## Observability for Non-Autonomous Systems

## Fabian Gabel

Hamburg University of Technology fabian.gabel@tuhh.de

Clemens Bombach

Technische Universität Chemnitz

clemens.bombach@mathematik.tu-chemnitz.de

Christian Seifert

Hamburg University of Technology

christian.seifert@tuhh.de

Martin Tautenhahn

Universität Leipzig

martin.tautenhahn@math.uni-leipzig.de

We study non-autonomous abstract Cauchy problems

$$\dot{x}(t) = A(t)x(t), \ y(t) = C(t)x(t), \ t > 0, \quad x(0) = x_0 \in X,$$

where  $A(t): D(A) \to X$  is a strongly measurable family of operators on a Banach space X and  $C(t) \in \mathcal{L}(X,Y)$  is a family of bounded observation operators from X to a Banach space Y.

For measurable subsets  $E \subseteq (0,T)$ , T > 0, we provide sufficient conditions such that the Cauchy problem satisfies a *final state observability estimate* 

$$||x(T)||_X \lesssim \left(\int_E ||y(t)||_Y^r dt\right)^{1/r}, \quad r \in [1, \infty),$$

where an analogous estimate holds for the case  $r = \infty$ .

An application of the above result to families of strongly elliptic differential operators A(t) and observation operators

$$C(t)u := \mathbf{1}_{\Omega(t)}u, \quad \Omega(t) \subseteq \mathbb{R}^d, u \in L^p(\mathbb{R}^d),$$

is presented. In this setting, we give sufficient and necessary geometric conditions on the family of sets  $(\Omega(t))$  such that the corresponding Cauchy problem satisfies a final state observability estimate.