an exact sequence of \( P \)-modules

\[
P' \xrightarrow{G} P^a \xrightarrow{F} P \to R \to 0,
\]

there exist constants \( a_F, a_G \) and \( b \), such that the following holds: If \( c \in \mathbb{N}_0 \), \( F' \) and \( G' \) are matrices whose entries are congruent to those of \( F \) and \( G \) modulo \( \mathfrak{a}^{a_F+c} \) and \( \mathfrak{a}^{a_G} \) respectively, and if \( F' \circ G' = 0 \), then there is an automorphism \( \Phi \) of \( P \) over \( S \) which is congruent to the identity modulo \( \mathfrak{a}^{a_F+c-b} \) and carries the ideal \( I' := \text{Im}(F') \) onto \( I = \text{Im}(F) \).

In particular, the theorem is valid if we choose \( a \) to be the following ideal \( H_I \), which can easily be computed from the given data.

**Definition 1.2.** Let \( \text{Jac}(F) \) denote the jacobian matrix of partial derivatives of \( F \). If \( A, B, C, D \) are subsets of indices, let \( G_{AB} \) and \( \text{Jac}(F)_{CD} \) denote the corresponding submatrices.

We define the ideal \( H_I \subset P \) to be generated by

\[
\{ \det(G_{AB}) \cdot \det(\text{Jac}(F)_{CD}) \mid \#A = \#B = p, \quad \#C = \#D = s - p \quad \text{and} \quad A \cup D = \{1, \ldots, s\} \}.
\]

The ideal \( H_I \) or rather \( H_I + I \) describes the nonsmooth locus of \( R \) over \( S \). Since the cotangent cohomology has support in the nonsmooth locus, a power of \( H_I \) annihilates \( T^1 \). Following Artin, [Art76, Part II], we outline a direct proof: Consider the complex

\[
R' \xrightarrow{G \otimes R} R^a \xrightarrow{\text{Jac}(F) \otimes R} R^a.
\]

Localizing at a prime \( p \supset I \) gives a split sequence iff \( H_I \subset p \). In this case the dual complex of (1) is also a split sequence. In particular it is exact. Now \( T^1(R/S) \) is the homology of this dual complex, so \( T^1(R/S) \) is annihilated by some power of \( H_I \).

The special case of Theorem 1.1 for an ideal defining the nonsmooth locus has already appeared, slightly modified, in [CS97, Th. 4.4]. However, our theorem is stronger, since the support of \( T^1 \) can be smaller than the nonsmooth locus, e.g. for rigid singularities.

Now we consider the special case that \( S \) is a field and \( a = m = (x_1, \ldots, x_n) \). Following Artin's proof for isolated singularities [Art69, Th. 3.8], we deduce the algebraizability of singularities with \( \dim_k T^1(R/k) < \infty \).

**Theorem 1.3.** Let \( k \) be any field. Let \( I \subset m \subset P = k[[x_1, \ldots, x_n]] \) be an ideal, \( R := P/I \) and \( \dim_k T^1(R/k) < \infty \). Let \( H = k\langle x_1, \ldots, x_n \rangle \) be the Henselization of the
polynomial ring at the maximal ideal \((x_1, \ldots, x_n)\), i.e., the ring of algebraic power series.

Then there is an ideal \(J \subset H\) and a formal automorphism \(\Phi\) of \(P\), which transforms the completion of \(J\) into \(I\):

\[
\Phi(\hat{J}) = I.
\]

**Proof.** The condition \(\dim_k T^1(R/k) < \infty\) is equivalent to \(m^a T^1(R/k) = 0\) for some constant \(a\). We choose a representation

\[
P^s \xrightarrow{G} P^r \xrightarrow{F} P \rightarrow R \rightarrow 0
\]

of \(R\). So if \(F = (f_i)\) and \(G = (g_{ij})\), we have generators \(f_1, \ldots, f_s\) of \(I\) and relations \(\sum_i f_i g_{ij} = 0\). The \(f_i\) and \(g_{ij}\) are solutions of the following system of equations in the unknowns \(Y_i, Y_{ij}\):

\[
\sum_{i=1}^s Y_i Y_{ij} = 0, \quad j = 1, \ldots, r.
\]

Now we make use of the Artin approximation theorem as stated in [KPR75, Satz 5.2.1, (4)]:

**Theorem 1.4.** (Artin Approximation Theorem) Let \(H = k(x_1, \ldots, x_n)\) be the Henselization of the polynomial ring at the maximal ideal \((x_1, \ldots, x_n)\). We assume \(y(x) \in P^N\) to be a solution of a system of polynomial equations in \(N\) variables over \(H\). Let \(k\) be any number. Then there is an algebraic solution \(y(x) \in H^N \subset P^N\), approximating the given solution up to order \(k\):

\[
y(x) \equiv y(x) \equiv 0 \mod m^k.
\]

Choosing \(k\) to be bigger than the constants \(a_F\) and \(a_G\) in the theorem, we are done.

By essentially the same proof as for Theorem 1.1 we obtain the following statement for finitely generated modules.

**Theorem 1.5.** Let \(M\) be a finitely generated module over \(P = k[[x_1, \ldots, x_n]]\) with \(a^a \text{Ext}^1(M, M) = 0\). Fix a representation

\[
P^r \xrightarrow{G} P^s \xrightarrow{F} P^t \rightarrow M \rightarrow 0
\]

of \(M\), where \(G\) and \(F\) are matrices with entries in \(P\). Then there are constants \(a_F\), \(a_G\) and \(b\) such that the following holds: If \(F'\) and \(G'\) are matrices whose entries are congruent to those of \(F\) and \(G\) modulo \(a_F^{ac}\) and \(a_G^{ac}\) respectively, \(c \in \mathbb{N}_0\) and if...
$F' \circ G' = 0$, then there is an automorphism of $P'$ which carries $\text{Im}(F')$ onto $\text{Im}(F)$. The automorphism is congruent to the identity modulo $a^{a_F+c-b}$.

**Corollary 1.6.** A finitely generated module over $P = k[[x_1, \ldots, x_n]]$ with the property $\dim_k \text{Ext}^1(M, M) < \infty$ is algebraic, i.e., the completion of a module over the ring of algebraic power series $H = k(x_1, \ldots, x_n)$.

**2. Proof of Theorem 1.1.** We denote the entries of the matrices $F$ and $G$ by $f_i$ and $g_{ij}$ respectively. The exact sequence

$$
P^* \xrightarrow{G} P^* \xrightarrow{F} I \xrightarrow{0}
$$

gives us an embedding of the normal module $N = \text{Hom}_P(I, R)$ into $R^*$:

$$
0 \rightarrow \text{Hom}_P(I, R) \xrightarrow{F^*} \text{Hom}_P(P^*, R) \xrightarrow{\cong} R^*,
$$

where $n \mapsto F^*(n) \mapsto (n(f_1), \ldots, n(f_s))$.

The entries of $F'$ and $G'$ are

$$
\begin{align*}
(2) & \quad f'_i = f_i + \phi_i, \quad \phi_i \in a^{a_F}, \\
(3) & \quad g'_{ij} = g_{ij} + \gamma_{ij}, \quad \gamma_{ij} \in a^{a_G},
\end{align*}
$$

with $a_F, a_G \geq 0$. We will give explicit lower bounds for $a_F$ and $a_G$ later on in the proof. We have assumed that

$$
0 = \sum f'_i g'_{ij} = \sum f_i g_{ij} + \sum \phi_i g_{ij} + \sum f_i \gamma_{ij} + \sum \phi_i \gamma_{ij}.
$$

The first summand is zero, the third is in the ideal $I = (f_1, \ldots, f_s)$ and the fourth is an element of $a^{a_F+a_G}$. So $\bar{n}(f_i) := (f'_i - f_i) = \phi_i$ defines a $P$-module homomorphism $\bar{n}: I \rightarrow P/(I + a^{a_F+a_G})$ with the property

$$
\bar{n}(f_i) = \phi_i + (I + a^{a_F+a_G}).
$$

We would like to find an element $n$ in the normal module $N$ of $R$, i.e., a homomorphism from $I$ to $R = P/I$, that induces $\bar{n}$.

**Proposition 2.1.** Let $P$ be any Noetherian ring, $a \subseteq P$ an ideal, and $\lambda: A \rightarrow B$ any homomorphism between finitely generated $P$-modules. Then there exists an integer $c = c(\lambda)$ with the following property: For all $x \in A$ and $p \in \mathbb{N}$ such that

$$
\lambda(x) \equiv 0 \mod a^{p+c} B
$$


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there exists an $\bar{x} \in A$ such that

$$\lambda(\bar{x}) = 0$$

and $\bar{x} \equiv x \mod a^p A$.

**Proof.** Consider the submodule $\text{Im}(\lambda) \subset B$. By the Artin-Rees lemma (cf. [Eis95]), there exists an integer $c$ such that

$$\text{Im}(\lambda) \cap a^{p+c} B = a^p (\text{Im}(\lambda) \cap a^c B).$$

So if $\lambda(x) \in a^{p+c} B$ we must have $\lambda(x) = \sum_i r_i n_i$ with $r_i \in a^p$ and $n_i = \lambda(m_i) \in \text{Im}(\lambda)$. Then $\bar{x} = x - \sum_i r_i m_i$ is just what we want. \hfill $\square$

Now we apply this proposition to the $P$-modules $A = \text{Hom}_P(P^s, P)$ and $B = \text{Hom}_P(P^r, R)$ and the homomorphism

$$\lambda: A \rightarrow B,$$

$$\phi \mapsto \phi \circ G \mod I.$$

Let's call the integer $c(\lambda)$ of the proposition $c_1$. Then we end up with $\bar{\phi}_i$ such that

$$\bar{\phi}_i \equiv \phi_i \mod a^{a_F + a_G - c_1}$$

with the property that

$$\sum \bar{\phi}_i g_{ij} \equiv 0 \mod I.$$

Hence these $\bar{\phi}_i$ describe an $n \in N = \text{Hom}(I, R)$ defined by

$$n(f_i) = \bar{\phi}_i + 1.$$

Let's assume we have chosen $a_G > c_1$. As $\bar{\phi}_i \equiv \phi_i \mod a^{a_F + a_G - c_1}$ by (4), this implies $\bar{\phi}_i \equiv \phi_i \mod a^{a_F}$ and since $\phi_i \in a^{a_F}$ by (2) this leads to

$$\bar{\phi}_i \in a^{a_F}.$$

We have embedded the normal module $N$ into $R^s$ by assigning to a homomorphism in $N$ the $s$ values on $f_1, \ldots, f_s$. So our $n$ from (5) is mapped into $a^{a_F} R^s$. Applying Proposition 2.1 to the embedding $N \rightarrow R^s$, we obtain an integer $c_2$ depending only on the embedding, such that

$$n \in a^{a_F - c_2} N.$$
Next, we want to find a derivation \( \theta \in \text{Der}_S(P, P) \), whose restriction to \( I \) induces \( n \). The cokernel of the map from \( \text{Der}_S(P, P) \) to \( N \) is by definition \( T^1(R/S) \). We have assumed \( \alpha^a T^1(R/S) = 0 \), so \( \alpha^a N \) is contained in the image of \( \text{Der}_S(P, P) \) under this map. So \( n \) is induced by some

\[
\theta \in \alpha^{a_F-a-c_2} \text{Der}_S(P, P).
\]

This means we have the equalities

\[
n(f_i) = \theta(f_i) + I
\]

and by (4) and (5) this implies

\[
\theta(f_i) \equiv \phi_i \mod I + \alpha^{a_F+aG-c_1}.
\]

But as by (7) \( \theta \in \alpha^{a_F-a-c_2} \text{Der}_S(P, P) \) and by (2) \( \phi \in \alpha^{a_F} \), we also know

\[
\theta(f_i) \equiv \phi_i \mod \alpha^{a_F-a-c_2}.
\]

Applying the Artin-Rees lemma once more we find an integer \( c_3 \) such that

\[
\alpha^{p+c_3} \cap I = \alpha^p(\alpha^{c_3} \cap I) \subset \alpha^p I.
\]

We have chosen \( a_G > c_1 \). So \( a_F - a - c_2 < a_F + a_G - c_1 \) and (10) implies \( \alpha^{a_F-a-c_2} \cap (I + \alpha^{a_F+aG-c_1}) \subset \alpha^{a_F-a-c_2-c_3} I + \alpha^{a_F+aG-c_1} \). Combining this with (8) and (9) we get:

\[
\theta(f_i) \equiv \phi_i \mod \alpha^{a_F-a-c_2-c_3} I + \alpha^{a_F+aG-c_1}.
\]

We use the derivation \( \theta \) to construct an automorphism \( \Phi_{a_F} \) of \( P = S[[x_1, \ldots, x_n]] \) by setting

\[
\Phi_{a_F}(x_m) := x_m - \theta(x_m).
\]

From (7) we deduce the two obvious inclusions

\[
\theta(a^k) \subset \alpha^{a_F-a-c_2+k-1}
\]

\[
\text{and} \quad \Phi_{a_F}(f) \equiv f - \theta(f) \mod a^{2(a_F-a-c_2)} \quad \forall f \in P.
\]

We first notice that

\[
\Phi_{a_F} \equiv Id_P \mod \alpha^{a_F-a-c_2}.
\]
Further

\[ \Phi_{aF}(f_i + \phi_i) = \Phi_{aF}(f_i) + \Phi_{aF}(\phi_i) \]
\[ \equiv f_i + (\phi_i - \theta(f_i)) - \theta(\phi_i) \pmod{a^{2(aF-a-c_2)}} \]
\[ \equiv f_i - \theta(\phi_i) \pmod{(a^{aF-a-c_2-c_3}I + a^{aF+c_2+c_1})} \]
\[ \equiv f_i \pmod{a^{2aF-a-c_2-1}}. \]

The first congruence follows from (13), the second from (11) and the third from (12). If we choose \( a_G \geq c_1 + 1 \) and \( a_F \geq \max\{2a+2c_2+1, a+c_2+2, a_G+a+c_2+c_3\} \), we get

\[ \Phi_{aF}(f_i + \phi_i) \equiv f_i \pmod{a^{aG}I + a^{aF+1}} \]
\[ \Leftrightarrow \Phi_{aF}(f_i + \phi_i) = f_i + \psi_i + \phi_i'' \]\n\[ \text{with } \psi_i \in a^{aG}I, \phi_i'' \in a^{aF+1}. \]

Consider the vector \((f_i + \psi_i)\). It can be written as \((f_1, \ldots, f_3) \circ (1 + \Psi_{aF})\), where \( \Psi_{aF} \) is a matrix with entries in \( a^{aG} \). Since \( P \) is \( a \)-complete, \( 1 + \Psi_{aF} \) is invertible and describes an automorphism of \( P^s \). Set \( \tilde{\Phi} := F \circ (1 + \Psi_{aF}) \) and \( \tilde{G} := (1 + \Psi_{aF})^{-1} \circ G \). We get a new representation of \( R \):

\[ \begin{array}{ccc}
P^r & \xrightarrow{\tilde{G}} & P^s \\
& \swarrow \searrow \nearrow & \\
& \equiv & F \\
& \swarrow \searrow & \\
P^s & \xrightarrow{\tilde{\Phi}} & P \\
& \nearrow \searrow & \\
& \equiv & \Phi_{aF} \\
& \swarrow \searrow & \\
& \equiv & P \\
\end{array} \]

We set \( G'' = G' \) and \( F'' = \Phi_{aF} \circ F' \):

\[ \begin{array}{ccc}
P^r & \xrightarrow{G' = G''} & P^s \\
& \searrow \nearrow & \\
& \equiv & F'' \\
& \searrow \nearrow & \\
P & \xrightarrow{F''} & P \\
& \nearrow \searrow & \\
& \equiv & \Phi_{aF} \\
& \swarrow \searrow & \\
P & \xrightarrow{\Phi_{aF}} & P \\
\end{array} \]

Then \( F'' \circ G'' = 0 \). We have shown that the entries of \( \tilde{F} \) are congruent to those of \( F'' \) modulo \( a^{aF+1} \). The entries of \( \tilde{G} \) are congruent to those of \( G \) modulo \( a^{aG} \), which in turn are congruent to those of \( G'' = G' \) modulo \( a^{aG} \), so the entries of \( \tilde{G} \) are congruent to those of \( G'' \) modulo \( a^{aG} \).

So we have improved the situation by raising \( a_F \) by one. Now we want to use induction on \( a_F \); to do this we have to check whether all those constants may be taken to be the same in the next step of our induction:

The constants \( a \) and \( c_3 \) only depend on \( a \) and \( I \).

The constant \( c_1 \) was found by applying Proposition 2.1 to the homomorphism

\[ \lambda: \Hom_P(P^s, P) \to \Hom_P(P^r, P) \]
\[ \phi \mapsto \phi \circ G \pmod{I}. \]
In the next step of our induction we will apply it to

\[ \lambda': \phi \mapsto \phi \circ (\text{Id} + \Psi_{af})^{-1} \circ G \mod I. \]

That is to say: Instead of at \( \lambda \) we will be looking at the composition of \( \lambda \) with the automorphism \((\text{Id} + \Psi_{af})^{-1})^*\) of \( \text{Hom}_P(P^s, P)\). Since the integer \( c_1 \) only depends on the image of \( \lambda \), it will be the same as before.

The last constant we have to consider is \( c_2 \). It was found by applying the Artin-Rees lemma to the submodule \( F^*(\text{Hom}(I, R)) \subset \text{Hom}_P(P^s, R) \cong R^s \). In the next step we will be considering the submodule \((1 + \Psi_{af})^*(F^*(\text{Hom}(I, R)))\) in \( \text{Hom}_P(P^s, R) \). But \((1 + \Psi_{af})^*\) is a \( P \)-module automorphism of \( \text{Hom}_P(P^s, R) \), so we can apply the following easy lemma:

**Lemma 2.2.** Let \( P \) be a ring, \( a \subset P \) an ideal, \( A \subset M \) two \( P \)-modules and \( \varphi \in \text{Aut}_P(M) \). If

\[ (15) \quad A \cap a^{p+c}M = a^p(A \cap a^cM) \]

for some integers \( p, c \in \mathbb{N} \), then

\[ (16) \quad \varphi(A) \cap a^{p+c}M = a^p(\varphi(A) \cap a^cM). \]

In particular, if \( P \) is noetherian and \( M \) finitely generated, the Artin-Rees lemma gives rise to the same constants when applied to the two submodules \( A \) and \( \varphi(A) \) of \( M \).

**Proof.** It is trivial to see that for any two submodules \( B_1, B_2 \) of \( M \) we have \( \varphi(B_1 \cap B_2) = \varphi(B_1) \cap \varphi(B_2) \) and also \( \varphi(a^pB_1) = a^p\varphi(B_1) \). So the left resp. right side of (15) gets mapped to the left resp. right side of (16). \( \square \)

Now let’s do the induction. \( \Phi_{af} \equiv \text{Id} \mod a^{a_{af} - a - c_2} \), so we have a limit \( \Phi = \cdots \circ \Phi_{af+1} \circ \Phi_{af} \), which is an automorphism of \( P \). In the same way we get a matrix \( \Psi \) with entries in some power of \( a \) such that \( (1 + \Psi) = \prod (1 + \Psi_{af+k}) \). By construction, \( \Phi(f_i + \phi_i) \) is the \( i \)th component of \( (f_1, \ldots, f_s) \circ (1 + \Psi) \), so \( \{f_i + \phi_i\} \) is being mapped to the generating system \( \{f_i \circ (1 + \Psi)\} \) of \( I \), hence \( \Phi(I') = I \). \( \square \)

**3. Proof of Theorem 1.5.** The proof is the same as for Theorem 1.1. We will only check that the condition \( a^d \text{Ext}^1(M, M) = 0 \) for modules is the analogue to the condition \( a^dT^1 = 0 \) we had before. We fix a presentation

\[ P^r \xrightarrow{G} P^s \xrightarrow{F} P^t \rightarrow M \rightarrow 0 \]

of \( M \) and consider a perturbation

\[ P^r \xrightarrow{G+\Gamma} P^s \xrightarrow{F+\Phi} P^t \]
which is an exact sequence. Then the $(t \times s)$-matrix $\Phi$ defines a homomorphism $\text{Im}(F) \cong P^s/(\text{Im}(G)) \xrightarrow{\Phi} (P^t/\text{Im}(F))/a^\infty$. We approximate this homomorphism by a homomorphism to $P^t/\text{Im}(F)$. Now the crucial point is to extend this homomorphism from $\text{Im}(F)$ to all of $P^t$. We begin with the exact sequence

$$0 \to \text{Im}(F) \to P^t \to M \to 0.$$ 

This gives us a long exact sequence which starts like this:

$$0 \to \text{Hom}(M, M) \to \text{Hom}(P^t, M) \to \text{Hom}(\text{Im}(F), M) \to \text{Ext}^1(M, M) \to \cdots.$$ 

So if $a^a \text{Ext}^1(M, M) = 0$, all homomorphisms in $a^a \text{Hom}(\text{Im}(F), M)$ can be extended to $P^t$. Finally we lift this extension from $\text{Hom}(P^t, M)$ to an automorphism $\Psi \in \text{Hom}(P^t, P^t)$. The automorphism $\text{Id}_{P^t} + \Psi$ is the analogue to the automorphism we have constructed above. The rest of the proof is exactly as for Theorem 1.1. It consists mainly of keeping track of the powers of $a$ up to which things vanish. We leave the details to the reader.

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